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Strategic Maritime Container Transport Design in Oligopolistic Markets

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Abstract

This paper considers the maritime container assignment problem in a market setting with two competing firms. Given a series of known, exogenous demands for service between pairs of ports, each company is free to design a liner service network serving a subset of the ports and demand, subject to the size of their fleets and the potential for profit. The model is designed as a three-stage complete information game: in the first stage, the firms simultaneously invest in their fleet; in the second stage, they individually design their networks and solve the route assignment problem with respect to the transport demand they expect to serve, given the fleet determined in the first stage; in the final stage, the firms compete in terms of freight rates on each origin-destination movement. The game is solved by backward induction. Numerical solutions are provided to characterize the equilibria of the game.

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Keywords: logistics; container assignment model; maritime network; monopoly; duopoly; sequential Games; Backward Induction.

1. Introduction

Over the last 50 years, containerization has grown to account nowadays for roughly 70% of total deep sea trade (by value) and it is now a key component of the global economy (UNCTAD, 2014). The maritime industry has long sought the development of a model that can represent the flow of containers through the global liner shipping network. Such a model would not only assist shipping lines in the design of their services, but would also be useful

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for port operations, governments and international organizations as it could inform investments, strategic and operational planning, and policy design. One of the earliest works to model maritime container flows is the Container World project (reviewed in Newton, 2008), while Perrin et al. (2008) developed one of the earlier macroscopic container assignment models. At the same time, much research has focused on the empty container management problem, which occurs due to the effects of trade imbalances, predominately (e.g., between Western and Asian markets).

Previous work by Bell et al. (2011 and 2013) focused on the development of a container flow assignment framework that acknowledges transshipment operations and capacity constraints in the ports and vessels involved. The resulting optimization algorithms remain linear in nature, are built around the frequency-based structure of liner shipping services and seek to minimize aggregate container travel durations or costs, respectively. Both techniques are capable of simultaneously addressing full and empty container flows – in the past the latter have mostly been examined in isolation and as part of the empty container repositioning problem. Given their linear nature they have modest computational requirements, therefore making them particularly attractive for application in large problem settings that involve hundreds of ports and services.

With a few exceptions, most studies on container flow assignment have relied on predetermined liner service structures, used as inputs to the container assignment problem. Agarwal and Ergun (2008) notably added a network design stage to the container assignment model. In their paper this is referred to as a cargo routing model, therefore reflecting the absence of empty container repositioning from their assignment model. More recently, Mulder and Decker (2014) have enhanced the methodology developed by Agarwal and Ergun. Both studies however take the perspective of a single shipping line or alliance.

The considered academic efforts have provided important results towards the representation of the forces that drive the global flows of containers. However, the existing models still fail to take into account some crucial features of the global shipping market such as: the elasticity of the transport demand to the economic conditions (e.g., travel time and fees) that prevail in the shipping industry at any given moment; and, on the supply side, the effects of competitive actions that are taken by groups of players (e.g., shipping liners or alliances) in the same shipping market.

As regards the effects of competition on network design and service provision, the economic theory of industrial organization highlights two possible outcomes (Tirole, 1988). If competing shipping firms (i.e., liners or alliances) provide services of similar quality (e.g., service reliability) and features (e.g., planned delivery time), competition would take the form of strong price war, reducing profits and, in some cases, preventing the market to find a stable configuration (or medium-term equilibrium). On the other hand, if competing shipping firms are able to determine some form of service differentiation they may reduce the strength of competition and, in this way, increase profits (which nevertheless will remain lower than in the monopolistic case). This case is potentially relevant for the shipping industry, given that firms are able to design different service networks that, in turn, imply diverse delivery times and service features[1]. The policy implications of the two scenarios are quite different: if competition is strong, no regulation of the shipping sector is desirable, given that the market tends to determine the lowest possible prices for shippers; conversely, if competition is weak, prices and profits would tend to increase, and a regulatory remedy could be desirable.

The model introduced in this paper seeks to address this gap in academic literature, by developing an algorithm that could be used to determine an optimal set of liner services, given the presence of a competing shipping firm. The resulting model has game-theoretic elements, whereby the network design strategy of each shipping liner or alliance reacts to the actions of its competitors. Container flow assignment is integral part of this process, as it is used to establish how the market would respond to the simultaneous provision of routes by competing parties.

The types of services considered in the model follow established liner shipping trends, where vessel services are composed by a looped sequence of port visits (commonly referred to as service strings or loops). The resulting model takes into account revenues deriving from a demand for transport services that is distributed between firms in accordance to service costs. The latter include fees charged by the firm operating the chosen service as well as the opportunity cost of travel times. Empty container repositioning is retained in the container assignment model.

The key actors of the model are two firms (i.e., shipping liners or alliances) that seek to maximize their profit by operating in a given region with known transport demands for full containers among a set of ports. To meet this

[1] We can read the considered scenarios in terms of a monopolistic competition model where the degree of substitutability of services provided by different firms is infinite (first scenaro) or finite (second scenario).

objective, they both seek to establish sets of liner services, each being a circular tour of ports with given frequency and capacity. Respective fleet sizes constrain the services that each firm is capable of offering.

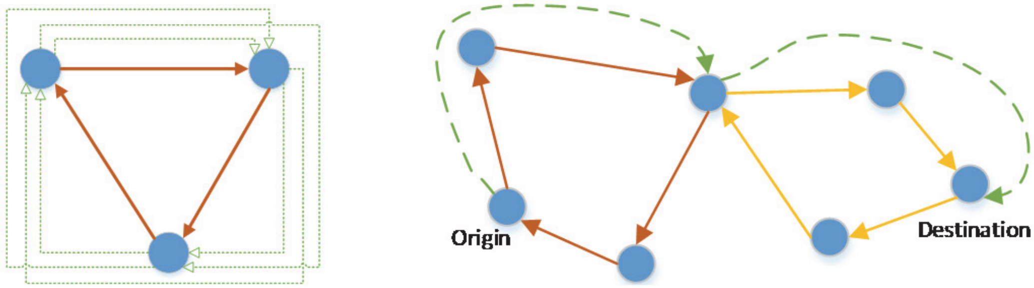

Fig. 1 – Left: A single service loop that consists of three port visits. Trip links are illustrated using red arrows while journey legs are illustrated using green arrows. Right: A multi-service constrainer trip that utilizes two intersecting service loops.

The model acknowledges the practice of transshipment in the maritime industry, as the possibility exists for a firm to satisfy demand between two ports that are not served by the same tour. This is achieved by identifying two services that include the origin and destination port, respectively, that also intersect at some intermediate location. In modelling terms, this is achieved by simultaneously representing each service in three forms:

- as a sequence of port visits (in a closed loop form);
- as a sequence of trip links (journeys between two adjacent ports in the service);
- as a sequence of journey legs (a pair of any two ports in a service, among which travel is possible).

In our analysis, container routes will consist of leg chains, belonging to potentially intersecting tours offered by a single or two competing firms. The relationships among ports, trip links and journey legs are illustrated in Figure 1.

While practical when no other options exist, container flows that include transshipment include additional handling operations, and are therefore likely to have longer journey times and total service costs when compared to a direct route between the same two ports (some exceptions may exist). Furthermore, storage limitations in transshipment ports may impose constraints to the amount of containers that may be temporarily stored between journey legs.

The model is designed as a complete-information sequential game, taking place over the following stages:

Stage 1: The firms simultaneously invest in their fleet;
Stage 2: The firms design their networks and solve the flow assignment problem with respect to the transport demand that they expect to serve, given the fleet sizes determined in the first stage;
Stage 3: The firms compete in terms of freight rates on each origin-destination movement, acknowledging the share of overall transport demand to be served by the competing party.

The game is solved using backward induction (i.e., incorporating the solution of later stages in previous ones), relying on the concept of Subgame Perfect Equilibrium (Gibbons, 1992)[2]. The model is solved for two cases: monopoly and duopoly. The main difference between these cases is the expression for revenues that captures the monopolistic or competitive behavior of each firm, respectively, and is worked out as solution of the third stage of the game. Given the complexity of the problem, we rely on numerical examples to characterize the equilibria of the sequential game.

The main contribution of the paper to the literature is both theoretical and methodological. At first, we address the issue of the effect of competition on network design and service provision in the shipping industry. From a methodological point of view, the main innovations are the use of advanced game-theoretic concept to address the issue of competition, while keeping the model tractable as well as the introduction of innovation in the manipulation

[2] Backward induction can be considered a generalization of Bellman's optimization principle (Fudenberg and Tirole, 1991). The latter is applied in dynamic optimization problems (hence, also in dynamic games where multiple agents choose their intertemporal strategies), while the first is applied also in static optimization problems.

of variables to address the resulting mixed linear-integer program.

Our theoretical and methodological setting considers a shipping industry where only one or two firms (i.e., liners or shipping alliances) operate. However, the framework can be straightforwardly extended to analyze oligopolistic shipping industries where more than two liners (or alliances) operate. Furthermore, the monopolistic and duopolistic cases provide appropriate frameworks to analyze competition in real-world regional shipping markets where we observe only one or two alliances.

The remainder of the paper is organized as follows. Section 2 describes the basic structure of the model; Section 3 and 4 analyze the monopolistic and duopolistic cases, respectively; Section 5 draws conclusions.

2. The Model

We consider a model of strategic network design with two competing firms (i.e., shipping liners or alliances); the list of all variables and parameters is in the Appendix. The map of ports – specifically, the distances between each pair of ports – and the specific features of each port $i \in I$ (where I is the set of existing ports or nodes) are exogenous, in particular the maximum throughput capacity (MQP_i). To keep the analysis as simple as possible, we consider a standard transport technology, thus the distance of the link $l \in L$ between two ports (where L is the set of all possible links between pair of ports) implies exogenous sailing time (STL_l).

The demand for maritime transport services between any pair of ports is assumed to be exogenous; but, quite naturally we assume that transport demand is decreasing (or non-increasing) in the total cost of transport borne by shippers. In particular, the total transport cost from the port of origin *r* to the port of destination *s* is the sum of the travel fees for full containers chosen by the shipping firm providing the transport service, q_{rs}^f, and the opportunity cost of travel (e.g., commodity depreciation) involved by the service, $COT\, u_{rs}$, where COT is the average opportunity cost per travel day for the shipper and u_{rs} is the travel time in days involved by the provided service.[3]

Continuously decreasing functions, commonly used in transport economics to represent market demand, would introduce non-linearities requiring a Mixed Integer Quadratically Constrained Quadratic Program (MIQCQP) formulation that is difficult to solve for larger problem instances. Thus, for the sake of tractability, we represent the container transport demand from *r* to *s* using a simple step-function:

- MQF_{rs}, for all $r \in O$ and $s \in D$ (where $O, D \subset I$ are the set of origin and destination ports, respectively), if the total transport cost borne by shippers is lower or equal than a given threshold MCT_{rs};[4]
- zero if the total transport cost is above MCT_{rs}.

Each firm would then seek to maximize its total profit Π, using the following objective function:

$$\Pi = TRP - \sum_{n \in N} (TCS_n + TCL_n + TCD_n) - TCC \tag{1}$$

The aggregate profit function Π is defined a composite of the various components of operational costs and revenues:

$$TRP = \sum_{r \in O} \sum_{s \in D} d_{rs} \tag{1.1}$$

$$TCS_n = \sum_{l \in L} LRR_{nl}\, p_n (STL_l\, CSR_l + CSP_l) \tag{1.2}$$

$$TCL_n = CHC_n\, p_n \sum_{a \in A_n} \left(x_{a+}^f + x_a^e\right) \tag{1.3}$$

$$TCD_n = CCD \left(\sum_{a \in A_n} \left(\left(x_{a+}^f + x_a^e\right) \sum_{l \in L} (LLR_{al}\, STL_l) \right) + w_{++}^f + w_+^e \right) \tag{1.4}$$

$$TCC = CRC\, CFS \tag{1.5}$$

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[3] In case a firm provides the service through different paths, we consider the average travel time, weighted by the container flows. This amounts to assume that shippers bear the risk of different actual delivery times.

[4] In economic terms, this threshold represents the "reserve price" above which the average shipper becomes uninterested in transport services. Of course, $MQF_{rs} = 0$ for all $r \in O$.

In equation 1.1, TRP is the total revenue earned across all container transport demands, with d_{rs} being the specific revenue obtained by the firm for the transport of full containers between r and s. The remaining terms in eq. 1 relate to various operational cost aspects, incurred by each potential route n.

Service costs for each route $n \in N$ (where N is the set of all possible routes) are provided by equation 1.2. Only services that are actually offered are considered (therefore $p_n = 1$). These can be decomposed into two components:

- A travel cost $STL_l\ CSR_l$ for each sailing link l that belongs to the route (therefore $LRR_{nl} = 1$), that would include bunker fuel, vessel maintenance, crew salaries and other costs that would be a function of link sailing time.
- Port visit costs CSP_l for the destination port of each link l in the route.

A container handling cost CHC_n would be incurred by containers travelling through each journey legs a, and would include the cost of loading and unloading operations on leg endpoints (ports). The presence of such costs would deter unnecessary transshipment whenever a feasible and direct service exists between two ports r and is captured by eq. 1.3. This cost would apply to both full and empty container flows (x^f_{a+} and x^e_a respectively), and is assumed to be the same for either class for the purposes of this study.

Container rental and cargo depreciation is captured by equation. 1.4, that are incurred by the periods STL_l that a container would spent aboard vessels over a journey leg a, as well as while waiting in a port before loading and in case of transshipment. (w^f_{++} and w^e_+ for full and empty containers, respectively). The parameter LLR_{al} is used to identify links in set L that belong to a leg a in subset A_n.

A simplified form of set summations for decisions variables x^f_{as}, x^e_a, w^f_{is} and w^e_i is used in eqs. 1.3 and 1.5. These conventions apply to similar expressions encountered later in this paper:

$$x^f_{a+} = \sum\nolimits_{s \in D} x^f_{as} \qquad w^f_{++} = \sum\nolimits_{i \in I} \sum\nolimits_{s \in D} w^f_{is} \qquad w^e_+ = \sum\nolimits_{i \in I} w^f_i$$

Fixed vessel operation costs (such as loan repayments, vessel insurance and administrative costs) are captured by eq.1.6, where CFS is the size of the firm's fleet, and CRC is the average vessel operation cost. As such, the following remarks are in order:

i. when a given route n is not operated (i.e., $p_n = 0$), only fixed costs associated to capacity deployed on that route are borne by the firm;
ii. assuming a standardized vessel size, used across all routes:
   a. $MRC_n = NCS\ NS_n$ is the maximum capacity (measured in terms of number of containers per week) that the firm deployed on route n, with NCS the number of containers that can be transported on each ship;
   b. $FSL_n = \frac{NS_n}{\sum_{a \in A\_n} \sum_{l \in L} LLR_{al} STL_l}$ is the frequency of sailing at route n;

The functional structure of the maritime container shipping market is represented by the following sequential game:

1) each firm deploys its maximum transport capacity for each route n, that in turn determines firm's fixed costs in the following stages of the game;
2) given firms' capacity constraints and expectations about the other firm's service offer, and the outcome of downstream (i.e., third stage) competition, each firm decides its network design (i.e., which routes are operated) and solves its container-flows assignment problem;
3) as an outcome of the second stage, container travel times and potential transport capacity limits (with respect to the potential demand)[5] between each pair of ports are determined for each firm, then firms

[5] Remark that capacity limits in the transport of containers between any two ports may arise because of the overall capacity on involved routes (that is fixed at the first stage of the game) is insufficient, given the solution of the assignment problem. In this case, firms just ration their services with respect to potential market demand.

compete *à la* Bertrand by fixing a fee for each full container transported between each pair of ports that are served by their networks.[6]

3. Maritime container transport design in monopolistic market

In this section, we consider (as a benchmark) the case of a single firm that is free to accommodate the entire transport demand in all ports. As usual in complete-information sequential games, the solution has to be found by backward induction (i.e., incorporating the solution of later stages in previous ones).

3.1 Theoretical analysis

Once the deployed capacity, network design and assignment have been chosen (by the monopolistic firm) it is possible to determine the admissible (maximum) flow of full containers, t_{rs}^{f}, and transport time, u_{rs}, for each pair of origin and destination ports.

The demand for transport from *r* to *s* will be considered as equal to MQF_{rs} only if $MCT_{rs} \geq q_{rs}^{f} + COT\, u_{rs}$. As such if transport time is too high (i.e., $u_{rs} > \frac{MCT_{rs}}{COT}$) no containers would be shipped from *r* to *s*, independently of the level of transport fee. Conversely, if $u_{rs} \leq \frac{MCT_{rs}}{COT}$, the monopolistic firm would charge the maximum travel fee that is compatible with a positive demand, i.e., $q_{rs}^{f} = MCT_{rs} - COT\, u_{rs}$.

As such, the revenue that the monopolist earns on the (potential) service from *r* to *s* can be characterized as:

$$d_{rs} = \begin{cases} (MCT_{rs} - COT\, u_{rs})t_{rs}^{f}, \; u_{rs} \leq \frac{MCT_{rs}}{COT} \\ 0, \; u_{rs} > \frac{MCT_{rs}}{COT} \end{cases} \tag{2}$$

where $t_{rs}^{f} \leq MQF_{rs}$.

Anticipating the impact of the market reaction on firm's revenue (equation 2), and given the deployed capacity on each route, the monopolist optimizes its network design and assignment problem by solving a mixed linear-integer program (MIP) that is characterized by the maximization of the profit function (1) subject to the following set of constraints (see the Appendix for notation):

$$\sum_{a\in A_i^+} x_{as}^{f} - \sum_{a\in A_i^-} x_{as}^{f} = b_{is}^{f} \text{ for all } i \in I, s \in D \tag{3.1}$$

$$\sum_{a\in A_i^+} x_{a}^{e} - \sum_{a\in A_i^-} x_{a}^{e} = -b_{i}^{e} \text{ for all } i \in I \tag{3.2}$$

$$x_{as}^{f} \leq \sum_{n\in N} FSL_n LR_{na} p_n w_{is}^{f} \text{ for all } a \in A_i^-, i \neq s \in I, s \in D \tag{3.3}$$

$$x_{a}^{e} \leq \sum_{n\in N} FSL_n LR_{na} p_n w_{i}^{e} \text{ for all } a \in A_i^-, i \in I \tag{3.4}$$

$$MQP_i \geq \sum_{a\in A_i^-}(x_{a+}^{f} + x_{a}^{e}) + \sum_{a\in A_i^+}(x_{a+}^{f} + x_{a}^{e}) \text{ for all } i \in I \tag{3.5}$$

$$MRC_n p_n \geq \sum_{a\in A_n}\sum_{s\in D} LLR_{an}(x_{a+}^{f} + x_{a}^{e}) \quad \text{for } n \in N, l \in L_n \tag{3.6}$$

$$b_{is}^{f} = \begin{cases} -t_{rs}^{f} \text{ if } i = r \in O \\ t_{+s}^{f} \text{ if } i = s \in D \\ 0 \text{ otherwise} \end{cases} \tag{3.7}$$

$$b_{i}^{e} = \begin{cases} t_{+i}^{f} \; if \; i = r \in I \cap D \\ -t_{+i}^{f} \; if \; i = r \in I \cap O \\ t_{+i}^{f} - t_{i+}^{f} \text{ if } i = r \in O \cap D \\ 0 \text{ otherwise} \end{cases} \tag{3.8}$$

$$d_{rs} = (MCT_{rs} - COT\, u_{rs})t_{rs}^{f} \geq 0 \tag{3.9}$$

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[6] The underlying assumption of Betrand's competition in the last stage of the game is that the service provided by competing firms is homogeneous. This is rather realistic for the maritime container transport industry, though some form of service quality differentiation may arise, which could bring to imperfect substitutability and monopolistic competition between the two firms.

$$t_{rs}^{f} \leq MQF_{rs} \quad (3.10)$$
$$x_{as}^{f} \geq 0 \text{ for all } a \in A, s \in D \quad (3.11)$$
$$x_{a}^{e} \geq 0 \text{ for all } a \in A \quad (3.12)$$
$$d_{rs} \geq 0 \text{ for all } r \in O, s \in D \quad (3.13)$$
$$t_{rs}^{f} \geq 0 \text{ for all } r \in O, s \in D \quad (3.14)$$
$$p_{n} \in \{0,1\} \text{ for all } n \in N \quad (3.15)$$

The constraints (3.3) and (3.4) feature a multiplication between decision variables. However, it is possible to linearize this constraint using the following transformations:

$$x_{as}^{f} \leq \sum_{n \in N} FSL_{n} LR_{na}\, g_{nis} \text{ for all } a \in A_{i}^{-}, i \neq s \in I, s \in D, \quad (3.3)$$

where:

$$g_{nis} \leq MDT_{i}^{f} p_{n} \quad (3.3.1)$$
$$g_{nis} \leq w_{is}^{f} \quad (3.3.2)$$
$$g_{nis} \geq w_{is}^{f} - MDT_{i}^{f}(1 - p_{n}) \quad (3.3.3)$$
$$g_{nis} \geq 0 \quad (3.3.4)$$

where MDT_{i} is the maximum dwell time of full containers at port *i*. Similarly, the constraint (3.4) can be linearized as follows:

$$x_{as}^{e} \leq \sum_{n \in N} FSL_{n} LR_{na}\, h_{ni} \text{ for all } a \in A_{i}^{-}, i \neq s \in I, s \in D \quad (3.4)$$
$$h_{ni} \leq MDT_{i}^{e} p_{n} \quad (3.4.1)$$
$$h_{ni} \leq w_{i}^{e} \quad (3.4.2)$$
$$h_{ni} \geq w_{i}^{e} - MDT_{i}^{e}(1 - p_{n}) \quad (3.4.3)$$
$$h_{ni} \geq 0 \quad (3.4.4)$$

where MDT_{i}^{e} is the maximum dwell time of empty containers at port *i*.

Given the solution of the second and third stages of the optimization, the total profit of the firm depends on the vector of the maximum capacity deployed on each route. Thus, the firm chooses the vector of capacity deployment maximizing its profit. This model possesses a sufficient amount of operational detail that would facilitate its applications to realistic scenarios, as illustrated by the numerical example presented in the following section.

3.2 Numerical analysis

Perhaps the most challenging part of problem instance definition for the above model is the generation of virtual network components for a given maritime service network and managing the LRR_{ln}, $LLR_{al}$and LR_{an} parameter sets that capture the structural and operational relationships between the two types of networks.

A custom tool (Delos) was developed (using the C# programming language) to simplify this process. The key features of this tool include a geospatial data management interface that facilitates the visual definition of maritime transport networks, and a database that captures operational aspects of the maritime transport market environment (including parameters that relate to companies, services, ports and market demands). The actual MILP model that was defined in the previous section was implemented using the OPL modelling language and solved using the CPLEX optimization engine. A custom C#/OPL interface was developed to link Delos with the final OPL model and manage the algorithm workflow.

To test the monopolistic version of the model we created a sample scenario focused on the Eastern Mediterranean liner shipping market (Figure 2) and several key ports in the region (Ancona, Antalya, Beirut, Benghazi, Cagliari, Gioia Tauro, Haifa, Heraklion, Istanbul, Izmir, Kavala, Limassol, Pireaus, Port Said, Thessaloniki, Tripoli and Venice). Five mid-range feeder services were defined, inspired from real market offerings. Liner service circuit durations spanned between six and nine days, therefore requiring either one or two vessels for the provision of weekly services – the latter is the case for services that exceed the seven-day-duration threshold. Several opportunities for transshipment were provided (mainly in the port of Piraeus).

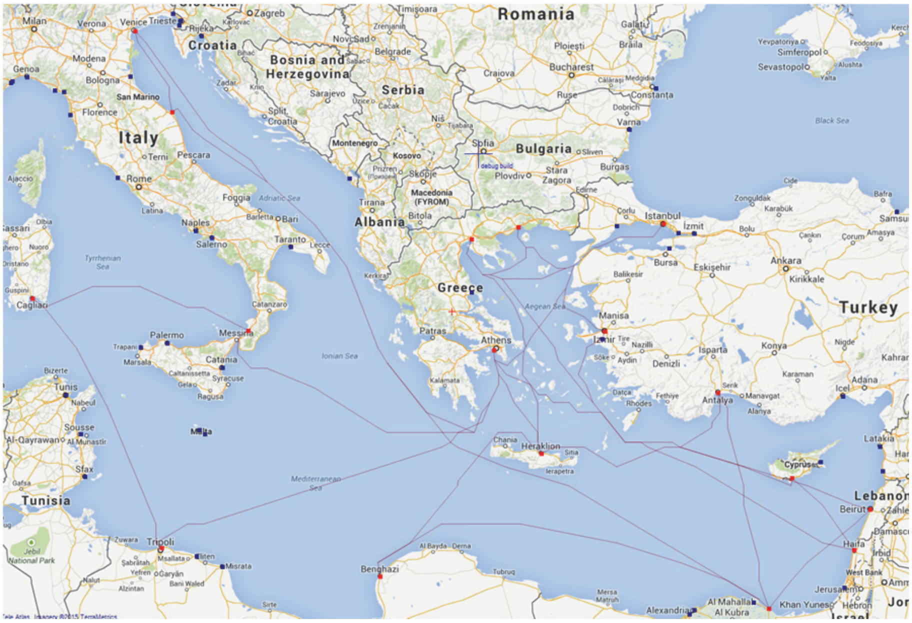

Fig. 2: Delos screenshot – The area covered and the routes included in the numerical example (Map data: Google Maps)

Table 1: Services included in the numerical example / Order of port calls

Service	Calls
Service 1	Piraeus→ Beirut → Port Said → Ancona → Venice → Piraeus
Service 2	Heraklion → Benghazi → Port Said → Antalya → Heraklion
Service 3	Piraeus → Tripoli → Cagliari → Gioia Tauro → Piraeus
Service 4	Thessaloniki → Istanbul → Izmir → Limassol → Haifa → Thessaloniki
Service 5	Piraeus → Beirut → Port Said → Ancona → Venice → Piraeus

Four transport service demands were defined in the model, namely Kavala → Cagliari, Thessaloniki →Izmir, Piraeus → Ancona and Antalya →Haifa (all 100 TEU/w). Two iterations of the model were required in order to reach a stable solution, with the firm opting to partially serve only two of the four market demands, namely Thessaloniki →Izmir (50 out of 100 TEU/w), Piraeus →Ancona (75 out of 100 TEU/w). Despite the MIP nature of the problem definition, individual problem instances were solved in less than 400ms using the CPLEX optimizer on a relatively modern computer workstation (Quad core Intel i7 2.8GHz processor and 16GB RAM).

Given the solution of the second and third stage, the total profit of the firm depends on the vector of the maximum capacity deployed on each route. Thus, the firm chooses the vector of capacity deployment maximizing its profit, taking into account the financial feasibility of service provision and limitations imposed by its available fleet. In the numerical example outlined above and given the design of the service network, a fleet of four vessels would have been sufficient to provide the entire set of services defined in the model. But, only two of the services were provided by the firm, even in cases that the firm fleet consisted of more than 2 vessels. Thus, by backward induction procedure, we know that in at the first stage of the game, the firm would choose to invest in two vessels only.

The outcome of this process can be justified from the fact that further service provision would diminish the potential profit of the firm; given the limited set of potential services and the amount of transport service demand in the market (as defined within this problem instance), the requirement for more complicated transshipment arrangements that would have been necessary to serve port pairs without any direct service links (Antalya → Haifa in this case). A different outcome would have been observed widening the set of potential services that the firm could select, thus making it more able to control the operating cost it faces to satisfy market demand.

4. Maritime container transport design in duopolistic market

We now consider the case of two firms competing in the maritime container market. We solve the game relying on the concept of Subgame Perfect Equilibrium. Hence, we proceed by backward induction.

4.1 Theoretical analysis

Once deployed capacity, network design and assignment are chosen by the competing firms, we obtain (similarly to the benchmark case), the admissible (maximum) flow of full containers, t_{rs}^{f}, and transport time, u_{rs}, for each firm and for its rival (i.e., t_{rs}^{f*} and u_{rs}^{*}) and any pair of origin and destination ports. As before, the demand for transport from *r* to *s* is positive (and equal to MQF_{rs}) only if any of the two competing firms is able to offer a sufficiently low total cost of transport (i.e., below MCT_{rs}).

Two kinds of equilibria may arise at this stage of the game on market of transport services from *r* to *s*:

- if $t_{rs}^{f} + t_{rs}^{f*} < MQF_{rs}$ there is an excess of demand over the supply of transport services offered by both firms;
- if $t_{rs}^{f} + t_{rs}^{f*} \geq MQF_{rs}$ there is an excess of supply over demand.

Let us consider a given firm (the same analysis holds true also for the rival firm). Any type of equilibrium involving non-zero demand flows for the firm may arise only if $MCT_{rs} \geq COT\, u_{rs}$. The revenue function of the firm is $d_{rs} = q_{rs}^{f}\, t_{rs}^{f}$. The following proposition characterizes the equilibrium of type *a)*:

Proposition 1. If equilibrium flows from *r* to *s* are such that $t_{rs}^{f} + t_{rs}^{f*} < MQF_{rs}$, then the equilibrium firm's fee necessarily is $q_{rs}^{f} = MCT_{rs} - COT\, u_{rs}$.

Proof. Remark that $q_{rs}^{f} > MCT_{rs} - COT\, u_{rs}$ would imply a zero demand for the firm. Assume, by contradiction, that $q_{rs}^{f} < MCT_{rs} - COT\, u_{rs}$. The rival firm cannot absorb any excess of demand, because it already operates at its optimally planned flow (as determined at stage two). Thus, the firm could increase its revenue, $d_{rs} = q_{rs}^{f}\, t_{rs}^{f}$, (hence its total profit) by raising the transport fee. And the proposition follows. ■

The implication of the previous argument is that the revenue function for each firm is exactly as in the monopolistic case analyzed in Section 3. Let us now consider the equilibrium of type *b)*:

Proposition 2. If equilibrium flows from *r* to *s* are such that $t_{rs}^{f} + t_{rs}^{f*} \geq MQF_{rs}$, then the equilibrium firm's fee necessarily is

$$q_{rs}^{f} = \begin{cases} COT\,(u_{rs}^{*} - u_{rs})t_{rs}^{f}, & u_{rs}^{*} \geq u_{rs} \\ 0, & u_{rs}^{*} < u_{rs} \end{cases} \tag{4}$$

Proof. By $t_{rs}^{f} > MQF_{rs} - t_{rs}^{f*}$ the actual flow of demand for transport from *r* to *s* could be lower than the optimally planned flow t_{rs}^{f}. The same argument applies to the rival firm. Given the excess of supply over demand, the equilibrium fees necessarily should imply that the total cost of transport from *r* to *s* is equal across them, i.e., $q_{rs}^{f} = q_{rs}^{f*} + COT\,(u_{rs}^{*} - u_{rs})$. Moreover, if $u_{rs}^{*} < u_{rs}$ the firm offer a worse service in terms of travel (opportunity cost of) time, however the minimum transport fee that it can charge is $q_{rs}^{f} = 0$ (earning a zero revenue, whatever the actual transport flow). The reverse holds if $u_{rs}^{*} \geq u_{rs}$, in particular: $q_{rs}^{f*} = 0$. In this case, the firm's equilibrium fee cannot be higher than $COT\,(u_{rs}^{*} - u_{rs}) \geq 0$ (otherwise it would lose demand flows to the benefit of the rival) nor lower than that (otherwise it could increase its revenue and profit by a small increase of the fee, given that the service provided by the rival is worse in terms of travel time). Thus the proposition follows. ■

A key feature of this model is that the most time-efficient firm can always attract all the traffic that it is able to serve (i.e., t_{rs}^{f}) by fixing its transport fee in such a way that the total transport cost for the shipper is slightly lower than the rival firm one. Having observed this we can conclude that the revenue function of the firm can be represented as follows:

$$d_{rs} = \begin{cases} COT\ (u_{rs}^{*} - u_{rs})t_{rs}^{f}, & u_{rs}^{*} \geq u_{rs} \\ 0, & u_{rs}^{*} < u_{rs} \end{cases} \quad (5)$$

where $t_{rs}^{f} \leq MQF_{rs}$.

Taking as given the network design and assignment solution of the rival, the firm anticipates the impact of the market reaction on its revenue, given the deployed capacity on each route. In particular, the firm will anticipate to ioerate on each service from r to s it as a monopolist (in which case revenue is given by equation 2) or as a duopolist (in which case revenue is given by equation 5).

The firm would thus optimize its network design and assignment problem by solving a mixed integer linear program whose objective is to maximize the same profit function (1) subject to the following set of constraints (see the Appendix for notation):

$$\sum_{a\in A_i^+} x_{as}^{f} - \sum_{a\in A_i^-} x_{as}^{f} = b_{is}^{f} \text{ for all } i \in I, s \in D \quad (6.1)$$

$$\sum_{a\in A_i^+} x_{a}^{e} - \sum_{a\in A_i^-} x_{a}^{e} = -b_{i}^{e} \text{ for all } i \in I \quad (6.2)$$

$$x_{as}^{f} \leq \sum_{n\in N} FSL_n LR_{na} p_n w_{is}^{f} \text{ for all } a \in A_i^-, i \neq s \in I, s \in D \quad (6.3)$$

$$x_{a}^{e} \leq \sum_{n\in N} FSL_n LR_{na} p_n w_{i}^{e} \text{ for all } a \in A_i^-, i \in I \quad (6.4)$$

$$MQP_i \geq \sum_{a\in A_i^-}(x_{a+}^{f} + x_{a}^{e} + x_{a+}^{f*} + x_{a}^{e*}) + \sum_{a\in A_i^+}(x_{a+}^{f} + x_{a}^{e} + x_{a+}^{f*} + x_{a}^{e*}) \text{ for all } i \in I \quad (6.5)$$

$$MRC_n p_n \geq \sum_{a\in A_n}\sum_{s\in D} LLR_{an}(x_{a+}^{f} + x_{a}^{e}) \quad \text{for } n \in N, l \in L_n \quad (6.6)$$

$$b_{is}^{f} = \begin{cases} -t_{rs}^{f} \text{ if } i = r \in O \\ t_{+s}^{f} \text{ if } i = s \in D \\ 0 \text{ otherwise} \end{cases} \quad (6.7)$$

$$b_{i}^{e} = \begin{cases} t_{+i}^{f}\ if\ i = r \in I \cap D \\ -t_{+i}^{f}\ if\ i = r \in I \cap O \\ t_{+i}^{f} - t_{i+}^{f} \text{ if } i = r \in O \cap D \\ 0 \text{ otherwise} \end{cases} \quad (6.8)$$

$$d_{rs} = \begin{cases} COT\ (u_{rs}^{*} - u_{rs})t_{rs}^{f} & if \quad t_{rs}^{f} + t_{rs}^{f*} \geq MQF_{rs} \\ (MCT_{rs} - COT\ u_{rs})t_{rs}^{f} & if \quad t_{rs}^{f} + t_{rs}^{f*} < MQF_{rs} \end{cases} \text{ for all } r \in O, s \in D \quad (6.9)$$

$$t_{rs}^{f} \leq MQF_{rs} \quad (6.10)$$

$$x_{as}^{f} \geq 0 \text{ for all } a \in A, s \in D \quad (6.11)$$

$$x_{a}^{e} \geq 0 \text{ for all } a \in A \quad (6.12)$$

$$d_{rs} \geq 0 \text{ for all } r \in O, s \in D \quad (6.13)$$

$$t_{rs}^{f} \geq 0 \text{ for all } r \in O, s \in D \quad (6.14)$$

$$p_n \in \{0,1\} \text{ for all } n \in N \quad (6.15)$$

The constraint (6.9) features a variable-conditional constraint. To solve this problem we adopt the following transformation, based on the introduction of indicator decision variables, $da_{rs} = 1$ and $da_{rs} = 0$, if $t_{rs}^{f} + t_{rs}^{f*} \geq MQF_{rs}$, and $da_{rs} = 0$ and $da_{rs} = 1$ if $t_{rs}^{f} + t_{rs}^{f*} < MQF_{rs}$, for all $r \in O, s \in D$. Thus, for all $r \in O, s \in D$, the constraint (6.9) can be substituted by:

$$da_{rs} = (1 - db_{rs}) \quad (6.9.1)$$

$$t_{rs}^{f} + t_{rs}^{f*} \leq MQF_{rs}\ (2 - db_{rs}) \quad (6.9.2)$$

$$d_{rs} = MCT_{rs}t_{rs}^{f} + COT\ u_{rs}^{*}t_{rs}^{f} - COT\ u_{rs}t_{rs}^{f} - MCT_{rs}t_{rs}^{f}da_{rs} - COTu_{rs}^{*}t_{rs}^{f}db_{rs} \quad (6.9.3)$$

However, the constraint (6.9.3) is affected by multiplication between decision variables. Hence, we linearize it as follows:

$$d_{rs} = MCT_{rs}t_{rs}^{f} + COT\, u_{rs}^{*}t_{rs}^{f} - COT\, u_{rs}t_{rs}^{f} - MCT_{rs}dc_{rs} - COTu_{rs}^{*}dd_{rs} \quad (6.9.3')$$
$$dc_{rs} \leq MQF_{rs}^{f}da_{rs} \quad (6.9.3.1)$$
$$dc_{rs} \leq t_{rs}^{f} \quad (6.9.3.2)$$
$$dc_{rs} \geq t_{rs}^{f} - MQF_{rs}^{f}(1 - da_{rs}) \quad (6.9.3.3)$$
$$dd_{rs} \leq MQF_{rs}^{f}db_{rs} \quad (6.9.3.4)$$
$$dd_{rs} \leq t_{rs}^{f} \quad (6.9.3.2)$$
$$dd_{rs} \geq t_{rs}^{f} - MQF_{rs}^{f}(1 - db_{rs}) \quad (6.9.3.3)$$

As before, also constraints (6.3) and (6.4) feature multiplication between decision variables that are linearized as follows:

$$x_{as}^{f} \leq \sum_{n \in N} FSL_{n}LR_{na}\, g_{nis} \text{ for all } a \in A_{i}^{-}, i \neq s \in I, s \in D, \quad (6.3)$$

where:

$$g_{nis} \leq MDT_{i}^{f}p_{n} \quad (6.3.1)$$
$$g_{nis} \leq w_{is}^{f} \quad (6.3.2)$$
$$g_{nis} \geq w_{is}^{f} - MDT_{i}(1 - p_{n}) \quad (6.3.3)$$
$$g_{nis} \geq 0 \quad (6.3.4)$$

where MDT_i is the maximum dwell time of full containers at port *i*. Similarly, the constraint (6.4) can be transformed as follows:

$$x_{as}^{f} \leq \sum_{n \in N} FSL_{n}LR_{na}\, h_{ni} \text{ for all } a \in A_{i}^{-}, i \neq s \in I, s \in D \quad (6.4)$$
$$h_{ni} \leq MDT_{i}^{e}p_{n} \quad (6.4.1)$$
$$h_{ni} \leq w_{i}^{e} \quad (6.4.2)$$
$$h_{ni} \geq w_{i}^{e} - MDT_{i}^{e}(1 - p_{n}) \quad (6.4.3)$$
$$h_{ni} \geq 0 \quad (6.4.4)$$

where MDT_i^e is the maximum dwell time of empty containers at port *i*.

At the first stage of the game, both firms are able to anticipate the effect of subgame equilibria (stages 2 and 3) on their outcome, depending on their strategic choices in terms of capacity limits. Again, because of the complexity of the sequential game, we rely on a numerical example to characterize the equilibrium.

4.2 Numerical example

The analysis can be conducted on the basis of an iterative process (Figure 3). As first, we assume that the rival firm behaves as a monopolist. Thus, we determine the outcome of the linear-integer program as in Section 3. These results are the first-iteration guess of the firm about the behavior of the rival firm, to run the linear-integer optimization program of the firm. The output of such a program is then used as second-iteration guess.

This algorithm was implemented again using a combination of the Delos maritime network design tool (used for data management and algorithm flow control) and IBM OPL CPLEX. The duopolistic version of the algorithm was tested on the same market environment that was used earlier in this study, with the key difference being that in this case two symmetrical shipping firms being allowed to operate and compete. In this context, Company 2 is referred to as the rival firm (represented with an asterisk) and Company 1 the other firm – both have the ability to allocate up to four vessels to accommodate the various transport demands in the market.

The algorithm would then allocate flows to among firms, taking into account the effects of competition in the structure of the revenue functions. A series of iterations are necessary before a stable solution is reached, with the flow of the algorithm summarized in Figure 3 below:

Up to three iterations on each step were required to obtain market allocations for each firm with a converged cargo flow time u_{rs}, with four overall iterations of the duopolistic case before a stable market allocation for both firms had been reached. The final market allocation was found to have a minor degree of sensitivity to the initial market share assumption for the competing firm.

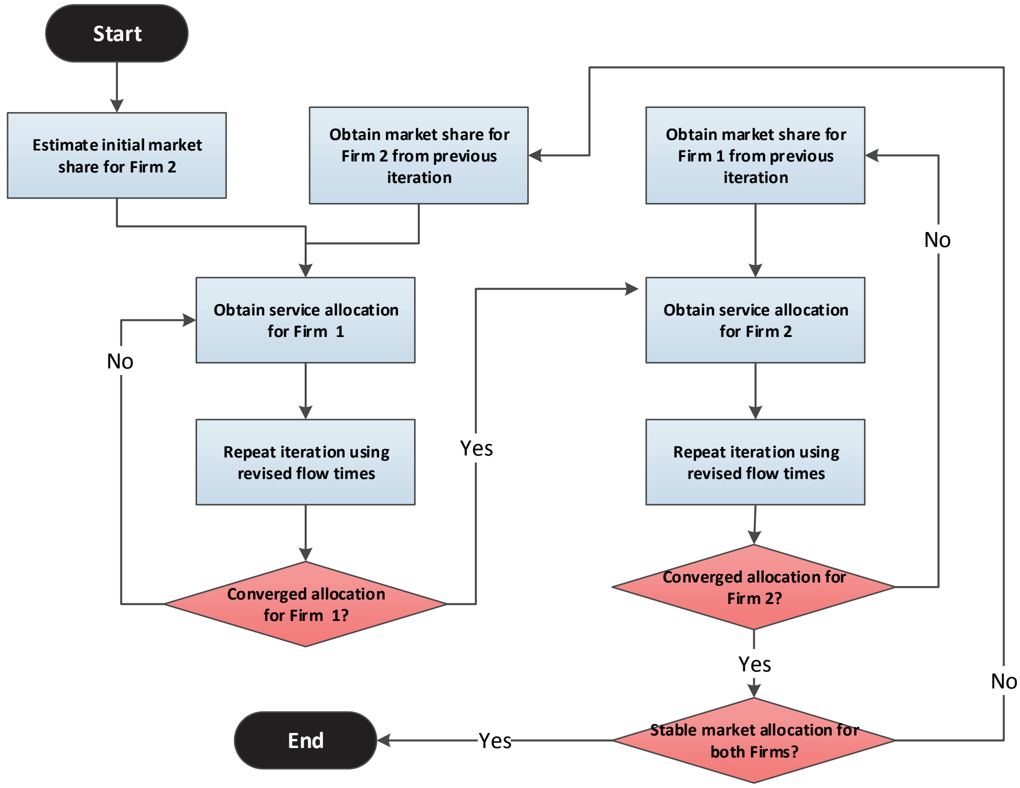

Fig. 3: Logic flow of the duopolistic assignment algorithm

While one of the potential companies of the competitive iteration process would have been an unstable assignment of the market share among the two firms, with a set of two (or more) allocations for each firm that would continuously alternate, and thus prohibiting the algorithm from terminating, such an outcome was not observed in any of the scenarios attempted.

Table 2: Market allocation in a duopolistic scenario with symmetrical competitors

Port Pair	Market Demand (TEU/week)	Firm 1 flow (TEU/week)	Firm 2 flow (TEU/week)	Unmet Demand (%)
KAVALA → CAGLIARI	100	75	0	25%
THES/NIKI → IZMIR	100	60	0	40%
PIRAEUS → ANCONA	100	0	70	30%
ANTALYA → HAIFA	100	0	0	100%

Assuming an initial market share of 40% for all cargo demands by Firm 2, the algorithm quickly converged into an asymmetric allocation, with Firm 1 opting to operate on Service 1, 3 and 4. On the other hand Firm 2 operated on Service 5. Given the two distinct strategies, there is evidence that the two competitors to concentrate on different parts of the market. The final outcome of the iteration process is summarized in Table 2.

At the first stage of the game, both firms are able to anticipate the effect of subgame equilibria (stages 2 and 3) on their outcome, depending on their strategic choices in terms of capacity limits. In the above competitive scenario, the two companies opted to allocate 2 and 3 vessels (respectively) in the region, therefore accommodating market demands for cargo transportation in all port pairs except Antalya → Haifa. As in the monopolistic case, the vessel capacity limit is not reached due to exceedingly large operational costs. The fact that firms do not serve all the available demand can be justified by the excessive cost that the available service set induce (mainly determined by transshipment between two linked services), one of which had already exhausted its available capacity (in the case of Firm 2) while serving the remaining market demands.

5. Conclusions and future work

This paper presents a sequential game-theoretic model to analyze network design, container assignment and service provision of shipping firms (or alliances) both when they operate in a set of ports as monopolist and when they compete with another rival firm. The model takes into account exogenous demands for container transport among ports of origin and destination which reacts to the total cost of transport, including the travel fee that is paid to the shipping firm and the opportunity cost of time (e.g., depreciation of shipped commodities) for the shipper. Because of the complexity of the theoretical model, we rely on a numerical algorithm to characterize the equilibrium of the game. We find that:

- the monopolist firm does not cover all possible market demand, because of the high cost of available services (i.e., possible routes that it can activate) mainly linked to transshipment. Moreover, the monopoly never satisfies all the existing demand in ports that are served through the chosen network.
- when a duopoly is considered, the scope for demand satisfaction improves. The firms tend to choose diversify network designs; one of the two competing firms uses all possible routes, while the other firm uses the same services than the monopolist.

The introduction of competitive behavior adds a new level of complexity to container flow assignment and liner service network design problems, therefore bringing them closer to real life settings. Duopoly (and monopoly) are a good approximation of the behavior of the shipping market in some regional areas of the world. Our model can be used as a framework for future research addressing the case of oligopolistic shipping markets with multiple players. Several other issues need to be investigated, such as what is the effect of widening the scope for network design (i.e., potential routes to be selected) on the possibility of firms to differentiate their networks and thus relax completion among them, and on the possibility for each firm to optimize the cost structure of their networks. The assessment of these issues in alternative set of ports can also shed some light on the role of vessel capacity limit (and investment costs). The theoretical model and the numerical algorithm we developed in this paper is sufficiently powerful to address these issues, which will be explored as part of future research.

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Appendix – Variable Definitions

Indices

a	for legs
b	for companies
l	for links
n	for routes
i	for ports
r	for origin ports
s	for destination ports

Sets

A	Legs
O	Origin Ports
D	Destination Ports
I	All Ports
N	Set of routes
L	Set of links

Subsets

A_i^+	Legs entering port i
A_i^-	Legs leaving port i
A_n	Legs on route n
N_a	Routes on leg a
L_n	Links on route n

Parameters

CSR_l	Operating cost for link l	\$/day
CSP_l	Port visit cost for link l	\$
CHC_n	Container handling cost for route n offered	\$/TEU
CCD	Cost per unit time per container	$\$/(\mathrm{TEU} \cdot \mathrm{week})$
COT	Opportunity cost of time	\$/week
MRC_n	Capacity of route n	TEU/week
MCT_{rs}	Maximum cost of travel (from port r to port s)	\$
MQF_{rs}	Maximum flow of full containers (from port r to port s)	TEU/week
MQP_i	Maximum throughput of port i	TEU/week
STL_l	Sailing time for link l	week
LRR_{nl}	1 if link l uses route n, and 0 otherwise	-----
LR_{an}	1 if leg a is on route n, 0 otherwise	-----
LLR_{al}	1 if leg a contains link l, 0 otherwise	-----
CFS	Fleet Size for current firm	vessels
RVR_n	Route Vessel Requirement	vessels
MDT_i^f / MDT_i^e	max empty and full dwell times at i	week
FSL_n	Frequency of sailings at route n	vessel/week

Decision Variables

t_{rs}^f	Flow of full containers (from port r to port s)	TEU/week
p_n	1 if route n is offered; 0 otherwise	-----
u_{rs}	Travel time for full containers (from port r to port s)	week
d_{rs}	Revenue for full containers from (from port r to port s)	\$
x_{as}^f	Flow of full containers on leg a en route to port s	TEU
x_a^e	Flow of empty containers on leg a	TEU
w_{is}^f	Dwell time at port i for full containers en route to port s	week
w_i^e	Dwell time at port i for all empty containers	week

Composite Expressions

TRP	Total revenue across all OD pairs	\$
TCS_n	Service operational costs for route n	\$
TCL_n	Leg operational costs for route n	\$
TCD_n	Container deployment costs for route n	\$
TCC_n	Vessel deployment costs for route n	\$