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Asymmetry and Leverage in GARCH models: A News Impact Curve perspective

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ABSTRACT
Models for conditional heteroskedasticity belonging to the GARCH class are now common tools in many economics and finance applications. Among the many possible competing univariate GARCH models, one of the most interesting groups allows for the presence of the so-called asymmetry or leverage effect. In our view, asymmetry and leverage are two distinct phenomena, both inspired by the seminal work of Black (1976). We propose definitions of leverage and asymmetry that build on the News Impact Curve, allowing to easily and coherently verify if they are both present. We show that several GARCH models are asymmetric but none is allowing for a proper leverage effect. Finally, we extend the leverage definition to a local leverage effect and show that the AGARCH model of Engle (1990) is coherent with the presence of local leverage. An empirical analysis completes the paper.

KEYWORDS
Conditional volatility models, GARCH models, Asymmetry, Leverage.

JEL CLASSIFICATION
C12; C13; C22; C52; C58; G32.

1. Introduction

Models for conditional heteroskedasticity belonging to the Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) class initiated by Engle (1982) are now common tools in many economics and finance applications. Within this class, the seminal specification of Bollerslev (1986) represents one of the most common choices, despite the still growing number of univariate specifications proposed within the econometrics literature (see for a survey, Bollerslev 2010). Among the many possible competing univariate GARCH models, one of the most interesting groups, from an empirical viewpoint, collects the specifications allowing for the presence of the so-called asymmetry and/or leverage effects, whose definition and relevance will be the focus of this paper. This group includes the Glosten-Jagannathan-Runkle (GJR) GARCH model of Glosten et al. (1993), the Threshold GARCH (TARCH) of Zakoian (1994), the Exponential GARCH (EGARCH) of Nelson (1991) and the Asymmetric Power ARCH (APARCH) of Ding et al. (1993).

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Recent contributions, McAleer (2014), McAleer and Hafner (2014) and Chang and McAleer (2017), made some interesting progresses in motivating the derivation of GARCH-type specifications from a Random Coefficient model (starting from the work of Tsay 1987), and, building on these theoretical contribution, derived regularity conditions for the existence of asymmetry and leverage, according to a common economic-based definitions of these two effects. McAleer (2014) tries to derive the conditions for asymmetry and leverage applying the same definition to competing models. Later, Chang and McAleer (2017) clarify the conditions for the existence of asymmetry in the EGARCH model of Nelson (1991), leading to a condition conflicting with the one included in McAleer (2014).

In this paper, after setting a few preliminary elements, we first review the characterization of asymmetry and leverage effects in terms of conditional variances and News Impact Curve (Engle and Ng 1993), proposing definitions of these two effects which differ from those in Chang and McAleer (2017). We believe that the definitions we put forward, in particular that for asymmetry, are more coherent with the economic intuition provided in Black (1976), and with the empirical interpretation of the asymmetry effect.

Building on our definitions, we verify if a group of selected GARCH specifications allow for the presence of asymmetry and leverage effects. We show that many models are inherently asymmetric while none is coherent with the leverage effect. Moreover, our conditions for the EGARCH model are coherent with those in (McAleer 2014) and are in conflict with (Chang and McAleer 2017). We show how the condition in (Chang and McAleer 2017) are, in our view, incoherent with the common interpretation of symmetry for the GARCH model of (Bollerslev 1986).

Finally, we provide a more flexible characterization of leverage, allowing for its existence only at the local level. We show that the Asymmetric GARCH (AGARCH) model of Engle (1990) allows for local leverage.

In verifying if GARCH models allow for leverage and/or asymmetry, we focus on the dynamic equation and on the conditions for positivity of the conditional variances since, in empirical studies, these elements are central. In fact, most GARCH models are commonly applied to real data even if the asymptotic properties of the estimators are unknown, or only partially available, or not rigorously proved. Consequently, we do not discuss the regularity conditions to ensure covariance stationarity and the asymptotic properties of estimators. These elements go beyond the scope of our paper.

The paper proceeds as follows. Section 2 introduces selected univariate GARCH specifications and the News Impact Curve. Section 3 discussed the characterization of asymmetry and leverage. Section 4 provides an empirical example on the presence of asymmetry and leverage in the US market and Section 5 concludes.

2. Preliminaries

2.1. *Selected Univariate GARCH*

To introduce a proper definition for asymmetry and leverage, we first present selected models using a consistent notation in order to highlight their commonalities. For all models we consider the simplest specifications, i.e. we limit all model orders to be equal to one (and to simplify the model acronyms, we do not report the model orders). For more general specifications we refer the reader to the papers introducing the various models.

Let x_t be a sequence of returns (the logarithmic difference of the asset price P_t) for a given financial instrument. We characterize the conditional mean as follows:

$$x_t = E(x_t|\Omega_{t-1}) + \varepsilon_t, \quad \varepsilon_t \equiv h^{\frac{1}{2}} z_t, \quad z_t \sim iid\,(0,1)\,, \tag{1}$$

where Ω_{t-1} is the information set at time $t-1$, h_t is the conditional variance, and z_t is an innovation term with zero mean and unit variance following an un-specified density.

The GARCH model of Bollerslev (1986) provides the following dynamic for the conditional variances

$$h_t = \omega + \beta h_{t-1} + \alpha \varepsilon_{t-1}^2. \tag{2}$$

To ensure the positivity and the stationarity of the conditional variance h_t, the parameters must satisfy the following restrictions: $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ and $\alpha + \beta < 1$ (see Bollerslev 1986). There is a general agreement in the literature on the absence of asymmetry and/or leverage effects in the GARCH model.

The GJR-GARCH model of Glosten et al. (1993) adds a component to the GARCH model:

$$h_t = \omega + \beta h_{t-1} + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 \mathbb{I}_{[\varepsilon_{t-1}<0]}, \tag{3}$$

where $\mathbb{I}\,(.)$ is the indicator function. The coefficients γ drives the presence/absence of asymmetry and/or leverage effects. Sufficient conditions for positivity of conditional variances h_t, require that $\omega \geq 0$, $\alpha \geq 0$, $\beta \geq 0$, $\gamma \geq 0$, while for covariance stationarity, under the additional assumption of symmetry for the density of $z_t = \varepsilon_t h_t^{-1}$, we need $\alpha + \beta + \frac{1}{2}\gamma < 1$.

The EGARCH model by Nelson (1991) defines the dynamic on the logarithm of the conditional variance,

$$\log h_t = \omega + \beta \log h_{t-1} + \alpha z_{t-1} + \kappa\left(|z_{t-1}| - E|z_{t-1}|\right). \tag{4}$$

Due to the logarithmic transformation, variances are positive by construction. Consequently, there are no restrictions on the parameters for positivity, but they must anyway satisfy restrictions ensuring covariance stationarity. In particular, a sufficient condition is that $|\beta| < 1$. In the EGARCH model, the presence of asymmetry and/or leverage depends α and κ.

The Threshold-GARCH (TGARCH) specification proposed by Zakoian (1994) models the conditional standard deviation with a dynamic that preserves the information on the shock sign, and thus potentially allowing for asymmetry and/or leverage. The conditional volatility follows:

$$h_t^{1/2} = \omega + \beta h_{t-1}^{1/2} + \alpha \varepsilon_{t-1} + \gamma |\varepsilon_{t-1}|. \tag{5}$$

Sufficient conditions for the positivity of conditional variances are $\omega > 0$, $\beta \geq 0$, $\gamma \geq 0$ and $-\gamma \leq \alpha \leq \gamma$.

The Asymmetric GARCH model (AGARCH), introduced in Engle (1990) and further discussed in Engle and Ng (1993), models the conditional variance as

$$h_t = \omega + \beta h_{t-1} + \alpha\left(\varepsilon_{t-1} - \varphi\right)^2, \tag{6}$$

and the asymmetry and/or leverage effect depends on the parameter φ. Moreover, Engle and Ng (1993) provide two alternative specifications of the shock dynamic defined in Equation 6. In the first, Nonlinear Asymmetric GARCH model (NGARCH), they add the interaction of the conditional standard deviation with φ, replacing $(\varepsilon_{t-1} - \varphi)^2$ with $\left(\varepsilon_{t-1} - \varphi h_{t-1}^{1/2}\right)^2$. In the second, the VGARCH model,[1] they replace the ε_{t-1} in 6 by the variance standardized shock z_t.

Finally, the Asymmetric Power ARCH (APARCH) model by Ding et al. (1993) can be viewed as a non-linear generalization of the GARCH, GJR-GARCH and TGARCH models. The parameter $\delta > 0$ drives the non linearity and represents the power of the conditional volatility over which we define the dynamic. The model reads as:

$$h_t^{\delta} = \omega + \beta h_{t-1}^{\delta} + \alpha \left(|\varepsilon_{t-1}| - \varphi \varepsilon_{t-1}\right)^{\delta}. \tag{7}$$

where $|\varphi| \leq 1$ and $\delta > 0$. An excellent discussion for necessary and sufficient moment conditions is provided by Ling and McAleer (2002).

2.2. *News Impact Curve*

The News Impact Curve (NIC), introduced by Pagan and Schwert (1990) for the GARCH model and then discussed for other models by Engle and Ng (1993), measures how news impact on the conditional variances. In the NIC framework, the shocks of the GARCH models, either ε_t or z_t depending on the specifications, are proxy of the news. We here provide the NIC for the previously mentioned GARCH models as the NIC will have a central role in the following section. As in Engle and Ng (1993), we refer to the GARCH as our benchmark, since it represent the most used specification in the literature (see, among many others, Bollerslev et al. 1992; Engle 1993; Hansen and Lunde 2005).

Let us denote by σ^2 the unconditional variance of ε_t, i.e. $\sigma^2 = E\left[\varepsilon_t^2\right] = E\left[h_t\right]$. Engle and Ng (1993) derive the NIC for the GARCH(1,1) model by replacing the lagged variance with the unconditional variance, and then let the NIC be a function of the shock only. Therefore, for the GARCH, the NIC equals:

$$NIC\left(\varepsilon_{t-1}\right) = \omega + \beta\sigma^2 + \alpha\varepsilon_{t-1}^2. \tag{8}$$

With a similar approach, several authors derive the NIC for other GARCH-type models. To report directly comparable NIC for several models, we always express the NIC in the variance scale, that is, the NIC monitors the impact of shocks on the variances, irrespective of the fact the dynamic is defined on the variances or on other powers of the conditional volatility. Table 1 collects the NIC for the models discussed in the previous subsection.

3. Asymmetry and leverage in GARCH models

According to Black (1976), the leverage effect is the negative correlation between the shocks on returns and the subsequent shocks on volatility. Consequently, after a negative returns shock we expect volatility to increase while after a positive shock on

[1] Engle and Ng (1993) do not explicitly specify the meaning of the letter V in the acronym of the VGARCH model.

Model	News Impact Curve
GARCH	$NIC\left(\varepsilon_{t-1}\right) = A_{(2,1)} + \alpha\varepsilon_{t-1}^{2}$
GJR-GARCH	$NIC\left(\varepsilon_{t-1}\right) = \begin{cases} A_{(2,1)} + \alpha\varepsilon_{t-1}^{2} & \varepsilon_{t-1} \geq 0, \\ A + \left(\alpha+\gamma\right)\varepsilon_{t-1}^{2} & \varepsilon_{t-1} < 0, \end{cases}$
EGARCH	$NIC\left(\varepsilon_{t-1}\right) = \begin{cases} B \times \exp\left[\frac{(\alpha+\kappa)}{\sigma}\varepsilon_{t-1}\right] & \varepsilon_{t-1} \geq 0, \\ B \times \exp\left[\frac{(\alpha-\kappa)}{\sigma}\varepsilon_{t-1}\right] & \varepsilon_{t-1} < 0, \end{cases}$
TARCH	$NIC\left(\varepsilon_{t-1}\right) = \begin{cases} A_{(1,2)} + 2A_{(1,1)}\left(\alpha+\gamma\right)\varepsilon_{t-1} + \left(\alpha+\gamma\right)^{2}\varepsilon_{t-1}^{2} & \varepsilon_{t-1} \geq 0, \\ A_{(1,2)} + 2A_{(1,1)}\left(\alpha-\gamma\right)\varepsilon_{t-1} + (\alpha-\gamma)^{2}\varepsilon_{t-1}^{2} & \varepsilon_{t-1} < 0, \end{cases}$
AGARCH	$NIC\left(\varepsilon_{t-1}\right) = A_{(2,1)} + \alpha\left(\varepsilon_{t-1} - \varphi\right)^{2}$
NGARCH	$NIC\left(\varepsilon_{t-1}\right) = A_{(2,1)} + \alpha\left(\varepsilon_{t-1} - \varphi\sigma\right)^{2}$
VGARCH	$NIC\left(\varepsilon_{t-1}\right) = A_{(2,1)} + \alpha\left(\frac{\varepsilon_{t-1}}{\sigma} - \varphi\right)^{2}$
APARCH	$NIC\left(\varepsilon_{t-1}\right) = \begin{cases} \left(A_{(\delta,1)} + \alpha\left(1-\gamma\right)^{\delta}\varepsilon_{t}^{\delta}\right)^{1/\delta} & \varepsilon_{t-1} \geq 0, \\ \left(A_{(\delta,1)} + \alpha\left(1+\gamma\right)^{\delta}\left(-\varepsilon_{t}\right)^{\delta}\right)^{1/\delta} & \varepsilon_{t-1} < 0. \end{cases}$

Table 1. News Impact Curves for selected GARCH models. To compress the notation we set $A_{(a,b)} = \left(\omega + \beta\sigma^{a}\right)^{b}$ and $B \equiv \sigma^{2\beta}\exp\left(\omega - \kappa E\left[|z_t|\right]\right)$.

returns we should observe a decrease in the volatility. This economic intuition represent the foundation for the existence of asymmetry in GARCH models, as stated in the works by Nelson (1991) and Glosten et al. (1993), and in all papers proposing GARCH-type models where the sign of the shocks has some relevance. Therefore, it might seem that leverage effect and asymmetry are two different names for the very same phenomenon. Even Engle and Ng (1993) use the two terms as synonyms, and end up with a preference for leverage only because it was "*...popular among researchers...*", see their footnote 1 at page 1752. We do not agree with such a viewpoint when focusing on the GARCH model class, and we prefer the interpretation of McAleer (2014) and Chang and McAleer (2017), where asymmetry and leverage are two distinct effects.

We highlight that the use of leverage as a synonym of asymmetry is quite common from an empirical viewpoint. Among others, we quote Ding et al. (1993), "*... The asymmetric response of volatility to positive and negative 'shocks' is well known in the finance literature as the leverage effect of the stock market returns [Black (1976)], which says that stock returns are negatively correlated with changes in return volatility - i.e. volatility tends to rise in response to 'bad news' (excess returns lower than expected) and to fall in response to 'good news' (excess returns higher than expected) [Nelson (1991)]. Empirical studies by Nelson (1991), Glosten, Jaganathan and Runkle (1989) and Engle and Ng (1992) show it is crucial to include the asymmetric term in financial time series models [for a detailed discussion, see Engle and Ng (1992)]).…*". It is clear that the leverage effect of Black (1976) is matched with the asymmetry of GARCH models. Further, we also refer to the work by Stavroyiannis (2017), that considers the disagreement in the GARCH literature and in specialized econometric software,[2] and analyses the constraints of the parameters which lead to the existence of asymmetry and leverage. The author shows that the approach used by software packages is not consistent with the Nelson-Cao inequality constraints for ensuring positivity of conditional variance. More recently, Engle and Siriwardane (2018) have introduced the Structural GARCH model where leverage is referred to the capital structure of a firm, a totally different concept with respect to the volatility asymmetry effect.
In this note we argument that the asymmetry and leverage effects within a GARCH model are not identical. To provide a complete characterization of both effect, we separately address the two, and propose to detect their presence by focusing on the News Impact Curve.

3.1. *Asymmetry and the News Impact Curve*

Building on the first proposals for GARCH models allowing for asymmetry (e.g. Nelson 1991; Glosten et al. 1993), a commonly accepted definition of asymmetry for GARCH models is the following:

Definition 1: *A GARCH model allows for asymmetry if positive and negative shocks of the same (absolute) size have a different* ***impact*** *on the variance (or volatility).*

We emphasize *impact* as, in our opinion, the interpretation of that word is crucial for the proper characterization of asymmetry. In fact, Chang and McAleer (2017) interpret

[2] The use of leverage and asymmetry as synonyms is also common in statistical and econometric software. See, among others, Matlab and Eviews.

impact in a literal (econometric) way, that is, by reading the coefficients of GARCH models as measures of impact of the right hand side variables on the left hand side variable. This clearly reminds the classical econometric interpretation of coefficients in linear equations, and lead to the use of derivatives to measure the impact of shocks.

However, such an interpretation lead to a clear incoherence for a reference model, the GARCH. Using the notation of the previous section, and applying the interpretation of Chang and McAleer (2017), the evaluation of asymmetry builds on the first order derivative of the conditional variance with respect to the shock ε_{t-1},

$$\frac{\partial \sigma_t^2}{\partial \varepsilon_{t-1}} = 2\alpha\varepsilon_{t-1}. \tag{9}$$

Therefore, according to the interpretation of Definition 1 given by Chang and McAleer (2017), the equality of the impact for positive and negative shocks implies that the derivatives for positive and negative shocks of the same (absolute) size must be identical.

Clearly, this is possible in the limiting case of $\alpha = 0$. However, under this case the model collapses to a constant (Francq and Zakoian 2011, Equation 2.46 at page 42). We thus rule out such a possibility. Assuming that $\alpha > 0$, and coherently with the constraints for variance positivity, symmetry is not possible (!) as the derivative depends on the shock. In fact, given two shocks, one positive and one negative, both with the same absolute size, and given the derivative evaluated at those two shocks, we will clearly obtain different values. For instance, for shocks of size $+1$ and -1, the derivatives will be 2α and -2α. Summarizing, under the characterization of asymmetry put forward by Chang and McAleer (2017), with a literal interpretation of Definition 1, the GARCH model seems to be always asymmetric (apart in the limiting case where we do not have heteroskedasticity). This is opposite to the common interpretation of the GARCH model as a symmetric specification. Consequently, if a problem is present, it must be in the definition adopted, or in the interpretation of the word *impact*.

We opt for an interpretation of the word *impact* in Definition 1 different from that in Chang and McAleer (2017). In fact, we believe that the it points at the change that shocks induce in the conditional variance or in the conditional volatility. To provide a clear characterization of asymmetry, we suggest to adopt a modified definition.

Definition 2: *A GARCH model allows for asymmetry if positive and negative shocks of the same size induce changes in the conditional variance (volatility) of different magnitude.*

Our definition is not a novel interpretation of the asymmetry, but rather a clarification of the interpretation of what asymmetry represents. On the one side, it is coherent with the leverage definition as in Black (1976) that also points at changes in the variance and not at the impact of shocks. On the other side, it is coherent with the commonly adopted interpretation of asymmetry. Finally, it is also coherent with the claim in Section 2.2 of McAleer (2014) stating that leverage is a special case of asymmetry.

If we move the focus toward the change that shocks induce on the conditional variances, our definition of asymmetry has a strong link with the NIC. In fact, according to our definition, the symmetry in the NIC (with respect to the Y axis) implies symmetry in the GARCH model. This is the same viewpoint adopted by Hentschel (1995) in discussing asymmetry in GARCH models.

Coming back to the GARCH example, the simple inspection of the NIC reported in Equation 8 allows verifying the presence of symmetry. In fact, positive and negative shocks of the same magnitude produce equal positive changes in the variances.

We might rationalize our previous comment with the following corollary to Definition 2, which exploits the fact that the NIC is a function of a single argument, the shock.

Corollary 1: *A GARCH model allows for asymmetry if, for all shocks θ, we have $NIC(\theta) \neq NIC(-\theta)$.*

By adopting Corollary 1, we can verify the conditions that model parameters must satisfy to have asymmetry.

For the GARCH we have $NIC(\theta) = A + \alpha\theta^2$. Consequently, $NIC(\theta) = NIC(-\theta)$ for all θ, and the GARCH is symmetric.

In the EGARCH model, the condition $NIC(\theta) = NIC(-\theta)$ requires $\frac{\alpha+\kappa}{\sigma}\theta = -\frac{\alpha-\kappa}{\sigma}\theta$. Therefore, the condition for asymmetry is that $\alpha \neq 0$. Note this is opposite to the claim of Chang and McAleer (2017) supporting the presence of asymmetry if $\kappa \neq 0$.[3]

Moving to the GJR-GARCH, $NIC(\theta) = NIC(-\theta)$ if and only if $\varphi \neq 0$. In this case, there is asymmetry if the coefficient associated with the sign of the shock is not null.

The TARCH models behaves as the EGARCH, and there is asymmetry if the coefficient multiplying the shock (not squared neither in absolute terms, thus maintaining the information on the sign) is not null. In fact, we have that $NIC(\theta) = NIC(-\theta)$ if $D(\alpha+\gamma)\theta + (\alpha+\gamma)^2\theta^2 = D(-\alpha+\gamma)\theta + (\alpha-\gamma)^2\theta^2$. The latter equality is satisfied for all θ if $\alpha = 0$, thus leading to symmetry. Note that there might exist specific parameter combinations satisfying the equality for specific values of θ.[4]

The AGARCH model is symmetric if $NIC(\theta) = NIC(-\theta)$ and thus when $(\theta-\varphi)^2 = (-\theta-\varphi)^2$. The latter is possible only when $\varphi = 0$. A similar condition applies to the NGARCH and VGARCH specifications.

Finally, in the APARCH model, the condition for symmetry $NIC(\theta) = NIC(-\theta)$ leads to the equality $(1-\gamma)^\delta\theta^\delta = (1+\gamma)^\delta\theta^\delta$ which is satisfied only when $\gamma = 0$. Therefore, the condition for asymmetry is $\gamma \neq 0$.

Results for higher order specifications can be derived with some algebra and might lead to increased flexibility in the conditions for asymmetry, even accounting for more flexible parameter bound for variance positivity (Nelson and Cao 1992). However, we believe the cases we report represent the vast majority of models that can be considered in empirical analyses.

[3]Chang and McAleer (2017) associate symmetry to equality of the first order derivative of the conditional variance equation with respect to the shock, evaluated for positive and negative shocks. Therefore, for the EGARCH model, the derivative equals $\frac{\partial \log h_t}{\partial z_{t-1}} = \alpha - sign(z_{t-1})\kappa$. Consequently, according to Chang and McAleer (2017), symmetry requires $\kappa = 0$.

[4]One can verify that by solving the equality imposing that parameters α and γ satisfy the conditions for positivity of conditional volatilities, but $\alpha < 0$. For instance by setting $\alpha = -\frac{1}{2\gamma}$.

3.2. *Leverage and the News Impact Curve*

As we already stated, we agree with McAleer (2014) that leverage should be used in accordance with the definition of Black (1976), while asymmetry represents a different phenomenon. In fact, if leverage is matched with a negative correlation between shocks and volatility, we expect to observe that positive shocks lead to a decrease of volatility and not an impact on volatility with a size smaller than the one of negative shocks. Therefore, we adopt the following characterization of leverage within a GARCH model.

Definition 3: *A GARCH model is coherent with the presence of the leverage effect if negative shocks lead to an increase in the variance while positive shocks lead to a decrease in the variance.*

The leverage effect is a special case of asymmetry as, under leverage, positive and negative shocks have by definition a different impact on the conditional variances. Note that, according to Definition 3, the leverage effect is not present in the GARCH model as positive and negative shocks have always a positive (i.e. increasing) effect on the conditional variances. This is a consequence of the positivity of the α parameter, commonly imposed within the sufficient conditions for variance positivity. If the model orders are larger than one, that is in a more general GARCH(P,Q), one should verify if, under the conditions of Nelson and Cao (1992), positive shocks are associated with decreases in the conditional variance. However, coherently with our choices in the case of asymmetry, we maintain the above specifications which are the most used in empirical analyses. In this regards, we verify the conditions for the existence of leverage effects in the other GARCH models described in the previous subsection.

We proceed by first noticing that, similarly to the case of asymmetry, the presence of leverage is also detectable by focusing on the NIC. According to Definition 3, it is now important to verify that the NIC decreases after a positive shock while it increases after a negative shock. Therefore, to verify the presence of leverage we suggest to analyse the behaviour of the NIC derivative with respect to the shock. The following Corollary gives a sufficient condition for leverage, according to our definition:

Corollary 2: *A GARCH models allows for leverage if* $\frac{\partial NIC(\varepsilon_{t-1})}{\partial \varepsilon_{t-1}} < 0$ *for all shocks* ε_{t-1}.

Corollary 2 provides a relatively simple condition to check, that is, the NIC is a monotone decreasing function of the shock. In the case of the GARCH model, the NIC is not a monotone decreasing function of the shock and the model does not allow for leverage. The NIC derivative equals $NIC'(\varepsilon_{t-1}) = 2\alpha\varepsilon_{t-1}$ and is positive for positive shocks (given that $\alpha \geq 0$).

For the GJR-GARCH model, the NIC not monotone decreasing in ε_{t-1} if $\alpha < 0$ and $\alpha + \gamma > 0$. As already noted by McAleer (2014), this is not coherent with the constraints for positivity of conditional variances. Therefore, the GJR-GARCH model does not allow for leverage.

The EGARCH model is more interesting. In fact, the sign of the NIC derivative depends on the derivative of the exponential arguments (see the EGARCH NIC in Table 1). The existence of leverage requires that $\frac{\alpha+\kappa}{\sigma} < 0$ and $\frac{\alpha-\kappa}{\sigma} < 0$. These conditions are satisfied if $\alpha < 0$ and $\alpha < \kappa < -\alpha$. Given that these conditions are not conflicting with usual parameters restrictions for covariance stationarity of the EGARCH, the model seems to allow for the presence of the leverage effect. However, as noted by McAleer (2014) and McAleer and Hafner (2014), in order to derive proper regularity conditions for the existence of the model, resorting to a more general Random Coefficient model, we must have both $\alpha > 0$ and $\kappa > 0$. Consequently, if we link the EGARCH to a random coefficient model, we are ruling out leverage.

For the TARCH model, the negativity conditions on the NIC derivative equal

$$\begin{cases} D(\alpha+\gamma) < -2(\alpha+\gamma)^2 \varepsilon_{t-1}, & \text{for } \varepsilon_{t-1} \geq 0, \\ D(\alpha-\gamma) < -2(\alpha-\gamma)^2 \varepsilon_{t-1}, & \text{for } \varepsilon_{t-1} < 0. \end{cases} \tag{10}$$

The positivity of the conditional variance is ensured by $\gamma > 0$ and $-\gamma < \alpha < \gamma$. Unlike the second condition, the first condition is never satisfied, due to the conditions for variance positivity. Consequently, the TARCH model does not allow for leverage.

Moving to the APARCH model, to allow for leverage, from the derivative of the NIC for positive shocks we can easily obtain the following condition

$$\frac{1}{\delta}\left(E + \alpha(1-\gamma)^{\delta} \varepsilon_{t-1}^{\delta}\right)^{\frac{1-\delta}{\delta}} \left(\delta\alpha(1-\gamma)^{\delta} \varepsilon_{t-1}^{\delta-1}\right) < 0, \quad \varepsilon_{t-1} > 0. \tag{11}$$

For a subset of all the possible values of δ (for even δ), the previous condition is satisfied if $\alpha < 0$ or $\gamma > 1$. However, both cases are ruled out by conditions needed to ensure the positivity of conditional variances. Similarly to the previous specifications, the model does not allow for leverage.

The most interesting case is the AGARCH model where the NIC derivative is equal to

$$NIC'(\varepsilon_{t-1}) = 2\alpha(\varepsilon_{t-1} - \varphi). \tag{12}$$

The derivative, though not monotone decreasing in ε_{t-1}, is anyway negative for $\varepsilon_{t-1} < \varphi$ when $\varphi > 0$. Therefore, we do have a decreasing NIC for positive shocks smaller than φ (when $\varphi > 0$). Similar cases realize for te NGARCH and the VGARCH models.

The AGARCH, NGARCH and VGARCH models are clearly peculiar cases, but they open the door for a more flexible characterization of the leverage effect. In this regard, we thus suggest the following definition:

Definition 4: *A GARCH model allows for local leverage if the* $\frac{\partial NIC(\varepsilon_{t-1})}{\partial \varepsilon_{t-1}} < 0$ *for a subset of* $\Theta = (0, +\infty)$ *where* Θ *is the positive fraction of the support of* ε_{t-1}.

According to Definition 4, the AGARCH model is thus consistent with local leverage.

Overall, all the GARCH models are not coherent with the presence of leverage effects while only the AGARCH and similar models allow for local leverage. In some cases, leverage might exist (see the EGARCH model) but associated with parameters sign and size not coherent with the proper derivation of the model from a more general random coefficient model. Local leverage might also exist for specifications with orders larger than one and for other GARCH models not discussed in this paper.

4. Empirical Analysis

We close the paper with an empirical analysis aiming at identifying the presence of asymmetry and/or leverage in the US market.[5] We consider the constituents of the S&P500 index at March 2018 and whose data are available from January 2003 at a daily frequency (426 assets). We estimate the models of order 1 described in the previous sections by considering, for comparison purposes, four specifications of the GARCH distribution errors:[6] the Normal, the Student's t, the GED and the Skew-t. Moreover, we perform the estimation in four sub-samples (2003-2006, 2007-2008, 2009-2012 and 2013-2018) for a total of $47,712$ estimated process. Table 4 reports the total number of assets classified by industry.[7]

Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
21	12	69	46	45	50	2	26	79	45	395

Table 2. Total number of estimated assets according by industry. The sectors are: Energy, Basic Materials, Industrials, Consumer Goods, Health Care, Consumer Services, Telecom, Utilities, Financials and Technology.

To compare the models, we select the most interesting period in terms of volatility which is represented by the Global Financial Crisis during the period (January)2007–(December)2008. We focus on the best performing models according to the various possible forms of innovation density as our interest is to compare asymmetry and leverage features of alternative specifications for the volatility equation.[8]

Table 3 reports the percentages of the best performing model during the crisis according to the BIC criteria and for a specific distribution for the errors. Regarding the Normal distribution, the best model for the total market (last column) and 5 sectors is the EGARCH model followed by the TARCH model (3 sectors). The results for the other distributions are pretty similar. In fact, the TARCH model results the best one for the market and 3 sectors (4 sectors for the SKEW-t) while the NGARCH is the second best model. Generally as expected, the models including asymmetry in the volatility perform better than the GARCH model. By contrasting results among different distributional hypotheses for the errors, we note that the results for the non-Gaussian cases are very close one to the other, and differ from those of the Normal case, a somewhat expected result.

We then evaluate the News Impact Curves building on the estimates coming from the Skew-t distribution.[9] Appendix B reports the descriptive statistics of the estimates. To compute the NICs, we set the parameters for the volatility equations equal to the mean of the estimates for the S&P500 market as reported in the second row of the last column in Tables B1-B4. Figure 1 reports News Impact Curves for the GARCH model (solid line) versus the (dashed line) GJR model (Upper-left panel),

[5]As our purpose is purely illustrative on the presence of asymmetry and/or leverage, we do not make model comparisons in terms of forecasts but only with respect to the in-sample fit.

[6]We do not include the VGARCH model here given its similarity with the AGARCH model in preliminary estimates.

[7]The full collection of assets includes 426 equities. However, we excluded from the analysis the assets (31) with problems of convergence in at least one of the estimated models. Hence, the final number of assets is equal to 395.

[8]For sake of completeness, we include in Appendix A the percentages of the best overall performing models for all the considered periods. A related study is that of Bampinas et al. (2018) which provides descriptive statistics on the conditional variances under alternative distributions estimated using the GARCH, GJR-GARCH and EGARCH models for the constituents of the S&P Composite 1500.

[9]The estimates of the parameters are very similar to those obtained by other distributions and are available upon request to the authors.

NORMAL

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	Market
GARCH	19.05%	8.33%	10.14%	13.04%	6.67%	6.00%	0.00%	7.69%	5.06%	8.89%	8.61%
GJR	0.00%	8.33%	5.80%	8.70%	8.89%	2.00%	**50.00%**	15.38%	11.39%	8.89%	8.10%
EGARCH	**33.33%**	8.33%	20.29%	**34.78%**	28.89%	**48.00%**	0.00%	0.00%	**31.65%**	**44.44%**	**30.38%**
TARCH	19.05%	**33.33%**	**28.99%**	23.91%	**31.11%**	22.00%	0.00%	19.23%	15.19%	20.00%	22.78%
AGARCH	0.00%	0.00%	4.35%	6.52%	8.89%	0.00%	0.00%	0.00%	6.33%	0.00%	3.80%
NGARCH	23.81%	**33.33%**	27.54%	10.87%	8.89%	20.00%	**50.00%**	**57.69%**	27.85%	13.33%	23.04%
APARCH	4.76%	8.33%	2.90%	2.17%	6.67%	2.00%	0.00%	0.00%	2.53%	4.44%	3.29%

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GARCH	28.57%	8.33%	13.04%	21.74%	8.89%	10.00%	0.00%	23.08%	17.72%	13.33%	15.44%
GJR	4.76%	8.33%	5.80%	4.35%	13.33%	4.00%	**50.00%**	19.23%	6.33%	11.11%	8.10%
EGARCH	14.29%	0.00%	17.39%	**36.96%**	22.22%	**32.00%**	0.00%	0.00%	**30.38%**	24.44%	23.54%
TARCH	14.29%	33.33%	**37.68%**	28.26%	**42.22%**	28.00%	0.00%	11.54%	12.66%	**35.56%**	**27.34%**
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%	0.00%	0.00%	2.53%	2.22%	1.01%
NGARCH	**38.10%**	**50.00%**	24.64%	6.52%	13.33%	24.00%	**50.00%**	**46.15%**	**30.38%**	11.11%	23.80%
APARCH	0.00%	0.00%	1.45%	2.17%	0.00%	0.00%	0.00%	0.00%	0.00%	2.22%	0.76%

GED

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	Market
GARCH	**28.57%**	16.67%	14.49%	19.57%	11.11%	10.00%	0.00%	19.23%	16.46%	15.56%	15.70%
GJR	4.76%	8.33%	7.25%	4.35%	11.11%	4.00%	**50.00%**	15.38%	7.59%	11.11%	8.10%
EGARCH	23.81%	0.00%	17.39%	28.26%	22.22%	**36.00%**	0.00%	0.00%	**32.91%**	28.89%	24.56%
TARCH	14.29%	33.33%	**34.78%**	**36.96%**	**42.22%**	32.00%	0.00%	11.54%	11.39%	**33.33%**	**27.85%**
AGARCH	0.00%	0.00%	1.45%	2.17%	0.00%	0.00%	0.00%	0.00%	2.53%	0.00%	1.01%
NGARCH	**28.57%**	**41.67%**	24.64%	8.70%	13.33%	18.00%	**50.00%**	**53.85%**	29.11%	8.89%	22.53%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.22%	0.25%

SKEW-t

GARCH	**38.10%**	8.33%	11.59%	23.91%	8.89%	12.00%	0.00%	34.62%	16.46%	13.33%	16.71%
GJR	4.76%	8.33%	5.80%	4.35%	13.33%	4.00%	**50.00%**	15.38%	6.33%	11.11%	7.85%
EGARCH	14.29%	8.33%	17.39%	**34.78%**	22.22%	**34.00%**	0.00%	0.00%	30.38%	20.00%	23.29%
TARCH	14.29%	33.33%	**40.58%**	28.26%	**42.22%**	28.00%	0.00%	11.54%	11.39%	**35.56%**	**27.59%**
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%	0.00%	0.00%	2.53%	0.00%	0.76%
NGARCH	28.57%	**41.67%**	23.19%	6.52%	13.33%	20.00%	**50.00%**	**38.46%**	**32.91%**	17.78%	23.04%
APARCH	0.00%	0.00%	1.45%	2.17%	0.00%	0.00%	0.00%	0.00%	0.00%	2.22%	0.76%

Table 3. Percentages of the best performing models using the BIC criteria for industry and the total market (last column) according to the type of distribution errors.

EGARCH (Upper-right panel), TARCH (middle-left panel), AGARCH (middle-right) and APARCH (lower panel). We report the NICs on the absolute values of ε_{t-1} in order to highlight the asymmetry. The NICs report the impact of the positive (negative) shocks as described by a dashed blue (red) line.[10] Clearly, the asymmetry is due to a greater effect of negative shocks on the increase of the variance with respect to the positive shocks. This asymmetry is more pronounced for the EGARCH model and less pronounced for the AGARCH model.

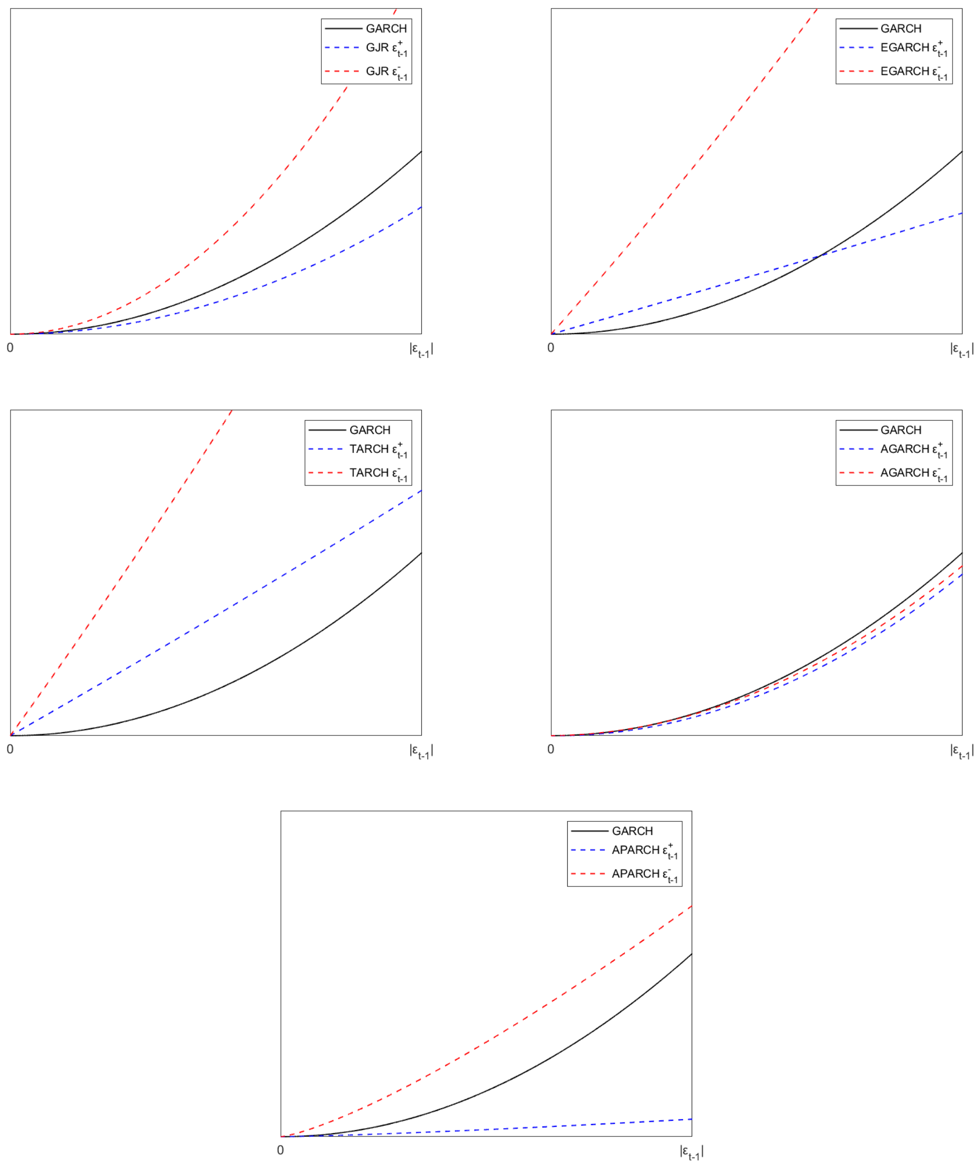

Figure 1. News Impact Curves for the GARCH model (solid line) versus the (dashed line) GJR model (Upper-left panel), EGARCH (Upper-right panel), TARCH (Middle-left panel), AGARCH (Middle-right) and APARCH (Lower-left panel).
Note: We report the NICs on the absolute values of ε_{t-1} in order to highlight the asymmetry and thus, the impact of the positive (negative) shocks which is described in the dashed blue (red) lines.

[10] We standardized the NICs for comparison purposes and hence the NICs of the AGARCH and NGARCH models are equivalents.

Finally, we report the percentages of assets showing asymmetry and leverage in Table 4 and Table 5 for each model and error distributions in all the four considered periods, respectively. Given the error distributions, the percentages do not substantially differ across models. Except for the APARCH model, the asymmetry showed in Table 4 results the highest for all the models during the period 2009-2012 which coincides with the end of the global crisis and the recovery of the US financial market. The EGARCH specification shows the highest percentage of asymmetry in all the periods, followed by the GJR-GARCH and the AGARCH specification. Regarding the leverage reported in Table 5, the percentage of leverage is very low the EGARCH specification for all the considered periods (close to $3-4\%$ in the first two subsamples and almost below 1% in the last two subsamples). The local leverage in the AGARCH model shows the highest percentage during the period 2009-2012 (above 70%) and the lowest percentage in the pre-crisis period (close to 30%). Overall, according to our definitions, asymmetry is a diffuse phenomena while the leverage appears only in limited number of cases. On the contrary, if we focus on the AGARCH model, we note that in a large fraction of cases there is an empirical evidence in favor of local leverage.

5. Conclusion

Starting from recent contributions on the proper parameter constraints associated with asymmetry and leverage in GARCH models, we propose here a unique treatment for these two effects. We clarify that, in our view, leverage and asymmetry are distinct phenomena, even if both originated by the seminal work of Black (1976).

In this regard, we propose definitions of leverage and asymmetry that build at the News Impact Curve and provide examples on how to verify these two definitions with selected GARCH models.

We show that several GARCH models are asymmetric but none is capable of showing leverage effects. We then extend the leverage definition to allow for local leverage effect, and show that the AGARCH model of (Engle 1990) is coherent with the presence of local leverage.

Finally, an empirical analysis on the S&P 500 markets identifies a diffused presence of asymmetry, and if we focus on AGARCH model estimates, a relevant occurrence of local leverage.

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	EGARCH	GJR-GARCH	TARCH	AGARCH	APARCH
asymmetry	$\alpha \neq 0$	$\gamma \neq 0$	$\alpha \neq 0$	$\gamma \neq 0$	$\gamma \neq 0$
			2003-2006		
Normal	78.23%	35.19%	39.75%	39.75%	43.04%
Student's	71.90%	24.81%	29.62%	31.65%	27.85%
GED	62.78%	25.06%	29.62%	32.91%	34.94%
SKEW-t	69.37%	23.29%	28.10%	28.86%	28.35%
			2007-2008		
Normal	88.35%	72.15%	35.19%	66.33%	62.28%
Student's	81.77%	62.03%	22.03%	62.53%	57.47%
GED	80.25%	64.81%	19.49%	60.76%	56.71%
SKEW-t	80.51%	60.51%	21.27%	62.28%	56.46%
			2009-2012		
Normal	94.43%	78.99%	48.35%	75.44%	62.78%
Student's	97.22%	75.70%	34.18%	74.43%	57.22%
GED	91.39%	73.92%	34.68%	74.18%	52.41%
SKEW-t	96.20%	75.95%	33.16%	74.94%	56.71%
			2013-2018		
Normal	92.66%	63.54%	43.54%	63.54%	63.29%
Student's	94.18%	60.25%	32.15%	65.06%	67.85%
GED	88.10%	55.19%	34.18%	58.99%	59.24%
SKEW-t	93.67%	62.03%	31.90%	64.30%	69.11%

Table 4. Percentages of assets in the market showing asymmetry under the different error distributions according to the type of models for the periods 2003-2006, 2007-2008, 2009-2012 and 2013-2018. The first row reports the asymmetry conditions discussed in Section 3.

	EGARCH			
leverage	$\alpha < 0$ and $\alpha < \kappa < -\alpha$			
	2003-2006	*2007-2008*	*2009-2012*	*2013-2018*
Normal	3.80%	5.32%	1.52%	0.76%
Student's	3.54%	4.05%	0.25%	0.51%
GED	2.78%	3.29%	1.27%	0.51%
SKEW-t	3.80%	3.29%	0.25%	0.51%
	AGARCH			
local leverage	$\gamma \neq 0$			
Normal	34.43%	65.06%	74.94%	58.73%
Student's	29.87%	62.53%	73.92%	64.56%
GED	29.62%	60.76%	74.18%	58.48%
SKEW-t	26.58%	62.03%	74.94%	64.30%

Table 5. Percentages of assets in the market showing leverage under the different error distributions according to the type of models for the periods 2003-2006, 2007-2008, 2009-2012 and 2013-2018. The first row in EGARCH (AGARCH) model reports the leverage (local leverage) condition discussed in Section 3.

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Appendix A. Model selection in the four considered periods according to the BIC criteria

In this section, we include the percentages of the best overall performing models during the four considered periods (2003-2006, 2007-2008, 2009-2012,2013-2018) by including all the specified distribution errors. The selection is done according to BIC criteria for each industry and for the total market (last column).

NORMAL

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	19.05%	0.00%	0.00%	0.00%	0.00%	2.00%	0.00%	7.69%	0.00%	0.00%	1.77%
GJR	9.52%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.51%
EGARCH	4.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.85%	1.27%	0.00%	0.76%
TARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AGARCH	4.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%
NGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

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	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	**33.33%**	16.67%	15.94%	**28.26%**	22.22%	16.00%	**100.00%**	**61.54%**	**36.71%**	**37.78%**	**29.11%**
GJR	4.76%	0.00%	2.90%	0.00%	4.44%	4.00%	0.00%	0.00%	1.27%	2.22%	2.28%
EGARCH	0.00%	**33.33%**	**33.33%**	23.91%	**33.33%**	**32.00%**	0.00%	19.23%	18.99%	35.56%	26.58%
TARCH	4.76%	16.67%	20.29%	13.04%	15.56%	12.00%	0.00%	0.00%	6.33%	6.67%	11.14%
AGARCH	0.00%	0.00%	1.45%	0.00%	0.00%	2.00%	0.00%	0.00%	0.00%	0.00%	0.51%
NGARCH	4.76%	8.33%	10.14%	0.00%	6.67%	0.00%	0.00%	3.85%	3.80%	0.00%	4.05%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

SKEWT

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	9.52%	8.33%	2.90%	6.52%	0.00%	0.00%	0.00%	0.00%	7.59%	0.00%	3.54%
GJR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.85%	1.27%	0.00%	0.51%
EGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.25%
TARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AGARCH	0.00%	8.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%
NGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.25%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

GED

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	Market
GARCH	0.00%	0.00%	4.35%	13.04%	8.89%	2.00%	0.00%	0.00%	11.39%	8.89%	6.84%
GJR	0.00%	0.00%	0.00%	0.00%	0.00%	8.00%	0.00%	0.00%	0.00%	0.00%	1.01%
EGARCH	4.76%	0.00%	4.35%	8.70%	0.00%	14.00%	0.00%	0.00%	3.80%	6.67%	5.32%
TARCH	0.00%	0.00%	2.90%	6.52%	8.89%	6.00%	0.00%	0.00%	2.53%	2.22%	3.80%
AGARCH	0.00%	8.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.51%
NGARCH	0.00%	0.00%	1.45%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.51%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%	0.00%	0.00%	0.00%	0.00%	0.25%

Table A1. Percentages of the best overall performing models during 2003-2006 by including all the specified distribution errors. The selection is done according to BIC criteria for each industry and for the total market (last column) .

NORMAL

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	14.29%	0.00%	1.45%	4.35%	0.00%	2.00%	0.00%	0.00%	1.27%	0.00%	2.03%
GJR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.85%	0.00%	0.00%	0.25%
EGARCH	**23.81%**	0.00%	2.90%	2.17%	2.22%	8.00%	0.00%	0.00%	1.27%	0.00%	3.54%
TARCH	0.00%	0.00%	2.90%	2.17%	0.00%	2.00%	0.00%	0.00%	2.53%	0.00%	1.52%
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.25%
NGARCH	9.52%	8.33%	4.35%	2.17%	2.22%	6.00%	0.00%	15.38%	10.13%	0.00%	5.82%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

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	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	4.76%	8.33%	10.14%	10.87%	8.89%	4.00%	0.00%	7.69%	11.39%	4.44%	8.35%
GJR	0.00%	8.33%	1.45%	2.17%	11.11%	4.00%	0.00%	11.54%	6.33%	11.11%	5.82%
EGARCH	4.76%	0.00%	17.39%	26.09%	15.56%	16.00%	0.00%	0.00%	**20.25%**	20.00%	16.46%
TARCH	9.52%	**33.33%**	**28.99%**	**21.74%**	**37.78%**	**28.00%**	0.00%	0.00%	10.13%	**35.56%**	**23.04%**
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.22%	0.25%
NGARCH	9.52%	25.00%	15.94%	4.35%	4.44%	14.00%	50.00%	7.69%	15.19%	8.89%	11.65%
APARCH	0.00%	0.00%	1.45%	2.17%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.51%

SKEWT

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	0.00%	0.00%	1.45%	6.52%	0.00%	2.00%	0.00%	11.54%	2.53%	11.11%	3.80%
GJR	0.00%	0.00%	1.45%	0.00%	2.22%	0.00%	50.00%	0.00%	0.00%	0.00%	0.76%
EGARCH	4.76%	0.00%	0.00%	6.52%	2.22%	8.00%	0.00%	0.00%	8.86%	4.44%	4.56%
TARCH	4.76%	0.00%	4.35%	2.17%	0.00%	0.00%	0.00%	7.69%	0.00%	0.00%	1.77%
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.25%
NGARCH	0.00%	8.33%	2.90%	0.00%	6.67%	0.00%	0.00%	**23.08%**	3.80%	0.00%	3.80%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

GED

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	Market
GARCH	4.76%	0.00%	0.00%	0.00%	0.00%	2.00%	0.00%	7.69%	2.53%	0.00%	1.52%
GJR	4.76%	0.00%	0.00%	2.17%	0.00%	0.00%	0.00%	3.85%	0.00%	0.00%	0.76%
EGARCH	0.00%	0.00%	0.00%	2.17%	4.44%	4.00%	0.00%	0.00%	0.00%	0.00%	1.27%
TARCH	0.00%	0.00%	2.90%	2.17%	2.22%	0.00%	0.00%	0.00%	0.00%	0.00%	1.01%
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
NGARCH	4.76%	8.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.76%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.22%	0.25%

Table A2. Percentages of the best overall performing models during 2007-2008 by including all the specified distribution errors. The selection is done according to BIC criteria for each industry and for the total market (last column) .

NORMAL

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.85%	3.80%	0.00%	1.01%
GJR	4.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	7.69%	3.80%	2.22%	1.77%
EGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
TARCH	0.00%	0.00%	1.45%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
NGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.53%	0.00%	0.51%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

STUDENT'S t

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	14.29%	8.33%	2.90%	10.87%	13.33%	14.00%	0.00%	38.46%	21.52%	8.89%	13.92%
GJR	**42.86%**	**25.00%**	17.39%	6.52%	8.89%	16.00%	0.00%	**19.23%**	8.86%	13.33%	14.43%
EGARCH	0.00%	16.67%	17.39%	15.22%	**28.89%**	**20.00%**	0.00%	0.00%	5.06%	26.67%	15.19%
TARCH	14.29%	8.33%	**18.84%**	**36.96%**	20.00%	18.00%	0.00%	11.54%	6.33%	**31.11%**	**18.73%**
AGARCH	4.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%
NGARCH	4.76%	16.67%	26.09%	13.04%	15.56%	18.00%	50.00%	7.69%	**24.05%**	13.33%	17.97%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

SKEWT

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
GARCH	4.76%	0.00%	0.00%	2.17%	0.00%	0.00%	0.00%	0.00%	6.33%	0.00%	1.77%
GJR	0.00%	0.00%	2.90%	4.35%	0.00%	2.00%	0.00%	3.85%	10.13%	0.00%	3.54%
EGARCH	0.00%	0.00%	1.45%	2.17%	0.00%	0.00%	0.00%	0.00%	0.00%	2.22%	0.76%
TARCH	4.76%	0.00%	5.80%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%
AGARCH	4.76%	0.00%	0.00%	0.00%	0.00%	2.00%	0.00%	0.00%	1.27%	0.00%	0.76%
NGARCH	0.00%	8.33%	5.80%	2.17%	0.00%	0.00%	50.00%	7.69%	5.06%	0.00%	3.29%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

GED

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	Market
GARCH	0.00%	0.00%	0.00%	4.35%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.51%
GJR	0.00%	0.00%	0.00%	2.17%	0.00%	6.00%	0.00%	0.00%	1.27%	0.00%	1.27%
EGARCH	0.00%	8.33%	0.00%	0.00%	6.67%	4.00%	0.00%	0.00%	0.00%	2.22%	1.77%
TARCH	0.00%	8.33%	0.00%	0.00%	6.67%	0.00%	0.00%	0.00%	0.00%	0.00%	1.01%
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
NGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table A3. Percentages of the best overall performing models during 2009-2012 by including all the specified distribution errors. The selection is done according to BIC criteria for each industry and for the total market (last column) .

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
NORMAL											
GARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
GJR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
EGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
TARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
NGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
STUDENT'S t											
GARCH	4.76%	0.00%	8.70%	8.70%	2.22%	16.00%	0.00%	30.77%	1.27%	6.67%	8.10%
GJR	19.05%	0.00%	2.90%	0.00%	4.44%	0.00%	0.00%	0.00%	5.06%	0.00%	3.04%
EGARCH	14.29%	25.00%	7.25%	19.57%	24.44%	24.00%	0.00%	0.00%	7.59%	15.56%	14.18%
TARCH	**19.05%**	**41.67%**	**62.32%**	**47.83%**	**55.56%**	**48.00%**	**50.00%**	3.85%	**26.58%**	**57.78%**	**43.54%**
AGARCH	0.00%	0.00%	2.90%	0.00%	0.00%	0.00%	0.00%	0.00%	5.06%	0.00%	1.52%
NGARCH	28.57%	0.00%	5.80%	2.17%	2.22%	4.00%	0.00%	3.85%	26.58%	2.22%	9.37%
APARCH	0.00%	8.33%	2.90%	4.35%	0.00%	2.00%	0.00%	0.00%	0.00%	0.00%	1.52%
SKEWT											
GARCH	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%	0.00%	3.85%	1.27%	0.00%	0.76%
GJR	4.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.51%
EGARCH	0.00%	0.00%	0.00%	2.17%	2.22%	0.00%	0.00%	0.00%	0.00%	0.00%	0.51%
TARCH	4.76%	8.33%	2.90%	2.17%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	1.52%
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
NGARCH	0.00%	0.00%	0.00%	2.17%	0.00%	0.00%	0.00%	0.00%	2.53%	0.00%	0.76%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

GED

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	Market
GARCH	0.00%	0.00%	0.00%	4.35%	2.22%	0.00%	50.00%	50.00%	7.59%	0.00%	5.82%
GJR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
EGARCH	4.76%	0.00%	0.00%	2.17%	0.00%	2.00%	0.00%	3.85%	3.80%	4.44%	2.28%
TARCH	0.00%	16.67%	4.35%	4.35%	4.44%	2.00%	0.00%	0.00%	7.59%	13.33%	5.57%
AGARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
NGARCH	0.00%	0.00%	0.00%	0.00%	2.22%	0.00%	0.00%	3.85%	2.53%	0.00%	1.01%
APARCH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table A4. Percentages of the best overall performing models during 2013-2018 by including all the specified distribution errors. The selection is done according to BIC criteria for each industry and for the total market (last column) .

Appendix B. Estimates of the GARCH models

In this Appendix, we include the estimates for the Skew-t distribution. Tables B1-B4 report the descriptive statistics for the parameters of the volatility equations while Tables B5-B7 report the descriptive statistics for the parameters of the Skew-t distribution for the considered models.

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
						GARCH					
						α					
%sign	90.48%	83.33%	75.36%	56.52%	62.22%	64.00%	100.00%	100.00%	88.61%	82.22%	76.46%
mean	0.0943	0.0994	0.0935	0.1102	0.1392	0.1199	0.1063	0.1389	0.1227	0.0899	0.1125
std	0.0289	0.0319	0.0515	0.0459	0.0739	0.0695	0.0025	0.0287	0.0421	0.0639	0.0545
						β					
%sign	100.00%	91.67%	100.00%	100.00%	97.78%	98.00%	100.00%	100.00%	100.00%	100.00%	99.24%
mean	0.9010	0.9024	0.9068	0.9053	0.8503	0.8916	0.8741	0.8329	0.8821	0.9048	0.8877
std	0.0344	0.0415	0.0553	0.0557	0.0998	0.0958	0.0050	0.0362	0.0455	0.0797	0.0703
						GJR					
						α					
%sign	9.52%	8.33%	10.14%	15.22%	4.44%	6.00%	0.00%	11.54%	18.99%	0.00%	10.13%
mean	0.0663	0.0813	0.0641	0.0961	0.0000	0.0427	-	0.0719	0.0968	-	0.0783
std	0.0002	0.0000	0.0559	0.0528	0.0000	0.0371	-	0.0628	0.0580	-	0.0551
						γ					
%sign	57.14%	66.67%	62.32%	43.48%	64.44%	52.00%	100.00%	76.92%	73.42%	46.67%	60.51%
mean	0.1170	0.1671	0.1386	0.1480	0.1973	0.1430	0.1121	0.1393	0.1497	0.1300	0.1486
std	0.0453	0.0181	0.0577	0.0613	0.0964	0.0475	0.0272	0.0298	0.0563	0.0475	0.0614
						β					
%sign	100.00%	100.00%	98.55%	95.65%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	99.24%
mean	0.9076	0.8990	0.9120	0.9027	0.8545	0.9113	0.9151	0.8511	0.8895	0.9202	0.8960
std	0.0405	0.0465	0.0450	0.0597	0.0822	0.0563	0.0123	0.0364	0.0434	0.0443	0.0568

Table B1. Summary statistics for the estimates of the GARCH and GJR models in the S&P 500 market. The table reports the percentage of significant coefficients (first row), the mean of the coefficients (second row) and the standard deviation of the coefficients (third row) over the sample period from January 2007 to December 2008 (daily frequency).

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
EGARCH											
α											
%sign	76.19%	91.67%	73.91%	78.26%	82.22%	68.00%	100.00%	96.15%	92.41%	73.33%	80.51%
mean	0.1559	0.1571	0.1354	0.1608	0.1568	0.1442	0.1395	0.2090	0.2112	0.1140	0.1645
std	0.0585	0.0636	0.0657	0.0907	0.0828	0.1068	0.0108	0.0497	0.0635	0.0885	0.0828
κ											
%sign	66.67%	91.67%	79.71%	63.04%	82.22%	80.00%	100.00%	80.77%	81.01%	71.11%	77.22%
mean	-0.0830	-0.0926	-0.1022	-0.0928	-0.1080	-0.0961	-0.0750	-0.0888	-0.1010	-0.0944	-0.0978
std	0.0371	0.0158	0.0351	0.0303	0.0387	0.0357	0.0170	0.0157	0.0409	0.0361	0.0354
β											
%sign	95.24%	100.00%	100.00%	97.83%	100.00%	100.00%	100.00%	100.00%	100.00%	97.78%	99.24%
mean	0.9833	0.9810	0.9776	0.9795	0.9686	0.9771	0.9812	0.9673	0.9844	0.9815	0.9782
std	0.0106	0.0089	0.0196	0.0150	0.0240	0.0381	0.0039	0.0089	0.0092	0.0172	0.0207
TARCH											
α											
%sign	33.33%	16.67%	8.70%	32.61%	17.78%	10.00%	0.00%	42.31%	34.18%	6.67%	21.27%
mean	0.0705	0.0847	0.0717	0.0835	0.0423	0.0866	-	0.0843	0.0880	0.0890	0.0796
std	0.0179	0.0419	0.0585	0.0354	0.0613	0.0770	-	0.0324	0.0424	0.0302	0.0442
γ											
%sign	66.67%	83.33%	85.51%	71.74%	80.00%	76.00%	100.00%	92.31%	78.48%	77.78%	79.24%
mean	0.1011	0.1197	0.1163	0.1074	0.1353	0.1112	0.0917	0.1072	0.1183	0.0977	0.1138
std	0.0304	0.0179	0.0379	0.0325	0.0403	0.0314	0.0150	0.0198	0.0433	0.0274	0.0362
β											
%sign	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	98.73%	100.00%	99.75%
mean	0.9109	0.9106	0.9188	0.9089	0.8972	0.9203	0.9207	0.8764	0.8913	0.9272	0.9074
std	0.0268	0.0314	0.0302	0.0383	0.0388	0.0438	0.0090	0.0301	0.0308	0.0287	0.0368

Table B2. Summary statistics for the estimates of the GARCH and GJR models in the S&P 500 market. The table reports the percentage of significant coefficients (first row), the mean of the coefficients (second row) and the standard deviation of the coefficients (third row) over the sample period from January 2007 to December 2008 (daily frequency).

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
AGARCH											
α											
%sign	85.71%	91.67%	82.61%	67.39%	71.11%	78.00%	100.00%	96.15%	91.14%	82.22%	82.03%
mean	0.0886	0.0971	0.0880	0.1040	0.1078	0.1070	0.0716	0.1185	0.1177	0.0787	0.1019
std	0.0244	0.0345	0.0493	0.0454	0.0573	0.0816	0.0125	0.0297	0.0427	0.0607	0.0534
φ											
%sign	47.62%	91.67%	76.81%	54.35%	60.00%	60.00%	50.00%	34.62%	67.09%	60.00%	62.28%
mean	0.0142	0.0134	0.0136	0.0103	0.0153	0.0115	0.0125	0.0090	0.0096	0.0176	0.0126
std	0.0066	0.0066	0.0040	0.0043	0.0067	0.0032	0.0000	0.0037	0.0031	0.0065	0.0053
β											
%sign	100.00%	100.00%	100.00%	97.83%	95.56%	94.00%	100.00%	100.00%	98.73%	97.78%	97.97%
mean	0.9019	0.8949	0.8983	0.9001	0.8672	0.8919	0.9043	0.8463	0.8775	0.9097	0.8880
std	0.0359	0.0413	0.0514	0.0558	0.0717	0.0892	0.0117	0.0340	0.0416	0.0767	0.0617
NGARCH											
α											
%sign	80.95%	91.67%	73.91%	78.26%	64.44%	70.00%	100.00%	96.15%	91.14%	75.56%	78.99%
mean	0.0916	0.0750	0.0690	0.0832	0.0871	0.0917	0.0600	0.1095	0.1008	0.0635	0.0862
std	0.0325	0.0390	0.0466	0.0496	0.0596	0.0887	0.0115	0.0309	0.0398	0.0733	0.0559
φ											
%sign	38.10%	91.67%	60.87%	52.17%	44.44%	56.00%	50.00%	53.85%	55.70%	40.00%	53.16%
mean	0.7041	1.0663	1.2171	1.0599	1.2115	1.1513	1.0383	0.5996	0.7067	1.2103	1.0129
std	0.3702	0.4028	0.3692	0.5124	0.4010	0.4524	0.0000	0.1768	0.3380	0.3554	0.4451
β											
%sign	95.24%	100.00%	100.00%	97.83%	95.56%	94.00%	100.00%	100.00%	98.73%	95.56%	97.47%
mean	0.8738	0.8539	0.8602	0.8721	0.8198	0.8577	0.8798	0.8269	0.8569	0.8751	0.8561
std	0.0523	0.0431	0.0571	0.0597	0.0807	0.0827	0.0047	0.0318	0.0558	0.0855	0.0670

Table B3. Summary statistics for the estimates of the GARCH and GJR models in the S&P 500 market. The table reports the percentage of significant coefficients (first row), the mean of the coefficients (second row) and the standard deviation of the coefficients (third row) over the sample period from January 2007 to December 2008 (daily frequency).

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
						APARCH					
						α					
%sign	52.38%	83.33%	76.81%	73.91%	73.33%	66.00%	50.00%	53.85%	68.35%	64.44%	68.86%
mean	0.0967	0.0812	0.0745	0.0944	0.0919	0.0797	0.0704	0.1198	0.1051	0.0691	0.0887
std	0.0281	0.0294	0.0259	0.0364	0.0353	0.0394	0.0000	0.0217	0.0408	0.0219	0.0358
						γ					
%sign	33.33%	75.00%	63.77%	56.52%	68.89%	52.00%	0.00%	26.92%	59.49%	57.78%	56.46%
mean	-0.7617	-0.8142	-0.8469	-0.7758	-0.7540	-0.8427	-	-0.4801	-0.6101	-0.8458	-0.7597
std	0.2252	0.2410	0.1970	0.2363	0.2663	0.2162	-	0.2654	0.2711	0.2065	0.2541
						β					
%sign	100.00%	100.00%	97.10%	100.00%	97.78%	100.00%	100.00%	92.31%	98.73%	100.00%	98.48%
mean	0.9145	0.9148	0.9216	0.9148	0.9012	0.9233	0.9191	0.8668	0.8984	0.9266	0.9106
std	0.0364	0.0338	0.0349	0.0445	0.0468	0.0464	0.0022	0.0277	0.0370	0.0344	0.0417
						δ					
%sign	71.43%	83.33%	76.81%	54.35%	84.44%	72.00%	100.00%	69.23%	63.29%	71.11%	70.63%
mean	1.5925	1.2966	1.1285	1.0889	1.2298	1.2850	1.8308	1.7848	1.4940	1.1514	1.3054
std	0.7025	0.7550	0.6253	0.4372	0.6834	0.5605	0.4959	0.8529	0.7362	0.5195	0.6687

Table B4. Summary statistics for the estimates of the GARCH and GJR models in the S&P 500 market. The table reports the percentage of significant coefficients (first row), the mean of the coefficients (second row) and the standard deviation of the coefficients (third row) over the sample period from January 2007 to December 2008 (daily frequency).

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
						GARCH					
						v					
%sign	61.90%	75.00%	94.20%	93.48%	95.56%	92.00%	100.00%	88.46%	91.14%	100.00%	91.39%
mean	8.8709	5.7794	84.4873	5.8807	4.4037	6.0833	8.6935	7.0681	7.0247	5.5086	20.2623
std	2.4919	1.6649	633.9476	2.2757	1.2487	2.2925	0.2742	1.9256	2.4707	1.4779	268.9976
						λ					
%sign	47.62%	25.00%	10.14%	19.57%	15.56%	12.00%	0.00%	42.31%	7.59%	8.89%	15.95%
mean	-0.1828	-0.1401	-0.1341	0.0297	-0.0493	0.1485	-	-0.1578	0.0247	-0.0842	-0.0682
std	0.0613	0.0432	0.0245	0.1419	0.1238	0.0304	-	0.0221	0.1923	0.1429	0.1413

Table B5. Summary statistics for the estimates of the Skew-t distribution for GARCH and GJR models in the S&P 500 market. The table reports the percentage of significant coefficients (first row), the mean of the coefficients (second row) and the standard deviation of the coefficients (third row) over the sample period from January 2007 to December 2008 (daily frequency).

	GJR										
	υ										
%sign	52.38%	83.33%	92.75%	84.78%	97.78%	84.00%	100.00%	76.92%	89.87%	97.78%	87.85%
mean	27.1752	6.0330	45.5862	5.9820	4.7875	6.4200	10.1153	7.0527	13.4054	5.8576	15.4500
std	61.1561	1.9701	314.0549	1.9570	1.6457	2.4330	2.1629	1.3898	47.6749	1.6928	136.9485
	λ										
%sign	47.62%	25.00%	7.25%	15.22%	15.56%	16.00%	0.00%	34.62%	6.33%	11.11%	14.94%
mean	-0.1810	-0.1555	-0.1430	0.0022	-0.0224	0.1462	-	-0.1559	0.0382	-0.0882	-0.0613
std	0.0644	0.0516	0.0229	0.1601	0.1429	0.0279	-	0.0235	0.2130	0.1275	0.1496
	EGARCH										
	υ										
%sign	57.14%	83.33%	89.86%	89.13%	93.33%	88.00%	100.00%	76.92%	88.61%	100.00%	88.10%
mean	173.2309	6.1039	266.0603	7.1517	4.6798	6.4230	8.8784	6.9414	381.7237	5.8912	133.7652
std	514.2273	2.1422	2045.6878	6.6109	1.4533	2.2747	1.1997	1.2887	2730.5930	1.5273	1500.6143
	λ										
%sign	33.33%	25.00%	7.25%	19.57%	15.56%	16.00%	0.00%	30.77%	12.66%	11.11%	15.70%
mean	-0.1872	-0.1492	-0.1490	0.0019	-0.0339	0.1217	-	-0.1582	0.0075	-0.0883	-0.0545
std	0.0615	0.0502	0.1194	0.1394	0.1496	0.0323	-	0.0208	0.1579	0.1276	0.1461
	TARCH										
	υ										
%sign	57.14%	83.33%	95.65%	89.13%	97.78%	90.00%	100.00%	76.92%	84.81%	100.00%	89.11%
mean	147.9077	6.2605	63.8541	6.5916	4.9425	6.6860	8.9915	7.3005	84.6279	5.9458	36.7673
std	480.7024	2.0309	464.8443	2.6504	1.7156	2.3882	1.0555	1.4771	449.1295	1.5855	294.5126
	λ										
%sign	38.10%	25.00%	8.70%	23.91%	13.33%	16.00%	0.00%	34.62%	13.92%	11.11%	16.96%
mean	-0.1805	-0.1526	-0.1004	0.0132	-0.0095	0.1489	-	-0.1545	-0.0109	-0.0875	-0.0473
std	0.0625	0.0444	0.0511	0.1478	0.1212	0.0241	-	0.0207	0.1974	0.1237	0.1392

Table B6. Summary statistics for the estimates of the Skew-t distribution for GARCH and GJR models in the S&P 500 market. The table reports the percentage of significant coefficients (first row), the mean of the coefficients (second row) and the standard deviation of the coefficients (third row) over the sample period from January 2007 to December 2008 (daily frequency).

	Energy	Bas-Mat.	Ind.	Cons-G.	Health-C.	Cons-Serv.	Tlc	Utilities	Fin.	Tech.	All
AGARCH											
v											
%sign	61.90%	83.33%	94.20%	89.13%	93.33%	86.00%	100.00%	76.92%	84.81%	95.56%	87.59%
mean	20.6577	5.9854	108.0032	6.1066	4.5912	6.1681	8.7914	7.0610	15.0041	5.7450	27.3646
std	41.5722	2.0892	820.7029	2.3500	1.5144	2.1431	0.7697	1.4106	63.8652	1.6172	356.8167
λ											
%sign	38.10%	25.00%	13.04%	19.57%	8.89%	12.00%	0.00%	38.46%	7.59%	11.11%	15.19%
mean	-0.1880	-0.1508	-0.1185	0.0062	-0.0566	0.1531	-	-0.1526	0.0078	-0.0875	-0.0698
std	0.0201	0.0190	0.0828	0.0500	0.1056	0.0388	-	0.0283	0.0510	0.0179	0.0518
NGARCH											
v											
%sign	57.14%	83.33%	91.30%	91.30%	93.33%	88.00%	100.00%	76.92%	87.34%	91.11%	87.34%
mean	32.7420	6.0982	53.7201	6.4782	4.7409	6.6796	9.6396	7.4031	33.7905	5.8356	21.2797
std	81.3142	2.0311	376.3645	2.6532	1.7648	2.6181	1.2992	1.5900	154.2714	1.7506	175.5332
λ											
%sign	42.86%	25.00%	11.59%	17.39%	11.11%	12.00%	0.00%	38.46%	11.39%	11.11%	15.95%
mean	-0.1827	-0.1603	-0.1351	-0.0089	-0.0901	0.1545	-	-0.1537	-0.0127	-0.0880	-0.0776
std	0.0631	0.0589	0.0299	0.1579	0.1196	0.0260	-	0.0224	0.1740	0.1242	0.1387
APARCH											
v											
%sign	52.38%	91.67%	91.30%	84.78%	97.78%	92.00%	100.00%	69.23%	86.08%	100.00%	87.85%
mean	917.6009	39.4397	39.0992	6.2670	4.9676	6.6620	10.0853	7.3143	85.4743	5.9761	57.6171
std	3012.9584	110.1590	258.0082	2.2321	1.7546	2.4466	2.2758	1.5327	469.3766	1.6289	585.2651
λ											
%sign	42.86%	25.00%	7.25%	21.74%	11.11%	14.00%	0.00%	34.62%	11.39%	8.89%	15.44%
mean	-0.1856	-0.1542	-0.1480	0.0157	0.0189	0.1574	-	-0.1563	-0.0091	-0.0761	-0.0543
std	0.0687	0.0518	0.0221	0.1478	0.1603	0.0302	-	0.0218	0.1817	0.1449	0.1535

Table B7. Summary statistics for the estimates of the Skew-t distribution for GARCH and GJR models in the S&P 500 market. The table reports the percentage of significant coefficients (first row), the mean of the coefficients (second row) and the standard deviation of the coefficients (third row) over the sample period from January 2007 to December 2008 (daily frequency).

Appendix C. Estimates of the GARCH models

For illustrative purposes, we include in Figure C1 the usual NICs in all the domain of ε_{t-1}.

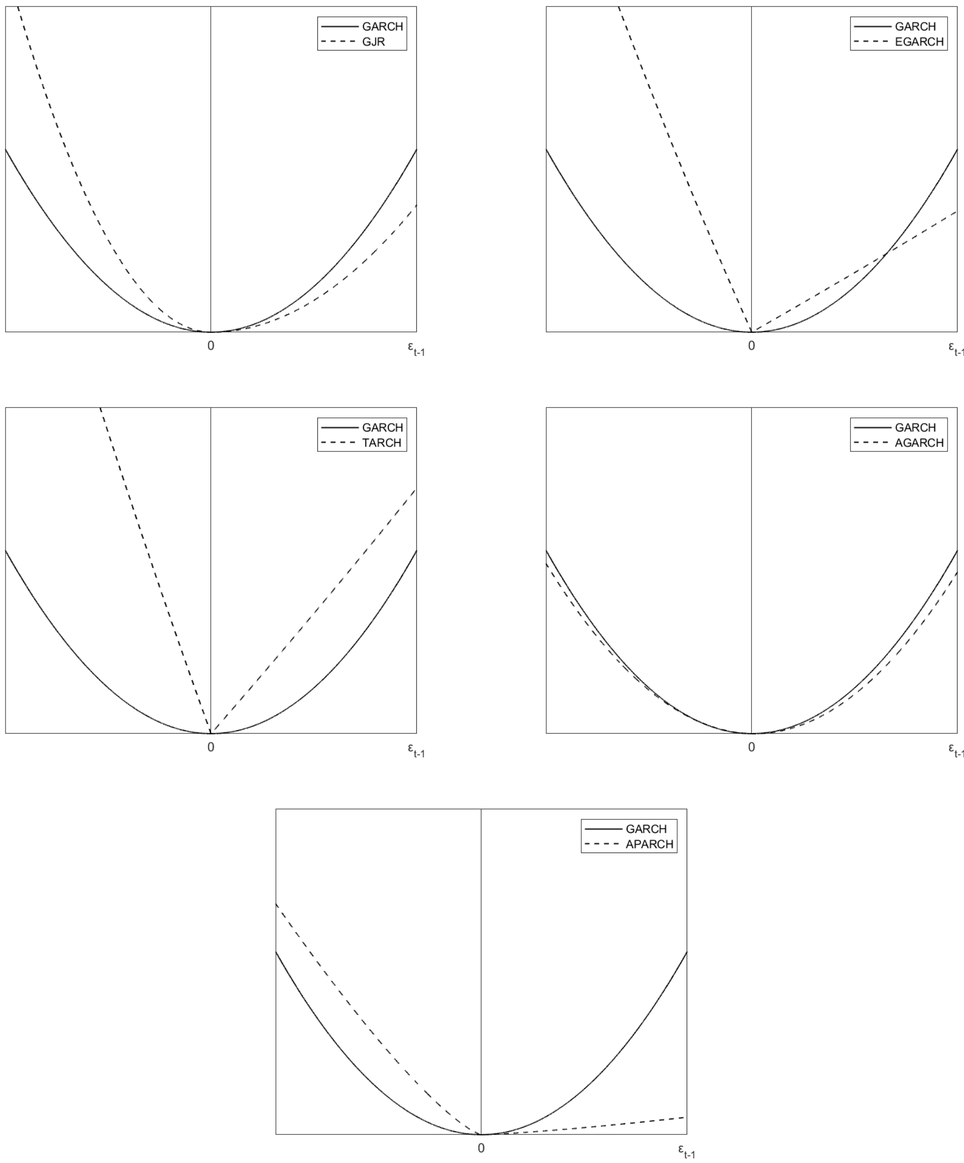

Figure C1. News Impact Curves for the GARCH model (solid line) versus the (dashed line) GJR model (Upper-left panel), EGARCH (Upper-right panel), TARCH (Middle-left panel), AGARCH (Middle-right) and APARCH (Lower-left panel).