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EXPERIMENTAL AND THEORETICAL ANALYSIS ON THE STICK-SLIP PROBLEM

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SUMMARY

One of the effect of the presence of the friction forces in mechanical systems is the stick-slip phenomenon. A numerical-experimental study of the vibration response of a single degree of freedom system, with Coulomb friction damping non-linearity, is described. Particular emphasis is placed upon the numerical description of the friction force. The mass behaviour in the range frequency in which two to ten stops per cycle occur are investigated and discussed. The comparision of the experimental results to those obtained from the simulations shows a good agreement.

1. INTRODUCTION

The friction phenomenon plays an important role in the dynamic of several mechanical systems, for example servomechanisms, control systems, machine tool slideway, position mechanism.

In a lot of cases the system behavior could be described adequately by the simple model of a single-degree-of-freedom system with a non-linear dry friction damper. The non-linearity is due, firstly, to the discontinuity of the damping force near the rest position and, secondly, to the dependence of the friction coefficient to the velocity of relative moving parts with contact.

One of the main effect observed, as a result of friction presence, is the "stick-slip phenomenon" which consists in an irregular motion with two or more stops per vibration cycle.

In common practice simple solutions of the governing equation (of forced vibrations of single-degree-of-freedom system without stops under combined viscous and Coulomb damping) rely upon the linearization of the describing equation by considering the energy dissipation and defining the "equivalent viscous damping" [1,2]. This approach gives an approximate solution valid only near the resonance of the system and when the mass moves continuously without stops.

Closed form analytical solutions of the equation of the motion are found for the systems shown in figure 1, with the assumption of constant and equal values of static and dynamic friction.

The systems analyzed have little differences. The system studied by Den Hartog [3] is excited harmonically by an external force applied on the mass with friction forces acting between it and the ground. He found out the condition for the stick-slip phenomenon with two stops per cycle.

Levitan [4] analyzed the continuous motion by using a Fourier series approximation for the Coulomb friction force.

Hundal [5] treated the continuous and two stops motion of a system, with harmonic displacement of the base, and the friction force acting between the mass and the ground.

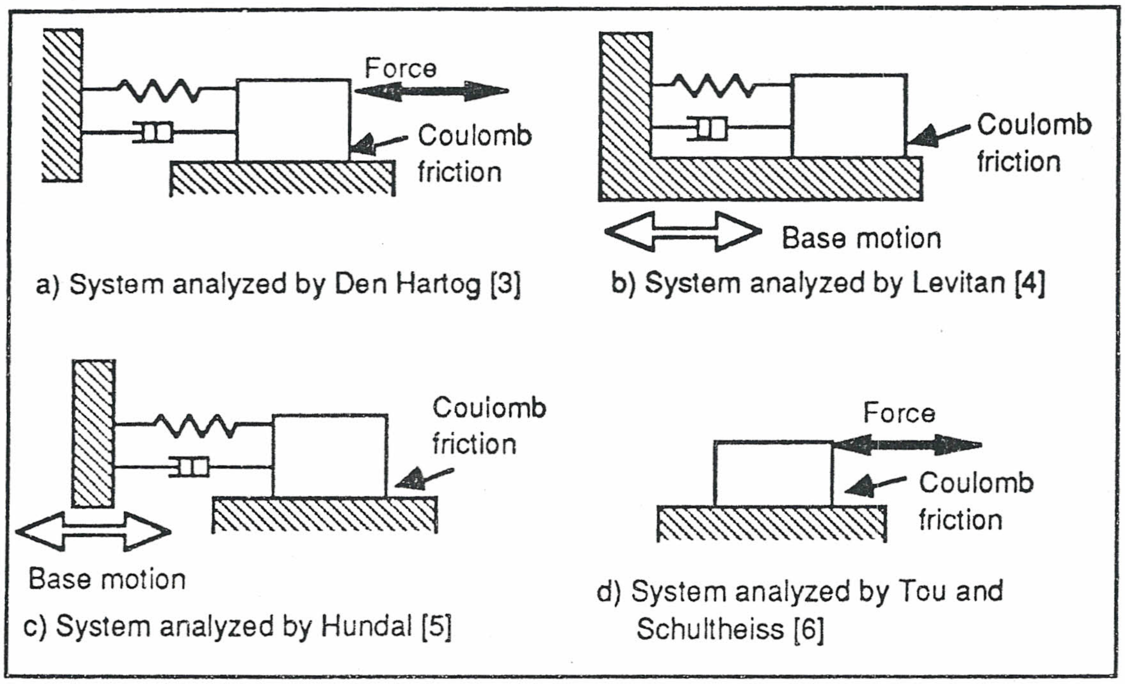


Fig. 1 System analyzed by different authors

The fourth model represented in figure 1 refers to a body, without spring and viscous damper, driven by an external harmonic force for which some interesting features of the motion are provided by Ton and Shultheiss [6]. They considered both static and dynamic friction and found a motion with stops if $2\pi > R/F > 2/\pi$, where R is the static friction force and F is the external sinusoidal one. This model is of interest because it is applicable for vibrating systems exicited in high frequency range where the spring force loses its importance.

All the analytical solutions refer to only continuous and two stops motion excluding the multi-stops phenomena.

Moreover, if different values between static and dynamic friction are considered, the problem becomes more complex and consequently a numerical approach is necessary.

A numerical analysis was performed by Marchis and Vatta [7] on the Den Hartog system by considering different values for static and dynamic friction. They showed that, in the frequency domain, the non-linearities effects are represented, in the response of the system, by higher order harmonics.

Marui and Kato [8] investigated analitically and experimentally the Hundal system. The linear forced vibration equation (between two continuous stops) are solved through the concept of "stopping region of motion". They found out the behaviour of the system depended on three non-dimensional parameters: frequency ratio, friction and viscous non-dimensional ratio.

The purpose of this paper is to investigate both numerically and experimentally the stick-slip phenomenon in the frequency range where multi-stops occur.

The experiments were carried out on a base excited system like Hundal's model. The numerical analysis was performed by taking into account the measured static and dynamic friction values. The frictional model was based on the hypothesis of local elastic deformation in the contact-points of the rubbing surfaces.

2. NUMERICAL TECHNIQUE

The analyzed system is shown in figure 2 and consists of a mass-spring-damper system, with one degreee of freedom, excited by the base motion, where :

- m is the global moving mass,
- k is the stiffness of one spring,
- c is the damping of the system as results from the measurements,
- x is the mass displacement,
- y is the base displacement given by $y = y_0 \sin(\omega t)$

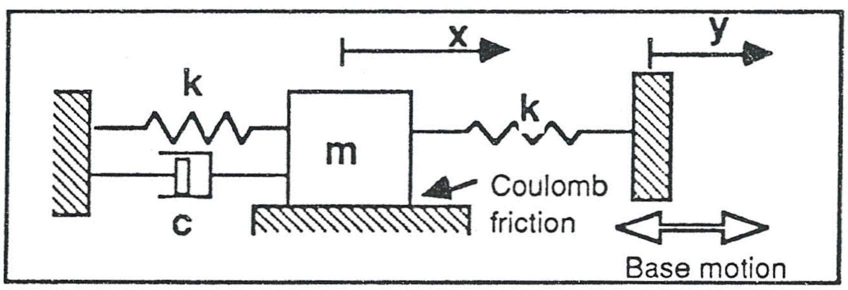


Fig. 2 System with harmonic excitation of the base

The resulting equation of the motion is:

$$m\ddot{x} + cx' + 2kx = -k y_0 \sin \omega t + F_{friction} \tag{1}$$

The possibility of integrating this equation depends on the knowledge of the friction force.

The aim was to use a model of friction force without discontinuities for null speed and with a continuous variation of the coefficient depending on the speed. The expression used for the friction force is:

$$F_{friction} = -\text{sign}(x') \left[(A/e^{B|x'|}) + C + D|x'| \right] \tag{2}$$

where x' is the relative speed between the mass and the guide surface and A,B,C,D are four constants related with the values of static and dynamic forces as follows

A + C = coefficient of static friction;
C = coefficient of dynamic friction (asintotic);

D = coefficient related to the increasing of force over a speed limit value;

B = coefficient related to the slope of the curve.

The use of a numeric method was necessary at this point. Some tests have been done in order to determine the most satisfyng method of integration: the Runge-Kutta of the fourth order, a predictor-corrector and a predictor-corrector method modified with iteraction of the prediction and correction coefficients.

From these tests it has been seen that, under the same value of time integration step, the three methods give results in good agreement each other, but the second and the third methods need more computation time. For a little number of steps per cycle the predictor-corrector methods and in particular the third method are more stable. Nevertheless it was decided to use the Runge-Kutta method because it achieves the desired degree of accuracy in a faster way.

The problem of describing the friction force can be investigated using the control of the difference between the absolute value of the friction force and the sum of the spring and dumper forces, as done by Marchis and Vatta [7].

An other way to resolve it, is by assuming the hipothesis of considering the friction force, near the rest position, as resulting by a local deformation.

This deformation can be considered of plastic model (theory of Bowden and Tabor[9]) or of elastic model (theory of Archard[10]).

In this work the following assumptions have been done in order to model the friction force:

1) - there is an elastic local deformation , modelled by a local spring, which acts only if the resulting force is less than the static friction force;

2) - out of this range the friction force acts as previously cleared.

The local stiffness K_l was adjusted in order to match the experimental results; a value equal to fifty times the spring stiffness was assumed. The elastic deformation force is calculated by:

$$F_l = k_l\,(x - x_{stop}) \tag{3}$$

where x_{stop} is the previous rest position determined numerically as the first value of the displacement for which the absolute value of the relative speed **x'** is pratically null (less than 10^{-6} m/s).

At this point only one control was necessary:

$$\begin{array}{llll} \text{if} & k_l(x-x_{stop})<A+C & \text{then} & F_{fric} = F_l = k_l\,(x - x_{stop}) \\ \text{if} & k_l(x-x_{stop})>A+C & \text{then} & F_{fric} = -\mathrm{sign}(x')\,[\,(A/e^{B|x'|}) + C+D|x'|\,] \end{array} \tag{4}$$

The experimental results have permitted to find out the most appropriate value of this four constants (A,B,C,D), which gave simulations in good agreement with the experimental measurements.

3. EXPERIMENTAL APPARATUS AND PROCEDURE

A scheme of the experimental apparatus purposely constructed for this work is shown in fig. 3; it is composed by three foundamental parts: the vibration exciter; the mechanical device to investigate the stick-slip phenomenon; the measurement system.

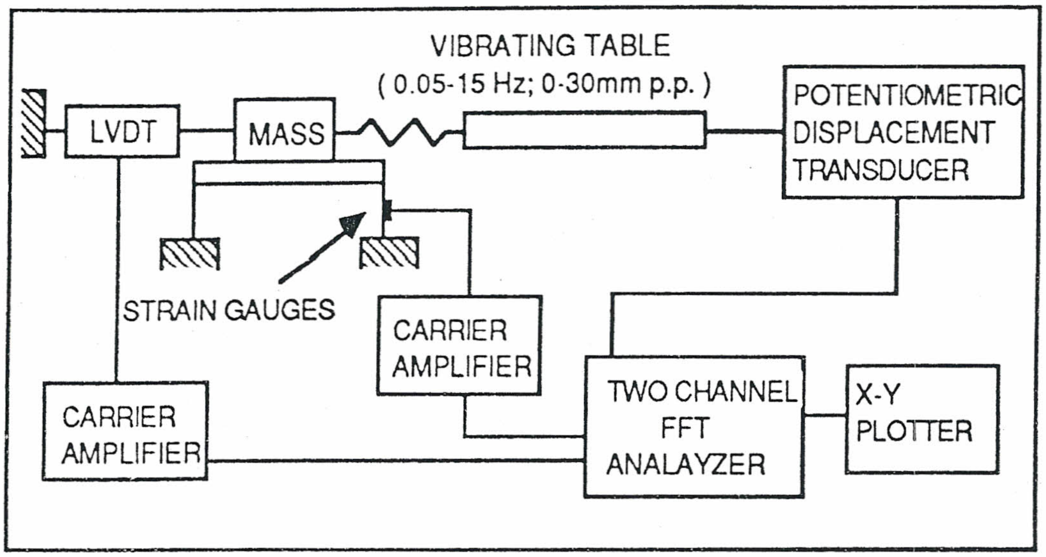


Fig. 3 Experimental apparatus

The vibration exciter is composed by:
- an oleodinamic motor-reducer which output shaft can rotate in the two opposite directions from zero to 900 r.p.m. ;
- a reduction-gear having an overall gear ratio of 1/19.3
- a Scotch mechanism to transform the rotating motion in a directional sinusoidal one; between this device and the reduction-gear a fly-wheel is mounted to uniform motion;
- a vibrating table drived by the Scotch mechanism.

This exciter can produce vibrations with amplitude until 30 mm peak to peak, in the frequency range from 0.05 to 15 Hz; the motion can be considered sinusoidal because it was verified that the amplitude of higher harmonics are less than 0.3 % with respect to the fondamental.

The mass m is connected with the vibrating table and with the frame by means of two springs; the friction and elastic forces lie on the same plane.

The viscous damping coefficient was determined by analyzing the free vibrations of the system without friction force.

The mass moves on a table connected to the frame by three cantilevers whose deformations, related to the friction force, were measured by strain gauges applied on their surfaces. The natural frequency of the friction measuring device is 10 times the natural frequency of the vibrating body.

The masurement system is constituted by:
- a potentiometric displacement transducer to measure;
 the exciting displacemant of the vibrating table;
- a LVDT displacement transducer to measure the displacement of the mass m;
- two carrier amplifiers: one for displacement signal, and one for frictional force signal;
- a low-pass filter for frictional force signal;
- a two channel FFT analyzer;
- a X-Y recorder.

First of all the main charateristics of the mechanical system, such as natural frequency of mass-spring system and damping ratio, were measured. The proofs were performed in order to evaluate the resulting motion of the mass and the frictional force for different exciting frequencies and displacements of exciting base. From these records it was determined the amplitude of mass displacement, the numbers of stops in a cycle and the amplitude of friction force.

The experimental analysis was performed in the following frequency range: from 0.08 Hz to a value just a little under the natural frequency of mass-spring system .

The grinded contact surfaces between the mass and the guide are made of steel with a maximum roughness of about 0.5 µm. The surfaces were lubrificated by means of a low-viscous mineral oil.

The physical characteristics of this experimental device are summarized in Table 1.

Table 1. Experimental device characteristics.

Vibrating mass	m=2.38 Kg	Spring stiffness k=1986 N/m
Natural frequency	ν = 4.6 Hz	Log. decrement = 0.207
Damping ratio	ζ = 4.6 Hz	(without dry friction)

4. RESULTS AND DISCUSSION

A typical example of recordings is shown in fig. 4 and 5, which represent the time response history in steady state conditions. In fig. 4 are plotted respectively the mass displacement and the base motion while fig. 5 shows the mass velocity and the friction force.

There is a little time shifting between the representations of fig. 4 and fig. 5 caused by the use of two different FFT (two channels) analyzers at the same time.

The velocity wave form was obtained using the derivation option of the analyzer.

The friction force wave form reachs its maximum at the corresponding starting point of the mass (at the end of dead band in the velocity curve). During the motion it is possible to see a decrease of the friction force value. The real static friction peak is higher than the one visible in fig. 5; the reasons are to be found in the time signal sampling of the signals of the analyzer. The variation of friction force is so quick that the instrument, with the selected full scale, cannot record it exactly. To avoid this problem it was necessary to low the exciting frequency and to use the zoom option.

The resulting mass displacement spectrum shows the presence of superior harmonic frequencies where the third and fifth spectral lines are of the most importance (see fig. 6).

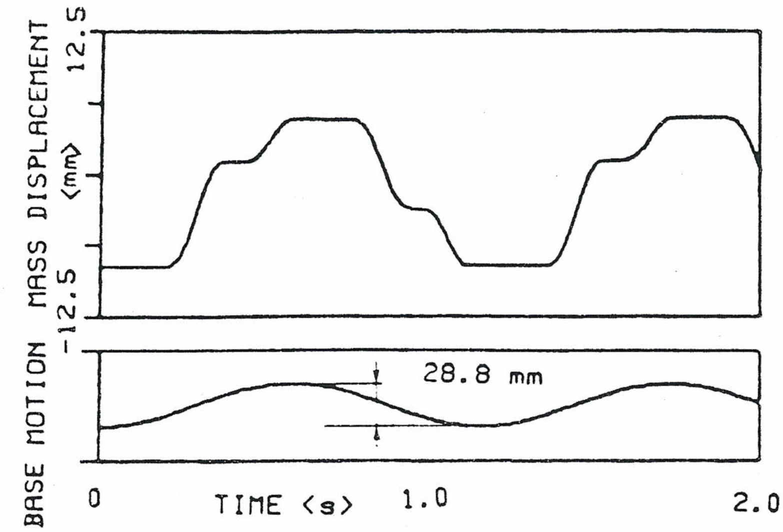


Fig. 4 Sample of recorded test data on mass displacement and base motion.

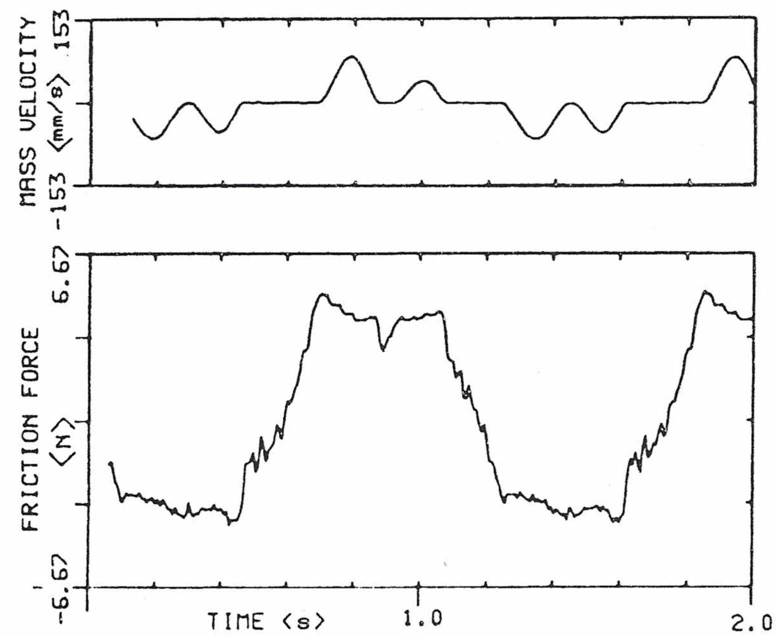


Fig. 5 Sample of recorded test data on mass velocity and friction force.

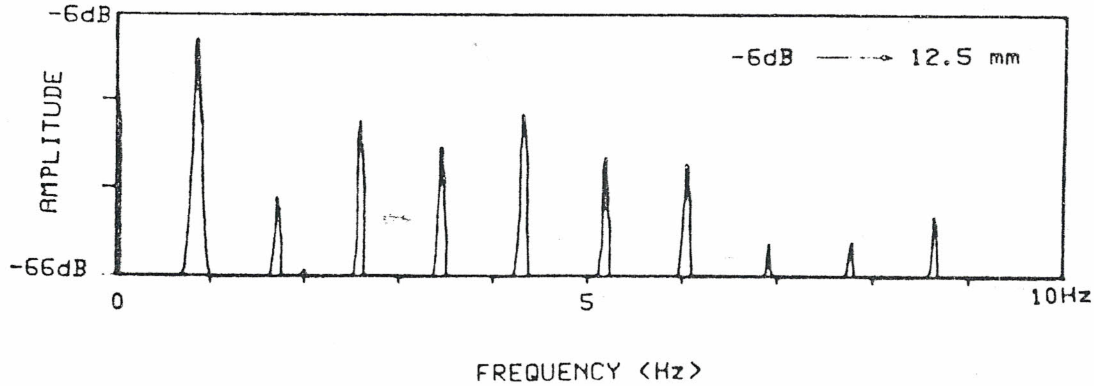


Fig. 6 Mass displacement spectrum of signal represented in fig. 4.

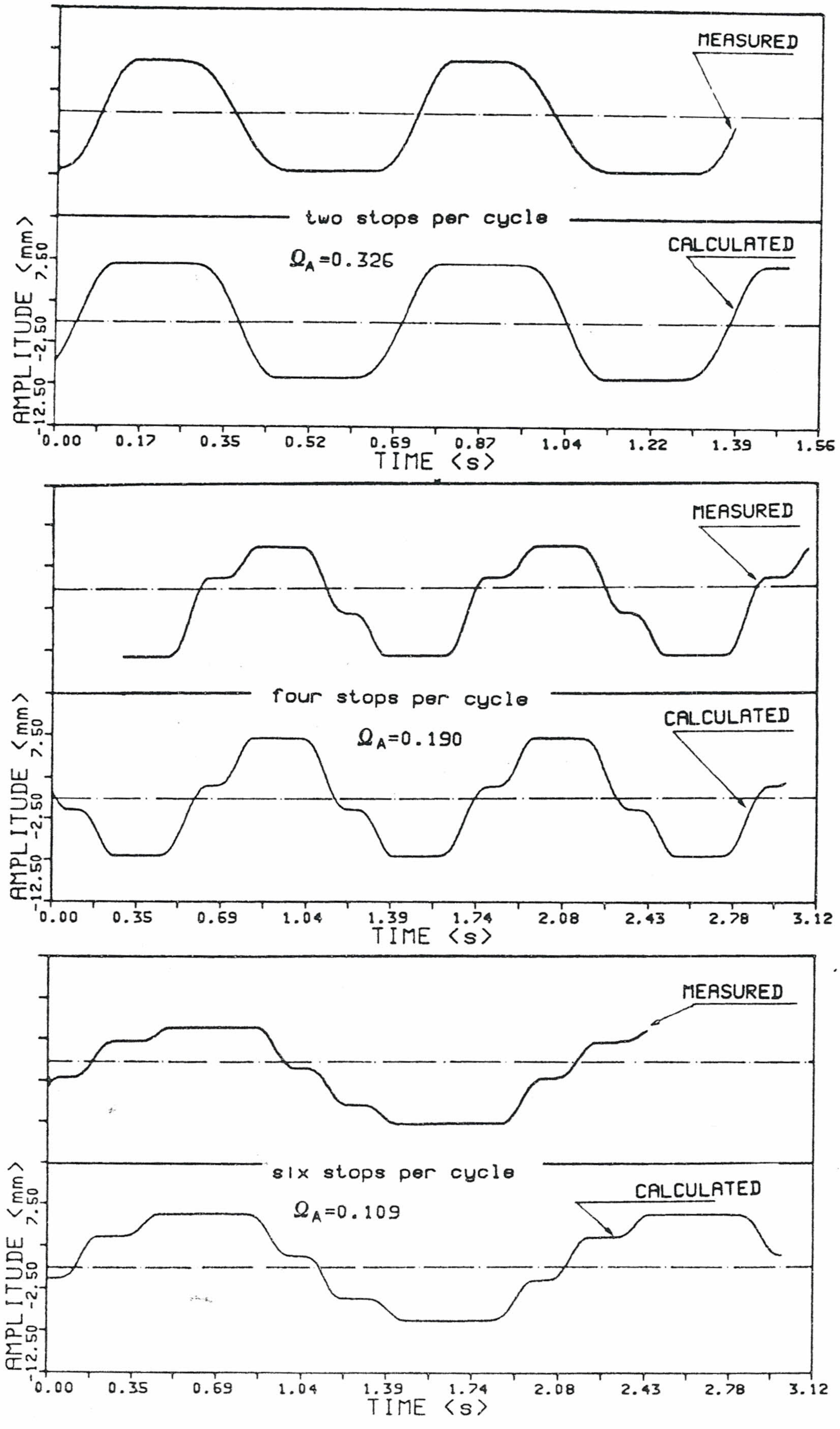
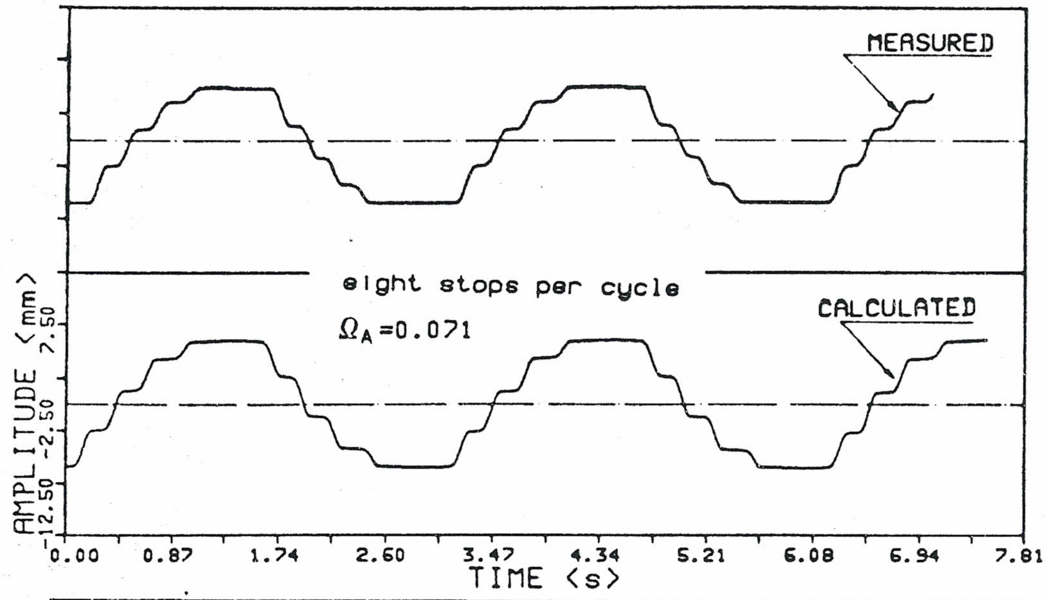


Fig. 7 Comparision of measured and calculated mass displacement for different exciting frequencies (Ω_A=0.326, Ω_A=0.190, Ω_A=0.109)



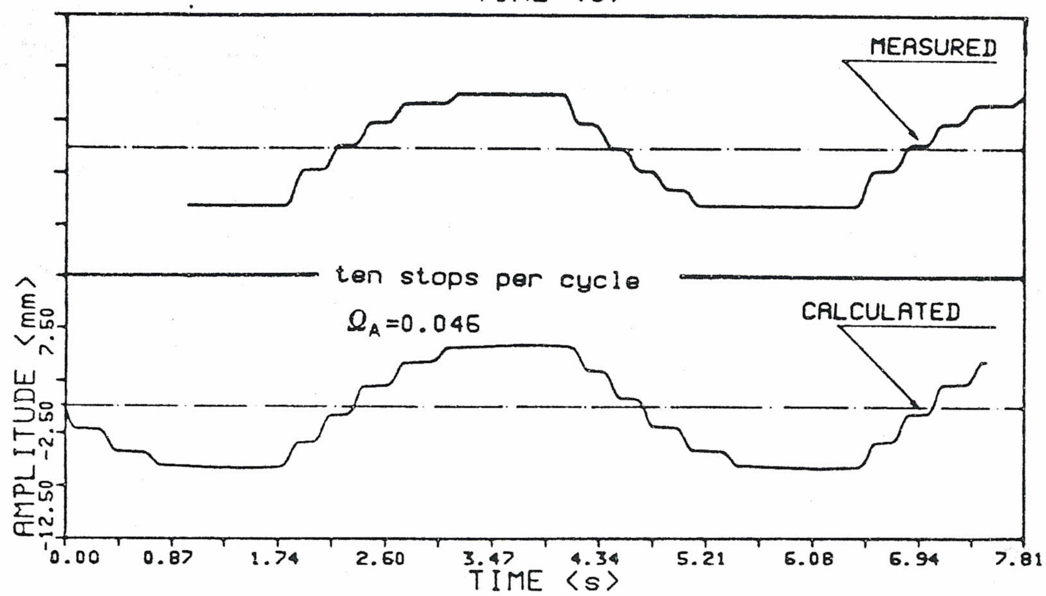


Fig. 8 Comparision of measured and calculated mass displacement for different values of the exciting frequency (Ω_A=0.071, Ω_A=0.046)

Comparing fig. 7 and fig. 8 , which represent the experimental and predicted steady-state time responses for different exciting frequencies, it can be seen that the number of stops per cycle is changing from two to ten stops decreasing the exciting frequency ratios. The calculated time wave forms are in good agreement with the experimental results; the little differences are due to the transient still present in the response (amplitude was calculated after 5-10 cycle) and to the approximation of the friction force (values of the friction coefficients : $A=F_{static}-F_{dinamic}$ B=100., $C=F_{dinamic}$, D=0.002).

To avoid it, it would be necessary to increase the integration time and therefore the computation time.

The exciting frequency has primary importance in the determination of the stops-number. As it can be seen in fig. 9, where three experimental curves, traced for three different values of exciting displacement y_0, are plotted, the stops-number depends on the exciting frequency. The change from one stopping region to another is in corrispondence with the local minimum of the curve. Increasing the exciting displacement amplitude, the change among the corresponding stopping regions occurs at a higher exciting frequency. The presence of odd stops per cycle is due to some non-symmetrical conditions in experimental device. The friction dry coefficients in the three cases are respectively:

case: $y_0/x_{static}=2.63$ $\mu_{friction}=0.376 \pm 0.015$

case: $y_0/x_{static}=3.51$ $\mu_{friction}=0.320 \pm 0.015$

case: $y_0/x_{static}=4.90$ $\mu_{friction}=0.487 \pm 0.024$

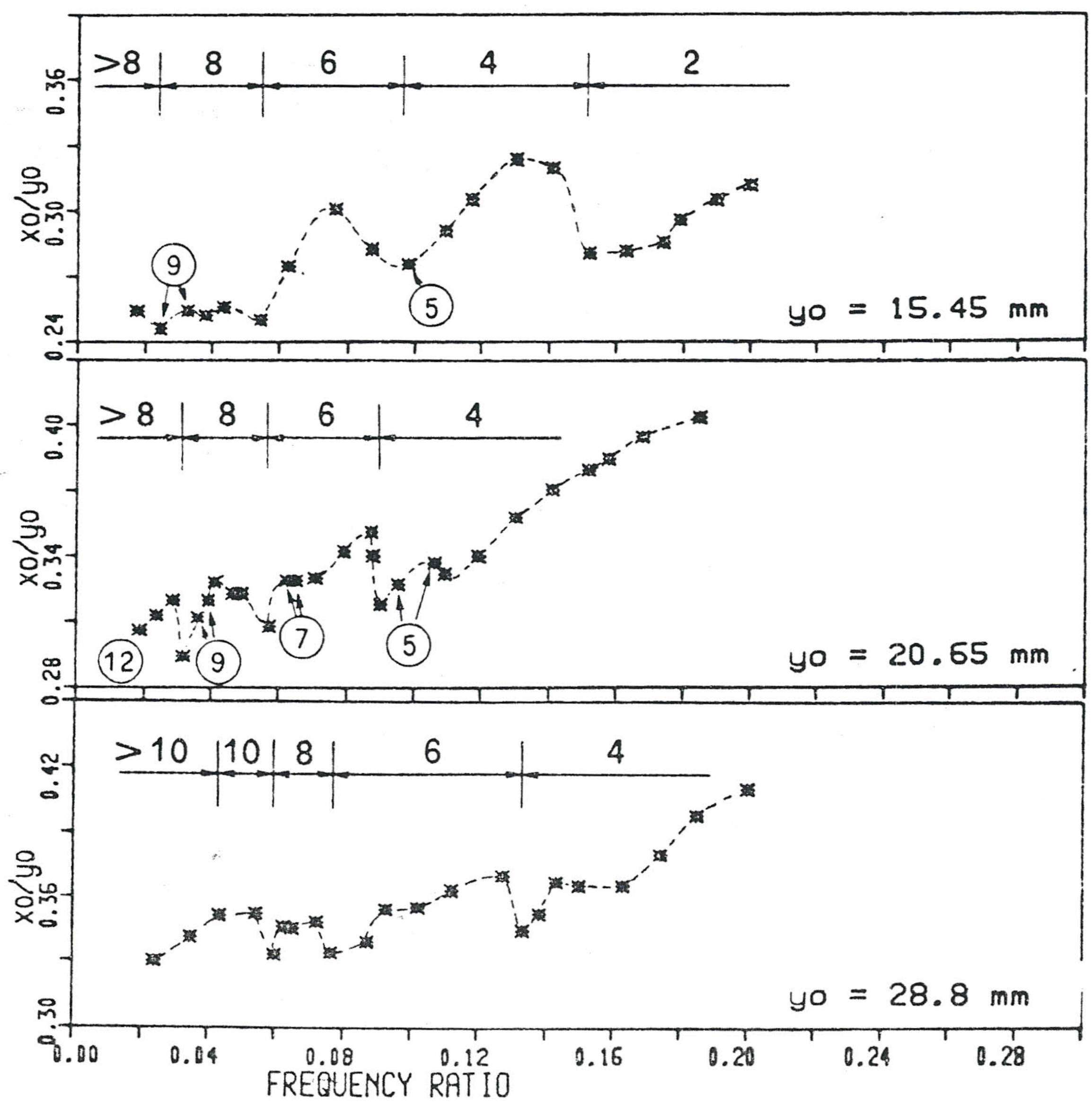


Fig. 9 Experimental dimensionless amplitude versus frequency ratio for different values of the exciting base amplitude.

The influence of dynamic to static friction forces ratio was studied numerically (fig. 10). In the plots of the figure the exciting displacement was kept costant (y0=20.65 mm peak to peak, y_0/x_{static}=3.51). Also in this case, as previously discussed, the presence of different stopping regions is clear. Increasing the friction ratio the change of stops number is better seen and occurs at higher frequencies.

For a low value of friction ratio and in presence of low exciting frequencies it becomes to be difficult to determine the stopping regions clearly. That can be seen in the lowest curve of fig. 10 in the change between 4 stops region and 6 stops region. An other reason to explain the oscillation of this curve is the presence of the transient in the calculated wave form. Considering the curves plotted in the middle of fig. 9 and 10 it is possible to judge the accuracy of the numerical model.

The experimental and numerical technique carried out will be extended in the future to systems having different characteristics, in particular different materials, roughness surfaces and different lubri cation conditions.

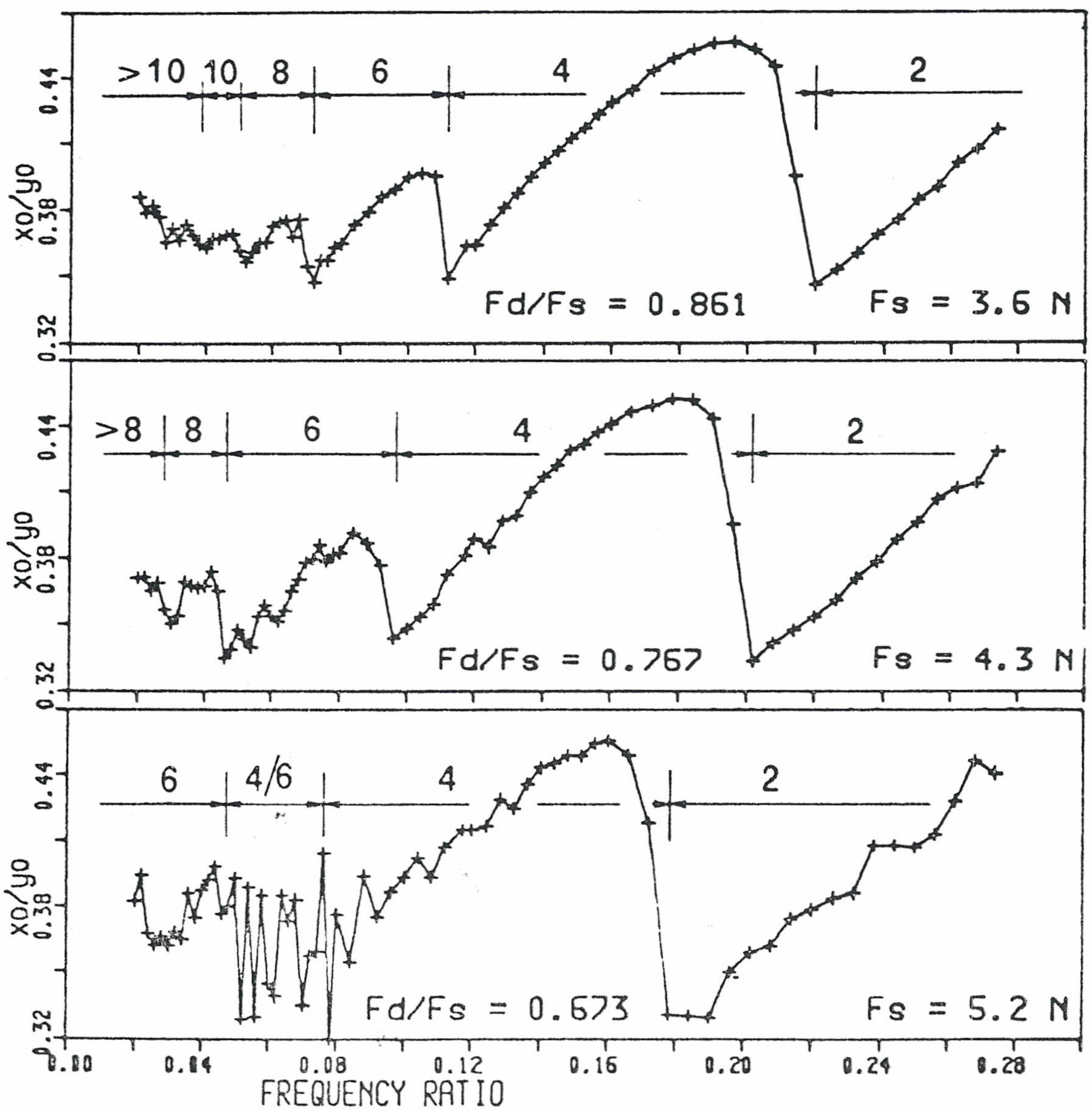


Fig. 10 Calculated dimensionless amplitude versus frequency ratio for various values of friction force. (Exciting base amplitude = 20.65 mm)

5. CONCLUSION

The numerical-experimental analysis has shown the characteristics of the stick-slip phenomenon for a single degree system with harmonic base excitation.

Some conditions, for the analyzed system, governing the transition from two stops to ten stops per cycle region have been found. The correlation among the vibration response amplitude , the stops number during each cycle , the exciting frequency and motion amplitude was established.

The experimental and numerical technique carried out will be extended in the future to systems having different characteristics, in particular different materials, roughness surfaces and different lubrification conditions.

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