

THE EFFECT OF PRIMORDIAL NON-GAUSSIANITY ON HALO BIAS

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ABSTRACT

It has long been known how to analytically relate the clustering properties of the collapsed structures (halos) to those of the underlying dark matter distribution for Gaussian initial conditions. Here we apply the same approach to physically motivated non-Gaussian models. The techniques we use were developed in the 1980s to deal with the clustering of peaks of non-Gaussian density fields. The description of the clustering of halos for non-Gaussian initial conditions has recently received renewed interest, motivated by the forthcoming large galaxy and cluster surveys. For inflationary-motivated non-Gaussianities, we find an analytic expression for the halo bias as a function of scale, mass, and redshift, employing only the approximations of high peaks and large separations.

Subject headings: cosmology: theory — galaxies: clusters: general — galaxies: halos — large-scale structure of universe

1. INTRODUCTION

Constraining primordial non-Gaussianity offers a powerful test of the generation mechanism of cosmological perturbations in the early universe. While standard single-field models of slow-roll inflation lead to small departures from Gaussianity, nonstandard scenarios allow for a larger level of non-Gaussianity (Bartolo et al. 2004 and references therein). The standard observables to constrain non-Gaussianity are the cosmic microwave background and large-scale structure. A powerful technique is based on the abundance (Matarrese et al. 2000; Verde et al. 2001; LoVerde et al. 2007; Robinson & Baker 2000; Robinson et al. 2000) and clustering (Grinstein & Wise 1986; Matarrese et al. 1986; Lucchin et al. 1988) of rare events such as dark matter density peaks as they trace the tail of the underlying distribution. These theoretical predictions have been tested against numerical N -body simulations (Kang et al. 2007; Grossi et al. 2007; Dalal et al. 2007). Dalal et al. (2007) showed that primordial non-Gaussianity affects the clustering of dark matter halos inducing a scale-dependent bias. This effect will be useful for constraining non-Gaussianity from future surveys which will provide a large sample of galaxy clusters over a volume comparable to the horizon size (e.g., DES, Pan-STARRS, PAU, LSST, DUNE, ADEPT, SPACE, DUO) or mass-selected large clusters samples via the Sunyaev-Zel’dovich effect (e.g., ACT, SPT), considered alone or via cross-correlation techniques (e.g., ISW, lensing).

Here we resort to results and techniques developed in the 1980s (Grinstein & Wise 1986; Matarrese et al. 1986; Lucchin et al. 1988) to extend this work and derive an accurate analytical expression for halo bias, in the presence of general non-Gaussian initial conditions, accounting for its scale, mass, and redshift dependence.

2. HALOS AS PEAKS OF THE DENSITY FIELD

Halo clustering is generally studied by assuming that halos correspond to regions where the (smoothed) linear density field exceeds a suitable threshold. This amounts to modeling the local halo number density as a theta (step) function

$$\rho_{h,R}(\mathbf{x}, z_f) = \theta[\delta_R(\mathbf{x}, z_f) - \Delta_c] = \theta[\delta_R(\mathbf{x}) - \delta_c(z_f)], \quad (1)$$

modulo a constant normalization factor which is irrelevant for the calculation of correlations. Here R denotes a smoothing radius which defines the halo mass M by $M = \Omega_{m,0} 3H_0^2/(8\pi G)(4/3)\pi R^3$, with $\Omega_{m,0}$ denoting the present-day matter density parameter, H_0 the present-day Hubble parameter, and G Newton’s constant. The threshold Δ_c is the linearly extrapolated overdensity for spherical collapse: it is 1.686 in the Einstein–de Sitter case, while it slightly depends on redshift for more general cosmologies (e.g., Kitayama & Suto 1996). The redshift z_f is the formation redshift of the halo, which for high-mass halos is very close to the observed redshift z_o . Hereafter we will thus make the approximation $z_f \approx z_o = z$. The second equality can be understood if we think of the density fluctuation as being time-independent while giving a redshift dependence to the collapse threshold $\delta_c(z_f) \equiv \Delta_c(z_f)/D(z_f)$. Here $D(z)$ denotes the general expression for the linear growth factor, which depends on the background cosmology. In particular $D(z) = (1+z)^{-1}g(z)/g(0)$ where $g(z)$ is the growth suppression factor for non Einstein–de Sitter universes.

For Gaussian initial conditions, one obtains (Kaiser 1984; Politzer & Wise 1984; Jensen & Szalay 1986)

$$\xi_{h,M}(r) = \exp\left[\frac{\nu^2}{\sigma_R^2} \xi_R(r)\right] - 1 \simeq \frac{\nu^2}{\sigma_R^2} \xi_R(r), \quad (2)$$

where σ_R is the rms. of the underlying dark matter fluctuation field smoothed on scale R , $\nu = \delta_c/\sigma_R$, and $\xi_{h,M(R)}$ denotes the correlation function of the halos of mass M corresponding to radius R . W_R denotes the top-hat function of width R and the definition of $\xi_R(r)$ is $\int d^3r' \xi(r') W_R^2(|\mathbf{r} - \mathbf{r}'|)$. In the second equality above we have expanded the exponential in series. The truncation of the series holds for separations $r \gg R$. Thus

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we obtain the well-known Kaiser's formula (Kaiser 1984) of a scale-independent bias:

$$\xi_{h,M}(r) = b_L^2 \xi_R(r), \quad (3)$$

where $b_L = \delta_c/\sigma_R^2$. Compared with the more refined relations in, e.g., Mo & White (1996) and Catelan et al. (1998), an additive term $1/\delta_c$ has been dropped as a consequence of the high-peak approximation in the first equality of equation (2).

Here the subscript ‘‘L’’ indicates that this should be considered as a Lagrangian bias, because all correlations and peaks considered here are those of the *initial* density field (linearly extrapolated until the present time). Making the standard assumptions that halos move coherently with the underlying dark matter, one can obtain the final Eulerian bias as $b_E = 1 + b_L$, using the techniques outlined in Efstathiou et al. (1988), Cole & Kaiser (1989), Mo & White (1996), and Catelan et al. (1998).

The two-point correlation function of regions above a high threshold has been obtained, for the general non-Gaussian case, in Grinstein & Wise (1986), Matarrese et al. (1986), and Lucchin et al. (1988):

$$\xi_{h,M}(|\mathbf{x}_1 - \mathbf{x}_2|) = -1 + \exp \left\{ \sum_{N=2}^{\infty} \sum_{j=1}^{N-1} \frac{\nu^N \sigma_R^{-N}}{j!(N-1)!} \xi^{(N)} \left[\begin{array}{c} \mathbf{x}_1, \dots, \mathbf{x}_1, \\ j \text{ times} \end{array} \mathbf{x}_2, \dots, \mathbf{x}_2 \right] \right\}. \quad (4)$$

As before, for large separations we can expand the exponential to first order. To leading order for non-Gaussianity of the type (Salopek & Bond 1990; Gangui et al. 1994; Verde et al. 2000; Komatsu & Spergel 2001)

$$\Phi = \phi + f_{\text{NL}} * (\phi^2 - \langle \phi^2 \rangle), \quad (5)$$

where $*$ denotes convolution, as in general f_{NL} may be scale and configuration dependent, but for constant f_{NL} it reduces to a simple multiplication. For simplicity, below we will carry out calculations assuming constant f_{NL} and will generalize our results at the end. Here Φ denotes Bardeen's gauge-invariant potential, which, on sub-Hubble scales reduces to the usual Newtonian peculiar gravitational potential, up to a minus sign. In the literature, there are two conventions for equation (5): the large-scale structure and the CMB one. Following the large-scale structure convention, here Φ is linearly extrapolated at $z = 0$. In the CMB convention Φ is instead primordial: thus $f_{\text{NL}} = g(z = \infty)/g(0)f_{\text{NL}}^{\text{CMB}}$.

In equation (5), ϕ denotes a Gaussian random field. For values of f_{NL} consistent with observations, we can keep terms up to the three-point correlation function $\xi^{(3)}$, obtaining that the correction to the halo correlation function, $\Delta\xi_h$ due to a nonzero three-point function is given by

$$\begin{aligned} \Delta\xi_h &= \frac{\nu_R^3}{2\sigma_R^3} [\xi_R^{(3)}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_2) + \xi_R^{(3)}(\mathbf{x}_1, \mathbf{x}_1, \mathbf{x}_2)] \\ &= \frac{\nu_R^3}{\sigma_R^3} \xi_R^{(3)}(\mathbf{x}_1, \mathbf{x}_1, \mathbf{x}_2). \end{aligned} \quad (6)$$

3. APPLICATION TO A LOCAL NON-GAUSSIAN MODEL

We want to find an expression for the correlation function

of the late-time halos which form from the dark matter overdensity.

In Fourier space, the present-time ($z = 0$) filtered linear overdensity δ_R is related to Φ by the Poisson equation:

$$\delta_R(\mathbf{k}) = \frac{2}{3} \frac{T(k)k^2}{H_0^2 \Omega_{m,0}} W_R(k) \Phi(\mathbf{k}) \equiv \mathcal{M}_R(k) \Phi(\mathbf{k}), \quad (7)$$

where $T(k)$ denotes the matter transfer function⁴ and $W_R(k)$ is the Fourier transform of $W_R(\mathbf{r})$. From equation (5) the definition of Φ is

$$\Phi(\mathbf{k}) = \phi(\mathbf{k}) + f_{\text{NL}} \int \frac{d^3k'}{(2\pi)^3} \phi(\mathbf{k}') \phi(\mathbf{k} - \mathbf{k}'), \quad (8)$$

whose bispectrum is

$$\begin{aligned} B_\phi(k_1, k_2, k_3) &= 2f_{\text{NL}} [P_\phi(k_1)P_\phi(k_2) \\ &\quad + P_\phi(k_2)P_\phi(k_3) + P_\phi(k_1)P_\phi(k_3)]. \end{aligned} \quad (9)$$

With these definitions the density bispectrum becomes $B_\delta(k_1, k_2, k_3) = \mathcal{M}_R(k_1)\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)B_\phi(k_1, k_2, k_3)$, where P_ϕ denotes the power-spectrum of the Gaussian field ϕ . The three-point function of equation (6) becomes

$$\begin{aligned} \xi^{(3)}(\mathbf{x}_1, \mathbf{x}_1, \mathbf{x}_2) &= \frac{1}{(2\pi)^6} \int d^3k_1 d^3k_2 2f_{\text{NL}} \mathcal{M}_R(k_1) \mathcal{M}_R(k_2) \mathcal{M}_R(|\mathbf{k}_1 + \mathbf{k}_2|) \\ &\quad \times [P_\phi(k_1)P_\phi(k_2) + 2P_\phi(k_1)P_\phi(|\mathbf{k}_1 + \mathbf{k}_2|)] e^{i(\mathbf{k}_1 + \mathbf{k}_2) \cdot (\mathbf{x}_1 - \mathbf{x}_2)} \end{aligned} \quad (10)$$

and Fourier transform of equation (10) becomes

$$\begin{aligned} \frac{2f_{\text{NL}}}{(2\pi)^2} \mathcal{M}_R(k) \int dk_1 k_1^2 \mathcal{M}_R(k_1) P_\phi(k_1) \\ \times \int_{-1}^1 d\mu \mathcal{M}_R(\sqrt{\alpha}) [P_\phi(\sqrt{\alpha}) + 2P_\phi(k)], \end{aligned}$$

where $\alpha = k_1^2 + k^2 + 2k_1 k \mu$.

4. RESULTS

We can now write an expression for the non-Gaussian contribution to the halo power spectrum. Equation (6) in Fourier space becomes

$$\Delta P_h(k) = b_{0,L}^2 4f_{\text{NL}} \delta_c P_{\phi\delta}(k) \mathcal{F}_R(k), \quad (11)$$

where we have used $b_{0,L} \equiv \delta_c/\sigma_R^2$, corresponding to the La-

⁴ The matter transfer function and the window functions cannot be neglected here: the initial conditions of eq. (5) are set out well before matter-radiation equality and the density field should be smoothed to define the halo mass.

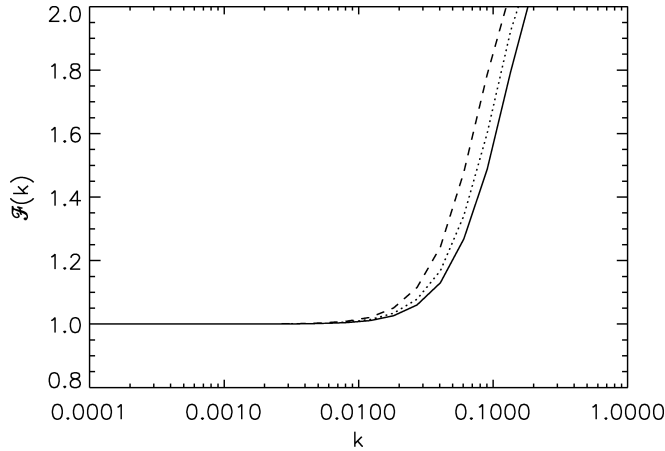


FIG. 1.—Function $\mathcal{F}_R(k)$ for three different masses: $1 \times 10^{14} M_\odot$ (solid curve), $2 \times 10^{14} M_\odot$ (dotted curve), $1 \times 10^{15} M_\odot$ (dashed curve).

grangian linear bias that the halos would have in the Gaussian case, $P_{\phi\delta}(k) \equiv \mathcal{M}_R(k)P_\phi(k)$ and

$$\mathcal{F}_R(k) = \frac{1}{8\pi^2\sigma_R^2} \int dk_1 k_1^2 \mathcal{M}_R(k_1) P_\phi(k_1) \times \int_{-1}^1 d\mu \mathcal{M}_R(\sqrt{\alpha}) \left[\frac{P_\phi(\sqrt{\alpha})}{P_\phi(k)} + 2 \right]. \quad (12)$$

The “form” factor $\mathcal{F}_R(k)$ is plotted as a function of k in Figure 1 for three different masses. Hereafter when needed we assume a flat LCDM cosmology: $\Omega_m = 0.27$, $\Omega_b h^2 = 0.042$, $n_s = 0.97$, $h = 0.73$, $\sigma_8 = 0.8$.

The expression for the halo power-spectrum can be rewritten in a more convenient form where we can also make the redshift dependence explicit:

$$P_h(k, z) = \frac{\delta_c^2(z) P_{\delta\delta}(k, z)}{\sigma_R^4 D^2(z)} \left[1 + 4f_{\text{NL}} \delta_c(z) \frac{P_{\phi\delta}(k) \mathcal{F}_R(k)}{P_{\delta\delta}(k)} \right],$$

where $P_{\delta\delta}^{\text{NL}}(k, z) = D^2(z) P_{\delta\delta}(k) = D^2(z) \mathcal{M}_R^2(k) P_\phi(k)$.

We can now define the Lagrangian bias b_L of the halos from $b_L^2 = P_h(k, z)/P_{\delta\delta}(k, z)$ and use $b_E = 1 + b_L$ to obtain the expression for the non-Gaussian halo bias

$$b_h^{\text{fNL}} = 1 + \frac{\Delta_c(z)}{\sigma_R^2 D^2(z)} \left[1 + 2f_{\text{NL}} \frac{\Delta_c(z)}{D(z)} \frac{\mathcal{F}_R(k)}{\mathcal{M}_R(k)} \right]. \quad (13)$$

Thus $b_h^{\text{fNL}} \simeq b_h(1 + \Delta b_h/b_h)$ where b_h denotes the halo bias for the Gaussian case and is always >1 . Here $\Delta b_h/b_h$ is $2f_{\text{NL}}$ times a redshift-dependent factor $\Delta_c(z)/D(z)$, plotted in Figure 2, times a k and mass dependent factor $\mathcal{F}_R(k)/\mathcal{M}_R(k)$, shown in Figure 3. Figures 2 and 3 enclose all the information needed to compute the effect of non-Gaussianity: the relative correction to b_h at a given redshift \hat{z} , scale \hat{k} and mass M , is obtained by reading the $\delta_c(\hat{z})$ from Figure 2, multiplying this by $\mathcal{F}_R(\hat{k})/\mathcal{M}_R(\hat{k})$ corresponding to mass M from Figure 3, and multiplying the result by $2f_{\text{NL}}$. For example, for $f_{\text{NL}} = 100$, $z = 1.0$, and $k = 0.06 \text{ Mpc}^{-1}$ and for virtually all interesting masses, the non-Gaussian correction to b_h would be a 50% effect.

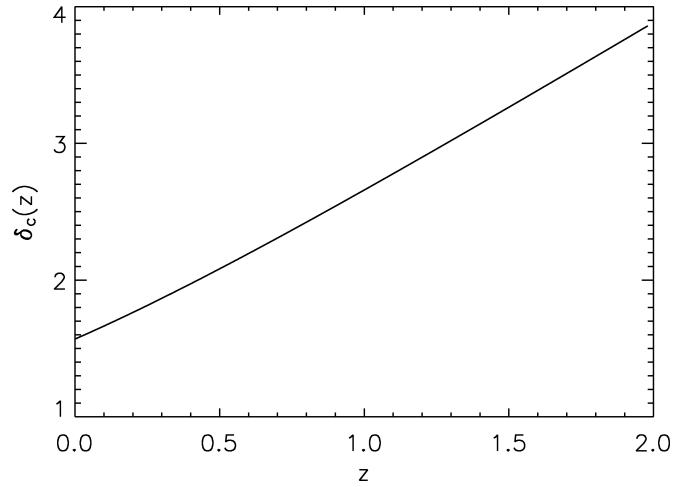


FIG. 2.—Redshift dependence of $\Delta b_h/b_h$ for the LCDM cosmology assumed. From eq. (13) one can see that the non-Gaussian correction to b_h can be factorized as a constant factor of 2 times a redshift-dependent factor (shown here) times a mass- and k -dependent factor (shown in Fig. 3). In particular, the redshift-dependent factor is $\delta_c(z) = \Delta_c(z)/D(z)$ as discussed in § 2.

5. DISCUSSION AND CONCLUSIONS

We have obtained an analytic expression for the bias of dark matter halos for non-Gaussian initial conditions. The only approximations used in our approach are: (1) high peaks, i.e., large values of $\nu = \delta_c(z)/\sigma_R$ (as in the original Kaiser’s formula), which essentially amounts to a limitation on the mass range over which one can apply this formula, and (2) large separation among the halos, which is the standard assumption allowing to use linear bias. While it is true that on large scales ($k \rightarrow 0$) the form factor, the transfer function, and the window function go to unity, on the scales of interest neglecting these terms may lead to errors on Δb_h and therefore on f_{NL} of the order of 100%.

The derivation presented here is based on analytical techniques which are known to be approximated even for applications to initially Gaussian perturbations (see for example the shortcoming of the standard Press-Schechter approach [Press

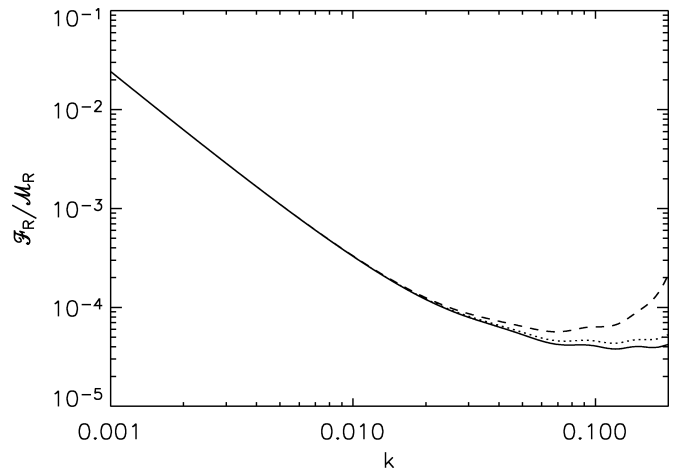


FIG. 3.—Scale dependence of $\Delta b_h/b_h$ —for the LCDM cosmology assumed—for three different masses: $1 \times 10^{14} M_\odot$ (solid curve), $2 \times 10^{14} M_\odot$ (dotted curve), $1 \times 10^{15} M_\odot$ (dashed curve). From eq. (13) one can see that the non-Gaussian correction to b_h can be factorized as a constant factor of 2 times a redshift-dependent factor (shown in Fig. 2) times a mass- and k -dependent factor (shown here). In particular, the mass dependence becomes important only at small scales.

& Schechter 1974]). The fractional corrections due to non-Gaussianity, such as the quantity $\Delta b_h/b_h$ studied here, are usually more robust (see, e.g., Verde et al. 2001; Sefusatti et al. 2007; Grossi et al. 2007). Nevertheless, it is highly desirable to validate this analytic approximation with N -body simulations with non-Gaussian initial conditions. Comparison of these analytical findings with simulations will be presented elsewhere (M. Grossi et al., in preparation).

An advantage of our approach is that it can be easily generalized to nonlocal and scale-dependent non-Gaussian models in which $B_\phi(k_1, k_2, k_3)$ is the dominant higher order correlation and has a general form, obtaining

$$\frac{\Delta b_h}{b_h} = \frac{\Delta_c(z)}{D(z)} \frac{1}{8\pi^2 \sigma_R^2} \int dk_1 k_1^2 \mathcal{M}_R(k_1) \times \int_{-1}^1 d\mu \mathcal{M}_R(\sqrt{\alpha}) \frac{B_\phi(k_1, \sqrt{\alpha}, k)}{P_\phi(k)}. \quad (14)$$

Modeling the clustering of hot and cold CMB spots for non-Gaussian initial conditions is a straightforward extension of this calculation (A. F. Heavens et al., in preparation).

We envision that this calculation will be useful for constraining non-Gaussianity from future surveys which will provide a large sample of galaxy clusters over a volume comparable to the horizon size (e.g., DES, PanSTARRS, PAU, LSST, DUNE, ADEPT, SPACE, DUO) or mass-selected (via the Sunyaev-Zel'dovich effect) large cluster samples (e.g., ACT, SPT).

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REFERENCES

- Bartolo, N., Komatsu, E., Matarrese, S., & Riotto, A. 2004, *Phys. Rep.*, 402, 103
- Catelan, P., Lucchin, F., Matarrese, S., & Porciani, C. 1998, *MNRAS*, 297, 692
- Cole, S., & Kaiser, N. 1989, *MNRAS*, 237, 1127
- Dalal, N., Dore, O., Huterer, D., & Shirokov, A. 2007, preprint (arXiv: 0710.4560)
- Efstathiou, G., Frenk, C. S., White, S. D. M., & Davis, M. 1988, *MNRAS*, 235, 715
- Gangui, A., Lucchin, F., Matarrese, S., & Mollerach, S. 1994, *ApJ*, 430, 447
- Grinstein, B., & Wise, M. B. 1986, *ApJ*, 310, 19
- Grossi, M., Dolag, K., Branchini, E., Matarrese, S., & Moscardini, L. 2007, *MNRAS*, 382, 1261
- Jensen, L. G., & Szalay, A. S. 1986, *ApJ*, 305, L5
- Kaiser, N. 1984, *ApJ*, 284, L9
- Kang, X., Norberg, P., & Silk, J. 2007, *MNRAS*, 376, 343
- Kitayama, T., & Suto, Y. 1996, *MNRAS*, 280, 638
- Komatsu, E., & Spergel, D. N. 2001, *Phys. Rev. D*, 63, 063002
- LoVerde, M., Miller, A., Shandera, S., & Verde, L. 2007, preprint (arXiv: 0711.4126)
- Lucchin, F., Matarrese, S., & Vittorio, N. 1988, *ApJ*, 330, L21
- Matarrese, S., Lucchin, F., & Bonometto, S. A. 1986, *ApJ*, 310, L21
- Matarrese, S., Verde, L., & Jimenez, R. 2000, *ApJ*, 541, 10
- Mo, H. J., & White, S. D. M. 1996, *MNRAS*, 282, 347
- Politzer, H. D., & Wise, M. B. 1984, *ApJ*, 285, L1
- Press, W. H., & Schechter, P. 1974, *ApJ*, 187, 425
- Robinson, J., & Baker, J. E. 2000, *MNRAS*, 311, 781
- Robinson, J., Gawiser, E., & Silk, J. 2000, *ApJ*, 532, 1
- Salopek, D. S., & Bond, J. R. 1990, *Phys. Rev. D*, 42, 3936
- Sefusatti, E., Vale, C., Kadota, K., & Frieman, J. 2007, *ApJ*, 658, 669
- Verde, L., Jimenez, R., Kamionkowski, M., & Matarrese, S. 2001, *MNRAS*, 325, 412
- Verde, L., Wang, L., Heavens, A. F., & Kamionkowski, M. 2000, *MNRAS*, 313, 141