ARTICLE IN PRESS
TFS-17967; No of Pages 11
Technological Forecasting & Social Change xxx (2014) xxx–xxx
Contents lists available at ScienceDirect
Technological Forecasting & Social Change

Heterogeneity in diffusion of innovations modelling: A few fundamental types

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ARTICLE INFO

Article history:
Received 4 February 2013
Received in revised form 16 December 2013
Accepted 12 February 2014
Available online xxxx

Keywords:
Latent heterogeneity of agents
Multimodality
Diffusion of innovations
Two-point distributions
Non-parametric Bayesian mixture models
Connectivity strength
Moore and von Neumann neighborhoods

ABSTRACT

Heterogeneity of agents in aggregate systems is an important issue in the study of innovation diffusion. In this paper, we propose a modelling approach to latent heterogeneity, based on a few fundamental types, which avoids cumbersome integrations with not easy to motivate a priori distributions. This approach gives rise to a discrete non-parametric Bayesian mixture model with a possibly multimodal distributional behaviour. The result is inspired by two alternative theories: the first is based on the Rosenblueth two-point distributions (TPD), and the second is related to Cellular Automata models. From a statistical point of view, the proposed reduction allows for the recognition of discrete heterogeneous sub-populations by assessing their significance within a realistic diffusion process. An illustrative application is discussed with reference to Compact Cassettes for pre-recorded music in Italy.

1. Introduction

Consumer response to innovation is a general issue in marketing research characterized by three main aspects: customer innovativeness, growth modelling of new products, and network externalities, as highlighted, for instance, in Hauser et al. [17], Meade and Islam [22], and Peres et al. [23]. Customer innovativeness is an individual property that expresses a willingness to adopt innovations and is usually considered a function of cultural, behavioural, demographic, and economic characteristics. As such, it has a natural variability among agents and over time. Due to costs and reliability problems related to collecting personal information, new product growth modelling, substantially beginning with Bass [2], is focused on an aggregate level. The cumulative adoption process is dynamically governed by two latent and separate forces: an external influence, associated to innovators, and an internal one, associated to imitators. The Bass model opened a new way in the characterization of new product life-cycle, despite some limitations due to an assumed homogeneity of agents and a uniform accessibility to innovations. Stability of its parameters over the cycle, over different countries, and over product categories brought into question some of its typical assumptions jointly with the need to introduce interaction or control variables in it. Bass et al. [3] is an outstanding answer for describing the effect of external time dependent control effects on diffusion. Moreover, the presence of phases in the life-cycle, such as take-off, slowdown and saddle, also required convenient modifications of basic model assumptions and related equations. For instance, the take-off of many products may be prevented by network externality effects. Network externality modelling (direct and indirect) is a relevant area of research, which may link individual and aggregate levels in innovation diffusion. Cellular Automata modelling, and related mean-field approximation, define a fruitful bridge between Complex Systems representations, usually based on simulative tools, and System Analysis based on aggregate descriptions through differential equations. See,

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http://dx.doi.org/10.1016/j.techfore.2014.02.023
0040-1625/

for instance, Guseo and Guidolin [12,13], where S-shaped growth curves (or their modified versions) emerge from social contagion or through the increasing affordability of heterogeneous consumers with a different willingness-to-pay. In [13], latent heterogeneity is micro-modelled through a threshold, which generates a dynamic market potential with a precise take-off effect when a sufficient critical mass is reached.

A micro-modelling of latent heterogeneity of agents was also proposed in Chatterjee and Eliashberg [6] through a mixing distribution in the definition of the adoption process. However, the direct application of the model was partially limited by its complex nature.

A comparison between Agent-Based models (AB) and differential equation models (DE) is examined in Rahmandad and Sterman [24] relaxing the homogeneity of agents and perfect mixing in network hypotheses. They examine a classical SEIR model (Susceptible, Exposed, Infective, Removed) by considering an AB representation as a 'real-world' reference and a DE as an inferential counterpart. Results of their simulations highlight strong effects when different network topologies are considered. Heterogeneity of agents appears less sensitive to variations. We may notice that all simulations are performed under a unimodality hypothesis, thus excluding possible alternative multimodal patterns.

Most studies on heterogeneity in innovation diffusion have generally focused on latent structures. A different contribution, among others, dealing with observed heterogeneity, based on duration models, is due to Sinha and Chandrashekaran [28].

In this paper, we focus our interest on the latent case: in particular, we suggest that the lack of homogeneity may be related to the different relationships among agents that generate stationary or dynamic networks in a complex system. In the Bass models, interactions among agents are assumed to be homogeneous over space and time. The corresponding word-of-mouth effect, WOM, is described through a share q of all possible interactions, i.e., $qF(t)\,(1-F(t))$, where $F(t)$ defines the relative cumulative number of adoptions (or adopters) at time t.

A first way to relax this assumption may be found in Easingwood et al. [7], where a simple modulation of interactions is proposed to act on the basic factor responsible of WOM, $F(t)$. Its exponential form induces an acceleration or delay of adoptions which is not uniform over time. The selected interaction component is $qF^{\delta}(t)\,(1-F(t))$. For $\delta < 1$, we have a rapid concentration of sales for increasing time t, and vice versa, a delay for $\delta > 1$. This non-uniform influence gives rise to an asymmetric behaviour as compared with the Bass model, though preserving a unimodal distribution of adoptions over time. In the past, analogous transformations of the basic interaction effects $F(t)\,(1-F(t))$ were introduced: see, for instance, Gompertz [11], Floyd [9], Sharif and Kabir [27], and Jeuland [18]. Previous expressions of heterogeneity are described through differential equations whose solutions are not always explicit.

Based on a mixture of special densities, a different approach in heterogeneity modelling is considered by Bemmaor [4] and Bemmaor and Lee [5]. The basic hypothesis expresses heterogeneity of agents by assuming that some parameters characterizing local Bass-like dynamics are stochastic over the current population. Starting, in particular, with a shifted-Gompertz distribution,

$$F(t|\eta, p+q) = \left(1-e^{-(p+q)t}\right)exp\left\{-\eta e^{-(p+q)t}\right\}, \tag{1}$$

where $(p+q)$ is considered fixed, the parameter η that defines agents' propensity to buy, is assumed gamma distributed, $\eta \sim G(\lambda = 1/\beta, A)$.

Based on the moment generating function, the marginal mixture is an immediate result:

$$\begin{aligned} F(t) &= \int_0^{\infty} F(t|\eta; p+q)\frac{1}{\Gamma(A)}\left(\frac{1}{\beta}\right)^A \eta^{A-1} e^{-\frac{\eta}{\beta}} d\eta \\ &= \left(1-e^{-(p+q)t}\right)/\left(1+\beta e^{-(p+q)t}\right)^A. \end{aligned} \tag{2}$$

For $A = 1$ and $\beta = q\,/\,p$, the standard Bass model results as a special barycentric case. Parameter $A > 0$ characterizes different asymmetries, even though $F(t)$ in Eq. (2) is always unimodal. Low levels of parameter A, $A < 1$, define homogeneous agents with a common propensity to buy and a corresponding acceleration of the adoption process. Vice versa, high levels of A, $A > 1$, denote heterogeneous agents with different propensities that determine a distributed delay of the adoptions.

Notice that previously introduced models, and in particular, Bass [2], Bemmaor and Lee [5], Easingwood et al. [7], among others, and further covariate dependent models, such as Bass et al. [3], consider the market potential as fixed, $m(t) = m$, over the whole life-cycle. A different possibility may be the definition of a more flexible market potential $m(t)$. In Guseo and Guidolin [12], for instance, a generic market potential $m(t)$ is introduced through Cellular Automata representations, and in particular, its dynamic is obtained by exploiting a latent evolving network of relationships that mimics the heterogeneity of agents over space and time. The proposed cumulative model is $m(t)\left(1-e^{-(p+q)\int_0^t x(\tau)d\tau}\right)/\left(1+\beta e^{-(p+q)\int_0^t x(\tau)d\tau}\right)$, and it may represent, for $x(t) = 1$, at most bimodal situations of the corresponding rate process not yielded through a classical mixture. Interpretations of both components are effective in applied contexts. See, in particular, Guseo and Guidolin [13,14].

An alternative perspective considers heterogeneity as characterized by a discrete number of different types or segments. In this sense, the hazard of the category process, $h(t) = f(t)\,/\,(1-F(t))$, is composed through different local hazards, $h_i(t) = f_i\,/\,(1-F_i(t))$, i=1, 2,···, k, where k is the number of separate sub-populations. The non-homogeneity of composed hazards recognizes a kind of clustering effect in the development of diffusion with different local dynamics. An example among others, based on a discrete mixture of Gompertz distributions, may be found in Robertson et al. [25] where information on separate segments is known through separate time series. A more interesting and common context would be the analysis of composed dynamics under an aggregate time series that does not distinguish the separate origins of adoption data.

The purpose of this paper is to deal with latent heterogeneity in innovation diffusion, but unlike previously cited papers, we focus our attention on modelling processes that do not have a unimodal behaviour, but rather a multimodal one. Such multimodality results from the co-existence of different sub-populations of adopters in the diffusion process. In doing so, we propose a reduced approach based on a few latent types avoiding difficult integrations through not easy to motivate a priori distributions. The results are obtained by following two alternative theories: the first is based on the Rosenblueth two-point distributions (TPD), and the second is related to Cellular Automata representations, and in particular to the description of clustered connectivity among agents. An illustrative application is discussed with reference to Compact Cassettes for pre-recorded music in Italy whose life-cycle was characterized by four significantly heterogeneous sub-populations with proportional dynamic parameters.

The paper is organized as follows. Section 2 introduces the general Bass model and the mixing device to control heterogeneity. Section 3 introduces the TPD formalism by Rosenblueth [26]. Section 4 proposes a simplified version of a four-point distribution representation with proportional dynamics in parallel sub-populations referred to a benchmark one. Section 5 provides the same results following von Neumann and Moore metrics in describing connectivity of neighboring clusters of agents. Section 6 extends the main results with further local asymmetric effects as in Bemmaor and Lee [5]. Section 7 is devoted to the application of the proposed model to the life-cycle of Compact Cassettes in Italy. Final remarks and discussion are included in Section 8.

2. The standard Bass model and heterogeneity of agents

The standard Bass model [2] refers to a homogeneous system. Instantaneous adoptions $z'(t)$ are modelled through an autonomous Riccati equation with further non-negativity constraints, i.e.,

$$z'(t)=\left(p+q\frac{z(t)}{m}\right)(m-z(t)),\qquad z(t_0)=z_0, \tag{3}$$

where $z(t)$ denotes the cumulative adoptions (sales), m is the asymptotic market potential, P represents the "external influence" usually describing the contribution of innovators, and q the "internal influence" related to the share imputed to imitators. Dividing by m both members of Eq. (3), we obtain an equivalent normalized equation where $y(t) = z(t) / m$:

$$y'(t)=(p+qy(t))(1-y(t)),\quad y(t_0)=y_0. \tag{4}$$

The solution process depends upon all involved parameters, p, q, and y_0, in a nonlinear form, $y(t|p, q, y_0)$. In a homogeneous system, parameters p and q may be interpreted as specific dynamic aspects of the population. In a more complex situation, p and q may be referred to as limited homogeneous sub-populations and considered as time independent random variables depicting heterogeneous agents with a specific density $\phi(p, q)$. The observable normalized process is, therefore, a mixture due to the latency of (p, q):

$$y_1(t)=\iint y(t|p,q,y_0)\phi(p,q)\ dp\ dq. \tag{5}$$

This approach apparently introduces an infinite two-dimensional class of (p, q) sub-populations with a bivariate density $\phi(p, q)$, which is usually unknown.

The solution $y(t|p, q, y_0)$ does not depend upon the deterministic or stochastic nature of the dynamic factors p and q, namely,

$$y(t|p,q,y_0)=\frac{1-\alpha p\, e^{-(p+q)(t-t_0)}}{1+\alpha q\, e^{-(p+q)(t-t_0)}},\qquad t\geq t_0, \tag{6}$$

and zero elsewhere in order to exclude negative behaviour in the diffusion of innovations context. The constant α is a function of the initial condition, $\alpha = (1 - y_0)/(p + qy_0)$. For $t_0 = 0$ and $y_0 = 0$, we have $\alpha = 1 / p$ obtaining the usual Bass solution (see Bass, 1969 [2]),

$$y(t)=\frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}},\qquad t\geq 0, \tag{7}$$

and zero elsewhere.

The observable process $y_1(t)$ in Eq. (5) depends upon a complex integration and usually the density $\phi(p, q)$ is not realistically known. Formally, it is a first moment of the random variable $y(t|p, q, y_0)$, and in this perspective, we can use the results by Rosenblueth [26], Karmeshu and Lara-Rosano [20], Karmeshu and Goswami [19], and Goswami and Karmeshu [21] in order to obtain the first moment of a function of a random variable through the TPD formalism based on few moments of two-point distributions. This approach is based on a minimal discretization of a general continuous random variable and allows for a corresponding approximate representation of Eq. (5), simplifying computations both theoretically and practically. In other words, the continuous density $\phi(p, q)$ is converted to a discretized version with equivalent low-order moments and a minimal discrete support. This approach strongly simplifies integration in Eq. (5).

3. TPD formalism

Let us consider a random variable X characterized by the moments μ_X, σ_X, and ν_X, mean, standard deviation and skewness, respectively. Following Rosenblueth [26], we may construct a two-point distribution with the same three moments based on the following density,

$$\psi(x)=P_X^-\,\delta(X-x^-)+P_X^+\,\delta\left(X-x^+\right), \tag{8}$$

where $\delta(\cdot)$ is a Dirac's delta function. The properties of the discrete distribution $(P_X^\pm, x^\pm)$ are,

$$\begin{aligned}1&=P_X^-+P_X^+\\ \mu_X&=P_X^-x^-+P_X^+x^+\\ \sigma_X^2&=P_X^-(x^--\mu_x)^2+P_X^+\left(x^+-\mu_x\right)^2\\ \nu_X\sigma_X^3&=P_X^-(x^--\mu_x)^3+P_X^+\left(x^+-\mu_x\right)^3\end{aligned} \tag{9}$$

and, therefore,

$$P_X^\pm=\frac{1}{2}\left[1\mp\frac{\nu_X}{\sqrt{4+\nu_X^2}}\right], \tag{10}$$

Please cite this article as: R. Guseo, M. Guidolin, Heterogeneity in diffusion of innovations modelling: A few fundamental types, Technol. Forecast. Soc. Change (2014), http://dx.doi.org/10.1016/j.techfore.2014.02.023

$$x^{\pm} = \mu_X + \frac{1}{2}\left(\nu_X \pm \sqrt{4+\nu_X^2}\right)\sigma_X. \tag{11}$$

If we consider the transformation $Y(X)$, we can determine its moments in a simple way, i.e.,

$$E(Y^r(X)) = \int Y^r(x)\psi(x)dx = P_X^-(y^-)^r + P_X^+\left(y^+\right)^r, \qquad r = 1, 2, \cdots, \tag{12}$$

where $y^{\pm} = Y(x^{\pm})$. Following again Rosenblueth [26], we can consider a real transformation of a bivariate random variable $Y(X_1, X_2)$ obtaining the subsequent density, avoiding independence assumptions of components X_1 and X_2,

$$\begin{aligned}\phi(X_1, X_2) = {} & P_{X_1X_2}^{--}\delta(X_1 - x_1^-)\delta(X_2 - x_2^-) + P_{X_1X_2}^{++}\delta\left(X_1 - x_1^+\right)\delta\left(X_2 - x_2^+\right) \\ & + P_{X_1X_2}^{-+}\delta(X_1 - x_1^-)\delta\left(X_2 - x_2^+\right) + P_{X_1X_2}^{+-}\delta\left(X_1 - x_1^+\right)\delta(X_2 - x_2^-).\end{aligned} \tag{13}$$

Notice that Eqs. (10) and (11) allow the determination of the univariate marginals X_1 and X_2 under the knowledge of the correspondent first moments up to order three μ_{X_1}, σ_{X_1}, ν_{X_1}, μ_{X_2}, σ_{X_2}, and ν_{X_2}.

The joint discrete distribution Eq. (13) may be determined by imposing a final constraint on the correlation coefficient ρ between X_1 and X_2. We do not perform such computations because they are difficult to implement and not relevant in the sequel. Nevertheless, we observe that the parametric dimension of density Eq. (13) is 7.

4. The observable adoption process

The observable adoption process $y_1(t)$ is defined through the integral in Eq. (5) as a first order moment of the solution Eq. (7) based on the distribution $\phi(p, q)$ that characterizes the agents' heterogeneity with reference to the dynamic parameters of a local Bass-like evolutionary process. We can discretize previous integral with the help of the seven-dimensional density Eq. (13) based on μ_p, σ_p, ν_p, μ_q, σ_q, ν_q and ρ, and obtain the following mixture model,

$$\begin{aligned}y_1(t) = {} & P_{p\ q}^{--}\frac{1 - e^{-(p^- + q^-)t}}{1 + \frac{q^-}{p^-}e^{-(p^- + q^-)t}} + P_{p\ q}^{++}\frac{1 - e^{-(p^+ + q^+)t}}{1 + \frac{q^+}{p^+}e^{-(p^+ + q^+)t}} \\ & + P_{p\ q}^{+-}\frac{1 - e^{-(p^+ + q^-)t}}{1 + \frac{q^-}{p^+}e^{-(p^+ + q^-)t}} + P_{p\ q}^{-+}\frac{1 - e^{-(p^- + q^+)t}}{1 + \frac{q^+}{p^-}e^{-(p^- + q^+)t}},\end{aligned} \tag{14}$$

which may be represented without reference to the seven original moments, i.e., it may be based on three parameters for the joint distribution, $P_{p\ q}^{-\ -}$, $P_{p\ q}^{+\ +}$, and $P_{p\ q}^{-\ +}$, and four parameters for the four separate minimal trajectory types implemented by the four distinct couples $p^{\pm}$, $q^{\pm}$.

Previous minimal representation of the aggregate observable process $y_1(t)$ may be reshaped with a further parameter in order to increase flexibility and simplify interpretation. This suggestion is due to the difficulty in establishing closed form solutions for the joint distribution $P_{p\ q}^{\pm\ \pm}$ as an explicit function of the seven moments.

To this end, we introduce a convenient parametrization for the sums $p^{\pm} + q^{\pm}$, which suggests the introduction of an invariance for the ratio $q^{\pm}/p^{\pm}$, in particular,

$$\begin{aligned}&\left(p^+ + q^+\right) \leftrightarrow \frac{q^+}{p^+} \Rightarrow (p+q) \leftrightarrow \frac{q}{p}; \quad P_1 = P_{p\ q}^{++} \\ &(p^- + q^-) \leftrightarrow \frac{q^-}{p^-} \Rightarrow a(p+q) \leftrightarrow \frac{q}{p}; \quad P_2 = P_{p\ q}^{--} \\ &\left(p^+ + q^-\right) \leftrightarrow \frac{q^-}{p^+} \Rightarrow b(p+q) \leftrightarrow \frac{q}{p}; \quad P_3 = P_{p\ q}^{+-} \\ &\left(p^- + q^+\right) \leftrightarrow \frac{q^+}{p^-} \Rightarrow c(p+q) \leftrightarrow \frac{q}{p}; \quad P_4 = P_{p\ q}^{-+}.\end{aligned} \tag{15}$$

This extended parametrization, which adds one more parameter, q / p, with respect to those involved in Eq. (14), allows for a more direct interpretation. We have a benchmark trajectory driven by two parameters, p and q, or the equivalent couple $(p + q)$, q / p. The other three define a slower behaviour for $a < b < c < 1$. Notice that the ratios $\frac{aq}{ap} = \frac{bq}{bp} = \frac{cq}{cp} = \frac{q}{p}$ are invariant.

We propose a common system with an internal benchmark controlled by the p, q dynamic factors. The parallel sub-populations share a scaled proportion of this common dynamic through types a, b, and c that modify the local speed of adoptions with reference to the benchmark.

The marginal observable adoption process Eq. (14) has, therefore, a modified approximate aspect with a simple form that does not require a direct reference to the marginal first few moments of (p, q) and related correlation coefficient ρ, namely,

$$\begin{aligned}y_1(t) = {} & P_1\frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p}e^{-(p+q)t}} + P_2\frac{1 - e^{-(p+q)at}}{1 + \frac{q}{p}e^{-(p+q)at}} \\ & + P_3\frac{1 - e^{-(p+q)bt}}{1 + \frac{q}{p}e^{-(p+q)bt}} + P_4\frac{1 - e^{-(p+q)ct}}{1 + \frac{q}{p}e^{-(p+q)ct}},\end{aligned} \tag{16}$$

where $\sum_{i=1}^{4} P_i = 1$.

In other words, we have imagined a population of agents characterized by possible infinite homogeneous cells with different dynamic factors (p, q) and with a time independent density $\phi(p, q)$. The mean behaviour of the observable normalized process Eq. (5), under local Bass-like dynamics, may be approximated, within the logic of the first three moments, by the discretized mixture of four local Bass 'witnesses' characterized in Eq. (16) by four specific parameters: the constant 1 for the benchmark process with 'relevance' P_1, and a, b and c for the coordinated local main components with relevance P_2, P_3 and P_4, respectively. This description is quite simple and efficient but does not explain why local sub-populations have different levels in the dynamic factors $(p\delta, q\delta)$, $\delta = 1, a, b, c$. In the following section we propose a possible motivation for a discrete differentiation through a few heterogeneous groups.

5. Topological and metric aspects of diffusion

When we describe an aggregate diffusion of innovation process we express its behaviour under latent environmental constraints which are generally unknown and locally concentrated. The environmental space is not homogeneous:

Cultural, behavioural, demographic and economic dimensions are often complementary and heterogeneous in different areas. The multidimensional structure of this space may be simplified through a qualitative lattice of contiguous cells based on a few dimensions (for instance two or three) with possibly internal non-accessible sub-regions. Under a fixed conventional neighborhood of a cell, we may measure the local strength of connectivity among cells through the cardinality of the surrounding cells. The emerging types are only a few. Let us consider, for example, agents or cells of a system distributed within a two-dimensional square lattice. In Cellular Automata representations, the Moore neighborhood of a central cell C is defined by 8 cells surrounding it (see Fig. 1).

This number is smaller for borderline cells (5 or 3). Table 1 represents a square lattice of cells. In the upper part of each cell we denote the connectivity strength through the cardinality of the Moore neighborhood based on Chebyshev distance 1.

A different approach is based on the von Neumann neighborhood, which considers only the four cells surrounding a central cell C in a two-dimensional square lattice. This neighborhood is equivalent to the set of cells at a Manhattan distance of 1 (see Fig. 1).

In the square lattice of Table 1, the number in the lower part of each cell represents the connectivity strength through the cardinality of the von Neumann neighborhood.

The proposed metrics define only three types of connectivity strength in different scales, (8, 5, 3) for the Moore neighborhood, and (4, 3, 2) for the von Neumann. If we extend Table 1 to a cubic lattice, we can distinguish four separate types: (26, 17, 11, 7) for the Moore approach and (6, 5, 4, 3) for the von Neumann.

Notice that there is a one-to-one correspondence between the different metric definitions. All cells which are equivalent for the Moore metric are exactly equivalent under the von Newmann. We may summarize the connectivity strength, as a proxy of the interaction among cells, through some parameters that compare dynamic effects with a benchmark. In a normalized version, we can state $\beta < \alpha < \gamma = 1$. For a two-dimensional square lattice see Table 2.

The diffusion process stemming from cells in a lattice may be specified for each of them. For instance, with reference to a local Bass-like dynamic we may express the cumulative behaviour of a cell (i, j) in the absolute scale as:

$$z_{ij}(t) = m_{ij}\frac{1-e^{-(p_{ij}+q_{ij})t}}{1+\frac{q_{ij}}{p_{ij}}e^{-(p_{ij}+q_{ij})t}}, \qquad t \geq 0, \tag{17}$$

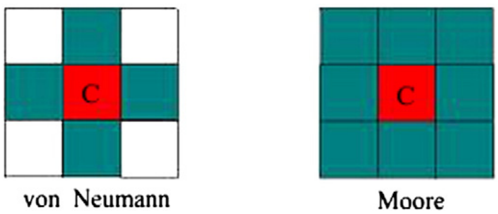

Fig. 1. Von Neumann neighborhood at a Manhattan distance of 1, and Moore neighborhood at a Chebyshev distance of 1.

Table 1
Connectivity strength based on the cardinality of the Moore neighborhood (upper figure) or on the cardinality of the von Neumann neighborhood (lower figure).

i \ j	1	2	3	4
1	3/2	5/3	5/3	3/2
2	5/3	8/4	8/4	5/3
3	5/3	8/4	8/4	5/3
4	3/2	5/3	5/3	3/2

and obtain an aggregate observable model defined by

$$z(t) = m\sum_{ij} P_{ij}\frac{1-e^{-(p_{ij}+q_{ij})t}}{1+\frac{q_{ij}}{p_{ij}}e^{-(p_{ij}+q_{ij})t}}, \qquad t \geq 0, \tag{18}$$

where $m_{ij} = mP_{ij}$ and $\sum_{ij} P_{ij} = 1$.

Even if model (18) is quite general, there are many difficulties in identifying it from a statistical point of view when we only observe $z(t)$. In this respect, it is much more plausible to detect only a smaller set of fundamental types in Eq. (18). Following the common classification based on connectivity strength in a square or cubic lattice, under Moore or von Neumann measures of interactions, we can select a benchmark with the full effect of local dynamic factors (p, q). The other few components may be scaled with coefficients less than $\gamma = 1$, i.e., $\beta < \alpha < 1$.

The reduced model based on fundamental types within a square lattice may be,

$$z(t) = m\left\{P_1\frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}} + P_2\frac{1-e^{-(p+q)\alpha t}}{1+\frac{q}{p}e^{-(p+q)\alpha t}} + P_3\frac{1-e^{-(p+q)\beta t}}{1+\frac{q}{p}e^{-(p+q)\beta t}}\right\}, \tag{19}$$

where $\sum_i^3 P_i = 1$ and $m_i = mP_i$, $i = 1, 2, 3$, are the local aggregate market potentials of the fundamental types. Under a cubic lattice, the fundamental types are four, both with Moore or von Neumann neighboring and related metrics.

Table 2
Connectivity strength equivalence between Moore and von Neumann neighboring. In a normalized version, we can state $\gamma = 1$ and $\beta < \alpha < 1 = \gamma$.

i	j			
	1	2	3	4
1	β	α	α	β
2	α	γ	γ	α
3	α	γ	γ	α
4	β	α	α	β

6. Further improvements

From a statistical point of view, the proposed theoretical reduction based on two different approaches – the TPD formalism and the Cellular Automata framework – depicts a more realistic diffusion dynamic with heterogeneous agents. In this case, we limit generality to fundamental types that may be introduced and tested.

Moreover, following Bemmaor [4], and Bemmaor and Lee [5], we may specify further aspects of heterogeneity that generate local asymmetric behaviour. The corresponding model may be easy to obtain, namely,

$$z_B(t) = m\left\{ P_1 \frac{1-e^{-(p+q)t}}{\left(1+\frac{q}{p}e^{-(p+q)t}\right)^{A_1}} + P_2 \frac{1-e^{-(p+q)\alpha t}}{\left(1+\frac{q}{p}e^{-(p+q)\alpha t}\right)^{A_2}} + P_3 \frac{1-e^{-(p+q)\beta t}}{\left(1+\frac{q}{p}e^{-(p+q)\beta t}\right)^{A_3}} \right\}. \tag{20}$$

Further extensions may incorporate exogenous intervention functions $x_i(t)$ mimicking the Generalized Bass model, GBM, by Bass et al. [3], obtaining, for a simple two component model, the following representation,

$$z_{GBM}(t) = m\left\{ P_1 \frac{1-e^{-(p+q)\int_0^t x_1(\tau)d\tau}}{\left(1+\frac{q}{p}e^{-(p+q)\int_0^t x_1(\tau)d\tau}\right)^{A_1}} + P_2 \frac{1-e^{-(p+q)\alpha\int_0^t x_2(\tau)d\tau}}{\left(1+\frac{q}{p}e^{-(p+q)\alpha\int_0^t x_2(\tau)d\tau}\right)^{A_2}} \right\}. \tag{21}$$

Moreover, market potential m or its local versions $m_i = mP_i$ may be generalized following Guseo and Guidolin [12] in order to take into account its dependence upon time. As an example, we may model it through a general communication process based on a network knowledge expansion:

$$m(t) = K\sqrt{\frac{1-e^{-(p_c+q_c)t}}{1+\frac{q_c}{p_c}e^{-(p_c+q_c)t}}}. \qquad t \geq 0. \tag{22}$$

Further aspects that may be considered are the local times. Different types may have different origins but this should be introduced with parsimony in order to balance complexity of the model with its empirical identification and estimate.

6.1. Network interactions

Heterogeneity discussed in this paper is mainly focused on local perfect communication among agents. In Guseo and Guidolin [12], some effort is spent in order to describe the evolving latent network of relevant knowledge that is necessary for adoption.

Following a static approach, Fibich and Gibori [8], introduced a one-dimensional network structure that strongly constrains the development of knowledge within a system. Every agent may receive relevant information only from a left-or right-positioned member of the system. The resulting limiting cumulative distribution function is defined by:

$$F_{FG}(t) = 1-e^{-(p+q)t+q\left(1-e^{-pt}\right)/p}. \tag{23}$$

If we introduce a first order approximation of e^{-pt}, i.e., $e^{-pt} = 1 - pt$, the approximate cumulative distribution of Fibich and Gibori equals a special monomolecular case:

$$\widetilde{F}_{FG}(t) = 1-e^{-pt}, \tag{24}$$

which explains the limited role of the q parameter in Eq. (23). When we imagine a limited local communication among agents of relative homogeneous sub-populations, we may substitute, partially or completely, the basic Bass-like kernel adopted in Eqs. (14), (16), and (19) with a Fibich–Gibori kernel as described in Eqs. (23) or (24).

7. Compact Cassette format for pre-recorded music in Italy

In this section, we present an application of the proposed modelling approach to a multimodal diffusion process in the music industry.

Music is not only a form of artistic expression, but also an important medium of social communication. It has always been a part of life for people in many cultures and historical periods. Music has been evolved with civilization through the creation of instruments and the development of new modes of performance and use. Musicians and singers have been performing for millennia in houses, under nomad tents, and in public spaces, such as squares, theaters, courts and concert halls. Today, music may be played live, or it may be recorded. In industrialized countries in the 20th century, listening to music through a recorded form became more common than experiencing live performances. This occurred thanks to the introduction and diffusion of many technological innovations in recording and reproduction, which also gave rise to a flourishing industrial sector connected with the creation and sale of recorded music. Even if the magnetic recording attempts date back to the end of the 18th century when Valdemar Poulsen patented the telegraphone in 1898, the commercial development grounded on mechanical reproduction began in 1887 with the phonograph by Thomas Edison, based on tin cylinders, and more importantly, the gramophone based on disks. A positive interaction between disks and the radio highly supported the diffusion of music. After World War II, microgroove technology of vinyl disks, long-playing records (33 rpm) and extended-play records (45 rpm) based on electromechanical players brought music to fruition for almost everybody.

Magnetic recording on tapes was developed in Germany. The AEG Magnetophon (1935) was systematically used in Hitlerian propaganda. After World War II, magnetic tape recording saw increased development even if this music technology never exceeded a limited market of customers with good technical skills.

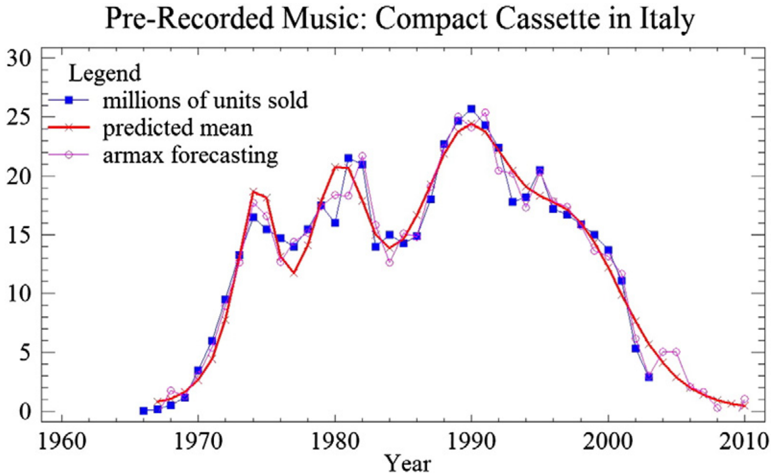

Fig. 2. Pre-recorded music in Italy. Compact Cassette in millions of units. Four heterogeneous Bass sub-populations with decreasing proportional dynamics and Bemmaor asymmetry correction.

In 1963, Philips introduced a prototype of the Compact Cassette in Europe, and in 1965, based on a patent open to compatibility, launched the new technology free of charge to manufacturers. At the world level, the Compact Cassette technology covered a long life-cycle (1965–1990) with continuous improvements in signal-to-noise performances. The new technology increased the market potential of pre-recorded music with a parallel expansion of personal recording and duplications. Piracy phenomena were concomitant aspects of its diffusion, with not necessarily negative effects due to illegal reproduction under copyright regulations (see, for instance, Givon et al. [10] for a first representation of the problem and further extensions in Guseo and Mortarino [15,16] for more general aspects of competition). During the 1980s, the cassettes' popularity grew further as a result of portable pocket recorders and high-fidelity players, such as Sony's Walkman launched in 1979. The Walkman defined the portable music market in the 1980s with sales overtaking those of LPs. The Compact Cassette has been defined 'the winner', the greatest commercial success in the history of audio reproduction (see Andriessen [1]). The reasons for this success may be derived from its technical features, which provided the possibility of enjoying music at any time and any place.

In Western Europe and America, the market for cassettes has declined since its peak in the late 1980s. In particular, the decline is connected with the introduction of the Compact Disk (CD), even though sales remained quite high for at least 10 years.

In this paper, we analyze the special case of the Italian market for Musicassettes. The time series, expressed in million of units sold (source: Musica e Dischi), covers a period from 1966 to 2003, and is characterized by a multimodal distribution over time (see Fig. 2). We argue that this multimodal pattern may be due to the presence of some fundamental sub-populations of adopters.

The proposed statistical model is a special version of Eq. (20), namely:

$$z(t) = mg\frac{1-e^{-(p+q)t}}{\left(1+\frac{q}{p}e^{-(p+q)t}\right)^{A}} + mc\frac{1-e^{-(p+q)\,ct}}{\left(1+\frac{q}{p}e^{-(p+q)\,ct}\right)^{A}} + mb\frac{1-e^{-(p+q)\,bt}}{\left(1+\frac{q}{p}e^{-(p+q)\,bt}\right)^{A}} + ma\frac{1-e^{-(p+q)\,at}}{\left(1+\frac{q}{p}e^{-(p+q)\,at}\right)^{A}}. \quad (25)$$

This model is based on the recognition of four fundamental types that define the basic sources of heterogeneity with a

Table 3
Pre-recorded music in Italy. Compact Cassette in millions of units. Four heterogeneous Bass sub-populations: a benchmark market potential *mg*, and further components *mc*, *mb*, and *ma*, a benchmark dynamic, *p*, *q* and corresponding decreasing proportional dynamic effects *c*, *b*, and *a*, and a common Bemmaor asymmetry correction, *A*. Marginal linearized asymptotic 95% confidence limits are in parentheses. $R^2 = 0.999932$ is a determination index based on cumulative data.

mg	*p*	*q*	*A*	*mc*	R^2
59.5168	0.00000226025	1.42974	0.465723	104.182	0.999932
(53.8131)	(0.00000010192)	(1.37609)	(0.402494)	(98.5084)	*SSE*
(65.2204)	(0.00000441859)	(1.48339)	(0.528952)	(109.856)	88.8903
c	*mb*	*b*	*ma*	*a*	*D–W*
0.598316	179.218	0.371327	209.727	0.280703	1.14617
(0.581892)	(165.332)	(0.359076)	(195.452)	(0.270973)	
(0.614739)	(193.105)	(0.383579)	(224.002)	(0.290432)	

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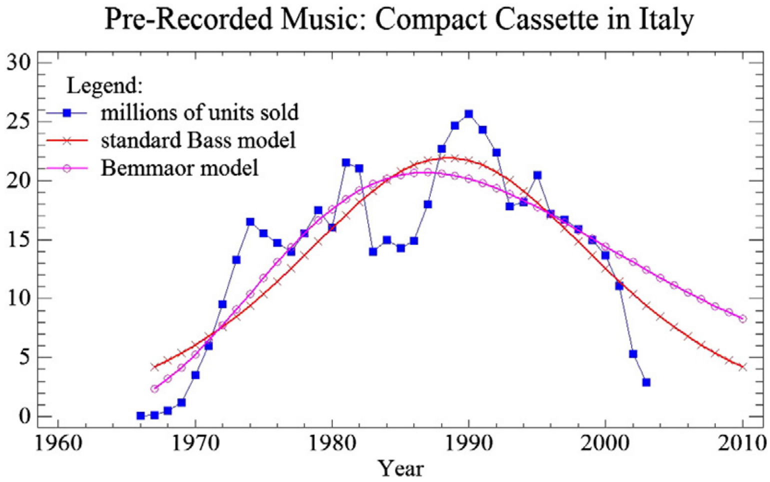

Fig. 3. Pre-recorded music in Italy. Compact Cassette in millions of units. Standard Bass and Bemmaor models.

common asymmetry Bemmaor effect referred to as standard Bass sub-models. For a statistical implementation, we embed model (25) in a nonlinear regression,

$$w(t) = z(t) + \varepsilon(t) \tag{26}$$

where $\varepsilon(t)$ is a residual noise factor and apply a two-step inferential procedure: a nonlinear least squares (NLS) for the estimation of parameters of Eq. (25), and a subsequent ARMAX (auto-regressive moving average with control variables X), in order to identify and estimate the structure of the residual noise $\varepsilon(t)$. Notice that ARMAX is implemented, in this case, with the NLS solution $\hat{z}(t)$ as a control variable.

NLS results are summarized in Table 3. Fig. 2 depicts the efficient description of the multimodal behaviour of Compact Cassette in Italy. The NLS global goodness-of-fit of the cumulative model (25) is very high, $R^2 = 0.999932$, in comparison with a single cycle standard cumulative Bass model with $R^2 = 0.997727$, and the cumulative Bemmaor model with $R^2 = 0.998373$ here not reported in extended form for brevity. For a graphical comparison of instantaneous Bass and Bemmaor models see Fig. 3.

The estimated sub-populations in Table 3 present increasing local market potentials, $mg = 59.5$, $mc = 104.2$, $mb = 179.2$, and $ma = 209.7$, and are temporally subsequent denoting an increasing delay expressed by the factor that multiplies time t in the exponential forms, 1 for the first component assumed as the benchmark, $c = 0.598$ for the second wave, $b = 0.371$ for the third, and finally $a = 0.281$. Notice that all marginal asymptotic confidence intervals are very small with coherent signs, and that the Durbin–Watson statistic may suggest an important structure in the residuals after the NLS estimation phase, D–$W = 1.14$. For a graphical description of global rate sales, and for the identification of four heterogeneous Bass-like sub-populations with decreasing proportional dynamics and local Bemmaor asymmetric correction, see Fig. 4.

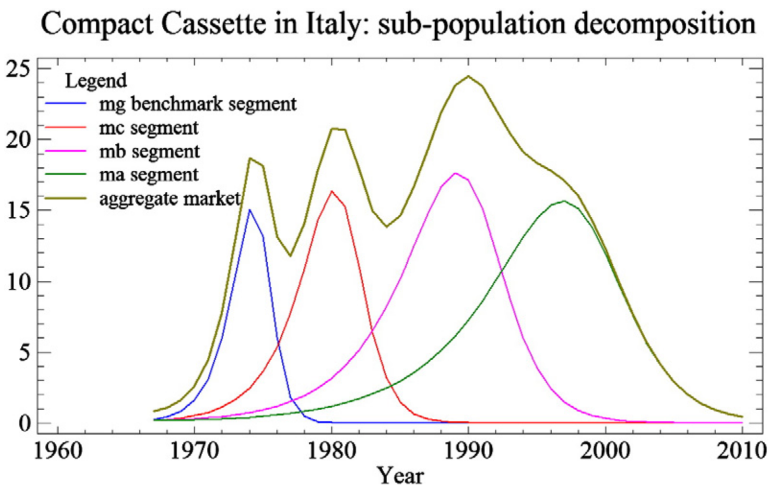

Fig. 4. Pre-recorded music in Italy. Compact Cassette in millions of units. Four heterogeneous Bass sub-populations with decreasing proportional dynamics and Bemmaor asymmetry correction: segments.

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Table 4
Pre-recorded music in Italy. Compact Cassette in millions of units. Discrete type heterogeneous Bass sub-populations with decreasing proportional dynamic effects: ARMAX (5,5) on autocorrelated residuals with the proposed model as input variable; *t*-statistics are in parentheses, *p* values are in square brackets.

AR (1)	AR (2)	AR (3)	AR (4)	AR (5)	Proposed model
0.542679	−0.188258	0.495087	−0.799886	0.545088	1.00223
(2.14569)	(−0.825715)	(1.78518)	(−3.51274)	(2.05299)	(340.363)
[0.041408]	[0.416479]	[0.085905]	[0.001642]	[0.050264]	[0.000000]
MA (1)	*MA* (2)	*MA* (3)	*MA* (4)	*MA* (5)	Mean
−0.155015	0.677711	0.868257	−0.366528	−0.614611	−0.811504
(−0.735962)	(3.47271)	(3.29165)	(−1.85001)	(−2.44636)	(−0.901945)
[0.468342]	[0.001818]	[0.002868]	[0.075710]	[0.021505]	[0.375366]

We may improve the description of model (26) by examining the NLS residuals $\hat{\varepsilon}(t) = w(t) - \hat{z}(t)$ with a convenient ARMAX model,

$$\Phi(B)[w(t) - \xi\hat{z}(t)] = \Theta(B)a_t, \tag{27}$$

in order to detect further autoregressive or moving average components that allow, jointly with NLS predicted values $\hat{z}(t)$, a better forecast for a short-term horizon. Results are summarized in Table 4 and confirm the proposed mean trajectory model (25) with a practically unitary coefficient, $\hat{\xi} = 1.00223$ and a very high *t*-statistic, $t = 340.4$.

In order to compare model (25) with a similar multimodal model we consider a restriction in Bemmaor asymmetric effect by imposing a neutral level, $A = 1$. The reduced model with 9 parameters attains a lower determination index under NLS, namely, $R_9^2 = 0.999879$ which has to be compared with $R_{10}^2 = 0.999932$ associated to the model (25) with 10 parameters. The squared partial correlation coefficient $\tilde{R}^2 = \left(R_{10}^2 - R_9^2\right)/\left(1 - R_9^2\right) = 0.438016$ presents a high value. The corresponding non-parametric *F*-ratio, which take into account the involved degrees of freedom between the nested models, is surely significant: $F = \tilde{R}^2(n-k)/\left(1-\tilde{R}^2\right) = 21.82$ (where $n = 38$ is the number of observations and $k = 10$ the number of parameter of the more complex model (25)). The Bemmaor effect is therefore relevant and cannot be omitted.

A further aspect in evaluating the stability and sensitivity of the proposed model, with reference to forecasting properties, is summarized in Table 5 where we report MAPE and RMSE indexes for both BM and model (25) for different time horizons by eliminating from the observed 38 data the last $h = 4$, 8, 12, and 16 time points. The results denote an alternate behaviour where, in particular model (25) performs better for $h = 12$ and $h = 16$.

If we explore the stability of the estimated response function we notice an exceptional robustness also with $h = 16$ excluded points over 38. Fig. 5 shows that the estimated trajectories are exactly superimposed. With a number of $h = 20$ excluded points – twenty years – the life-cycle does not have sufficient data to discover the multimodalities of the non-observed second period of time. This stress test confirms that the proposed model (25) is quite realistic.

In Fig. 6, we present the cumulative behaviour of the four heterogeneous Bass-like sub-populations with decreasing proportional dynamics and local Bemmaor asymmetry correction. This cumulative overlapping effect of delayed segments gives evidence to the presence of different sub-populations of adopters or adoptions. The first generation of adoptions, with an estimated market potential of about 60 million units sold, represented the most innovative component of the process and peaked in 1974. In particular, we may argue that this first wave is due to high fidelity fans, interested in the technological innovation of Compact Cassette and related players, which were quite expensive in the initial stages.

During the period 1974–1980, microgroove technology, based on extended-play records (45 rpm) and long-playing records (33 rpm), contrasted the expansion of the Compact Cassette support that reached, with a second wave, a market potential of about 104 million units sold. After 1980, the 45 rpm records declined within a few years. The LPs were able to maintain a good level of sales for 10 years although declined rapidly after 1990. Sales of pre-recorded music Compact Cassettes exceeded LP sales in Italy for the first time in 1986. A possible interpretation of this second sub-population, with a higher market potential, is connected with the wide adoption of cassette players for home listening and the parallel diffusion of the car audio culture. In fact, one of the greatest merits of cassettes with respect to vinyls is to have mobilized music listening. The decline in sales occurring after 1980 may be imputed to the concurrent phenomenon of home reproduction of albums using blank cassettes.

The third wave, which peaked in 1990, had a market potential of about 180 millions of units sold, and may be reasonably connected with the introduction and massive diffusion in Italy of the Walkman, which individualized music listening, becoming a lifestyle symbol. The introduction of CD supports in 1983 did not produce an immediate effect on Compact Cassette sales. After 1990, its expansion became much stronger, implying a decreasing behaviour in cassette

Table 5
Pre-recorded music in Italy. Compact Cassette in millions of units. Four discrete type heterogeneous Bass sub-populations with decreasing proportional dynamic effects compared with standard Bass model BM in terms of MAPE for $h = 4$, 8, 12, and 16, and RMSE.

	MAPE		RMSE	
h	BM	4-Types	BM	4-Types
4	0.83629	1.62261	5.05638	10.82289
8	2.70209	7.11721	14.3084	41.00707
12	13.5159	2.58781	71.5425	13.07708
16	32.3352	12.1660	166.331	78.887514

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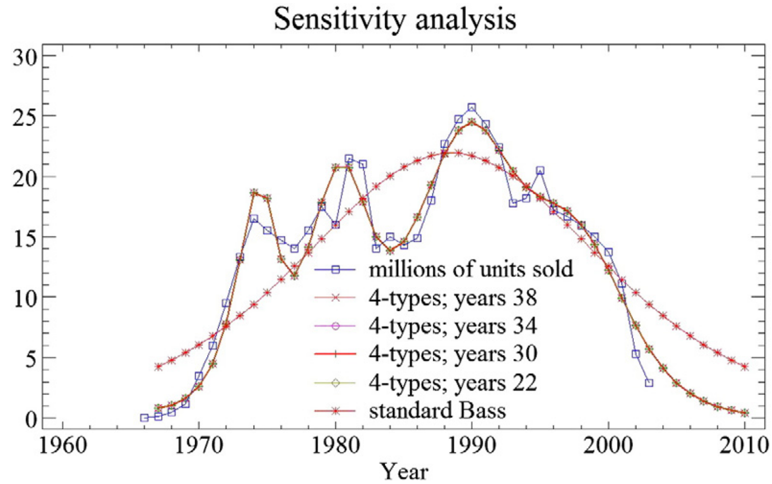

Fig. 5. Pre-recorded music in Italy. Compact Cassette in millions of units. Sensitivity analysis of the four types of heterogeneous Bass sub-populations with decreasing proportional dynamics and Bemmaor asymmetry correction. The estimated NLS trajectories for 38, 34, 30, and 22 observed points with $h = 0, 4, 8$ and 16, are practically indistinguishable.

sales, with a partial revival due to the fourth large generation peaked in 2000 with about 210 million units sold. This fourth generation may be interpreted as an inertial effect: cassettes remained popular for specific applications, such as car audio, for many years, being more resistant to dust, heat and shocks than the competitor (CD).

As we have seen, the life-cycle of the Compact Cassette in Italy has been characterized by a multimodal behaviour, which we interpreted as the result of the co-existence of four heterogeneous populations. It is worth observing that the same user may have been part of different populations over time; for instance, he may have listened to cassettes at home first, then while driving, and finally with the Walkman. So, what qualifies heterogeneity is a different mode of usage, which appears strongly related to the diffusion of new technologies for music listening.

8. Final remarks and conclusions

Heterogeneity of agents and network structure may affect the dynamics of the diffusion of an innovation. In this paper, we chose an aggregate and sufficiently flexible multimodal description of heterogeneity, which allows for an efficient and cost-effective prediction process. As discussed in the Introduction, heterogeneity in innovation diffusion is not limited to individual properties of agents, but has much to do with individual access to information, and thus with relationships, WOM effects and network structures. Heterogeneity may also affect the market potential structure and the corresponding adoption process. In the literature, we find different solutions based on micro-modelling approaches. In Chatterjee and Eliashberg [6], heterogeneity is modelled within the adoption process, while in Guseo and Guidolin [12] it generates a dynamic market potential.

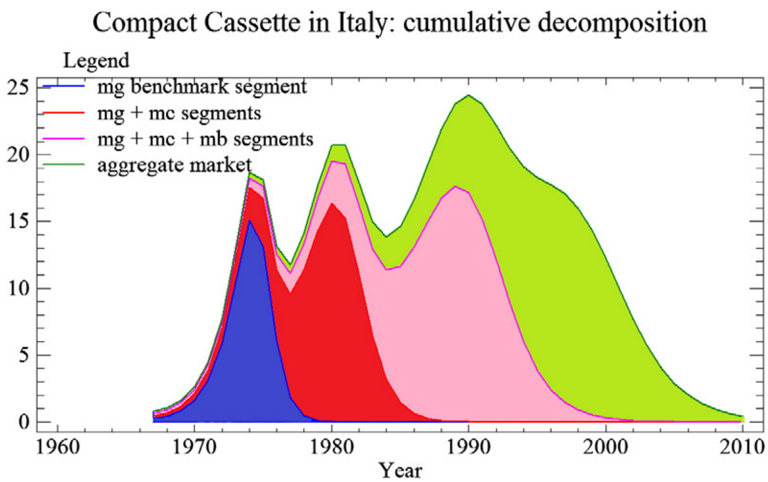

Fig. 6. Pre-recorded music in Italy. Compact Cassette in millions of units. Four heterogeneous Bass sub-populations with decreasing proportional dynamics and Bemmaor asymmetry correction: cumulative segments.

In this paper, we consider heterogeneity at the adoption level and propose a general composite population with random dynamic factors under an unknown mixing distribution. The assumed conditional kernel is a Bass-like distribution, which may be modified under different conditions. The proposed discrete mixture is based on Rosenblueth TPD formalism with some extensions in order to achieve easy-to-implement models. In particular, the multimodal model (16) is based on a benchmark and few other parallel types. A similar result is obtained following connectivity criteria based on von Neumann and Moore metrics. Bemmaor-like improvements may be obtained by accommodating local asymmetric behaviour Eq. (20).

We have applied the proposed multimodal model to the life-cycle of the Compact Cassette format for pre-recorded music in Italy. The obtained decomposition in parallel waves is statistically significant and quite interesting for substantive interpretations. Residual components around the proposed model may be adequately described through an ARMAX model (27) in order to improve short-term forecasting. Comparisons with other models and sensitivity tests confirm the significance of the proposed model.

Further improvements and extensions may be studied in the future in order to take into account, especially with reference to the fourth wave, some effects introduced by the successive technologies.

Acknowledgments

The authors would like to express their gratitude to the anonymous reviewers for helpful comments and suggestions.

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