Uncertainty Shocks and the Great Recession: Nonlinearities Matter*

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Abstract

We show that a nonlinear DSGE framework estimated on moments (impulse responses) specific to the great recession implies a peak response of the cyclical component of output to an uncertainty shock 50% larger than the one predicted by the very same model estimated on moments implied by a standard linear VAR. As an implication, the DSGE framework estimated on impulse responses generated with the nonlinear VAR assigns to the 2008Q4 uncertainty shock a contribution of about 60% as regards the output loss experienced by the US economy during and after the great recession. The same DSGE framework estimated by matching the impulse responses of the linear VAR severely underestimates such contribution.

Keywords: Uncertainty shock, nonlinear IVAR, nonlinear DSGE framework, minimum-distance estimation, great recession.

JEL codes: C22, E32, E52.

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1 Introduction

The largest spike of the VIX - a proxy for financial uncertainty - recorded so far at quarterly frequencies occurred in 2008Q4, in the midst of the great recession.[1] Was such a spike a relevant driver of the US business cycle? We address this question by estimating the state-of-the-art, policy-relevant Basu and Bundick (2018) DSGE framework via the Bayesian impulse response matching function approach popularized by Christiano, Trabandt, and Walentin (2011). This approach considers the impulse responses generated with a linear VAR as "data", and requires the DSGE framework to match those impulse responses. But a linear VAR, while offering a good approximation of the dynamic responses to an identified macroeconomic shock in normal times, is ill-suited to describe extreme events such as the great recession. Evidence of nonlinear effects of uncertainty shocks over the business cycle has recently been provided by Caggiano, Castelnuovo, and Groshenny (2014), Caggiano, Castelnuovo, and Figueres (2017), Caggiano, Castelnuovo, and Figueres (2020), and Caggiano, Castelnuovo, and Nodari (2020). This evidence could be due to the presence of the zero lower bound (in the case of the great recession, see Caggiano, Castelnuovo, and Pellegrino (2017)), heightened financial stress (Alessandri and Mumtaz (2019)), occasionally binding constraint on downward wage adjustment (Cacciatore and Ravenna (2018)), and confidence-related effects (Pellegrino, Ravenna, and Züllig (2020)).

The key contribution of this paper is to estimate the Basu and Bundick (2018) model with impulse responses generated by a nonlinear VAR framework designed to capture the possibly larger reaction of real activity to an uncertainty shock during the great recession. Operationally, we: i) estimate linear and nonlinear VARs over a period involving the great recession; ii) show that the nonlinear VAR is preferred by the data to its (encompassed) linear version; iii) use moments generated by either VARs to estimate a version of Basu and Bundick's (2018) nonlinear DSGE framework which allows (but not necessarily require) uncertainty shocks to drive the business cycle; iv) use the estimated DSGE frameworks (i.e., the same framework estimated first with the moments generated by the linear VAR, then with those generated by the nonlinear VAR) to predict the output response to the 2008Q4 uncertainty shock.

[1] The VIX is a measure of market's expectation of 30-day forward-looking volatility produced by the Chicago Board Options Exchange (CBOE). Quarterly realizations of the VIX (computed as within-quarter averages of daily observations) are available at https://fred.stlouisfed.org/series/VIXCLS . The value recorded by the VIX in 2008Q4 is 58.60. Just to give a sense on the magnitude of that peak, the average value of the VIX in the 1990Q1-2020Q2 sample is 19.37.

The DSGE model estimated with great recession "data" predicts that about 60% of the cumulative output loss occurred during and after the great recession could be attributed to the 2008Q4 uncertainty shock. The version of the DSGE model estimated with moments generated by the linear VAR points instead to a 37% contribution. Our findings call for the employment of nonlinear frameworks for studying extraordinary episodes such as the great recession.

The paper develops as follows. Section 2 presents our non-linear VAR model, the identification strategy we use, (informally) the DSGE framework, and documents the VAR and DSGE impulse responses. Section 3 focuses on the response of output to the 2008Q4 uncertainty shock according to our estimated DSGE frameworks. Section 4 concludes.

2 The real effects of uncertainty shocks: Evidence

2.1 Nonlinear VAR

Reduced-form nonlinear VAR. We follow Pellegrino (2017, 2018) and Caggiano, Castelnuovo, and Pellegrino (2017) and focus on the following nonlinear Interacted VAR (IVAR):

$$\mathbf{Y}_t = \boldsymbol{\alpha} + \sum_{j=1}^{L} \mathbf{A}_j \mathbf{Y}_{t-j} + \left[\sum_{j=1}^{L} \mathbf{c}_j \ln VXO_{t-j} \times \Delta \ln GDP_{t-j} \right] + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim d(0, \boldsymbol{\Omega}) \tag{1}$$

where $\mathbf{Y}_t$ is the $(n \times 1)$ vector of the endogenous variables, $\boldsymbol{\alpha}$ is the $(n \times 1)$ vector of constant terms, $\mathbf{A}_j$ are $(n \times n)$ matrices of coefficients, $\boldsymbol{\eta}_t$ is the $(n \times 1)$ vector of error terms whose variance-covariance (VCV) matrix is $\boldsymbol{\Omega}$, and $d(\cdot)$ is the distribution of the residuals. The interaction term in brackets makes an otherwise standard VAR a non-linear IVAR model. For each lag j, such interaction term includes a $(n \times 1)$ vector of coefficients $\mathbf{c}_j$, a measure of uncertainty $\ln VXO_t$, and an indicator of the business cycle $\Delta \ln GDP_{t-j} \equiv \ln GDP_{t-j} - \ln GDP_{t-j-1}$, which is the quarter-on-quarter growth rate of real GDP. The interaction term $\ln VXO_{t-j} \times \Delta \ln GDP_{t-j}$ parsimoniously captures the potentially state-contingent effects of a shock to $\ln VXO_{t-j}$ (i.e., an uncertainty shock) conditional on the state of the business cycle, which is proxied by the growth rate of real GDP. Hence, our IVAR allows uncertainty shocks to have different effects in normal times vs. during the great recession.

Data. We model the vector $\mathbf{Y}_t = [\ln VXO, \ln GDP, \ln C, \ln I, \ln H, \ln P, R]'$, where VXO denotes the stock market S&P 100 implied volatility index, GDP per capita GDP, C per capita consumption, I per capita investment, H per capita hours worked, P the price level, and R the policy rate. The variables in this vector are those used by Basu and Bundick (2018) in their linear VAR analysis.[2] We estimate our IVAR model with four lags, sample: 1962Q3-2017Q4. Given that the VXO is unavailable before 1986, we follow Bloom (2009) and splice it with the within-month volatility of S&P500 daily returns, which has displayed an extremely high correlation with the VXO since 1986. The sample includes the 2008Q4-2015Q4 zero lower bound period. We then work with the shadow rate constructed by Wu and Xia (2016) to account for the effects of unconventional policy responses to financial uncertainty shocks.

Evidence in favor of the nonlinear VAR. A standard likelihood-ratio test favors our IVAR specification against the linear VAR model (which is nested in our IVAR model in case of the overall exclusion of the interaction terms from model (1)). In particular, the LR test suggests a value for the test statistic $\chi_{28} = 61.99$, which allows us to reject the null hypothesis of linearity at any conventional statistical level in favor of the alternative of our I-VAR model (p-value $<< 0.01$).

Identification. Let the system of contemporaneous relationships mapping reduced form residuals $\boldsymbol{\eta}_t$ and structural shocks $\mathbf{e}_t$ be described as $\boldsymbol{\eta}_t = \boldsymbol{B}\mathbf{e}_t$, $\mathbf{e}_t \sim d(0, \boldsymbol{I_n})$, where $\boldsymbol{B}$ is a matrix featuring n^2 elements. Given that the reduced form covariance matrix $\boldsymbol{\Omega}$ features only $n(n{+}1)/2$ restrictions, we have to impose further restrictions to identify the effects of the structural shocks $\mathbf{e}_t$ on the endogenous variables $\mathbf{Y}_t$. Without such further restrictions, infinitely many solutions satisfy the covariance restrictions $\boldsymbol{\Omega} = \boldsymbol{B}\boldsymbol{B}'$. We collect these solutions into the set $\mathcal{B} = \{\boldsymbol{B} = \boldsymbol{P}\boldsymbol{Q} : \boldsymbol{Q} \in \mathcal{O}_n, diag(\boldsymbol{B}) \geqslant 0, \boldsymbol{\Omega} = \boldsymbol{B}\boldsymbol{B}'\}$, where $\mathcal{O}_n$ is the set of $(n \times n)$ orthonormal matrices (i.e., $\boldsymbol{Q}\boldsymbol{Q}' = \boldsymbol{I_n}$), and $\boldsymbol{P}$ is the unique lower-triangular Cholesky factor with non-negative diagonal elements, i.e., $\boldsymbol{\Omega} = \boldsymbol{P}\boldsymbol{P}'$. The set $\mathcal{B}$ is constructed by simulating one million different $\boldsymbol{B}$ - which imply one million unconstrained shocks $\mathbf{e}_t(\boldsymbol{B}) = \boldsymbol{B}^{-1}\boldsymbol{\eta}_t$, $t = 1, ..., T$ - via the algorithm proposed by Rubio-Ramírez, Waggoner, and Zha (2010).

All models in $\mathcal{B}$ are mathematically coherent with the data, but not all of them are equally credible from an economic standpoint. We impose restrictions directly on the shocks $\mathbf{e}_t(\boldsymbol{B})$ to work out the set of economically admissible solutions $\overline{\mathcal{B}}$. Following Lud-

[2] Their VAR also features the presence of money. Adding money implies no changes in our empirical results. The definition and construction of the variables common to our investigations is exactly the same as in Basu and Bundick (2018).

vigson, Ma, and Ng (2019), we work with two types of restrictions, i.e., event constraints and external variable constraints. In short, event constraints require uncertainty shocks to be large in selected dates such as, e.g., the Black Monday (1987Q4), 9/11 (2001Q3), the great recession (2008Q4), and so on, in correspondence of which financial uncertainty skyrocketed. In particular, we require financial uncertainty shocks to be larger than the 75th percentile of the empirical distribution of the realizations of financial uncertainty shocks $\mathbf{e}_{FUt}(\boldsymbol{B})$ in 1987Q4 and 2008Q4, and larger than the 50th percentile in all other dates. The selected dates are an updated version of those selected by Bloom (2009) and Ludvigson, Ma, and Ng (2019) - they can be found in our companion paper (Pellegrino, Castelnuovo, and Caggiano (2020)). External variable constraints, which are imposed to further narrow down the set of retained structural models, required the correlation between $\mathbf{e}_{FUt}(\boldsymbol{B})$ and the aggregate stock market returns (growth rate of the real price of gold) to be below (above) the median of its empirical density. Generalized impulse responses (GIRFs) à la Koop, Pesaran, and Potter (1996) are computed to account for both the endogenous response of the growth rate of per capita GDP, i.e., our conditioning variable, to the uncertainty shock and the feedback this reaction can imply on the dynamics of the economy. Given that, in a nonlinear framework, impulse responses are initial condition-specific, we compute the dynamic reaction of $\mathbf{Y}_t$ to an uncertainty shock during and in the aftermath of the great recession by considering the specific initial history $\boldsymbol{\varpi}_{t-1} = \{\mathbf{Y}_{t-1}, ..., \mathbf{Y}_{t-4}\}$, where $t-1 = 2008Q3$, that corresponds to the quarter before the remarkable uncertainty spike in 2008Q4. Hence, the IVAR GIRFs $\widehat{\boldsymbol{\psi}^i}$ for the great recession are computed by iterating forward the system starting from the initial condition $\boldsymbol{\varpi}_{2008Q3}$. As regards the size of the shock δ, we impose a 4.4 standard deviation shock, which is the median size of the uncertainty shock in $t = 2008Q4$ among all retained shocks series.

2.2 DSGE model: Description and estimation

Description. We work with a version of the Basu and Bundick (2018) model.[3] The model extends an otherwise standard New Keynesian model to consider the macroeco-

[3]This framework differs from the Basu and Bundick (2017) one along two dimensions. First, it features households' preferences that do not imply any asymptotic response of output to an uncertainty shock when the labor supply elasticity tends to one (an issue affecting households' preferences in Basu and Bundick (2017)). Second, if features external habit formation in consumption, which we add to improve the fit of the DSGE as far as the response of consumption to an uncertainty shock is concerned. A fully detailed presentation of the model can be found in Basu and Bundick (2017, 2018) and Pellegrino, Castelnuovo, and Caggiano (2020).

nomic effects of an increase in the second moment of the preference shock process (a shock to households' discount rate). The model is able to generate a fall in real activity after a jump in uncertainty thanks to precautionary savings (which lead to a fall in consumption and, given that output is demand-driven because of sticky prices, to a fall in output) and sticky prices (which prevent aggregate prices to fully adjust downward in response to the fall in real wages driven by the drop in labor demand, lower investment because of the lower return to capital, and lower hours because of the weaker labor demand). Notably, in the model, intermediate firms issue equity shares, and the model allows us to compute the response of the conditional variance of the return on equity to an uncertainty shock, which can be compared with the VIX in our VAR to assess the DSGE model's ability to track the evolution of uncertainty after the shock. We work with a third-order approximation of the nonlinear DSGE model (Andreasen (2012)), which is solved via perturbation techniques (Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012)).

Estimation. We estimate the vector $\boldsymbol{\zeta}^i = \left[\rho_{\sigma^a}, \sigma, b, \phi_K, \phi_P, \rho_\pi, \rho_y\right]$ of structural parameters (with i to index "normal times" vs. "great recession" parameters). These parameters are the persistence of the second moment preference shock ρ_{σ^a}, the household risk aversion parameter σ, the consumption habit formation parameter b, the parameter regulating investment adjustment costs ϕ_K, the parameter regulating price adjustment costs ϕ_P, and the parameters of the Taylor rule ρ_π, ρ_y. We calibrate the prior means with the values in Basu and Bundick's (2018) analysis, and we use diffuse priors. For the habit formation parameter and the parameters of the Taylor rule, we use the priors employed by Christiano, Trabandt, and Walentin (2011). The remaining parameters of the model are calibrated as in Basu and Bundick (2018). Table 1 reports our prior densities and our estimates. Figure 1 displays the performance of the DSGE framework in replicating the VAR facts. In brief: i) the response of real activity is significantly larger (from a statistical and economic standpoints) during the great recession. In particular, the peak response of output is 50% larger than in normal times; that of consumption 32% larger; investment and hours display responses that are two and a half and two times larger than in normal times, respectively; ii) the estimated DSGE framework goes a long way in matching the VAR dynamics both when we consider a linear VAR (which is what the literature typically does) and when we focus on the great recession "data" (i.e., the VAR impulse responses computed by considering observations until 2008Q3 as initial conditions). As suggested by Table 1, and confirmed by simulations documented in Pellegrino, Castelnuovo, and Caggiano (2020), the three elements behind the amplified reaction of real activity to an uncertainty shock during the great recession are: i) higher risk aversion (in line with Schildberg-Horisch (2018), who documents a solid support by the experimental literature on an increase in risk aversion during the 2007-09 recession); ii) higher investment adjustment costs (in line with Lanteri (2018) and Dibiasi (2018), who propose evidence consistent with a countercyclical degree of reallocation frictions); iii) and a stronger policy response to output growth.[4]

3 2008Q4 uncertainty shock: Nonlinearities matter

Endowed with our estimated DSGE framework(s) (one estimated by matching the facts produced with the linear VAR, and the other one with those implied by our nonlinear VAR framework), we quantify the output loss attributable to the 2008Q4 uncertainty shock. Given that our DSGE framework does not feature any trending process, we follow Basu and Bundick (2017) and compare the DSGE-implied response of output with the cyclical evolution of output as described by the CBO output gap. Figure 2 contrasts the evolution of the output gap in the 2008Q4-2014Q3 period (after that, the impulse response of the DSGE model conditional on linear VAR moments is back to zero) with that of the response of output predicted by the estimated DSGE framework conditional on the linear VAR impulse responses and that predicted by the DSGE framework estimated with great recession data. Evidently, the latter assigns a larger role to the 2008Q4 uncertainty shock, with a predicted cumulative output loss which amounts to 62% of the one in the actual data. Differently, the DSGE framework estimated on moments generated by the linear VAR predicts a 37% contribution of the 2008Q4 uncertainty shock.

4 Conclusion

A nonlinear DSGE framework estimated with moments generated via a nonlinear VAR framework supported by the data assigns about 60% of the overall output lost by the US economy during and after the great recession. The same exercise, conducted with the DSGE model estimated by matching impulse responses generated with a linear VAR, points to a contribution of the uncertainty shock of "just" 37%. This finding suggests

[4] In Pellegrino, Castelnuovo, and Caggiano (2020), we show that our results are robust to controlling for a proxy of first moment financial shocks.

that, when one investigates extreme events such as the great recession, nonlinearities matter.

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		Priors	Posteriors	
			Linear VAR	Great Recession
Parameter	Interpretation	D(mean, std)	Mode, std	Mode, std
ρ_{σ^a}	Unc. shock, persistence	B(0.77, 0.10)	**0.64,** 0.03	**0.65,** 0.03
b	Habit formation parameter	B(0.75, 0.15)	**0.64,** 0.06	**0.66,** 0.04
ϕ_K	Investment adjustment costs	G(3.92, 2)	**2.29,** 0.50	**3.21,** 0.60
ϕ_P	Price adjustment costs	G(240, 40)	**236.78,** 32.26	**282.10,** 33.54
ρ_π	Taylor rule parameter, inflation	IG(1.5, 0.25)	**1.05,** 0.01	**1.05,** 0.01
ρ_y	Taylor rule parameter, output growth	G(0.2, 0.15)	**0.20,** 0.04	**0.28,** 0.05
σ	Risk aversion (fixed labor supply, no habits)	G(100, 60)	**385.90,** 50.45	**533.04,** 59.16
RRA	Risk aversion (endogenous labor supply, habits)		**104.85**	**144.96**

Table 1: **DSGE model: Average evidence vs. Great recession.** Values estimated conditional on both the linear VAR impulse responses and on the IVAR impulse responses in 2008Q4. Standard deviations estimated conditional on a Laplace approximation of the posterior density. Risk aversion in the model (RRA) computed by considering endogenous labor supply and habits as in Swanson (2018).

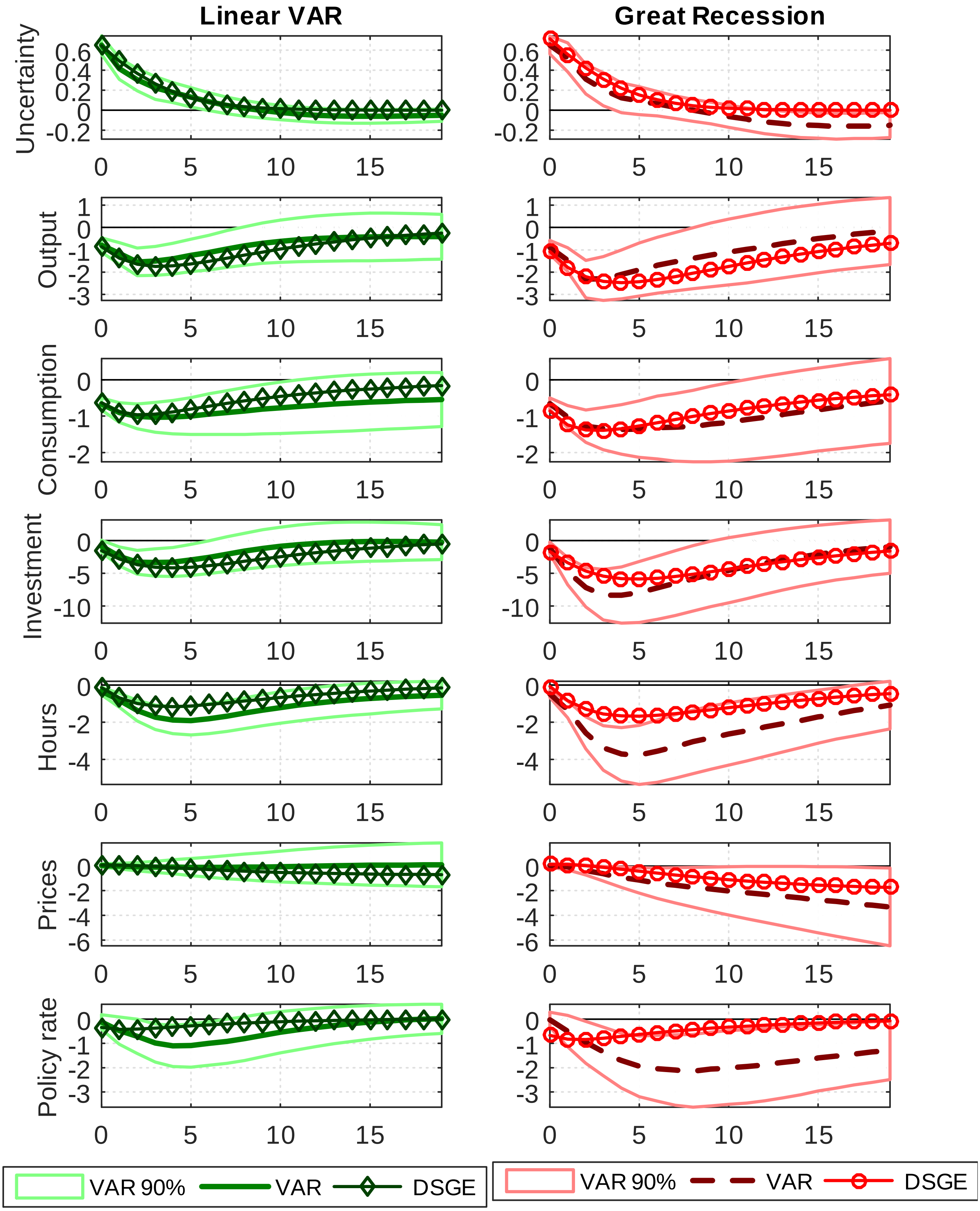

Figure 1: **DSGE and VAR impulses responses to an uncertainty shock: linear vs. great recession.** Solid lines with squares: Estimated responses of the DSGE model. Solid green line: linear VAR impulse responses, with 90% confidence bands. Dashed red lines: IVAR impulse reponses for the great recession, with 90% confidence bands. Sample: 1962Q3-2017Q4. VAR estimated with four lags.

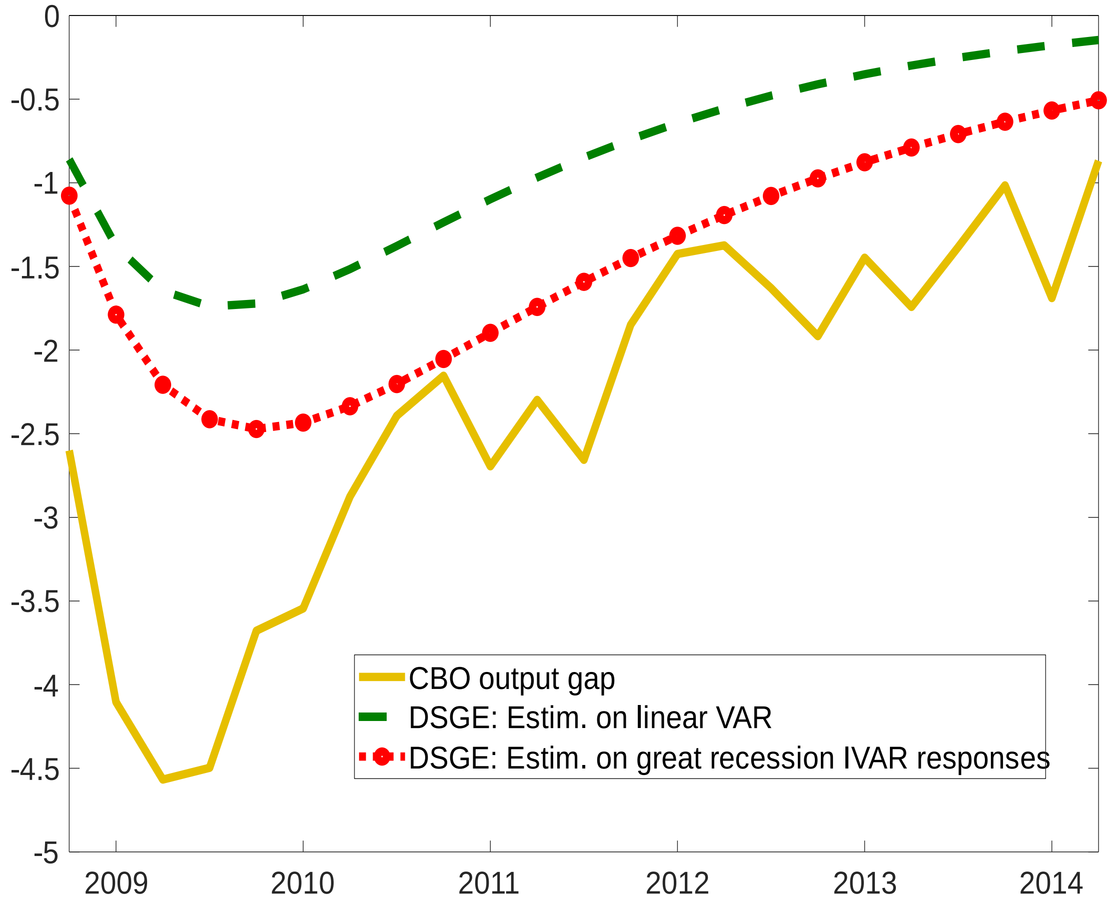

Figure 2: **Great Recession: Contribution of the 2008Q4 uncertainty shock.** CBO output gap normalized to zero in 2008Q3. "DSGE: Estimation on linear VAR" indicates the response of output according to the DSGE model estimated by matching the linear VAR impulse responses to the 2008Q4 uncertainty shock. "DSGE: Estim. on great recession IVAR responses" indicates the response of output according to the DSGE model estimated by matching the nonlinear IVAR impulse responses to the 2008Q4 uncertainty shock.