

A real-time groundwater management model using data assimilation

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[1] This study develops a groundwater management model for real-time operation of an aquifer system. A groundwater flow model is allied with a nudging data assimilation algorithm that reduces the forecast error, minimizes the risk of system failure, and improves management strategies. The nudging algorithm treats the unknown private pumping as an additional sink term in the groundwater flow equation and provides a consistently physical interpretation for the identification of pumping rates. The system response due to pumping and injection is represented by a response matrix that is generated by the influence coefficient method. The response matrix (with a much smaller dimension) is used as a reduced model and is embedded directly in the management model as a part of the constraint set. Additionally, the influence coefficient method is utilized to include the nudging effect in the reduced model. The management model optimizes the monthly operation for 12 months into the future and determines the optimal strategy using the information provided by nudging. The management model is updated at the beginning of each month when new head observations and pumping data become available. We also discuss the utility, accuracy, and efficiency of the proposed management model for real-time operation.

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1. Introduction

[2] Groundwater simulation models have been coupled with optimization models to assist in decision making for groundwater management. Gorelick [1983], Yeh [1992], Wagner [1995], and Ahlfeld and Mulligan [2000] provided a comprehensive review of simulation-optimization approaches to groundwater management. As documented in the literature, simulation-optimization approaches require the inclusion of a groundwater simulation model within the constraint set of the optimization (management) model. In general, there are two approaches to including a simulation model in an optimization model. The first approach, known as the embedding approach, embeds the discrete approximations of the groundwater governing equation directly in the constraint set of the optimization model. Yeh [1975] used this approach to solve the inverse problem of parameter identification governed by a two dimensional partial differential equation. Gharbi and Peralta [1994] used the embedding approach for sustainable groundwater quantity and quality management of complex nonlinear aquifers. However, this approach becomes infeasible when the set of discretized equations is very large. To alleviate the dimensionality problem inherent to the embedding approach, McPhee and Yeh [2008] replaced a groundwater flow model governed by a partial dif-

ferential equation with a simple reduced model governed by an ordinary differential equation. McPhee and Yeh [2008] achieved model reduction with empirical orthogonal functions, i.e., principal components. They demonstrated that the reduced model, with a considerably smaller dimension, in many instances captured the dominating characteristics of the original full-scale model and was accurate and sufficient for management purposes.

[3] The second approach to link simulation with optimization utilizes the response matrix. In this approach, the response matrix with a much smaller dimension replaces the simulation model and is included directly in the constraint set. The response matrix is formed by influence coefficients that can be obtained by perturbing the groundwater system with a small change of each decision variable (pumping and recharge), one at a time, at a specified location and time to evaluate the sensitivity of the state variable with respect to such change [Becker and Yeh, 1972; Maddock, 1972; Ahlfeld and Heidari, 1994]. Louie et al. [1984] developed a multiobjective optimization procedure using the constraint linear programming method, in which simulation is linked to optimization via the response matrix. Reichard [1995] developed a stochastic simulation optimization model for groundwater-surface water management. The uncertainty of surface water supplies was incorporated explicitly in the stochastic model and the response matrix was used to link simulation with optimization. More recently, Chiu et al. [2010] developed an optimal pump and recharge strategy for a planned conjunctive use project to remove the high nitrate concentration in a groundwater basin in southern California. The response matrix approach was used to link a flow and transport simulation model with optimization.

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[4] Most of the published work on groundwater management has been predominately for planning purposes. The problem of real-time groundwater management has received much less attention. In addition, insufficient and inaccurate pumping rates from private wells may lead to misguided management decisions. In this study, we propose a real-time management model with receding horizon control (RHC) and data assimilation (DA) to assist in decision making for groundwater management.

[5] We adopt RHC [Kwon and Han, 2005] to incorporate forecast information and improve management strategies. The RHC concept of using deterministic forecasting for optimization with frequent updating is similar to the methodology proposed by Becker and Yeh [1974] for optimizing the real-time operation of a complex reservoir system. Data assimilation (DA) is a technique that can be used to update the state variable as well as parameters in the simulation model when new observations are available, thus improving forecast accuracy. A few studies in the literature combine DA with optimization for parameter estimation [Vrugt et al., 2005, 2006] and irrigation schedule estimation [Wang and Cai, 2007]. Cheng et al. [2009] proposed an algorithm based on Newtonian relaxation (nudging) to estimate the unknown pumping rates from private extraction wells using a calibrated groundwater simulation model and hydraulic head observations. As typical of any data assimilation algorithm, nudging exploits the dynamics of the pumping rates and frequent observations of the system state (head) to resolve the identification problem. As demonstrated by Cheng et al. [2009], the estimation is particularly accurate in case of linear models characterized by time-independent uncertainty. In that study, pumping data of private wells were estimated using the nudging algorithm. For real-time applications, partial pumping data may be available and must be taken into account. Additionally, the missing pumping information must be estimated.

[6] In this paper, we extend the nudging algorithm of Cheng et al. [2009] to incorporate the available pumping information for updating. We combine the nudging algorithm with the response matrix for forecasting and updating in the overall management model. For a linear system, such as a confined aquifer, the influence coefficients remain unchanged; hence, the response matrix can be included directly in the constraint set and optimization can be carried out without iteration.

[7] Pumping activity from local private wells is a key factor that affects the management decision. Even though well locations generally are known, the pumping rate as a function of time is often not available, particularly for private wells [Cheng et al., 2009]. In California, for example, “property rights do not require land owners in non-adjudicated basins to measure their groundwater pumping rates and publicly disclose them” [Ruud et al., 2004, p. 52]. Even though local extraction well owners report their pumpage, the frequency and accuracy of reporting is generally insufficient. In light of this reality the goals of this study are (1) to develop a proper approach for incorporating available pumping data (that may be insufficient) and (2) to develop a real-time, sequential management model to assist in decision making for groundwater management. We apply the proposed management model to the Aquifer Storage and Recovery (ASR) Project of the Las Posas Groundwater Basin in southern California. The adapted nudging algorithm is used to estimate the missing pumping rates of private wells. We then use the estimated pumping rates in a real-time groundwater

management model to improve management decisions of the ASR well fields.

2. Management Model

2.1. The Framework of the Management Model With Receding Horizon Control

[8] The proposed management model seeks to determine the optimal operation of the ASR wells. As in any groundwater system, the ASR wells operate along with other forcing terms, e.g., recharge and other pumping wells. We employ the RHC algorithm, in which the system operation is optimized with given forecast information for a finite control time horizon (CTH) that comprised several stress periods. For a given initial condition and initial forecast information, the management model optimizes the operation over the specified CTH, but only the optimized decision for the immediate stress period is adopted. The CTH then is moved into the future by one stress period and the process is repeated when new information becomes available at the end of the succeeding stress period. The CTH is a moving window over which optimization is applied repeatedly.

[9] Clearly the reliability of the optimized strategy depends on forecast accuracy. For this reason we include the DA algorithm developed by Cheng et al. [2009], appropriately extended to handle partially available information, to improve the accuracy of withdrawal rates of pumping wells other than ASR wells. At the end of each stress period, when new head observations and pumping data become available, the DA algorithm is deployed to estimate the missing pumping rates and update the state variable, i.e., the head distribution. The updated head distribution then is used as the initial condition for the simulation model that evaluates the new forecast data.

2.2. Management Model Formulation

[10] We consider the discharge rates as positive in case of injection and negative in case of withdrawal (pumping). Well capacities and water demand and availability are considered to be positive values.

[11] The objective function of the management model is to minimize the costs of injection and pumping plus the cost of deficit, i.e., the portion of the demand that is not satisfied by the subsurface resource. In the standard cost minimization formulation, all cost coefficients are assumed to be positive. The general formulation of the management model can be written as

Objective function

$$\min Z = \sum_{j=1}^{N_{ASR}} \sum_{t=1}^{N_T} \left[C_{j,t}^{in} \cdot Q_{j,t}^{in} - C_{j,t}^p \cdot Q_{j,t}^p \right] + \sum_{t=1}^{N_T} C_t^d \cdot \text{deficit}_t \quad (1)$$

subject to

Water availability

$$\sum_{j=1}^{N_{ASR}} \sum_{t=1}^{N_T} Q_{j,t}^{in} \cdot \Delta t \leq V^A \quad (2)$$

Well capacity

$$0 \leq Q_{j,t}^{in} \leq Q_j^{in,cap} \quad j = 1, \dots, N_{ASR}; t = 1, \dots, N_T \quad (3)$$

$$0 \leq |Q_{j,t}^p| \leq Q_j^{p,cap} \quad j = 1, \dots, N_{ASR}; t = 1, \dots, N_T \quad (4)$$

Demand

$$0 \leq \sum_{j=1}^{N_{ASR}} |Q_{j,t}^p| \cdot \Delta t \leq D_t \quad t = 1, \dots, N_T \quad (5)$$

Head constraints

$$h_{i,t}^{\min} \leq h_{i,t} \leq h_{i,t}^{\max} \quad i = 1, \dots, N_C; t = 1, \dots, N_T \quad (6)$$

Deficit

$$\text{deficit}_t = D_t - \sum_{j=1}^{N_{ASR}} |Q_{j,t}^p| \cdot \Delta t \quad t = 1, \dots, N_T \quad (7)$$

Well constraint

$$Q_{j,t} = b'_{j,t} \cdot Q_{j,t}^{in} + b''_{j,t} \cdot Q_{j,t}^p \quad t = 1, \dots, N_T \quad (8)$$

$$b'_{j,t} + b''_{j,t} = 1 \quad t = 1, \dots, N_T \quad (9)$$

Groundwater simulation model

$$H(\bullet) = 0 \quad (10)$$

where $C_{j,t}^{in}$ is the unit injection cost of ASR well j for the t th stress period, $C_{j,t}^p$ is the unit pumping cost of ASR well j for the t th stress period, C_t^d is the unit cost of deficit for the t th stress period, $Q_{j,t}^{in}$ is the injection rate of ASR well j during the t th stress period, $Q_{j,t}^p$ is the pumping rate of ASR well j during the t th stress period, $Q_{j,t}$ is the total discharge rate of ASR well j during the t th stress period, V^A is the total water volume available for injection during the CTH, $Q_j^{p,cap}$ is the pumping capacity of ASR well j , $Q_j^{in,cap}$ is the injection capacity of ASR well j , D_t is the demand during the t th stress period, deficit_t , a positive variable, is the water supply deficit during the t th stress period, $b'_{j,t}$ and $b''_{j,t}$ are 0 and 1 binary variables, Δt is the duration of the stress period, N_T is the total number of stress periods during the CTH, N_{ASR} is the total number of ASR wells, N_C is the total number of selected locations where hydraulic head constraints are imposed (these may include ASR wells, pumping wells, observation wells, etc.), and $H(\bullet) = 0$ represents the groundwater simulation model.

3. Simulation Models

3.1. Groundwater Simulation Model With Nudging Data Assimilation

[12] Equation (10) denotes the governing equation and the associated initial and boundary conditions of a groundwater simulation model. For saturated flow in a confined aquifer the governing equation can be written as [Bear, 1988; Willis and Yeh, 1987]

$$S_s \frac{\partial h}{\partial t} = \text{div}[K \nabla h] + q(\vec{x}, t) \quad (11)$$

where S_s is the specific storage (L^{-1}), $h(\vec{x}, t)$ is the hydraulic head (L), t (T) is time, K is the hydraulic conductivity tensor ($L T^{-1}$), $q(\vec{x}, t)$ (T^{-1}) represents the source or sink term, and $\vec{x} = [x, y, z]^T$ (L) is the vector of three-dimensional spatial coordinates. Using the algorithm proposed by Cheng *et al.* [2009], we add a forcing term $q'(\vec{x}, t)$, or nudging term, to the governing equation to account for the unknown pumping:

$$S_s \frac{\partial h}{\partial t} = \text{div}[K \nabla h] + q(\vec{x}, t) + q'(\vec{x}, t) \quad (12)$$

To estimate the nudging term, the nudging algorithm converts the mismatch of hydraulic head between the model forecast and the observation into the pumping rate increment or decrement. In the following we use N_o to denote the number of observation wells and N_w the number of pumping wells for which we wish to estimate the discharge rate. For each pumping well j at the update time t_u , we calculate the pumping rate increment or decrement as

$$\Delta q_{u,j} = \sum_{i=1}^{N_o} w_{i,j}^* \frac{1}{r_{i,j,1}} w'_{i,j} d_{u,i} \quad (13)$$

where $w_{i,j}^*$ is the weighting coefficient; $w'_{i,j}$ is the distribution ratio; $r_{i,j,1} = \partial h_{i,t} / \partial q_{j,t}$ is the one-stress-period influence coefficient relating hydraulic head at observation well i to the pumping rate at well j evaluated at time t_u ; and $d_{u,i}$ is the mismatch of hydraulic head between model forecast and observation, also referred to as the innovation residual, recorded at observation well i . Let $\vec{x}_i = [x_i, y_i, z_i]^T$ be the spatial coordinate of observation well i , $\vec{x}_j = [x_j, y_j, z_j]^T$ be the spatial coordinate of pumping well j , and $D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ be the relative distance in a horizontal plane between the two wells. Accordingly, we can formulate [Cheng *et al.*, 2009]

$$w_{i,j} = w(\vec{x}_i, \vec{x}_j, t) = w_1(D_{i,j}) w_2(z_i, z_j) w_3(t) \quad (14)$$

$$w_1(D_{i,j}) = \max \left\{ \left(R_i^2 - D_{i,j}^2 \right) / \left(R_i^2 + D_{i,j}^2 \right), 0 \right\} \quad (15)$$

$$w_2(z_i, z_j) = \begin{cases} 1, & z_i = z_j \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

$$w_3(t) = \begin{cases} 1, & t = t_u \\ 0, & \text{otherwise} \end{cases} \quad (17)$$

$$w'_{i,j} = \frac{w_{i,j}}{\sum_{j=1}^{N_w} w_{i,j}} \quad (18)$$

$$w_{i,j}^* = \frac{w_{i,j}}{\sum_{i=1}^{N_o} w_{i,j}} \quad (19)$$

where R_i is the radius of influence for observation well i , which defines the range of pumping information collected for

the observation well. The definition of $w_3(t)$ implies that the nudging term is active only at the update time. If some pumping data are available from private wells, the distribution ratio w' and the weighting coefficient w^* need to be recalculated. For example, in a given stress period, there are three private wells (P1, P2, and P3) pumping in the field, but if the pumping rate of P1 is available, then we only need to estimate the pumping rates of P2 and P3. In this situation, we calculate the distribution ratio and weighting coefficient using only the locations of P2 and P3. We assume well locations are known and that the vertical influence on aquifer head due to pumping rate mismatch is confined within one computational layer.

[13] In this study, we use MODFLOW [Harbaugh et al., 2000] as the full simulation model to perform the sensitivity analysis and generate influence coefficients.

3.2. The Reduced Groundwater Simulation Model

[14] We use the response matrix, referred to as the reduced model, to replace the full groundwater simulation model and directly include it in the optimization model. We use Taylor series expansion to relate head (hydraulic head) and stress. The stress vector $\mathbf{q} = \{Q_{j,t}\}$, $j = 1, \dots, N_p$, where N_p is the number of pumping wells in the system, includes stresses at each selected location (j) for each management period (stress period) (t). The head at time T is a function of the current and past conditions (head distribution and forcings). A Taylor series expansion about an initial policy \mathbf{q}_0 can be expressed as [Louie et al., 1984; Ahlfeld and Mulligan, 2000]

$$h_{i,T}(\mathbf{q}) = h_{i,T}^0(\mathbf{q}_0) + \sum_{j=1}^{N_p} \sum_{t=1}^{N_T} \frac{\partial h_{i,T}}{\partial q_{j,t}}(\mathbf{q}_0) \cdot (Q_{j,t} - Q_{j,t}^0) + \dots \quad (20)$$

We ignore the high-order terms and use the symbol $r_{i,j,T}$ to represent the influence coefficient $\frac{\partial h_{i,T}}{\partial q_{j,t}}$. This influence coefficient can be obtained from a sensitivity analysis and represents the rate of change of head at location i at time $t = T$ in response to a unit change of stress at pumping well j at time $t = 1$ using $Q_{j,t}^0$ as reference. In equation (20) we implicitly assume that the influence coefficient is zero whenever $t > T$. The reduced model yields

$$h_{i,T} = h_{i,T}^0 + \sum_{j=1}^{N_p} \sum_{t=1}^{N_T} (Q_{j,t} - Q_{j,t}^0) \cdot r_{i,j,T-t+1} \quad (21)$$

where i is a selected location that contains either an observation well, a pumping well, an ASR well, or a selected control point; j represents a forcing location such as a pumping well or an injection well; $Q_{j,t}$ is the pumping rate; $Q_{j,t}^0$ is the initial pumping rate or the pumping rate at the end of the previous period; $h_{i,T}$ is the head for location i during stress period T ; $h_{i,T}^0$ is the initial head for location i during stress period T ; t is a time index that defines the pumping influence in the time domain; and N_p is the number of wells with known discharges, e.g., the ASR wells and private pumping wells with reported pumping rates. It should be mentioned that the model starts from time zero. The forcing during the first stress period is $Q_{j,t=1}$, and $Q_{j,t=0}$ has no physical meaning. For convenience, we let $Q_{j,t=0} = 0$.

[15] The reduced model is directly embedded in the optimization model as a part of the constraint set for the entire

CTH. As mentioned before, for real-time management, the optimization model is applied at the beginning of each stress period over the CTH, and then the CTH moves one stress period forward for the succeeding optimization run. Figure 1 shows the time indices adopted in equation (20) and the proposed real-time management model. In equation (20), t indicates the time from the beginning of the CTH ($t = 1$) through the end of the CTH ($t = N_T$). The time T represents any reference time point for the head distribution within the CTH.

3.3. The Reduced Simulation Model With Nudging

[16] Because of linearity of the governing equation, the nudging-estimated pumping rates can be incorporated directly into the reduced simulation model using the principle of superposition. The nudging component is treated as an additional term in the reduced simulation model. We assume here that there are N_p wells having known discharge rates, and we wish to estimate the discharge rates at a number N_w of wells. The resulting reduced model can be expressed as

$$h_{i,T} = h_{i,T}^0 + \sum_{j=1}^{N_p} \sum_{t=1}^{N_T} (Q_{j,t} - Q_{j,t}^0) \cdot r_{i,j,T-t+1} + \sum_{j=1}^{N_w} \sum_{t=1}^{N_T} (Q_{j,t} - Q_{j,t}^0) \cdot r_{i,j,T-t+1} \quad \forall i \quad (22)$$

where $Q_{j,t}$ is the unknown pumping rate to be identified by nudging, and $Q_{j,t}^0$ represents known pumping rates.

[17] In general, the nudging algorithm is activated when observation data are available. Once missing pumping rates are estimated via equation (13), the state variable (head) is updated accordingly. The head before and after the update at a selected location has the following relationship:

$$h_{i,tu}^+ = h_{i,tu}^- + \sum_{j=1}^{N_w} (Q_{j,t}^+ - Q_{j,t}^-) \cdot r_{i,j,1} \quad \forall i \quad (23)$$

where superscripts minus and plus represent before- and after-the-update processes. For real-time management, the pumping rate before the update, $Q_{j,t}^-$, is the forecast pumping rate. The pumping rate after the update, $Q_{j,t}^+$, is the reported pumping rate or the nudging-estimated pumping rate. The pumping rate difference between $Q_{j,t}^-$ and $Q_{j,t}^+$ is calculated via equation (13) using the head difference between the model forecast and the head observation. The head response due to the pumping rate difference is evaluated using equation (23). Equation (23) also indicates that the proposed nudging algorithm updates the state variable, the head distribution, using the estimated pumping rates.

4. Schematic Representation of the Management Model and Nudging Update Procedure

[18] The nudging algorithm is used to estimate the missing pumping rates and update head distribution. The algorithm is activated before each stress period when observations are available. The estimated pumping rates and the updated head distribution are combined with the forecasted information to invoke the real-time management model. Here we provide a step-by-step procedure.

[19] 1. Assemble necessary information for the management model and the nudging algorithm. First, a calibrated ground-

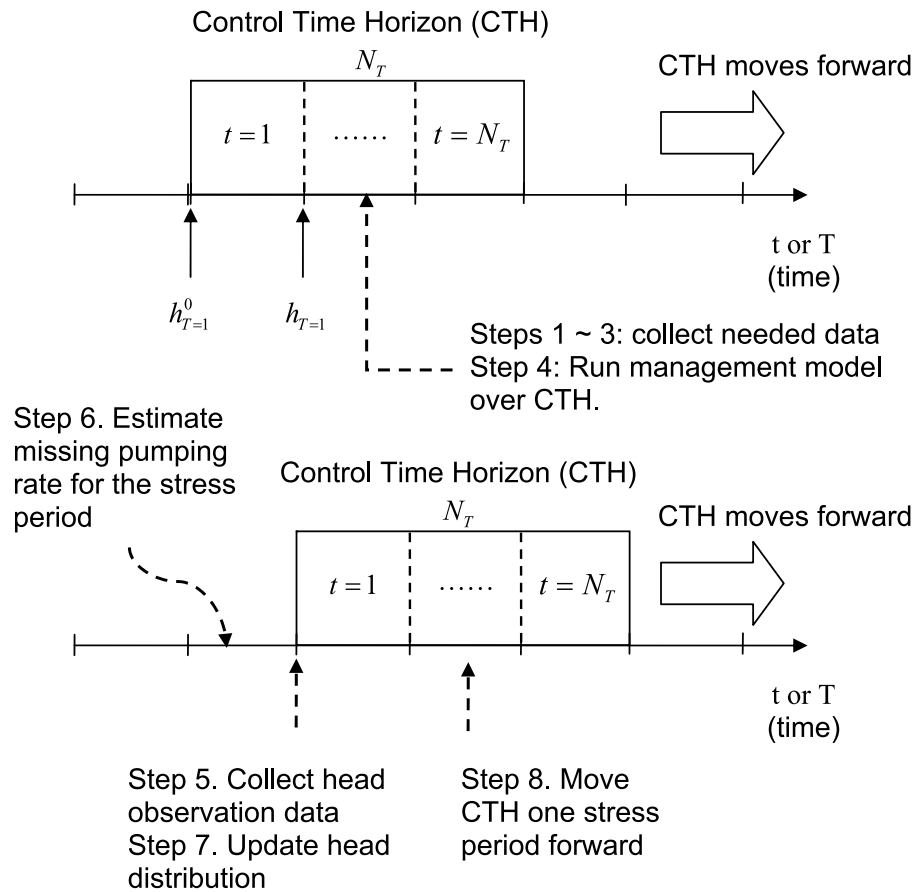


Figure 1. The proposed real-time management model and time indices adopted.

water simulation model is necessary for calculating influence coefficients. Second, identify the locations of pumping and observation wells. This information will be used to calculate the distribution ratios and weighting coefficients.

[20] 2. Collect the forecast pumping information and the initial condition. The forecast information is used in the management model. If no information is available, zero pumping rates or long-term average values can be assumed.

[21] 3. Calculate influence coefficients and build the reduced groundwater simulation model using the response matrix approach. If necessary, the influence coefficients can be updated at the beginning of each optimization run.

[22] 4. Run the management model with RHC using the updated initial condition and forecast information in order to obtain an optimal management strategy for the CTH. The results obtained from the management model represent the optimal management strategy for the entire CTH. However, only the optimized decision is adopted for the immediate stress period $t = 1$ to guide the operation.

[23] 5. Collect head observation data at the end of each stress period T (the update time t_u) and pumping data during stress period T (prior to the update time).

[24] 6. Estimate missing pumping rates for stress period T . The distribution ratio w' and the weighting coefficient w^* are calculated on the basis of the locations of observation wells and pumping wells that have reported pumping data.

[25] 7. Perform the nudging update of the head distribution using the estimated pumping rates. Updating is carried out at the end of the first stress period $t = 1$ of the CTH, when new observations become available.

[26] 8. Move the CTH one stress period forward and go to step 1.

5. Study Area: Las Posas Groundwater Basin, California

[27] We apply the proposed management model with a nudging algorithm to optimize the real-time operation of the Aquifer Storage and Recovery (ASR) wells of the Las Posas Groundwater Basin in southern California. Via the ASR wells, surplus surface water is injected and stored into groundwater aquifers and then recovered (extracted) for water supply when needed. ASR programs have been used to improve surface and groundwater conjunctive use in many field applications including Las Vegas [Thomas *et al.*, 2000], Texas [Sheng, 2005], and Australia [Rinck-Pfeiffer *et al.*, 2000].

[28] The Calleguas Municipal Water District (CMWD) manages the Las Posas groundwater basin, California. CMWD operates a conjunctive use project in eastern Ventura County. The Las Posas ASR project stores treated surplus water underground in a deep confined aquifer (305 m (1000 ft)) within the Calleguas Creek Watershed so that the stored water will be available for later use.

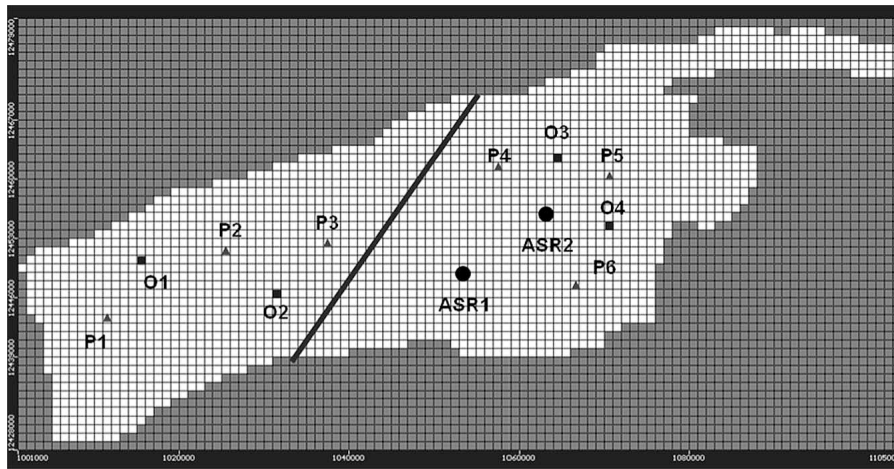


Figure 2. Planar view of the Las Posas Basin.

Presently over 10^8 m^3 (90,000 acre-feet) of water has been stored in the system.

[29] CMWD is working in partnership with the Metropolitan Water District of Southern California (MWD) on the Las Posas Basin ASR Project. Both agencies benefit from the ASR project. For MWD, the project provides water that can be delivered to its member agencies during dry years, allowing MWD to balance supplies and demands as well as providing reliability for emergencies and water quality events for 18 million southern California residents. For CMWD, the project provides a reliable source of water when imported supplies are limited because of scheduled maintenance shutdowns, drought, earthquakes, or other emergencies. The ASR project currently has 18 wells in two well fields. The wells are spatially separated as much as possible to minimize near-well interference. They are 240–365 m (800–1200 ft) deep and perforate the Fox Canyon Aquifer.

[30] We use true field data and MODFLOW to develop a 3-D model for the Las Posas groundwater basin. The developed 3-D full model is then reduced using the influence coefficient method. The accuracy of the reduced model is verified in section 5.1. To test the proposed methodology, we synthetically generate data on a real groundwater system. Predefined pumping rates are imposed on the real aquifer geometry to generate the “true” head distribution that provides the observed head for the case study. The difference between the estimated value and the true value, including pumping rates and head distribution, is used to evaluate the performance of the proposed methodology. In the case study, we assume the simulation model is perfect and the only source of uncertainty is the missing pumping information. In case I, we discuss the pumping estimation accuracy associated with pumping data availability. In case II, we apply the proposed methodology to real-time groundwater management for the Las Posas groundwater basin.

5.1. System Setting

[31] Figure 2 shows the planar view of the modeled groundwater basin. The simulation model consists of three confined layers and is defined aurally by 51×104 square cells, each with an area of $304.8 \times 304.8 \text{ m}^2$ ($1000 \times 1000 \text{ ft}^2$).

We distribute six private wells (P1 to P6) and four observation wells (O1 to O4) on both sides of the fault and use 12 months as the CTH. Figure 2 shows the spatial locations of the pumping and observation wells in addition to two ASR well fields (cluster of wells) within the calibrated MODFLOW model. The northeast-southwest line represents the fault which, as noted, acts as a hydraulic barrier practically separating the aquifer system into two parts. The radius of influence for the nudging algorithm is set at $R = 7620 \text{ m}$ (25,000 ft, equivalent to 25 cells). This setting implies that the identification of the pumping rates for private wells P1, P2, and P3 is based on the measurements taken at observation wells O1 and O2, while the measurements from O3 and O4 are used for identifying the pumping rates for wells P4 to P6.

[32] The influence coefficients are generated by MODFLOW at the beginning of each CTH. For a strictly confined aquifer system, the governing is linear; hence, the influence coefficients are invariant during the optimization. The accuracy of the reduced model solution will be verified by comparing it with the solution of the full model.

5.2. Verification of the Reduced Model

[33] The accuracy of the reduced simulation model is assessed on a transient test case based on a 24 month simulation run. All eight wells, composed of the two ASR wells and the six private wells, pump at the same rate during each month. However, the pumping rate changes dynamically with time. The pumping rate starts from $2832 \text{ m}^3/\text{d}$ ($100,000 \text{ ft}^3/\text{d}$) during the first month, decreases uniformly at each month by $283 \text{ m}^3/\text{d}$ ($10,000 \text{ ft}^3/\text{d}$) (10% of the initial value) for three months and then returns to the starting rate in three months with a uniform increment rate of $283 \text{ m}^3/\text{d}$ ($10,000 \text{ ft}^3/\text{d}$). This pattern is repeated during the 24 months.

[34] We compare the simulation results between the full simulation model (MODFLOW) and the reduced simulation model (response matrix). The error is defined as

$$\text{Error}^{i,t} = \frac{|h_{\text{MODFLOW}}^{i,t} - h_{\text{Response Matrix}}^{i,t}|}{\text{TD}^{i,t}} \times 100\% \quad (24)$$

where $h^{i,t}$ is the calculated hydraulic head at location i for time t and $\text{TD}^{i,t}$ is the total drawdown at location i for time t .

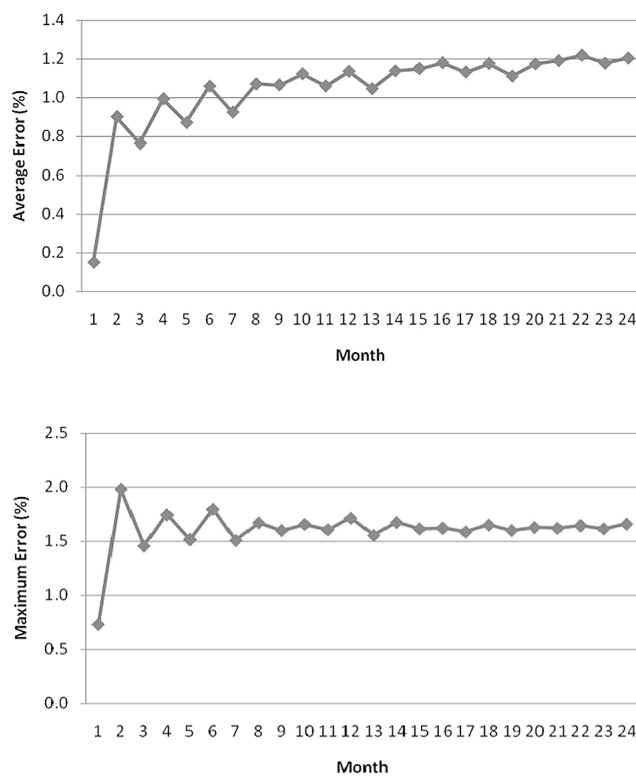


Figure 3. Comparison between the full model (MODFLOW) and the reduced model (response matrix).

The average error is defined as the average value of errors for the 12 selected locations, including two ASR wells, six private wells, and four observation wells. The maximum error is the largest error among the 12 locations.

[35] Figure 3 presents the comparison of the simulation results. Figure 3 (top) shows the spatially averaged error, while Figure 3 (bottom) shows the maximum error. The error percentage converges to 1.2% for the average error and 1.6% for the maximum error. As can be seen, the discrepancy between the MODFLOW solution and the reduced simulation model solution is negligible.

5.3. Case Study I: Evaluation of Pumping Estimation Accuracy Associated With Pumping Data Availability

[36] In this case study, we test the ability of the nudging algorithm to incorporate the available pumping data. No management model is considered. We assume that partial pumping information is available and the missing pumping rates are estimated using nudging along with the available pumping data. These estimated rates are used in the forecast simulation in the succeeding periods. We assume that pumping rates of ASR wells are given: well field 1 pumps at a constant rate of $2832 \text{ m}^3/\text{d}$ ($100,000 \text{ ft}^3/\text{d}$) during first 6 months, with pumping stopped from the seventh through twelfth months; well field 2 has no injection during the first seven months, but injection starts from the eighth month and continues through the twelfth month at a constant rate of $1699 \text{ m}^3/\text{d}$ ($60,000 \text{ ft}^3/\text{d}$). Figure 4 shows the pumping rate cycle as a function of time for each of the six private wells. The pumping rate for each well starts at $2832 \text{ m}^3/\text{d}$ ($100,000 \text{ ft}^3/\text{d}$) during the first month,

decreases uniformly each month by $283 \text{ m}^3/\text{d}$ ($10,000 \text{ ft}^3/\text{d}$) (10% of the initial value) for three months and then returns to the starting value over three months with constant increments of $283 \text{ m}^3/\text{d}$ ($10,000 \text{ ft}^3/\text{d}$).

[37] Four scenarios of pumping data availability are studied: 0% (case A), 10% (case B), 25% (case C), and 50% (case D). We assume all reported pumping information is available at the end of each stress period (month). The location and timing of available data are randomly selected. We also assume that no pumping information is available during the first month, and we initialize the nudging algorithm with a zero starting guess. Figure 5 shows the average error for pumping estimation. Note that the pumping estimation error is 100% before the first nudging update. From the numerical results we observe that the pumping estimation error is reduced by more than 90% at the end of the first stress period, with much slower error reduction during the remaining portion of the simulation. We observe also that increasing the available information leads to improved pumping rate estimations, and that the estimation errors tend to decrease with the simulation time, with the exception of case A, where no information is available. In cases B, C, and D, the average pumping estimation error is reduced to below 1% after 12 months, with estimation errors decreasing with time. We can attribute this reduction to the effectiveness of the nudging algorithm and the dissipative nature of the governing equations. The results are consistent with those reported by Cheng *et al.* [2009], where nudging was used in combination with a full instead of reduced model, showing the accuracy and robustness of the proposed DA in recovering the true pumping rates from observations.

5.4. Case Study II: Real-Time Groundwater Management With Nudging Algorithm for the Las Posas Groundwater Basin

[38] We now apply the proposed real-time management model with the nudging algorithm to the optimal operation of the ASR well fields in the Las Posas Groundwater Basin. The updated head distribution provides the initial condition for the management model. The estimated pumping rates are incorporated into the management model to improve the operation of the ASR well fields. For purposes of demon-

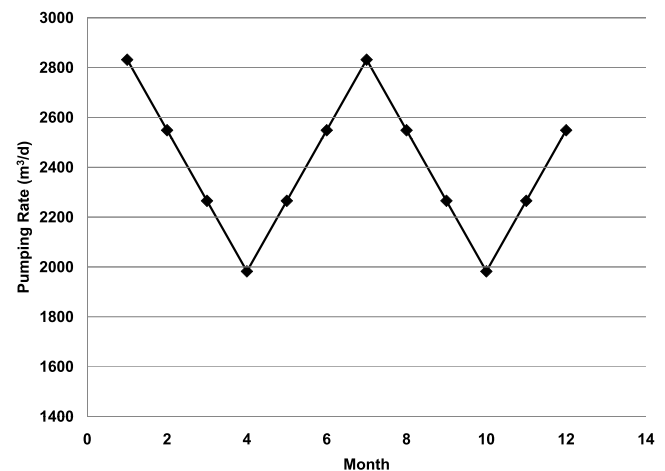


Figure 4. Pumping scenario of private wells.

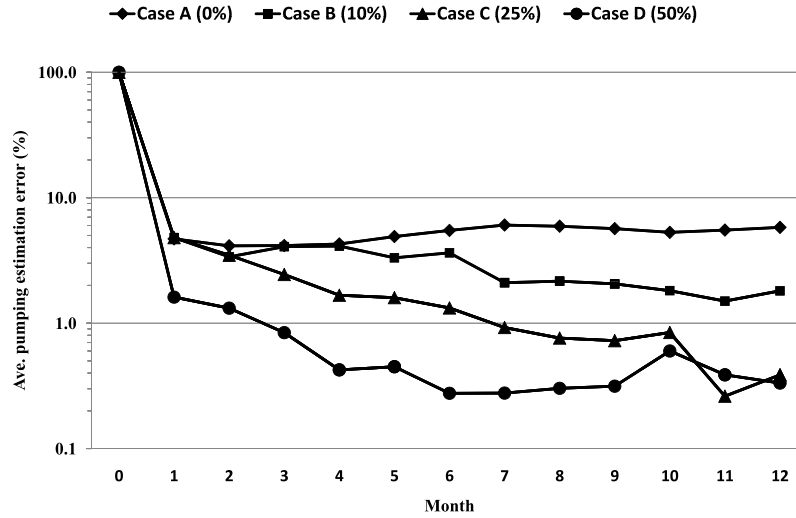


Figure 5. Average pumping estimation error for different pumping data availability.

stration, we only consider the injection scenario for the ASR well fields; hence, the term $Q_{j,m}^p$ drops out in the general formulation and no binary variables are required. The simplified objective function becomes

$$\min Z = \sum_{j=1}^{N_{ASR}} \sum_{m=1}^{N_T} [C_{j,m}^{in} \cdot Q_{j,m}^{in}] \quad (25)$$

The optimization is subject to constraints (2), (3), (6), and (10). Since the objective of this case is to maximize the total injection volume in the aquifer, we multiply the objective function by (-1) to convert the minimization problem to a maximization problem. The injection rates at the two ASR well fields are decision variables and we use a CTH period of 12 months. Consequently, we optimize the injection rates and the injection schedule within the CTH subject to predefined constraints. Note that the length of the CTH should be determined on the basis of forecast availability. It should be at least one stress period. We assign a large value to water availability for injection. The total injection capacity is set at 2832 m³/d (100,000 ft³/d). Upper and lower bounds of head constraints (equation (6)) are imposed at observation wells 3 and 4 as well as the two ASR well fields.

[39] For comparison purposes, we consider two additional scenarios: global optimum and real-time management without DA. The global optimum of the injection scenario is obtained from a deterministic optimization model with a planning horizon of 12 months using perfect information regarding private well pumping. For the real-time management without DA scenario, we simply run the real-time groundwater management scenario using only available pumping information without DA updating.

[40] The forecast of pumping rates for this scenario is obtained by randomly perturbing the true pumping rates of private wells as follows:

$$Q_{j,t}^{forecast} = 0.7 \times Q_{j,t}^{true} + N(0; 283 \text{ m}^3/\text{d}) \quad (26)$$

where $N(\mu; \sigma^2)$ represents a normal distribution with mean μ and standard deviation σ .

[41] Figures 6–8 report the results of our simulations. Figure 6 shows that the use of nudging to obtain a more accurate model forecast is essential to arrive at an accurate estimate of the optimal injection policy. The injection rate policy calculated with the proposed approach is characterized by only about a 1% discrepancy compared to the global optimal solution. On the other hand, the optimal policy calculated on the basis of inaccurate forecast (wrong estimates of private pumping rates) yields a largely suboptimal solution, with a time behavior that diverges from the reference solution and differences in injection rates that increase with time. We notice that both nudging and no-nudging simulations produced the same solution at the first month because both cases use the same forecast information. After applying the nudging algorithm, at the end of the first month the nudging solution incorporates the new information

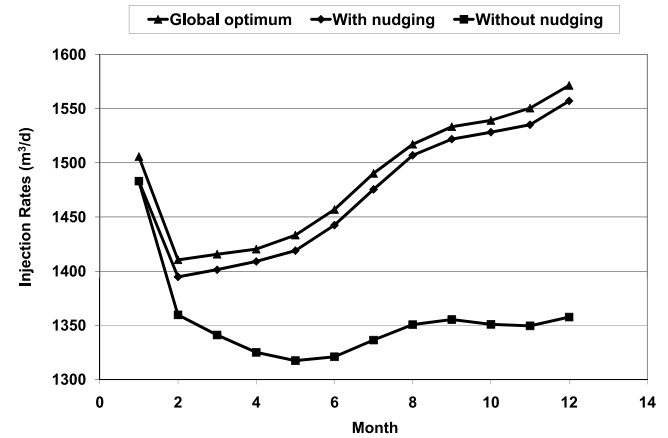


Figure 6. Injection rates of Aquifer Storage and Recovery (ASR) Project well field 1 (ASR1).

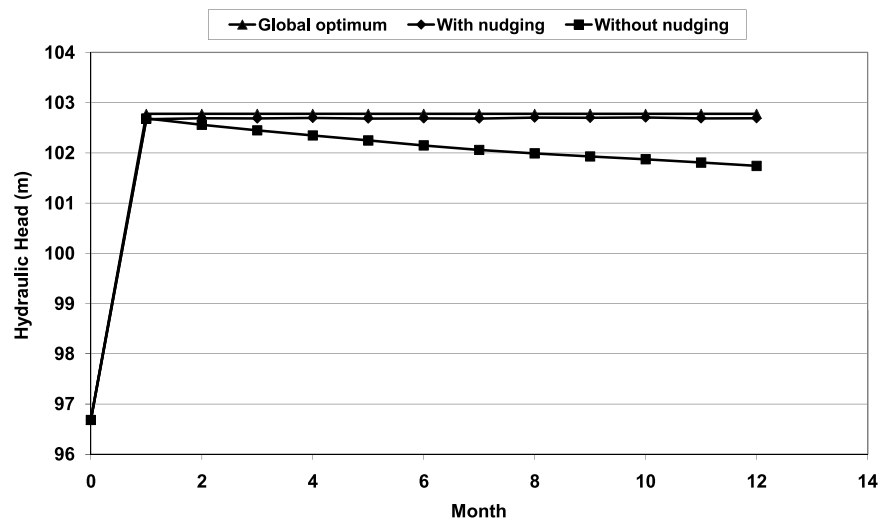


Figure 7. Hydraulic head at ASR well field 1 (ASR1).

coming from the observations, and the increased accuracy in the estimated pumping rates for the private wells greatly improves the management strategy. The injection rates at ASR well field 2 show the same values for all three solutions (global optimum, with nudging and without nudging), which correspond to the maximum imposed injection rate. However, the heads differ substantially. Figures 7 and 8 show the behavior over time of heads at ASR well fields 1 and 2, respectively. The results indicate that the head difference between the global optimal solution and the proposed model solution is smaller than 0.03 m for all time periods in both ASR well fields, while the head difference between the global optimal solution and the no-nudging solution increases over time. This increase is much less pronounced in ASR well field 2, a result directly related to the fact that all solutions dictate that maximum pumping is to be sustained. While the difference between the reference and nudging solutions is negligible in this case, the no-nudging solution displays a small discrepancy. This is due to the errors in the forecast because of inaccurate pumping rates of surrounding private wells that have an influence on the head solution at the ASR well field.

6. Conclusions

[42] We have developed a real-time, monthly management model with data assimilation that incorporates insufficient pumping data to assist in the operation of a groundwater basin. The nudging algorithm was used to estimate the missing pumping rates from private wells on the basis of head observations. In this paper we extended the nudging algorithm to incorporate multiperiod forecast pumping information and partially available pumping data. We used the receding horizon control with frequent updating for optimization, together with variable distribution ratios and weighting coefficients in the nudging method employed to estimate the missing pumping information. In principle, our methodology combines a real-time management model with a nudging algorithm to improve forecast accuracy, and can be coupled with any groundwater simulation model. Further-

more, in this study we used the response matrix approach to obtain a reduced simulation model and directly embedded it in the management model as a part of the constraint set. In addition, we also included the estimated pumping rates into the reduced simulation model as additional nudging terms. The hydraulic head distribution was updated by the nudging scheme with a notable improvement in forecast accuracy. Finally, we applied the proposed technique to a real world problem where we sought to make real-time decisions concerning water injection for the ASR project in the Las Posas Groundwater Basin in Ventura County, California. Two case studies (cases I and II) were performed. The results from case I showed an improvement in pumping estimation when we incorporated the available pumping data. The results from case II showed that the proposed real-time groundwater management model allied with the nudging algorithm can improve the operation of the groundwater system under consideration. The results demonstrate that the proposed approach reduces the forecast error, minimizes the risk of system failure, and improves management strategies.

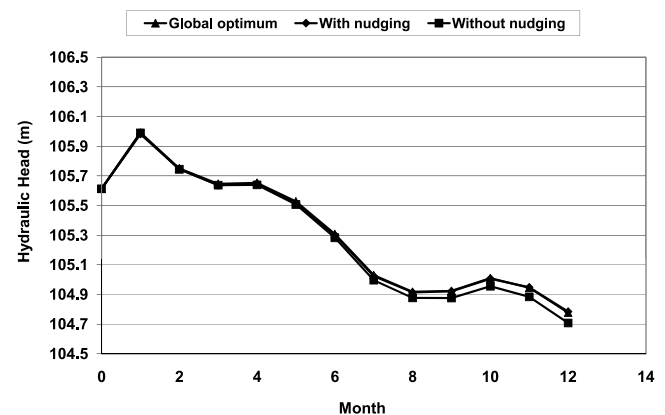


Figure 8. Hydraulic head at ASR well field 2 (ASR2).

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