On the ontological commitment to mereology

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Abstract

In *Parts of Classes* [1991] David Lewis argues that, like logic, but unlike set theory, mereology is "ontologically innocent". *Prima facie*, Lewis' innocence thesis seems to be ambiguous. On one side, he seems to argue that, given certain objects Xs, referring to their sum is ontologically innocent because there is not a *new* entity as referent of the expression "the sum of the Xs". So, talking of the sum of the Xs would simply be a different way of talking of the Xs, *looking at them as a whole*. However, on the other side, Lewis' innocence is not understood as a mere linguistic use, where sums are not reified. He himself claims that the innocence of mereology is different from that of plural reference, where the reference to some objects does not require the existence of a single entity picking up them in a whole. In the case of plural quantification "we have many things, in no way do we mention one thing that is the many taken together". Instead, in the mereological case: "we have many things, we do mention one thing that is the many taken together, but this one thing is nothing different from the many" ([1], 87). But, due to the fact that Lewis explicitly uses sums as outright objects, we think that Lewis' innocence thesis cannot be understood but in the sense that, even if the sum of the Xs is a well determined object, distinct from the Xs, the existence of such an object is to be necessarily accepted from whom which has already accepted the existence of the Xs. In other words, committing oneself to the existence of the Xs would be an implicit commitment to

some other entities and – among them – the sum of the Xs. On the other hand, the existence of the set of the Xs would not be implicitly guaranteed by the existence of the Xs.

The aim of the paper is to argue that – for a certain use of mereology, weaker than Lewis' one – an innocence thesis similar to that of plural reference is defendable. In order to give a definite account of plural reference, we use the idea of a plural choice. Then, we propose a *virtual theory of mereology*, where the role of individuals is played by plural choices of atoms. A choice is not an authentic object, its existence is merely potential and it consists in the act of performing it. Accordingly, in order to interpret a formal first order mereological language, as Goodman *calculus of individuals* (CG), we introduce a potential semantic of plural choices. We argue that our development of virtual mereology, grounded on the notion of plural choice, is ontologically innocent in a way completely analogous to that of plural reference: our claim is that mereological sums – unlike atoms – are not real objects. Referring to a sum of atoms is nothing but a way of referring to certain atoms. Our approach is adequate to interpret a first order mereological language. It is inadequate for Lewis' mereology, because his plural quantification on *all* objects is incompatible with our notion of plural choice, where just atoms are capable of being chosen.

1. Introduction

In *Parts of Classes* [1] David Lewis argues that, like logic, but unlike set theory, mereology is "ontologically innocent". *Prima facie*, Lewis' innocence thesis seems to be ambiguous. On one side, he seems to argue that, given certain objects Xs, referring to their sum is ontologically innocent because there is not a *new* entity as referent of the expression "the sum of the Xs". So, talking of the sum of the Xs would simply be a different way of talking of the Xs, *looking at them as a whole*. However, on the other side, Lewis' innocence is not understood as a mere linguistic use, where sums are not reified. He himself claims that the innocence of mereology is different from that of plural reference, where the reference to some objects does not require the existence of a single entity picking up them in a whole. In the case of plural quantification "we have many things, in no way do we mention one thing that is the many taken together". Instead, in the mereological case: "we have many things, we do mention one thing that is the many taken together, but this one thing is nothing different from the many" ([1], 87). But, due to the fact that Lewis explicitly uses sums as outright objects, we think that Lewis' innocence thesis cannot be understood but in the sense that, even if the sum of the Xs is a well determined object, distinct from the Xs, the existence of such an object is to be necessarily accepted from whom which has already accepted the existence of the Xs. In other words, committing oneself to the existence of the Xs would be an implicit commitment to some other entities and – among them – the sum of the Xs. On the other hand, the existence of the set of the Xs would not be implicitly guaranteed by the existence of the Xs.

The aim of the paper is to argue that – for a certain use of mereology, weaker than Lewis' one – an innocence thesis similar to that of plural reference is defendable. In order to give a definite account of plural reference, we use the idea of a plural choice. Then, we propose a *virtual theory of mereology*, where the role of individuals is played by plural choices of atoms. A choice is not an authentic object, its existence is merely potential and it consists in the act of performing it. Accordingly, in order to interpret a formal first order mereological language, as Goodman *calculus of individuals* (CG), we introduce a potential semantic of plural choices. We argue that our development of virtual mereology, grounded on the notion of plural choice, is ontologically innocent in a way completely analogous to that of plural reference: our claim is that mereological sums – unlike atoms – are not real objects. Referring to a sum of atoms is nothing but a way of referring to certain atoms. Our approach is adequate to interpret a first order mereological language. It is inadequate for Lewis' mereology, because his plural quantification on *all* objects is incompatible with our notion of plural choice, where just atoms are capable of being chosen.

We will consider some attempts, present in the literature, to defend the ontological innocence of mereology. Such attempts suggest the possibility of a fictional ontological commitment to mereological sums. Then, we will develop a conception of a fictional ontological commitment which is adequate to a virtual (and so ontologically innocent) of CG.

2. *Arguments for a fictional ontological commitment*

We will consider some arguments that, however certain individuals X are given, try to deny the additional existence of the sum of the x, by introducing a weak use of sum, according to which the latter is not an outright individual. The claim that the commitment to the existence of the fusion of the X is not a further commitment beyond the existence of the X is maintained in a fictional way by arguing in favor of the identification of the fusion of several individuals with the individual themselves. This identification seems to be suggested by the following Lewis' argument:

(P1) Composition – a many-one relation – is like identity.

(P2) The commitment to sums is already presupposed in the acceptance of the objects that are summed.

(P3) Nothing could be considered more ontologically innocent than the request to accept something identical to things already accepted.

(C) Mereology is ontologically innocent.

Lewis' argument rests on the thesis (P1) of *composition as identity*. However, Lewis criticizes the following strong version of the (StrongCom):

(StrongCom) The predicate "are" used for the composition relation is literally the plural for of the "is" of identity.

Formally:

$$\forall X\ \forall y\ ((y \text{ is the sum of the } X) \rightarrow y = X)$$

One of Lewis' argument against (StrongCom) concerns the the indiscernability of identical (InId) i.e.:

$$(\text{IdIn})\ \forall x\ \forall y\ (x = y \rightarrow \forall F\ (Fx \leftrightarrow Fy))$$

where the third universal quantification is of second order and "F" is a predicative variable. "Even though – Lewis argues – the many and the one are the same portion of Reality, and the character of that portion is given once and for all whether we take it as many or take it as one, still we do not really have a generalized principle of indiscernability of identicals… What is true of the many is not exactly what is true of the one. After all they are many while it is one" [Lewis 1991: 87].

Consider the following example. Suppose that the number of the X is *n*, where *n* >1. Then, the plural predicate "…are exactly *n*" should apply – given (InId) – to *y* too, but the number of *y* is one.
This argument shows how in Lewis' conception the sum of several individuals is an outright single object, but the presence of such a single object would not undermine the ontological innocence of mereology.
Lewis argues for the innocence of mereology grounding it on a weak reading of (P1) the weak one according to which:

(WeakCom) The predicate "are" used for the composition relation is *analogous* to the plural form of the "is" of identity.

In a previous paper we have criticized also this second reading of composition as identity [Carrara & Martino 2007]. We think that Lewis' above argument is better reformulated within a fictional conception. An example of that might be, at least for mereological questions, Baxter's formulation who in [Baxter 1988] agues for a way to maintain (InId) and (StrongCom) is introducing two kinds of identity, a *strict* and a *popular* one. Baxter gives the following exemplification of the above distinction: "Suppose a man owned some land which he divides into six parcels. Overcome with enthusiasm for [the denial of composition as identity] he might try to perpetrate the following scam. He sells off the six parcels while retaining ownership of the whole. That way he gets some cash while hanging on to his land. Suppose the six buyers of the parcels argue that they jointly own the whole and the original owner now owns nothing. Their argument seems right. But it suggests that the whole was not a seventh thing" [Baxter 1988: 579].
A justification of (StrongCom) is to argue that to strictly count the many is to loosely count the one.

(BT) The whole is the many parts counted as one thing [Baxter 1988: 579].

Even if Baxter argues that (BT) does not deny the existence of the whole, but just the *additional* existence of the whole, it seems to us that this popular mood does not reify the whole. Baxter's example demonstrates a *weak* use of the sum, not involving the existence of it as an entity. It seems to be a use of sums similar to the one of sets in a sentence as:

(1) The set of the Germans trekking on the Plose has cardinality six hundreds.

A sentence one can reformulate without the introduction of the notion of set, saying that:

(2) The number of the Germans trekking on the Plose is six hundreds.

Likewise, the sentence:

(3) I have seen a flock of seven bee eaters

can be rewritten in this way:

(4) I have seen seven bee eaters.

so that (4) does not involve that “flock” stands for a certain specific entity.
Obviously, such arguments are inadequate to defend Lewis’ realistic conception of mereology. They suggest, however, the possibility of a fictional conception of mereological sums.
In what follows we will develop such a conception which will be called virtual grounded on the notion of *arbitrary plural choice.*

3. *Arbitrary plural choices for a second order logic*

We start by making explicit a certain notion of arbitrary reference which is implicitly presupposed both by first and second order logic.
First order logic implicitly presupposes the possibility of singular reference to any individual of the universe of discourse. That can be shown by analyzing the quantification rules of the system of natural deduction for first and second order logic.
The *introduction rule* of the universal quantifier (I$\forall$) allows the inference of $\forall x \mathrm{A}x$ form the premise Ab in the usual way.

.
.
.
(n) Ab

(n+1) $\forall x \mathrm{A}x$

Where ‘b’ is a arbitrary name or (a free variable) not occurring in any assumption on which Ab depends. The soundness of the rule is grounded on the consideration that if one has proved that *b* enjoys the property $\lambda x\mathrm{A}x$, without any specific piece of information on *b*, then any individual enjoys the property in question. The above remark clearly presupposes that *b* can denote any individual. Similarly, for the elimination rule of the existential quantifier (E$\exists$).
So, the logical use of the quantification presupposes the ideal possibility of singularly referring to any individual.
The problem arises: how can one refer to any whatever individual? Perhaps, one could think, by means of some characterizing property. Unfortunately, that involves a problematic universe of properties suitable for characterizing any individual. Since this option involves the general notion of the property of individuals it seems to be inappropriate to first order logic. Besides, it faces the problem of how to refer to an arbitrary property. Therefore, it seems that the notion of reference to an arbitrary individual, which is presupposed by fist order logic is more primitive than any notion of linguistic reference. We think that the most appropriate idealization for justifying arbitrary reference should be grounded on the ideal possibility of direct access to any individual. We will invoke an ideal agent who is suppose to be able by means of an arbitrary act of choice to isolate any individual. In such a conceptual frame the *introduction rule* of the universal quantifier (I$\forall$) is justified in the following way.
Let us imagine an *ideal chooser* who arbitrarily chooses an individual *b*, about which we have no information at all. If just reasoning on *b* we are able to conclude that it enjoys $\lambda x\mathrm{A}x$, since, as far as we know *each* individual could be the chosen one, we can conclude that any individual has the chosen property and infer $\forall x\mathrm{A}x$.
As far as to second order logic, we argue that such a logic presupposes the possibility to simultaneously *refer* to some individuals. In fact, the semantics of second order logic quantifies over every subset of the individuals’ domain. The problem is now how to refer to an arbitrary set of individuals.
Consider the fact that everything we know of a certain set of individuals is that it is determined by its elements, the reference to a set is not realizable with a simultaneous reference to its elements. The comprehension principle of the second order logic:

(CP) $$\exists X\forall x(Xx \leftrightarrow A(x))$$

in the usual set-theorethical interpretation, where the second order variables X range over every subset of the individuals’ domain, rests on the assumption that the individuals satisfying the formula A(x) form a set:

(CP1) *The individuals designated by any plural choice form a set.*

(CP) is evident in a set-theoretical conception according to which, however certain individuals are given, they determine a certain set.
The problem arises: how such individuals are given?
One could think there is a property they share. But this strategy is unsatisfactory due to the notorious criticism to the impredicative definitions. In fact, the formula A(x) could contain some second order quantification. In this case the property expressed by the formula A(x) is defined in terms of the totality of the sets of the individuals. If every set presupposes the existence of a property which is able to characterize its elements the property $\lambda x \mathrm{A}x$ is circularly defined in terms of the totality of the properties of individuals. The circularity could be avoided by means of some *reducibility axiom a la Russell*: one could assume, for every property definable in second order logic, the existence of an elementary coextensive property (i.e. expressible without second order quantification). In this way quantifying over all sets would implicitly presuppose only quantification over such elementary properties. On the other way these are not necessarily expressible in the logical language and their nature seems to be highly problematic. Consider, for example, the well known criticism to the *Reducibility Axiom*. [1]
For these reasons we propose an alternative approach: the idea is that the members of a set are not isolated by means of a property, but by means of *an act of an arbitrary plural choice*. To the purpose let us extend the idealization of singular choices to that of plural choices. Imagine that there is an agent for every individual. For the sake of simplicity we will identify the individuals with the agents themselves. A plural choice consists of an arbitrary simultaneous choice by each agent of one of numbers 0, 1. The agents whose choice is 1 are the individuals designated by the choice in question. The crucial principle of set theory can be formulated as follows:

(SET) *The individuals designated by any plural choice are the members of a suitable set.*

(SET) supplies evidence (in a non circular way) to the comphrension principle (CP): since, in an act of plural choice, each agent can

[1] For a critisim to the Reducibility Axiom see Copi 1971. Instead, for a defence see J. Myhill 1979.

arbitrarily choose 0 or 1, it is certainly possible that a plural choice be performed, in which the agents choosing 1 are precisely those satisfying the formula A(x); so these agents, in virtue of (SET), determine a set.
Boolos in [Boolos 1984, 1985] has argued for the ontological innocence of the second order monadic logic by proposing an interpretation grounded on the notion of plural quantification. In Boolos interpretation second order variables do not range over sets of individuals, but over individuals plurally. In contrast, first order variables range over individuals singularly.
Boolos basic idea consists in interpreting the atomic formulas of form

Xy

as

y is one of the X,

the existential formulas of form

∃X…

as

There are some individuals X such that ...

The universal quantifier ∀X is expressible in terms of the existential one in the usual way. Boolos' proposal has been criticized in various ways: one wonders if speaking of pluralities of individuals is just a rough manner of speaking of sets.
The idea of plural choices can clarify Boolos' proposal. For, the project of reinterpreting second order logic without involving second order entities, as sets or properties, can be realized by reasoning in terms of acts of choice. Instead of assuming sets, involving the assumption of a very entity for any plural choice, i.e. the set of the designated individuals, we may use, as it were, a virtual theory of sets in which the set-theoretical language is paraphrasable in terms of plural choices.
Precisely we introduce the following semantics of plural choices.

Let ϕ be a second order monadic formula whose free first order variables are among $x_1,\ldots,x_m$ and free second order variables among $X_1,\ldots,X_n$. Consider, for each variable x_i, a singular choice x_i^* (i=1,…,

m) and, for each variable X_j, a plural choice X_j^* (j=1,..., n). We will inductively define the truth value of ϕ relative to the choices $x_1^*,\ldots,x_m^*$; $X_1^*,\ldots,X_n^*$. We will state only the clauses for atomic formulas and for second order quantifiers, the others being as usual:

1. if $\phi \equiv X_j\, x_i$, it is true if the individual designated by choice x_i^* is designated by choice X_j^*;
2. if $\phi \equiv \forall Y\psi$, it is true if, however a plural choice Y^* is performed, ψ is true relative to choices $x_1^*,\ldots,x_m^*$, $X_1^*,\ldots,X_n^*$, Y^*;
3. if $\phi \equiv \exists Y\psi$, it is true if it is possible to perform a plural choice Y^* in such a way that ψ turns out to be true relative to choices $x_1^*,\ldots,x_m^*$, $X_1^*,\ldots,X_n^*$, Y^*.

The above semantics is ontologically innocent because the acts of choice are not objects. An act of choice satisfying certain conditions, as the three conditions above specified, is not an entity existing or not in itself, but an action which might or might not be performed. Accordingly, the interpretation of quantifiers is merely potential. That does not undermine the validity of classical logic, since the obtaining or not of a plural choice satisfying clause (3) is completely determined from which individuals are available. Everything is perfectly determined because it is perfectly determined the domain of the individuals.
A relevant aspect of the difference between act of choices and outright sets is that sets, as individuals, are capable of being chosen in turn. In contrast, it would be meaningless to perform a simultaneous choice of infinitely many acts of choice, because the latter are never simultaneously available. For this reason, the semantics of plural choices cannot be extended to a logic of order higher than the second. Let alone it might used for interpreting general set theory as *ZF* set theory. The above limitations contribute to show that the semantics of arbitrary choices is not an expedient for introducing sets surreptitiously.
In the next section we will show that the semantics of arbitrary choices, suitably modified, can be used for developing visual mereology.

4. *Virtual mereology (VM). An informal exposition*

Imagine an infinite domain Σ of agents which will also be said mereological atoms. A plural choice consists in the simultaneous choice of an atom by each agent. The chosen atoms are said to be designated by the plural choice. Two plural choices c_1 and c_2 are equivalent if they designate the same atoms. As already observed, acts of choice are not individuals. Neverthless, we can make them play the role of mereological individuals: we will call individuals the acts of choice and say that equivalent choices are the same individual. We will treat the equivalence relation between choices as the identity relation between individuals. A speech about individuals is to be understood as an extensional speech about plural choices, i.e. a speech identifying equivalent choices. If a choice designate a unique atom it will be identified with the atom itself. If every atom designated by choice c_1 is also designated by choice c_2, we say that individual c_1 is part of individual c_2, in symbols $c_1 < c_2$.

So, we obtain the fundamental relation of mereology. A mereological property P is a law which with every plural choice associates one of the values 0,1 in an extensional way (i.e. in such a way that the same value is associated with equivalent choices). Similarly, a binary relation is an extensional law that with every ordered pair of plural choices associate one of values 0,1. We say that object c enjoys property P if the later associates value 1 to choice c. Similarly, we say that objects c_1, c_2 are related by relation R if R associates 1 to the pair of choices c_1, c_2.

Define the sum of c_1 and c_2, in symbol $c_1 + c_2$ the individual whose atoms are all those of $c1$ and all those of $c2$. This sum exists since it is certainly possible a choice whose designated atoms are precisely those designated by $c1$ and those designated by $c2$. Similarly, if $c1$ and $c2$ share at least one atom, then there exists their product $c1.\ C2$ whose atoms are the ones common to $c1$ and $c2$.

More generally, if P is a property enjoyed by some objects, then there exists the sum of all objects enjoying P, i.e. the object built up by all atoms, such that each of them is part of at least one object enjoying P. In fact, for any atom a it is a well determine fact if a plural choice c is possible or not such that a is designated by c and c enjoys c. It is therefore certainly possible a choice, which will be indicated by $\sigma x P x$, whose designated atoms are precisely the ones satisfying the above condition.

Similarly, if at least one object satisfies P, and at least one atom is part of every object satisfying P, then there exists the product of all objects enjoying P, $\pi x P x$, i.e. the object built up by all atoms common to all objects satisfying P.

Of course, since, as already observed for the virtual set theory, the acts of choice are conceived of as performable in time, we can speak of individuals of virtual mereology only in a mere potential sense. To say that at least one object exists satisfying property P amounts to saying that a choice c satisfying P is performable. To say that every object enjoys property P amounts to saying that, however a plural choice c is performed, it will enjoy property P. Again, we can observe that such a potential interpretation does not undermine the validity of classical logic. For, the possibility or not of performing a plural choice satisfying certain conditions is well determined by the objects available in the domain.

One might wonder if, and in what sense, one can speak of the cardinality of the universe of virtual objects, and compare it with that of the universe of atoms. There is a merely potential sense in which one might claim that there are more individuals than atoms. It is possible, by means of a single plural choice to determine infinitely many objects. Precisely, one can produce, by a single plural choice, a family of objects indicised by atoms. If c is a plural choice of ordered pairs of atoms, and i is the first component of a pair designated by c, we shall indicate by c_i the object whose atoms are the second components of the pairs designated by c whose first component is i. So, we get the family $\{c_i\}_{i\in I}$ of objects c_i indicised by the first components I of pairs designated by c.

Suppose, for the sake of simplicity, that there are denumerably many atoms and indicate them by the natural numbers. We can obtain, by means of a single plural choice, a family of objects $c_i \in$ N. By using the well known method of diagonalization, one can prove the existence of an object different from all objects of the family. Precisely, call P the property which is enjoyed by an atom if and only if it is an atom of c_i. It is certainly possible to perform a choice c whose designated atoms are exactly the ones enjoying P (disregarding the case in which all c_i are the universal object, which is to be treated separately in an obvious way).To such a choice corresponds an object different form all c_i. So, we can say that the objects constitute a non denumerable infinity in the sense that, however a succession of objects is determined by means of a plural choice, it is possible to determine an object different from all components of that succession. In other words, it is impossible to give simultaneous actual existence to all possible objects, nor the fact that the objects are merely virtual allows to speak of them as if they were all actually existent. For, the mere virtuality of objects, as here understood, consists in the possibility of translating the speech of objects in terms of plural choices. The ideal assumption of denumerably many agents grounds the possibility of simultaneously choosing infinitely many atoms, but it is by no means conceivable a simultaneous performance of all possible acts of choice.

5. *The calculus of individuals with atoms (CG)*

The above exposition constitutes a virtual interpretation of the axiomatic theory of mereology developed by Goodman in *The structure of Apparence* called “calculus of individuals” (CG) where the overlap relation is assumed as primitive. In the present paper we consider a version of CG with atoms, without sets, where the part relation is primitive (<), while the other mereological notions and identity (=) are defined in terms of the part relation.
Precisely, we introduce the following definitions:

(CGDef.1) $x \text{ o } y =_{df.} \exists z\, (z < x \wedge z < y)$ (x overlaps y)

(CGDef.2) $x = y =_{df.} x < y \wedge y < x$ (x is identical to y)

(CGDef.3) $At(x) = \forall y\, (y < x \rightarrow y = x)$ (x is an atom)

Two non overlapping individuals are said to be one to the other *discrete*:

(CGDef.4) $x \rfloor y =_{df.} \neg (x \mathbf{o}\, y)$ (x is discrete from y);

(CGDef.5) $x \ll y =_{df.} (x < y \wedge \neg(y < x))$ (x is proper part of y)

Finally, let us define the *sum* or *fusion* of the individuals satisfying a formula F .

(CGDef.6) $\forall y\, (Aty \rightarrow (y < z \leftrightarrow \exists x\, (Fx \wedge y < x)))$ (z is the sum of the F)

Axioms of (CG):

(CGA1) Any set of logical axioms for first order predicate calculus without identity.

(CGA2) $x < x$

(CGA3) $x < y \wedge y < z \rightarrow x < z$

(CGA4) $\forall z\ (\mathrm{At}z \wedge z < x \rightarrow z < y) \rightarrow x < y$

(CGA5) $\forall z\ (z\ \mathbf{o}\ x \leftrightarrow z\ \mathbf{o}\ y) \rightarrow (A \rightarrow A[y//x])$

where Ay/x is any formula obtained from A by replacing some occurrence of x free in A with an occurrence of y.

(CGA6) If the formula Fx is satisfied by some individuals, there is one and only one individual which is the sum of the F.

Such an individual will be denoted by σxFx. In particular, taking for Fx the formula:

$$(x = y \vee x = z)$$

we get the sum of y and z which will be simply indicated by y + z. Finally, it is easily seen that, if the F share a common part, there exists the product of the F defined as the individual built up by all atoms shared by all F. We shall indicate this product by πx Fx; in case of two individuals y and z simply by y . z. It follows from (CGA4) that:

(Teor1) $\forall x\, \exists y\ (\mathrm{At}y \wedge y < x)$ (each individual has an atomic part)

Proof. Suppose, by way of contradiction. that *x* is an individual without atoms. It follows from (CGA4): $\forall z\ (\mathrm{At}z \wedge z < x \rightarrow z < y) \rightarrow x < y$ that x is part of every y, so x has no proper parts. Therefore, it is itself an atom, what is absurd (QED).

6. *A formal semantics for VM*

Let L be the language of first order predicate logic with just the primitive binary predicate <.
We will define for L the interpretation of plural arbitrary choices (IPAC) by suitably modifying the above interpretation for second order logic.
Let us imagine denumerably many agents Σ which will be identified with mereological atoms. A plural choice *c* consists in the simultaneous arbitrary choice of any individual by each agent. An individual is *designated* by plural choice *c* if it is at least chosen by one agent. So, a plural choice determines the plurality of designated individuals. Define (IPAC) by fixing the truth-conditions of the L-sentences.
Let $A[x_1,\ldots, x_n]$ a formula whose free variable are the shown ones. For each variable x_i consider a plural choice c_i (i= 1,…, n). We will inductively define the truth conditions of $A[x_1,\ldots, x_n]$ relative to choices $c_1,\ldots,c_n$

(i) an atomic formula $x_1<x_2$ is truth relative to choices c_1, c_2 if the individuals designated by c_1 are designated by c_2.

(ii) Usual clauses for propositional connectives.

(iii) $\forall x_i\, A[x_1,\ldots, x_i,\ldots, x_n]$ is truth relative to choices $c_1 ,\ldots,c_{i-1}, c_{i+1},\ldots,c_n$ if, however a plural choice c_i relative to x_i is performed, the formula $A[x_1,\ldots, x_i,\ldots, x_n]$ is truth relative to choices $c_1,\ldots c_i,\ldots c_n$.

(iv) $\exists x_i\, A[x_1,\ldots, x_i,\ldots, x_n]$ is truth, relative to choices $c_1 ,\ldots,c_{i-1}, c_{i+1},\ldots,c_n$ if it is possible to perform a choice c_i for x_i such that $A[x_1,\ldots, x_i,\ldots, x_n]$ is truth relative to choices $c_1,\ldots c_i,\ldots c_n$.

By induction on the complexity of formulas one recognizes that the above clauses determine the truth-vale of every formula relative to the plural choices associated with the free variables. For the atomic

formulas this fact directly follows by observing that a plural choice designates a well determined plurality of individuals. For the formulas whose principal logical constant is a propositional connective, the truth value is determined in an obvious way from the values of the components. As far to quantifiers, suppose that Ax (assuming for the sake of simplicity only free variable) has, relative to every plural choice, a well determined value. Then, it is well determined whether, however the plural choices performed, the formula turns out to be true or it is possible to perform a choice falsifying it. So, it is well determined the value of $\forall xA[x]$. Similarly for $\exists xA[x]$.
Notice that the possibility of performing a choice verifying A[x] is not to be understood in an epistemic sense, but in an alethic sense: i.e. it is not required the possibility that the agents follow some strategy in order to obtain some goal. A plural choice is always constituted by arbitrary acts of choice, independent one from the other, performed by the agents. The possibility of a plural choice verifying A[x] is to be understood in the sense that it might happen that the choices, purely random choices, performed by the singular agents verify A[x].
In particular, the truth value of every sentence (closed formula) will be well determined. It follows that our interpretation is in agreement with the laws of classical logic.
Introduce for IPAC the mereological terminology. We say that a variable x, relative to a plural choice *c*, denotes an object. In particular, if *c* designates a unique individual, we say that x denotes an atom which we identify with the designated agent. We say that the object denoted by x relative to choice c is identical to the object denoted by y relative to choice c' if the individuals designated by c are the same individuals designated by c'. We say that x is part of y if every individual designated by c is designated by c'.
So, the ontological commitment reduces just to atoms. Speaking of compound objects is only a way of speaking *facon de parler*. The singular L-terms pretend to denote in virtue of the choice acts associated with them.

7. VM as a model of CG

Consider the axioms of CG: (CGA1) Any set of logical axioms for first order predicate calculus without identity.

(CGA2) $x < x$

(CGA3) $x < y \wedge y < z \rightarrow x < z$

(CGA4) $\forall z\, (\mathrm{At}z \wedge z < x \rightarrow z < y) \rightarrow x < y$

(CGA5) $\forall z\, (z\, \mathbf{o}\, x \leftrightarrow z\, \mathbf{o}\, y) \rightarrow (A \rightarrow A[y//x])$

(CGA6) If the formula Fx is satisfied by some individual, there is one and only one individual which is the sum of the F.

(CGA1) – (CGA4) are trivially verified in (MV). We limit ourselves to verify (CGA5) and (CGA6).

As to (CGA5) observe that from the antecedent of the conditional – $\forall z\, (z\, \mathbf{o}\, x \leftrightarrow z\, \mathbf{o}\, y)$ – restricting *z* to the atoms, it follows that *x* and y have the same atoms so that they are identical, and so one can substitute one to the other. As to (CGA6), let Fx be a formula satisfied by at least one object. Since every individual is built up from atoms there is at least one atom which is part of an individual satisfying Fx. Besides, as observed above, the formula determines which individuals (which plural choices) satisfy it. Therefore, for all atoms it is perfectly determined whether it is part of at least one individual satisfying Fx. So, it is possible a plural choice whose designated atoms are precisely those satisfying the condition in question. Such a choice represents the object σxFx, which is obviously the unique one. VM is inadequate to interprete Lewis' mereology because the latter uses reference and plural quantification over individuals and it assumes the existence of the sum of an arbitrary plurality of individuals (see Lewis 1991, 1993). In contrast, our virtual mereology VM allows only plural choices of atoms, not plural choices of arbitrary individuals. Accordingly, our justification of (CGA6) exploits essentially the fact that the object whose sum is claimed to exist are determined by a formula of the language (they are not arbitrary chosen).

8. Conclusion

In the present paper we try to take seriously Lewis' idea, and not only of Lewis, according to which the mereological sums should be identifiable with the objects composing them. We have shown how this idea, though objectionable in Lewis' use of mereology, suggests a virtual interpretation of a weaker mereology formalisable at first order level. Such an interpretation, based on the notion of plural arbitrary choice, attributes to mereological sums a merely fictional existence, enlightening in a precise, though limited, way of understanding the ontological innocence of mereology.

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