

Supplementary Material to  
“Guaranteed In-Control Control Chart Performance  
with Cautious Parameter Learning”

Giovanna Capizzi and Guido Masarotto  
Department of Statistical Sciences,  
University of Padua,  
Italy,  
e-mail: giovanna.capizzi@unipd.it

Tables S1–S3 contain a tabulation of the critical values  $L_\infty$  and  $\Delta_L$ . The provided values should be large enough in many practical cases. In particular, observe that as  $\Delta_L$  is approximately linear in  $1/\sqrt{m}$ , a value of  $\Delta_L$  corresponding to a Phase I sample size not included in the tabulation can be obtained by interpolation/extrapolation on that scale. For example, a practitioner wanting to implement a Shewhart  $X$  chart with a nominal  $ARL_0$  equal to 250 and starting with a Phase I sample of size  $m = 150$  can use the value

$$\Delta_L = 0.267 + (0.183 - 0.267) \frac{\frac{1}{\sqrt{150}} - \frac{1}{\sqrt{100}}}{\frac{1}{\sqrt{200}} - \frac{1}{\sqrt{100}}} = 0.2143$$

which should be compared with the value  $\Delta_L = 0.2137$  obtained solving equation (6) of the paper with  $H = 1000$ .

Table S1: Control limits constants for the Shewhart  $X$  control chart with cautious parameter learning ( $A = 1.5$ ,  $B = 50$ ,  $\alpha = 0.1$ ,  $\beta = 0.05$ ). The values of  $\Delta_L$  have been computed solving equation (6) of the paper with  $H = 1000$ .

$ARL_0$	$L_\infty$	$\Delta_L$				
		$m = 50$	$m = 100$	$m = 200$	$m = 300$	$m = 500$
100	2.576	0.489	0.302	0.197	0.152	0.110
250	2.878	0.469	0.267	0.183	0.149	0.114
500	3.090	0.412	0.220	0.150	0.126	0.103
1000	3.291	0.348	0.165	0.115	0.099	0.083
2500	3.540	0.318	0.114	0.076	0.068	0.060
5000	3.719	0.393	0.136	0.100	0.093	0.088

Table S2: Control limits constants for the EWMA control chart with cautious parameter learning ( $A = 1.5$ ,  $B = 50$ ,  $\alpha = 0.1$ ,  $\beta = 0.05$ ). The values of  $\Delta_L$  have been computed solving equation (6) of the paper with  $H = 1000$ .

$\lambda$	$ARL_0$	$L_\infty$	$\Delta_L$				
			$m = 50$	$m = 100$	$m = 200$	$m = 300$	$m = 500$
0.1	100	2.148	0.796	0.441	0.242	0.167	0.107
	250	2.546	0.902	0.510	0.291	0.209	0.138
	500	2.814	0.945	0.497	0.283	0.209	0.146
	1000	3.059	0.892	0.471	0.259	0.197	0.140
	2500	3.352	0.944	0.413	0.226	0.172	0.128
	5000	3.557	1.062	0.522	0.322	0.262	0.206
0.2	100	2.360	0.671	0.380	0.222	0.162	0.108
	250	2.719	0.744	0.427	0.250	0.189	0.132
	500	2.962	0.783	0.412	0.250	0.189	0.136
	1000	3.187	0.801	0.391	0.224	0.174	0.128
	2500	3.460	0.816	0.340	0.193	0.151	0.115
	5000	3.652	0.921	0.443	0.290	0.234	0.189
0.3	100	2.453	0.613	0.359	0.219	0.162	0.113
	250	2.791	0.710	0.405	0.240	0.184	0.132
	500	3.023	0.724	0.389	0.235	0.182	0.133
	1000	3.238	0.740	0.355	0.212	0.164	0.124
	2500	3.501	0.764	0.315	0.186	0.147	0.110
	5000	3.688	0.855	0.418	0.276	0.226	0.183
0.5	100	2.534	0.555	0.346	0.213	0.162	0.115
	250	2.851	0.628	0.371	0.232	0.182	0.130
	500	3.071	0.655	0.363	0.228	0.176	0.131
	1000	3.277	0.665	0.329	0.205	0.163	0.122
	2500	3.531	0.720	0.294	0.178	0.137	0.106
	5000	3.712	0.791	0.391	0.262	0.218	0.176

Table S3: Control limits constants for the CUSUM control chart with cautious parameter learning ( $A = 1.5$ ,  $B = 50$ ,  $\alpha = 0.1$ ,  $\beta = 0.05$ ). The values of  $\Delta_L$  have been computed solving equation (6) of the paper with  $H = 1000$ .

$k$	$ARL_0$	$L_\infty$	$\Delta_L$				
			$m = 50$	$m = 100$	$m = 200$	$m = 300$	$m = 500$
0.5	100	3.502	1.826	1.001	0.583	0.431	0.288
	250	4.389	2.429	1.318	0.744	0.560	0.399
	500	5.071	2.907	1.394	0.809	0.621	0.444
	1000	5.757	3.275	1.496	0.819	0.612	0.454
	2500	6.669	3.831	1.521	0.776	0.597	0.458
	5000	7.361	5.257	2.072	1.205	0.971	0.759
0.75	100	2.481	1.249	0.705	0.420	0.315	0.222
	250	3.080	1.593	0.854	0.523	0.400	0.289
	500	3.538	1.766	0.910	0.541	0.424	0.310
	1000	3.999	1.997	0.934	0.531	0.418	0.309
	2500	4.608	2.343	0.880	0.504	0.393	0.299
	5000	5.070	2.986	1.212	0.765	0.620	0.495
1.0	100	1.874	0.919	0.534	0.336	0.252	0.177
	250	2.323	1.190	0.651	0.404	0.312	0.225
	500	2.665	1.319	0.676	0.419	0.326	0.244
	1000	3.009	1.369	0.674	0.409	0.318	0.235
	2500	3.467	1.590	0.633	0.375	0.292	0.221
	5000	3.814	2.029	0.867	0.559	0.463	0.375
1.5	100	1.131	0.641	0.393	0.247	0.193	0.135
	250	1.466	0.757	0.447	0.278	0.219	0.159
	500	1.708	0.834	0.455	0.279	0.224	0.165
	1000	1.942	0.888	0.436	0.268	0.211	0.159
	2500	2.246	0.951	0.402	0.244	0.192	0.148
	5000	2.473	1.193	0.546	0.365	0.303	0.246