Optimal advertising in a segmented market under different media choices

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Abstract

Segmentation is a core strategy in modern marketing but, to the best of our knowledge, it is not considered in most dynamic advertising models. In this paper we aim at filling such a gap and we present a dynamic advertising model which includes market segmentation. First, we model the goodwill evolution in a segmented market under the assumption that the decision maker may choose independently the advertising intensity directed to each different segment. Then, we assume that the decision maker has to use a single medium, which reaches several segments with different effectiveness. We obtain the explicit solutions of the relevant optimal control problems. These results permit us to compare the two different contexts and to obtain a preference index for advertising media.

Keywords Advertising; Segmentation; Media

1 Introduction

Segmentation is a core strategy in modern marketing and an active research topic (see e.g. the editorial in [12] and the chapters of marketing books [14, Chapter 9] and [13, Chapter 10]). A market may be segmented in several ways and at various degrees. The maximum degree corresponds to one-to-one or customised marketing. At the opposite end we find mass marketing, where the whole market is considered as a unique segment (see [14, Chapter 9]). For the sake of simplicity, mass marketing is implicitly assumed in most dynamic advertising models (see [20], [4] and the references therein), but the hypothesis that the customers are not differentiated is often not verified empirically. In this paper we aim at filling such a gap and present a dynamic advertising model which includes market segmentation, following the lines of [1], [2]. The context of our model is different from the one of [1], [2], because we consider product sales and profits as distributed over time, whereas in the cited papers the profit only depends on the system state at a unique, terminal, time. We focus on nonpersonal communication channels, and in particular to the media, which may have various specifications, i.e. print, network, electronic, or display (see e.g. [13, p.576]). Actually, as a by-product of practical importance, we prove that the introduction of the segmentation hypothesis in a dynamic advertising model is relevant in the medium selection problem.

The starting point of our analysis is the classic Nerlove-Arrow's [18] advertising capital model. Even if the model is almost 50 years old, it is still a basic element of a variety of advertising models which are used both in empirical studies, through its discrete time version which is known from [15], [22] (see e.g. [16]), and in theoretical ones, as documented e.g. in [11, Section 3.5], [21], and [17].

Given a segmented market, we assume that the growth of goodwill (or awareness) in each market segment depends linearly on the advertising effort, while goodwill decays due to forgetting of the advertised brand. Hence, in order to describe the goodwill evolution in the market, we need one ordinary differential equation for each market segment. The ideal setting should allow the firm to advertise toward each segment independently, in order to use the most suitable marketing strategies to meet the different characteristics (e.g. demographic, geographical, behavioral, ... (see e.g. [14, Chapter 9]) of the consumers belonging to each segment. Nevertheless, in practice, the decision maker is bound to use some advertising media as television, newspapers, web sites, which hit different market segments with the same message, but with varying effectiveness. Hence, while selecting an advertising medium, a decision maker has to consider that the impact of the advertising action may be different for each segment.

The paper is organised as follows. In Section 2 we model the goodwill evolution in a segmented market under the assumption that the decision maker may choose independently the advertising intensity directed to each different segment, by choosing the activation levels of some segment-specific media; then we characterise the profit maximizing activation levels. In Section 3 we assume that the decision maker has to use a single medium which reaches several segments with different effectiveness; we characterise the optimal activation intensity of the medium. In Section 4, using the optimal value found in Section 3, we obtain a preference index to choose, among different advertising media, the most profitable one. Finally, in Section 5 we compare the solutions for a one-to-one advertising system to the one for a single medium advertising system. We propose some scenarios in order to understand when one-to-one advertising should be preferred to one-medium advertising. In order to make the paper more readable, all mathematical computations are postponed to the Appendix.

2 Segment-specific media

Let the consumer population be partitioned into groups (segments), each one specified by the value $a \in A$ of a suitable parameter *(segmentation attribute)*. Here A is a finite set. Typically, a small number of segments – two or three – is considered in practice. Let $G_a(t)$ represent the stock of goodwill of the product at time t, for the (consumers in the) a segment. We refer to the definition of *goodwill* given by Nerlove and Arrow [18] to describe the variable which summarises the effects of present and past advertising on the demand; the goodwill needs an advertising effort to increase, while it is subject to a spontaneous decay. Here we assume that the goodwill evolution satisfies the set of independent ordinary differential equations

$$\dot{G}_a(t) = w_a(t) - \delta\, G_a(t), \qquad a \in A, \tag{1}$$

where $\delta > 0$ represents the goodwill depreciation rate in all segments and $w_a(t)$ is the effective advertising intensity at time t directed to the segment a. We identify $w_a(t)$ with the *activation level* of a medium specifically devoted to the communication toward the segment a. Moreover, we assume we know the goodwill level at the initial time for all segments,

$$G_a(0) = \alpha_a \geq 0. \tag{2}$$

For each fixed value of the parameter $a \in A$, i.e. for each segment, the dynamics of the goodwill given by the linear equation (1) is essentially the same as the one proposed in [18]. Hence we are assuming that the decider may control an advertising process with such a high segment-resolution as to be able to reach each segment with any desired intensity. In order to stress such hypothetical ability of the decision maker to choose the advertising intensities w_a, $a \in A$, independently, we call one-to-one advertising the situation under study.

As in the original Nerlove–Arrow's model we want to find an advertising flow which maximises the profit of the firm in the long run. Here we consider a vector control function, the segment-distributed advertising intensity with components $w_a(t) \geq 0$, $a \in A$, and want to maximise the discounted profit functional

$$J \;=\; \int_0^\infty \sum_{a\in A} \left[r\beta\sigma_a G_a(t) - \frac{k_a}{2}\, w_a^2(t) \right] e^{-\rho t}\, \mathrm{d}t\,, \tag{3}$$

where r, β, $\rho > 0$, and k_a, $w_a > 0$, for all $a \in A$. In particular, ρ is the discount factor; r is the marginal profit, gross of advertising costs; $\beta\sigma_a$ is the marginal demand of goodwill from segment a; and $k_a w_a^2/2$ is the cost intensity associated with the advertising intensity w_a, i.e. with the activation level w_a of the medium oriented to the segment a. Moreover, we assume here that $\sum_{a\in A} \sigma_a = 1$, so that σ_a represents the percentage of sales given by the market segment a in the special situation in which the goodwill value is the same for all segments, $G_a \equiv \hat{G}$. We observe that we might also let $\sigma_a = 0$ for some segment a, but the goodwill of those segments would be irrelevant for the firm profit, then the positivity assumption on $(\sigma_a)_{a\in A}$ is not restrictive.

The above assumptions make the model particularly tractable and allow us to obtain closed form solutions for the segment-dependent profit problem. Moreover, such assumptions are quite reasonable and commonplace in the marketing literature.

In fact, on one hand, the assumption of linear dependence of sales on goodwill is equivalent to that of a linear response function to advertising (see e.g. [3, p.172, footnote 6]), and it is in agreement with the more general and widely used assumption that the product demand is an increasing and concave function of goodwill (see e.g. [18], [6], [20]).

On the other hand, the assumption that the advertising cost is an increasing and convex function of advertising intensity, i.e. of activation level, is in agreement with [5]; more specifically, the simple quadratic form $k_a w_a^2/2$ is frequently used in marketing models (see e.g. [16]), in particular those developed in the framework of the differential games theory (see e.g. [3], [9], [10]), [7], [8])).

Under the present assumptions the objective functional is additive in each segment, and the decision maker can choose the advertising intensity in each segment independently. Hence, in order to solve the profit maximisation problem, we need to solve more simply one profit maximisation problem for each single segment. From the computations described in the Appendix, with the substitutions $\phi = r\beta\sigma_a$, $\psi = k_a$, $\xi = 1$, and $\omega = \alpha_a$, we obtain that the optimal activation level for each segment is constant:

$$w_a^*(t) = \frac{r\beta}{(\delta+\rho)} \frac{\sigma_a}{k_a}\,. \tag{4}$$

These results are well known (they represent an application of the golden rule: "marginal costs equal to marginal revenue") and are in agreement with those described in [20] and [4]. The optimal value of (3) is the sum of all optimal values of the single segment problems and we obtain:

$$J^* = \frac{r\beta\bar{\alpha}}{\delta+\rho} + \frac{r^2\beta^2}{2\rho\,(\delta+\rho)^2} \sum_{a\in A} \frac{\sigma_a^2}{k_a}\,, \tag{5}$$

where

$$\bar{\alpha} = \sum_{a\in A} \sigma_a\alpha_a \geq 0\,. \tag{6}$$

3 A single medium

Actually, it may be difficult and perhaps expensive to plan an advertising campaign using a set of media which hit independently each segment. In practice, often the decision maker has to use a medium which reaches several segments with segment-variable effectiveness. Let us consider the decision maker using a single advertising medium and let $u(t)$ be the activation level of the advertising process. We assume that the goodwill evolution in the different segments is driven by the medium activation level $u(t)$ (the control function) according to the motion equations and initial conditions (1) and (2), where the effective advertising intensities are now

$$w_a(t) = \gamma_a u(t)\,, \qquad a \in A. \tag{7}$$

We assume that $\gamma_a \geq 0$, $a \in A$ and $\sum_{a\in A} \gamma_a > 0$. We call $(\gamma_a)_{a\in A}$ the medium *(segment-)spectrum*. Its components, γ_a, $a \in A$, provide the different effectiveness of the advertising medium on the market segments.

The segment-dependent profit problem requires to find a medium activation level function $u(t) \geq 0$, in order to maximise the firm discounted profit given by the functional

$$\mathbb{J} \;=\; \int_0^\infty \left[r\beta \sum_{a\in A} \sigma_a G_a\,(t) - \frac{k}{2}\,u^2(t) \right] e^{-\rho t}\,\mathrm{d}t\,, \tag{8}$$

where $ku^2/2$ is the advertising cost intensity associated with the medium activation level u, and the cost factor k is positive, $k > 0$. Precisely, k is the marginal advertising cost intensity at the unit activation level.

We call *aggregate goodwill* the weighted mean of the segment goodwill values $G_a\,(t)$, with weights $\sigma_a\,,\ a \in A$:

$$\bar{G}(t) \;=\; \sum_{a\in A} \sigma_a G_a\,(t)\;. \tag{9}$$

It is, up to the constant factor β, the aggregate demand for the good (the assumption that sales depend linearly on the goodwill is essential in this step). In view of the definition (9), we can write the objective functional (8) more simply as

$$\mathbb{J} \;=\; \int_0^\infty \left[r\beta\bar{G}(t) - \frac{k}{2}\,u^2(t) \right] e^{-\rho t}\,\mathrm{d}t\,. \tag{10}$$

Moreover, from equations (1), (7), and (2) we obtain that

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{G}(t) = \bar{\gamma}u\,(t) - \delta\bar{G}(t), \qquad \bar{G}\,(0) = \bar{\alpha}\,, \tag{11}$$

where

$$\bar{\gamma} \;=\; \sum_{a\in A} \sigma_a \gamma_a\,. \tag{12}$$

Using the results from the Appendix (with $\phi = r\beta$, $\psi = k$, $\xi = \bar{\gamma}$, $\omega = \alpha_a$) we obtain that there exists a unique and constant optimal activation level for the medium:

$$u^*(t) = \frac{r\beta}{(\delta+\rho)}\,\frac{\bar{\gamma}}{k}\,. \tag{13}$$

Moreover, the optimal value of the objective functional is

$$\mathbb{J}^* \;=\; \frac{r\beta\bar{\alpha}}{\delta+\rho} \;+\; \frac{r^2\beta^2}{2\rho(\delta+\rho)^2}\,\frac{\bar{\gamma}^2}{k}\,. \tag{14}$$

We observe that $\bar{\gamma} > 0$, because of our assumptions, so that $u^*(t) > 0$ too: it is optimal to advertise at all times and the optimal policy is even. In fact, the optimal medium activation level is proportional, with a positive factor, to the medium *mean target coverage* $\bar{\gamma}$. The optimal profit is a strictly increasing function of $\bar{\gamma}$ and of the demand factor β, whereas it is a strictly decreasing function of the decay parameter δ, the discount factor ρ, and the activation cost factor k.

4 Advertising medium selection problem

Let us assume that a decision maker has some media available, each one with a segment spectrum which is assessed as acceptable with respect to the target spectrum. Sometimes it makes sense for the decision maker to look for only one advertising medium which can give the maximum profit among those single media. This is particularly true in the case that the media segment spectra do not differ much from one another. In other words, we want to answer the question: "given some different advertising media, which one is the best?" A problem of this kind has been discussed in [2] for the special situation af a new product introduction. The results of the previous section provide us with a precise answer in the special case of goodwill-linear demand and quadratic activation costs.

We are concerned essentially for the comparison of two advertising media. Let a "Newspaper" medium and a "Television" medium be characterised by the activation cost parameters k_N, k_T, and the segment-spectra $\left(\gamma_a^N\right)_{a\in A}$, $\left(\gamma_a^T\right)_{a\in A}$. From (14) we obtain the associated optimal profits $\mathbb{J}_N^*$, $\mathbb{J}_T^*$, and we observe that

$$\mathbb{J}_N^* \geq \mathbb{J}_T^* \quad \Leftrightarrow \quad \frac{\left(\sum_{a\in A} \sigma_a \gamma_a^N\right)^2}{k_N} \geq \frac{\left(\sum_{a\in A} \sigma_a \gamma_a^T\right)^2}{k_T}\,. \tag{15}$$

We have obtained that the ratio of the squared mean target coverage to the cost factor may be used as a *medium preference index* to solve the medium selection problem. The result is formally rather the same as that presented in [2]: this is not surprising, but it needed to be proved, as here the advertising problem is different from the one in [2].

5 One-medium vs one-to-one advertising

In view of the analysis developed in Sections 2 and 3, a question naturally arises: "when one-to-one advertising should be preferred to one-medium advertising?" In order to answer it, we need to compare the performances of optimal advertising policies for a set of single segment media with those for one medium which has a wide spectrum.

Let us denote by J^* the optimal value (5) of the one-to-one advertising case (i.e. using the set of single segment media), and by $\mathbb{J}^*$ the optimal value (14) of the one-medium advertising case (i.e. using a wide spectrum medium), then

$$J^* \geq \mathbb{J}^* \quad \Leftrightarrow \quad \sum_{a\in A} \frac{\sigma_a^2}{k_a} \;\geq\; \frac{\left(\sum_{a\in A} \sigma_a \gamma_a\right)^2}{k} \;=\; \frac{\bar{\gamma}^2}{k}\,. \tag{16}$$

As a first comparison scenario, let us assume that the *(segment-)spectrum* of the single medium is constant over all the segments, and precisely that $\gamma_a = 1$ for all $a \in A$. Under this hypothesis, it makes sense to assume further that $k_a = k/n$ for all $a \in A$. The comparison rationale is that both the wide spectrum medium and the above set of single segment media allow the decision maker to obtain a unit effective advertising intensity $w_a = 1$, for each segment a, at the same cost intensity $k/2$. In this case, the wide spectrum medium mean target coverage is $\bar{\gamma} = 1$, and the r.h.s. inequality in (16) is

$$\frac{n}{k} \sum_{a \in A} \sigma_a^2 \;\geq\; \frac{1}{k}\,, \tag{17}$$

which is always true, because $\sum_{a \in A} \sigma_a^2 - 1/n \;=\; \sum_{a \in A} (\sigma_a - 1/n)^2 \;\geq\; 0$. Moreover the inequality is strict if $\sigma_a \neq 1/n$ for some a. In this scenario, one-to-one advertising performs better that one-medium advertising.

As a second and more general scenario, let us assume that the wide spectrum advertising medium has the segment spectrum γ_a, $a \in A$, and that all the single segment media of the one-to-one advertising system have the same cost factor, $k_a \;=\; k'$, $a \in A$. Then the one-to-one advertising process has a better performance than the one-medium advertising process if and only if the cost factor k' is not too large:

$$k' \;\leq\; \frac{\sum_{a \in A} \sigma_a^2}{\bar{\gamma}^2}\, k\,. \tag{18}$$

The inequality (18) provides a threshold (upper bound) for the cost parameters of single segment media, to be proposed in the advertising tools market as an alternative to a wide spectrum medium already available. The threshold is proportional to the cost parameter of the wide spectrum medium, and it is as smaller as its mean target coverage is larger.

As a third scenario, let the (available) single segment media of the one-to-one advertising system have cost factors k_a, $a \in A$, possibly variable with a, and let an alternative medium be available too, with segment spectrum γ_a, $a \in A$, and cost factor k. Then the one-medium advertising process has a better performance than the one-to-one advertising process if and only if the former cost factor k is not too large:

$$k \;\leq\; \bar{\gamma}^2 \left(\sum_{a \in A} \frac{\sigma_a^2}{k_a} \right)^{-1}\,. \tag{19}$$

The inequality (19) provides a threshold (upper bound) for the cost parameter of the wide spectrum medium, to be proposed in the advertising tools market as an alternative to a set of single segment media already available. In particular we observe that the threshold is an increasing function both of the cost parameters of single segment media and of the mean target coverage of the wide spectrum medium.

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Appendix: The basic optimal control problem

The data of the optimization problem are

$$\begin{aligned} J &= \int_0^\infty e^{-\rho t} \cdot \left\{ \phi \cdot x(t) - \frac{\psi}{2} \cdot u^2(t) \right\} \mathrm{d}t, \\ \dot{x}(t) &= \xi \cdot u(t) - \delta \cdot x(t), \\ x(0) &= \omega \geq 0. \end{aligned}$$

We observe that the hypotheses of the necessary conditions theorem for the infinite horizon optimal control problem [19, p.244, Th.16] are satisfied. The current value Hamiltonian is

$$H^c(x, u, \lambda) = \phi \cdot x - \psi \cdot u^2/2 + \lambda \cdot (\xi \cdot u - \delta \cdot x).$$

Hence the optimal control is

$$u^* = \lambda \cdot \xi/\psi.$$

After integration we obtain that the adjoint variable is

$$\lambda(t) = \phi/(\delta + \rho).$$

Therefore the optimal goodwill evolution is

$$x^*(t) = \omega e^{-\delta t} + \left(1 - e^{-\delta t}\right) \phi \xi^2 / \delta \psi (\delta + \rho),$$

and the optimal value is

$$J = \frac{\phi \omega}{\rho + \delta} + \frac{\phi^2 \xi^2}{2 \psi \rho (\delta + \rho)^2}.$$

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