On the Efficiency of Partial Information in Elections

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Abstract

We study the relation between the electorate's information about candidates' policy platforms during an election, and the subsequent provision of inefficient local public goods (pork) by the elected government. More information does not always lead to better outcomes. We show that the equilibrium outcome is efficient if voters are not very well informed; whereas, if the electorate is well informed, electoral competition leads candidates to provide inefficient pork in all equilibria.

Keywords: Elections, information, inefficiency, pork, campaigns.

During electoral campaigns, candidates running for office make policy proposals to woo voters. Voters pay only limited attention to electoral campaigns and as a result they do not become fully informed about the policies proposed by the candidates. We study the relation between the information acquired by the voters, and the policies that the candidates announce during the campaign and execute once in office. In particular, we explain the effect of voters' information on the provision of socially inefficient particularistic public goods (pork).

A particularistic or local public good provides a benefit only to the members of a single district or group. If the costs of provision are spread across society at large by general taxation, voters in each district want their own particularistic public good to be provided, while they prefer the public good in any other district not to be provided. Because voters enjoy the benefits of the public good provided to their own district fully, while they only pay a fraction of the cost of any given public good, they care more about the provision of their own good than about the non-provision of the public good in any one other given district. This leads politicians to promote inefficient policies that result in the over provision of particularistic public goods.

We find that a more informed electorate does not obtain a more efficient policy outcome. In fact, if inefficient local public goods provide a benefit that is at least two thirds of their cost, the efficient policy –no provision of pork– can be sustained in equilibrium if and only if the electorate is *not* well informed. If the electorate is better informed, all equilibria are inefficient, even if candidates also care about social efficiency along with their desire to gain office.

Examples of policies that distribute targeted goods of dubious efficiency abound: we highlight farm subsidies, military procurement, and some infrastructure investment in the United States and Europe.

Farm subsidies distribute between $12 billion and $25 billion every year in the United States, and around €55 billion per year in the European Union (almost one half of the European Union budget),[1] distorting the market and creating an aggregate welfare loss.

Parochial interests also trump efficiency in contracts for military equipment. Congress spent $65 billion from 1991 to 2009 on the F-22 jet fighter, designed to counter a Soviet threat and found unsuitable by the Air Force for any combat mission in the post-Soviet world; controversy surrounds the plans to develop two parallel versions of the engine for the F-35 fighter at an additional cost of $3 billion. President Obama declared purchases of the F-22 aircraft an "inexcusable waste" and the second engine for the F-35 an "unnecessary and extravagant expense."

These projects gain political support based on the funding and jobs they bring to specific districts, irrespective of their merit as a cost-efficient mean to satisfy military needs. Procurement decisions degenerate into contests for pork that serve as local jobs programs. Companies disperse production among multiple districts to maximize political support: Boeing promised to "create up to 50,000 jobs in 40 states" in its $40 billion contract to replace the US Air Force refueling tankers, which was awarded in 2011 after nine years of delays and controversies about fraud and waste. Building the F-22 that the Air Force has never used provided 25,000 jobs in 44 states.[2] In Europe, production of the European Typhoon jet fighter was assigned to countries in proportion to their procurement orders and not based on any measure of efficiency. And yet

[1] Source: US Department of Agriculture, http://www.ers.usda.gov/briefing/farmincome/govtpaybyfarmtype.htm and the European Union portal at http://ec.europa.eu/agriculture/publi/capexplained/cap_en.pdf

[2] For the F-22 story, see "Obama wins crucial Senate vote on F-22", New York Times, July 21st, 2009; for the F-35, "House votes to end alternate jet engine program", New York Times, February 16th, 2011. For the Boeing claim on the tanker, "Boeing Submits Final NewGen Tanker Proposal" at www.boeing.mediaroom.com.

the dispersion of production among many districts that makes a project politically viable is precisely the cause of large cost overruns and delays (Younossi, Stem Lourell and Lussier [74]). An audit on the European Typhoon conducted by the United Kingdom's Ministry of Defense, which alone has spent over £20 billion on the project, agrees, identifying "the inefficient commercial and managerial arrangements on the project as the root cause of much of the cost escalation and schedule slippage on the project."[3]

With regard to investment in public infrastructure, an egregious example in the United States was the practice (halted in 2011) of approving earmarks, which allocate funds to local projects avoiding the scrutiny and debate of regular appropriations. The total cost of these projects ascended to $16 billion in 2010.[4] Aside from earmarks, the cost effectiveness of other projects such as high speed rail lines, both existing ones in Europe (Ginés and Inglada [31]) and future ones in the United States[5] appears to be rather questionable.[6]

We investigate whether this inefficient allocation of funds occurs because voters are not sufficiently informed about issues that do not concern them directly. Most voters are not farmers or aviation industry workers, and they do not follow political campaigns closely enough to know each candidate's funding plans for farming or aircraft purchases, or other projects that are not directly relevant to them. To the extent that voters remain unaware of policy details on issues

[3]See the Report by the Comptroller and Auditor General, HC 755, session 2010–2011, 2 march 2011. These "inefficient commercial arrangements" were such that the left wings were built in Italy, the right ones in Spain, the front of the plane in the United Kingdom and the back in Germany.

[4]Taxpayers for Common Sense (www.taxpayer.org) defines earmarks as "legislative provisions that set aside funds within an account for a specific program, project, activity, institution, or location. These measures normally circumvent merit-based or competitive allocation processes."

[5]See Edward Glaeser, "Running the numbers on High-Speed Trains", in the Economix blog at the New York Times, August 4, 2009.

[6]For instance, in June 2011, Spanish state-controlled rail operator RENFE suspended service along its newest high speed line, after noting that its average daily passenger volume was nine passengers. See "Spain cuts high speed 'ghost train' " from *The Telegraph*, 28 June 2011.

that do not concern them directly, legislators have no incentives to pursue efficiency gains by eliminating programs that generate targeted benefits and diffused costs, even if the aggregate costs surpass the localized benefits.

This intuition, while true, is incomplete. A society with a perfectly informed electorate can also suffer from inefficient policies that favor organized lobbies and special interests (Grossman and Helpman [34]). Does more information at least alleviate the problem? Does a more informed electorate obtain less inefficient policy outcomes?

To answers these questions, we find the equilibrium policy proposals in an election with two parties in a Westminster democracy with multiple districts. Candidates compete by proposing to implement local public good projects that are inefficient for society, but beneficial to the district in which they are developed. The party that wins in most districts implements its policy proposal. The model also fits presidential elections, where the two agents competing for votes are two individual politicians; we refer to the two agents contesting the elections as candidates in both cases.

Voters care most about policies that directly affect their districts, and the time or effort they can devote to learn about policy proposals is limited. As a result, they become better informed about proposals that affect their districts than about projects in other districts that only affect them indirectly through general taxation. We capture this uneven attention to policy proposals by assuming that voters are informed about proposals in their own district, while they only observe proposals for other districts with some probability. This probability is our measure of the electorate's level of information. Voters who remain uninformed about proposals in other districts form beliefs about outside proposals based on the proposals they observe in their districts.

We first find that if pork is very inefficient, an equilibrium with no pork exists for any level of information possessed by the electorate. We say pork is "very" inefficient if it is so inefficient that pork provision to any majority of districts makes this majority worse off even though the costs are shared equally by all districts in society, including those that did not get pork. With three districts, pork is very inefficient if it provides less than 67cts of benefit per unit of cost; with n districts, less than $\frac{n+1}{2n}$ units of benefit per unit of cost.

Our main results concern pork that is inefficient, but not very inefficient. The efficient policy outcome cannot always be sustained in equilibrium. Too much information affects incentives perversely, as follows.

If the electorate is poorly informed, and voters face uncertainty about policy platforms, the efficient policy with no pork can be sustained in equilibrium; in fact, this is the unique equilibrium if the electorate has some, but not too much information and at least one candidate cares at least infinitesimally about the efficiency of her policy proposal, and not only about winning. In contrast, if the electorate is very well informed, the competition among districts to be in the winning coalition and between candidates to attract votes induces an inefficient overprovision of public goods: all equilibria are inefficient, even if one or both candidates care about the efficiency of their policy proposals, and not just about winnint the election.

The unsettling conclusion is that some degree of voters' ignorance is necessary to obtain efficient policies in equilibrium: an intermediate level of information is optimal.

Seminal contributions to the literature on distributive politics by Weingast [72], Shepsle and Weingast [67], Weingast, Shepsle and Johnsen [73] and Niou and Ordeshook [56] analyze the provision of local inefficient public goods as the result of a legislative bargaining game, and predict a "pork for everybody" outcome. Legislators commit to a norm of universalism by which every district gets its own inefficient project, rather than letting a minimal-winning majority distribute public goods only to the districts in this majority. An objection to this seminal theory is that it cannot explain why legislators do not embrace instead a Pareto-superior universalist norm by which no inefficient local public goods are ever provided. In fact, if legislators do not commit to any norm, Ferejohn, Fiorina and McKelvey [29] and Baron [8] show that only a minimal winning majority of districts benefit from the provision of inefficient projects. More recent developments study how pork provision is affected by the number of districts in the polity (Baqir [7], Primo and Snyder [59], Chen and Malhotra [18]), by term limits (Herron and Shotts [36]), or by the distribution of legislators' preferences for local or collective public goods (Volden and Wiseman [71]).[7]

A stream of economic theories explain targeted redistribution as the equilibrium outcome of a game in which candidates compete in elections (Cox and McCubbins [21], Lindbeck and Weibull [42]; Myerson [54]; Dixit and Londregan [23] and [24]; Lizzeri and Persico [43]; Chari, Jones and Marimon [17]; the survey by Persson and Tabellini [58]; and more recent work by Milesi-Ferreti, Perotti and Rostagno [51]; Roberson [61]; Fernandez and Levy [30]; Hirano, Snyder and Ting [37]; Smith and Bueno de Mesquita [69] and Huber and Ting [38]). These theories assume that citizens are fully informed about the policy proposals made by candidates. The assumption is unrealistic. The empirical literature on voter behavior has conclusively established that in practice voters have at best a sketchy idea of these policy proposals (Campbell, Converse, Miller

[7] Drazen and Izetzki [25] depart from most of the literature to argue that pork is welfare-improving when the agenda-setter uses it as a costly signal to inform other legislators.

and Stokes [16]; Bartels [10] and Alvarez [2]).

At the opposite extreme, Grosser and Palfrey [33] assume that citizens do not know anything about the candidates. Baron [9] and Gul and Pesendorfer [35] assume that some voters are fully informed, while others are uninformed about policy proposals. Glaeser, Ponzetto and Shapiro [32] assume that each voter becomes either informed or uninformed about the policy proposal of each candidate separately. The assumption that a voter is either fully informed or fully uninformed is stark: voters in large elections possess some information about candidates' policy proposals and they use their limited information to decide how to vote (Bullock [13]). Instead of studying fully informed or fully uninformed voters, it is more appropriate to build theories of elections with imperfectly informed voters. McKelvey and Ordeshook [47], [48] and [49] assume that voters infer candidates' policy positions indirectly from polls and endorsements. Snyder and Ting [70] argue that party labels serve as cues to provide some information to voters who do not directly know the candidates' preferences. Assuming voters face uncertainty about candidates' preferences, Dhami [22] studies general redistribution from rich to poor agents. Bernhard, Dubey and Hughson [12] introduce exogenous, fixed transfers from districts with a junior representative to districts with a senior representative. All these models deal with ideological preferences on the real line.[8] Aidt and Shvets [1] analyze the principal-agent problem of selecting high type politicians and motivating them to exert effort: they do not study the strategic formulation of policy proposals, assuming instead that effort and type translate mechanically into funds for local public goods. Their main finding is that under term limits, politicians exert less effort in

[8] Other models consider electorates that are not perfectly informed about the state of the world (Austen-Smith [5], Feddersen and Pesendorfer [27] and [28]) or about candidates' competence or character (Krishna and Morgan [41], Kartik and McAfee [39]), or allow candidates to deviate from their campaign proposals once in office (Banks [6] and Callander and Wilkie [14]). These models are more distantly related because they do not deal with distributive policies, and assume that voters are perfectly informed about candidates' announcements.

their last term in office.

The closest reference is the theory of redistributive transfers to partially uninformed voters by Gavazza and Lizzeri [44]. We depart from Gavazza and Lizzeri [44] in several respects. Our model is simpler, capturing the insights we deem relevant with fewer variables and fewer parameters. Gavazza and Lizzeri [44] assume that candidates maximize their expected share of votes, which implies that candidates are purely opportunistic with no policy concerns, and the electoral rule is proportional representation. We assume that the electoral rule is district by district plurality rule so that the candidate who wins most districts wins the election (as in India, United States, United Kingdom, Canada, Australia, and many others, see Cox [20]) and while we assume that candidates care about winning office, we allow for the possibility that they may also care about the policies they enact once elected; furthermore, we allow for heterogeneity in candidate motivation: one candidate may care a lot about policy efficiency, while the other cares little, or not at all. An important technical difference is that under standard solution concepts, Gavazza and Lizzeri's [44] theory delivers an indeterminate prediction with multiple equilibria, and they select among these equilibria using passive beliefs, which we find theoretically difficult to justify. Without resorting to questionable selection among equilibria, we obtain a clear comparative statics based on equilibrium uniqueness for some parameter values: the policy outcome under full information delivers lower aggregate social welfare than the policy outcome obtained if the electorate is partially uninformed. This result stands in sharp contrast to Gavazza and Lizzeri's [44], whose model collapses to a two-period version of Lindbeck and Weibull [44] and hence predicts full efficiency if the electorate is fully informed.

Our contribution is the first electoral theory of pork provision that applies to electoral systems with single-member districts and recognizes that voters have only partial information about candidates' policy proposals. It obtains sharp results based on standard equilibrium concepts and allowing candidates to have heterogeneous and hybrid motivations, caring both for winning office and for the efficiency of the policies they enact.

1 The Model

We consider a society partitioned into three subsets, with one representative voter $i \in \{a, b, c\}$ in each subset. We refer to these subsets as districts, but they could also be population groups of similar size divided by ethnicity, age, profession or class. The restriction to $n = 3$ is for ease of exposition: the theory and results generalize to any odd number of districts n.[9]

Two candidates A and B compete for election. Let $J \in \{A, B\}$ denote an arbitrary candidate and let $-J$ denote the other candidate, so that $\{J, -J\} \equiv \{A, B\}$. We assume that candidates and voters are fully strategic, rational agents who evaluate lotteries according to standard expected utilities.

The policy space consists on whether or not to provide a public good in each district.[10] A strategy for each candidate consists on proposing a policy in the policy space. Let $S^J = S^{-J} = \{0, 1\}^3$ be the strategy set of each candidate. Let $s^J = (s_a^J, s_b^J, s_c^J) \in \{0, 1\}^3$ be a strategy by candidate J, where $s_i^J = 1$ indicates that J proposes to provide the public good in district i and $s_i^J = 0$ indicates that J proposes not to provide it. Let s_{-i}^J denote the proposals for the other

[9] We present the theory and prove the results for the general n case in an Appendix.

[10] The theory is robust if we enrich the policy space to include an ideological dimension over which voters have single-peaked preferences, or a quantity g to invest on the provision of a public good that benefits all districts equally: both candidates would converge to the median on the ideological issue and would choose the welfare-maximizing quantity of the collective public good. The electoral outcome would depend on the candidates' proposals for provision of local public goods as analyzed in this paper.

two districts, not including i.

The timing is simple:

One – Candidates announce their policy proposals simultaneously.

Two – Nature determines how much information voters get about these proposals, as detailed below.

Three – An election is held, where voters simultaneously choose to vote for A, for B or to abstain.

Four – The candidate who receives most votes, or a randomly selected candidate in case of a tie, wins the election and implements her policy proposal.

Each public good brings a benefit β to the voter in the district where it is provided and no benefit to the other districts. The cost of each public good is identical across districts and is normalized to one. All the costs of providing local public goods are equally borne by all voters, regardless of which districts actually receive their local public good.

We assume that local public goods are inefficient, so no district would like to provide its own public good if it had to bear its full cost, but we assume that projects are sufficiently efficient so that, ceteris paribus, each voter prefers to have the public good provided in her district given that the district bears only $\frac{1}{3}$ of the cost of provision. This means that $\beta \in \left(\frac{1}{3}, 1\right)$.[11]

Voter $i's$ preference order over policies depends on β. If $\beta \in \left(\frac{2}{3}, 1\right)$, the preference order of

[11] If $\beta < 1/3$, the policy decision is simple, since all voters agree that it is best not to implement any project.

voter a over the eight possible policy outcomes is

$$\{1,0,0\} \succ_a \{1,1,0\} \sim_a \{1,0,1\} \succ_a \{0,0,0\} \succ_a \{1,1,1\} \succ_a \{0,0,1\} \sim_a \{0,1,0\} \succ_a \{0,1,1\}. \tag{1}$$

If $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$,

$$\{1,0,0\} \succ_a \{0,0,0\} \succ_a \{1,1,0\} \sim_a \{1,0,1\} \succ_a \{0,0,1\} \sim_a \{0,1,0\} \succ_a \{1,1,1\} \succ_a \{0,1,1\}. \tag{2}$$

Voters' preferences over candidates depend exclusively on the expected utility of the candidates' proposals. Each voter i uses all the information available to her about the policy proposal of candidate J to calculate the expected utility of candidate J to voter i. If candidate J proposes to provide the public good in district i, then voter i's utility of $J's$ proposal is equal to $\beta - \frac{1}{3}k$ where $k \in \{0,1,2,3\}$ is the total number of public goods proposed by candidate J. If candidate J does not propose to provide the public good in district i, then voter i's expected utility is equal to $-\frac{1}{3}k$. The expectation is over k, which may be unknown to the voter.

We assume that voters vote for the candidate with the highest expected payoff, that is, voters are sequentially rational (Kreps and Wilson [40]). We assume that voters do not use weakly dominated strategies. This rules out equilibria in which all voters vote for the same candidate even though some voters prefer the losing candidate's proposal. If the expected payoffs of both candidates for voter i coincide given voter $i's$ beliefs, voter i is indifferent about the candidates. We assume that in this case she abstains, unless abstention has been eliminated as a weakly dominated strategy.

We consider two possible motivations for a candidate: we assume that candidates care about winning office, and we allow but do not require that candidates also care about the efficiency of the policies they enact once in office. We say that a candidate is purely office motivated if winning office is her exclusive concern: her utility is one if she wins, zero if she loses. We say that a candidate is efficiency concerned if she values winning, but she also cares about the efficiency of the policies she would enact once in office; these candidates prefer to win proposing and implementing more efficient policies. Let k the number of projects in the proposal of candidate J.

Candidate $J's$ preferences are represented by the utility function:

$$\begin{aligned} U_J(k) &= \frac{1}{1+\alpha_J(1-\beta)k} \text{ if } J \text{ wins the election, where } \alpha_J \in \mathbb{R}_+. \quad (3) \\ U_J(k) &= 0 \text{ otherwise.} \end{aligned}$$

While we let parameter α_J take any non-negative value, two values are focal for us: $\alpha_J = 0$, which corresponds to purely office motivated candidates, and $\alpha_J = 1$, which yields the simplest functional form of the utility function of any efficiency concerned candidate. We pay additional attention to candidates with these parameter values. The term $(1-\beta)k$ measures the inefficiency of a policy that implements k projects, and the utility that an efficiency concerned candidate experiences if she wins proposing such policy decreases in this term.

We allow for heterogeneity in candidates' motivations, i.e. we allow for any $(a_A, \alpha_B) \in \mathbb{R}^2_+$. All previous electoral theories on distributive politics or local public good provision assume that candidates have homogenous motivations: either all candidates care only about winning (Person and Tabellini [58]), or they all care about both winning and policy outcomes (McKelvey and Riezman [50]). Roemer [62], Aragones and Palfrey [3] and Callander [15] introduce heterogenous motivations to the classical one dimensional spatial model.[12] To the best of our knowledge, our theory is the first to allow for heterogenous candidate motivations in an electoral theory of distributive politics or pork provision.

The solution concept we use is a Perfect Bayesian Equilibrium in which no agent uses weakly dominated strategies. We fully characterize the set of pure strategy equilibria, and for parameter values such that the efficient outcome with no pork cannot be sustained as an equilibrium outcome, we study mixed strategy equilibria to find the best (least inefficient) outcome that can be sustained in a pure or mixed strategy equilibrium. Let $p^J = (p_a^J, p_b^J, p_c^J) \in \{0,1\}^3$ be the policy actually proposed (action taken) by J, and let $p = (p^A, p^B)$.

We analyze how voters modify their behavior in response to changes in their information about candidates' proposals. We assume that each voter is well informed about candidates' proposals for the project in the voter's district, but it is harder for a voter to keep track of information about projects in other districts. Note that the theory is not about strategic communication on the part of candidates: in our model, given any policy proposal to any district by any candidate J, either candidate J or her opponent candidate $-J$ has an incentive to publicize the content of that proposal to the other districts. In particular, if $p_a^J = 0$, then J would like this be known by voters in districts b and c, while if $p_a^J = 1$ then $-J$ would like this to be known by b and c. If we allowed for strategic communication, any given piece of information would be strategically announced either by A or by B; we assume directly that all information

[12] See as well Saporiti, Drouvelis and Vriend [63].

is announced. The problem is not that candidates do not communicate; rather, the problem is that even though all the relevant information is announced, voters do not pay careful attention, and as a result they may not apprehend information that does not directly pertain to their district.

We parameterize how informed is the electorate by a single parameter π, which captures the probability that all voters are perfectly informed. Nature determines whether candidates' policy proposals become common knowledge or not. Policy proposals become common knowledge with probability $\pi \in [0, 1]$. With probability $1 - \pi$, each voter $i \in \{a, b, c\}$ only observes what candidates commit to do in her district, and she is completely unaware of what candidates promise in the other districts. Notice that the extreme case $\pi = 1$ corresponds to the standard model with perfect information, and $\pi = 0$ corresponds to an electorate in which voters only have local information about the proposals for their district, and never learn about the proposals to provide public goods in other districts.[13]

If Nature makes proposals public knowledge, voters observe p^A and p^B and compare the two proposals and vote for the one they prefer, according to the preference order (1) or (2).

If proposals do not become public knowledge, each voter i remains unaware about what each candidate proposes in districts other than i : Voter i only observes either $p_i^J = 0$ or $p_i^J = 1$ for each candidate J, that is, voter i observes $(p_i^A, p_i^B) \in \{0, 1\} \times \{0, 1\}$, so that voter i has four information sets in which to make a decision, and three possible actions (vote A, vote B or abstain) in each of these sets.

[13] As we discuss below on a section on extensions, qualitative results are robust if we let each district become informed with independent probability $\pi_i = \pi$, or if we relax the assumption that voters are always informed about proposals in their district.

Under this informational structure, in which voters have imperfect information, an equilibrium must describe strategies for voters and candidates, and beliefs for voters. Beliefs along the equilibrium path must be correct. Beliefs off the equilibrium path must assign all the probabilities to undominated strategies if any such strategy is consistent with the information possessed by the agent.[14]

The strategy pair (s^A, s^B) determines the information (p_i^A, p_i^B) received by each voter i. This information, together with beliefs, determine the expected utility for i if A or B wins, which in turn determines agent $i's$ vote and therefore, aggregating over all three agents, it determines the electoral outcome and the payoffs to A and B.

The set of equilibria depend on the efficiency of the public goods measured by β, and on how informed the electorate is, measured by π.

2 Results

We say that public goods are very inefficient if they are worth less than two thirds of their total cost. If local public goods are very inefficient, any minimal winning coalition of districts prefers a policy outcome in which none of the districts in the coalition receive their local public good, rather than an outcome in which all the districts in the coalition receive it, keeping fixed the policy for districts outside the coalition. Our first result concerns these goods. If voters are poorly informed, there are multiple equilibria and the efficient equilibrium is one of them.[15] If

[14] If no undominated strategy is consistent with the information possessed by the agent, then we let the agent hold any beliefs over the entire strategy set.

[15] This multiplicity of equilibria is an instance of a more general result well known in the multilateral contract literature: when one of many downstream agents does not receive the offer she expects from the upstream player and she does not observe the offers made to other downstream players, the downstream player's beliefs about the other offers are not pinned down by sequential equilibrium (Segal [65], McAfree and Schwartz [46]).

voters are sufficiently well informed, the outcome is efficient: no local public goods are provided in the unique equilibrium. The result is robust to either assumption on the motivation of candidates, whether they exclusively seek to win office, or whether they also have about policy outcomes.

Claim 1 *For any* $(\alpha_A, \alpha_B) \in \mathbb{R}_+^2$*, any* $\pi \geq 0$ *and any* $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$*, there exists an equilibrium in which both candidates propose to provide no pork, and if* $\pi > \frac{1}{2}$*, it is the unique equilibrium.*

We relegate to the Appendix section all the proofs, together with more extensive results including a full characterization of the set of equilibria.

Inefficient equilibria exist if the electorate is poorly informed; even the least efficient policy in which all districts receive pork can be obtained as an equilibrium outcome. This least efficient equilibrium is the one that would be selected using *passive beliefs* as a selection criterion. A voter holds passive beliefs if her posterior about the candidates' offers to other districts does not change when she observes an out-of-equilibrium offer to her district. We find little theoretical or empirical support for such selection criterion: in our context we find it more plausible that voters more readily believe that a candidate plans to reduce spending across the board if the candidate does indeed reduce spending in the budget items that the voter can observe. We therefore do not use this selection criterion: if multiple equilibria exist for some parameters, we characterize the set of all equilibria (see the Appendix).

Our main interest and focus is on local public goods that are inefficient, but not too inefficient, so that their benefit/cost ratio is closer to one. If local public goods provide enough benefit so that a minimal winning majority of districts would like to provide the public goods to itself making all districts pay for the cost through general taxation, then the efficient outcome with no provision of these inefficient goods cannot always be sustained in equilibrium, not even if both candidates care about efficiency.

Proposition 2 *Assume that both candidates are efficiency concerned ($\alpha_A = \alpha_j = 1$) and $\beta \in \left(\frac{2}{3}, 1\right)$.*

If $\pi \leq \frac{2-\beta}{6-4\beta}$, there exist multiple pure equilibria, including the efficient one;

If $\pi \in \left(\frac{2-\beta}{6-4\beta}, \frac{3-2\beta}{2}\right]$, there is a unique pure strategy equilibrium in which both candidates propose the efficient policy; and

If $\pi > \frac{3-2\beta}{2}$ there is no pure strategy equilibrium, there exists a mixed strategy equilibrium, and in expectation at least 9/7 projects are implemented.

If voters are poorly informed, they are unlikely to know the true proposals at the time of voting, and their votes more often than not depend on their beliefs: very different equilibria can be sustained given voters' beliefs such that any voter observing a deviation in her districts believes the worst about the deviating candidate. Equilibria in which both candidates propose zero projects and in which both candidates propose to provide an inefficient local public good to every district exist and are sequential (Kreps and Wilson [40]), trembling hand perfect (Selten [66]) and rationalizable (Pearce [57] and Battigali [11]).

These and other equilibria are sustained for any $\beta \in \left(\frac{2}{3}, 1\right)$ by the following voters' beliefs off the equilibrium path: a voter who observes a deviation in her district believes that the deviating candidate has proposed to provide the public goods in the other two districts. Given these beliefs, no voter votes for a deviating candidate unless information is fully revealed. It follows that if the probability that full information is revealed is low, deviating is never profitable. With a poorly informed electorate, comparative statics on β show that if pork becomes more inefficient –if β decreases–, the efficient equilibrium can be sustained by a larger set of off-equilibrium beliefs: the efficient equilibrium holds if each voter who observes a deviation believes that the deviator offers pork to at least $3\beta - 1$ other districts. In future work we plan to test in experiments which, if any, of the multiple equilibria do subjects play.[16]

An increase from low to intermediate information is at least weakly beneficial because it makes the efficient equilibrium unique: if the probability that voters become informed is close to one half, candidates with hybrid motivations would be willing to deviate to the efficiency policy and win only if their deviation is observed by the electorate, as long as this probability is close enough to one half. This willingness to deviate, even if deviating means winning with probability less than one half, destroys all the inefficient equilibria. By a similar argument, candidates are not willing to deviate from the efficient equilibria by offering pork to a minimal winning majority of two districts if the probability that this deviation is observed is only slightly greater than one half.

If the probability that citizens are informed is sufficiently high, the efficient equilibrium breaks down. For π sufficiently large, the unique equilibrium which emerges is an equilibrium in mixed strategies where in expectation at least 9/7 projects are implemented. The result is more intuitive than it might initially appear to be. Suppose that both candidates play pure strategies. At least one of the two candidates wins the election with probability at most $\frac{1}{2}$.

[16] We conjecture that subjects will *not* play equilibrium strategies consistent with passive beliefs; rather, our conjecture is that subjects will play most frequently the equilibrium that can be sustained by a largest set of off-equilibrium beliefs.

However, if citizens are fully informed, simple majority generates a Condorcet cycle: for any pure strategy, there exists another pure strategy that defeats it.[17] Each candidate could deviate to the strategy that defeats her opponent's strategy, and in this manner win whenever Nature reveals the policy proposals, which occurs with probability π; hence if π is sufficiently high, the initial (arbitrary) strategy pair cannot be supported in equilibrium, because even a candidate who also cares about efficiency would deviate to an inefficient policy that attracts a minimal winning majority of two districts with sufficiently high probability. Therefore, as shown in figure 1, the efficient outcome can only be obtained if citizens are *not* very well informed.

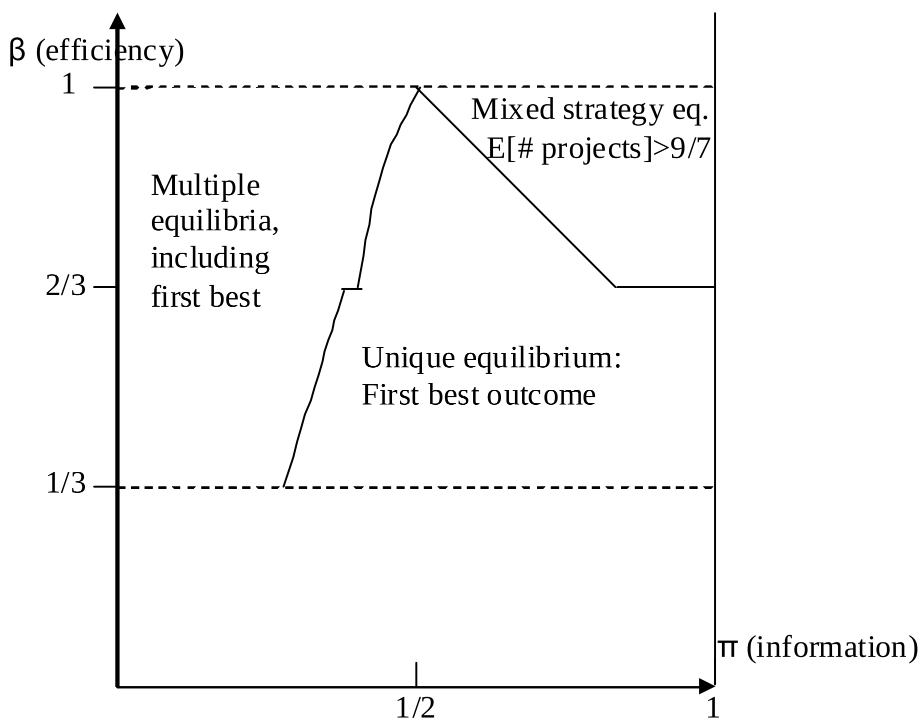

Figure 1: Equilibria and policy outcomes with efficiency concerned candidates.

[17] Proposing zero projects is defeated by proposing two, which is defeated by proposing one of those two, which is defeated by proposing all three, which is defeated by proposing zero.

If public goods are inefficient but not too inefficient, some degree of voters' ignorance is necessary to sustain an efficient outcome. Too much information is strictly detrimental, as it destroys the efficient equilibrium and leads to inefficient outcomes that reduce the ex-ante welfare of each voter. The intuition behind this distressing result is straightforward. Candidates, even those concerned about efficiency, care about winning the election: we assume that they prefer to win with certainty with a bad policy than to lose with certainty with a good one. If citizens are well informed, the efficient proposal cannot be part of an equilibrium because it is defeated by any proposal that provides the public good to two districts. If full information is revealed with sufficiently high probability, the unique equilibrium is in mixed strategies and the inefficient public good in each district is provided with positive probability.

The efficient equilibrium in which both candidates propose the policy $(0,0,0)$ can only exist if candidates are not tempted to deviate by offering pork to two districts. Lack of information makes this deviation unfruitful if voters do not believe it when they cannot verify it: voters who do not observe the true proposals and believe that candidates who offer pork in one district offer pork in every district do not support a candidate who offers them pork. Voters who are skeptical about candidates who promise to favor them and not others with pork, together with the inability for candidates to credibly announce that they favor a particular subset of districts sustain the efficient equilibrium. Campaign promises to carve out a minimal winning coalition are not credible if the electorate is poorly informed, but they are credible and they destroy the efficient equilibrium, if voters are well informed.

The qualitative result in Proposition 2 is robust if candidates' concern for efficiency vary in intensity across candidates. It is also robust if only one candidate is efficiency concerned, or if candidate's types (α_J) are private information. If the electorate is poorly informed, there are multiple equilibria including the efficient one; if π is in an intermediate interval, the efficient equilibrium is the unique equilibrium; and if the electorate is very well informed (π is above the intermediate interval), the equilibrium is inefficient, in mixed strategies. Too much information strictly reduces aggregate welfare. The logic for the uniqueness result is the same with one efficiency concerned candidate or with motivations that are private information as it is with two efficiency concerned candidates: if π is below but close to one half, an efficiency concerned candidate would deviate to a more efficient policy and would win if the deviation is observed by all voters, destroying any equilibrium but the efficient one. While this result holds as soon as at least one candidate cares at least infinitesimally about the efficiency of the policy she enacts, the range of parameter pairs (β, π) for which the efficient equilibrium can be sustained, and the set of parameter pairs for which it is the unique pure equilibrium, expand with α_J. This is intuitive: as candidates care more about efficiency, it becomes easier to sustain the efficient equilibrium, and harder to sustain any inefficient equilibrium. Nevertheless, for any α_J, there is some $\beta < 1$ such that if the electorate is sufficiently well informed, all equilibria are inefficient. The following proposition details this result.

Proposition 3 *Let $(\alpha_A, \alpha_B) \in \mathbb{R}^2_+$, and without loss of generality assume $\alpha_A \leq \alpha_B$. Then, for any $\beta \in \left(\frac{2}{3}, 1\right)$,*

a) if $\pi \leq \frac{1+\alpha_B(1-\beta)}{2+4\alpha_B(1-\beta)}$, multiple equilibria exist, including the efficient one in which both candidates propose zero pork,

b) if $\frac{1+\alpha_B(1-\beta)}{2+4\alpha_B(1-\beta)} < \pi \leq \frac{1}{2} + \alpha_A(1-\beta)$, the efficient equilibrium is the unique pure equilibrium, and *c) if* $\frac{1}{2} + \alpha_A(1-\beta) < \pi$*, all equilibria are in mixed strategies and inefficient.*

Efficiency can be sustained as an equilibrium outcome if the electorate is not very informed, irrespective of candidates' motivations. Efficiency is guaranteed to be the outcome of the unique equilibrium if information π is at an intermediate interval; this interval increases in size with candidates' concerns for efficiency. Only in the limit case in which $\alpha_A = a_B = 0$ so that neither candidate cares about anything but winning office, the intermediate interval disappears (its limits converge to $\frac{1}{2}$) and the efficient equilibrium is not unique for any π.

3 Extensions and Robustness

We consider several extensions and generalizations to the model, to check that the intuition underpinning our results is robust.

Our main generalization is to analyze a society with an arbitrary number of districts. While the case with three districts suffices to convey the intuition of the results in the clearest manner, we show that results are robust to a society with an arbitrary odd number n of districts. The relevant efficient cutoff is no longer 66.7cts of benefit per unit of cost, but rather, $\frac{n+1}{2n}$ units of benefit per unit of cost. As we prove in the Appendix section, the result in Claim 1 generalizes to the case with n districts: if $\beta < \frac{n+1}{2n}$, the efficient equilibrium with no pork can be supported in equilibrium for any epsilon, and is the unique equilibrium if $\pi > \frac{1}{2}$, regardless of whether candidates care only about winning, or also about efficiency.

The results in Propositions 2 and 3 also generalize, and are increasingly relevant as the efficiency threshold $\beta = \frac{n+1}{2n}$ above which the propositions apply decreases in n from 66.7cts of benefit per unit of cost for $n = 3$ to (asymptotically) 50cts of benefit per unit of cost for large n. In a society with a large number of districts, and projects that deliver more than 50cts of benefit per unit of cost, the efficient outcome with no pork can only observed if voters are not very well informed.

Beyond the generalization to an arbitrary number of districts, the most relevant extensions and variations to the model that we have analyzed are the following:

a) A relaxation of the assumption that voters are perfectly informed about projects in their own district, assuming instead that each voter is informed about her district with greater probability than about other districts.

b) A theory with asymmetric districts, so that one district is more informed or less informed than the others.

c) A model in which each district becomes informed with independent probability π_i.

d) A model in which candidates' motivations are private, unobserved by the other candidate or by voters.

e) A citizen-candidate model in which each candidate is biased toward her own district so that she favors allocating pork to that district.

f) A model in which candidates are policy motivated so that they care for the policy outcome even when they lose the election.

We show that our results are robust to these alternative variations: if $\beta \in (2/3, 1)$, the efficient equilibrium can only be sustained if citizens are poorly informed about proposals for projects outside their district, while if $\beta \in (1/3, 2/3)$, it can always be sustained; furthermore, if candidates are efficiency concerned, there is an interval of intermediate levels of information such that the efficient equilibrium is unique. The set of parameter values for which each equilibrium exists vary with each variation of the model, but the qualitative results are robust.

A precise formulation and proof of each of the claims regarding extensions a) through f) is available from the authors, along with a proof that all the equilibria are extensive form rationalizable (Pearce [57] and Battigali [11]), and a characterization of the set of trembling hand perfect (Selten [66]) and proper (Myerson [53]) equilibria for the case with two purely office motivated candidates.

4 Discussion

We have developed a theory on the provision of inefficient particularistic goods to an imperfectly informed electorate. We argue that citizens are better informed about government expenditures in their own district (which they favor) than about government expenditure in other districts (which they oppose). We analyze how this informational bias affects the provision of socially inefficient local public goods.

If particularistic goods are very inefficient, returning less than $\frac{n+1}{2n}$ units of benefit per unit of cost, the efficient policy with no provision of pork can be supported in equilibrium, whether or not voters are fully informed.

If particularistic goods are inefficient but provide more than $\frac{n+1}{2n}$ units of benefit per unit of cost, the equilibrium outcome depends on the information held by the electorate:

a) If voters are poorly informed, many different strategies and levels of provision of inefficient goods, including the efficient zero provision, can be sustained in equilibrium.

b) If at least one candidate also cares about efficiency and not only about winning office, there is an intermediate range of information for which the unique pure strategy equilibrium is the efficient one. A higher level of information destroys this equilibrium, as each candidate is then always able to best respond to any pure strategy by the other candidate by crafting a proposal that is more beneficial to a simple majority of districts.

c) If voters are very well informed, there is no pure strategy equilibrium because majority preferences exhibit a Condorcet cycle: a majority prefers to provide goods to no district rather than to all districts, and to provide to two districts over no district; a different majority prefers to provide the good to only one of those two districts instead of to both of them; and yet a different majority prefers to provide goods to all three districts instead of to just that one district, completing the cycle. Equilibria are in mixed strategies, and inefficient.

We therefore find that too much information makes every voter ex-ante worse off.

In a survey on the role of the media, Stromberg and Prat [60] argue that an electorate that ignores what is the state of the world may become worse off if it gains information about candidates' actions but not about outcomes, because it makes candidates' pander by choosing actions that match the prior of the voter about the right action to take (Maskin and Tirole [45], Ashworth and Shotts [4]). In their framework, the electorate would always be better off learning about outcomes. We identify a novel channel by which information hurts the electorate: without any uncertainty about the state of the world, an informed electorate leads candidates to defeat the efficient policy by proposing inefficient policies designed to benefit a simple majority of districts, which ex-ante makes every voter worse off.[18].

[18] More distantly related theories show a negative effect of information on welfare in the context of risk sharing in general equilibrium (Schlee [64]) or in beaty contests (Morris and Shin [52]).

Our results have normative implications with regard to voter education: making the electorate fully informed does not suffice and in fact harms the prospects of obtaining efficient policies from the political process. The classic solution to restore efficiency –to fund local public goods with locally raised revenue– appears more promising. In the words of Adam Smith [68]:

> "When high roads, bridges, canals, etc. are in this manner made and supported by the commerce which is carried on by means of them, they can be made only where that commerce requires them, as consequently where it is proper to make them... A magnificent high road cannot be made through a desert country where there is little or no commerce" (p. 683).

Appendix

We provide the proofs of Claim 1, Proposition 2 and Proposition 3, along with complete characterizations of the set of pure strategy equilibria, and of mixed strategy equilibria for the range of parameters for which there is no pure equilibrium, and the generalizationf of the theory to any arbitrary number of districts, in an Appendix in a separate file.

References

[1] Toke S. Aidt and Julia Shvets. 2011. "Distributive politics and electoral incentives: Evidence from seven US state legislatures." CESifo Working Paper Series No. 3405.

[2] Michael R. Alvarez. 1997. *Information and Elections.* University of Michigan Press.

[3] Enriqueta Aragones and Thomas R. Palfrey. 2005. "Electoral competition between two candidates of different quality: the effects of candidate ideology and private information," in D.Austen-Smith and J.Duggan (eds.) *Social Choice and Strategic Decisions* (Berlin, Germany: Springer) 93–112.

[4] Scott Ashworth and Kenneth W. Shotts. 2010. "Does informative media commentary reduce politician's incentives to pander? *Journal of Public Economics* 94(11-12): 838–847.

[5] David Austen-Smith. 1992. "Explaining the vote: constituency constrains on sophisticated voting." *American Journal of Political Science* 36(1): 68–95.

[6] Jeffrey S. Banks. 1990. "A model of electoral competition with incomplete information." *Journal of Economic Theory* 50(2): 309–325.

[7] Rezi Baqir. 2002. "Districting and government overspending." *Journal of Politics* 110(6): 1318–1354.

[8] David P. Baron. 1991. "Majoritarian incentives, pork barrel programs and procedural control." *American Journal of Political Science* 35(1): 57–90.

[9] David P. Baron. 1994. "Electoral competition with informed and uninformed voters." *American Political Science Review* 88(1): 33–47.

[10] Larry M. Bartels. 1986. "Issue voting under uncertainty: An empirical test." *American Journal of Political Science* 30: 709–728.

[11] Pierpaolo Battigalli. 1997. "On rationalizability in extensive games." *Journal of Economic Theory* 74(1): 40–61.

[12] Daniel Bernhardt, Sangita Dubey and Eric Hughson. 2004. "Term limits and pork barrel politics." *Journal of Public Economics* 88: 2383–2422.

[13] John G. Bullock. 2011. "Elite influence on public opinion in an informed electorate." *American Political Science Review* 105(3): 496–515.

[14] Steven Callander and Simon Wilkie. 2007. "Lies, damned lies, and political campaigns." *Games and Economic Behavior* 60(2): 262–286.

[15] Steven Callander. 2008. "Political motivations." *Review of Economic Studies* 75(3): 671–697.

[16] Angus Campbell, Philip E. Converse, Warren E. Miller and Donald E. Stokes. 1980. *The American Voter.* University of Chicago Press.

[17] V.V. Chari, Larry E. Jones and Ramon Marimon. 1997. "The economics of split-ticket voting in representative democracies." *American Economic Review* 87(5): 957–976.

[18] Jowei Chen and Neil Malhotra. 2007. "The law of k/n : The effect of chamber size on government spending in bicameral legislatures." *American Political Science Review* 101(4): 657–676.

[19] In-Koo Cho and David M. Kreps. 1987. "Signaling games and stable equilibria." *Quarterly Journal of Economics* 102(2): 179–222.

[20] Gary W. Cox. 1997. *Making Votes Count.* Cambridge Univ. Press.

[21] Gary W. Cox and Mathew D. McCubbins. 1986. "Electoral politics as a redistributive game." *Journal of Politics* 48 (2):370–89.

[22] Sanjit Dhami. 2003. "The political economy of redistribution under asymmetric information." *Journal of Public Economics* 87(9-10): 2067–2103.

[23] Avinash Dixit and John Londregan. 1995. "Redistributive politics and economic efficiency." *American Political Science Review* 89(4): 856–866.

[24] Avinash Dixit and John Londregan. 1996. "The determinants of success of special interests in redistributive politics." *Journal of Politics* 58: 1132–1155.

[25] Allan Drazen and Ethan Izetzki. 2011. "Kosher pork." NBER working paper 16667.

[26] Jon X. Eguia and Antonio Nicolò. 2011. "On the distribution of particularistic goods," in *Political Economy and Institutions,* eds. Caballero and Schofield, Springer.

[27] Timothy J. Feddersen and Wolfgang Pesendorfer. 1996. "The swing voter's curse." *American Economic Review* 86(3): 408–424.

[28] Timothy J. Feddersen and Wolfgang Pesendorfer. 1999. "Abstention in elections with asymmetric information and diverse preferences." *American Political Science Review* 93(2): 381–398.

[29] John Ferejohn, Morris Fiorina and Richard D. McKelvey. 1987. "Sophisticated voting and agenda independence." *American Journal of Political Science* 31(1): 169–193.

[30] Raquel Fernández and Gilat Levy. 2008. "Diversity and redistribution." *Journal of Public Economics* 92(5-6): 925–943.

[31] Ginés de Rus and Vicente Inglada. 1997. "Cost-benefit analysis of the high-speed train in Spain." *Annals of Regional Science* 31(2): 175–188.

[32] Edward L. Glaeser, Giacomo A. M. Ponzetto and Jesse M. Shapiro. 2005. "Strategic extremism: why Republicans and Democrats divide on religious values." *Quarterly Journal of Economics* 120(4): 1283–1330.

[33] Jens Grosser and Thomas Palfrey. 2010. "Candidate entry and political polarization: an anti-median voter theorem." Caltech HSS working paper 1343.

[34] Gene M. Grossman and Elhanan Helpman. 2002. *Special Interest Politics.* MIT Press.

[35] Faruk Gul and Wolfgang Pesendorfer. 2009. "Partisan politics and election failure with ignorant voters." *Journal of Economic Theory* 144(1): 146–174.

[36] Michael C. Herron and Kenneth W. Shotts. 2006. "Term limits and pork." *Legislative Studies Quarterly* 31(3): 383–403.

[37] Shigeo Hirano, James M. Snyder and Michael M. Ting. 2009. "Distributive Politics with Primaries." *Journal of Politics* 71(4): 1467–1480.

[38] Gregory A. Huber and Michael M. Ting. "Redistribution, pork and elections." SSRN working paper 1468143.

[39] Navin Kartik and R. Preston McAfee. 2007. "Signaling character in electoral competition." *American Economic Review* 97(3): 852–870.

[40] David M. Kreps and Robert Wilson. 1982. "Sequential equilibrium." *Econometrica* 50(4): 863–894.

[41] Vijay Krishna and John Morgan. 2010. "Overcoming ideological bias in elections." *Journal of Political Economy* 119(2): .183–211.

[42] Assar Lindbeck and Jörgen W. Weibul. 1987. "Balanced-budget redistribution as the outcome of political competition." *Public Choice* 52: 273–297.

[43] Alessandro Lizzeri and Nicola Persico. 2001. "The provision of public goods under alternative electoral incentives." *American Economic Review* 91(1): 225–239.

[44] Alessandro Lizzeri and Alessandro Gavazza. 2009. "Transparency and economic policy." *Review of Economic Studies* 76(3): 1023–1048.

[45] Eric Maskin and Jean Tirole. 2004. "The politician and the judge: Accountability in government." *American Economic Review* 94(4): 1034–1054.

[46] R. Preston McAfee and Marius Schwartz. 1994. "Opportunism in multilateral vertical contracting: nondiscrimination, exclusivity and uniformity." *American Economic Review* 84(1): 210–230.

[47] Richard D. McKelvey and Peter Ordeshook. 1985. "Elections with limited information: A fulfilled expectations model using contemporaneous poll and endorsement data as information sources." *Journal of Economic Theory* 36(1): 55–85.

[48] Richard D. McKelvey and Peter Ordeshook. 1985. "Sequential elections with limited information." *American Journal of Political Science* 29(3): 480–512.

[49] Richard D. McKelvey and Peter Ordeshook. 1985. "Information, electoral equilibria, and the democratic ideal." *Journal of Politics* 48(4): 909–937.

[50] Richard D. McKelvey and Raymond Riezman. 1992. "Seniority in legislatures." *American Political Science Review* 86(4): 951–965.

[51] Gian M. Milesi-Ferretti, Roberto Perotti and Massimo Rostagno. 2002. "Electoral systems and public spending." *Quarterly Journal of Economics* 117(2): 609–657.

[52] Stephen Morris and Hyun Song Shin. 2002. "Social value of public information." *American Economic Review* 92(5): 1521–1534.

[53] Roger B. Myerson. 1978. "Refinements of the Nash equilibrium concept." *International Journal of Game Theory* 7: 73–80.

[54] Roger B. Myerson. 1993. "Incentives to cultivate favored minorities under alternative electoral systems." *American Political Science Review* 87(4): 856–869.

[55] Roger B. Myerson. 1997. *Game Theory: Analysis of conflict.* Harvard Univ. Press.

[56] Emerson M.S. Niou and Peter C. Ordeshook. 1985. "Universalism in Congress." *American Journal of Political Science* 29(2): 246–258.

[57] David G. Pearce. 1984. "Rationalizable strategic behavior and the problem of perfection." *Econometrics* 52(4): 1029–1050.

[58] Torsten Person and Guido Tabellini. 2000. *Political Economics.* MIT Press.

[59] David M. Primo and James M. Snyder. 2008. *Journal of Politics* 70(2): 477–486.

[60] Andrea Prat and David Strömberg. 2011. "The political economy of mass media." CEPR working paper 8246.

[61] Brian Roberson. 2008. "Pork-barrel politics, targetable policies, and fiscal federalism. *Journal of the European Economic Association* 6(4): 819–844.

[62] John E. Roemer. 1999. "The democratic political economy of progressive income taxation." *Econometrica* 67: 1–19.

[63] Alejandro Saporiti, Michalis Drouvelis and Nicolaas J. Vriend. 2011. "Political motivations and electoral competition: Equilibrium analysis and experimental evidence." Working paper.

[64] Edward E. Schlee. 2001. "The value of information in the context of risk sharing arrangements." *American Economic Review* 91(3): 509–524.

[65] Ilya Segal. 1999. "Contracting with externalities." *Quarterly Journal of Economics* 114(2): 337–388.

[66] Reinhard Selten. 1975. "Reexamination of the perfectness concept for equilibrium points in extensive games." *International Journal of Game Theory* 4: 25–55.

[67] Kenneth A. Shepsle and Barry R. Weingast. 1981. "Political preferences for the pork barrel: A generalization." *American Journal of Political Science* 25(1): 96–111.

[68] Adam Smith. 1776. *The Wealth of Nations*. The Modern Library, New York.

[69] Alastair Smith and Bruce Bueno de Mesquita. "Contingent Prize Allocation and Pivotal Voting." Forthcoming, *British Journal of Political Science.*

[70] James M. Snyder and Michael M. Ting. 2002. "An informational rationale for political parties." *American Journal of Political Science* 46(1): 90–110.

[71] Craig Volden and Alan E. Wiseman. 2007. "Bargaining in legislatures over particularistic and collective goods." *American Political Science Review* 101(1): 79–92.

[72] Barry R. Weingast. 1979. "A rational choice perspective on congressional norms." *American Journal of Political Science* 23(2): 245–262.

[73] Barry R. Weingast, Kenneth A. Shepsle and Christopher Johnsen. 1981. "The political economy of benefits and costs: A neoclassical approach." *Journal of Political Economy* 89(4): 642–664.

[74] Obaid Younossi, David E. Stem, Mark A. Lorell and Frances M. Lussier. 2005. *Lessons Learned from the F/A-22 and the F/A-18E/F Development Programs.* Rand Corporation.

5 Appendix

This file contains an appendix to the manuscript "On the Efficiency of Partial Information in Elections", submitted for consideration for publication in the *Journal of Politics.*

This online Appendix continues the exposition contained in the body of the paper, and it uses definitions, notation, results and cross-references to other sections in the manuscript.

In this Appendix we provide the following:

a) A complete characterization of equilibria in pure strategies for any $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$ and any $\pi \in (0, 1]$, or for any $\beta \in \left(\frac{2}{3}, 1\right)$ and any $\pi \in (0, 1]$, for any pair $(\alpha_A, \alpha_B) \in \mathbb{R}^2_+$. Claim 1, the main part of Proposition 2 and Proposition 3 follow as corollaries of these characterizations. We extend the characterization to the special case $\pi = 0$, which sustains additional equilibria, for $(\alpha_A, a_B) \in \{(0,0), (0,1), (1,0), (1,1)\}$.

b) A solution for symmetric mixed strategy equilibria, which serves to complete the proof of Proposition 2, for $(\alpha_A, a_B) \in \{(0,0), (1,1)\}$.

c) A generalization to a society with an arbitrary odd number n of districts, proving that the results in the body of the paper are robust.

While the proofs of the full characterizations are long and tedious, most of the complication arises from the consideration of the non-generic case $\pi = 0$ and from the analysis of the mixed strategy equilibria. Readers who are satisfied with a streamlined proof of the main results for $\pi > 0$ can skip ahead to the much more concise proofs we provide for the general n case in subsection 5.5.

Additional results discussed in section 3 with generalizations and extensions to alternative assumptions on candidates' motivations, asymmetric districts, alternative informational assumptions, or different solution concepts are available from the authors upon request.

5.1 Preliminaries

We introduce notation that shortens the proofs of our results.

For each candidate $J \in \{A, B\}$, the set of pure strategies S^J consists of the following eight 3-dimensional vector. $s_1 = (0,0,0)$, $s_2 = (1,0,0)$, $s_3 = (0,1,0)$, $s_4 = (0,0,1)$, $s_5 = (1,1,0)$,

$s_6 = (1,0,1),\ s_7 = (0,1,1),\ s_8 = (1,1,1)$.

It is useful to classify strategy pairs in classes of strategic equivalence, as follows:

Let $S_1 = \{(s_1, s_1)\}$; $S_2 = \{(s_1, s_2), (s_1, s_3), (s_1, s_4), (s_2, s_1), (s_3, s_1), (s_4, s_1)\}$;

$S_3 = \{(s_1, s_5), (s_1, s_6), (s_1, s_7), (s_5, s_1), (s_6, s_1), (s_7, s_1)\}$; $S_4 = \{(s_1, s_8), (s_8, s_1)\}$;

$S_5 = \{(s_2, s_2), (s_3, s_3), (s_4, s_4)\}$; $S_6 = \{(s_2, s_3), (s_2, s_4), (s_3, s_4), (s_3, s_2), (s_4, s_2), (s_4, s_3)\}$; $S_7 = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), (s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$;

$S_8 = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), (s_7, s_2), (s_6, s_3), (s_5, s_4)\}$;

$S_9 = \{(s_2, s_8), (s_3, s_8), (s_4, s_8), (s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

$S_{10} = \{(s_5, s_5), (s_6, s_6), (s_7, s_7)\}$; $S_{11} = \{(s_5, s_6), (s_5, s_7), (s_6, s_7), (s_6, s_5), (s_7, s_5), (s_7, s_6)\}$;

$S_{12} = \{(s_5, s_8), (s_6, s_8), (s_7, s_8), (s_8, s_5), (s_8, s_6), (s_8, s_7)\}$; $S_{13} = \{(s_8, s_8)\}$.

Note that $S = S^A \times S^B = \prod_{k=1}^{13} S_k$ is the set of all possible candidates' pure strategy pairs, and $s = (s^A, s^B)$ is a strategy profile in pure strategies. Within each class S_k, it suffices to show whether a given $s \in S_k$ can be supported in equilibrium to establish whether all or no $s \in S_k$ can be supported in equilibrium. For each candidate J, let

$$\Delta S^J = \left\{ \sigma^J = (\sigma_1^J, ..., \sigma_8^J) : \sigma_k^J \geq 0 \text{ and} \sum_{k=1}^{8} \sigma_k^J = 1 \right\}$$

be the set of all probability distribution over S^J and let $\sigma^J \in \Delta S^J$ be a mixed strategy for candidate J, so that σ_k^J is the probability that candidate J assigns to pure strategy s_k.

To simplify notation, and given that voters' strategies are straightforward when full information is revealed, in all the analysis below we implicitly assume that if Nature fully reveals the policy proposals, voters vote according to their preferences and abstain when indifferent. This allows us to focus our analysis of voters on the branches of the game in which Nature does not reveal the full information so that voters face uncertainty.[19] For each voter $i \in \{a, b, c\}$, let $s^i : \{0,1\} \times \{0,1\} \longrightarrow \{A, B, \emptyset\}$ be a behavioral strategy for voter i, which is a function that maps each information set of the voter when Nature does not reveal the policy proposals

[19] We stress that this simplifies notation, but does not change our behavioral assumption that voters are fully strategic and rational. In the branches of the game with full information the voters' decision problem can be solved by simple domination arguments, and we directly anticipate and impose the outcome that follows from the unique undominated solution.

fully, into an action by the voter. A complete strategy for the voter specifies s^i, and the actions to be taken when information is fully revealed. We also express s^i as a vector $s^i = (s^i_1, ..., s^i_4)$, where s^i_k is the action chosen under the k-th information set according to the following order: $\{(0,0),(0,1),(1,0),(1,1)\}$.

The belief of voter i, denoted δ_i, is a vector $\delta_i = (\delta_i^A, \delta_i^B)$, where $\delta_i^J = (\delta_i^J(0), \delta_i^J(1))$ for each $J \in \{A, B\}$, and $\delta_i^J(p_i^J)$ is the probability distribution over the set of strategies played by candidate J that voter i holds as a belief after observing p_i^J. For $p_i^J \in \{0, 1\}$, let $\omega_k^{i,J}(p_i^J)$ be the sum of weights assigned by $\delta_i^J(p_i^J)$ to the set of strategies where J proposes to carry out k projects in districts other than i.

Let $v : S \longrightarrow \{A, B, \emptyset\}^3$ be the list of votes by voters $\{a, b, c\}$ as a function of candidates' strategies, given some specific beliefs.

Because the number of players is finite, and each player has a finite number of possible strategies, the game is finite, and hence an equilibrium (possibly in mixed strategies) exists (see for instance Myerson [55] pg 177).

All equilibria that exist for $\pi \in \left(0, \frac{1}{2}\right]$, also exist if $\pi = 0$, but additional equilibria exist if $\pi = 0$, so we study this case separately to prove each result.

Lemma 4 *If $\pi > 0$, in any pure strategy equilibrium both candidates propose to carry out the same number of projects and the election is tied.*

Proof. Consider any strategy profile such that candidate J wins with probability less than $\frac{1}{2}$. Since in equilibrium voters hold correct beliefs, the probability that J wins conditional on full information being revealed, or not revealed, is less than $\frac{1}{2}$ in each case. In pure strategies, the probability of victory is in the set $\{0, \frac{1}{2}, 1\}$ so if it is less than $\frac{1}{2}$, it is zero. Deviating to $s^J = s^{-J}$, candidate J ties the election if full information is revealed, so the probability of winning is at least $\frac{\pi}{2}$. ■

5.2 Characterization of Pure Equilibria with $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$

Under the assumption that projects are very inefficient, $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$, we first characterize the set of pure strategy equilibria with two office-motivated candidates, then with two efficiency concerned candidates, and finally with one purely office-motivated and one efficiency concerned. Claim 1 holds immediately as a corollary of these full characterization results.

5.2.1 Office Motivated Candidates

Claim 5 *Assume* $(\alpha_A, \alpha_B) = (0,0)$ *and* $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. *For any* $\pi \in \left(0, \frac{1}{2}\right)$, *an equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 5, 6, 11, 13\}$; *for* $\pi = \frac{1}{2}$, *an equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 5, 6, 10, 11, 13\}$; *while for any* $\pi \in \left(\frac{1}{2}, 1\right]$ *there is a unique equilibrium, in which candidates propose* (s_1, s_1).

Proof. To sustain equilibria, assume that off-equilibrium path beliefs in cases S_1, S_5, S_6, S_{11} and S_{13} are such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observe a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Suppose $\pi \leq \frac{1}{2}$. Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If full information is revealed there is not a different proposal that can defeat s_1. Hence there are no profitable deviations for any $\pi \in [0, 1]$.

S_2 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter abstains. If J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$

and J loses the election. If $\pi > \frac{1}{2}$ then if candidate J deviates and proposes $s^J = s_1$ then she wins with probability π and therefore the deviation is profitable.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. If $\pi > \frac{1}{2}$ then candidate A wins with probability π deviating to $s^A = s_1$

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. Suppose $\pi < \frac{1}{2}$. If candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \pi > \frac{1}{2}$. Suppose $\pi = \frac{1}{2}$, for any deviation played by candidate J, she wins with probability $\frac{1}{2}$ and loses with the same probability. Hence there is not a profitable deviation. If $\pi > \frac{1}{2}$, candidate J wins when full information is revealed deviating to $s^J = s_1$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_6) = (\emptyset, A, B)$. Suppose $\pi \leq \frac{1}{2}$. It suffices to check that A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Suppose $\pi > \frac{1}{2}$ if candidate J deviates to $s^J = s_1$ she wins the election with probability π.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} : All voters abstain and the election is tied. Suppose $\pi \leq \frac{1}{2}$. If candidate J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. Suppose $\pi > \frac{1}{2}$; if candidate J deviates to $s^J = s_1$ she wins the election with probability π. ■

Claim 6 *Assume $\pi = 0$, $(\alpha_A, \alpha_B) = (0,0)$ and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin S_{10} \cup S_{12}$.*

Proof. First note that $(s^A, s^B) \in S_{10}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates and proposes s_8 voters a and b beliefs are unaffected, while voter c votes for candidate J. Therefore candidate J wins the election.

Similarly, $(s^A, s^B) \in S_{12}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), voters a and b vote A, while voter c votes B. Suppose A deviates to s_8. Voters a and b do not observe the deviation, and continue to vote A, while voter c now abstains. Hence now A wins the election by a greater margin.

All the other strategy profiles are sustained in equilibria by the following beliefs. Beliefs' over equilibrium strategies are correct. Out-of-equilibrium beliefs are such that given the equilibrium proposal s_i^J, then $\omega_2^{i,J}(1 - s_i^J) = 1$ for both $J \in \{A, B\}$ and for $i \in \{a, b, c\}$. These are most pessimistic beliefs that a voter can have regarding candidates' strategy when she observes a deviation.

S_1 : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (A, B, B)$, and any voter who observes a deviation votes against the candidate who deviates.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, every voter votes for A and continues to vote for A after any deviation by B.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. In equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter abstains, and votes against any candidate who deviates.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. Every voter votes against any deviating candidate.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Voters a and c do not vote for B after any deviation by B, and voter b does not vote for A after any deviation.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$, and given any deviation by J, no voter changes her vote from voting for $-J$ to abstention or voting for J.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A and continue to do so after any deviation by B.

S_{10} : Not an equilibrium as shown above.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_g) = (\emptyset, A, B)$, and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Not an equilibrium as shown above.

S_{13} : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates. ■

5.2.2 Efficiency Concerned Candidates

Claim 7 *Assume $(\alpha_A, \alpha_B) \in \mathbb{R}^2_+$ and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. Without loss of generality let $\alpha_A \leq a_B$. An equilibrium in which candidates use the strategy pair (s_1, s_1) exists for any $\pi \geq 0$; an equilibrium in which candidates use the strategy $(s^A, s^B) \in S_k$ for $k \in \{5, 6\}$ exists if and only if $\pi \leq \frac{1}{2}\frac{1}{1+\alpha_B(1-\beta)}$; an equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{11}$ exists if and only if $\pi \leq \frac{1}{2}\frac{1}{1+2\alpha_B(1-\beta)}$; an equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{13}$ exists if and only $\pi \leq \frac{1}{2}\frac{1}{1+3\alpha_B(1-\beta)}$; there are no other pure strategy equilibria for any $\pi > 0$.*

Proof. Without loss of generality, assume $\alpha_A \leq \alpha_B$. As before, to sustain equilibria, we assume off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If full information is revealed there is not a different proposal that can defeat s_1. Hence there are no profitable deviation for all $\pi \in [0,1]$.

S_2 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. If candidate J deviates, she loses the election when information is not fully revealed. Hence the most profitable deviation is $s^J = s_1$ because J wins the election when information is fully revealed and the proposal is efficient. Candidate J prefers to deviate if and only if

$$\pi > \frac{1}{2}\frac{1}{1 + \alpha_J(1-\beta)}$$

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given voters' beliefs, $v(s_2, s_3) = (A, B, \emptyset)$. If J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. For the same argument as above, the best deviation for an efficiency concerned candidate J is $s^J = s_1$. Candidate J prefers to deviate if and only if

$$\pi > \frac{1}{2}\frac{1}{1 + \alpha_J(1-\beta)}$$

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates to $s^J = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate J wins with probability $1-\pi$. Candidate J prefers to deviate if and only if

$$(1-\pi)\frac{1}{1+\alpha_J 3(1-\beta)} > \frac{1}{2}\frac{1}{1+2\alpha_J(1-\beta)}$$
$$\pi < \frac{1+\alpha_J(1-\beta)}{2+4\alpha_J(1-\beta)}$$

Consider the deviation $s^J = s_1$. Candidate J only wins when information is fully revealed, and therefore the deviation is profitable if and only if

$$\pi > \frac{1}{2}\frac{1}{1+2\alpha_J(1-\beta)}, \text{ or}$$
$$\pi > \frac{1}{2+4\alpha_J(1-\beta)}$$

Since $\frac{1}{2+4\alpha_J(1-\beta)} < \frac{1+\alpha_J(1-\beta)}{2+4\alpha_J(1-\beta)}$ there is always a profitable deviation.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_6) = (\emptyset, A, B)$. If J deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, J loses the election. If J deviates to $s^J = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, J loses the election. It follows that the most profitable deviation is $s^J = s_1$ since candidate J wins the election when information is fully revealed and the proposal is efficient. Candidate J prefers to deviate to s_1 if and only if

$$\pi > \frac{1}{2}\frac{1}{1+2\alpha_J(1-\beta)}.$$

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} : All voters abstain and the election is tied. If candidate J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. If candidate J deviates $s^J = s_1$, she wins the election when information is fully revealed, and this proposal is efficient. Therefore candidate J prefers to deviate to s_1 if and only if

$$\pi > \frac{1}{2}\frac{1}{1+3\alpha_J(1-\beta)}. \tag{4}$$

■

Claim 8 *Assume $\pi = 0$, $(\alpha_A, \alpha_B) = (1,1)$ and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin S_{10}$.*

Proof. First note that $(s^A, s^B) \in S_{10}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates and proposes s_8 voters a and b beliefs are unaffected, while voter c votes for candidate J for all possible beliefs over candidate J strategy. Therefore candidate J wins the election. This deviation is profitable if and only if $\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$ which holds for all $\beta < 1$.

All the other strategy profiles are sustained in equilibria by the following beliefs. Beliefs' over equilibrium strategies are correct. Out-of-equilibrium beliefs are such that given the equilibrium proposal s_i^J, then $\omega_2^{i,J}(1 - s_i^J) = 1$ for both $J \in \{A, B\}$ and for $i \in \{a, b, c\}$.

S_1 : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (A, B, B)$, and any voter who observes a deviation votes against the candidate who deviates.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, every voter votes for A and continues to vote for A after any deviation by B.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. In equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter abstains, and votes against any candidate who deviates.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. Every voter votes against any deviating candidate.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Voters a and c do not vote for B after any deviation by B, and voter b does not vote for A after any deviation.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$, and given any deviation by J, no voter changes her vote from voting for $-J$ to abstention or voting for J.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A and continue to do so after any deviation by B.

S_{10} : Not an equilibrium as shown above.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_g) = (\emptyset, A, B)$, and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. By deviating candidate B cannot increase the number of votes he gets, due to voters' beliefs. For candidate A, the unique strategy that increases A's vote margin is s_8 because voter c would abstain, but an office motivated candidate does not find this deviation profitable.

S_{13} : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates. ■

5.2.3 One Purely Office Motivated, One Efficiency Concerned Candidate

Claim 9 *Assume* $\alpha_{-J} = 0$, $\alpha_J \in \mathbb{R}_{++}$ *and* $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. *There exists a cutoff function* $\pi(\alpha_J, \beta) : \mathbb{R}_{++} \times \left(\frac{1}{3}, \frac{2}{3}\right) \longrightarrow \left(0, \frac{1}{2}\right)$ *such that:*

If $\pi \in [0, \pi(\alpha_J, \beta)]$, *there exist multiple pure equilibria (detailed in the proof);*

If $\pi \in (\pi(\alpha_J, \beta), 1]$ *there is a unique pure strategy equilibrium, in which candidates propose* (s_1, s_1).

Proof. Without loss of generality, let assume that candidate A is office motivated, i.e. $\alpha_A = 0$ and $\alpha_B > 0$. The set S_2 contains profiles which are not strategically equivalent because candidate B is not indifferent, for instance, between winning with proposals s_1 or s_2. We must partition the set S_2 into the following subsets of profiles which are equivalent for both players, $S_2' = \{(s_1, s_2), (s_1, s_3), (s_1, s_4), \}$ and $S_2'' = \{(s_2, s_1), (s_3, s_1), (s_4, s_1)\}$. Similary, let

$S_3' = \{(s_1, s_5), (s_1, s_6), (s_1, s_7)\}$ and $S_3'' = \{(s_5, s_1), (s_6, s_1), (s_7, s_1)\}$;

$S_7' = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$ and

$S_7'' = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$;

$S_8' = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S_8'' = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$;

$S_9' = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S_9'' = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$; and

$S_{12}' = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S_{12}'' = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$.

To sustain equilibria, assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied. No deviation can defeat s_1, neither if information is revealed nor if it is not, so the office motivated candidate cannot gain by deviating. The efficiency concerned candidate has an even lesser incentive to deviate because s_1 is the efficient proposal. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Hence no candidate has profitable deviations for all $\pi \in [0, 1]$.

S_2' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_2'' = Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Ruled out by Lemma 4.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (B, B, B)$. Ruled out by Lemma 4.

(s_1, s_8) : Every voter votes for A. Ruled out by Lemma 4.

(s_8, s_1) : Every voter votes for B. Ruled out by Lemma 4

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Suppose first that $\pi \leq \frac{1}{2}$. Given (s_2, s_2), every voter abstains. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Consider candidate B. If candidate B deviates, B loses the election when information is not fully revealed. When information is revealed the most profitable deviation for candidate B is $s^B = s_1$, since B wins the election and the proposal is efficient. The deviation $s^B = s_1$ is profitable if and only if

$$\pi > \frac{1}{2}\frac{1}{1+\alpha_B(1-\beta)}$$

Suppose now $\pi > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Suppose $\pi \leq \frac{1}{2}$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Any deviation makes the candidate loses the election when information is not fully revealed. When information is revealed the most profitable deviation for candidate B is $s^B = s_1$, since she wins the election and the proposal is efficient. The deviation $s^B = s_1$ is profitable if and only if

$$\pi > \frac{1}{2}\frac{1}{1+\alpha_B(1-\beta)}.$$

Suppose now $\pi > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. Ruled out by Lemma 4.

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A. Ruled out by Lemma 4.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, B, B)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Suppose first that $\pi \leq \frac{1}{2}$. Given (s_5, s_5), all voters abstain. If the office motivated candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \pi > \frac{1}{2}$. Suppose now $\pi > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\pi \leq \frac{1}{2}$. Given (s_5, s_6), $v(s_5, s_6) = (\emptyset, A, B)$. The office motivated candidate A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Consider candidate B who is efficiency concerned. Deviating to $s^B = s^8$ is clearly unprofitable. By deviating candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_1$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed. Candidate B prefers to deviate to s_1 if and only if

$$\pi > \frac{1}{2} \frac{1}{1 + 2\alpha_B(1 - \beta)}$$

Suppose now $\pi > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable

S'_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S''_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. Ruled out by Lemma 4.

S_{13} : All voters abstain and the election is tied. Suppose $\pi \leq \frac{1}{2}$. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Consider candidate B who is efficiency concerned. By deviating candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_1$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed. Candidate B prefers to deviate to $s^B = s_1$ if and only if

$$\pi > \frac{1}{2}\frac{1}{1+3\alpha_B(1-\beta)} \tag{5}$$

Hence there is a profitable deviation for candidate B if the previous condition holds. Suppose now $\pi > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable. ■

Claim 10 *Assume $\pi = 0$, $(\alpha_A, \alpha_B) \in \{(0,1),(1,0)\}$, and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin S_{10} \cup S_{12'}$ where $S'_{12} = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$.*

Proof. Supose without loss of generality that candidate A is office motivated. We assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : In equilibrium the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S'_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S''_2 = Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Given voters' beliefs, candidate A cannot win the election by deviating and candidate B cannot increase her vote margin by deviating.

S'_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S''_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (B, B, B)$. No candidate has a profitable deviation given voters' beliefs.

(s_1, s_8) : Every voter votes for A. No candidate has a profitable deviation given voters' beliefs.

(s_8, s_1) : Every voter votes for B. No candidate has a profitable deviation given voters' beliefs.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. No candidate has a profitable deviation given voters' beliefs.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_7' = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$

and $S_7'' = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. No candidate has a profitable deviation given voters' beliefs.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_8' = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S_8'' = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. If candidate B deviates to any strategy with less projects loses the election.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_9' = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S_9'' = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate B deviates to any strategy with less projects loses the election.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates to s_8 wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Then $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. No candidate has a profitable deviation given voters' beliefs.

Let $S_{12}' = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S_{12}'' = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S_{12}' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If candidate A deviates to s_8, then voter c abstains and therefore candidate A wins with greater margin.

S''_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. No candidate has a profitable deviation given voters' beliefs.

S_{13} : In equilibrium the election is tied; a candidate who deviates to any strategy different than s_8 loses the election. ■

5.2.4 Completion of Proof of Claim 1

Claims 5, 6, 7, 8, 9 and 10 and their proofs characterize the set of pure strategy equilibria for any candidates' motivations $(\alpha^A, \alpha^B) \in \{0,1\}^2$ for any $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$ and any $\pi \in [0,1]$. To conclude the proof of Claim 1 we show in the following lemma that there is no equilibrium in mixed strategies when $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$ and $\pi > \frac{1}{2}$.

Lemma 11 *For any $(\alpha_i, \alpha_j) \in \{0,1\}^2$ and any $\beta \in (\frac{1}{3}, \frac{2}{3})$, if $\pi > \frac{1}{2}$ there is no equilibrium in which a candidate proposes to provide a public good with positive probability.*

Proof. Suppose candidate J plays with positive probability any strategy σ^J different than s_1 and candidate $-J$ plays s_1. If J deviates to s_1 she ties the election when information is revealed, so her utility deviating to s_1 is at least $\frac{\pi}{2}$. If $p^J \neq p_1$ and full information is revealed, candidate J loses the election. If information is not fully revealed, any i such that $p_i^J = 0$ votes for $-J$. If information is not fully revealed and $p_i^J = 1$, voter i votes for J if and only if $\omega_1^{i,J}(1) + 2\omega_2^{i,J}(1) > 3\beta - 1$. In order to win the election, candidate J must satisfy this condition for at least two voters; without loss of generality let them be a and b. But in order for this condition to hold for $i = a$, it must be that $\sigma_2^J > 0$ and in order for it to hold for b, it must be $\sigma_3^J > 0$, and yet neither s_2 nor s_3 is a best response for J, given that J loses choosing s_2 or s_3 regardless of whether information is revealed or not. So J cannot win or tie the election unless $p^J = p_1$. So J should deviate to s_1.

Suppose candidate J plays with positive probability any strategy $\sigma^J \neq s_1$ and candidate $-J$ plays any strategy $\sigma^{-J} \neq s_1$, and assume without loss of generality that in this equilibrium candidate J wins with probability at most one half. If J deviates to s_1 and information is fully revealed, J wins the election. Hence by deviating, J wins with probability $\pi > \frac{1}{2}$ so J is better off deviating to s_1. ■

5.3 Characterization of Pure Equilibria for $\beta \in \left(\frac{2}{3}, 1\right)$

In this subsection we first characterize the set of pure strategy equilibria with two office-motivated candidates, then with two efficiency concerned candidates, and finally with one purely office-motivated and one efficiency concerned, always assuming that projects are not very inefficient. Proposition 3 follows as a corollary from these characterizations, as does the main statement in Proposition 2.

5.3.1 Office Motivated Candidates

Proposition 12 *Assume* $(\alpha_A, \alpha_B) = (0, 0)$ *and* $\beta \in \left(\frac{2}{3}, 1\right)$. *For any* $\pi \in \left(0, \frac{1}{2}\right)$, *an equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 11, 13\}$; *for* $\pi = \frac{1}{2}$, *an equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 10, 11, 13\}$; *and for any* $\pi \in \left(\frac{1}{2}, 1\right]$ *there is no equilibrium in pure strategies.*

Proof. S_1 :Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$ and full information is not revealed, J loses the election. If $\pi \leq \frac{1}{2}$, no gain when full information is revealed can compensate for this loss. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Suppose now that $\pi > \frac{1}{2}$. If candidate A deviates and proposes $s^A = s_5$, she wins the election with probability π and therefore the deviation is profitable.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. By Lemma 4, this cannot occur in equilibrium.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Every voter votes for A. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate J deviates to $s^J = s_8$ wins the election both in case the information is revealed and in case it is not.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$ voters a and c vote for A both in case full information is revealed and in case it is not revealed. Hence by deviating, A wins with for sure.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. Suppose first that $\pi < \frac{1}{2}$. If candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \pi > \frac{1}{2}$. If $\pi = \frac{1}{2}$ then voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_2^{i,J}(0) = 1$ for $i = a, b$ and $J = A, B$ make the election tied and if candidate J deviates to any $s^J \neq s_5$, then J wins the election with probability at most $\frac{1}{2}$ and with complementary she loses the election. Hence no deviation is profitable. If $\pi > \frac{1}{2}$ then candidate A can win the election with probability π deviating to $s^A = s_2$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given $(s^A, s^B) = (s_5, s_6)$, beliefs such that $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$ support an equilibrium in which $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. Suppose $\pi \leq \frac{1}{2}$. It suffices to check that A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. If $\pi > \frac{1}{2}$ candidate A can win the election with probability π deviating to $s^A = s_2$.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} :Voter strategy $s^i = (\emptyset, B, A, \emptyset)$ for each voter i and beliefs such that $\omega_2^{i,J}(0) = \omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied. Suppose $\pi \leq \frac{1}{2}$; if candidate J deviates to any strategy $s^J \neq s_8$ and full information is not revealed, J loses the election. Suppose $\pi > \frac{1}{2}$; candidate J wins the election probability π deviating to $s^J = s_1$. ■

Claim 13 *Assume* $\pi = 0$, $(\alpha_A, \alpha_B) = (0, 0)$ *and* $\beta \in \left(\frac{2}{3}, 1\right)$. *An equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 2, 3, 4, 8, 11, 13\}$.

Proof. For each strategy pair class, we find whether an element of the class can be sustained in equilibrium. To sustain equilibria, we assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, $v(s_1, s_5) = (B, B, A)$. Given the beliefs, neither candidate can improve her electoral outcome by deviating.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs, in equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If A deviates to $s^A = s_6$,only voter c observes the deviation so $v(s_6, s_2) = (\emptyset, \emptyset, A)$ and candidate A wins the election.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$, only voter c observes the deviation, so $v(s_6, s_2) = (A, B, A)$ and candidate A wins the election.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. If B deviates to $s^B = s_7$, only voter c observes the deviation, so $v(s_2, s_7) = (A, B, B)$ and candidate B finds the deviation profitable.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. In equilibrium in which $v(s_2, s_7) = (A, B, B)$. It is easy to check that no candidate can improve her electoral outcome with any deviation.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate A deviates to s_8, voters b and c either vote for candidate A or abstains, depending upon their beliefs, and therefore candidate A wins the election (voter $a's$ beliefs do not change).

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates and proposes s_8, beliefs of voters a and b are unaffected, while voter c votes for candidate A for all possible beliefs over candidate $A's$ strategy. Therefore candidate A wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. In equilibrium $v(s_5, s_6) = (\emptyset, A, B)$ and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If A deviates to s_8, voter c either votes for A as well, or abstains, hence A is better off deviating because A increases the vote margin.

S_{13} : In equilibrium the election is tied, and if candidate J deviates to any strategy $s^J \neq s_8$, J loses the election. ■

5.3.2 Efficiency Concerned Candidates

The special case $\alpha_A = \alpha_B = 1$ corresponds to Proposition 2.

Proposition 14 *Assume $(\alpha_A, \alpha_B) \in \mathbb{R}^2_{++}$, and $\beta \in \left(\frac{2}{3}, 1\right)$. Without loss of generality let $\alpha_A \leq a_B$. The equilibrium in which candidates use the strategy pair (s_1, s_1) exists if and only if $\pi \leq \frac{1+2\alpha_A(1-\beta)}{2}$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_5$ exists if and only if $\left(\pi \leq \frac{1}{2+2\alpha_B(1-\beta)} \text{ and } \alpha_A \geq \frac{1}{1-\beta}\right)$; the equilibrium in which candidates use the strategy*

$(s^A, s^B) \in S_{10}$ exists if and only if $\pi = \frac{1+\alpha_B(1-\beta)}{2+4\alpha_B(1-\beta)}$ and $a_A = \alpha_B$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{11}$ exists if and only if $\pi \leq \frac{1+\alpha_B(1-\beta)}{2+4\alpha_B(1-\beta)}$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{13}$ exists if and only if $\pi \leq \frac{1}{2+6\alpha_B(1-\beta)}$; and there is no other pure strategy equilibrium.

Proof. Without loss of generality, assume $\alpha_A \leq \alpha_B$. As before, to sustain equilibria, we assume off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$ and full information is not revealed, J loses the election. If $\pi < \frac{1}{2}$, no hypothetical gain when full information is revealed can compensate for this loss. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Suppose that $\pi \geq \frac{1}{2}$. If candidate J deviates to $s^J \in \{s_2, s_3, s_4, s_8\}$, J loses the election with a more inefficient proposal. If candidate J deviates to $s^J = s_5$ (or to $s^J = s_6$, or $s^J = s_7$) wins the election with probability π (when full information is revealed) and therefore the deviation is profitable if and only if

$$\pi \frac{1}{1 + 2\alpha_J(1 - \beta)} > \frac{1}{2}, \text{ or}$$
$$\pi > \frac{1 + 2\alpha_J(1 - \beta)}{2}.$$

Therefore, (s_1, s_1) can be sustained in equilibrium if and only if

$$\pi \leq \frac{1 + 2\alpha_J(1 - \beta)}{2} \text{ for } J \in \{A, B\}.$$

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. By Lemma 4, this cannot occur in equilibrium.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs such that $\omega_2^{i,B}(0) = 1$ for all $i \in \{a, b, c\}$, every voter votes for A. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate J deviates to $s^J = s_8$, then candidate J wins the election. The deviation is profitable if only if

$$\begin{aligned} \frac{1}{1+3\alpha_J(1-\beta)} &> \frac{1}{2}\frac{1}{1+\alpha_J(1-\beta)} \\ 1 &> \alpha_J(1-\beta). \end{aligned}$$

If candidate J deviates to $s^J = s_1$, she wins if the deviation is observed, hence the deviation is profitable if and only if

$$\pi > \frac{1}{2+2\alpha_J(1-\beta)}.$$

Hence the (s_2, s_2) can only be sustained if $\alpha_A \geq \frac{1}{1-\beta}$ and $\pi \leq \frac{1}{2+2\alpha_B(1-\beta)}$.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. If candidate A deviates to $s^A = s_6$, then candidate A wins the election. The deviation is profitable since for all β, we have that

$$\frac{1}{1+2\alpha_A(1-\beta)} > \frac{1}{2}\frac{1}{1+\alpha_A(1-\beta)}.$$

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Consider the deviation $s^J = s_8$. If information is not fully revealed candidate J wins the election because voter c votes for J. If information is fully revealed candidate J loses the election. Hence, candidate J prefers to deviate if and only if

$$(1-\pi)\frac{1}{1+3\alpha_J(1-\beta)} > \frac{1}{2}\frac{1}{1+2\alpha_J(1-\beta)}, \text{ or } \pi < \frac{1+\alpha_J(1-\beta)}{2+4\alpha_J(1-\beta)}.$$

Consider the deviation $s^J = s_2$. If information is not fully revealed candidate J loses the election because voter b votes for J. If information is fully revealed candidate J wins the election. Hence, candidate J prefers to deviate if and only if

$$\pi\frac{1}{1+\alpha_J(1-\beta)} > \frac{1}{2}\frac{1}{1+2\alpha_J(1-\beta)}, \text{ so } \pi > \frac{1+\alpha_J(1-\beta)}{2+4\alpha_J(1-\beta)}.$$

Hence there is always profitable deviation for all $\pi \neq \frac{1+\alpha_J(1-\beta)}{2+4\alpha_J(1-\beta)}$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. If candidate J deviates to $s^J \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, J loses the election. If J deviates to $s^J = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, J loses the election. It follows that the best deviation for an efficiency concerned candidate is s_2 since candidate J wins the election when information is fully revealed and minimizes the number of proposed projects (if J proposes s_1 she loses the election). Candidate J prefers to deviate to $s^J = s_2$ if only if

$$\pi\frac{1}{1+\alpha_J(1-\beta)} > \frac{1}{2}\frac{1}{1+2\alpha_J(1-\beta)}, \text{ so } \pi > \frac{1+\alpha_J(1-\beta)}{2+4\alpha_J(1-\beta)}. \tag{6}$$

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} : If candidate J deviates to any strategy $s^J \neq s_8$ and full information is not revealed, J loses the election. Hence the best deviation for an efficiency concerned candidate is s_1 because J wins the election when information is fully revealed and the proposal is efficient. Candidate J prefers to deviate to s_1 if only if

$$\pi > \frac{1}{2+6\alpha_J(1-\beta)}. \tag{7}$$

■

It follows that for any $(\alpha_A, \alpha_B) \in \mathbb{R}^2_{++}$, any $\beta \in \left(\frac{2}{3}, 1\right)$ and any $\pi \in \left(\max\left\{\frac{1+\alpha_B(1-\beta)}{2+4\alpha_B(1-\beta)}, \frac{1}{2+6\alpha_B(1-\beta)}\right\}, \frac{1}{2} + c\right.$ the unique candidate equilibrium is (s_1, s_1), and since for any $(\alpha_A, \alpha_B) \in \mathbb{R}^2_{++} \in \mathbb{R}_{++}$ and any $\beta \in \left(\frac{2}{3}, 1\right)$, $\frac{1+\alpha_B(1-\beta)}{2+4\alpha_B(1-\beta)} < \frac{1}{2}$ and $\frac{1}{2+6\alpha_B(1-\beta)} < \frac{1}{2} < \frac{1}{2} + \alpha_A(1-\beta)$, it follows that the uniqueness interval is never empty. Notice as well that the interval's size increases in (α_A, α_B).

Claim 15 *Assume $\pi = 0$, $(\alpha_A, \alpha_B) = (1,1)$ and $\beta \in \left(\frac{2}{3}, 1\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \in S_k$ for some $k \in \{1, 2, 3, 4, 8, 11, 12, 13\}$.*

Proof. To sustain equilibria, we assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, $v(s_1, s_5) = (B, B, A)$. Given the beliefs, neither candidate can improve her electoral outcome by deviating.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs, in equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If A deviates to $s^A = s_6$,only voter c observes the deviation so $v(s_6, s_2) = (\emptyset, \emptyset, A)$ and candidate A wins the election. Candidate A prefers to deviate if and only if $\frac{1}{1+2(1-\beta)} > \frac{1}{2}\frac{1}{1+(1-\beta)}$which holds for all $\beta < 1$.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$, only voter c observes the deviation, so $v(s_6, s_2) = (A, B, A)$ and candidate A wins the election. As proved above, all candidate candidate A finds the deviation profitable.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. If B deviates to $s^B = s_7$, only voter c observes the deviation, so $v(s_2, s_7) = (A, B, B)$ and candidate B finds the deviation profitable.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. In equilibrium in which $v(s_2, s_7) = (A, B, B)$. It is easy to check that no candidate can improve her electoral outcome with any deviation.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate A deviates to s_8, voters b and c abstain and therefore candidate A wins the election (voter $a's$ beliefs do not change), so candidate $A's$ finds profitable to deviate.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates and proposes s_8, beliefs of voters a and b are unaffected, while voter c votes for candidate A for all possible beliefs over candidate $A's$ strategy. Therefore candidate A wins the election. Candidate A finds profitable to deviate if and only if $\frac{1}{1+(1-\beta)3} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$ which holds for all $\beta < 1$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. In equilibrium $v(s_5, s_6) = (\emptyset, A, B)$ and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. By deviating candidate B cannot increase the number of votes he gets, due to voters' beliefs. For candidate A, the unique strategy that increases A's vote margin is s_8 because voter c would abstain, but an efficiency concerned candidate does not find this deviation profitable.

S_{13} : In equilibrium the election is tied, and if candidate J deviates to any strategy $s^J \neq s_8$, J loses the election. ■

5.3.3 One Purely Office Motivated, One Efficiency Concerned Candidate

Proposition 16 *Assume $\alpha_J \in \mathbb{R}_+$, $\alpha_{-J} = 0$ and $\beta \in \left(\frac{2}{3}, 1\right)$. There exist a cutoff function $\pi_1(\beta)$ such that $0 < \pi_1(\beta) < \frac{1}{2}$ and:*

if $\pi \in [0, \pi_1(\beta)]$, there exist multiple pure equilibria;

if $\pi \in (\pi_1(\beta), \frac{1}{2}]$, there is a unique pure strategy equilibrium in which both candidates propose the efficient policy; and

if $\pi > \frac{1}{2}$ there is no pure strategy equilibrium.

Proof. Without loss of generality, let assume that candidate A is office motivated, $\alpha_A = 0$, $\alpha_B > 0$.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. If $\pi \leq \frac{1}{2}$, no gain when full information is revealed can compensate for this loss, and it is straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium if $\pi \leq \frac{1}{2}$. If $\pi > \frac{1}{2}$, if A deviates to $s^A = s_5$ she wins the election with probability π (when full information is revealed) and therefore such deviation is profitable for the office motivated candidate A.

The set S_2 contains profiles which are not strategically equivalent when candidates have different motivations. We must partition the set S_2 into the following subsets: $S_2' = \{(s_1, s_2), (s_1, s_3), (s_1, s_4)\}$ and $S_2'' = \{(s_2, s_1), (s_3, s_1), (s_4, s_1)\}$.

S_2' : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_2'' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Ruled out by Lemma 4.

Similary we partition S_3 into $S_3' = \{(s_1, s_5), (s_1, s_6), (s_1, s_7)\}$ and $S_3'' = \{(s_5, s_1), (s_6, s_1), (s_7, s_1)\}$.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (A, A, B)$. Ruled out by Lemma 4.

(s_1, s_8) : Every voter votes for A. Ruled out by Lemma 4.

(s_8, s_1) : Every voter votes for B. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate A deviates to $s^J = s_8$, she wins the election whether or not full information is revealed.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$ both in case full information is revealed and in case it not revealed, A wins the election. Hence the deviation is profitable for all $\pi \geq 0$.

Let $S_7' = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7)\}$

and $S_7'' = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. Ruled out by Lemma 4.

Let $S_8' = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S_8'' = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$.

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. Ruled out by Lemma 4.

Let $S_9' = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S_9'' = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$.

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. Ruled out by Lemma 4.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, B, A)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. Suppose first that $\pi < \frac{1}{2}$. If candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \pi > \frac{1}{2}$. Suppose that $\pi > \frac{1}{2}$. If A deviates to $s^A = s_2$ then A wins the election when information is fully revealed. Hence the deviation is profitable. If $\pi = \frac{1}{2}$, candidate B can deviate to $s^B = s_1$ and win when information is revealed.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\pi \leq \frac{1}{2}$. Given $(s^A, s^B) = (s_5, s_6)$, beliefs such that $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$ support an equilibrium in which $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. Consider first the office motivated candidate A. It suffices to check that A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Consider now candidate B who, by assumption, is efficiency concerned. Deviating to $s^B = s^8$ is clearly unprofitable. Playing any other deviation candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_2$ because it minimizes the inefficiency subject to winning when information is fully revealed (playing $s^B = s_3$ gives the same payoff as s_2). Candidate B prefers to deviate to s_2 if and only if

$$\pi\frac{1}{1+\alpha_B(1-\beta)} > \frac{1}{2}\frac{1}{1+2\alpha_B(1-\beta)}, \text{ so } \pi > \frac{1+\alpha_B(1-\beta)}{2+4\alpha_B(1-\beta)}. \tag{8}$$

Hence there is a profitable deviation for candidate B if the previous condition holds.

Let $S'_{12} = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S''_{12} = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S'_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. Ruled out by Lemma 4.

S''_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. Ruled out by Lemma 4.

S_{13} : Let voter strategy $s^i = (\emptyset, B, A, \emptyset)$ for each voter i and beliefs such that $\omega_2^{i,J}(0) = \omega_2^{i,J}(1) = 1$ for any voter i. Suppose first that $\pi \leq \frac{1}{2}$. In equilibrium the election is tied; consider the office motivated candidate A. If candidate A deviates to any strategy $s^A \neq s_8$ and full information is not revealed, A loses the election. Consider the efficiency concerned candidate B. If information is not fully revealed, candidate B loses the election if she deviates. If information is fully revealed, the unique profitable deviation is $s^B = s_1$. Candidate B prefers to deviate if and only if

$$\pi > \frac{1}{2+6\alpha_B(1-\beta)}. \tag{9}$$

Suppose that $\pi > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, then A wins the election when full information is revealed. Hence the deviation is profitable. ■

Claim 17 *Assume* $\pi = 0$, $(\alpha_A, \alpha_B) \in \{(1,0), (0,1)\}$, *and* $\beta \in \left(\frac{2}{3}, 1\right)$. *An equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 2, 3, 4, 8, 11, 13\}$ *or* $(s^A, s^B) \in \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$.

Proof. Supose without loss of generality that candidate A is office motivated. We assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1-s_i^J)=1$ for each $i \in \{a,b,c\}$ and $J \in \{A,B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i=(\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2' : Assume without loss of generality that $(s^A, s^B)=(s_1,s_2)$. Given (s_1,s_2), $v(s_1,s_2)=(B,A,A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S_2'' = Assume without loss of generality that $(s^A, s^B)=(s_2,s_1)$. Given (s_2,s_1), $v(s_2,s_1)=(A,B,B)$. Given voters' beliefs, candidate A cannot win the election by deviating and candidate B cannot increase her vote margin by deviating.

S_3' : Assume w.l.o.g. that $(s^A, s^B)=(s_1,s_5)$. Given (s_1,s_5), $v(s_1,s_5)=(B,B,A)$. Candidate A cannot win the election deviating to any strategy $s_A \neq s_1$. Candidate B is efficiency concerned so he might be interested in deviating to reduce the number of projects he proposes. By deviating to s_2, the election is tied: candidate B prefers to deviate if and only if $\frac{1}{2}\frac{1}{1+(1-\beta)} > \frac{1}{1+2(1-\beta)}$, but this condition never occurs. By deviating to s_1 candidate B loses the election so this deviation is never profitable.

S_3'' : Assume w.l.o.g. that $(s^A, s^B)=(s_5,s_1)$. Given (s_5,s_1), $v(s_5,s_1)=(A,A,B)$. No candidate has a profitable deviation given voters' beliefs.

(s_1,s_8) : Every voter votes for A. No candidate has a profitable deviation given voters' beliefs.

(s_8,s_1) : Every voter votes for B. No candidate has a profitable deviation given voters' beliefs.

S_5 : Assume w.l.o.g. that $(s^A, s^B)=(s_2,s_2)$. Given (s_2,s_2), every voter i abstains. If candidate A deviates to s_8 wins the election.

S_6 : Assume w.l.o.g. that $(s^A, s^B)=(s_2,s_3)$. Given (s_2,s_3), $v(s_2,s_3)=(A,B,\emptyset)$. If A deviates to $s^A=s_6$ both in case full information is revealed and in case it not revealed, A wins the election.

Let $S_7' = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$

and $S_7'' = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. No candidate has a profitable deviation given voters' beliefs.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_8' = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S_8'' = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. If candidate B deviates to any strategy with less projects loses the election.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_9' = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S_9'' = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate B deviates to any strategy with less projects loses the election.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates to s_8 wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Then $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. No candidate has a profitable deviation given voters' beliefs.

Let $S_{12}' = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S_{12}'' = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S_{12}' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If candidate A deviates to s_8, then voter c abstains and therefore candidate A wins with greater margin.

S_{12}'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. No candidate has a profitable deviation given voters' beliefs, because candidate B is efficiency concerned and does not find deviation s_8 profitable even if he would win with a greater vote margin.

S_{13} : In equilibrium the election is tied; if a candidate deviates to any strategy different than s_8 loses the election. ■

5.4 Symmetric Mixed Strategy Equilibria

We characterize the set of symmetric mixed strategy equilibria for the set of parameters such that the efficient policy (no pork) cannot be sustained in equilibrium, first given office motivated candidates, then given efficiency concerned candidates with $\alpha_A = \alpha_B = 1$. Let $s_L = (0, 1/3, 1/3, 1/3, 0, 0, 0, 0)$ and $s_H = (0, 0, 0, 0, 1/3, 1/3, 1/3, 0)$ be the special mixed strategies that consist, respectively, on proposing exactly one project and randomizing which one, and proposing exactly two projects and randomizing which two.

5.4.1 Office Motivated Candidates

Assume $(\alpha^A, \alpha^B) = (0, 0)$. From Claim 1 and Proposition 3, the set of parameters values for which the efficient outcome cannot be sustained in equilibrium is $\left\{(\pi, \beta) : \pi \in \left(\frac{1}{2}, 1\right] \text{ and } \beta \in \left(\frac{2}{3}, 1\right)\right\}$. Assume that parameter values are within this set. The next claims characterize the set of symmetric mixed equilibria for this range of parameter values.

Claim 18 *If $\pi \in \left(\frac{1}{2}, \frac{3}{4}\right)$, candidate strategies (s_H, s_H) are supported in equilibrium. Conversely, in any symmetric equilibrium, $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$ for $J \in \{A, B\}$.*

Proof. First we prove that (s_H, s_H) can be supported in equilibrium. Suppose (s_H, s_H) is played, for any candidate J and voter i, if i does not observe the full proposals, beliefs are such that $\omega_1^{i,J}(1) = \omega_2^{i,J}(0) = 1$ and hence $s^i(1, 0) = A$, $s^i(0, 1) = B$ and $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$. Therefore, given that (s_H, s_H) is played as expected by voters, either all voters abstain if $p^A = p^B$ or one voter votes for A, one for B and one abstains if $p^A \neq p^B$; in either case the election is tied and the probability that each candidate is elected is $\frac{1}{2}$, so the expected payoff for each candidate is $\frac{1}{2}$. Suppose A deviates to s_1, A loses $0 - 2$ if Nature does not reveal p, and A loses $1 - 2$ if Nature reveals p. Suppose A deviates to $s_k \in \{s_2, s_3, s_4\}$. If Nature reveals p, with probability $\frac{2}{3}$ A wins and with probability $\frac{1}{3}$ A loses; while if p is not revealed, A loses for sure. Hence, A wins with probability $\frac{2}{3}\pi$, the expected utility deviating is $\frac{2}{3}\pi$ and the deviation is profitable if and only if $\frac{2}{3}\pi > \frac{1}{2}$, that is, $\pi > \frac{3}{4}$. Suppose A deviates to $s_k \in \{s_5, s_6, s_7\}$. Then A achieves the same expected electoral outcomes and utilities as not deviating. Suppose A deviates to s_8. If Nature reveals p, A loses $1-2$. If Nature does not reveal p, then A wins $2-1$. But Nature reveals p with probability π, hence the expected utility for A is $1-\pi$ which is less than $\frac{1}{2}$ for any $\pi > \frac{1}{2}$. Hence, there is no profitable deviation.

Second, we prove that in any symmetric equilibrium both candidates propose two projects.

Suppose (σ^J, σ^J) is part of a symmetric equilibrium. Since the equilibrium is symmetric, for any $i \in \{a, b, c\}$, if i does not observe p, then $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$ and furthermore, for $k \in \{0, 1\}$, $s^i(k, 1-k) = A$ if and only if $s^i(1-k, k) = B$. Suppose $s^i(1, 0) \neq A$, so $s^i(0, 1) \neq B$. Then s_8 is not a best response and is not played in equilibrium. But if s_8 is not played, the expected payoff for i if A wins is strictly higher than if B wins given $(p_i^A, p_i^B) = (1, 0)$, so by assumption $s^i(1, 0) = A$, a contradiction. Thus, it must be $s^i(1, 0) = A$ and $s^i(0, 1) = B$. Then, given that $\pi \in (\frac{1}{2}, \frac{3}{4})$, for any strategy σ^{-J}, a best response by J must propose two projects. Thus no strategy that proposes any other number of projects can be used in a symmetric equilibrium.

■

Claim 19 *If $\pi \in \left[\frac{3}{4}, \frac{11+\sqrt{61}}{20}\right]$, there is a symmetric mixed strategy equilibrium, in which*

$$\sigma^J = \left(0, \frac{2\pi-1}{10\pi-3}, \frac{2\pi-1}{10\pi-3}, \frac{2\pi-1}{10\pi-3}, \frac{1}{10\pi-3}, \frac{1}{10\pi-3}, \frac{1}{10\pi-3}, \frac{4\pi-3}{10\pi-3}\right)$$

and if $\pi \in \left(\frac{3}{4}, \frac{11+\sqrt{61}}{20}\right)$, this is the unique symmetric equilibrium. The expected number of projects is weakly larger than $\frac{5}{3}$ and converges to $\frac{5}{3}$ as $\pi \to \frac{3}{4}$.

If $\pi \in \left[\frac{11+\sqrt{61}}{20}, 1\right]$, there is a symmetric mixed strategy equilibrium, in which

$$\sigma^J = \left(\frac{4\pi-3}{10\pi-3}, \frac{1}{10\pi-3}, \frac{1}{10\pi-3}, \frac{1}{10\pi-3}, \frac{2\pi-1}{10\pi-3}, \frac{2\pi-1}{10\pi-3}, \frac{2\pi-1}{10\pi-3}, 0\right)$$

and if $\pi \in \left(\frac{11+\sqrt{61}}{20}, 1\right)$, this is the unique symmetric equilibrium. The expected number of projects is weakly larger than $\frac{9}{7}$ and converges to $\frac{9}{7}$ as $\pi \to 1$.

Proof. Let (σ^A, σ^B) be a symmetric candidate strategy profile so $\sigma^A = \sigma^B$. Since the candidates' strategies are symmetric, voters' strategies must be such that $s^i(k,k) = \varnothing$ for $k \in \{0,1\}$, and $\{s^i(1,0) = A$ and $s^i(0,1) = B\}$ or $\{s^i(1,0) = B$ and $s^i(0,1) = A\}$ or $\{s^i(1,0) = \varnothing$ and $s^i(0,1) = \varnothing\}$ for every voter i. Suppose not $\{s^i(1,0) = A$ and $s^i(0,1) = B\}$. Then, given any strategy σ^{-J}, candidate J obtains a greater expected payoff playing s_1 than playing s_8, and a strictly greater payoff if $\sigma_8^{-J} > 0$. Therefore, in a symmetric mixed equilibrium, $\sigma_8^J = 0$. Then, it follows that for any voter i who observes $p_i^J = 1$ and $p_i^{-J} = 0$, the expected payoff for voter i is greater if J wins, thus by assumption, i votes J. Therefore, $s^i(1,0) = A$ and $s^i(0,1) = B$.

Next we prove that in any symmetric mixed strategy equilibrium, $\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} > 0$. Suppose not. Notice that given $\pi > \frac{3}{4}$, $s^i(1,0) = A$ and $s^i(0,1) = B$, if $-J$ proposes one project and J proposes two projects, in expectation J wins the election more often, whereas if J proposes two and $-J$ proposes zero or three, J wins more often. So if $-J$ never proposes one project, proposing two projects in expectation defeats any other proposal with probability more than one half. Then any best response by J to σ^{-J} with $\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} = 0$ must be such that $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$, which in turns means that any best response by J must be such that $\sigma_2^J + \sigma_3^J + \sigma_4^J = 1$, a contradiction.

Similarly, in any symmetric mixed strategy equilibrium, $\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} > 0$. Suppose not. Any best response by J must be such that $\sigma_2^J + \sigma_3^J + \sigma_4^J = 0$, but then the best response by $-J$ must be $\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} = 1$, a contradiction.

Therefore, in any symmetric mixed strategy equilibrium, both candidates propose one project, and two projects, with positive probability. But then, it must be that $\sigma_2^J = \sigma_3^J + \sigma_4^J$ and $\sigma_5^J = \sigma_6^J = \sigma_7^J$. Given that the randomization between districts, subject to choosing a number of projects, assigns equal weight to all districts, we can reduce the strategic problem to that of assigning weights to strategies s_1, s_8, s_L, s_H. The payoff matrix is as follows:

$$\left\{\begin{array}{ccccc} & s_1 & s_L & s_H & s_8 \\ s_1 & \frac{1}{2},\frac{1}{2} & \pi, 1-\pi & 0,1 & \pi, 1-\pi \\ s_L & 1-\pi, \pi & \frac{1}{2},\frac{1}{2} & \frac{2}{3}\pi, 1-\frac{2\pi}{3} & 0,1 \\ s_H & 1,0 & 1-\frac{2\pi}{3}, \frac{2\pi}{3} & \frac{1}{2},\frac{1}{2} & \pi, 1-\pi \\ s_8 & 1-\pi, \pi & 1,0 & 1-\pi, \pi & \frac{1}{2},\frac{1}{2} \end{array}\right\}$$

An equilibrium with $\sigma_1^J > 0, \sigma_L^J > 0, \sigma_H^J > 0$ and $\sigma_8^J = 0$ must satisfy

$i)$ $\frac{1}{2}\sigma_1^J + \pi\sigma_L^J = (1-\pi)\sigma_1^J + \frac{1}{2}\sigma_L^J + \frac{2}{3}\pi(1-\sigma_1^J - \sigma_L^J)$
$ii)$ $\frac{1}{2}\sigma_1^J + \pi\sigma_L^J = \sigma_1^J + (1-\frac{2\pi}{3})\sigma_L^J + \frac{1}{2}(1-\sigma_1^J - \sigma_L^J)$
$iii)$ $\frac{1}{2}\sigma_1^J + \pi\sigma_L^J \geq (1-\pi)\sigma_1^J + \sigma_L^J + (1-\pi)(1-\sigma_1^J - \sigma_L^J)$.
Dropping the superindex and simplifying $i)$ we obtain
$0 = \frac{2}{3}\pi + \frac{1}{2}\sigma_1 + \frac{1}{2}\sigma_L - \frac{5}{3}\pi\sigma_1 - \frac{5}{3}\pi\sigma_L,$
$(10\pi - 3)\sigma_1 = 4\pi + (3 - 10\pi)\sigma_L$

$$\sigma_1 = \frac{4\pi}{10\pi - 3} - \sigma_L.$$

Dropping the superindex and simplifying $ii)$ we obtain $\pi\sigma_L = \frac{1}{2}\sigma_L - \frac{2}{3}\pi\sigma_L + \frac{1}{2}$ and with simple computation

$$\sigma_L = \frac{3}{10\pi - 3}.$$

Thus

$$\sigma_1 = \frac{4\pi - 3}{10\pi - 3}$$

Then simplifying inequality $iii)$:

$$\frac{1}{2}\sigma_1 + \pi\sigma_L \geq \sigma_1 - \pi\sigma_1 + \sigma_L + 1 - \sigma_1 - \sigma_L - \pi + \pi\sigma_1 + \pi\sigma_L$$
$$\frac{1}{2}\sigma_1 \geq 1 - \pi$$

and substituting the value of σ_1 :

$$\frac{4\pi - 3}{20\pi - 6} \geq 1 - \pi$$

The previous expression holds as an equality for $\pi = \frac{11+\sqrt{61}}{20}$ (for $\pi \in (\frac{2}{3}, 1)$. So for $\pi > \frac{11+\sqrt{61}}{20}$, the above mix is an equilibrium, with expected number of projects $\frac{3}{10\pi-3} + 2(1 - \frac{3}{10\pi-3} - \frac{4\pi-3}{10\pi-3}) = \frac{12\pi-3}{10\pi-3}$. The probability of proposing two projects is $\frac{10\pi-3-4\pi}{10\pi-3} = \frac{6\pi-3}{10\pi-3}$ which converges to $\frac{9}{7}$ as π converges to 1. The initial assumption that voters vote $s^i(1,0) = A$ and $s^i(0,1) = B$ is supported because $\sigma_8 = 0$ and $\beta > 2/3$.

If instead $\pi < \frac{11+\sqrt{61}}{20}$, this equilibrium does not exist. We look then for a fully mixed equilibrium, which must satisfy:

i) $\frac{1}{2}\sigma_1 + \pi\sigma_L + \pi(1-\sigma_1-\sigma_L-\sigma_H) = (1-\pi)\sigma_1 + \frac{1}{2}\sigma_L + \frac{2}{3}\pi\sigma_H$

ii) $\frac{1}{2}\sigma_1 + \pi\sigma_L + \pi(1-\sigma_1-\sigma_L-\sigma_H) = \sigma_1 + (1-\frac{2\pi}{3})\sigma_L + \frac{1}{2}\sigma_H + \pi(1-\sigma_1-\sigma_L-\sigma_H)$

iii) $\frac{1}{2}\sigma_1 + \pi\sigma_L + \pi(1-\sigma_1-\sigma_L-\sigma_H) = (1-\pi)\sigma_1 + \sigma_L + (1-\pi)\sigma_H + \frac{1}{2}(1-\sigma_1-\sigma_L-\sigma_H)$

Simplifying the first equation

$\pi = \frac{1}{2}\sigma_1 + \frac{1}{2}\sigma_L + \frac{5}{3}\pi\sigma_H$

i) $3\sigma_1 + 3\sigma_L + 10\pi\sigma_H = 6\pi$

Simplifying the second equation

$\pi\sigma_L = \frac{1}{2}\sigma_1 + (1-\frac{2\pi}{3})\sigma_L + \frac{1}{2}\sigma_H$

$0 = \frac{1}{2}\sigma_1 + (1-\frac{5\pi}{3})\sigma_L + \frac{1}{2}\sigma_H$

ii) $3\sigma_1 + (6-10\pi)\sigma_L + 3\sigma_H = 0$

Simplifying the third equation

$\pi = \frac{1}{2}\sigma_L + \frac{1}{2}\sigma_H + \frac{1}{2}$

iii) $\sigma_L + \sigma_H = 2\pi - 1$

From *iii)*

$$\sigma_H = 2\pi - 1 - \sigma_L.$$

From *i)* $-$ *ii)*

$$3\sigma_L + 10\pi\sigma_H - (6-10\pi)\sigma_L - 3\sigma_H = 6\pi$$

$$(10\pi - 3)(\sigma_H + \sigma_L) = 6\pi$$

$$\sigma_H = \frac{6\pi}{10\pi - 3} - \sigma_L.$$

The two equalities together imply that

$$\frac{6\pi}{10\pi - 3} = 2\pi - 1$$

$$\pi = \frac{1}{20}\sqrt{61} + \frac{11}{20}.$$

Which means that the fully mixed equilibrium is non-generic. Consider equilibria such that $\sigma_1 = 0$, so that candidates mix between proposing one, two and three projects. An equilibrium with these characteristics requires:

$$\begin{Bmatrix} \frac{1}{2}, \frac{1}{2} & \pi, 1-\pi & 0, 1 & \pi, 1-\pi \\ 1-\pi, \pi & \frac{1}{2}, \frac{1}{2} & \frac{2}{3}\pi, 1-\frac{2\pi}{3} & 0, 1 \\ 1, 0 & 1-\frac{2\pi}{3}, \frac{2\pi}{2} & \frac{1}{2}, \frac{1}{2} & \pi, 1-\pi \\ 1-\pi, \pi & 1, 0 & 1-\pi, \pi & \frac{1}{2}, \frac{1}{2} \end{Bmatrix}$$

$i)$ $\pi\sigma_L + \pi(1-\sigma_L-\sigma_H) \leq \frac{1}{2}\sigma_L + \frac{2}{3}\pi\sigma_H$

$ii)$ $\frac{1}{2}\sigma_L + \frac{2}{3}\pi\sigma_H = (1-\frac{2\pi}{3})\sigma_L + \frac{1}{2}\sigma_H + \pi(1-\sigma_L-\sigma_H)$

$iii)$ $\frac{1}{2}\sigma_L + \frac{2}{3}\pi\sigma_H = \sigma_L + (1-\pi)\sigma_H + \frac{1}{2}(1-\sigma_L-\sigma_H)$.

Simplifying the three expressions:

i) $\pi \leq \frac{1}{2}\sigma_L + \frac{5}{3}\pi\sigma_H$

ii) $(\frac{5\pi}{3} - \frac{1}{2})(\sigma_L + \sigma_H) = \pi$

iii) $(\frac{5}{3}\pi - \frac{1}{2})\sigma_H = \frac{1}{2}$.

From iii) and ii)

$(\frac{5\pi}{3} - \frac{1}{2})\sigma_L = \pi - \frac{1}{2}$, so $\sigma_L = \frac{6\pi-3}{10\pi-3}$ and $\sigma_H = \frac{3}{10\pi-3}$ and thus $\sigma_8 = \frac{4\pi-3}{10\pi-3}$.

We then check that the first inequality is satisfied:

$$\pi \leq \frac{1}{2}\frac{6\pi-3}{10\pi-3} + \frac{5}{3}\pi\frac{3}{10\pi-3}$$

$$2\pi(10\pi-3) \leq 6\pi - 3 + 10\pi$$

$$20\pi^2 - 22\pi + 3 \leq 0$$

which is true for $\pi \in (\frac{3}{4}, \frac{11+\sqrt{61}}{20})$. The expected number of projects in this equilibrium is $\frac{18\pi-6}{10\pi-3}$ which converges to $\frac{5}{3}$ as $\pi \to \frac{3}{4}$ and then increases but stays below $\frac{7}{4}$. Finally, there cannot be a symmetric mixed strategy equilibrium such that $\sigma_1 = \sigma_8 = 0$, because if $\sum_{k=2}^{7} \sigma_k^J = 1$, any best response by $-J$ must be such that $\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} = 1$, which, as shown above, cannot hold in a symmetric mixed strategy equilibrium with $\pi > \frac{3}{4}$. ■

5.4.2 Efficiency Concerned Candidates

Assume $(\alpha^A, \alpha^B) = (1,1)$. From Claim 1 and Proposition 3, the set of parameters values for which the efficient outcome cannot be sustained in equilibrium is $\left\{(\pi,\beta) : \pi \in \left(\frac{3}{2}-\beta, 1\right] \text{ and } \beta \in \left(\frac{2}{3}, 1\right)\right\}$. Assume that parameter values are within this set. The next claims characterize the set of symmetric mixed equilibria for this range of parameter values, and establish that the expected number of projects in these equilibria is at least $\frac{9}{7}$, completing the proof of Proposition 2.

Claim 20 *If* $\pi \in \left(\frac{3}{2}-\beta, \frac{3}{4}\frac{2-\beta}{3-2\beta}\right)$, *candidate strategies* (s_H, s_H) *are supported in equilibrium. Conversely, in any symmetric equilibrium,* $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$ *for* $J \in \{A, B\}$.

Proof. First we prove that (s_H, s_H) can be supported in equilibrium. Given (s_H, s_H) is played, for any candidate J and voter i, if i does not observe the full proposals, beliefs are such that $\omega_1^{i,J}(1) = \omega_2^{i,J}(0) = 1$ and hence $s^i(1,0) = A$, $s^i(0,1) = B$ and $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$. Therefore, given that (s_H, s_H) is played as expected by voters, either all voters abstain if $p^A = p^B$ or one voter votes for A, one for B and one abstains if $p^A \neq p^B$; in either case the election is tied and the probability that each candidate is elected is $\frac{1}{2}$, so the expected payoff for each candidate is $\frac{1}{2}\frac{1}{3-2\beta}$.

Suppose A deviates to s_1, A loses $0-2$ if Nature does not reveal p, and A loses $1-2$ if Nature reveals p. Suppose A deviates to $s_k \in \{s_2, s_3, s_4\}$. If Nature reveals p, with probability $\frac{2}{3}$ A wins and with probability $\frac{1}{3}$ A loses; while if p is not revealed, A loses for sure. Hence, A wins with probability $\frac{2}{3}\pi$, the expected utility deviating is $\frac{2}{3}\frac{\pi}{2-\beta}$ and the deviation is profitable if and only if $\frac{2}{3}\frac{\pi}{2-\beta} > \frac{1}{2}\frac{1}{3-2\beta}$, that is, $\pi > \frac{3}{4}\frac{2-\beta}{3-2\beta}$. Suppose A deviates to $s_k \in \{s_5, s_6, s_7\}$. Then A achieves the same expected electoral outcomes and utilities as not deviating. Suppose A deviates to s_8. If Nature reveals p, A loses $1-2$. If Nature does not reveal p, then A wins $2-1$. But Nature reveals p with probability π, hence the expected utility for A is $(1-\pi)\frac{1}{4-3\beta}$ which is less than $\frac{1}{2}\frac{1}{3-2\beta}$ for any $\pi > \frac{1}{2}$. Hence, there is no profitable deviation.

Second, we prove that in any symmetric equilibrium both candidates propose two projects.

Suppose (σ^J, σ^J) is part of a symmetric equilibrium. Since the equilibrium is symmetric, for any $i \in \{a, b, c\}$, if i does not observe p, then $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$ and furthermore, for $k \in \{0,1\}$, $s^i(k, 1-k) = A$ if and only if $s^i(1-k, k) = B$. Suppose $s^i(1,0) \neq A$, so $s^i(0,1) \neq B$.

Then s_8 is not a best response and it is not played in equilibrium. But if s_8 is not played, the expected payoff for i if A wins is strictly higher than if B wins given $(p_i^A, p_i^B) = (1, 0)$, so by assumption $s^i(1, 0) = A$, a contradiction. Thus, it must be $s^i(1, 0) = A$ and $s^i(0, 1) = B$. Then, given that $\pi \in (\frac{3}{2} - \beta, \frac{3}{4}\frac{2-\beta}{3-2\beta})$, for any strategy σ^{-J}, a best response by J must propose two projects. Thus no strategy that proposes any other number of projects can be used in a symmetric equilibrium. ■

Claim 21 *There exists an increasing function $\pi(\beta)$ such that*

if $\pi \in \left(\max\left\{\frac{3}{2} - \beta, \frac{3}{4}\frac{2-\beta}{3-2\beta}\right\}, \pi(\beta)\right)$, there is a unique symmetric mixed strategy equilibrium, in which $\sigma^J = (\alpha_1, \alpha_1, \alpha_1, \alpha_2, \alpha_2, \alpha_2, 1 - 3\alpha_1 - 3\alpha_2)$ and the expected number of projects is $\frac{18\pi - 6}{10\pi - 3}$; and

if $\pi \in (\pi(\beta), 1)$, there is a unique symmetric mixed strategy equilibrium, in which $\sigma^J = (0, \beta_1, \beta_1, \beta_1, \beta_2, \beta_2, \beta_2, 1 - 3\beta_1 - 3\beta_2)$ and the expected number of projects is $\frac{12\pi - 3}{10\pi - 3} \geq \frac{9}{7}$ and which converges to $\frac{9}{7}$ as $\pi \to 1$.

We find the exact functional form of $\pi(\beta)$ and the weights of the mixed strategies as part of the proof.

Proof. Let (σ^A, σ^B) be a symmetric candidate strategy profile so $\sigma^A = \sigma^B$. Since the candidates' strategies are symmetric, voters' strategies must be such that $s^i(k, k) = \varnothing$ for $k \in \{0, 1\}$, and $\{s^i(1, 0) = A$ and $s^i(0, 1) = B\}$ or $\{s^i(1, 0) = B$ and $s^i(0, 1) = A\}$ or $\{s^i(1, 0) = \varnothing$ and $s^i(0, 1) = \varnothing\}$ for every voter i. Suppose not $\{s^i(1, 0) = A$ and $s^i(0, 1) = B\}$. Then given any strategy σ^{-J}, candidate J obtains a greater expected payoff playing s_1 than playing s_8, and a strictly greater payoff if $\sigma_8^{-J} > 0$. Therefore, in a symmetric mixed equilibrium, $\sigma_8^J = 0$. Then, it follows that for any voter i who observes $p_i^J = 1$ and $p_i^{-J} = 0$, the expected payoff for voter i is greater if J wins, thus by assumption, i votes J. Therefore, $s^i(1, 0) = A$ and $s^i(0, 1) = B$.

Next we prove that in any symmetric mixed strategy equilibrium, $\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} > 0$. Suppose not. Notice that given $\pi > \frac{3}{4}$ and $s^i(1, 0) = A$ and $s^i(0, 1) = B$, if candidate $-J$ proposes one project and candidate J proposes two, in expectation J wins the election more often, whereas if J proposes two and $-J$ proposes zero or three, J wins more often. So if candidate $-J$ never proposes one project, proposing two projects in expectation defeats any other proposal with probability more than one half. Then, any best response by candidate J to σ^{-J} with $\sigma_2^{-J}+\sigma_3^{-J}+\sigma_4^{-J}=0$ must be such that $\sigma_5^J+\sigma_6^J+\sigma_7^J=1$, which in turns means that any best response by J implies $\sigma_2^J+\sigma_3^J+\sigma_4^J=1$, a contradiction.

Similarly, in any symmetric mixed strategy equilibrium, $\sigma_5^{-J}+\sigma_6^{-J}+\sigma_7^{-J}>0$. Suppose not. Any best response by J must be such that $\sigma_2^J+\sigma_3^J+\sigma_4^J=0$, in which in turns implies that the best response by $-J$ is $\sigma_5^{-J}+\sigma_6^{-J}+\sigma_7^{-J}=1$.

Therefore, in any symmetric mixed strategy equilibrium, both candidates propose one project, and two projects, with positive probability. But then, it must be that $\sigma_2^J=\sigma_3^J+\sigma_4^J$ and $\sigma_5^J=\sigma_6^J=\sigma_7^J$. Given that the randomization among districts (subject to choosing a number of projects) assigns equal weight to all districts, we can reduce the strategic problem to that of assigning weights to strategies s_1, s_8, s_L, s_H. The payoff matrix is as follows:

$$\left\{\begin{array}{ccccc} & s_1 & s_L & s_H & s_8 \\ s_1 & \frac{1}{2},\frac{1}{2} & \pi, 1-\pi & 0, \frac{1}{3-2\beta} & \pi, \frac{1-\pi}{4-3\beta} \\ s_L & \frac{1-\pi}{2-\beta},\pi & \frac{1}{2(2-\beta)},\frac{1}{2(2-\beta)} & \frac{2\pi}{3(2-\beta)},\frac{3-2\pi}{3(3-2\beta)} & 0,\frac{1}{4-3\beta} \\ s_H & \frac{1}{3-2\beta},0 & \frac{3-2\pi}{3(3-2\beta)},\frac{2\pi}{3(2-\beta)} & \frac{1}{2(3-2\beta)},\frac{1}{2(3-2\beta)} & \frac{\pi}{3-2\beta},\frac{1-\pi}{4-3\beta} \\ s_8 & \frac{1-\pi}{4-3\beta},\pi & \frac{1}{4-3\beta},0 & \frac{1-\pi}{4-3\beta},\frac{\pi}{3-2\beta} & \frac{1}{2(4-3\beta)},\frac{1}{2(4-3\beta)} \end{array}\right\}$$

Dropping the superindex (so as to use the Maple feature of Scientific Workplace to solve the equations), a symmetric equilibrium strategy with $\sigma_1^J>0, \sigma_L^J>0, \sigma_H^J>0$ and $\sigma_8^J=0$ must satisfy

i) $\frac{1}{2}\sigma_1+\pi\sigma_L=\frac{1-\pi}{2-\beta}\sigma_1+\frac{1}{2(2-\beta)}\sigma_L+\frac{2\pi}{3(2-\beta)}(1-\sigma_1-\sigma_L)$

ii) $\frac{1}{2}\sigma_1+\pi\sigma_L=\frac{1}{3-2\beta}\sigma_1+\frac{3-2\pi}{3(3-2\beta)}\sigma_L+\frac{1}{2(3-2\beta)}(1-\sigma_1-\sigma_L)$

iii) $\frac{1}{2}\sigma_1+\pi\sigma_L\geq\frac{1-\pi}{4-3\beta}\sigma_1+\frac{1}{4-3\beta}\sigma_L+\frac{1-\pi}{4-3\beta}(1-\sigma_1-\sigma_L)$.

Solving we obtain

$$\sigma_1=\frac{4\pi+(3-16\pi+6\beta\pi)\sigma_L}{10\pi-3\beta}$$

and

$$\sigma_L = \frac{3-6(1-\beta)\sigma_1}{22\pi - 12\beta\pi - 3} =$$

$$\sigma_L = \frac{3-6(1-\beta)\frac{4\pi+(3-16\pi+6\beta\pi)\sigma_L}{10\pi-3\beta}}{22\pi - 12\beta\pi - 3}$$

$$\sigma_L = -\frac{-9\beta+6\pi+24\beta\pi}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18}$$

which as $\beta \to 1$ converges to $\sigma_L = -\frac{-9+6\pi+24\pi}{60\pi-100\pi^2-9} = \frac{3(10\pi-3)}{(10\pi-3)^2} = \frac{3}{10\pi-3}$ as it ought to.

Also,

$$\begin{aligned} \sigma_1 &= \frac{4\pi-(3-16\pi+6\beta\pi)\frac{-9\beta+6\pi+24\beta\pi}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18}}{10\pi-3\beta} \\ \sigma_1 &= -\frac{88\pi^2-60\pi+18\beta\pi-48\beta\pi^2+9}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18} \end{aligned}$$

which as $\beta \to 1$ converges to $\sigma_1 = \frac{4\pi-3}{10\pi-3}$ as it ought to.

Then simplifying inequality iii),

$$\frac{1}{2}\sigma_1 + \pi\sigma_L \geq \frac{1-\pi}{4-3\beta}\sigma_1 + \frac{1}{4-3\beta}\sigma_L + \frac{1-\pi}{4-3\beta}(1-\sigma_1-\sigma_L)$$

$$\sigma_1 \geq \frac{6\pi(1-\beta)\sigma_L - 2(1-\pi)}{3\beta-4}$$

$$-\frac{88\pi^2-60\pi+18\beta\pi-48\beta\pi^2+9}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18} \geq \frac{-6\pi(1-\beta)\frac{-9\beta+6\pi+24\beta\pi}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18} - 2(1-\pi)}{3\beta-4}$$

$$\frac{6\pi(1-\beta)\frac{-9\beta+6\pi+24\beta\pi}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18} + 2(1-\pi)}{3\beta-4} - \frac{88\pi^2-60\pi+18\beta\pi-48\beta\pi^2+9}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18} = 0$$

$$\frac{48\pi-9\beta-304\pi^2+440\pi^3+48\beta\pi+24\beta\pi^2-240\beta\pi^3}{(3\beta-4)(10\pi-3)(3\beta+22\pi-12\beta\pi-6)} = 0$$

$$48\pi-9\beta-304\pi^2+440\pi^3+48\beta\pi+24\beta\pi^2-240\beta\pi^3 = 0$$

The solution, solved by Mathematica, is a cumbersome expression that simplifies to the desired $\pi > \frac{11+\sqrt{61}}{20}$ for $\beta = 1$.

The probability of proposing two projects is

$$1+\frac{-9\beta+6\pi+24\beta\pi+88\pi^2-60\pi+18\beta\pi-48\beta\pi^2+9}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18}=\frac{3\left(44\pi^2-24\pi+8\beta\pi-24\beta\pi^2+3\right)}{(10\pi-3)\left(3\beta+22\pi-12\beta\pi-6\right)}$$

and the expected number of projects is $\frac{6\left(44\pi^2-24\pi+8\beta\pi-24\beta\pi^2+3\right)}{(10\pi-3)(3\beta+22\pi-12\beta\pi-6)}-\frac{-9\beta+6\pi+24\beta\pi}{9\beta+126\pi-220\pi^2-66\beta\pi+120\beta\pi^2-18}=\frac{1}{10\pi-3}\left(12\pi-3\right)=\frac{12\pi-3}{10\pi-3}$, which converges to $\frac{9}{7}$ as π converges to 1. The initial assumption that voters vote $s^i(1,0)=A$ and $s^i(0,1)=B$ is supported because $\sigma_8=0$ and $\beta>2/3$.

If instead π is below the cutoffs, candidates mix between proposing one, two and three projects. An equilibrium with these characteristics requires:

$$\left\{\begin{array}{lllll} & s_1 & s_L & s_H & s_8\\ s_1 & \frac{1}{2},\frac{1}{2} & \pi,1-\pi & 0,\frac{1}{3-2\beta} & \pi,\frac{1-\pi}{4-3\beta}\\ s_L & \frac{1-\pi}{2-\beta},\pi & \frac{1}{2(2-\beta)},\frac{1}{2(2-\beta)} & \frac{2\pi}{3(2-\beta)},\frac{3-2\pi}{3(3-2\beta)} & 0,\frac{1}{4-3\beta}\\ s_H & \frac{1}{3-2\beta},0 & \frac{3-2\pi}{3(3-2\beta)},\frac{2\pi}{3(2-\beta)} & \frac{1}{2(3-2\beta)},\frac{1}{2(3-2\beta)} & \frac{\pi}{3-2\beta},\frac{1-\pi}{4-3\beta}\\ s_8 & \frac{1-\pi}{4-3\beta},\pi & \frac{1}{4-3\beta},0 & \frac{1-\pi}{4-3\beta},\frac{\pi}{3-2\beta} & \frac{1}{2(4-3\beta)},\frac{1}{2(4-3\beta)} \end{array}\right\}$$

$i)$ $\pi\sigma_L+\pi(1-\sigma_L-\sigma_H)\leq\frac{1}{2(2-\beta)}\sigma_L+\frac{2\pi}{3(2-\beta)}\sigma_H$

$ii)$ $\frac{1}{2(2-\beta)}\sigma_L+\frac{2\pi}{3(2-\beta)}\sigma_H=\frac{3-2\pi}{3(3-2\beta)}\sigma_L+\frac{1}{2(3-2\beta)}\sigma_H+\frac{\pi}{3-2\beta}(1-\sigma_L-\sigma_H)$

$iii)$ $\frac{1}{2(2-\beta)}\sigma_L+\frac{2\pi}{3(2-\beta)}\sigma_H=\frac{1}{4-3\beta}\sigma_L+\frac{1-\pi}{4-3\beta}\sigma_H+\frac{1}{2(4-3\beta)}(1-\sigma_L-\sigma_H)$.

Simplifying the first inequality expressions, we get $\pi\leq\frac{1}{2(2-\beta)}\sigma_L+\frac{(8-3\beta)}{3(2-\beta)}\pi\sigma_H$.

From ii)

$$\sigma_L=\frac{12\pi-6\beta\pi+(14\beta\pi+6-3\beta-24\pi)\sigma_H}{20\pi-10\beta\pi-3}$$

From iii)

$$\sigma_H=-\frac{(3\beta+6\sigma_L-6\beta\sigma_L-6)}{3\beta+28\pi-18\beta\pi-6}$$

So,

$$\begin{aligned}\sigma_L &= \frac{12\pi-6\beta\pi-(14\beta\pi+6-3\beta-24\pi)\frac{(3\beta+6\sigma_L-6\beta\sigma_L-6)}{3\beta+28\pi-18\beta\pi-6}}{20\pi-10\beta\pi-3}\\ \sigma_L &= \frac{9\beta+108\pi-168\pi^2-60\beta\pi+108\beta\pi^2-18}{18\beta+174\pi-280\pi^2-114\beta\pi+180\beta\pi^2-27}\end{aligned}$$

which simplifies to

$$\sigma_L = \frac{9 + 108\pi - 168\pi^2 - 60\pi + 108\pi^2 - 18}{18 + 174\pi - 280\pi^2 - 114\pi + 180\pi^2 - 27} = \frac{6\pi - 3}{10\pi - 3}$$

when $\beta = 1$ as desired. Also,

$$\begin{aligned} \sigma_H &= -\frac{3\beta - 6 + 6(1-\beta)\sigma_L}{3\beta + 28\pi - 18\beta\pi - 6} = -\frac{3\beta - 6 + 6(1-\beta)\frac{9\beta+108\pi-168\pi^2-60\beta\pi+108\beta\pi^2-18}{18\beta+174\pi-280\pi^2-114\beta\pi+180\beta\pi^2-27}}{3\beta + 28\pi - 18\beta\pi - 6} \\ \sigma_H &= \frac{1}{10\pi - 3}\frac{24\pi + 6\beta\pi - 9}{6\beta + 28\pi - 18\beta\pi - 9} \end{aligned}$$

which simplifies to

$$\frac{1}{10\pi - 3}\frac{24\pi + 6\pi - 9}{6 + 28\pi - 18\pi - 9} = \frac{3}{10\pi - 3}$$

and, the expected number of projects in this equilibrium is:

$$\begin{aligned} & 2\frac{1}{10\pi - 3}\frac{24\pi + 6\beta\pi - 9}{6\beta + 28\pi - 18\beta\pi - 9} + \frac{9\beta + 108\pi - 168\pi^2 - 60\beta\pi + 108\beta\pi^2 - 18}{18\beta + 174\pi - 280\pi^2 - 114\beta\pi + 180\beta\pi^2 - 27} \\ & +3(1 - \frac{24\pi + 6\beta\pi - 9 - (9\beta + 108\pi - 168\pi^2 - 60\beta\pi + 108\beta\pi^2 - 18)}{(10\pi - 3)(6\beta + 28\pi - 18\beta\pi - 9)}) \\ = & \frac{18\pi - 6}{10\pi - 3}. \end{aligned}$$

■

5.5 The Theory with n Districts

Let N be a society with n districts, where each district has one voter and one project associated to it. Let $i \in N$ be an arbitrary voter or district. Let A and B be two candidates who are not one of the n voters and are not associated to any district.

Let $S^J = \{0,1\}^n$ be the set of feasible pure strategies for each candidate J. Let $(s^A, s^B) \in S^A \times S^B = S$ be an arbitrary candidates' strategy pair.

We consider candidates who are both purely office motivated, or both efficiency concerned with $\alpha_A = \alpha_B = 1$. First we present some preliminary results, which are useful in both cases.

Lemma 22 *In any pure strategy equilibrium, both candidates win with equal probability.*

Proof. Consider any candidates' strategy pair (s^A, s^B) such that candidate J wins with probability less than $\frac{1}{2}$. Since in equilibrium voters hold correct beliefs, the probability that J wins conditional on full information being revealed, or not revealed, is less than $\frac{1}{2}$ in each case. In pure strategies, the probability of victory is in the set $\{0, \frac{1}{2}, 1\}$ so if it is less than $\frac{1}{2}$, it is zero. Deviating to $s^J = s^{-J}$, candidate J ties the election if full information is revealed, so the probability of winning is at least $\frac{\pi}{2}$. Since for both types, office motivated and efficiency concerned, to win with positive probability gives a positive payoff, then there is no equilibrium where a candidate wins with zero probability. ■

Corollary 23 *Assume $\beta \neq \frac{k}{n}$ for any integer k. In any pure strategy equilibrium both candidates propose to carry out the same number of projects.*

Proof. By lemma 22, both candidates win with equal probability. If $\beta \neq \frac{k}{n}$ and candidates propose to implement a different number of projects, then no voter is indifferent between the two candidates, and hence no voter abstains, so that either A or B win for sure, which is a contradiction. ■

Lemma 24 *Assume $\beta \neq \frac{k}{n}$ for any integer k. In any pure strategy equilibrium, $s^i(0,0) = s^i(1,1) = \emptyset$, $s^i(1,0) = A$ and $s^i(0,1) = B$ for any voter $i \in N$.*

Proof. $s^i(0,0) = s^i(1,1) = \emptyset$ follows directly from lemma 22. Note that if voter i observes $(s_i^A, s_i^B) = (1,0)$, by lemma 22 it means that B proposes to carry out one more project than A in districts other than i. So i prefers A. A symmetric argument holds to show that $s^i(0,1) = B$. ■

Consider the following out-of-equilibrium beliefs.

B1: Given an equilibrium strategy pair (s^A, s^B), let the out of equilibrium beliefs of any voter $i \in N$ be such that $\delta_i^J(1 - s_i^J)$ assigns probability 1 to strategy $(1 - s_1^J, s_{-i}^J)$ where $s_{-1}^J = \{1\}^{n-1}$, that is, voter i believes candidate J proposes to carry out a project in every other district.

Lemma 25 *If a candidate strategy pair (s^A, s^B) cannot be supported in an equilibrium with out of equilibrium beliefs $B1$, then it cannot be supported in equilibrium.*

Proof. Suppose that (s^A, s^B) and beliefs $B1$ are not an equilibrium but there exists a system of beliefs that sustains (s^A, s^B) as an equilibrium strategy profile. If (s^A, s^B) is an equilibrium strategy pair, it follows that there are no profitable deviation for any candidate. Consider, without loss of generality, any deviating proposal $\tilde{s}^A \neq s^A$ by candidate A. For each voter $i \in N$ who observes a deviation the utility attached at strategy $\tilde{s}^A$ under beliefs $B1$ is equal or lower than the utility attached to strategy s^A under the original beliefs. Therefore if there were no profitable deviation under the original beliefs there are no profitable deviation when each voter i has beliefs $B1$. ■

5.5.1 Purely Office Motivated Candidates

Proposition 26 *Assume* $\pi \in \left(0, \frac{1}{2}\right)$ *and* $\beta \neq \frac{k}{n}$ *for any integer* k. *A candidates' strategy pair* (s^A, s^B) *can be supported in equilibrium if and only if* $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B = x$ *and one of the two following conditions holds:*

i) $x < (1-\beta)n$; *or*

ii) $x > \frac{n+1}{2}$ *and* $s_i^A + s_i^B \geq 1$ *for any* $i \in N$.

Proof. By corollary 23, in equilibrium $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B$. Let $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B = x$ and assume $x < (1-\beta)n$. Consider four subsets of voters: $N_{x,y} = \{i \in N : (s_i^A, s_i^B) = (x, y)\}$ for $x, y \in \{0, 1\}$. In equilibrium, each $i \in N_{00} \cup N_{11}$ abstains, $i \in N_{10}$ votes A and $i \in N_{01}$ votes B. Without loss of generality, consider deviations by A. Suppose A deviates, proposing $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{00}$. With probability $1 - \pi > \frac{1}{2}$, full information is not revealed, and given out of equilibrium beliefs $B1$, voter i expects utility $\beta - 1$ if A wins, and utility $-\frac{x}{n}$ if B wins. Note that $x < (1-\beta)n$ implies $\frac{x}{n} < 1 - \beta$ so voter i votes for B and thus candidate A loses the election.

Suppose A deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{01}$; whether or not full information is revealed, voter i prefers B and continues to vote for B, while other voters, if they observe the deviation, reduce their expected utility from a victory by A, so A gains nothing by deviating.

Suppose A deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{10} \cup N_{11}$. With probability $1 - \pi > \frac{1}{2}$, full information is not revealed, and given out of equilibrium beliefs $B1$, it follows that voter i

votes for B and A loses the election.

Assume $x \in \left((1-\beta)n, \frac{n+1}{2}\right)$. Suppose A deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{00}$. With probability $1-\pi > \frac{1}{2}$ full information is not revealed, and under out of equilibrium beliefs $B1$, voter i expects utility $\beta - 1$ if A wins, and utility $-\frac{x}{n}$ if B wins; since $\beta - 1 > -\frac{x}{n}$, it follows i prefers A and votes for A and A wins the election. Hence there is no equilibrium with out of equilibrium beliefs $B1$, but then by lemma 25, there is no equilibrium.

Assume $x > \frac{n+1}{2}$ but $N_{00} \neq \emptyset$. Suppose A deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{00}$. With probability $1-\pi > \frac{1}{2}$ full information is not revealed, and under out of equilibrium beliefs $B1$, voter i expects utility $\beta - 1$ if A wins, and utility $-\frac{x}{n}$ if B wins; since $\beta - 1 > -\frac{x}{n}$, it follows i prefers A and votes for A and A wins the election. Hence there is no equilibrium with out of equilibrium beliefs $B1$, but then by lemma 25, there is no equilibrium.

Assume $x > \frac{n+1}{2}$ and $N_{00} = \emptyset$. Suppose A deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{01}$; whether or not full information is revealed, voter i prefers B and continues to vote for B and other voters either do not observe the deviation or if they do, they now have a lower expected utility from A winning, so A gains nothing by deviating. Suppose A deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{10} \cup N_{11}$. With probability $1 - \pi > \frac{1}{2}$, full information is not revealed, and given out of equilibrium beliefs $B1$, voter i votes for B and A loses the election. ■

Proposition 27 *Assume $\pi \in (\frac{1}{2}, 1]$. If $\beta > \frac{n+1}{2n}$, there is no equilibrium in pure strategies; if $\beta < \frac{n+1}{2n}$, there is a unique equilibrium, in which both candidates propose zero projects.*

Proof. Consider first the case $\beta > \frac{n+1}{2n}$ and suppose there is an equilibrium in pure strategy. By corollary 23 let k be the number of projects proposed in equilibrium by each candidate. Suppose without loss of generality that candidate A proposes in districts $1, 2, ..., k$. Consider first that $k < \frac{n+1}{2}$. Candidate B can win the election when full information is revealed, that is with probability $\pi > \frac{1}{2}$, proposing a project in districts $k+1, k+2, ..., n$ and no project in district $i \leq k$. Consider now the case with $k \geq \frac{n+1}{2}$. Candidate B can propose a project in districts $1, 2, ..., k-1$ and no project in the other districts. All voters $1, 2, ...k-1$ and voters with $i \geq k+1$ (if any) vote for candidate B when full information is revealed and therefore candidate B wins the election with probability $\pi > \frac{1}{2}$. Consider now the case $\beta < \frac{n+1}{2n}$. If B deviates and proposes zero projects, then B wins the election when full information is revealed. In fact, if $1 \leq k < \frac{n+1}{2}$ and full information is revealed all voters to which candidate A does not propose a project vote for B. If $k \geq \frac{n+1}{2}$ each voter $i \leq k$ votes for B, because if A is elected voter i gets $\beta - \frac{k}{n}$, while voter i gets 0 if B is elected; hence each voter $i \leq k$ votes for B if $\beta < \frac{k}{n}$ which holds by assumption. Finally suppose both candidates propose no projects. If candidate J deviates and proposes some projects loses the election both when full information is not revealed (by assumption **B1)** and when information is revealed, because, if $\beta < \frac{n+1}{2n}$, any minimum majority coalition formed by $\frac{n+1}{2}$ districts prefers zero projects than $k = \frac{n+1}{2}$ implemented. ■

5.5.2 Efficiency Concerned Candidates

Assume $(\alpha_A, \alpha_B) = (1, 1)$.

Proposition 28 *Assume $\pi \in \left(0, \frac{1}{2}\right)$ and $\beta \neq \frac{k}{n}$ for any integer k. A candidates' strategy pair (s^A, s^B) can be supported in equilibrium if and only if $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B = x$ and one of the two following conditions holds:*
i) $x < (1-\beta)n$ and $\pi \leq \frac{1}{2}\frac{1}{(1-\beta)x+1}$; or
ii) $x > \frac{n+1}{2}$, $s_i^A + s_i^B \geq 1$ for any $i \in N$ and iia) $\pi < 1 - \frac{(n-1)(1-\beta)}{(1-\beta)x+1}$ if $\beta > \frac{n+1}{2n}$; iib) $\pi < \frac{1}{2}\frac{1}{(1-\beta)x+1}$ if $\beta < \frac{n+1}{2n}$.

Proof. By corollary 23, in equilibrium $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B$. Let $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B = x$ and assume $x < (1-\beta)n$. Consider four subsets of voters: $N_{x,y} = \{i \in N : (s_i^A, s_i^B) = (x, y)\}$ for $x, y \in \{0, 1\}$. In equilibrium, $i \in N_{00} \cup N_{11}$ abstains, $i \in N_{10}$ votes A and $i \in N_{01}$ votes B. Without loss of generality, consider deviations by A. In the proof of proposition 26 we already showed that there do not exist profitable deviations for a office motivated candidate such that more projects are proposed in the deviation. Therefore, these are not profitable deviations in case the candidate is efficiency concerned, too. Consider any deviation such that $x' < x$ projects are proposed. Given, voters' beliefs, this deviation can be profitable only if full information is revealed. It follows that the most profitable deviation for an efficiency concerned candidate is to propose zero project. This deviation is profitable if and only if

$$\pi > \frac{1}{2}\frac{1}{(1-\beta)x+1}.$$

Assume $x \in \left((1-\beta)n, \frac{n+1}{2}\right)$ or $x > \frac{n+1}{2}$ but $N_{00} \neq \emptyset$. Suppose A deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{00}$. Since $x > (1-\beta)n$ then voter i votes for A. This deviation is profitable if and only if

$$(1-\pi)\frac{1}{(1-\beta)(x+1)+1} > \frac{1}{2}\frac{1}{(1-\beta)x+1}, \text{ or}$$

$$\begin{aligned} \frac{\pi}{(1-\beta)(x+1)+1} &< \frac{1}{(1-\beta)(x+1)+1} - \frac{1}{2}\frac{1}{(1-\beta)x+1}, \\ \pi &< \frac{(1-\beta)(x-1)+1}{2(1-\beta)x+2} \equiv \pi'. \end{aligned}$$

Suppose A deviates to offer zero projects. This deviation is profitable if and only if

$$\pi > \frac{1}{2}\frac{1}{(1-\beta)x+1} \equiv \pi''.$$

For all x, $\pi' \geq \pi''$ and there is always a profitable deviation. Hence there is no equilibrium with out of equilibrium beliefs $B1$, but then by lemma 25, there is no equilibrium.

Assume $x > \frac{n+1}{2}$ and $N_{00} = \emptyset$. Suppose A deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{01}$; whether or not full information is revealed, voter i prefers B and continues to vote for B and other voters either do not observe the deviation or if they do, they now have a lower expected utility from A winning, so A loses by deviating since she is proposing more projects. When full information is not revealed, and given out of equilibrium beliefs $B1$, each voter i who observes a deviation from candidate A votes for B. So candidate A can only find a deviation that is profitable when full information is revealed. Consider first the case with $\beta > \frac{n+1}{2n}$. The most profitable deviation is such that candidate A proposes x' projects such that $\tilde{s}_i^A = 1$ only if $\tilde{s}_i^B = 1$

and $(n-x)+x' = \frac{n+1}{2}$ that is $x' = \frac{2x+1-n}{2}$. This deviation is profitable if and only if

$$\begin{aligned} \pi \frac{1}{(1-\beta)x'+1} &> \frac{1}{2}\frac{1}{(1-\beta)x+1}, \text{ or} \\ \pi &> 1 - \frac{(n-1)(1-\beta)}{(1-\beta)x+1}. \end{aligned}$$

If $\beta < \frac{n+1}{2n}$, then the most profitable deviation is to propose zero project. This deviation is profitable if and only if

$$\pi > \frac{1}{2}\frac{1}{(1-\beta)x+1}.$$

■

Proposition 29 *Assume $\pi \in \left(\frac{1}{2}, 1\right]$ and $\beta \neq \frac{k}{n}$ for any integer k. If $\beta < \frac{n+1}{2n}$ then the candidates' strategy pair (s^A, s^B) is supported in equilibrium if and only if $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B = 0$. If $\beta > \frac{n+1}{2n}$ then the candidates' strategy pair (s^A, s^B) is supported in equilibrium if and only if $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B = 0$ and $\pi < \frac{(1-\beta)(n+1)+2}{4}$. If $\beta > \frac{n+1}{2n}$ and $\pi > \frac{(1-\beta)(n+1)+2}{4}$ there is no equilibrium in pure strategies.*

Proof. First of all it is easy to check that there is no equilibrium in pure strategies with $x > 0$ if $\pi > \frac{1}{2}$; in fact there is always a profitable deviation when full information is revealed such that fewer projects are implemented. Suppose then that both candidates proposes zero projects. If $\beta < \frac{n+1}{2n}$, there is no profitable deviation because the minimum winning coalition prefers zero projects to have all their projects implemented (and no other project implemented). If $\beta > \frac{n+1}{2n}$ then candidate J who proposes $\frac{n+1}{2}$ projects wins the election when full information is revealed. This deviation is profitable if and only if

$$\pi \frac{1}{(1-\beta)\frac{n+1}{2}+1} > \frac{1}{2}, \text{ or } \pi > \frac{(1-\beta)(n+1)+2}{4}.$$

■