# Oscillations of Atmospheric Neutrinos with a Large Neutrino Telescope.

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**Summary.** — The sensitivity of a large water Čerenkov neutrino telescope in detecting possible oscillations $(\nu_\mu \rightarrow \nu_\tau)$ and $(\nu_\mu \rightarrow \nu_e)$ of atmospheric neutrinos is studied, taking also into account the Wolfenstein, Mikheyev and Smirnov matter effects. The large water Čerenkov device is able to detect with high accuracy upward-going muons with energies greater than 10 GeV. After one year running time the search for $\nu$ oscillations will reach the following sensitivity for the oscillation parameters: $\Delta m^2 > 3 \cdot 10^{-3}\,(\text{eV}^2)$, for $\sin^2 2\theta = 1$, $\sin^2 2\theta > 4 \cdot 10^{-2}$, for $\Delta m^2 > 0.1\,(\text{eV}^2)$ for the oscillation $(\nu_\mu \rightarrow \nu_\tau)$ and $\delta m^2 > 4 \cdot 10^{-3}\,(\text{eV}^2)$, for $\sin^2 2\theta = 1$, $\sin^2 2\theta > 8 \cdot 10^{-3}$, for $\delta m^2 = 6 \cdot 10^{-2}\,(\text{eV}^2)$ for the oscillation $(\nu_\mu \rightarrow \nu_e)$. These limits are well below the current accelerator and reactor limits. It is also stressed the importance of such a measurement in the context of solar $\nu$ puzzle.

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## 1. – Introduction.

The atmospheric neutrinos, $\nu_\mu$, $\nu_e$, are produced by the interactions of the cosmic rays in the atmosphere, via the subsequent decay of pions, kaons and muons. Only a little fraction of neutrinos interacts in the rock around the detector and gives rise to the measured flux of upward-going muons at the detector. This flux is of the order of $2 \cdot 10^{-9}$ muons/(m$^2$s sr), for energies $E_\mu > 2\,\text{GeV}$.

Neutrino oscillations may change the neutrino flavour on their way to the detector thus modifying the upward-going muon flux. Therefore it is possible to test neutrino oscillations by comparing the observed upward-going muon flux with the flux obtained by calculating the atmospheric-neutrino products. In this analysis 2 kinds of oscillations are studied:

*a*) the oscillation $(\nu_\mu \rightarrow \nu_\tau)$,

*b*) the oscillation $(\nu_\mu \rightarrow \nu_e)$.

In case *a*) it has been assumed, as a first approximation, that $\nu_\tau$ are «sterile», *i.e.* cannot give muons passing through the detector. In fact, the $\nu_\tau$ charged-current interactions in the rock produce upward-going muons in the direction of the parent neutrino only about 20% of the times (from $\tau$ decay) and these muons have, on the average, only 1/6 of the neutrino energy.

In case *b*) the Wolfenstein-Mikheyev-Smirnov matter effects [1], [2], which affect neutrino penetrating the Earth, have been also taken into account.

## 2. – Characteristics of the detector.

The neutrino telescope we have considered is a large water Čerenkov detector designed to investigate neutrino astronomy with a size of $(300 \times 100 \times 50)\,\mathrm{m}^3$, (mass = 1500 kton). Figure 1 shows a lay-out of the apparatus [3]. The detector is

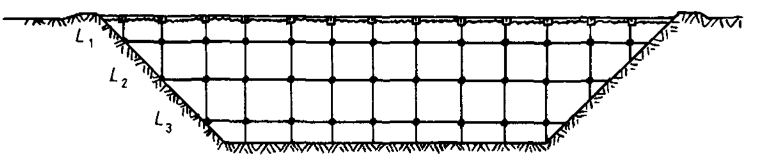


Fig. 1. – Lay-out of a large water Čerenkov neutrino telescope (adapted from ref. [3], not in scale).

divided into 3 horizontal layers, $L_1$, $L_2$, $L_3$, optically isolated and instrumented with downward looking photomultipliers (PMTs), sketched as circles. Čerenkov light from relativistic charged particles passing through water is emitted at an angle of 42° from the track direction and hits PMTs, which form a grid in each layer (a spacing of 16.7 m between two layers and a distance of 6 m between two PMTs is assumed). Discrimination between upward-going muons and downward-going muons is obtained by means of Čerenkov light directionality and from the time of flight through the detector's three layers.

This kind of water Čerenkov neutrino telescope is able to detect upward-going muons coming from the rock with zenith angle $\theta_z$ (vertical muons have $\cos\theta_z = -1$), which cross the whole detector, with an angular resolution $\Delta\theta_z < 1°$, assuming PMTs with resolution time $\sigma_\tau \simeq 4\sqrt{N_{\mathrm{pe}}}\,\mathrm{ns}$, where $N_{\mathrm{pe}}$ is the number of collected photoelectrons. The directions of these muons are reconstructed with an algorithm based on the time and position of hit PMTs [4]. For vertical directions, upward-going muons passing the whole detector produce an amount of Čerenkov light that hits about 60 PMTs. The energy threshold $E_{\mathrm{th}}$ for these events is

$$E_{\mathrm{th}} = \frac{10\,\mathrm{GeV}}{|\cos\theta_z|}. \qquad (1)$$

Stopping muons, which are «seen» from a smaller number of PMTs, are measured with a lower accuracy.

Such a large detector has an average detection area of about $50.000\,\mathrm{m}^2$ which permits large statistics and, in ultimate analysis, determines its good sensitivity in testing neutrino oscillations.

## 3. – The flux of upward-going muons.

The upward-going muon flux in the detector, $\mathrm{d}\Phi_\mu/\mathrm{d}\Omega$, coming from atmospheric neutrino interactions, is given by the expression

$$\frac{\mathrm{d}\Phi_\mu}{\mathrm{d}\Omega} = \int_{E_{\mathrm{th}}} P(E_\nu)\frac{\mathrm{d}\Phi_\nu^{\mathrm{det}}}{\mathrm{d}E_\nu\,\mathrm{d}\Omega}\mathrm{d}E_\nu\,, \tag{2}$$

where $\mathrm{d}\Phi_\nu^{\mathrm{det}}/\mathrm{d}E_\nu\,\mathrm{d}\Omega$ is the neutrino spectrum at the detector and $P(E_\nu)$ is the probability that a neutrino with energy $E_\nu$ interacts in the rock outside the detector and produces a muon, with energy $E_\mu > E_{\mathrm{th}}$, passing through the detector. The evaluation of expression (2) depends strongly on assumptions about three physical processes:

1) The atmospheric neutrino flux.

2) Weak-interaction cross-section between muon neutrino and rock.

3) Muon energy loss in the rock.

Here it was used:

1) The atmospheric neutrino flux calculated by Gaisser [5] in the range $0.1 < E_\nu < 100\,\mathrm{GeV}$ combined with the flux calculated by Volkova [6] in the range $E_\nu > 100\,\mathrm{GeV}$. The most important components of the calculation of the neutrino flux are:

*a*) the energy dependence of the primary cosmic rays at the top of the atmosphere,

*b*) the details of particle production in the atmosphere.

Both of them must be known over several orders of magnitude in energy. Considering the uncertainties in the primary spectrum and the hadronic interactions, the overall uncertainty in the absolute normalization of the neutrino flux is at least of order $\pm 20\%$. Therefore this value is assumed as the indetermination of the absolute normalization of the muon flux.

2) The differential cross-section in the parton model for the lepton-production process $(\nu_\mu \mathcal{N} \to \mu X)$

$$\frac{\mathrm{d}\sigma_\nu}{\mathrm{d}x\,\mathrm{d}y} = \frac{G_F^2 m E_\nu}{\pi}\frac{M_W^4}{(2mxyE_\nu + M_W^2)^2}(q(x) + s(x) + [\bar{q}(x) - \bar{s}(x)](1-y)^2), \tag{3}$$

where $y = 1 - E_\mu/E_\nu$, $2mxyE_\nu = |Q^2|$, $m$ is the nucleon mass and $q(x) = u(x) + d(x) + s(x)$ is the quark distribution function, without scaling violations, given by Eichten *et at.* [7]. The corresponding espression for scattering of an $\bar{\nu}$ involves the interchange $q \leftrightarrow \bar{q}$ and $s \leftrightarrow \bar{s}$ in eq. (3). Although the $Q^2$ independence of the structure functions is broken at energy $E_\nu \simeq 100\,\mathrm{GeV}$ [8], Gaisser and Grillo [9] have demonstrated that (for muons with energy $E_\mu > 2\,\mathrm{GeV}$):

*a*) The QCD evolution of the distribution function is of pratical importance only for $E_\nu > 10^5\,\mathrm{GeV}$. In fact only from this energy up the probability $P(E_\nu)$ varies in a considerable way.

*b*) The QCD evolution is always negligible for power-law neutrino spectra $E_\nu^{-\gamma}$

with indices $\gamma$ greater than 2.2. This is the case of atmospheric neutrino spectrum where $\gamma$ is about 3 [9].

3) The muon energy loss in the rock of the form

$$-\frac{dE_\mu}{dX} = a + bE_\mu, \tag{4}$$

with $a = 2\,\text{MeV/(g/cm}^2)$, $b = 3.9 \cdot 10^{-6} (\text{g/cm}^2)^{-1}$, where the first term is due to ionization energy loss and the second term to bremsstrahlung and photonuclear interactions.

The expected upward-going muon flux at the detector is shown in fig. 2.

In the range $[\cos\theta_z = -1,\ \cos\theta_z = -0.2]$, since the $\nu$ flux increases as $\cos\theta_z$ gets larger, the muon flux increases too. In fact, pions, kaons and muons travel larger distances in the atmosphere, so their decay probability gets bigger. For $\cos\theta_z \to 0$ the threshold rise is the cause of a fall in the muon flux.

In expression (2) it is assumed, as a first approximation, that the zenith angle of the upward-going muon is also the zenith angle of the neutrino. This is true for high-energy thresholds. In general, the deviation $\Delta\phi$ between the neutrino direction and that of the observed upward-going muon flattens the angular distribution in the range $[\cos\theta_z = -1,\ \cos\theta_z = -0.2]$.

A Monte Carlo program was performed in order to estimate this effect. A sample of 10000 upward-going muons produced by atmospheric neutrinos, with energies

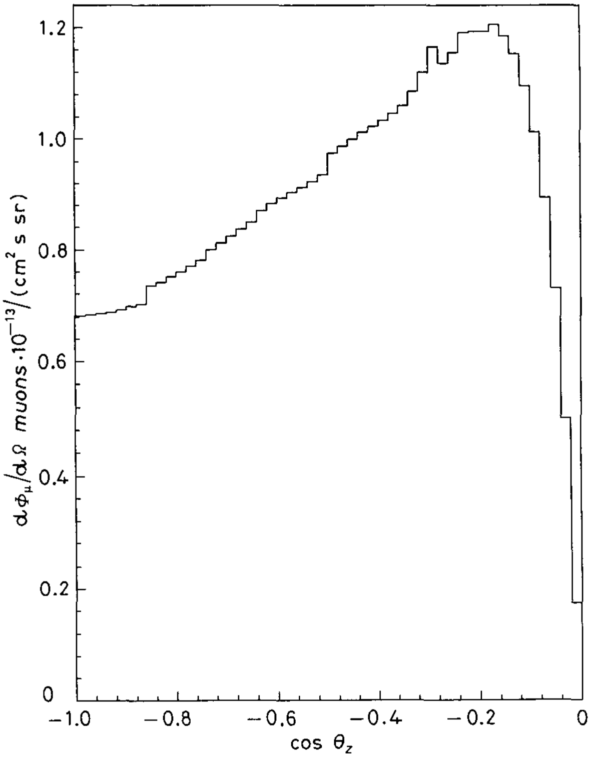


Fig. 2. – Zenith-angle distribution of the upward-going muon flux.

$E_\mu > E_{\rm th}$, was generated, giving an average deviation

$$\langle \Delta\phi \rangle = (0.8 \pm 1.0)\ \text{degrees}. \tag{5}$$

On the basis of this result the change in $d\Phi_\mu/d\Omega$ is estimated to be less than 2%.

## 4. – Test of neutrino oscillations.

In the hypothesis of neutrino oscillations ($\nu_\mu \to \nu_\tau$) the atmospheric neutrino flux at the detector in eq. (2) is replaced by an oscillating neutrino flux

$$\frac{d\Phi_\nu^{\rm det}}{dE_\nu\, d\Omega} = \frac{d\Phi_{\nu_\mu + \bar\nu_\mu}}{dE_\nu\, d\Omega}(1 - P(\nu_\mu \to \nu_\tau)). \tag{6}$$

$P(\nu_\mu \to \nu_\tau)$ is the oscillation probability given by

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2\left(\pi \frac{L}{l_v}\right), \tag{7}$$

where $\theta$ is the vacuum mixing angle. $P(\nu_\mu \to \nu_\tau)$ is the probability that a $\nu_\mu$ produced with energy $E_\nu$(GeV) becomes a $\nu_\tau$ after travelling a distance $L$(km). Here $l_v$ is the oscillation length in vacuum given by

$$l_v = \frac{4\pi E_\nu}{\delta m^2} = 2.5 \frac{E(\text{GeV})}{\delta m^2(\text{eV}^2)}(\text{km}), \tag{8}$$

where $\delta m^2 = m_2^2 - m_1^2$ is the square mass splitting.

Similarly, in the case of the ($\nu_\mu \to \nu_e$) resonant oscillation, the oscillating neutrino flux is

$$\frac{d\Phi_\nu^{\rm det}}{dE_\nu\, d\Omega} = \frac{d\Phi_{\nu_\mu}}{dE_\nu\, d\Omega}(1 - P(\nu_\mu \to \nu_e)) + \frac{d\Phi_{\nu_e}}{dE_\nu\, d\Omega} P(\nu_\mu \to \nu_e) + \tag{9}$$

$$+ \frac{d\Phi_{\bar\nu_\mu}}{dE_\nu\, d\Omega}(1 - \bar P(\bar\nu_\mu \to \bar\nu_e)) + \frac{d\Phi_{\bar\nu_e}}{dE_\nu\, d\Omega} \bar P(\bar\nu_\mu \to \bar\nu_e),$$

where $P(\nu_\mu \to \nu_e)$, $\bar P(\bar\nu_\mu \to \bar\nu_e)$ are obtained by numerically integrating the Wolfenstein equations of evolution [1], taking into account the variable density of the Earth [10]. Approximated expressions for $P(\nu_\mu \to \nu_e)$, $\bar P(\bar\nu_\mu \to \bar\nu_e)$ may be derived by considering neutrinos which propagate in two distinct media with nearly constant density: the core and the mantle;

*a*) in the core the density varies from $\rho = 12.5(\text{g/cm}^3)$ to $\rho = 9.9(\text{g/cm}^3)$,

*b*) in the mantle the density varies from $\rho = 5.5(\text{g/cm}^3)$ to $\rho = 2.8(\text{g/cm}^3)$.

In a medium with constant electron density $N_e$ the oscillation probability

$P(\nu_\mu \to \nu_e)$ is described by an expression analogue to (7) with $\theta$, $l_v$ replaced by $\theta_m$, $l_m$ [1], [2]:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - l_v/l_0)^2}, \tag{10}$$

$$l_m = \frac{l_v}{\sqrt{\sin^2 2\theta + (\cos 2\theta - l_v/l_0)^2}}, \tag{11}$$

$$l_0 = \frac{2\pi}{\sqrt{2}G_F N_e} = \frac{1.6 \cdot 10^4}{\rho(\mathrm{g/cm^3})Z/A}\,(\mathrm{km}), \tag{12}$$

where $Z$ is the atomic number, $A$ the mass number and $l_m$ is the oscillation length in matter. $l_0$ takes into account the $\nu_e e \to \nu_e e$ interaction of $\nu_e$ due to the charged-current elastic scattering (W exchange) on electrons.

In the Earth, the mean density is $\rho \simeq 6$ (g/cm³), $Z/A \simeq 1/2$, $l_0 \simeq 6000$ km, namely $l_0$ is of the order of the Earth radius. For $l_v \ll l_0$ matter effects can be ignored, for $l_v \gg l_0$ the amplitude (10) is highly reduced (also in the case of maximal mixing, $\sin^2 2\theta = 1$), while the amplitude (10) reaches its maximum (also in the case of small mixing angles) for

$$l_v = l_0 \cos 2\theta. \tag{13}$$

The importance of this case was discovered by Mikheyev and Smirnov who refer to condition (13) as the resonant amplification of neutrino oscillations by matter [2].

For the case of antineutrinos $\bar\nu_\mu$, $\bar\nu_e$, the expressions (10), (11), (12), are also valid, but the sign of $l_0$ is reversed. Thus the resonance condition (13) can be satisfied either by neutrinos only (if $\delta m^2 > 0$) or by antineutrinos only (if $\delta m^2 < 0$).

In order to evaluate the detector sensitivity in testing neutrino oscillation hypothesis, two angular distributions of the muon flux are compared for 10 values of $\cos\theta_z$. The first distribution, $(\mathrm{d}\Phi_\mu/\mathrm{d}\Omega)^{\mathrm{exp}}$, refers to the hypothesis of absence of $\nu$ oscillation and is obtained directly from eq. (2) using the expected atmospheric neutrino flux calculed by Gaisser and Volkova. The second distribution, $(\mathrm{d}\Phi_\mu/\mathrm{d}\Omega)^{\mathrm{osc}}$, in the hypothesis of presence of $\nu$ oscillation, is calculated by replacing in eq. (2) the expected neutrino flux with the oscillating neutrino flux given by expressions (6) or (9).

Constraints on oscillation parameters $\sin^2 2\theta$ and $\delta m^2$ are established by means of the likelihood-ratio test, with the likelihood function $L = L(\sin^2 2\theta, \delta m^2)$ defined by

$$L = \prod_{i=1}^{10} \int \mathrm{d}\alpha \frac{1}{2\pi\sigma_i\sigma_\alpha} \exp\left[\frac{-[(\mathrm{d}\Phi_\mu/\mathrm{d}\Omega)_i^{\mathrm{exp}} - \alpha(\mathrm{d}\Phi_\mu/\mathrm{d}\Omega)_i^{\mathrm{osc}}]^2}{2\sigma_i^2}\right] \exp\left[-\frac{[1-\alpha]^2}{2\sigma_\alpha^2}\right], \tag{14}$$

where $\sigma_i$ is the statistical error of $(\mathrm{d}\Phi_\mu/\mathrm{d}\Omega)^{\mathrm{exp}}$ and $\alpha$ is the absolute normalization of $(\mathrm{d}\Phi_\mu/\mathrm{d}\Omega)^{\mathrm{osc}}$. To take into account uncertainties in the absolute normalization, a Gaussian distribution with standard error $\sigma_\alpha = 0.2$, is assumed.

As it is known, the statistic

$$t = -2\ln \frac{L(\sin^2 2\theta,\, \delta m^2)}{\max[L(\sin^2 2\theta,\, \delta m^2)]} \tag{15}$$

has a distribution that, asymptotically, is a $\chi^2$ distribution with 2 degrees of freedom. Therefore a given pair of parameters $(\sin^2 2\theta, \delta m^2)$ is excluded with 90% c.l. if

$$2\ln[(\sin^2 2\theta,\, \delta m^2)] < 2\ln(\max[L(\sin^2 2\theta,\, \delta m^2)]) - 4.6. \tag{16}$$

## 5. – Results and conclusion.

With a large water Čerenkov neutrino telescope, as described in sect. 2, in 1 year running time and an energy threshold $E_{th} = 10\,\text{GeV}/|\cos\theta_z|$, it is possible to reach the sensitivity for ν-oscillations as illustrated in fig. 3 and 4, where the results, at 90% cl., are displayed in the plane $(\sin^2 2\theta,\ \delta m^2)$.

Figure 3 refers to the resonant oscillation $(\nu_\mu \to \nu_e)$, while the region illustrated in fig. 4 belongs to the oscillation $(\nu_\mu \to \nu_\tau)$. The regions of the excluded values for the parameters are:

$$\delta m^2 > 4\cdot 10^{-3}\,(\text{eV}^2), \qquad \text{for } \sin^2 2\theta = 1$$

$$\sin^2 2\theta > 8\cdot 10^{-3}, \qquad \text{for } \delta m^2 = 6\cdot 10^{-2}\,(\text{eV}^2)$$

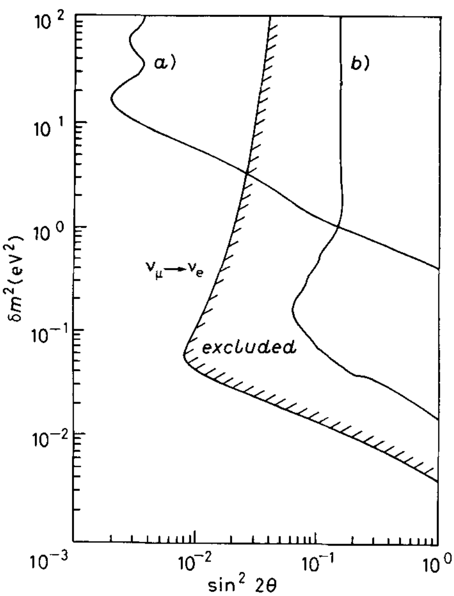


Fig. 3.

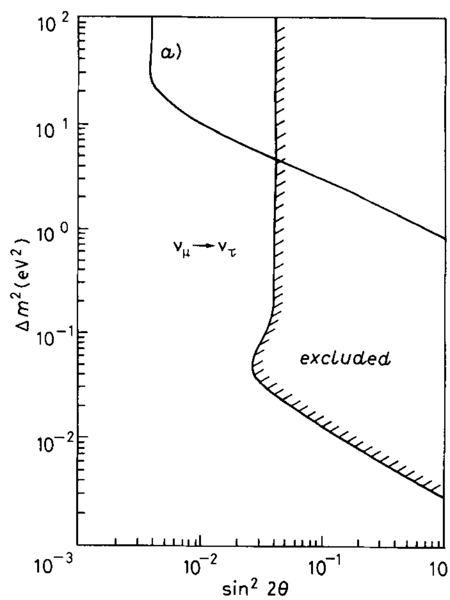


Fig. 4.

Fig. 3. – Region in the parameter space $(\sin^2 2\theta,\ \delta m^2)$ that can be excluded at 90% c.l. in a running time of 1 y for the oscillation $(\nu_\mu \to \nu_e)$.

Fig. 4. – Region in the parameter space $(\sin^2 2\theta,\ \Delta m^2)$ that can be excluded at 90% c.l. in a running time of 1 y for the oscillation $(\nu_\mu \to \nu_\tau)$.

for the oscillation $(\nu_\mu \rightarrow \nu_e)$ and

$$\Delta m^2 > 3 \cdot 10^{-3} (\mathrm{eV}^2), \qquad \text{for } \sin^2 2\theta = 1$$
$$\sin^2 2\theta > 4 \cdot 10^{-2}, \qquad \text{for } \Delta m^2 > 0.1 (\mathrm{eV}^2)$$

for the oscillation $(\nu_\mu \rightarrow \nu_\tau)$.

In fig. 3 and 4 these results are compared with the current experimental limits obtained from accelerators (curve *a*)) and reactors (curve *b*)). It appears that the limits reachable with such a large water Čerenkov detector are well below the present accelerator and reactor limits.

Furthermore it is worthwhile to stress the importance of such a measurement also in the context of the solar $\nu$ puzzle.

The essence of this phenomenon is the discrepancy between the event rate in the $^{37}Cl \rightarrow {}^{37}Ar$ solar-neutrino experiment [11] $(2.1 \pm 0.3)$ SNU [12] and the predicted rate [13] $(5.3 \div 10.5)$ SNU of the Standard Solar Model (SSM). This discrepancy has recently been confirmed by the Kamiokande II experiment [14], sensitive only to $^8B$ neutrinos with energies $E_\nu > 9$ MeV, which reports a value for the $\nu_e$ flux (from $\nu_e e$ elastic scattering) that is less then about half the expected event rate:

$$\frac{\text{Kam-II data}}{\text{SSM}} = 0.46 \pm 0.13 (\text{stat.}) \pm 0.08 (\text{syst.}).$$

Such a result is consistent with the data obtained by the $^{37}Cl$ detector in essentially the same period [15]. There are many possible explanations for the discrepancy (see ref. [16]). Recently, Mikheyev and Smirnov [2] have proposed an elegant new solution which examines the effects of matter on $\nu_e$ oscillations. The basic idea is that $\nu_e$ emerging from the center of the Sun are transformed into $\nu_\mu$ or $\nu_\tau$ as a result of the enhancement of the oscillation mechanism in the Sun. The analysis of these effects on the existing solar-neutrino experiments, taking properly into account the uncertainties in the SSM [17], points out that the constraints on the oscillation parameters by the actual limits are rather weak and cannot yet discriminate between these two limiting cases:

*a*) Only the $^8B$ solar-neutrino flux is reduced. This solution corresponds to $m^2_{\nu_i} - m^2_{\nu_e} \simeq 10^{-4}\,\mathrm{eV}^2$, where $i = \mu$ or $\tau$; the initial $\nu_e$ are adiabatically converted to $\nu_\mu$ or $\nu_\tau$ as they pass through the resonance region in the Sun with probability $\simeq 1$.

*b*) All contributions to the total neutrino flux are reduced. This solution corresponds to $m^2_{\nu_i} - m^2_{\nu_e} = (10^{-7} \div 10^{-5})\,\mathrm{eV}^2$; here the adiabatic approximation breaks down and the conversion probability is less than unity.

Experimental data favour solution *b*) but cannot yet rule out solution *a*). However if the possibility of solution *b*) is considered, interpreted as $(\nu_\mu \rightarrow \nu_e)$ oscillations in the Sun, the see-saw mechanism [18], with the relation between the lepton masses:

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = (m_e)^2 : (m_\mu)^2 : (m_\tau)^2 \tag{17}$$

would suggest for $m_{\nu_\tau}$ a value of the order $10^{-1} \div 1$ eV or $m^2_{\nu_\tau} - m^2_{\nu_\mu} = 10^{-2} \div 10\,\mathrm{eV}^2$. In this context, the sensitivity of a large water Čerenkov neutrino telescope in detecting possible $(\nu_\mu \rightarrow \nu_\tau)$ oscillations of atmospheric neutrino permits to search $(\nu_\mu \rightarrow \nu_\tau)$ oscillations with squared mass differences $m^2_{\nu_\tau} - m^2_{\nu_\mu} = (10^{-2} \div 10)\,\mathrm{eV}^2$ in a large region of the parameters space.

* * *

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