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Study of Giant Pairing Vibrations with Neutron-Rich Nuclei*

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Abstract—We investigate the possible signature of the presence of giant pairing states at an excitation energy of about 10 MeV via two-particle transfer reactions induced by neutron-rich weakly bound projectiles. Performing particle–particle RPA calculations on ^{208}Pb and BCS + RPA calculations on ^{116}Sn, we obtain the pairing strength distribution for two-particle addition and removal modes. Estimates of two-particle transfer cross sections can be obtained in the framework of the macroscopic model. The weak-binding nature of the projectile kinematically favors transitions to high-lying states. In the case of the (^{6}He, ^{4}He) reaction, we predict a population of the Giant Pairing Vibration with cross sections of the order of a millibarn, dominating over the mismatched transition to the ground state. © *2003 MAIK "Nauka/Interperiodica"*.

1. PAIRING FIELD AND REACTION MECHANISMS

1.1. Introduction

Nuclei in interaction with external fields display a wide variety of collective vibrations known as giant resonances, associated with various degrees of freedom and multipolarities. The giant isovector dipole resonance and the giant isoscalar quadrupole resonance are the most studied examples in this class of phenomena. A particular mode that is associated with vibrations in the number of particles was predicted in the 1970s [1] and discussed, under the name of Giant Pairing Resonance, in the middle of the 1980s in a number of papers [2]. This phenomenon, despite some early efforts aimed at resolving some broad bump in the high-lying spectrum in (p, t) reactions [3], is still without any conclusive experimental confirmation. For a discussion, in particular, in connection with two-particle transfer reactions, on many aspects of pairing correlations in nuclei, we refer to a recent review [4].

We have studied the problem of collective pairing modes at high-excitation energy in two-neutron transfer reactions with the aim to prove the advantage of using an unstable beam as a new tool to enhance the excitation of such modes [5]. The main point is that, with standard available beams, one is faced with a large energy mismatch that strongly hinders the excitation of high-lying states and favors the transition to the ground state of the final system. Instead, the optimum Q-value condition in the (^{6}He, ^{4}He) stripping reaction suppresses the ground state and should allow the transition to the energy region of 10–15 MeV. We have performed particle–particle RPA calculations on lead and BCS + RPA on tin, as paradigmatic examples of normal and superfluid systems, evaluating the response to the pairing operator. Subsequently the two-neutron transfer form factors have been constructed in the framework of the macroscopic model [6] and used in DWBA computer codes. We have estimated cross sections of the order of some millibarns, dominating over the mismatched transition to the ground state. Recently we added similar calculations on other much studied targets to give some guide for experimental work.

1.2. The Giant Pairing Vibrations (GPV)

The formal analogy between particle–hole and particle–particle excitations is very well established both from the theoretical side [7] and from the experimental side for what concerns low-lying pairing vibrations around closed shell nuclei and pairing rotations in open shells. The predicted concentration of strength of an $L = 0$ character in the high-energy region (8–15 MeV for most nuclei) is understood microscopically as the coherent superposition of $2p$ (or $2h$) states in the next major shell above the Fermi level. We have roughly depicted the situation in Fig. 1. In closed shell nuclei, the addition of a pair of particles (or holes) to the next major shell, with a total energy $2\hbar\omega$, is expected to have a high degree of collectivity. Also, in the case of open shell nuclei, the same is expected for the excitation of a pair of particles with $2\hbar\omega$ energies.

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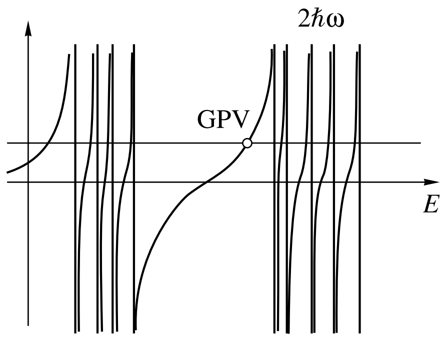

Fig. 1. Raw picture of the dispersion relation. The two bunches of vertical lines represent the unperturbed energy of a pair of particles placed in a given single-particle-energy level. The graphical solution of the secular equation is the intersection of the horizontal line with the curves. The GPV is the collective state relative to the second major shell.

2. DETAILS OF CALCULATIONS

For normal nuclei, the Hamiltonian with a monopole strength interaction reads

$$H = \sum_j \epsilon_j a_j^\dagger a_j - 4\pi G P^\dagger P, \tag{1}$$

where P annihilates a pair of particles coupled to zero total angular momentum.

Getting rid of all the technicalities of the solution of the pp-RPA equations (which may be found in the already cited work by the author), we merely state that the pairing phonon may be expressed as a superposition of $2p$ (or $2h$) states with proper forward and backward amplitudes (X_n and Y_n). The pair transfer strength, which is a measure of the amount of collectivity of each state n, is given by

$$\beta_{Pn} = \sum_j \sqrt{2j+1}[X_n(j) + Y_n(j)]. \tag{2}$$

This quantity is plotted in Fig. 2 for the removal (Fig. $2a$) and addition mode (Fig. $2b$). In the same figure are reported the pairing strength parameters for the states of ^{116}Sn. To obtain these last quantities for superfluid spherical nuclei, one has to rewrite the Hamiltonian according to the BCS transformation and solve more complex RPA equations. In this case, the pairing strength for the addition of two particles is given, for each state n, by

$$\beta_P(2p) = \sum_j \sqrt{2j+1}\langle n|[a_j^\dagger a_j^\dagger]_{00}|0\rangle \tag{3}$$
$$= \sum_j \sqrt{2j+1}[U_j^2 X_n(j) + V_j^2 Y_n(j)],$$

where U and V are the usual occupation probabilities. The amount of collectivity is a clear signal of the structural existence of GPV in the high-lying energy region. We also report in Fig. 3 a number of analogous results for other commonly studied targets with the aim of giving some indications to experimentalists on the reasons why we think that lead and tin are some of the most promising candidates. We have studied two isotopes of calcium with closed shells. Even if the absolute magnitudes of β_P is lower, it is worthwhile to note that some enhancement is seen in the more neutron-rich ^{48}Ca with respect to ^{40}Ca. An important role in this change is certainly due to the different shell structure of the two nuclei as well as to the scheme that we implemented to obtain the set of single-particle levels. The latter is responsible for the collectivity of the removal modes in both Ca isotopes and also for the difficulty in finding a collective state in the addition modes. We also display results for ^{90}Zr, where the strength is much more fragmented and the identification of the GPV is more difficult. In the work of Bes and Broglia [7], estimates for the energy of the pairing resonance are given as $68/A^{1/3}$ MeV and $72/A^{1/3}$ MeV for normal and superfluid systems, respectively. Our figures follow roughly these prescriptions based on simple arguments (and much better grounded in the case of normal nuclei), as evident from Table 1.

Table 1. Comparison of the position (in MeV) of GPV between our calculation and the Bes and Broglia estimate

Nucleus	Our calculation	Bes and Broglia estimate
Sn	12.68	14.76
Pb	11.81	11.47

3. MACROSCOPIC MODEL FOR TWO-PARTICLE TRANSFER REACTIONS

The starting point of the macroscopic model for two-particle transfer reactions is to push further the analogy of the vibrations of the nuclear surface with the vibrations across different mass partitions. If one imagines an idealized space in which a discrete coordinate (the number of particles of the system) labels different sections of the space, it is plausible to give an interpretation of pairing modes as back and forth oscillations in the number of particles. The role of macroscopic variable in this game is played by the quantity ΔA, which is the difference in mass from the initial mass partition. Exploiting the analogy with inelastic modes leads us to construct a macroscopic guess for the pairing transition density $\delta\rho_P$ modeled on the surface transition density $\delta\rho_S$:

$$\delta\rho_S = \frac{\partial\rho}{\partial\alpha}\alpha = \frac{\partial\rho}{\partial r}R_0\alpha, \tag{4}$$

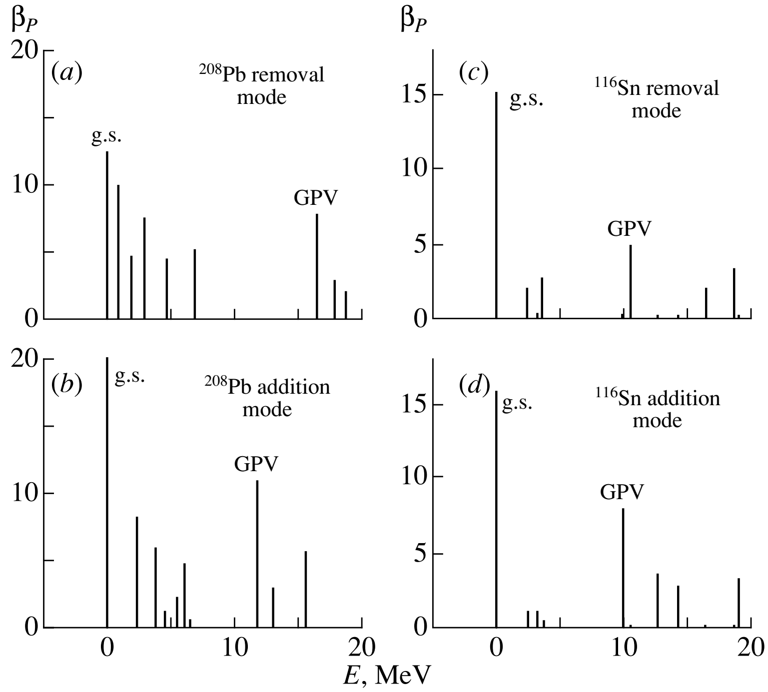

Fig. 2. Pairing response for removal and addition mode in ^{208}Pb and ^{116}Sn. The ground-state transition and the candidate for the GPV are marked.

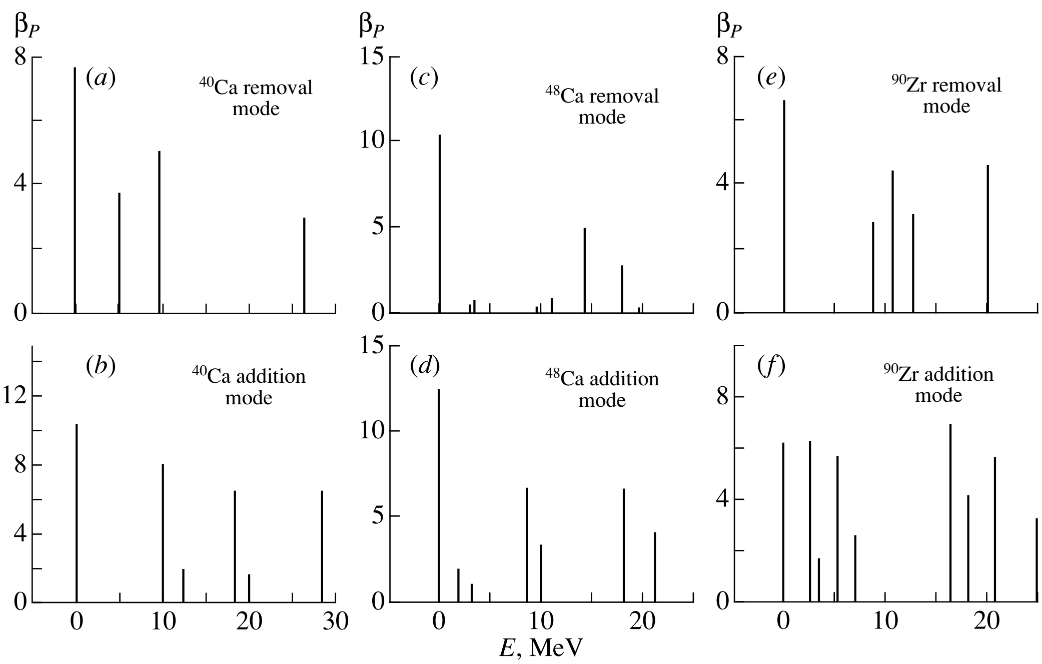

Fig. 3. Pairing response for removal and addition mode in ^{40}Ca, ^{48}Ca, and ^{90}Zr.

$$\delta\rho_P = \frac{\partial\rho}{\partial\Delta A}\Delta A = \left(\frac{R_0}{3A}\right)\frac{\partial\rho}{\partial r}\Delta A. \qquad (5)$$

One usually identifies α with the deformation parameter β_S, and the formal analogy suggests the correspondence with a pairing deformation parameter $\beta_S \Leftrightarrow \beta_P/(3A)$. This scheme implies the assumption that nuclear density is saturated and that a change in the number of particles is strictly related to a change of volume. The two-particle transfer form factors may

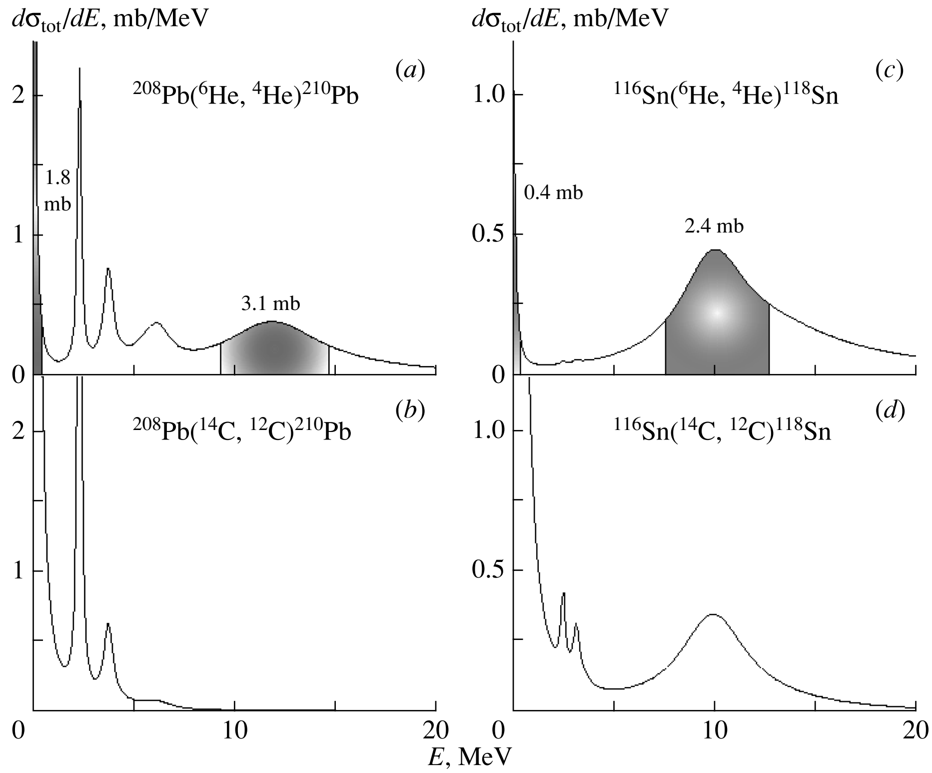

Fig. 4. Differential cross sections as function of the excitation energy. The shaded areas for the (^{6}He, ^{4}He) reactions allows a comparison between the transition to the ground states and to the GPVs. Note that the vertical scale is changed in Sn with respect to Pb.

then be connected to the ion–ion potential $U(r)$ as

$$F_P(r) = \left(\frac{\beta_P}{3A}\right) R_0 \frac{\partial U(r)}{\partial r}. \quad (6)$$

This formalism has been applied to many low-energy aspects of two-particle transfer reactions [8, 9]. Certainly, the macroscopic approach is liable to improvements when one turns to a microscopic description, but the predictions may be considered robust enough to give order-of-magnitude evaluations.

Table 2. Cross sections (in mb) for ground-state and GPV transitions obtained with the DWBA code Ptolemy [the target (column) and projectile (row) are specified]

	^{14}C → ^{12}C	^{6}He → ^{4}He
^{116}Sn → ^{118}Sn$_{g.s}$	19.4	0.4
^{208}Pb → ^{210}Pb$_{g.s}$	15.3	1.8
^{116}Sn → ^{118}Sn$_{GPV}$	0.14	2.4
^{208}Pb → ^{210}Pb$_{GPV}$	0.04	3.1

4. RESULTS FOR Pb, Sn, AND OTHER TARGETS

DWBA calculations have been performed for two-neutron transfer reactions on the two cited targets either with usually available beams (^{14}C, ^{12}C) or with new unstable ones (^{6}He, ^{4}He). The last reaction has been chosen since it has optimal matching conditions: the Q values for the transition to the ground states of both targets are strongly positive, with the consequence of Q values to the GPV close to the optimum Q value ($Q_{\rm opt} \sim 0$ MeV). This should favor the excitation of the pairing mode, while the situation with carbon beam is reversed, having large (and negative) Q values for the high-lying energy region and small Q values for the low-lying region. In Table 2, we report the angle-integrated cross sections obtained with standard DWBA computer codes.

These cross sections have been derived for sharp states, and we refer to the numbers in the last table when speaking of order-of-magnitude estimates. Obviously, cross sections in the high-lying energy region have a finite (and large) width that should be inserted for a more realistic description of the spectrum. We have chosen a simple scheme that gives a Lorentzian distribution with a width that grows quadratically with the excitation energy, $\Gamma = kE_x^2$, with k adjusted to give a width of 4 MeV for the GPV. This might seem rather arbitrary since there is no reason for an a priori assignment of this quantity. We have been brought to this simple prescription because other collective states (of different nature) lying in the same energy region display similar values for their width, and it is reasonable to assume some rule to narrow the low-energy states and to broaden the high-energy ones.

5. FINAL REMARKS

The final achievements for the four reactions studied in detail are presented in Fig. 4, where the areas corresponding to the cross sections given above have been shaded to give a feeling of the relative magnitudes of the transition to the ground states and to the GPVs. It is worthwhile to note that, in the case of Pb, there is a considerable gain in using unstable beams, while in Sn it is much less evident. One sees the need for unstable helium when one compares the magnitude for the pairing resonance in the (c) and (d) panels with the peak at zero energy: in the first panel, the transition to the ground state is extremely hindered.

A ^{6}He beam is currently available (or it will be available in the very near future) in many radioactive ion-beam facilities around the world, and the calculations that we have presented could allow planning for future experiments aimed at studying the not yet completely unraveled role of pairing interaction in common nuclei, using exotic weakly bound nuclei as useful tools.

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