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Global/local hybridization of the DIRECT algorithm for derivative-free global optimization in ship design

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Abstract A simulation-based derivative-free global optimization of the hull-form of a military vessel is presented, aimed at the reduction of the resistance in calm water. The objective function is of the black-box type, computationally expensive and characterized by noise and non-smoothness. The presence of local minima cannot be excluded a priori. For these reasons, the use of derivative-free, global, `DIRECT`-type algorithms is proposed. Specifically, a recent hybridization of the `DIRECT` algorithm by local minimization is applied, and compared with a novel strategy for the management of the local searches. Comparative results are reported on a set of well-known and well-established test problems and on the simulation-based hull-form optimization problem.

Keywords Global optimization · Local Search · `DIRECT`-type algorithm · Simulation-based design optimization · Ship design

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1 Introduction

Simulation-based design (SBD) optimization is an essential part of the design of complex engineering systems, since the conceptual and early design stages. SBD optimization has been widely applied to diverse engineering fields, such as aerospace [30, 22, 8], automotive [10, 37, 16] and naval [29, 9, 20] design. SBD optimization methodologies are constantly developed, with large computational simulations managed to assess the performance of a design and evaluate the relative merit of design alternatives. The constant search for improvements in SBD optimization focuses on its three essential elements: (i) the simulation capabilities and the associated accuracy of the analysis, (ii) the definition and management of the design variables through the design and optimization process, and (iii) the efficiency and robustness of the optimization algorithm.

Within SBD optimization, a non-convex nonlinear programming problem of the form

$$\min_{x \in \mathcal{D}} f(x) \tag{1}$$

is solved, where $\mathcal{D} = \{x \in \mathbb{R}^n : l_i \leq x \leq u_i, i = 1, \dots, n\}$ is the design variables domain, and the objective function $f(x)$ represents the performance of the engineering system and is usually of the black-box type, with values provided by computationally-expensive computer simulations. In order to achieve an optimal design decision, a global minimum of $f(x)$ on the feasible domain $\mathcal{D}$ is sought, which is motivated by the following considerations:

- typical objective functions are almost ever non-convex and characterized by the presence of noise and non-smoothness, so that local minimization packages could likely be trapped in local minimum points;
- the ever advancing work of design engineers have lead to the production of near optimal configurations so that the margin for improvements is rapidly narrowing, and the possibility that further improvements could come from local optimization methods is likely getting small.

Problem (1) is very general and, as such, encompasses many real-world applications arising in many different fields, like computational mathematics, physics and engineering. Herein, the optimal design of a military ship hull is used as an engineering test case, based on computational fluid dynamics (CFD) simulations. The focus is on the optimization algorithm, as an essential element for the success of the overall SBD optimization process.

The objective of the present study is the efficient solution of problem (1) by hybridization of `DIRECT`-type algorithms [19, 11, 38, 1, 15, 5, 12, 23, 18, 13, 14, 6], which are well-behaved deterministic algorithms for global optimization and have been successfully applied to solve diverse real-world problems. Specifically, the focus is on the `DIRMIN` algorithm (proposed in [23] and, in a derivative-free context, in [24]) which is an enriched version of the `DIRECT` algorithm by local minimization. A modification of `DIRMIN` (which is hereafter referred to as `DIRMIN-2`) is proposed for further investigations, consisting in an enhanced strategy for the management of the local searches.

In order to investigate the performance of `DIRMIN` and `DIRMIN-2`, forty six test functions with a level of complexity and dimensionality similar to typical optimization problems in ship design are used. The sensitivity of the algorithms' performance to the local minimization activation trigger and the local minimization tolerance is studied. Data and performance profiles [28], along with three absolute metrics [32], are assessed and used to identify the most significant algorithms' setting parameters. The most promising implementations are applied to the ship SBD optimization problem.

The SBD application pertains to the hull-form optimization of a USS Arleigh Burke-class destroyer, namely the DDG-51. The DTMB 5415 model, an open-to-public early concept of the DDG-51, is used in the current study. The DTMB 5415 model has been widely investigated through towing tank experiments [33, 26, 17] and SBD studies, including hull-form optimization [3, 34, 21]. Lately, the DTMB 5415 has been selected as the test case for the SBD activities within the NATO STO AVT-2014 "Assess the Ability to Optimize Hull Forms of Sea Vehicles for Best Performance in a Sea Environment", aimed at multi-objective design optimization for multi-speed reduced resistance and improved seakeeping performance. Herein, a single-speed single-objective SBD example is presented, aimed at the reduction of the total resistance in calm water at 18 kn, corresponding to a Froude number (Fr) equal to 0.25. An orthogonal representation of the shape modification is used for efficient SBD optimization [7]. Specifically, two sets of orthogonal functions are used for the modification of the hull and the sonar dome shapes, and controlled by a total of six design variables. The constraints include fixed displacement and length, along with a $\pm 5\%$ maximum variation of beam and draft. The solver used is a linear potential flow code [2], allowing for the evaluation of the wave resistance by transversal wave cut [35]. The resistance due to friction is estimated by a local approximation based on flat-plate theory [31].

2 Global/local hybridizations of the DIRECT algorithm

In this section, the `DIRMIN` algorithm is first recalled, as proposed in [25], and then a new `DIRECT`-type algorithm, namely `DIRMIN-2`, is described which is better suited for the ship hull design application of concern.

The `DIRMIN` Algorithm proposed in [25] is described below.

Algorithm DIRMIN

$\mathcal{H}_1 = \{D\}$, $c =$ center of D, $f_{min} = f(c)$, $X_{min} = \{c\}$, tol, $kmax$, $k = 1$

Repeat

(S.1) identify the potentially-optimal hyperrectangles $\mathcal{P}_k$ in $\mathcal{H}_k$

(S.2) for all centroids c^i of hyperrectangles in $\mathcal{P}_k$ perform a local minimization and record the best function value f_{ml}

(S.3) subdivide the potentially-optimal hyperrectangles to build $\mathcal{H}_{k+1}$

(S.4) evaluate f in the centers of the new hyperrectangles

(S.5) $f_{min} = \min\{f(c) : c \in C_k, f_{ml}\}$, $X_{min} = \{x \in D : f(x) = f_{min}\}$, $k = k + 1$

C_k is the set of centroids c of the hyperrectangles in $\mathcal{H}_k$

Until (stopping criterion satisfied)

Return f_{min}, X_{min}

Note that the local minimizations at Step (S.2) are performed by using the derivative-free local optimization algorithm for bound constrained problems proposed in [27]. As showed in [25], the DIRMIN algorithm can be quite efficient. However, when the evaluation of the objective function is computationally expensive and therefore the budget of function evaluations is limited, DIRMIN might miss the global solution or a point sufficiently close to it. For this reason, a more efficient variant of DIRMIN is proposed. Rather than performing the local minimizations starting from the centroids of all the potentially-optimal hyperrectangles, a single local minimization is performed starting from the best point produced by dividing the potentially-optimal hyperrectangles. This strategy should result in a more efficient algorithm, which is less demanding in terms of number of function evaluations and preserves a good capability to find global solutions.

In details, the new algorithm DIRMIN-2 is defined as follows.

Algorithm `DIRMIN-2`

$\mathcal{H}_1 = \{D\}$, c = center of D, $f_{min} = f(c)$, $X_{min} = \{c\}$, tol, $kmax$, $k = 1$

Repeat

(S.1) identify the potentially-optimal hyperrectangles $\mathcal{P}_k$ in $\mathcal{H}_k$

(S.2) subdivide the potentially-optimal hyperrectangles and to build $\mathcal{H}_{k+1}$.

(S.3) let $\tilde{c} = \arg\min\{f(c) : c \in C(\mathcal{S}_k)\}$,
where $\mathcal{S}_k$ is the set of newly produced hyperrectangles and
$C(\mathcal{S}_k)$ the set of the centroids in $\mathcal{S}_k$.

(S.3) perform a local minimization starting from $\tilde{c}$ and
let f_{ml} be the final function value.

(S.4) evaluate f in the centers of the new hyperrectangles

(S.5) $f_{min} = \min\{f(c) : c \in C(\mathcal{H}_k), f_{ml}\}$, $X_{min} = \{x \in D : f(x) = f_{min}\}$, $k = k + 1$

Until (stopping criterion satisfied)

Return f_{min}, X_{min}

It may be stressed that, differently from `DIRMIN` (that performs as many local searches as the number of identified potentially-optimal hyperrectangles), `DIRMIN-2` performs a single local minimization per iteration. More in particular, the single local minimization starts from the best point found after subdividing the potentially-optimal hyperrectangles at the current iteration. The rationale behind this choice hinges on considering the subdivision of potentially-optimal hyperrectangles as a crude kind of local search, which can be improved by the use of a more sophisticated and efficient local minimization algorithm.

3 Evaluation metrics

The metrics used to assess the performance of `DIRMIN` and `DIRMIN-2` are recalled in the following.

3.1 Performance and data profiles

In order to evaluate the relative performance of the proposed algorithms, the procedure in [28] is used. The following convergence condition is applied:

$$f(x_0) - f(x_k) \geq (1 - \tau)(f(x_0) - f_L) \tag{2}$$

where $0 \leq \tau \leq 1$ is a suitably chosen tolerance and f_L is the smallest function value obtained by any solver within the same maximum computational budget.

The main concepts needed to formally define data and performance profiles are recalled in the following. Let $\mathcal{A}$ be a set of n_a algorithms, and $\mathcal{P}$ a set of $|\mathcal{P}|$ problems and a performance measure $m_{p,a}$ (e.g. in our case, the number of function evaluations). The performance on problem p by algorithm a is compared with the best performance by any algorithm on this problem, using the following *performance ratio*

$$r_{p,a} = \frac{m_{p,a}}{\min\{m_{p,a} \ : \ a \in \mathcal{A}\}}.$$

Thus, a first measure of the performance of algorithm a is defined by the *performance profile*:

$$\rho_a(\alpha) = \frac{1}{|\mathcal{P}|}\text{size}\{p \in \mathcal{P} \ : \ r_{p,a} \leq \alpha\},$$

which approximates the probability for algorithm $a \in \mathcal{A}$ that the performance ratio $r_{p,a}$ is within a factor $\alpha \in R$ of the best possible ratio. The convention $r_{p,a} = \infty$ is used when algorithm a fails to satisfy the convergence test (2) for problem $p \in \mathcal{P}$.

A further measure of the algorithms' performance is given by the percentage of problems that can be solved (for a given tolerance τ) within a certain number of function evaluations. To this aim, let us define $t_{p,a}$ the number of function evaluations needed for algorithm a to satisfy (2) for a given tolerance τ, then the percentage of problems solved with ν function evaluations is the so called *data profile*:

$$d_a(\nu) = \frac{1}{|\mathcal{P}|}\text{size}\{p \in \mathcal{P} \ : \ \frac{t_{p,a}}{n_p + 1} \leq \nu\},$$

where n_p is the number of variables in $p \in P$. If the convergence test (2) cannot be satisfied within the assigned computational budget, we set $t_{p,a} = \infty$.

3.2 Absolute metrics

Three absolute performance criteria are used to further assess the algorithms and defined as follows [32]:

$$\Delta_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - x_i^\star}{R_i}\right)^2}, \quad \Delta_f = \frac{f_{\min} - f_{\min}^\star}{f_{\max}^\star - f_{\min}^\star}, \quad \Delta_t = \sqrt{\frac{\Delta_x^2 + \Delta_f^2}{2}} \qquad (3)$$

Δ_x is a normalized Euclidean distance between the minimum position found by the algorithm (x_{min}) and the analytical minimum position ($x_{min}^\star$). Δ_f is the associated normalized distance in the image space, $f_{\min}$ is the minimum found by the algorithm, $f_{\min}^\star$ is the analytical minimum, and $f_{\max}^\star$ is the analytical maximum of the function f in the research space. Δ_t is a combination of Δ_x and Δ_f and used for an overall assessment.

4 Optimization problems

The problems used to evaluate the performance of `DIRMIN` and `DIRMIN-2` are presented in the following.

4.1 Analytical test problems

Forty six analytical test problems are used, as summarized in Tab. 1. They include simple unimodal, highly-complex multimodal and not differentiable problems, with dimensionality ranging from two to six. The dimensions of the problems considered are consistent with typical applications in shape optimization for ship hull-form design, which typically have less than ten variables (see, e.g., [3, 34, 32, 21, 4, 7]).

Table 1 Analytical test problems

$f_k(x)$	**Name**	**Dimension** n	**Bounds** $[x_{min}, x_{max}]^{i,\ldots,n}$	**Optimum** min $f(x)$
$f_1(x)$	Sphere	2	$[-5, 4]^n$	0.000
$f_2(x)$	Freudenstein-Roth	2	$[-5, 5]^n$	0.000
$f_3(x)$	Ackley	2	$[-5, 4]^n$	0.000
$f_4(x)$	Three-Hump Camel Back	2	$[-5, 4]^n$	0.000
$f_5(x)$	Six-Hump Camel Back	2	$[-2.5, 2.5]^i$, $[-1.5, 1.5]^j$	-1.032
$f_6(x)$	Quartic	2	$[-10, 10]^n$	-0.352
$f_7(x)$	Beale	2	$[-4.5, 4.5]^n$	0.000
$f_8(x)$	Schubert penalty 1	2	$[-10, 10]^n$	-186.731
$f_9(x)$	Schubert penalty 2	2	$[-10, 10]^n$	-186.731
$f_{10}(x)$	Booth	2	$[-10, 10]^n$	0.000
$f_{11}(x)$	Matyas	2	$[-9, 7]^n$	0.000
$f_{12}(x)$	Goldstein-Price	2	$[-2, 2]^n$	3.000
$f_{13}(x)$	Bukin n.6	2	$[-15, -5]^i$, $[-3, 3]^j$	0.000
$f_{14}(x)$	Rosenbrock	2	$[-100, 100]^n$	0.000
$f_{15}(x)$	Schaffer n.2	2	$[-100, 90]^n$	0.000
$f_{16}(x)$	Schaffer n.6	2	$[-100, 90]^n$	0.000
$f_{17}(x)$	Easom	2	$[-100, 100]^n$	-1.000
$f_{18}(x)$	Test Tube Holder	2	$[-10, 10]^n$	-10.872
$f_{19}(x)$	Treccani	2	$[-5, 4]^n$	0.000
$f_{20}(x)$	Tripod	2	$[-100, 100]^n$	0.000
$f_{21,22}(x)$	Exponential	2, 4	$[-9, 7]^n$	-1.000
$f_{23,24}(x)$	Styblinski-Tang	2, 4	$[-5, 5]^n$	$-39.166 \cdot n$
$f_{25,26}(x)$	Cosine Mixture	2, 4	$[-1, 0.5]^n$	$-0.100 \cdot n$
$f_{27,28}(x)$	Hartman n.3, n.6	3, 6	$[0, 1]^n$	-3.860, -3.320
$f_{29,30}(x)$	5^n loc. minima (Levy)	2, 5	$[-10, 10]^n$	0.000
$f_{31,32}(x)$	10^n loc. minima (Levy)	2, 5	$[-10, 10]^n$	0.000
$f_{33,34}(x)$	15^n loc. minima (Levy)	2, 5	$[-5, 5]^n$	0.000
$f_{35,36}(x)$	Griewank	2, 5	$[-9, 7]^n$	0.000
$f_{37,38}(x)$	Alpine	2, 5	$[-9, 7]^n$	0.000
$f_{39,40}(x)$	Multi Modal	2, 5	$[-1, 0.5]^n$	0.000
$f_{41,42}(x)$	Dixon-Price	2, 5	$[-1, 1]^n$	0.000
$f_{43}(x)$	Colville	4	$[-10, 10]^n$	0.000
$f_{44}(x)$	Shekel n.5	4	$[0, 10]^n$	-10.153
$f_{45}(x)$	Shekel n.7	4	$[0, 10]^n$	-10.403
$f_{46}(x)$	Shekel n.10	4	$[0, 10]^n$	-10.536

4.2 SBD optimization of the DTMB 5415

Figure 1 shows the geometry of a 5.720 m length DTMB 5415 model used for towing tank experiments, as seen at CNR-INSEAN [33]. The main particulars of the full scale model and test conditions are summarized in Tab. 2 and

Table 2 DTMB 5415 main particulars (full scale)

Description	Symbol	Unit	Value
Displacement	D	tons	8,636
Lenght between perpendiculars	LBP	m	142.0
Beam	B	m	18.90
Draft	T	m	6.160
Longitudinal center of gravity	LCG	m	71.60
Vertical center of gravity	VCG	m	1.390

Table 3 Test conditions

Description	Symbol	Unit	Value
Speed	U	kn	18.00
Water density	ρ	kg/m^3	998.5
Kinematic viscosity	ν	m^2/s	$1.09 \cdot 10^{-6}$
Gravity acceleration	g	m/s^2	9.803

3, respectively. The objective function is the total resistance, R_T, in calm water at Fr equal to 0.25. A six dimensional design space is considered. Design modifications are defined in terms of orthogonal functions, $\boldsymbol{\psi}_j$ $(j = 1, ..., 6)$, defined over surface-body patches as

$$\begin{cases} \boldsymbol{\psi}_j(\xi,\eta) := \alpha_j \sin\left(\dfrac{p_j \pi \xi}{A_j} + \phi_j\right) \sin\left(\dfrac{q_j \pi \eta}{B_j} + \chi_j\right) \mathbf{e}_{k(j)} \\ (\xi,\eta) \in [0; A] \times [0; B] \end{cases} \tag{4}$$

where the coefficient α_j is the corresponding (dimensional) design variable; p_j and q_j define the order of the function in ξ and η direction respectively; ϕ_j and χ_j are the corresponding spatial phases; A_j and B_j define the patch dimension; $\mathbf{e}_{k(j)}$ is a unit vector. Modifications may be applied in x, y or z direction ($k(j) = 1, 2,$ or 3 respectively).

Fig. 1 A 5.720 m length model of the DTMB 5415 (CNR-INSEAN model 2340)

Specifically, four orthogonal functions and design variables are used for the hull, whereas two functions/variables are used for the sonar dome, as summarized in Tab. 4. The corresponding functions used for shape modification are shown in Figs. 2 and 3. Upper and lower bounds used for dimensional (α_j) and non-dimensional ($x_j = 2(\alpha_j - \alpha_{j,\min})/(\alpha_{j,\max} - \alpha_{j,\min}) - 1$) design variables are included in Tab. 4. Geometric constraints include fixed length between

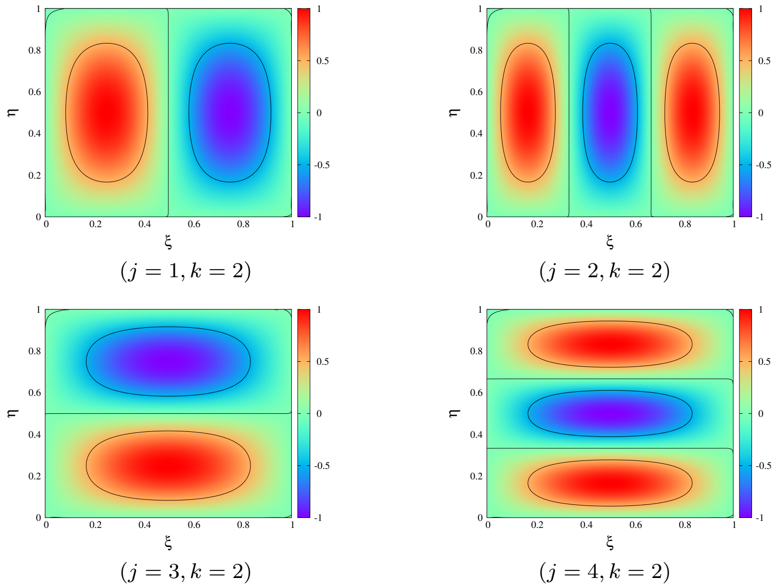


Fig. 2 Orthogonal functions $\psi_j(\xi, \eta), j = 1, ..., 4$, for the hull modification

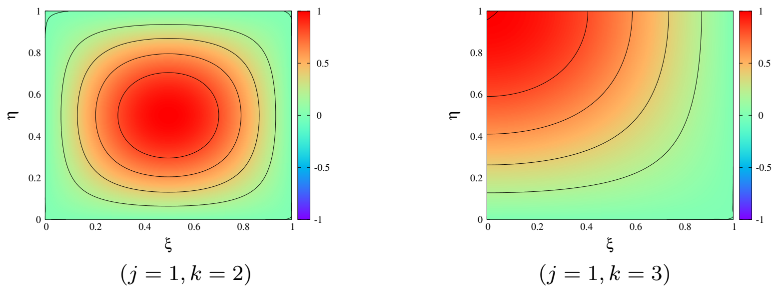


Fig. 3 Orthogonal functions $\psi_j(\xi, \eta), j = 5,\ 6$, for the sonar dome modification

perpendiculars (LBP) and fixed displacement (D), with beam (B) and draft (T) varying between ±5% of the original value.

Simulations are conducted using the code WARP (WAve Resistance Program), developed at CNR-INSEAN. Wave resistance computations are based on linear potential flow theory; details of equations, numerical implementation and validation of the numerical solver are given in [2]. The wave resistance is evaluated with the transverse wave cut method [35], whereas the frictional resistance is estimated using a flat-plate approximation, based on the local Reynolds number [31]. Simulations are performed for the right demi-hull, taking advantage of symmetry about the xz plane. The computational domain for the free surface is defined within 1 hull length upstream, 3 lengths downstream and 1.5 lengths aside, as shown in Fig. 5. The associated panel grid used (Fig. 5) is summarized in Tab. 5 and guarantee solution convergence. The

Table 4 Orthogonal functions parameters, for shape modification

Description	j	p_j	ϕ_j	q_j	χ_j	$k(j)$	$\alpha_{j,\min}$	$\alpha_{j,\max}$	$x_{j,\min}$	$x_{j,\max}$
Hull modification	1	2.0	0	1.0	0	2	-2.0	2.0	-1.0	1.0
	2	3.0	0	1.0	0	2	-2.0	2.0	-1.0	1.0
	3	1.0	0	2.0	0	2	-1.0	1.0	-1.0	1.0
	4	1.0	0	3.0	0	2	-1.0	1.0	-1.0	1.0
Sonar dome modification	5	1.0	0	1.0	0	2	-0.6	0.6	-1.0	1.0
	6	0.5	$\pi/2$	0.5	0	3	-1.0	1.0	-1.0	1.0

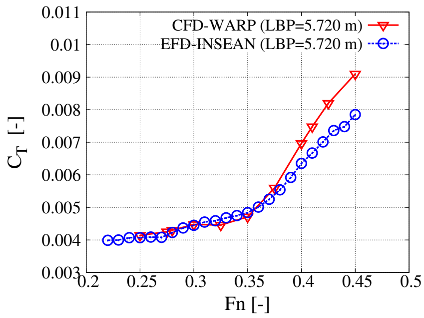


Fig. 4 Total resistance coefficient ($C_T = 0.5 R_T/\rho U^2 S$) in calm water versus Fr: comparison of WARP results with experiments

Table 5 Panel grid used for WARP

Hull	Free surface			Total
	Upstream	Hull side	Downstream	
150 × 30	30 × 44	30 × 44	90 × 44	11k

validation of CFD analyses performed by WARP for the original hull versus experimental data collected at CNR-INSEAN is shown in Fig. 4, showing a reasonable trend.

5 Numerical results

This section shows the propaedeutic studies, conducted on the analytical test problems, followed by the simulation-based design optimization of the DTMB 5415 hull-form.

A limit on the maximum number of function evaluations is imposed, equal to $b(n) = 256n$. `DIRMIN` and `DIRMIN-2`'s local minimization is used when the number of function evaluations reaches the activation trigger $\gamma b(n)$, with $\gamma \in (0, 1)$. The local minimization proceeds until either the number of function evaluations exceeds $b(n)$ or the stepsize Δ falls below a given tolerance β. The latter parameters γ and β are considered herein as $\gamma = c \cdot 10^{-1}$, with

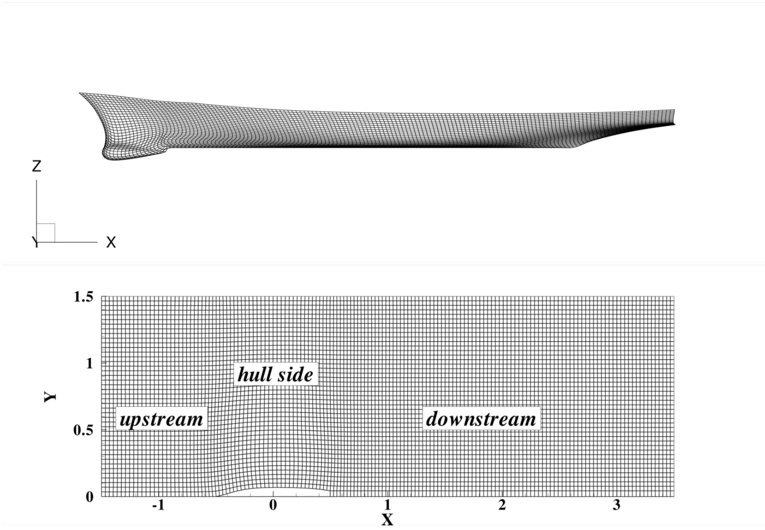


Fig. 5 Computational panel-grid

$c \in \{1, 3, 5\}$ and $\beta = 10^{-d}$, with $d \in \{2, 3, 4\}$. In the following, a specific algorithm's setting is referred to as per Tab. 6.

Table 6 Algorithms' setup ID

γ \ β	10e-2	10e-3	10e-4
0.1	1	2	3
0.3	4	5	6
0.5	7	8	9

5.1 Propaedeutic studies on analytical test problems

Test-problems results are presented in the following and used to define the most promising setups for `DIRMIN` and `DIRMIN-2`.

In Fig. 6, data and performance profiles are reported to compare the nine setups of `DIRMIN`. As it can be easily seen, the setup 3 (i.e. $\gamma = 0.1, \beta = 10^{-4}$) is the most efficient. Indeed, the corresponding data profile raises more steeply than the other ones, and the performance profile is the one starting with the highest value. However, another performance that has to be carefully taken into consideration when evaluating the behavior of an optimization algorithm is the robustness, that is the reliability exhibited by the method in locating the solution with a sufficiently good precision. This information can be extracted from data and performance profiles. In particular, the most robust codes are

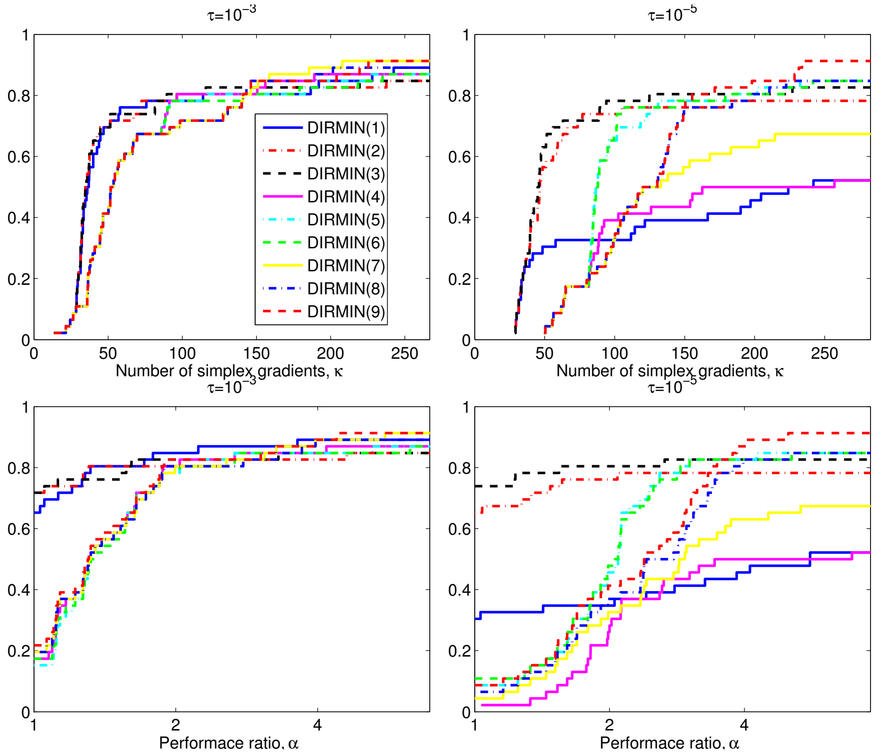


Fig. 6 Data and performance profiles for the nine versions of algorithm `DIRMIN`

the one associated with the curves that reach the highest level. With respect to robustness, from Fig. 6 emerges that the best setup is number 9 (i.e. $\gamma = 0.5, \beta = 10^{-4}$).

In Fig. 7, data and performance profiles for the nine setups of `DIRMIN-2` are shown. Following a similar analysis as that for `DIRMIN`, the most efficient version of `DIRMIN-2` is the setup 3 (i.e. $\gamma = 0.1, \beta = 10^{-4}$), whereas the most robust versions are those corresponding to setups 6 (i.e. $\gamma = 0.3, \beta = 10^{-4}$) and 9 (i.e. $\gamma = 0.5, \beta = 10^{-4}$).

Figures 8a and 8b, show the performance of `DIRMIN` and `DIRMIN-2` in terms of the absolute metrics Δ_x, Δ_f and Δ_t. Average values are presented, conditional to γ and β and respectively. Figure 8c show the relative variance σ_r^2 [32] of Δ_x, Δ_f and Δ_t respectively, for γ and β. β is found the most significant parameter for `DIRMIN`, while γ is the most important for `DIRMIN-2`. Nevertheless, γ and β are important and should be considered carefully. Figure 8d show the performance of the whole set of `DIRMIN` and `DIRMIN-2`'s setups, in terms of Δ_x, Δ_f and Δ_t respectively. The best performing setup is 9 for `DIRMIN` and 6 for `DIRMIN-2`. This is consistent with the nature of the algorithms. Indeed, when the local searches are executed, the cost per iteration in terms of number of function evaluations is higher in `DIRMIN` than `DIRMIN-2`. Accordingly, it is

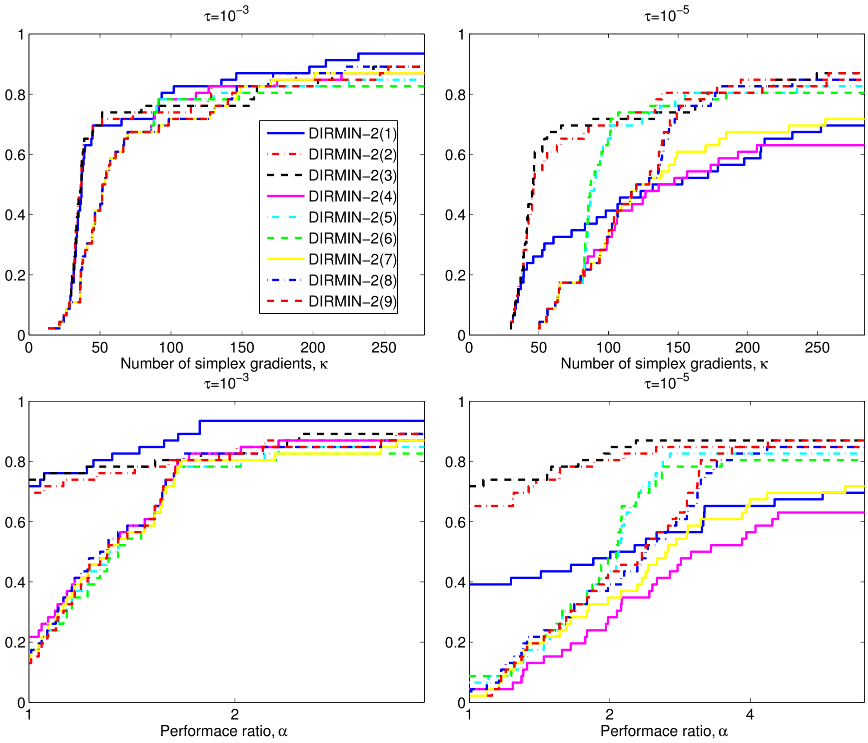
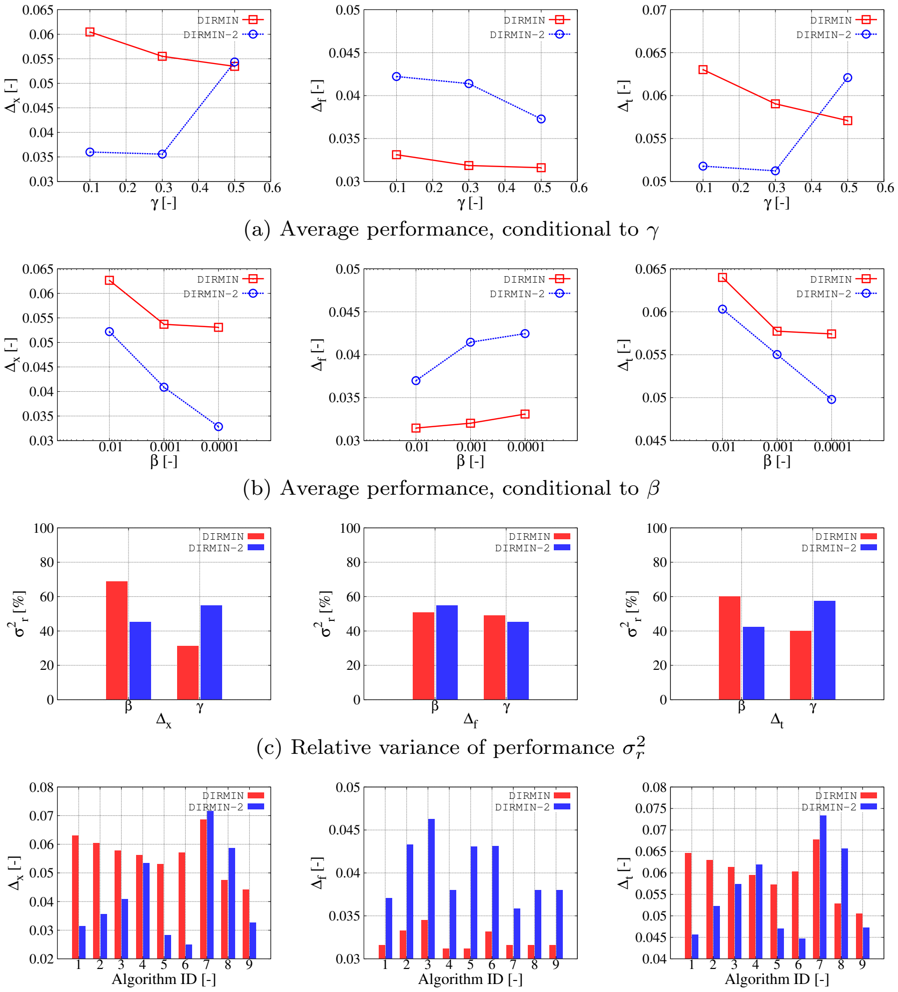


Fig. 7 Data and performance profiles for the nine versions of algorithm DIRMIN-2

reasonable that the best setting of DIRMIN corresponds to a value of $\gamma = 0.5$, which means that the local minimizations are used when the number of function evaluations reaches the 50% of the available budget $b(n)$. On the contrary, for DIRMIN-2 it is more convenient to activate the local minimization as soon as the number of function evaluations reaches the 30% of the available budget $b(n)$, which corresponds to a value of $\gamma = 0.3$.

Finally, the performance of DIRMIN(3) and (9) and DIRMIN-2(3) and (6) are compared. Figure 9 shows the comparison in terms of data and performance profiles. Profiles corresponding to DIRMIN(3) and DIRMIN-2(3) are close to each other. Even though DIRMIN-2(3) is slightly inferior in terms of efficiency, it is significantly better in terms of robustness. As expected, DIRMIN(9) and DIRMIN-2(6) have a poor performance in terms of efficiency. Nevertheless, DIRMIN(9) is the most robust method overall. The newly proposed method DIRMIN-2(3) has a robustness performance close to that of DIRMIN(9), while preserves a good efficiency. Overall, DIRMIN-2(3) is the most promising method, combining efficiency and robustness.



(a) Average performance, conditional to γ

(b) Average performance, conditional to β

(c) Relative variance of performance σ_r^2

(d) Average performance, conditional to `DIRMIN` and `DIRMIN-2` setup

Fig. 8 Algorithms' performance for the test problems

5.2 SBD optimization of the DTMB 5415

A preliminary sensitivity analysis for each design variable is shown in Fig. 10, showing the relative percent resistance reduction ($\varDelta_{obj}$) with respect to the parent hull. Unfeasible designs are not reported. Changes in $\varDelta_{obj}$ are found significant, revealing a reduction of the total resistance at Fr $= 0.25$ close to 10%.

The SBD optimization procedure achieves a reduction of the objective function by 15.9 and 16.2%, using `DIRMIN`(3) and `DIRMIN`(9) respectively, and a reduction by 16.2 and 16.1%, using `DIRMIN-2`(3) and `DIRMIN-2`(6) respec-

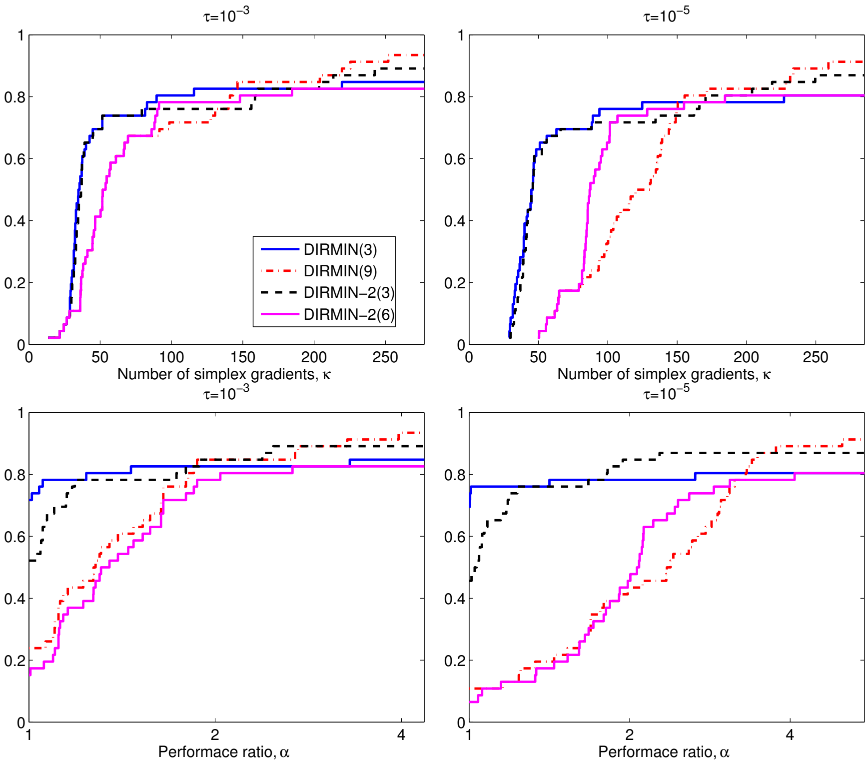


Fig. 9 Data and performance profiles comparing `DIRMIN`(3) and (9) with `DIRMIN-2`(3) and (6)

tively. The convergence history of the objective function towards the minimum is shown in Fig. 11a, confirming the efficiency and robustness of the newly proposed method `DIRMIN-2`(3). Figure 11b presents the values of the corresponding optimal design variables, showing appreciable differences. Figure 12 shows the optimized shapes compared to the original. The reduction of the total resistance is consistent with the reduction of the wave elevation patterns, both in terms of transverse and diverging Kelvin waves, as shown in Fig. 13. Finally, Fig. 14 shows the pressure field on the optimized hulls compared to the original, showing a better pressure recovery towards the stern. A summary of the results is presented in Tab. 7.

6 Conclusions

A simulation-based derivative-free global optimization of the hull-form design of a military vessel (namely the DTMB 5415 model) has been shown, based on global/local hybridizations of the `DIRECT` algorithm by local search. Two hybridization algorithms (namely `DIRMIN` and the novel `DIRMIN-2`) have been presented and tested on a set of forty six well-known analytical problems and

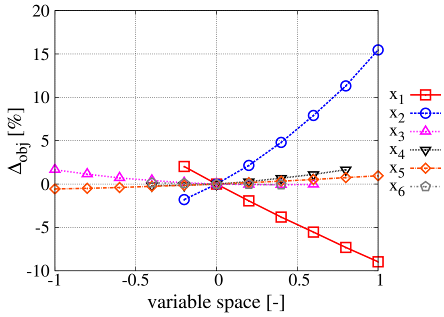
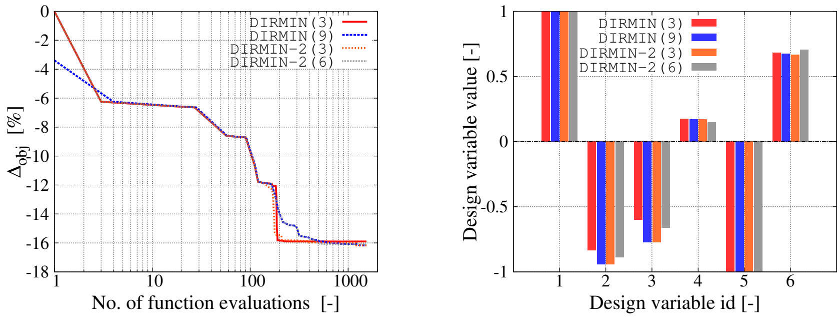


Fig. 10 Sensitivity analysis of design variables for the DTMB 5415 optimization



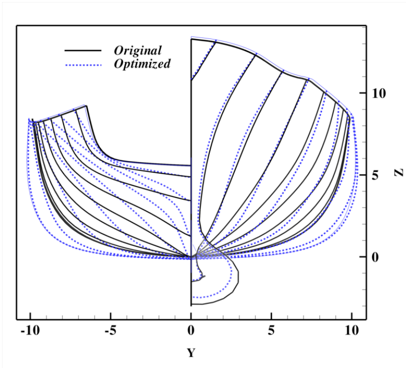
(a) (b)

Fig. 11 Objective function convergence history (a) and optimal design variables (b)

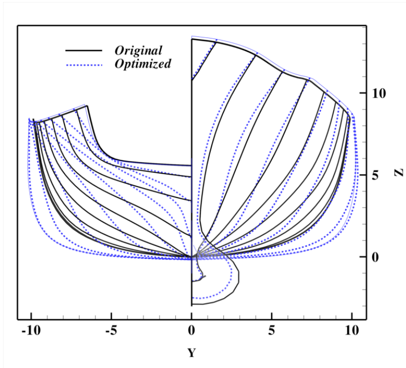
Table 7 Summary of optimization results for DTMB 5415

	Design variables (non-dimensional)						$R_T \times 10^5 (N)$	
`DIRMIN`	x_1	x_2	x_3	x_4	x_5	x_6	value	$\Delta_{obj}(\%)$
3	1.000	-0.834	-0.599	0.174	-1.000	0.682	2.888	-15.9
9	1.000	-0.944	-0.774	0.172	-0.997	0.674	2.878	-16.2
`DIRMIN-2`	x_1	x_2	x_3	x_4	x_5	x_6	value	$\Delta_{obj}(\%)$
3	1.000	-0.944	-0.774	0.172	-0.998	0.667	2.878	-16.2
6	1.000	-0.890	-0.663	0.148	-0.996	0.706	2.882	-16.1

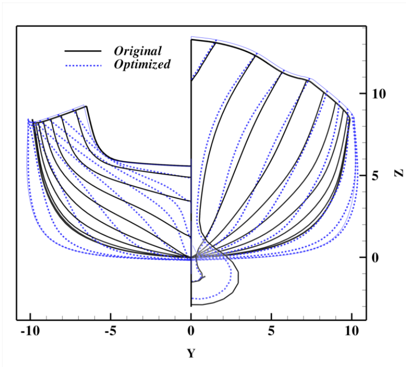
on the ship design problem. The two algorithms differ in the local search management. In particular, while `DIRMIN` executes the local search starting from the centroids of all the potentially-optimal hyperrectangles, `DIRMIN-2` performs a single local minimization starting from the best point produced by dividing the potentially-optimal hyperrectangles. The latter method results in a more efficient algorithm, with beneficial effects on the overall computational cost, which is a considerable advantage especially when the objective function is evaluated through computer simulations.



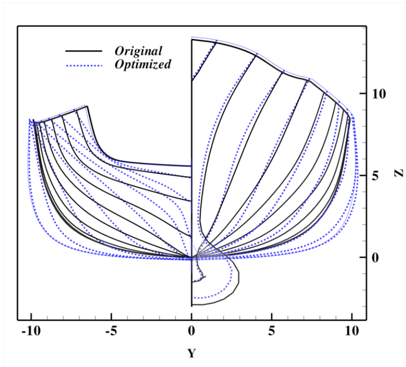
(a) DIRMIN(3)



(b) DIRMIN(9)



(c) DIRMIN-2(3)

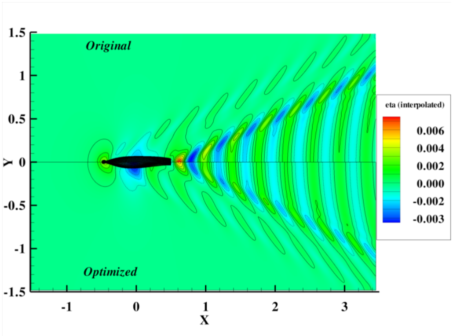


(b) DIRMIN-2(6)

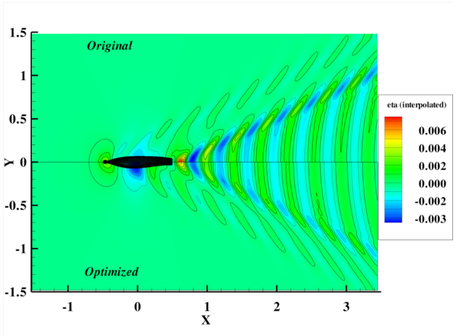
Fig. 12 Optimal hull-form shapes compared with the original

Nine diverse setups of the algorithms have been investigated, varying the local search activation trigger γ and the local search tolerance β. Data and performance profiles, along with absolute evaluation metrics, have been used to identify the most promising algorithm with respect to efficiency and robustness. The results on the test problems and the ship design optimization have been found consistent. Specifically, the novel algorithm DIRMIN-2, with a low value of both γ and β, has been found the most promising method in terms of the trade-off between efficiency and robustness.

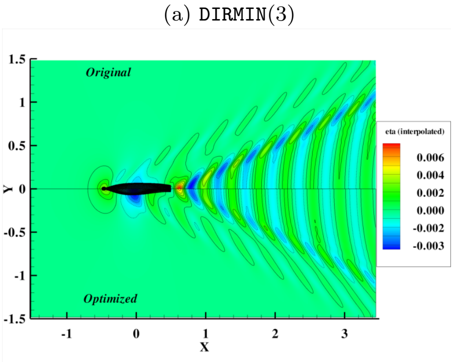
The hull-form optimization has achieved a reduction of the resistance in calm water at 18 kn close to 16%. Design variants have been produced using a set of orthogonal functions, defined over body-surface patches for both the hull and the sonar dome. The simulations have been conducted using a potential-flow solver, with a viscous correction based on a local flat-plate approximation. The effects of the optimization are visible in the wave elevation pattern produced by the optimized design, consisting in a reduction of both the transverse and the diverging Kelvin waves. The optimized hull also shows a better pressure recovery astern.



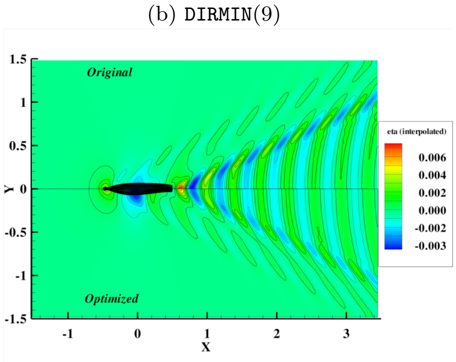
(a) DIRMIN(3)



(b) DIRMIN(9)

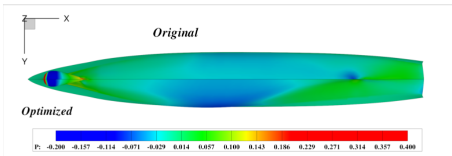


(c) DIRMIN-2(3)

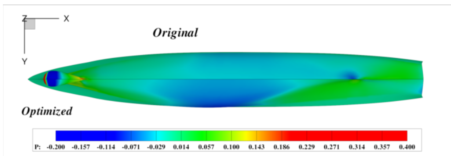


(b) DIRMIN-2(6)

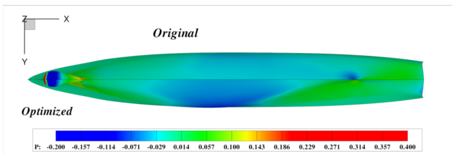
Fig. 13 Wave elevation patterns produced by the optimal hull-form shapes at Fr = 0.25 compared with the original



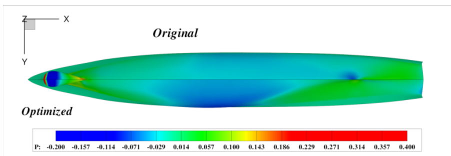
(a) DIRMIN(3)



(b) DIRMIN(9)



(c) DIRMIN-2(3)



(b) DIRMIN-2(6)

Fig. 14 Pressure field on the optimal hull-form shapes at Fr = 0.25 compared with the original

Future work include the extension of global/local hybridization of the DIRECT algorithm to multiobjective problems, along with the application of higher fidelity solvers (such as Reynolds-averaged Navier-Stokes equations solvers) and design optimization based on static/dynamic metamodels [36].

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