FREQUENCY DEPENDENT WAVESHAPING

Giovanni De Poli

C.S.C. Istituto di Elettrotecnica ed Elettronica - Università di Padova,
Italy

ABSTRACT

Non linear techniques are used more and more commonly in musical sound synthesis Waveshaping allows to produce periodic sounds, whose spectra depend on the input amplitude, but not on the input frequency, as the process is memory-less. This variation can be achieved only acting on the control parameters. When an automatic dependence is required, a non-linear transformation with memory has to be employed. An efficient method to realize it is presented in this work. The output signal is composed by the sum of some polynomial waveshapings, each applied to progressively delayed values of the input sinusoidal. The resulting static and dynamic spectra are deeply analyzed. Designing criteria and three different implementations are discussed both from a theoretical and a practical point of view.

0. INTRODUCTION

Synthesis techniques can be described as belonging to two types (De Poli 1983). The first type, called the direct technique, produces the signal directly from the given data. Fixed-waveform synthesis and additive synthesis are examples of this type. The second type of synthesis, is called the transformation technique. This procedure can be considered in two steps: generation of one or more simple signals called input signals and their transformation.

In the last few years nonlinear transformation techniques, especially instantaneous ones, have been used increasingly. In these transformations the value of the output signal depends only on the value of the input signal at that instant, with a time-invariant relationship. It may be compared to a signal from a distorting amplifier: not all frequencies are treated in the same way, and new spectral components not present in the input signal are introduced. Thus the input signal is distorted or shaped by such a processor; hence the terms non linear distortion or waveshaping for this synthesis technique. The function describing the transformation is called the shaping function.

It is noteworthy that a variety of output waveforms may be obtained simply by varying the amplitude and /or the dc offset of the input signal. Thus different ranges of the shaping function are used. Moreover, because the function is time invariant, it need be computed only once and can be stored in a table. This implementation is therefore particularly efficient. Le Brun (1979) and Arfib (1979) recently studied the case of a polynomial shaping function with a sinusoidal input signal. Reinhard (1981) extended these results to an input sum of two sinusoidal signals of different frequencies.

Even though introduced from a different point of view, the Moorer (1976) discrete summation synthesis technique can also be seen as a transformation of this kind, as Le Brun (1979) has pointed out. The shaping function is not a polynomial but has a real pole. Similar formulas have also been suggested by Lehmann and Brown (1976). The case where the shaping function is rational (that is. when it is a ratio between two polynomials) and the input signal sinusoidal has been studied by De Poli (1984).

Till now not much research (Reinhard) has been carried out in the application to music of non-linear transformation with memory. In this paper a method to realize it, is presented. As it will be seen, this technique is analyzable with the help of linear transformation theory.

In the next section the basic theory (Le Brun, Arfib, De Poli 1981) of simple waveshaping is briefly summarized. Then it is applied to frequency dependent waveshaping.

1. BASIC NOTIONS ON WAVESHAPING

Let us consider the discrete signal:

1) $$x(n) = \cos 2\pi f_o nT = \cos \lambda_o n$$

of frequency f_o and normalized angular frequency $\lambda_o = 2\pi f_o T = 2\pi f_o/F_c$. We apply this sequence to

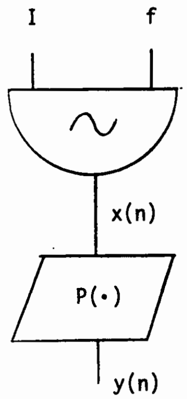

fig. 1

the system of fig. 1 (non-linear amplifier) where P(•) is an instantaneous polynomial distorting function. The output sequences y(n) can be expressed as

2) $y(n) = P(x(n))$.

To analyze the spectrum, it is convenient to develop the shaping function P(•) as the sum of Chebyshev polynomials. The Chebishev polynomial T_i of degree i is defined as $T_i(x) = \cos(i \arccos x)$ or $T_i(\cos\theta) = \cos i\theta$ if $x = \cos\theta$. From this definition it is evident that if is used as a shaping function only the i-th harmonic is obtained at the output.

With M given as the degree of the shaping polynomial P(•) and d_k given as its k-th coefficient, we obtain

3) $P(x) = \sum_{k=0}^{M} d_k x^k = \sum_{i=0}^{M} h_i T_i(x)$

by developing p(x) as the sum of Chebyshev polynomials.

The conversion from one base (power series) to the other (Chebyshev polynomials) can be expressed with matrices. Let $\underline{d}$ be the column vector containing the coefficients d_k and $\underline{h}$ the column vector containing the coefficients h_i, the relation

4) $\underline{d} = \underline{B} \cdot \underline{h}$

holds, where $\underline{B}$ is the square matrix (0:M, 0:M) containing in the columns the Chebyshev polynomial coefficients. The j-th column of $\underline{B}$ contains the coefficients of T_j

5) $T_j(x) = \sum_{i=0}^{M} b_{ij} x^i$

It holds also that

6) $\underline{h} = \underline{A} \cdot \underline{d}$

where $\underline{A} = \underline{B}^{-1}$ is a square matrix (0:M, 0:M) whose j-th column contains the component of x^j expressed in terms of $T_i(x)$, as

7) $x^j = \sum_{i=0}^{j} a_{ij} T_i(x)$

The output signal is

8) $y(n) = P(\cos(\lambda_o n)) = \sum_{i=0}^{M} h_i \cos(i\, \lambda_o n)$

Let us consider the case of dynamic spectra by analyzing the effect of an input amplitude variation for the system of fig. 1. Then

9) $x(n) = I \cdot \cos(2\pi f_o Tn) = I \cos \lambda_o n$

where I is a modulation index. The output of our system is still a periodic signal with M harmonics where M is the degree of the polynomial P(•). But the harmonic amplitudes are modulated by subsequent powers of the index I. The vector $\underline{h}$ now is given by

10) $\underline{h}(I) = \underline{A} \cdot \underline{C}(I) \cdot \underline{d} = \underline{A} \cdot \underline{C}(I) \cdot \underline{B} \cdot \underline{H}(1)$

where $\underline{C}(I)$ is a diagonal matrix of (0:M, 0:M) containing the powers of I ($c_{ii} = I^i$, $c_{ij} = 0$ $i \neq j$).

Infact, the effect is analogous to changing the polynomial P(•) by multiplying the i-th coefficient by I^i. In matrix terms, the new coefficient vector $\underline{d}^*$ results

11) $\underline{d}^* = \underline{C}(I) \cdot \underline{d}$

which, substituted in (6), gives (10). The amplitude of the i-th harmonic results

12) $h_i(I) = \sum_{j=0}^{M} a_{ij} d_j I^i$

Since $\underline{A}$ is a triangular superior matrix, each i-th harmonic amplitude is a polynomial in I of degree M. The coefficients of this polynomial are given by the product of the i-th row of $\underline{A}$ by the coefficients of the shaping polynomial P(•). Moreover in the even (or odd) harmonics are present only the even (or odd) powers of degree not less than i. Each column of the matrix $\underline{A}$ describes the contribution to the spectrum of every d_j coefficient.

The resulting spectrum of simple waveshaping depends only on the variation of the modulation index I.

2. ANALYSIS OF FREQUENCY DEPENDENT WAVE-SHAPING

Let us consider the system in fig. 2, where the output y is the addition of different polynomial distortions of progressively delayed input values. So we have N+1 polynomials $P_j(x)$ ($j = 0,\ldots,N$) of M degree.

As seen above (3), each polynomial can be expres-

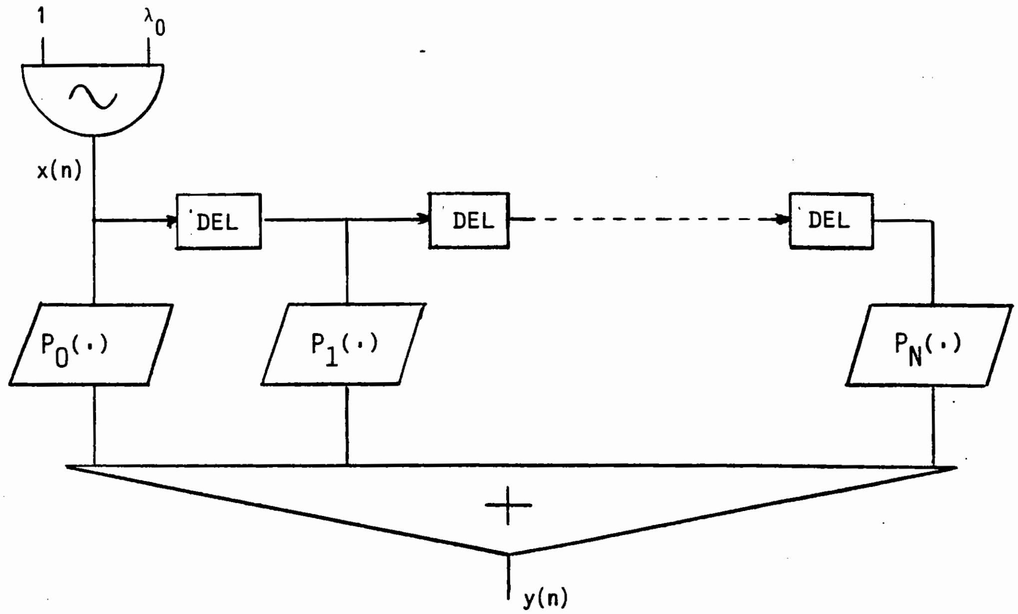

fig. 2

sed as the sum of Chebyshev polynomials

13) $P_j(x) = \sum_{o k}^{M} d_{kj} x^k = \sum_{o i}^{M} h_{ij} T_i(x)$

where 4) is valid for each polynomial P_j. Thus if $\underline{D}$ is the matrix (0:M, 0:N) with the j-th column formed by the coefficients of the polynomial P_j and if $\underline{H}$ is the matrix (0:M, 0:N) with the j-th column formed by the coefficients of the expansion of P_j with the Chebyshev polynomials, it results that

14) $\underline{D} = \underline{B} \cdot \underline{H}$

15) $\underline{H} = \underline{A} \cdot \underline{D}$

where $\underline{A}$ and $\underline{B}$ are the matrices defined in the preceding paragraph. The output signal is:

16) $y(n) = \sum_{o j}^{N} P_j(x(n-j))$

and using 13) we obtain

17) $y(n) = \sum_{o j}^{N} \sum_{o k}^{M} d_{kj} x^k(n-j) =$

$= \sum_{o j}^{N} \sum_{o i}^{M} h_{ij} T_i(x(n-j)).$

Exchanging the summation order

18) $y(n) = \sum_{o i}^{M} \sum_{o j}^{N} h_{ij} T_i(x(n-j)).$

Since $T_i(x(n))$ is the i-th harmonic of x(n) and calling

19) $x_i(n) = T_i(x(n)) = \cos 2\pi i f_o nT = \cos i \lambda_o n$

where f_o is the fundamental frequency, it results that

20) $y(n) = \sum_{o i}^{M} \sum_{o j}^{N} h_{ij} x_i(n-j) = \sum_{o i}^{M} y_i(n)$

where

21) $y_i(n) = \sum_{o j}^{N} h_{ij} x_i(n-j)$

This last expression can be interpreted as the output sequence of a finite impulse response filter (FIR) whose input is the sequence $x_i(n)$. Thus the system of fig. 2 is equivalent to that of fig. 3, i.e. to a generator of harmonics, each of which passes through a different FIR filter. The impulsive response of the i-th FIR filter is given by the coefficients h_{ij} with j=0,...,N in the i-th row of $\underline{H}$. The generated signal is given by the sum of the sinusoidal signals y_i 21) of frequency $i \cdot f_o$. Their phase and amplitude depend on the coefficients h_{ij}.

The frequency response of the i-th filter is

22) $H_i(\lambda) = |H_i(\lambda)| \cdot \exp(\phi_i(\lambda)) =$

$= \sum_{o k}^{N} h_{ik} e^{-jk\lambda}$

Thus, with $\lambda_o = 2\pi f_o T$

23) $y_i(n) = |H_i(i\lambda_o)| \cos[i\lambda_o n + \phi_i(i\lambda_o)]$

The produced signal is periodic with M harmonics.

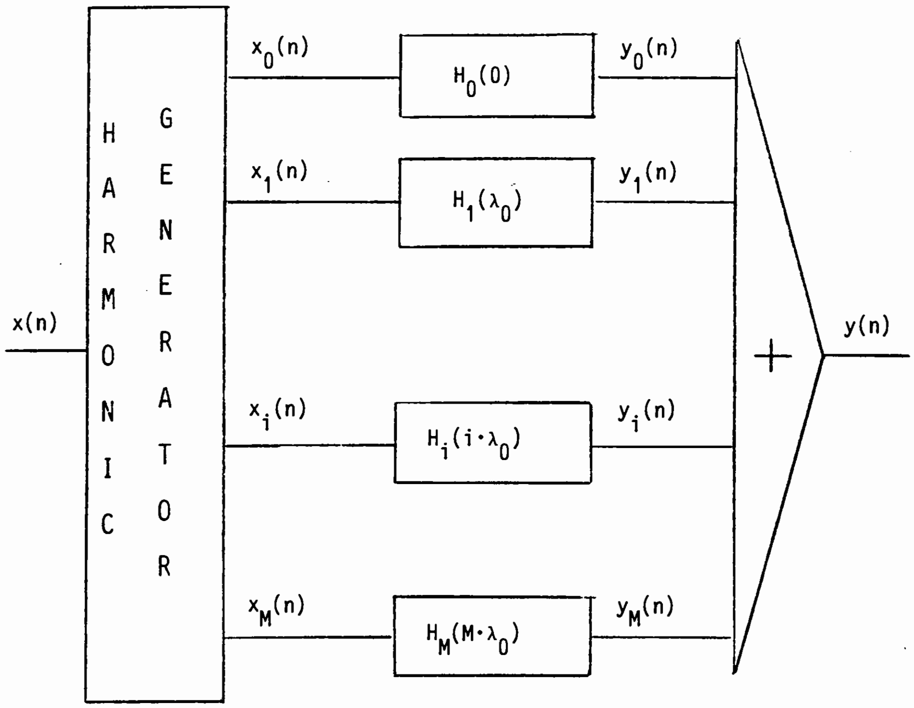

fig. 3

The amplitude and the phase of each harmonic depend on the frequency f_o of the sinusoidal input.

This technique allows us to obtain periodic spectra which are automatically modified when the fundamental frequency varies.

3. DESIGN OF FREQUENCY DEPENDENT WAVE-SHAPING

We have analyzed the effect of the distortions of a progressively delayed input. Let us see now how to apply these considerations in the synthesis. For this purpose we examine two examples.

Let us suppose we want to control the number of significant harmonics of a spectrum, or its bandwidth as a function of signal frequency (i.e. design a frequency dependent low pass filter). For example if we want to obtain a sound with 10 significant harmonics when the signal frequency is 100 Hz, and 6 when the signal frequency is 300 Hz and with linearly varying bandwidth, we have (dark line in fig. 4)

24) $$b(f) = 1000 + \frac{1800-1000}{300-100}(f-100) = 600+4\cdot f$$

With normal waveshaping or an oscillator, the harmonic number would not depend on the frequency, and the bandwidth would be simply proportional to the frequency (thin line in fig. 4)

25) $$b(f) = c\cdot f$$

If this last signal were filtered with a normal low-pass filter, the bandwidth would be constant and we would have (dashed line in fig. 4)

26) $$b(f) = f_c$$

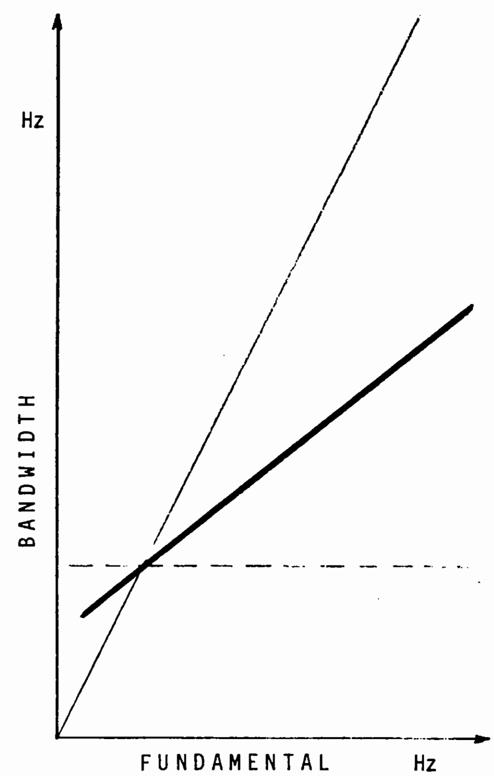

fig. 4

The normal procedure allows us to obtain only constant bandwidth or constant harmonic number. It is possible to obtain different behaviours only by varying the filter coefficients or the waveshaping modulation index. These modifications are often difficult to determine. But with the method proposed in the present work this problem can be easily solved. In fact, it is possible to define a different filter for each harmonic and design it according to the following procedure.

The cut-off frequency of the n-th harmonic filter is obtained from the intersection between the straight line

27) $y = n \cdot f$

with the function $y = b(f)$.

In our example the system gives

28) $y = \dfrac{600}{1-4/n}$;

The first four harmonics should not be filtered and are present in only one polynomial. The cut-off frequencies of the others, from the fifth to the tenth, will be 3000, 1800, 1400, 1200, 1080, 1000 respectively. Designing such filters completely determines the polynomials. Notice that the fundamental varies within a fixed range, between f_{min} and f_{max}. Foldover should definitely be avoided, so

29) $f_{max} \leq F_c/M$

where M is the number of harmonics produced and F_c the sampling rate.

The frequency range of the n-th harmonic is $[n \cdot f_{min}, n \cdot f_{max}]$. The filters should be designed with constraints defined within these intervals.

Fig. 5a shows the frequencies of the various harmonics and the desired bandwidth as a function of the fundamental frequency of the sound. As can be seen, the intersections give the cut-off frequency of each harmonic filter (fig. 5b). With a sampling

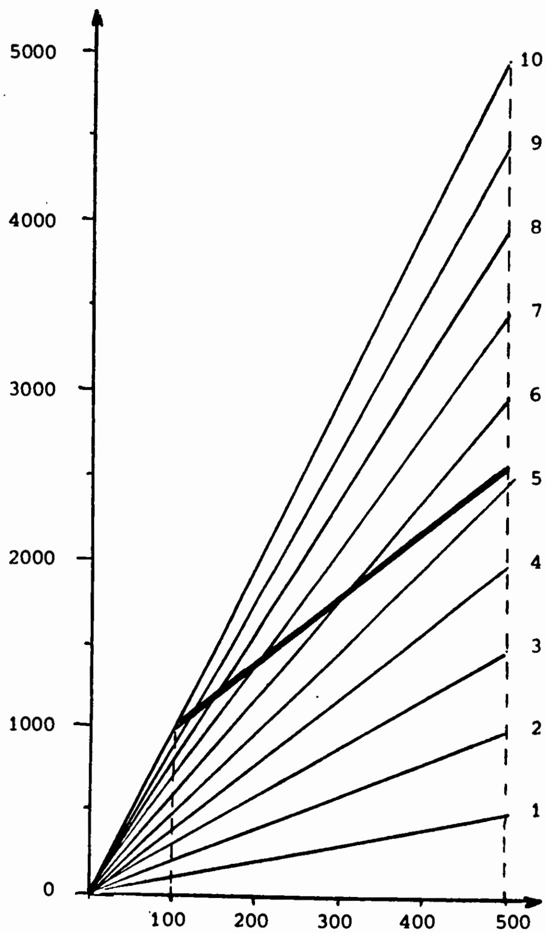

fig. 5a

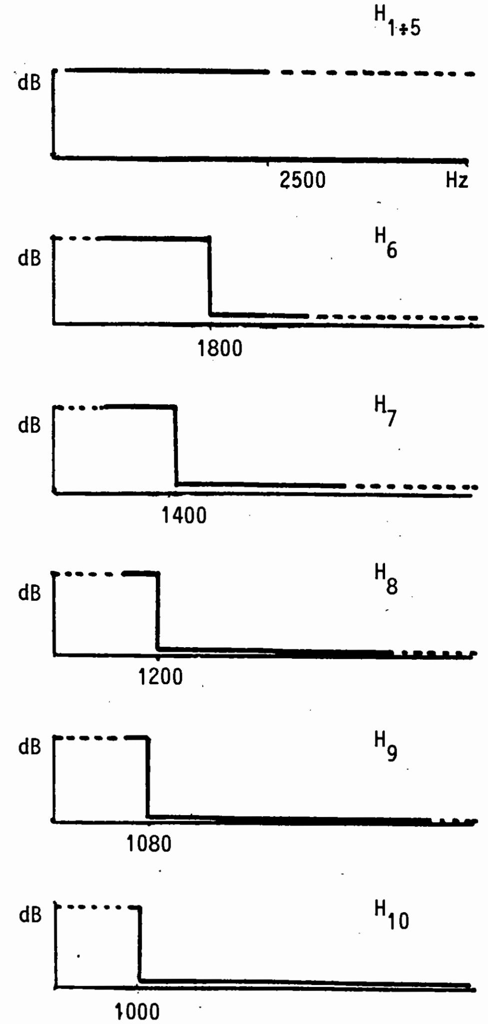

fig. 5b

rate of 10 kHz, the fundamental is in the 100 500 Hz range. So the part to design is included between the two verticals lines of abscisse f_{min}=100 Hz and f_{max}=500 Hz (fig. 5a). The intersection with the straight line of the harmonic 27) gives the frequency range of each filter. The external parts, which are not used, are represented with a dashed line in fig. 5b. For example the range of the sixth harmonic is 600÷3000 Hz. Notice that fig. 5b represents the ideal transfer function of the low pass filters. In the design an opportune transition band has to be provided.

Let us examine a second esample. We want to design a waveshaping which generates formants varying with the fundamental, as occurs in the singing voice (Sundberg). In fig. 6a the spectral envelope surface is shown though level curves. Here too, the intersections between the straight lines representing the different harmonics and the curves give the desired frequency responses of each harmonic filter. In fig. 6b the frequency responses of the sixth and of the tenth harmonic filter are shown.

F.I.R. filters can be designed employing different techniques. For linear phase filters the classic McClellan (1973) program can be used. For our purposes it has been modified (Volonnino) to allow the design of filters as those shown in fig. 6b A monitor has been realized which allows us to specify the spectra desired at different fundamentals. The filters obtained permit us to generate varia-

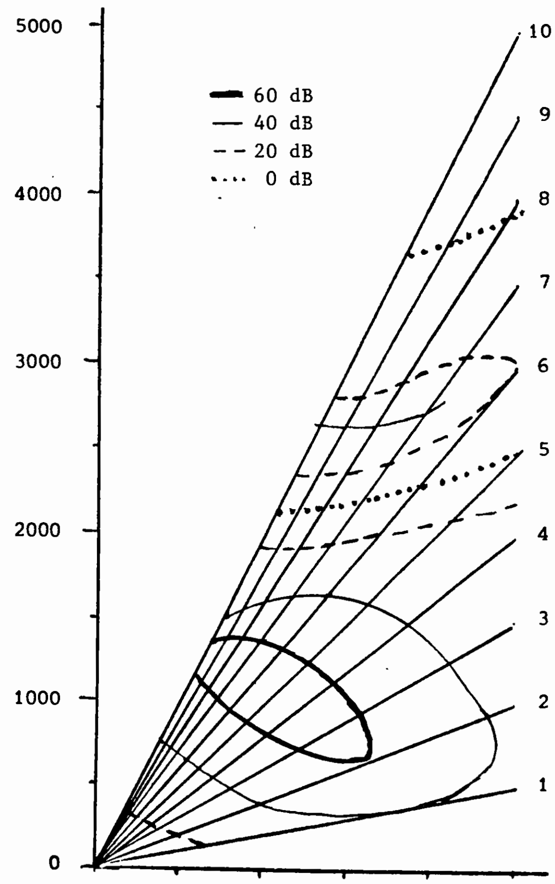

fig. 6a

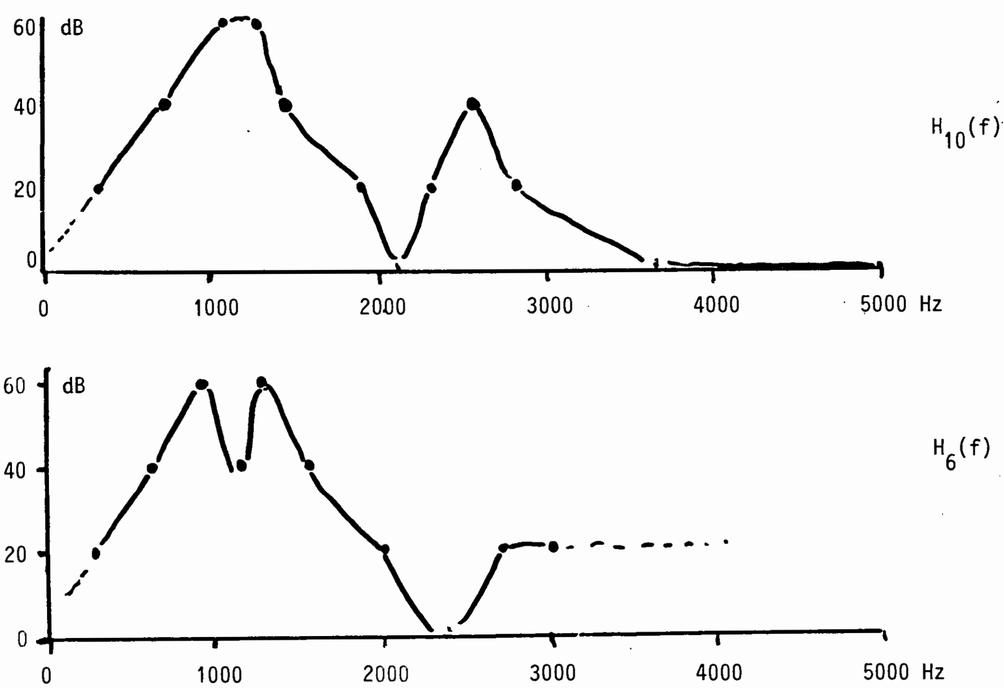

fig. 6b

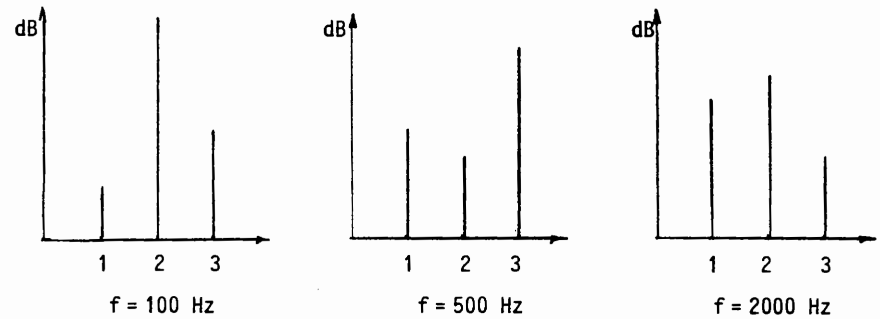

fig. 7a

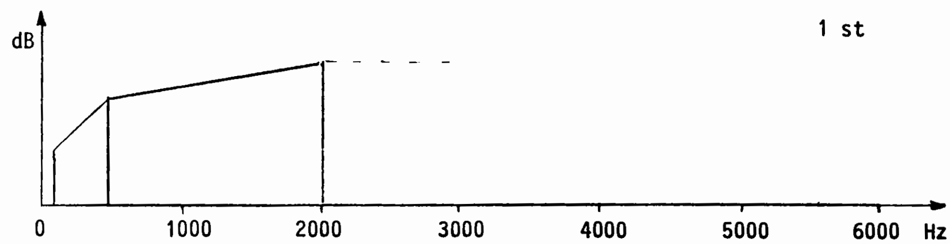

fig. 7b

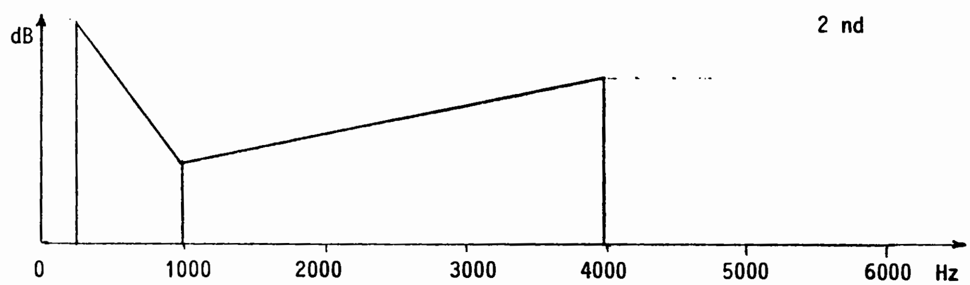

fig. 7c

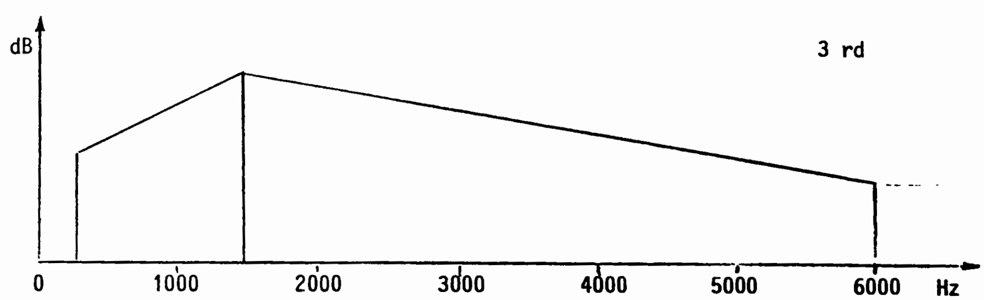

fig. 7d

ble spectra, by, interpolating the given values.

In fact the specification of the spectral surface (as in fig. 6a) is not always easy neither very suitable to computer. Instead in some cases we have reference spectra for some fundamentals (e.g. for each octave or fifth), which can be designed analyzing instrumental sounds. The aim is to produce sounds in the entire fundamental range passing smoothly through the refefence spectra. This can be obtained designing a filter, for each harmonic whose transfer function is a linear interpolation between the given values. For example we want a sound with three harmonics, whose amplitudes are respectively 30,60,40 db at 100 Hz, 35,30,55 dB at 500 Hz and 45,50,35 dB at 2000 Hz (fig. 7a). For the first harmonic, we have to design a filter which is the line linear interpolation between $H_1(100)$=30 dB, $H_1(500)$ =35 dB and $H_1(2000)$=45 dB (fig. 7b) for the second $H_2(200)$=60 dB, $H_2(1000)$=30 dB, $H_2(4000)$=50 dB (fig. 7c); for the third $H_3(300)$=40 dB, $H_3(1500)$=55 dB, $H_3(6000)$=35 dB (fig. 7d).

The coefficients of linear phase FIR filters are symmetric or antisymmetric. The same symmetry holds also for the shaping polynomials. In fact it result respectively that $P_0(x)=P_N(x)$, $P_1(x)=P_{N-1}(x)$ etc. or $P_0(x)=-P_N(x)$, $P_1(x)=-P_{N-1}(x)$ etc. Thus only half of the polynomials has to be stored. Applying these criteria, the following polynomials have been designed for the frequency dependent low pass filter of fig. 5 (N=13):

$$P_0(x)=P_{13}(x)=0.0131+1.065x+0.9938x^2-11.31x^3-13.406x^4+31.837x^5+44.42x^6-32.88x^7-54.377x^8+11.18x^9+22.36x^{10}$$

$$P_1(x)=P_{12}(x)=0.133+0.37x-0.805x^2-5.91x^3-14.66x^4+26.97x^5+71.05x^6-43.31x^7-99.94x^8+22.13x^9+44.26x^{10}$$

$$P_2(x)=P_{11}(x)=0.163+3.94x+0.216x^2-35.23x^3-32.52x^4+96.08x^5+136.03x^6-105.02x^7-184.5x^8+40.36x^9+80.72\,x^{10}$$

$$P_3(x)=P_{10}(x)=0.199+0.429x+0.462x^2-12.74x^3-45.49x^4+68.38x^5+194.4x^6-115.87x^7-270.2x^8+60.45x^9+120.9x^{10}$$

$$P_4(x)=P_9(x)=0.085+3.093x+4.029x^2-41.57x^3-71.09x^4+146.3x^5+270.4x^6-189.5x^7-366.5x^8+81.86x^9+163.7x^{10}$$

$$P_5(x)=P_8(x)=0.033+1.038x+6.785x^2-24.09x^3-87.7x^4+118.35x^5+319.6x^6-191.9x^7-432.9x^8+97.58x^9+195.1x^{10}$$

$$P_6(x)=P_7(x)=-0.156+3.91x+6.77x^2-46.34x^3-100.15x^4+164.8x^5+362.1x^6-229.37x^7-484.8x^8+109.2x^9+218.4x^{10}$$

It is possible also to design minimum phase FIR filters. The length of the filter can be reduced to 2/3 maintaining the same performance. However, since the coefficients are no longer symmetric, all the polynomials have been stored. The number of operations is reduced, but the memory necessary for shaping tables increases. It should be noticed that a good approximation of the required behaviour needs rather long filters and thus many polynomials. In general one should come to a compromise between the desired approximation and the computational efficiency.

4. DYNAMIC CASE

When the input amplitude varies (fig. 8) the output spectrum changes, as in normal waveshaping.

If the input signal is given by 9), the relation 15), in analogy with 10), becomes

30) $\underline{H}(I) = \underline{A}\ \underline{C}(I)\ D = \underline{A}\ \underline{C}(I)\ \underline{B}\ \underline{H}(1)$

The i-th row of $\underline{H}(I)$ contains the impulse response coefficients of the filter of the i-th harmonic. In practice, for i even (or odd), it is a combination of the impulsive response of even (or odd) superio filters (e.g. if i=4, it is a combination of the 4th, 6th, 8th... filters). Thus it is possible to determine the dynamic behaviour, but apart from special cases, it is not easy to have an intuitive idea a priori.

5. PHASE DEPENDENT WAVESHAPING

It should be noticed that the input signal to the different shaping polynomials in fig. 2 are sinusoidal progressively delayed signals, which means progressively phase displaced. In fact with $\lambda = 2\pi fT$, normalized angular frequency, and with the input signal to the first shaper given by 1), the k-th input is

31) $\cos(n-k)\lambda = \cos(n\lambda-k\lambda) = \cos(n\lambda+\phi_k)$

which in respect to the first is phase displaced by

32) $\phi_k = -k\lambda$.

It is possible to adopt a different scheme for the synthesis (fig. 9). Instead of generating a single sinusoidal signal and then delaying it, many, one for each shaper and properly phase displaced 32), are generated. When $\lambda = 2\pi f_0 T$ nothing changes and the result is still 20). In this case it is possible to act on the phase independently of the input frequency. This corresponds to varying λ in the filter frequency response 22) independently of the fundamental.

Thus the harmonic amplitudes and the resulting spec

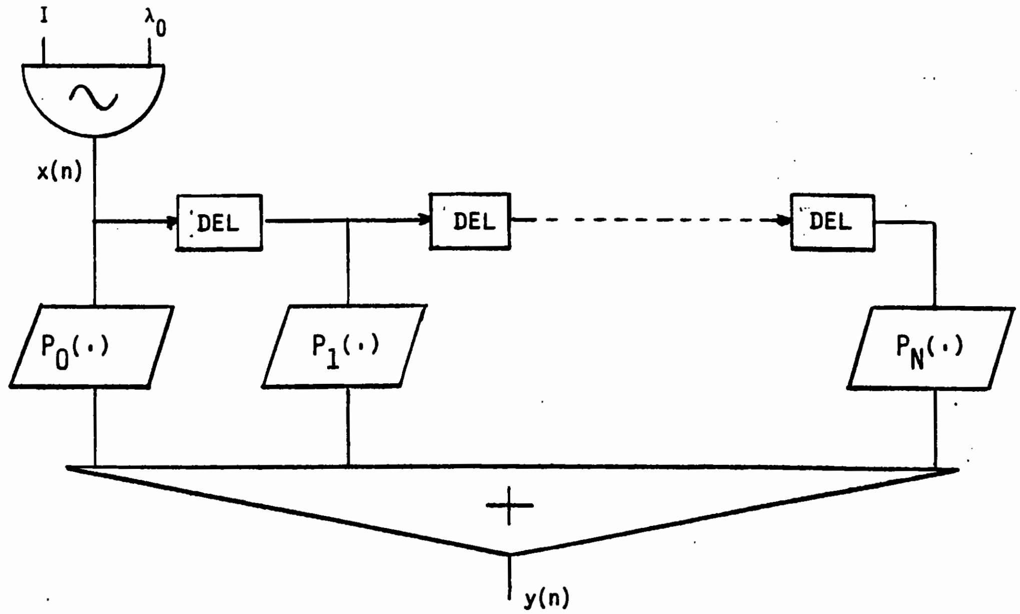
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fig. 8

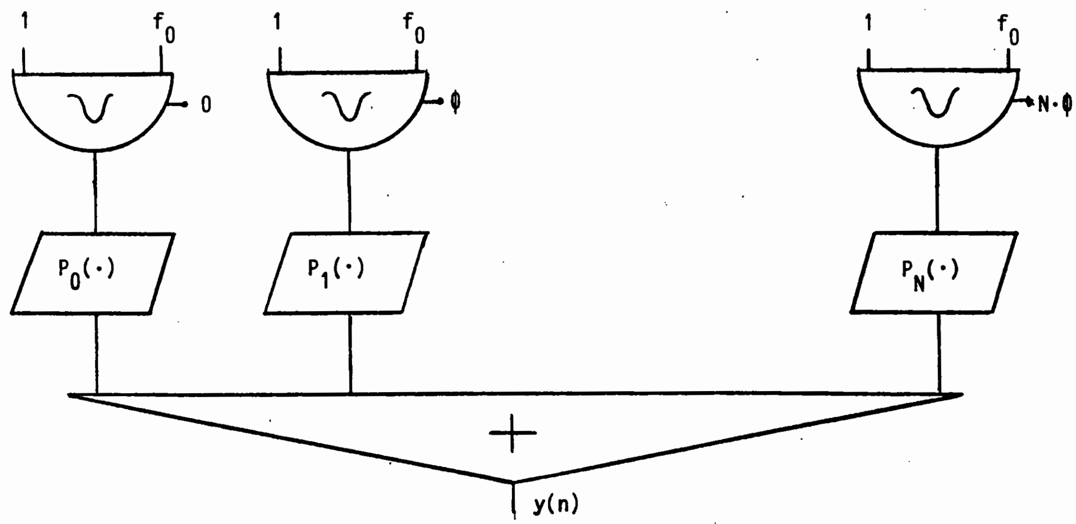
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fig. 9

trum can vary without any changes in the fundamental. Or, if λ remains constant, the fundamental varies without affecting the spectrum. In the first example above a change of λ modifies the signal bandwidth independently of the frequency.

In conclusion, if $\phi_k = -k\lambda$ the 22) become

33) $$H_i(\lambda) = \sum_{o}^{N} h_{ik} e^{-jk\lambda}$$

34) $$y_i(n) = |H_i(i\cdot\lambda)| \cdot \cos\{2\pi f_o T n + \arg[H_i(i\cdot\lambda)]\}$$

which give the amplitude and phase of the resulting i-th harmonic.

If the phase displacement ϕ_k of every oscillator is chosen arbitrarily, not proportional to λ, the relation 34) becomes

35) $$y_i(n) = |G_i| \cdot \cos[2\pi f_o Tn + \arg(G_i)]$$

where

36) $$G_i = \sum_{o}^{N} h_{ik} e^{-j\phi_k}$$

This last case corresponds to have FIR filters with unequal delays.

The dynamic behaviour is still given by 30) if the oscillator amplitudes is always given by a single variable value. In the phase-dependent waveshaping, each input amplitude can be varied separately. If α_k is the k-th oscillator amplitude, the element h_{ij} of the matrix $\underline{H}$ describing the filters becomes

37) $$h_{ij} = \sum_{jm}^{M} a_{im} \alpha_j^m d_{mj}$$

If we are interested only in the dynamic behaviour determined by the phase displacement and not in that produced by the input amplitude, then it is no longer necessary to shape sinusoidal inputs in order to obtain periodic signals. All we have to do is add simple oscillators having the resulting waveforms (fig. 10). Thus if h_{ij} is the generic element of the required matrix $\underline{H}$, the j-th oscillator waveform is

38) $$F_j(x) = \sum_{o}^{M} h_{ij} \cdot \cos(i\,\omega\, x)$$

which is the sum of M cosine harmonics of amplitude h_{ij}.

When synthesis programs like MUSIC V (Mathews) are used, attention must be payed to the implicit amplitude normalization of waveforms. In practice the value of each oscillator amplitude has to be equal to the waveform maximum, in absolute value. All the oscillators, have the same frequency and initial phase $\phi_k = -k\lambda$, computed in sampling increment.

6. CONCLUSION

The use of non-linear techniques for sound synthesis, although powerful and efficacious, is often restrained by theoretical and practical problems. This is true particularly for the choice of parameters and of dynamic control functions. Till now not much research has been carried out in the application to music of non-linear transformation with memory.

This work contributes toward a better knowledge and a wider utilization of such techniques. A simple realization, analyzable with the help of linear transformation theory, is suggested. The relations of produced spectra with parameters and dynamic controls have been deeply examined both from a theoretical and a practical point of view.

The choice of shaping polynomial coefficient is

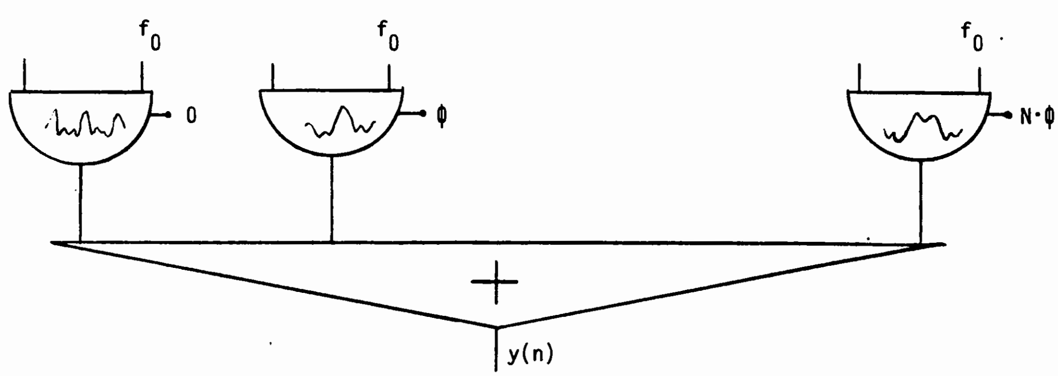

fig. 10

not intuitive. An extension of McClellan's filter design program has been realized for this purpose. Obviously multiplying the output signal by a sinu soidal carrier, the spectrum can be shifted, obtaining harmonic or inharmonic sounds. In conclusion this technique provides further variation possibility to normal waveshaping in an efficient way and aims to be a step toward a more extensive use of non-linear network to sound synthesis.

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