

QUANTUM MEASUREMENT IN A FAMILY OF HIDDEN-VARIABLE THEORIES¹

Giulio Peruzzi and Alberto Rimini

*Dipartimento di Fisica Nucleare e Teorica
Università di Pavia
Via Bassi 6, 27100 Pavia, Italy*

Received July 18, 1996

The measurement process for hidden-configuration formulations of quantum mechanics is analysed. It is shown how a satisfactory description of quantum measurement can be given in this framework. The unified treatment of hidden-configuration theories, including Bohmian mechanics and Nelson's stochastic mechanics, helps in understanding the true reasons why the problem of quantum measurement can successfully be solved within such theories.

Key words: quantum measurement, hidden variable, Bohmian mechanics, pilot-wave theory, stochastic mechanics.

1. INTRODUCTION

According to the so-called Copenhagen interpretation supporting the standard formulation of quantum mechanics, it is admitted that we can identify two different situations: the undisturbed evolution of a (quantum) micro-system (\mathcal{S}) and the measurements, i. e., the interactions of \mathcal{S} with suitable (classically described) macroscopic devices (\mathcal{A}). In the standard formulation, the ordinary time evolution of the state of \mathcal{S} is governed by the Schrödinger equation. This deterministic evolution is suspended when a measurement is performed. The acts of measurement are governed via special rules (*probability rule* and *reduction postulate*) having a stochastic character. Measurements have usually many possible outcomes and in general the knowledge of the state of the system *before the measurement* does not determine the actual outcome but only a probability distribu-

¹ Supported in part by the Istituto Nazionale di Fisica Nucleare.

tion for the various different outcomes (*probability rule*). The state of the system *after the measurement* is determined by the actual outcome (*reduction postulate*). The a priori acceptance of the dualism of the physical situations is logically a necessary condition for the acceptance of the dualism of the evolution principles.

One of the big difficulties connected with such forms of dualism in quantum mechanics is that the distinction between the quantum system and the classically described measuring apparatus, even though easy in practice, is not sharply defined in principle and this introduces a basic vagueness into the fundamental physical theory. The theory of quantum measurement tries to get rid of the dualism assuming the Schrödinger equation as the sole principle of evolution for the composite system $S + A$. We shall briefly review, in Sec. 2, the hard difficulties met by this programme, giving also a résumé of the families of theories which overcome these difficulties without losing anything as regards the description of the quantum-mechanical phenomenology. The hidden-configuration theories, which include de Broglie-Bohm's theory [1] and Nelson's stochastic mechanics [2,3], are one of such families. As described in Sec. 3 some results of Davidson [4] and of Bohm and Hiley [5] allow to present in a unified way the whole family. The measurement process in this framework will be analysed in Sec. 4. In the last Section we shall present some concluding remarks.

The theory of quantum measurement in the de Broglie-Bohm theory is due to Bell [6,7] and Bohm and Hiley [8]. Recently, it has been revisited by Dürr, Goldstein and Zanghi [9] in the framework of Bohmian mechanics, and by Rimini [9] through a simple but significant example.

On the other side, the situation of the theory of quantum measurement in the stochastic formulation of quantum mechanics is more involved. Whereas Blanchard, Golin and Serva [11] consider the problem of quantum measurements without including the apparatus A , the stochastic mechanical treatment of the whole $S + A$ is given by Goldstein [12] and Blanchard, Cini and Serva [13]. It is stated, in both the latter papers, that the solution of the problem of quantum measurement in stochastic mechanics is based on the decoherence of the wave function and on the role of hidden configuration variables. While we completely agree with the treatment of Goldstein, we find some interpretative uncertainties in the point of view of Blanchard, Cini and Serva.

A totally different approach, based on the theory of stochastic variational principles and their discrete generalizations, is described by Guerra [14]. Starting from a single variational principle, one can simulate, together with critical processes reducing to the usual ones of stochastic mechanics (in the continuous limit), also processes simulating the formation of a mixture, in analogy with the quantum

measurement collapse. The physical implications for the theory of quantum measurement, and, before that, a physical justification for the assumed form of the stochastic Lagrangian, are still open problems.

In the present paper, we adopt a unified formulation of hidden-configuration theories and show that a satisfactory treatment of quantum measurement can be given in the framework of such a formulation. We think that the unified treatment helps in understanding the true reasons why the problem of quantum measurement can successfully be solved within such theories.

2. THE PROBLEM OF QUANTUM MEASUREMENT

Let us consider the measurement process including the measuring apparatus \mathcal{A} together with the measured system \mathcal{S} in the description. Different outcomes of the measurement correspond to different final pointer positions and consequently to different, macroscopically distinguishable states of \mathcal{A} and $\mathcal{S} + \mathcal{A}$.

In the standard formulation the state is identified with the state vector and the correspondence between the outcome and the state is enforced by the reduction postulate. Let Ψ^i and Ψ^f be the state vectors of $\mathcal{S} + \mathcal{A}$ before and after the measurement, respectively. For definiteness and simplicity let us consider ideal measurements (this is not essential for the arguments which follow). The evolution from Ψ^i to Ψ^f will be of the form

$$\Psi^i = \sum_m c_m \varphi_m \alpha^i \longrightarrow \Psi^f = \begin{cases} \varphi_1 \alpha_1^f, & \text{with probability } |c_1|^2, \\ \varphi_2 \alpha_2^f, & \text{with probability } |c_2|^2, \\ \dots\dots\dots & \dots\dots\dots \end{cases} \tag{1}$$

with obvious meaning of symbols. The output of the measurement is not uniquely determined, that is the final situation is a mixture represented by the statistical ensemble made up with the various outputs with their proper weights. This is the consequence of the probability rule. When the measurement is completed, we will get one and only one definite outcome which corresponds to a definite state of $\mathcal{S} + \mathcal{A}$. If we immediately repeat the same measurement on the same system we shall obtain with certainty (i. e., with probability equal to one) the same outcome.

The program of the theory of quantum measurement, however, is to assume the Schrödinger equation as the sole principle of evolution for the composite system $\mathcal{S} + \mathcal{A}$. Because of the linearity of the Schrödinger equation, the evolution from Ψ^i to Ψ^f will be of

the form

$$\Psi^i = \sum_m c_m \varphi_m \alpha^i \xrightarrow{\text{S.E.}} \Psi^f = \sum_m c_m \varphi_m \alpha_m^f. \quad (2)$$

The final state Ψ^f in Eq. (2) is a superposition of macroscopically distinguishable states, a pure state, not a mixture. In such a state all outcomes coexist, no matter of the fact that $\mathcal{S} + \mathcal{A}$ is macroscopic. This *inconceivable state* is the rub leading to the familiar paradoxes associated with quantum measurement, such as Schrödinger's cat [15] or Wigner's friend [16].

A first family of attempts to overcome these difficulties is provided by theories based on what can be called *ensemble interpretation*. This consists essentially in the following three statements:

- (A) the state vector Ψ describes an ensemble of identically prepared physical systems, (it does not describe an individual system);
- (B) the state vector Ψ always evolves according to the Schrödinger equation;
- (C) the link between the state vector and the results of experiments is given by the standard probability rule.

These theories can be seen as the most immediate extensions of standard quantum mechanics, in that they develop the program of the theory of quantum measurement referring only to the standard formalism plus some extra assumptions and/or some crucial remarks.

The goal of the theories of measurement based on the ensemble interpretation is the proof that the final situations in (1) and (2) are equivalent. Before proceeding, we must make statement (C) more precise. Let us assume that the probabilities to which this statement refers are the probabilities of the outcomes of any (realized or realisable) measurement on the considered physical system, at any chosen instant of time. Proving the equivalence of the final situations in (1) and (2) means proving that the outcomes of any further measurement realisable on the ensemble of the systems $\mathcal{S} + \mathcal{A}$ are the same in the two cases. This, clearly, involves the need of suitable limitative assumptions on the measurements which can actually be performed. Different limitative assumptions correspond to different theories in the family of those based on the ensemble interpretation. Starting with different limitative assumptions, all these theories, in practice, converge in the proof that the interference effects between different terms in (2) cannot be experimentally revealed. Such a result can be called *effective incoherence*. We note that, in this type of approach, even though the measurement apparatus is included in the description, one does not avoid the reference to further measurements to be performed at successive times. Therefore the vague concept of

“measurement” is not removed at the level of principles, and this fact is detrimental to the claim of descriptive precision.

A particular subfamily of theories of measurement based on the ensemble interpretation makes use of the practically inescapable interaction of macroscopic systems with their environment, \mathcal{E} (cf. the work of Joos and Zeh [17] and the works of Gottfried [18]). In this case the limitative assumption consists in the recognition of the practical inaccessibility by means of a measurement of all the degrees of freedom of \mathcal{E} . This is the weakest limitation, in the sense that no assumption on the measurability of $S + \mathcal{A}$ is necessary. For this reason, we make reference in the subsequent discussion to this approach, having in mind that the same problems of descriptive precision and, as we shall see, of consistency arise anyhow. The evolution in Eq. (2) is then replaced by

$$\Psi^i = \sum_m c_m \varphi_m \alpha^i \eta^i \xrightarrow{\text{S.E.}} \Psi^f = \sum_m c_m \varphi_m \alpha_m^f \eta_m^f, \quad (3)$$

where η denotes the states of \mathcal{E} (or of its part inaccessible to measurement). If the states η_m^f are and remain practically orthogonal, the effective incoherence of the different terms in Ψ^f immediately follows. In other words, the ensemble corresponding to the mixture and the one corresponding to the pure state cannot be distinguished by measurements which do not involve also \mathcal{E} .

However, besides the problem of precision mentioned above, a problem of consistency arises. Let the measurement in Eq. (3) be performed at the time t_1 and the same measurement be repeated at the time $t_2 > t_1$ by means of a similar apparatus \mathcal{A}' . The evolution will now be

$$\begin{aligned} \Psi^i = \sum_m c_m \varphi_m \alpha^i \alpha'^i \eta^i &\xrightarrow{\text{S.E.}} \Psi_{t_1} = \sum_m c_m \varphi_m \alpha_m^f \alpha'^i \eta_m^f, \\ &\xrightarrow{\text{S.E.}} \Psi_{t_2} = \sum_m c_m \varphi_m \alpha_m^f \alpha'^f \eta_m^f, \end{aligned} \quad (4)$$

with obvious meaning of symbols. The quantum probability law can then be applied in two different ways. First, to the state vector after the first measurement (Ψ_{t_1}) to calculate the probability distribution of the second measurement. In this case, if one turns her/his the attention to the (selected) subensemble for which, say, the result l has been obtained, one gets a *wrong answer*. Second, to the state vector after the second measurement (Ψ_{t_2}) to calculate the probability of a third measurement consisting in the joint reading of the pointers of apparatuses \mathcal{A} and \mathcal{A}' . In this case one gets the *correct*

answer that the first and the second measurements gave the same result. Such an inconsistency is produced because in the description of the system after a measurement there is no trace of the outcome obtained for a specific element of the ensemble, or, at least, for the elements of a (selected) sub-ensemble of the original ensemble: every element is always described by the whole Ψ_t . A similar inconsistency is obtained, even without performing selections, in the case of a finite ensemble [19].

We can summarize the above discussion in the following way. Let us consider the statement

(CSR) there is a correspondence between the outcome of a measurement and the description (state) of the system $\mathcal{S} + \mathcal{A}$ after the measurement.

Let us call this proposition the *Common Sense Requirement* (CSR). The standard formulation, including the reduction principle, satisfies CSR. On the other hand, if we pretend that Schrödinger's equation be the sole principle of evolution, we get the inconceivable state, i. e., a description of $\mathcal{S} + \mathcal{A}$ after the measurement in which there is no counterpart of the individual outcomes, so that CSR is not satisfied. Adding limitative assumptions on measurability does not make CSR satisfied, causing the type of inconsistency discussed above.

Statement (C) can be made precise also in a way different from that proposed above. Let us assume that the probabilities to which statement (C) refers are the probabilities that the situations described in the various terms in the wave function Ψ_t (say the wave functions appearing in Eq. (4)) come true. At first sight, this new interpretation could appear a slight modification of the previous one. On the contrary, in this way, statement (A) is radically changed: the pair (Ψ, m) — the wave function and the branch label, i. e., the specification of the term in the wave function — describes an individual system. One may question whether such an additional element of description is introduced in a satisfactory way, but, as a matter of fact, the branch label is there and it is used to describe individual systems. Both problems of precision (no reference to the concept of measurement is there at the level of principles) and consistency (in fact CSR is satisfied) are solved, but we are far away the conceptual framework of the ensemble interpretation. Actually, specifying the statement (C) as above leads to the so called *consistent histories* interpretation of quantum mechanics [20,21,22]. The details of such an interpretation are beyond the aim of this paper.

There are two other ways of recovering CSR maintaining a single principle of evolution: *hidden variables theories* and *reduction theories*.

In hidden-variables theories, like the de Broglie-Bohm theory

and Nelson's stochastic mechanics, the evolution of Ψ is always governed by the Schrödinger equation, but, contrary to the standard formulation, Ψ does not exhaust the description of the system. A new variable is added to Ψ : this (hidden) variable evolves in such a way that its value after the measurement is in correspondence with the outcome.

In reduction theories, like the GRW theory [23], the evolution of Ψ , which completely specifies the state, is changed assuming a new principle of evolution. This is essentially equivalent to the Schrödinger equation in ordinary situations but it incorporates reduction in the measurement situations: the state vector collapses stochastically in accordance with quantum mechanical probabilities.

3. HIDDEN-CONFIGURATION THEORIES

In hidden-configuration theories, the state of an N -particle system at time t is described by the pair (\mathbf{x}, Ψ) , where $\mathbf{x}(t) = (x_1(t), \dots, x_N(t))$ is the point in configuration space, \mathbb{R}^{3N} , and $\Psi(\mathbf{x}, t)$ is a vector in the quantum Hilbert space. If the configuration of the system or of part of it is measured at a certain time, the system or its part are found as specified by the value of \mathbf{x} at that time. The time evolution of the pair (\mathbf{x}, Ψ) is given by

$$i\hbar \frac{\partial \Psi}{\partial t} = H \Psi, \quad (5)$$

$$d\mathbf{x} = b(\mathbf{x}, t) dt + \sqrt{\alpha} d\mathbf{w}. \quad (6)$$

Equation (5) is the Schrödinger equation with Hamiltonian H given by

$$H = -\frac{\hbar^2}{2m} \Delta + V$$

(we consider for simplicity spinless particles), where $\Delta/m = \sum_i \Delta_i/m_i$. For all regions where Ψ is different from zero we can write it in the form

$$\Psi(\mathbf{x}, t) = \exp [R(\mathbf{x}, t) + iS(\mathbf{x}, t)], \quad (8)$$

with R and S time dependent real functions on \mathbb{R}^{3N} . In Eq. (6) $\mathbf{w}(\mathbf{x}, t) = (w_1(x_1, t), \dots, w_N(x_N, t))$ is a set of N independent Wiener processes in \mathbb{R}^3 such that

$$\overline{dw_{ij}} = 0, \quad \overline{(dw_{ij})^2} = \frac{\hbar}{m_i}, \quad j = 1, 2, 3, \quad (9)$$

and α is a positive real constant. Then, for any α , $\mathbf{x}(t)$ is a Markov process in \mathbb{R}^{3N} characterized by the diffusion constants²

$$\nu_i = \alpha \frac{\hbar}{m_i} \quad (10)$$

and by the forward drift $\mathbf{b} = (b_1, \dots, b_N)$. We shall not discuss the possible physical origins of these fluctuations. In analogy with the use in Bohmian mechanics, Eq. (6) can be called the *guidance equation*. If $\rho(\mathbf{x}, t)$ represents the probability density of the stochastic process \mathbf{x} , the backward drift \mathbf{b}^* is given by

$$\mathbf{b}^* = \mathbf{b} - \alpha \hbar \frac{1}{\rho} \frac{\nabla}{m} \rho, \quad (11)$$

and the current density $\mathbf{j}(\mathbf{x}, t) = (j_1, \dots, j_N)$ by

$$\mathbf{j} = \frac{1}{2}(\mathbf{b} + \mathbf{b}^*)\rho = \mathbf{b}\rho - \frac{1}{2}\alpha\hbar\frac{1}{\rho}\frac{\nabla}{m}\rho, \quad (12)$$

with $\nabla/m = (\nabla_1/m_1, \dots, \nabla_N/m_N)$. ρ and \mathbf{j} satisfy the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0. \quad (13)$$

Let us assume that the probability density for the stochastic process ρ is given at some initial time $t = t_0$ by

$$\rho(\mathbf{x}, t) = |\Psi(\mathbf{x}, t)|^2 = \exp[2R(\mathbf{x}, t)]. \quad (14)$$

This relation remains true at all times if the quantum continuity equation

$$\frac{\partial \exp(2R)}{\partial t} + \nabla \cdot \hbar \left[\left(\frac{\nabla}{m} S \right) \exp(2R) \right] = 0 \quad (15)$$

coincides with the continuity equation of the stochastic process (13). This condition implies

$$\mathbf{j} = \hbar \left(\frac{\nabla}{m} S \right) \exp(2R), \quad (16)$$

² In general, one can consider for each kind of particles $\nu_i = \alpha_i \hbar / m_i$. The results below remain valid in the general case, while the choice of a unique α simplifies the notation.

i. e., assuming, for the forward drift \mathbf{b} appearing in Eq. (6),

$$\mathbf{b} = \hbar \frac{\nabla}{m} (\alpha R + S). \quad (17)$$

One may raise the objection that, as already noted just above Eq. (8), the formulation of quantum mechanics via the diffusion process described by (6) with \mathbf{b} and \mathbf{j} given by (17)³ and (16) is well defined only for the regions of the configuration space where Ψ is different from zero and sufficiently regular. The drift \mathbf{b} , for example, could have singularities on the nodal surfaces of Ψ or in correspondence of possible singular points of the potential V , where Ψ has singularities. It is proved (see [26,3] for Nelson's mechanics, and in general for quantum stochastic processes like those described here, and see [27] for Bohmian mechanics) that for a general class of potentials (including the case of N -particle Coulomb interaction with arbitrary charges and masses) the dynamics of hidden-configuration theories is well defined. Roughly speaking, the relation between Ψ and the diffusion process is formulated in such a way that if a particle trajectory starts in the complement of the nodal set and of the set of singularities of Ψ it does not leave this region with probability equal to one. As a consequence \mathbf{b} and \mathbf{j} can be considered equal to zero on the nodal set and on the set of singularities of Ψ .

Assuming that all measurements can be reduced to position measurements,⁴ the hidden-configuration theories and standard quantum mechanics are experimentally indistinguishable. In fact, hidden-configuration theories and standard quantum mechanics predict the same probabilities of finding the system in any configuration at any time, if $\rho = |\Psi|^2$ is satisfied at the initial time.

Even though the presentation is completely different, the key idea of this section, that is the construction of a family of stochastic theories equivalent to standard quantum mechanics, is due, as

³ Equation (17) can be also written in the form

$$\mathbf{b} = \frac{\hbar}{m} \left(\alpha \operatorname{Re} \frac{\nabla \Psi}{\Psi} + \operatorname{Im} \frac{\nabla \Psi}{\Psi} \right).$$

⁴ This assumption is called "sufficiency of position measurement" by Goldstein [12]. In fact, if a variable does not commute with position, its measurement is achieved by a time-delayed position measurement. This has been firstly remarked by Feynman and Hibbs [24] and applied in the framework of the path integral formulation of quantum mechanics. The same result, in the context of the stochastic interpretation of quantum mechanics, can be found in Ghirardi, Omero, Rimini and Weber [25].

said above, to Davidson. It is easily seen that for $\alpha = 0$ Bohmian mechanics and for $\alpha = 1$ Nelson's stochastic mechanics are obtained.

We remark that, in the formulation adopted here, the existence of the Ψ field which evolves according with the Schrödinger equation is assumed *ab initio*. For such a reason, the problem of equivalence between the Madelung hydrodynamic equations and the Schrödinger equation, discussed by Wallstrom [28], is bypassed. Correspondingly, any reference to quantum potentials or to Newton's equation is eliminated.

It is important to note, in preparation for the next section, that if Ψ splits into two (or more) parts, $\Psi = \Psi_1 + \Psi_2$, in such a way that the Schrödinger evolution keeps the two terms separated in configuration space during a certain time, then

$$\rho = \rho_1 + \rho_2 \quad (18)$$

and

$$\mathbf{j} = \mathbf{j}_1 + \mathbf{j}_2, \quad (19)$$

where ρ_1, \mathbf{j}_1 and ρ_2, \mathbf{j}_2 are nonzero, respectively, in two domains in configuration space D_1 and D_2 disjoint during the considered time. According to Eqs. (14) and (16), for the purpose of the evolution of a system whose \mathbf{x} lays, say, in D_1 the term Ψ_2 can consistently be dropped. A similar decomposition

$$\mathbf{b} = \mathbf{b}_1 + \mathbf{b}_2. \quad (20)$$

holds for the drift \mathbf{b} . It follows from the very general results quoted above the (almost sure) impossibility of the configuration of evolving in such a way to cross the boundaries of the disjoint supports.

4. THEORY OF MEASUREMENT IN HIDDEN-CONFIGURATION THEORIES

We consider the quantum measurement process in the framework of hidden-configuration theories through a simple but significant example [19,10], in which a coarse measurement of the position of the particle S is performed by a pair \mathcal{A} of detectors (see Fig. 1). For simplicity, we shall indicate by \mathcal{B} the pair \mathcal{A} of detectors plus the environment \mathcal{E} . In hidden-configuration theories, the state of the system $S + \mathcal{B}$ is given by

$$((\mathbf{x}, \mathbf{y}), \Psi(\mathbf{x}, \mathbf{y})),$$

where \mathbf{x} is the position of the particle, \mathbf{y} is the configuration of the particles in \mathcal{B} , and $\Psi(\mathbf{x}, \mathbf{y}) = \sum_i c_i \psi_i(\mathbf{x}) \beta_i(\mathbf{y})$ is the wave function

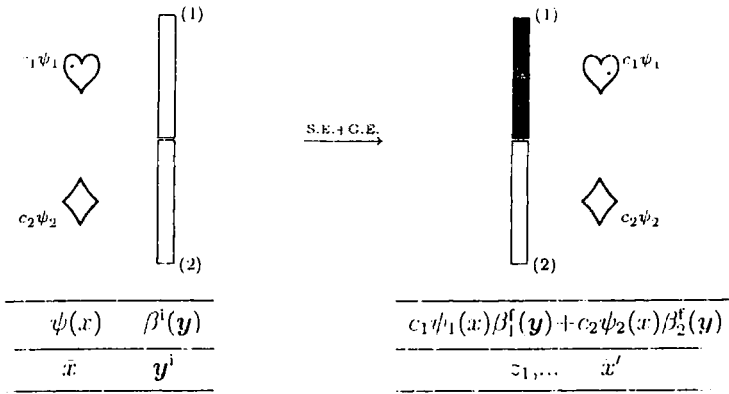


Fig. 1. The evolution of the system $S + B$ during the coarse position measurement according to hidden-configuration theories. Between the lines at the bottom, the complete descriptions of the system $S + B$ before and after the measurement are indicated. The dot in the term $c_1\psi_1$ of ψ represents the actual positions \bar{x} , \bar{x}' of the particle.

of the entire system. A subset z among the coordinates y plays the role of pointer variable of the detectors. The value, say, $z = z_1$ ($= z_2$) corresponds to the first (second) detector having revealed the particle. Let the particle wave function before the measurement be the superposition of two packets ψ_1 and ψ_2 , each hitting one detector. The evolution of the wave function of the system $S + B$ during the measurement is ruled by the Schrödinger equation and is of the type (3), i. e., in the present notation,

$$\begin{aligned} \Psi^i &= (c_1\psi_1(x) + c_2\psi_2(x))\beta^i(y) \xrightarrow{\text{S.E.}} \Psi^f \\ &= c_1\psi_1(x)\beta_1^f(y) + c_2\psi_2(x)\beta_2^f(y), \end{aligned} \tag{21}$$

where we have omitted to indicate the trivial time evolution bringing the particle through the detectors. In (21) β^i , β_1^f and β_2^f are the wave functions of B before the measurement, after the measurement if the particle wave function were ψ_1 alone, and after the measurement if the particle wave function were ψ_2 alone, respectively. β_1^f and β_2^f are strongly peaked around $z = z_1$ and $z = z_2$, respectively. The evolution of the configuration of $S + B$ is ruled by the guidance

equation and can be written

$$\bar{x}, \mathbf{y}^i \xrightarrow{\text{G.E.}} \begin{cases} \bar{x}', \bar{z}^f = z_1, \dots \text{if } \bar{x} \text{ lies within } \psi_1, \\ \bar{x}', \bar{z}^f = z_2, \dots \text{if } \bar{x} \text{ lies within } \psi_2, \end{cases} \quad (22)$$

where \bar{x} and \bar{x}' are the initial and final positions of the particle \mathcal{S} for the particularly considered member of the ensemble (the prime is intended to describe the effect of the diffusion present in Eq. (6)), \mathbf{y}^i is the initial configuration of \mathcal{B} , and \bar{z}^f is the final value of the pointer variable of the detectors. The wave function Ψ^f is a superposition of two terms disjoint in configuration space. After the measurement, each member of the ensemble of systems $\mathcal{S} + \mathcal{B}$ is described by Ψ^f (the same for all members) and by the coordinates $\bar{x}', \bar{z}^f, \dots$ (particular values for a particular member). The wave function Ψ^f is no more inconceivable because it is only a part of the complete state and CSR is satisfied. It is obvious that, if the measurement is repeated by a second apparatus similar to \mathcal{A} possibly contained in \mathcal{B} , corresponding results are obtained for each member of the ensemble.

As already noted, the two terms in the wave function Ψ^f immediately after the measurement have supports disjoint in configuration space. Because of the enormous complexity of the system \mathcal{B} (see, e.g., [8]), this condition will remain true for all subsequent times with overwhelming probability, in spite of any possible attempt to rejoin the two terms. This circumstance, in conjunction with the assumption that any measurement is ultimately a position measurement, is the form taken by effective incoherence in hidden-configuration theories and it ensures that the statistical predictions of these theories as regards any possible subsequent measurement coincide practically with those of standard quantum mechanics.

One further point remains to be discussed. The condition $\rho = |\Psi|^2$ is satisfied for the originally considered ensemble \mathbf{E} after the measurement as it is before the measurement, according to the discussion in the previous section. However, after the measurement, \mathbf{E} can be split into two parts, \mathbf{E}_1 and \mathbf{E}_2 , depending on whether the particle has been found in one detector or the other. Considering e.g. \mathbf{E}_1 , the systems contained in it are described, according to the discussion above, by the wave function $\Psi = c_1 \Psi_1 + c_2 \Psi_2 = c_1 \psi_1(x) \beta_1^f(\mathbf{y}) + c_2 \psi_2(x) \beta_2^f(\mathbf{y})$, but the probability distribution is $\rho = |\Psi_1|^2$ corresponding to Ψ_1 alone. Therefore, the ensemble \mathbf{E}_1 , whose consideration is perfectly legitimate, violates the condition $\rho = |\Psi|^2$. This fact does not give rise to contradictions, because the condition $\rho = |\Psi_1|^2$ for the systems in \mathbf{E}_1 remains true when time passes even maintaining that the wave function is the whole Ψ . Indeed, the dynamics of each system $\mathcal{S} + \mathcal{B}$ is specified by the initial value of the

wave function Ψ together with the Schrödinger equation governing its evolution and by the initial value of the configuration (x, \mathbf{y}) together with the guidance equation governing (stochastically, if $\alpha \neq 0$) its evolution, the drift appearing in the latter equation being in turn determined by Ψ . Assumptions on the distribution ρ of configurations at a given time in a given ensemble have nothing to do with the fundamental dynamics and only serve the practical purpose of making statistical predictions about the ensemble. The role of such assumptions is similar to the role of the choice of statistical ensembles in classical statistical mechanics. If an ensemble is put together using informations about configurations, as in the case of the ensemble E_1 above, it is natural and obligatory taking into account such informations assuming $\rho = |\Psi_1|^2$. Then the agreement with the known phenomenology described by standard quantum mechanics is ensured by the fact that this condition is conserved in time and the consistency of the theory by the fact that such a conservation holds even using as the wave function the whole Ψ , because Ψ_1 and Ψ_2 remain forever separated in configuration space and because of the remark at the end of the previous section. In conclusion, each individual physical system is described by the values of the coordinates of all particles in the system and by a wave function which never suffered any reduction.

5. CONCLUSION

The content of the previous sections is to be compared with that of similar works. As already stated in the Introduction, the theory of quantum measurement in the framework of Bohmian mechanics has already been developed along the same lines followed here by Bell [6,7], Bohm and Hiley [8], and Dürr, Goldstein and Zanghì [9]. It has also been retraced on the basis of the same simple example used in Sec. 4 by one of the present authors [10].

The theory of quantum measurement in the framework of Nelson's stochastic mechanics has been worked out in a paper by Blanchard, Cini and Serva [13], and, previously, by Goldstein [12]. The arguments used by Blanchard, Cini and Serva are very similar to those developed here, but they are presented in a form which implies that the use of Nelson's stochastic mechanics is crucial to get a satisfactory description of measurement. We think, on the contrary, that what is crucial is solely the addition to the wave function of a new element of description (the configuration), which allows to satisfy the proposition referred to above as Common Sense Requirement. As we have seen, this is possible for all formulations of quantum mechanics of the family introduced in Sec. 3. Even though the paper of Goldstein refers only to Nelsonian mechanics, that point is clear

in his presentation.⁵

The unified treatment of the measurement process in Sec. 4 for all formulations of the family described in Sec. 3 helps in the clear identification of the true crucial point allowing the elaboration of a satisfactory theory of quantum measurement. Furthermore it seems to us that, at this point, one should address the question whether the introduction of a diffusion is really worthwhile, having realized that it is not essential to solve the problems of precision and consistency of quantum mechanics.⁶

REFERENCES

1. D. Bohm, *Phys. Rev.* **85** (1952), pp. 166 and 180.
2. E. Nelson, *Phys. Rev.* **150** (1966), p. 1079.
3. E. Nelson, *Quantum Fluctuations* (Princeton University Press, Princeton, 1985).
4. M. Davidson, *Lett. Math. Phys.* **3**, 271 (1979).
5. D. Bohm and B.J. Hiley, *Phys. Rep.* (Review section of *Phys. Lett.*) **172**, 93 (1989).
6. J.S. Bell, Cern Preprint Th 1424, 1971; *Epist. Lett.*, July 1978, p. 1.
7. J.S. Bell, in *Quantum Gravity 2*, C. Isham, R. Penrose, and D. Sciama, eds. (Clarendon Press, Oxford, 1981), p. 611.
8. D. Bohm and B.J. Hiley, *Found. Phys.* **14**, 255 (1984).
9. D. Dürr, S. Goldstein, and N. Zanghì, *J. Stat. Phys.* **67**, 843 (1992).
10. A. Rimini, in *Waves and Particles in Light and Matter*, A. van der Merwe and A. Garuccio, eds. (Plenum, New York, 1994), p.

⁵ There is no relation between the theory of measurement in the framework of hidden configuration theories presented in Sec. 4 and the reduction-like processes hypothesized by Guerra [14] in the framework of discrete generalization of stochastic variational principles. Even though it is not explicitly stated, the theory described there concerns the quantum system alone (the system we have called \mathcal{S}), while in Section 4 here the system is $\mathcal{S} + \mathcal{A} + \mathcal{E}$, and in this sense a theory of measurement is developed. As we have seen, in such a context no reduction has to be introduced, and a satisfactory description of the measurement process is nevertheless obtained. May be that the reduction-like process, which is possibly present in the formalism of Guerra, can be used to describe the effective reduced dynamics for the quantum system \mathcal{S} alone.

⁶ Nevertheless, it may reveal useful to exploit the well developed mathematical methods of probability theory and stochastic processes in quantum mechanics, in particular in its path integral formulation.

603.

11. Ph. Blanchard, S. Golin, and M. Serva, *Phys. Rev. D* **34**, 3732 (1986).
12. S. Goldstein, *J. Stat. Phys.* **47**, 645 (1987).
13. Ph. Blanchard, M. Cini, and M. Serva, in *Ideas and Methods in Quantum and Statistical Physics*, S. Alberverio, ed. (Cambridge University Press, Cambridge, 1992).
14. F. Guerra, in *The Foundations of Quantum Mechanics - Historical Analysis and Open Questions*, C. Garola and A. Rossi, eds. (Kluwer Academic, Dordrecht, 1995), p. 339.
15. E. Schrödinger, *Naturwiss.* **23**, 844 (1935); English translation in *Proc. Am. Philos. Soc.* **124**, 323 (1980).
16. E.P. Wigner, in *The Scientist Speculates*, I.J. Good, ed. (Basic Books, New York, 1962); reprinted in E.P. Wigner, *Symmetries and Reflections* (Indiana University Press, Bloomington, Indiana, 1967).
17. E. Joos and H.D. Zeh, *Z. Phys. B* **59**, 223 (1985).
18. K. Gottfried, *Quantum Mechanics* (Benjamin, New York, 1966); *Phys. Worlds* **4** (10), 34 (1991).
19. O. Nicosini and A. Rimini, in *Symposium on the Foundations of Modern Physics 1990*, P. Lahti and P. Mittelstaedt, eds. (World Scientific, Singapore, 1991), p. 280.
20. R. Griffiths, *J. Stat. Phys.* **36**, 219 (1984).
21. R. Omnès, *Rev. Mod. Phys.* **64**, 339 (1992).
22. M. Gell-Mann and J.B. Hartle, *Phys. Rev. D* **47** 3345 (1993).
23. G.C. Ghirardi, A. Rimini, and T. Weber, *Phys. Rev. D* **34**, 470 (1986); **36**, 3287 (1987).
24. R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
25. G.C. Ghirardi, C. Omero, A. Rimini, and T. Weber, *Riv. Nuovo Cimento* **1** (3), 1 (1978).
26. E.A. Carlen, *Comm. Math. Phys.* **94**, 293 (1984).
27. K. Berndl, D. Dürr, S. Goldstein, G. Peruzzi, and N. Zanghì, *Comm. Math. Phys.* **173**, 647 (1995).
28. T.C. Wallstrom, *Phys. Rev. A* **49**, 1613 (1994).