

On the Logical Adequacy of Identity Criteria

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Abstract

From a logical point of view, identity criteria should mirror the identity relation in being reflexive, symmetrical, and transitive. However, the relation representing the identity condition fails to be transitive in many cases. We consider the proposals given so far to give logical adequacy to inadequate identity conditions. We focus on the most refined proposal and expand its formal framework by taking into account two further aspects that we consider essential in the formal treatment of identity criteria: *contexts* and *granular levels*.

1. Introduction: on the logical adequacy of identity criteria

In a realistic approach to ontology it is common to claim that we accept entities as real components of the world if and only if they belong to sorts for which identity criteria can be clearly stated. Identity criteria offer the conditions for determining when two individuals belonging to some sort K are identical.

Among the possible formulations of identity criteria, we consider the following:

$$(IC) \quad \forall x \forall y \in D (f(x) = f(y) \leftrightarrow R(x, y))$$

It is assumed that there is a domain of individuals D and a function f such that $f(D)$ constitutes a sort of individuals K . R represents the condition under which x and y are said to be identical. In the left side of the biconditional in (IC), there is an identity relation, which is an equivalence relation. Consequently, the relation R on the right side of the biconditional must be an equivalence relation, too. Unfortunately, as has been observed in the philosophical debate about identity criteria, some relations considered as candidates for R often fail to be transitive. The following is an example of transitivity failure of R (see [Williamson (1986)]):

Let x , y , and z range over colour samples and f be the function that maps colour samples to perceived colours. A plausible candidate for R might be the relation of perceptual indistinguishability. It is easy to verify, though, that such an R is not necessarily transitive: It might happen that x is indistinguishable in colour from y and y from z , but x and z can be perceived as different in colour.

The example shows how some relations that are intuitively plausible candidates to be identity conditions do not meet the logical constraint that (IC) demands. However, instead of refusing this kind of plausible but inadequate identity criteria, it has been suggested to approximate the relation R whenever it is not transitive. That means that, given a non-transitive R , we can obtain equivalence relations that approximate R by some operations.

Specifically, in this paper we analyze cases where the relation R which the identity condition consists of is not transitive and we seek for a way to obtain equivalence relations approximating R as much as possible.

Some approaches on how to approximate identity criteria have been suggested by [Williamson (1986)] and [De Clercq and Horsten (2005)]. The aim of this paper is to present an improvement of De Clercq and Horsten's approach.

2. Closer approximations to identity conditions

[Williamson (1986)] suggests giving up the requirement for the identity condition to be both necessary and sufficient. Given a non-transitive R , let R_1, R_2, \dots, R_n be equivalence relations that approximate R . Among them, we want to find the relation R_i that best approximates R . Williamson's proposal is to apply one of the following approaches:

Approach from above: Consider the smallest (unique) equivalence relation R^+ such that $R \subseteq R^+$.

Approach from below: Consider the largest (not unique) equivalence relation R^- such that $R^- \subseteq R$.

Adopting the approach from above, you get a relation R^+ that is a sufficient identity condition. On the contrary, if you adopt the approach from below, you obtain a relation R^- that is a necessary identity condition. How can you choose between the two approaches? According to Williamson, there are non transitive relations R that are clearly necessary identity conditions (e.g., the relation of perceptual indistinguishability is considered a necessary identity condition for colours) and non transitive relations R that are clearly sufficient identity conditions (e.g., some forms of mental continuity are considered sufficient identity conditions for persons). So, when you deal with a non transitive, necessary identity condition, you apply the approach from below, otherwise you apply the approach from above.

De Clercq and Horsten claim that there are not always good reasons to decide whether you must take a necessary or a sufficient identity condition R . They consider a third option: to give up both the necessity and the sufficiency of the identity condition and to search for an overlapping relation R^\pm that is neither a super- nor a sub-relation of R . Such an overlapping relation has the advantage of being closer to R than either R^+ or R^- . To understand why, consider how De Clercq and Horsten determine which relation R_i among the approximations R_1, \dots, R_n of a non transitive relation R is the closest (or best) approximation with respect to R . First, they call *revision* any adding or removing of an ordered pair to or from R ; second, they count the number of revisions made to get each approximation R_1, \dots, R_n from R : such a number is called *degree of unfaithfulness*.

Then, they state that a relation R_i is the best approximation with respect to R iff its degree of unfaithfulness is lower than that of all the other approximations of R .

Consider the following example. Let D be a domain of objects:

$$D = (a, b, c, d, e).$$

Assume there is a candidate relation R , reflexive and symmetric, for the identity condition for the individuals of D . Assume that for each element x of D , xRx . When R holds between two different objects x and y , we denote this as \overline{xy} . Let R on D be the following:

$$R = (\overline{ac}, \overline{ad}, \overline{bc}, \overline{bd}, \overline{cd}, \overline{de}).$$

R is not an equivalence relation. In fact, it fails to be transitive. For instance, R holds between a and d and between d and e , but it does not hold between a and e .

Apply, firstly, Williamson's approach from above. We obtain the smallest equivalence relation R^+ such that it is a superset of R , i.e.:

$$R^+ = (\overline{ab}, \overline{ac}, \overline{ad}, \overline{ae}, \overline{bc}, \overline{bd}, \overline{be}, \overline{cd}, \overline{ce}, \overline{de}).$$

Apply, secondly, the approach from below. We get a relation R^- that is not unique. For instance, one of the largest equivalence relations that are subsets of R is the following:

$$R^- = (\overline{bc}, \overline{bd}, \overline{cd}).$$

Apply, then, the overlapping approach. You obtain the following relation:

$$R^{\pm} = (\bar{ab}, \bar{ac}, \bar{ad}, \bar{bc}, \bar{bd}, \bar{cd}).$$

R^+ is obtained by adding four ordered pairs to R , R^- by removing three ordered pairs, and R^{\pm} by adding one ordered pair and removing another one. The degree of unfaithfulness of R^+ is 4, the degree of R^- is 3, the degree of R^{\pm} is 2. The latter is the lowest degree of unfaithfulness. Thus, R^{\pm} is closer to R than R^+ and R^- . That means that with R^{\pm} , you stay closer to your intuitive identity condition R , because R^- modifies R less than R^+ .

3. Refinement of the overlapping approach

If a relation R is not transitive and then, according to De Clercq and Horsten, possibly neither necessary nor sufficient, then it can occur that R holds between two objects a and b in a given situation, but in other situations R does not hold between the same objects a and b . We claim that such a variation of R with respect to a and b occurs both when we consider a and b in different contexts (i.e. in contexts containing a different number of objects) and when we consider a and b from different levels of observation. Consider the following two examples concerning perceived colours:

1. You see two monochromatic colour samples, A and B, and you do not see any difference with respect to their colour. Accepting that the identity condition for perceived colours is perceptual indistinguishability, you claim that A-colour is identical to B-colour. Now, add two further monochromatic colour samples, C and D, such that they are perceptually distinguishable. However, A is indistinguishable from C and B from D. In such a scenario, you can accept to revise your previous identity judgment and say that A-colour is not identical to B-colour.

2. You see two colour samples A and B from a distant point of view such that you are not able to distinguish A-colour from B-colour; so, you say that A-colour is identical to B-colour. Now you get closer to A and B and see a difference between them. So, you revise your previous judgment and say that A-colour is not identical to B-colour.

Our proposal is to integrate the notions of contexts and granular levels with De Clercq and Horsten's formal treatment of approximating relations. Informally, our suggestion is as follows: Given a context, i.e. a set of elements of a domain, each granular level provides a relation R for the elements of that context; however, if we fix a granular level of observation, R can hold between two objects in a context and not hold between the same objects in a different context.

Let L be a formal language consisting of:

- individual constant symbols: \bar{a}, \bar{b}, \dots (there is a constant symbol for each element of the domain);
- individual variables: x_0, x_1, x_2, \dots (countably many);
- two-place predicate symbols P_1, P_2, \dots ; and
- usual logical connectives with identity, quantifiers.

The set of terms consists of individual constants and individual variable symbols. The formulas can be defined in the standard way.

Consider now the following interpretation of L . Let D_K be a fixed, non-empty domain of objects that we assume to belong to some sort K . A context o is defined as a subset of the domain D_K . So, the set of all contexts O in D_K is the powerset of D_K :

$$O = \wp(D_K).$$

Consider now a binary relation R (a two-place predicate). Assume that R is reflexive and symmetric, but not (always) transitive. R pairs the elements in each context $o \in O$ that are indistinguishable in some respect. For instance, in the case of color samples, R gives rise to a set of ordered pairs, each of them consisting of elements that are indistinguishable with regard to their (perceived) color.

Consider the behavior of R across granular levels. Take the following context with three elements: $o = (a, b, c)$. Depending on the granular level you are, one of the following scenarios can occur:

1. R gives rise to three ordered pairs.
2. R gives rise to two ordered pairs.
3. R gives rise to one ordered pair.
4. R does not give rise to any ordered pair.

In 1, we are in a coarse-grained level; in 4, in a very fine-grained level; and in 2 and 3, in some intermediate granular level. The extension of R for each context $o \in O$ varies across granular levels. Now, call *granular structure* a structure M consisting of the domain D_K and a binary relation R ; formally, $M = \langle D_K, R \rangle$. We assume that there is at least one granular structure for each granular level. Consider again the scenarios 1–4. There are very coarse granular structures with an R that behaves as in 1, some refined granular structures with an R that behaves as in 4, and other granular structures with an R that behaves as in 2 or 3.

Now, consider the behaviour of R across contexts. Given a granular structure, say M_1 , consider two contexts: $o = (a, b, c)$, $o' = (a, b, c, d)$. Suppose that M_1 has a relation R such that $R_o = (\bar{a}b, \bar{b}c)$ and $R_{o'} = (\bar{a}b)$. You can observe that R holds between b and c in o , but it does not hold between them in o' . So, fixed a granular structure, the extension of R can vary across contexts.

If, according to some granular structure, the relation R fails to be transitive with respect to some context $o \in O$, then the formal framework given by De Clercq and Horsten is applied. For instance, consider again M_1 . Its relation R is not transitive in context o . Thus, an equivalence overlapping relation R^\pm can be defined for R relatively to o . In contexts where R is not transitive, R^\pm denotes a relation that differs from R in that it adds and/or removes some ordered pairs to or from R .

4. Conclusion

In this paper we have tried to show how the overlapping approach proposed by De Clercq and Horsten can be improved. Before determining the closest approximation to R , we suggest fixing a context and a granular level of observation, since R can vary along those two variables. If, according to a granular structure M_i , R fails to be transitive in a context, you can build the closest approximation to R for that context in M_i .

References

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