Quantum properties of the heterotic five-brane

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A B S T R A C T

We find that the conjectured heterotic $SO(32)$ five-brane sigma model develops necessarily k-anomalies, and we investigate their form. We show that these anomalies can be absorbed by modifications of the superspace constraints, that satisfy automatically the modified Bianchi-identity $dH_7 = X_8$ of $N = 1$, $D = 10$ supergravity. The k-anomalies induce in particular a quantum deformation of the torsion constraint $T^a_{\alpha\beta} = 2\gamma^a_{\alpha\beta}$.

1. Introduction

There are a few brane-excitations of M-theory that are still waiting for a low energy description in terms of a classical supersymmetric action. Examples are the systems of multiple D-branes [1] and $M2$-branes [2], while among the single branes the most prominent missing classical action is that of the heterotic five-brane. For the $SO(32)$ heterotic five-brane the field content has been determined in [3], while for the $E_8 \times E_8$ five-brane even the field content is still unknown. Despite the fact that these classical five-brane actions are not explicitly known, the believe in their existence is still alive. The main motivations for this credo are that they *exist* as excitations, in that they are the S-duals of the corresponding fundamental heterotic strings, and that all other five-brane excitations, the $NS5$-brane [4], the $D5$-brane [5] and the $M5$-brane [6,7] admit, actually, supersymmetric – better, k-symmetric – local classical actions.

The interest in generic p-brane σ-models stems from their deep relationship with the dynamics of the background field theory in which they are embedded: a brane can move consistently only in a supergravity target-space whose dynamics is properly constrained, i.e. whose fields satisfy "effective" equations of motion. In the case of string σ-models there are several methods to derive this effective dynamics. In the supersymmetric Green–Schwarz approach it can be determined through the β-function method, relying on conformal invariance [8], or, alternatively, through the k-anomaly method [9–12], see below for details. These two methods are related since, in a certain sense, the square of a k-transformation amounts to a conformal transformation [13]. Unfortunately, the efficiency of these methods is reduced by the absence of a manifest covariant quantization procedure for the Green–Schwarz string. On the other hand, the manifestly supersymmetric pure-spinor approach [14] does entail neither k-symmetry nor conformal invariance, and the role of the k-anomaly and conformal methods is played by the requirement of nihilpotency of a certain BRST charge Q.

For p-branes with $p > 1$, instead, the pure spinor approach is not available and, moreover, these objects are not conformally invariant. Under these circumstances k-symmetry – and the k-anomaly method – regain their fundamental roles for the derivation of the effective target-space dynamics. In this Letter we enforce this method for the (conjectured) heterotic $SO(32)$ five-brane σ-model, to analyze its relation with $N = 1$, $D = 10$ supergravity in superspace, as the low energy approximation of heterotic string theory. From a phenomenological point of view this string theory appears still most promising due to the presence of non-Abelian gauge fields already in ten dimensions; for the $SO(32)$-theory see, e.g., [15–17]. From this point of view the heterotic five-brane gains its interest from its direct relationship with the one-loop corrected modified Bianchi identity of the seven-form,

$$dH_7 = \beta' X_8, \tag{1.1}$$

where X_8 is given in (1.3) and $1/\beta'$ is the brane tension. An open problem in string theory regards, indeed, the supersymmetrization of this Bianchi identity in superspace. For recent progress in this direction see [18] and [19]. One purpose of this Letter is to envis-

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age an – at least in principle – systematic approach to attack this problem.

Since the low energy dynamics of the heterotic five-brane is known only in its geometric sector – the one described by $x^m(\sigma)$ and $\vartheta^\mu(\sigma)$ – the analysis of this Letter will rely essentially only on the symmetries this brane is supposed to have. The results of the Letter are summarized as follows: (1) the conjectured heterotic $SO(32)$ five-brane σ-model entails necessarily k-anomalies, whose building block we determine explicitly; (2) the cancellation of these anomalies requires a modification of the "classical" superspace constraints of H_7 and of the torsion T^A; (3) the Wess–Zumino consistency condition fulfilled by the k-anomalies ensures that the so modified H_7 and T^A satisfy automatically the modified Bianchi identity (1.1) and the torsion Bianchi identity $DT^A = E^B R_B^A$. A fundamental point of our analysis is that the target-space polynomial X_8 implies necessarily the presence of several k-anomalies; (4) the k-anomalies induce, in particular, non-vanishing deformations of first order in β' of the dimension-zero torsion,

$$T_{\alpha\beta}^a = 2\gamma_{\alpha\beta}^a + o(\beta'),$$

and of the constraints of H_7, in agreement with [19,22]. The steps (2) and (3) represent the basic ingredients of the "k-anomaly method".

An explicit evaluation of the k-anomalies of the heterotic five-brane would thus lead to explicit and consistent expressions for the modified constraints. These constraints are usually "determined" solving superspace Bianchi identities, and we present such a "minimal" solution [19] in Section 4.1, see Eqs. (3.11), (4.7), (4.13), (4.14). The reliability of this minimal solution could thus be tested upon comparison with the constraints derived through the k-anomaly method. Eventually the results of this Letter provide a concrete reason for why one should insist on the validity of (1.1) in *superspace*.

In a certain sense the present Letter represents a generalization of the analysis of [9], from the heterotic string to the heterotic five-brane: with this respect the main difference between strings and five-branes is that for the heterotic string the first order deformations (in that case in α') of the classical constraints of T^A and H_3 are all *vanishing*.

1.1. Gauge anomaly cancellation

What is known about the heterotic $SO(32)$ five-brane are its gauge group $\mathcal{G} = SO(32) \times SU(2)$, and its $d = 6$, $N = 1$ supersymmetric field content [3]: the "geometric" sector is made out of a hypermultiplet that is singlet under $\mathcal{G}$, described by the fields (x^m, ϑ^μ), and the "heterotic" sector is made out of an $SU(2)$ Super-Yang–Mills multiplet, and of a hypermultiplet belonging to the representation $(32, 2)$ of $\mathcal{G}$. The fermions of the hypermultiplets and of the Yang–Mills multiplet have opposite chirality in $d = 6$, and the total anomaly polynomial $2\pi I_8$ has been computed in [20], see [21] for a preliminary analysis,

$$I_8 = X_8 + (X_4 + \chi_4)Z_4. \tag{1.2}$$

Here X_8 and X_4 are the standard target-space polynomials (related to the ten-dimensional Green–Schwarz anomaly polynomial through $X_{12} = X_4 X_8$),

$$X_8 = \frac{1}{192}\left(trR^4 + \frac{1}{4}(trR^2)^2 - trR^2 trF^2 + 8trF^4\right),$$

$$X_4 = \frac{1}{4}(trR^2 - trF^2), \tag{1.3}$$

and

$$\chi_4 = \frac{1}{8}\varepsilon^{ABCD}T^{AB}T^{CD},$$

$$Z_4 = \frac{1}{48}(trR^2 - 2trT^2 - 24trS^2). \tag{1.4}$$

R, F, S and T are the two-form curvatures, divided by 2π, associated respectively to $SO(1,9)$, $SO(32)$, $SU(2)$ and to the $SO(4)$-normal bundle. As shown in [20], the gauge anomaly (1.2) can be cancelled via the inflow mechanism through the local WZ-term on the five-brane worldvolume,

$$\frac{1}{\beta'}\int B_6 + \frac{1}{\alpha'}\int B_2 Z_4, \tag{1.5}$$

where B_6 and B_2 are the dual gauge potentials of $N = 1$, $D = 10$ supergravity. The curvature associated to B_2 satisfies the Bianchi identity $dH_3 = \alpha' X_4$. The peculiar form of the polynomial (1.2), that allows this cancellation mechanism, emerges through a number of cancellations that seem miraculous, supporting thus strongly the hypothesis that a consistent low energy effective action for the heterotic $SO(32)$ five-brane should exist.

In this Letter we will be concerned only with the irreducible part of (1.2), i.e. with X_8, in that the interpretation of the factorized term of (1.2) requires presumably the knowledge of the detailed dynamics of the heterotic sector.

2. Super five-brane σ-model: action and symmetries

In this section we present the k-invariant super five-brane σ-model in a ten-dimensional supergravity background, that we consider as the building block describing the geometric sector of the heterotic five-brane. We use this model to deduce the form of the symmetries and the BRST algebra that should characterize also the heterotic five-brane. In the next section we will then use this algebra to analyze the σ-model one-loop effective action of the heterotic five-brane. So, strictly speaking, the analysis of this section applies to the *super* five-brane.

The action of the super five-brane, rescaled by β', is standard,

$$I[Z] = \int\left(\sqrt{g}\,d^6\sigma + B_6\right). \tag{2.1}$$

The supercoordinate field is $Z^M(\sigma) = (x^m(\sigma), \vartheta^\mu(\sigma))$, $m = 0, \ldots, 9$, $\mu = 1, \ldots, 16$, and $g = -\det g_{ij}$, where the induced metric is $g_{ij} = V_i^a V_j^b \eta_{ab}$, and $V_i^A = \partial_i Z^M E_M^A(Z)$. $A = \{a, \alpha\}$ stands for a ten-dimensional vector index and a sixteen-dimensional spinor index, and $i, j = 0, 1, \ldots, 5$ are worldvolume indices that will be raised and lowered with the metric g_{ij}. The second term in (2.1) is the pullback on the worldvolume of the superspace six-form $B_6(Z)$. Target superspace zehnbein, connection, torsion and curvature are denoted respectively by $E^A = dZ^M E_M^A$, Ω_A^B, $T^A = dE^A + E^B \Omega_B^A = \frac{1}{2}E^B E^C T_{CB}^A$, $R_A^B = d\Omega_A^B + \Omega_A^B \Omega_C^B$. We define the zero-order superspace constraints for $H_7 = dB_6$ and T^A as,

$$\widetilde{T}_{\alpha\beta}^a = 2\gamma_{\alpha\beta}^a, \qquad \widetilde{T}_{b\alpha}^a = 0, \qquad \widetilde{T}_{\alpha\beta}^\gamma = 0, \tag{2.2}$$

$$\widetilde{H}_{\alpha\beta a_1 \cdots a_5} = -2(\gamma_{a_1 \cdots a_5})_{\alpha\beta}, \qquad \widetilde{H}_{\alpha a_1 \cdots a_6} = 0, \tag{2.3}$$

where it is understood that the components of $\widetilde{H}_7$ with more than two spinor indices vanish. This choice of constraints is convenient in that the dilaton φ appears in the field strength dB_2, but not in dB_6. The constraints (2.2), (2.3) entail a solution of the Bianchi identities $DT^A = E^B R_B^A$, $dH_7 = 0$, that describes pure $N = 1$, $D = 10$ supergravity.

The equations of motion following from (2.1) for x^m and ϑ^μ respectively are,

$$D_i V_a^i = -\frac{1}{6!\sqrt{g}} \varepsilon^{i_1 \cdots i_6} V_{i_1}^{A_1} \cdots V_{i_6}^{A_6} (dB_6)_{aA_6 \cdots A_1}, \tag{2.4}$$

$$\left[\gamma^j (1-\gamma) \right]_\beta^\alpha V_j^\beta = 0. \tag{2.5}$$

The derivative D_i in (2.4) is covariant w.r.t. $SO(1,9)$ Lorentz-transformations and $d=6$ diffeomorphisms. In (2.5) we introduced the 16×16 matrices,

$$\gamma_i = V_i^a \gamma_a, \qquad \gamma = \frac{1}{6!\sqrt{g}} \varepsilon^{i_1 \cdots i_6} \gamma_{i_1 \cdots i_6}, \qquad \gamma^2 = 1. \tag{2.6}$$

The matrices $\frac{1}{2}(1 \pm \gamma)$ are thus projectors.

2.1. BRST-symmetry

The action (2.1) is invariant under $d=6$ diffeomorphisms, with an anticommuting ghost field c^i, and under k-transformations, with a spinorial commuting ghost field k^α,

$$\delta Z^M = c^i \partial_i Z^M + \Delta^\alpha E_\alpha{}^M, \qquad \Delta^\alpha \equiv \frac{1}{2}(1+\gamma)^\alpha{}_\beta k^\beta. \tag{2.7}$$

While under diffeomorphisms $I[Z]$ is invariant for arbitrary background fields, under k-transformations it is invariant only if these fields are suitably constrained. This feature becomes manifest if one realizes that the variation of (2.1) under (2.7) can be written as,

$$\delta I = \int d^6\sigma \left(\sqrt{g}\, V_a^i V_i^B \Delta^\alpha \left(T_{\gamma B}^a - \widetilde{T}_{\gamma B}^a \right) \right. \\ \left. - \frac{1}{6!} \varepsilon^{i_1 \cdots i_6} V_{i_1}^{A_1} \cdots V_{i_6}^{A_6} \Delta^\alpha (dB_6 - \widetilde{H})_{\alpha A_1 \cdots A_6} \right). \tag{2.8}$$

Hence, I is invariant if one imposes on T^A and dB_6 the constraints (2.2), (2.3). To derive (2.8) one has to use the variation,

$$\delta V_i^A = c^j \partial_j V_i^A + \partial_i c^j V_j^A + D_i \Delta^A - V_i^B L_B{}^A + V_i^B \Delta^C T_{CB}^A, \tag{2.9}$$

where,

$$\Delta^a \equiv 0, \qquad L_a{}^b = \Delta^\gamma \Omega_{\gamma a}{}^b, \qquad L_\alpha{}^\beta = \frac{1}{4} L_{ab} (\gamma^{ab})_\alpha{}^\beta,$$

$$L_a{}^\alpha = 0 = L_\alpha{}^a.$$

It is a characteristics of k-transformations that the associated BRST-algebra closes only on-shell, and that it is infinitely reducible, in the sense that it requires "ghosts for ghosts". The transformations of the ghost fields that lead to an on-shell nihilpotent BRST-operator – $\delta^2 = 0$ – can be determined to be,[1]

$$\delta c^i = -c^j \partial_j c^i + \Delta \gamma^i \Delta, \tag{2.10}$$

$$\delta k^\alpha = -c^i \partial_i k^\alpha - k^\beta L_\beta{}^\alpha - (\Delta \gamma^i \Delta) V_i^\alpha \\ + (V_i \gamma_a \Delta) \left[\gamma \gamma^i (\gamma^a - V_j^a \gamma^j) \right]^\alpha{}_\beta k^\beta + \frac{1}{2}(1-\gamma)^\alpha{}_\beta k_2^\beta, \tag{2.11}$$

where $(V_i \gamma_a \Delta) \equiv V_i^\alpha (\gamma_a)_{\alpha\beta} \Delta^\beta$ etc., and k_2^β indicates the second generation ghost, with ghost number two. The above transformations follow from the requirement $\delta^2 Z^M = 0$, upon enforcing the equation of motion (2.5) for ϑ^μ. The presence of the term $\frac{1}{2}(1-\gamma)k_2$ follows essentially from the definition of the spinor Δ in (2.7), that determines k only modulo "left-handed" fermions. On the other hand, this term is *needed* in δk, because otherwise the operator δ would not square to zero on k. For completeness

we list the transformation laws for the whole tower of ghosts k_n, $n = 1, 2, \ldots, k_1 \equiv k$, although their explicit form will not be needed in what follows. Imposing $\delta^2 k_n = 0$ for all n, iteratively and suppressing spinor indices, one obtains,[2]

$$\delta k_n = \frac{1}{2} \delta \gamma (k_n - \delta k_{n-1}) + \frac{1}{2}(1 \pm \gamma) k_{n+1}, \quad n \geqslant 2, \tag{2.12}$$

where the sign is $+$ $(-)$ for n even (odd). The transformation of the matrix γ in (2.6) is computed to be,

$$\delta \gamma = \gamma \gamma^i (\gamma_a - V_a^j \gamma_j) \delta V_i^a,$$

where δV_i^a is given in (2.9). The above transformation laws ensure also that $\delta^2 c^i = 0$. Notice also the identity $V_i^a \Delta \gamma^i \Delta = \Delta \gamma^a \Delta$, implied by $\gamma \Delta = \gamma$.

Despite the fact that the above formulae look rather complicated, the transformation law for Δ^α turns out to be rather simple, and fortunately it is all we shall need below,

$$\delta \Delta^\alpha = -c^i \partial_i \Delta^\alpha - \Delta^\beta L_\beta{}^\alpha - (\Delta \gamma^i \Delta) V_i^\alpha. \tag{2.13}$$

2.2. Quantization

As anticipated above, the quantization of the super five-brane in a *flat* target superspace – as the quantization of the string in Green–Schwarz formulation – encounters two main difficulties: (1) the BRST-algebra closes only on shell and, (2) k-symmetry is infinitely reducible, requiring an infinite number of ghost fields. In the case of the Green–Schwarz string these problems force eventually a non-covariant quantization scheme, to truncate the infinite tower of ghosts. For a generic p-brane the first difficulty can, actually, be overcome employing the Batalin–Vilkovisky approach [23], that allows to quantize systems with open gauge algebra, while the second difficulty represents still an open problem. On the contrary, for a p-brane in a *curved* target superspace, as our σ-model (2.1), one can employ the background field method combined with a normal coordinate expansion, and in this framework one can overcome both difficulties, as outlined in [21]. More precisely, one can impose on the external classical fields the equations of motion, avoiding thus the rather cumbersome Batalin–Vilkovisky approach, and one can furthermore impose a *covariant* background gauge on the quantum fields, in which the infinite ghosts do not propagate. The practical outcome is that in this way one can define perturbatively a quantum effective action $\Gamma = I + \beta' I_1 + o(\beta'^2)$, that – in absence of anomalies – would satisfy the on-shell relation $\delta \Gamma = 0$.

3. k-anomalies

We assume now that the quantum effective action Γ of the heterotic five-brane entails the BRST-symmetry constructed in the previous section. This functional defines then a local anomaly through,

$$\delta \Gamma = \mathcal{A}, \tag{3.1}$$

that, thanks to $\delta^2 = 0$, satisfies the on-shell Wess–Zumino consistency condition,

$$\delta \mathcal{A} = 0, \tag{3.2}$$

that will play a fundamental role in what follows. In the following we will work at the one loop order, i.e. at first order in β'. In

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[1] Our operator δ acts from the right.

[2] This recursive relation holds if one shifts the ghost k_2 in (2.11) to absorb from δk all terms proportional to $(1-\gamma)$, i.e. such that $\delta k = \frac{1}{2}(1+\gamma)\delta k + \frac{1}{2}(1-\gamma)k_2$.

the rest of the Letter we will show that the anomaly $\mathcal{A}$ is necessarily non-vanishing, that (3.2) implies that this anomaly can be cancelled by modifying the superspace constraints (2.2), (2.3) at first order in β', and that these modifications realize a consistent solution of the superspace Bianchi identities,

$$DT^A = E^B R_B{}^A, \qquad dH_7 = \beta' X_8. \tag{3.3}$$

The anomaly has to be a local functional of ghost-number one, i.e. linear in k^α and c^i. Since diffeomorphism anomalies can be traded for local Lorentz-anomalies, that have already been determined in (1.2), it is sufficient to consider $\mathcal{A}$ linear in k^α. Moreover, since the variations δZ^M involve k^α only through Δ^α, $\mathcal{A}$ is linear in Δ^α. In analogy with [11] we make the Ansatz,

$$\begin{aligned}
\mathcal{A} &= -\int d^6\sigma \left(\sqrt{g}\, V_a^i V_i^B \Delta^\gamma S_{\gamma B}^a \right. \\
&\quad \left. + \frac{1}{6!}\varepsilon^{i_1\cdots i_6} V_{i_1}^{A_1}\cdots V_{i_6}^{A_6}\Delta^\gamma W_{\gamma A_6\cdots A_1}\right) \\
&\equiv \mathcal{A}_1 + \mathcal{A}_2,
\end{aligned} \tag{3.4}$$

where we introduced the targetspace superforms, of first order in β',

$$S^a = \frac{1}{2}E^C E^B S_{BC}^a, \qquad W_7 = \frac{1}{7!}E^{A_7}\cdots E^{A_1} W_{A_1\cdots A_7}. \tag{3.5}$$

Notice that the purely vectorial components of these fields do not enter (3.4). We do not assert that (3.4) is the most general form of the anomaly, but we will show that on the heterotic five-brane anomalies of this kind are necessarily present. As we will see, the forms S^a and W_7 represent the quantum corrections to the classical constraints (2.2) and (2.3) for T^a and H_7 respectively.

3.1. Wess–Zumino consistency condition

We impose now the Wess–Zumino consistency (3.2) on (3.4). $\delta\mathcal{A}_2$ is most easily computed noting that one has $\mathcal{A}_2 = -\int i_\Delta W_7$, where i_Δ denotes the inner product of a superform with the vector $\Delta^M \equiv \Delta^\alpha E_\alpha{}^M$. Since $\mathcal{A}_2$ is invariant under $d=6$ diffeomorphisms, the variation (2.7) reduces to $\delta Z^M = \Delta^M$, that corresponds formally to a superdiffeomorphism in $D=10$, and for a superdiffeomorphism-BRST transformation on a generic p-superform Φ we have,

$$\delta(i_\Delta\Phi) = \frac{1}{2}(i_\Delta i_\Delta d + d i_\Delta i_\Delta)\Phi.$$

This leads to,

$$\begin{aligned}
\delta\mathcal{A}_2 &= -\frac{1}{2}\int i_\Delta i_\Delta dW_7 \\
&= -\frac{1}{2}\int d^6\sigma \frac{1}{6!}\varepsilon^{i_1\cdots i_6} V_{i_1}^{A_1}\cdots V_{i_6}^{A_6}\Delta^\alpha\Delta^\beta (dW_7)_{\alpha\beta A_6\cdots A_1}.
\end{aligned} \tag{3.6}$$

The evaluation of the variation of $\mathcal{A}_1$ is more cumbersome, and requires explicit use of (2.9), (2.13), as well as of the equations of motion (2.4), (2.5). Our operator δ is nihilpotent, indeed, only on-shell.[3] In particular, (2.4) has to be used because δV_i^β, see (2.9), contains $D_i\Delta^\beta$, and an integration by parts gives then rise to $D_i V^{ia}$. Eventually, after a rather long calculation one obtains,

$$\begin{aligned}
\delta\mathcal{A}_1 &= -\frac{1}{2}\int d^6\sigma \left[3\sqrt{g}\, V_a^i V_i^C \Delta^\alpha \Delta^\beta \left(D_{[C} S_{\alpha\beta)}^a + \widetilde{T}_{[\alpha\beta}^D S_{DC)}^a\right)\right. \\
&\quad \left. - \frac{28}{6!}\varepsilon^{i_1\cdots i_6} V_{i_1}^{A_1}\cdots V_{i_6}^{A_6}\Delta^\alpha \Delta^\beta S_{[\alpha\beta}^c \widetilde{H}_{cA_6\cdots A_1)}\right].
\end{aligned} \tag{3.7}$$

The terms in the round brackets in (3.7) are graded antisymmetrized in $\alpha\beta C$, and in (3.7) there is a graded antisymmetrization over $\alpha\beta A_6\cdots A_1$. Eventually the total anomaly must satisfy $\delta\mathcal{A}_1 + \delta\mathcal{A}_2 = 0$, and one could argue that the terms proportional to $\varepsilon^{i_1\cdots i_6}$ and the ones proportional to $V_a^i V_i^C$ should vanish separately. However, there can be a migration between these two types of terms. Indeed, given an arbitrary two-superform S^γ with components,

$$S^\gamma = \frac{1}{2}E^B E^C S_{CB}^\gamma, \tag{3.8}$$

one can prove the identity,

$$\begin{aligned}
&\frac{28}{6!}\varepsilon^{i_1\cdots i_6} V_{i_1}^{A_1}\cdots V_{i_6}^{A_6}\Delta^\alpha \Delta^\beta S_{[\alpha\beta}^\gamma \widetilde{H}_{\gamma A_6\cdots A_1)} \\
&= 3\sqrt{g}\, V_a^i V_i^C \Delta^\alpha \Delta^\beta S_{[\alpha\beta}^D \widetilde{T}_{DC)}^a.
\end{aligned} \tag{3.9}$$

It can be seen that this is, actually, the unique way of transforming a term of the $\varepsilon^{i_1\cdots i_6}$-type into one of the $V_a^i V_i^C$-type. Adding and subtracting (3.9) from (3.7), (3.7), $\delta\mathcal{A} = \delta\mathcal{A}_1 + \delta\mathcal{A}_2$ can eventually be written as,

$$\begin{aligned}
\delta\mathcal{A} &= -\frac{1}{2}\int d^6\sigma \left[3\sqrt{g}\, V_a^i V_i^C \Delta^\alpha \Delta^\beta \left(D_{[C} S_{\alpha\beta)}^a\right.\right. \\
&\quad \left. + \widetilde{T}_{[\alpha\beta}^D S_{DC)}^a + S_{[\alpha\beta}^D \widetilde{T}_{DC)}^a\right) \\
&\quad - \frac{1}{6!}\varepsilon^{i_1\cdots i_6} V_{i_1}^{A_1}\cdots V_{i_6}^{A_6}\Delta^\alpha\Delta^\beta \\
&\quad \left. \times \left(28 S_{[\alpha\beta}^D \widetilde{H}_{DA_6\cdots A_1)} - (dW_7)_{\alpha\beta A_6\cdots A_1}\right)\right].
\end{aligned} \tag{3.10}$$

3.2. k-anomaly cancellation and Bianchi identities

At first order in β' the variation of the quantum effective action $\Gamma = I + \beta' I_1$ amounts to,

$$\delta\Gamma = \delta I + \mathcal{A},$$

where $\mathcal{A}$ is given in (3.4). The classical action I of the heterotic five-brane – that must be a completion of (2.1) – is unknown. But since it must be invariant once T^a and dB_6 satisfy the classical constraints (2.2), (2.3), we assume that δI is still given by (2.8). The requirement of anomaly cancellation $\delta\Gamma = 0$, demands then that the torsion constraints for T^a have to be corrected according to,

$$T_{\gamma B}^a = \widetilde{T}_{\gamma B}^a + S_{\gamma B}^a, \tag{3.11}$$

and that one has to impose the constraints $\widehat{H}_7 = \widetilde{H}_7$ to the modified curvature seven-superform,

$$\widehat{H}_7 \equiv dB_6 + W_7 \quad \Rightarrow \quad d\widehat{H}_7 = dW_7. \tag{3.12}$$

However, these new identifications for the constraints are consistent only if they satisfy the relevant Bianchi identities. But this is guaranteed – and this is the fundamental point – by the consistency condition $\delta\mathcal{A} = 0$, given (3.10). In fact, defining, see (3.8),

$$T_{\gamma B}^\alpha = \widetilde{T}_{\gamma B}^\alpha + S_{\gamma B}^\alpha, \tag{3.13}$$

and remembering that $\widetilde{T}_{BC}^A$ and $\widetilde{H}_7$ satisfy the Bianchi identities $DT^a = E^b R_b{}^a$ and $dH_7 = 0$ at zero order in β', up to first order in β' (3.10) can be recast into,

$$\delta\mathcal{A} = -\frac{1}{2}\int d^6\sigma\left[3\sqrt{g}V_a^i V_i^C \Delta^\alpha \Delta^\beta \left(DT^a - E^b R_b{}^a\right)_{\alpha\beta C}\right.$$
$$\left. -\frac{1}{6!}\varepsilon^{i_1\cdots i_6}V_{i_1}^{A_1}\cdots V_{i_6}^{A_6}\Delta^\alpha\Delta^\beta(d\widetilde{H}_7 - dW_7)_{\alpha\beta A_6\cdots A_1}\right]. \tag{3.14}$$

Notice that the term involving the curvature $R_b{}^a$ does, actually, drop out from this expression, since $R_A{}^B$ is Lie-algebra valued, i.e. $R_\beta{}^a = 0 = R_b{}^\alpha$, $R_{ab} = -R_{ba}$. To examine the content of the identity $\delta\mathcal{A} = 0$ we define the three-superform,

$$K^a \equiv DT^a - E^b R_b{}^a = \frac{1}{6}E^B E^C E^D K^a{}_{DCB}.$$

Choosing in the first line of (3.14) the index $C = \gamma$, the condition $\delta\mathcal{A} = 0$ implies first of all that K^a vanishes in the sector with three spinorial indices,

$$K^a{}_{\alpha\beta\gamma} = 0. \tag{3.15}$$

Choosing instead $C = c$, in the sector with two spinorial indices and one vector index we obtain,

$$K^{(ac)}{}_{\alpha\beta} = 0, \tag{3.16}$$

because $V_a^i V_{ic}$ is symmetric in a and c. But, as shown in [10], the conditions (3.15), (3.16) are precisely the ones under which *a consistent first order solution of the whole set of Bianchi identities $DT^A = E^B R_B{}^A$ can be found.*

Similarly, defining the closed eight-superform $K_8 \equiv d\widetilde{H}_7 - dW_7$, the vanishing of the second line of (3.14) implies that K_8 vanishes in all sectors with at least two spinorial indices,

$$K_{\alpha\beta A_1\cdots A_6} = 0. \tag{3.17}$$

Actually, in the sector with two spinorial and six vectorial indices the vanishing of the second line of (3.14) implies,

$$K_{\alpha\beta a_1\cdots a_6} = \left(\gamma^b\right)_{\alpha\beta}A_{ba_1\cdots a_6} + (\gamma_{b[a_1\cdots a_4})_{\alpha\beta}B^b{}_{a_5 a_6]},$$

where A and B are arbitrary completely antisymmetric tensors. However, the A-tensor can be eliminated by choosing appropriately the vectorial components $W_{a_1\cdots a_7}$ of W_7, and the B-tensor can be eliminated by choosing appropriately the vectorial components $S^a{}_{bc}$ of S^a, see (3.5). This is possible because both these components do not enter into (3.4). Since K_8 is a closed superform, (3.17) guarantees that K_8 vanishes identically, see, e.g., [19,24]. This means that also the Bianchi identity $d\widehat{H}_7 = dW_7$ in (3.12) admits a consistent solution, once one imposes on $\widehat{H}_7$ the constraints (2.3).

In conclusion, the consistency condition $\delta\mathcal{A} = 0$ ensures that the modified curvatures (3.11), (3.12), (3.13), necessary for anomaly cancellation, satisfy the required superspace Bianchi identities.

4. The *k*-anomaly of the heterotic five-brane and a coupled cohomology

In this section we show that anomalies of the type $\mathcal{A}_1$ as well as those of the type $\mathcal{A}_2$ are necessarily present in the heterotic five-brane quantum effective action $\Gamma = I + \beta' I_1$, and determine their form.

To this end we remember that Γ is also plagued by the gauge anomalies (1.2), in particular by the anomaly due to the *target-space induced* polynomial X_8, that represents a gauge-anomaly $\mathcal{A}_G$ associated to the group $G \equiv SO(1,9) \times SO(32)$. If we call the corresponding nihilpotent BRST operator δ_G, $\delta_G^2 = 0$, and use the standard transgression formalism $X_8 = d\omega_7$, $\delta_G\omega_7 = d\omega_6^1$, we have,

$$\delta_G\Gamma = \mathcal{A}_G = \beta'\int\omega_6^1. \tag{4.1}$$

But since *k*-transformations preserve G, we have also the operatorial identity $\delta\delta_G = -\delta_G\delta$. Inserting (3.1) and (4.1) in the identity $(\delta + \delta_G)^2\Gamma = 0$, one obtains then the "coupled cohomology problem",

$$\delta\mathcal{A} = 0, \qquad \delta_G\mathcal{A}_G = 0, \qquad \delta_G\mathcal{A} + \delta\mathcal{A}_G = 0. \tag{4.2}$$

While the first two relations are automatically satisfied, the third identity gives us new information about the *k*-anomaly $\mathcal{A}$. First of all, since $\mathcal{A}_G$ is not invariant under *k*-transformations, $\mathcal{A}$ is necessarily *non-vanishing*. Moreover, $\mathcal{A}_G$ is known and so we can elaborate the third identity in (4.2) to get a concrete information on $\mathcal{A}$. Since on (the pull-back of) a targetspace form Φ the *k*-transformations $\delta Z^M = \Delta^M$ act as the Lie-derivative $\delta\Phi = (i_\Delta d + di_\Delta)\Phi$, we have,

$$\delta\mathcal{A}_G = \beta'\int\delta\omega_6^1 = \beta'\int i_\Delta d\omega_6^1$$
$$= \beta'\int i_\Delta\delta_G\omega_7 = \delta_G\left(\beta'\int i_\Delta\omega_7\right). \tag{4.3}$$

The third relation in (4.2) becomes then $\delta_G(\mathcal{A} + \beta'\int i_\Delta\omega_7) = 0$, meaning that we have,

$$\mathcal{A} = -\beta'\int i_\Delta\omega_7 + \mathcal{A}_0, \tag{4.4}$$

where $\mathcal{A}_0$ is an ($SO(32)$ and Lorentz)-*invariant k*-anomaly. Comparing with the general form (3.4) and recalling that $\mathcal{A}_2 = -\int i_\Delta W_7$, we see that we must have,

$$W_7 = \beta'\omega_7 - Y_7, \tag{4.5}$$

where Y_7 is an *invariant* super-form of order β'. From (4.5) we conclude in particular that $\mathcal{A}_2$ is non-vanishing. Given this form of W_7 we may rewrite (3.12) as,

$$\widehat{H}_7 + Y_7 = dB_6 + \beta'\omega_7, \qquad d(\widehat{H}_7 + Y_7) = \beta' X_8. \tag{4.6}$$

We have thus derived (1.1), with the identification,

$$H_7 = \widehat{H}_7 + Y_7. \tag{4.7}$$

Since the constraints for $\widehat{H}_7$ are the classical ones given in (2.3), we see that Y_7 *represents the quantum corrections to the H_7-constraints.*

Given (4.6), also $\mathcal{A}_1$ must be non-vanishing. The reason for this is that, as shown in [25], as long as one imposes in $D = 10$, $N = 1$ supergravity the dimension-zero-torsion constraint $T^a_{\alpha\beta} = 2\gamma^a_{\alpha\beta}$, the *invariant* seven-superform is necessarily *closed*: this is in contrast to (4.6), that requires instead $d(\widehat{H}_7 + Y_7) \neq 0$. The validity of (4.6) implies thus that the dimension-zero-torsion in (3.11) gains necessarily a non-vanishing deformation $S^a_{\alpha\beta}$. Also $\mathcal{A}_1$ is, therefore, non-vanishing. Eventually, as we will see in the next section, the solution of (4.6) requires on top of this also a non-vanishing Y_7.

4.1. Comparison with a minimal superspace solution

Recently a first order "minimal" solution of (1.1), or equivalently (4.6), has been proposed in [19]. The author uses a different set of superspace constraints, but since different choices of constraints are related by (more or less complicated) field redefinitions, all our conclusions above hold true. We present now a third set of constraints – a slight "deformation" of the ones in [19] – for reasons that will be explained in a moment. We replace (2.2) and (2.3) by,

$$\widetilde{T}^a_{\alpha\beta} = 2\gamma^a_{\alpha\beta}, \qquad \widetilde{T}^a_{b\alpha} = 0,$$

$$\widetilde{T}^\gamma_{\alpha\beta} = 2\delta^\gamma_{(\alpha}\lambda_{\beta)} - (\gamma^a)_{\alpha\beta}(\gamma_a)^{\gamma\delta}\lambda_\delta, \tag{4.8}$$

$$\widetilde{H}_{\alpha\beta a_1\cdots a_5} = -2e^{-2\varphi}(\gamma_{a_1\cdots a_5})_{\alpha\beta},$$

$$\widetilde{H}_{\alpha a_1\cdots a_6} = -2e^{-2\varphi}(\gamma_{a_1\cdots a_6})_\alpha{}^\beta\lambda_\beta, \tag{4.9}$$

where φ is the dilaton, and $\lambda_\alpha = D_\alpha\varphi$. In this framework (2.1) is replaced by $I[Z] = \int(e^{-2\varphi}\sqrt{g}\,d^6\sigma + B_6)$, and the anomaly $\mathcal{A}_1$ in (3.4) by $\mathcal{A}_1 = -\int(e^{-2\varphi}\sqrt{g}\,V^i_a V^B_i \Delta^\gamma S^a_{\gamma B})d^6\sigma$, while the expression of $\mathcal{A}_2$ remains the same. The main advantage of this choice consists in the fact that at zero-order in β' it entails the "symmetric" parametrizations for the $SO(32)$- and $SO(1,9)$-curvatures F and R, see [25],

$$R_{\alpha\beta cd} = 0 = F_{\alpha\beta}, \qquad R_{a\alpha cd} = 2(\gamma_a)_{\alpha\beta}T^\beta_{cd},$$

$$F_{a\alpha} = 2(\gamma_a)_{\alpha\beta}\chi^\beta,$$

where T^α_{ab} is the gravitino field strength and χ^α is the gluino. As a consequence the components of X_8 in (1.3) with more than four spinor indices vanish. Eq. (4.6) is then automatically satisfied in the sectors with more than four spinor indices, if one sets the components of Y_7 with more than two spinor indices to zero. In the sector with four spinor indices (4.6) is, instead, non-trivial and reads,

$$\left(2\gamma^a_{\alpha\beta} + S^a_{\alpha\beta}\right)\left(-2e^{-2\varphi}(\gamma_{abcde})_{\gamma\delta} + Y_{\gamma\delta abcde}\right)$$
$$= 8\beta'(\gamma_b)_{\alpha\varphi}(\gamma_c)_{\beta\varepsilon}(\gamma_d)_{\gamma\rho}(\gamma_e)_{\delta\sigma}C^{\varphi\varepsilon\rho\sigma}, \tag{4.10}$$

where antisymmetrization over $bcde$ and symmetrization over $\alpha\beta\gamma\delta$ are understood, and we have inserted the parametrization (3.11) for $T^a_{\alpha\beta}$. The completely antisymmetric tensor $C^{\varphi\varepsilon\rho\sigma}$ is given by,

$$C^{\varphi\varepsilon\rho\sigma} = tr\left(T^\varphi T^\varepsilon T^\rho T^\sigma\right) + \frac{1}{4}tr\left(T^\varphi T^\varepsilon\right)tr\left(T^\rho T^\sigma\right)$$
$$- tr\left(T^\varphi T^\varepsilon\right)tr\left(\chi^\rho\chi^\sigma\right) + 8tr\left(\chi^\varphi\chi^\varepsilon\chi^\rho\chi^\sigma\right), \tag{4.11}$$

where T^α stands for the matrix valued spinor field T^α_{ab}, and antisymmetrization over $\varphi\varepsilon\rho\sigma$ is understood. Notice that at order zero in β' (4.10) is identically satisfied, while at first order in β' it amounts exactly to the vanishing of the second line in (3.10) in the sector proportional to four powers of V^α_i (remember that $dW_7 = \beta'X_8 - dY_7$). Due to complete antisymmetry the general decomposition of (4.11) is,

$$C^{\varphi\varepsilon\rho\sigma} = (\gamma_{a_1a_2a_3})^{[\varphi\varepsilon}(\gamma_{b_1b_2b_3})^{\rho\sigma]}$$
$$\times \left(A^{a_1a_2a_3b_1b_2|b_3} + \eta^{a_1b_1}B^{a_2a_3|b_2b_3}\right), \tag{4.12}$$

where the tensors $A^{abcde|f}$ and $B^{bc|ef}$ span respectively dimension 1050 and 770 irreducible representations of $SO(1,9)$. Using the identity,

$$(\gamma_{[bc})_{(\alpha}{}^{(\varphi}(\gamma_{de)})_{\beta)}{}^{\varepsilon)}$$
$$= -\frac{1}{12}\left[(\gamma^g)_{\alpha\beta}(\gamma_{gbcde})^{\varphi\varepsilon} + (\gamma^g)^{\varphi\varepsilon}(\gamma_{gbcde})_{\alpha\beta}\right]$$
$$+ \frac{1}{3}(\gamma_{bcde})_{(\alpha}{}^{(\varphi}\delta_{\beta)}{}^{\varepsilon)},$$

and performing the (a bit cumbersome) gamma matrix algebra, the right-hand side of (4.10) can then be written as,

$$(\gamma_b)_{\alpha\varphi}(\gamma_c)_{\beta\varepsilon}(\gamma_d)_{\gamma\rho}(\gamma_e)_{\delta\sigma}C^{\varphi\varepsilon\rho\sigma}$$
$$= \frac{1}{10}(\gamma_{bcdeg})_{\alpha\beta}(\gamma_{a_1a_2a_3a_4a_5})_{\gamma\delta}A^{a_1a_2a_3a_4a_5|g}$$

$$+ (\gamma^g)_{\alpha\beta}\left[36(\gamma_{bca_1a_2a_3})_{\gamma\delta}A_{deg}{}^{a_1a_2|a_3}\right.$$
$$+ 24(\gamma^h)_{\gamma\delta}A_{bcdeg|h} + 16(\gamma_{bcd}{}^{a_1a_2})_{\gamma\delta}B_{eg|a_1a_2}\big],$$

where (anti)symmetrizations are understood. Consequently (4.10) admits the solution,

$$S^a_{\alpha\beta} = -\frac{2\beta'}{5}e^{2\varphi}(\gamma_{b_1b_2b_3b_4b_5})_{\alpha\beta}A^{b_1b_2b_3b_4b_5|a}, \tag{4.13}$$

$$Y_{\alpha\beta a_1a_2a_3a_4a_5} = 16\beta'\left(15(\gamma^{cde}{}_{[a_1a_2})_{\alpha\beta}A_{a_3a_4a_5]cd|e}\right.$$
$$+ 6(\gamma^b)_{\alpha\beta}A_{a_1a_2a_3a_4a_5|b}$$
$$+ 10(\gamma^{bc}{}_{[a_1a_2a_3})_{\alpha\beta}B_{a_4a_5]|bc}\big). \tag{4.14}$$

Notice that the right-hand sides of these formulae are quartic in the fermions. Formula (4.13) has been derived for the first time in [19], where it has also been shown that the remaining components of S^a, S^α and Y_7 are consistently determined by the Bianchi identities (3.3), but their expressions are presumably much more complicated. Actually, the *general* solution of (4.10) allows also for terms in $S^a_{\alpha\beta}$ that belong to the irreducible representations $1_a \oplus 45 \oplus 54 \oplus 210_a$, and correspondingly for terms in $Y_{\alpha\beta a_1a_2a_3a_4a_5}$ that belong to $1_b \oplus 45 \oplus 54 \oplus 210_a \oplus 210_b$. However, all these tensors can be eliminated through field redefinitions of E^a, E^α, and B_6, see [27]. From the anomaly point of view these terms can be seen to correspond to trivial k-anomalies, that can be absorbed subtracting local counterterms from Γ.

If one trusts in this minimal solution, the present Letter makes the testable (once a consistent formulation has been found) prediction, that the heterotic five-brane carries the k-anomalies (3.4), where the forms W_7 and S^a are given by (4.5), (4.13) and (4.14), $S^a_{\alpha\beta}$ and $Y_{\alpha\beta a_1a_2a_3a_4a_5}$ being thus particular fourth order polynomials in χ^α and T^α_{ab}.

5. Concluding remarks

Whereas it is by no means clear that the low energy dynamics of the heterotic five-brane can be described by a local σ-model, there are at least two indications in favor of this assumption. The first is that the gauge anomalies cancel, and the second is that there exists a first order superspace solution of the associated Bianchi identity (1.1). What we have shown in this Letter is that a k-symmetric σ-model, together with its one-loop k-anomalies, are perfectly consistent with this solution. The other main reason in favor of such a model is obviously the general duality paradigm, that foresees five-branes as S-duals of strings.

With respect to the heterotic string σ-model [10], from the results of our analysis the following main differences between strings and five-branes arise. For the string the k-anomaly is again a sum of two terms like in (3.4), where W_7 is replaced by a three-form $W_3 = \alpha'\omega_3 - Y_3$, analogous to (4.5). This leads to $H_3 \equiv \widehat{H}_3 + Y_3 = dB_2 + \alpha'\omega_3$, entailing the modified Bianchi identity $dH_3 = \alpha'X_4$, where $d\omega_3 = X_4$. But in that case the three-form Y_3 vanished at first order in α', $Y_3 = o(\alpha'^2)$, in that no one-loop k-anomaly of this kind was revealed. The H_3-constraints received, therefore, no corrections at first order in α'. Similarly, in the case of the string also the anomaly $\mathcal{A}_1$ in (3.4) turned out to be zero at first order in α', $S^a = o(\alpha'^2)$, so that also the constraints for T^a received no first order corrections in α'. There exists, actually, a consistent all order solution of the Bianchi-identity $dH_3 = \alpha'X_4$ [26], that maintains the dimension-zero-constraint $T^a_{\alpha\beta} = 2\gamma^a_{\alpha\beta}$ at all orders in α'. In the case of the heterotic five-brane, instead, Y_7 as well as S^a are already non-vanishing at first order in β'.

The main open problem regarding the heterotic five-brane is the absence of a complete classical σ-model action, that de-

scribes also the heterotic sector. From the point of view of the present Letter another open problem regards the (cancellation of the) k-anomaly associated necessarily – via coupled cohomology as in (4.2) – to the factorized gauge-anomaly $(X_4 + \chi_4)Z_4$. But since this anomaly is not a purely "target-space" anomaly, its form can be investigated probably only once also the dynamics of the heterotic sector is explicitly known.

The method presented in this Letter can presumably be applied also to the $M5$-brane in $D = 11$. In that case the classical action is complicated by the presence of the self-interacting chiral two-form on the worldvolume, but this time, in addition to the anomaly polynomial $X_8^{(M5)}$ [28], also the complete k-invariant classical action is known [6,7]. This allows for a "true" analysis, and no longer for only a "conjectured" one, that could in particular shed new light on non-minimal $N = 1$, $D = 11$ supergravity [29].

Acknowledgements

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