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Variations and Control of Thrust and Mixture Ratio in Hybrid Rocket Motors

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Abstract

Hybrid rocket motors have several attracting characteristics like simplicity, low cost, safety, reliability, environmental friendliness. In particular, hybrid rockets can provide complex and flexible thrust profiles not possible with solid rockets in a simpler way than liquid rockets, controlling only a single fluid. Unfortunately, the drawback of this feature is that the mixture ratio cannot be directly controlled but depends on the specific regression rate law. Therefore, in the general case the mixture ratio changes with time and with throttling. Thrust could also change with time for a fixed oxidizer flow. Moreover, propellant residuals are generated by the mixture ratio shift if the throttling profile is not known in advance. The penalties incurred could be more or less significant depending on the mission profile and requirements. In this paper, some proposed ways to mitigate or eliminate these issues are recalled, quantitatively analysed and compared with the standard case. In particular, the addition of energetic additives to influence the regression rate law, the injection of oxidizer in the post-chamber and the altering-intensity swirling-oxidizer-flow injection are discussed. The first option exploits the pressure dependency of the fuel regression to mitigate the shift during throttling. The other two techniques can control both the mixture ratio and thrust (at least in a certain range) at the expense of an increase of the architecture complexity. Moreover, some other options like pulse width modulation or multi-chamber configuration are also presented. Finally, a review of the techniques to achieve high throttling ratios keeping motor stability and efficiency is also discussed.

Keywords: Hybrid rockets, throttling, mixture ratio control

Nomenclature

a, n, m	regression rate law coefficients
A_p, A_b	port, burning area
c^*	characteristic velocity
G_{ox}	oxidizer mass flux
L_f	fuel grain length
$\dot{m}_f$	fuel mass flow
$\dot{m}_{ox\ p}$	port oxidizer mass flow
$\dot{m}_{ox\ aft}$	aft-injected oxidizer mass flow
$\dot{m}_{ox\ tot}$	total oxidizer mass flow
$\dot{m}_{ox\ ax}$	axial oxidizer mass flow
$\dot{m}_{ox\ tan}$	tangential oxidizer mass flow
p_c	chamber pressure
$\dot{r}$	regression rate
res_f, res_{tot}	fuel, propellant residuals
S_{min}, S_{max}	min, max swirl number
S_e, S_g	effective, geometric swirl number
φ_{min}, φ_{max}	min, max mixture ratio
ρ_f	fuel density

Acronyms/Abbreviations

SOFT	Swirling Oxidizer Flow Type
A-	Altering-intensity
AOIM	Aft-end Oxidizer Injection Method
HAST	High Altitude Supersonic Target
PWM	Pulse Width Modulation

1. Introduction

Hybrid rockets have been studied since the '30s but have never come into fruition and their research has been limited compared to the commonly employed solid and liquid technologies [1-6]. Nowadays, a shift of focus from pure performance to a broader range of aspects including simplicity, reliability, cost, safety, flexibility, ease-of-use/production, environmental friendliness, has brought recently a renewed interest toward hybrid propulsion, which potentially bring several advantages compared to traditional liquid and solid propulsion [7]. In particular, one often claimed advantage of hybrid propulsion is to guarantee a similar level of energy management of a liquid rocket (deep throttling and multiple stop-restart on demand), which is much higher than commonly possible with solids, together with a significantly reduced complexity and improved safety compared to liquids [8].

Several demonstrations of this capability have been performed along the years, starting from Moore [9], continuing with the work at UTC related to target drones (Sandpiper, HAST) and upper stages [10-12], followed by AMROC experience [13-18] and several NASA related programs [19-22]. A more detailed summary of hybrid rocket throttling history is given by Peterson [23, 24].

Recently, remarkable demonstrations have been performed by Purdue [25], USU [23, 24], NAMMO [26, 27] and UNIPD [28-32]. Moreover, a program for a Mars soft-lander demonstrator based on hybrid propulsion has been funded by the European community [33-36].

On the whole, throttling ratio as high as 10:1 have been successfully achieved keeping high levels of combustion stability and efficiency.

However, a well-known negative peculiarity of hybrid rockets is the shift of mixture ratio with time and throttling. This is in contrast with liquids, where both propellants mass flows are controlled, and solids, where the mixture ratio is defined in the propellant grain casting process. On the contrary, the specific hybrid rocket behaviour is related to the fact that only the oxidizer flow is controlled and the regression of the fuel follows a nonlinear dependency on the oxidizer flux:

$$\dot{m}_f = \rho_f \dot{r} A_b = \rho_f a G_{ox}^n p_c^m A_b \qquad (1)$$

which, in turn, varies also with the port area:

$$G_{ox} = \dot{m}_{ox\,p} / A_p \qquad (2)$$

Several ideas have been proposed in order to be able to control the mixture ratio both in time and during throttling, like the addition of an aft-injection of further oxidizer (AOIM) [2, 37] and the recently proposed A-SOFT configuration by Shimada et al. [38-43] (see Fig. 1).

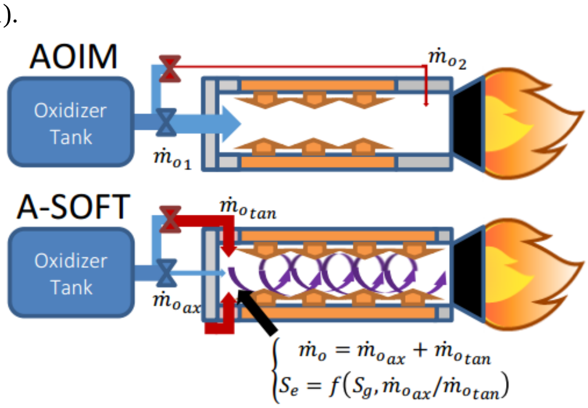

Fig. 1. AOIM and A-SOFT configurations. Figure taken from Ozawa and Shimada [43].

The objective of this paper is to recall and revaluate all the main techniques to control hybrid rockets mixture ratio and compare them to classical hybrid behaviour.

The paper is organized in the following way. In the second chapter the classical behaviour (included a pressure dependency) is considered and the penalties related to throttling are evaluated. In the third chapter the AOIM technique is analysed. In the fourth chapter the A-SOFT is investigated. The two techniques are then compared in chapter five. In the sixth chapter other options are briefly discussed. Finally, a review of the techniques to achieve high throttling ratios keeping motor stability and efficiency is also presented.

2. Classical behaviour and penalties evaluation

It is well known that in a classical hybrid rocket the mixture ratio varies with time and throttling, as described in [6, 44]. For a constant oxidizer flow and a cylindrical port, if the regression rate exponent is 0.5 the mixture ratio is constant with time, if it is higher the mixture ratio increases, while the opposite occurs for lower values. For non-cylindrical port (e.g. triangular, trapezoidal) typically used in wagon wheel multiport grains, the neutral point shifts below 0.5, with the extreme of a neutral burning area grain (e.g. some stars shapes) where the neutral point is n=0 and the hybrid thrust behaviour is, otherwise, always regressive.

In case of throttling, if n is equal to one no mixture ratio shift occurs, while for lower values (as in typical hybrids) the fuel mass flow variation with the oxidizer is sublinear. However, as shown in [28], while a value of n equal to one guarantees no direct penalties, a value of n equal to 0.5 can be favourable also for throttling as performance (e.g. thrust level) become independent from the throttling history. In [44] it was demonstrated that throttling down from the optimal point provides the most linear behaviour. Moreover, for the same time spent at a certain throttle level, the maximum amount of total impulse is generated at maximum thrust, so it makes sense to have the optimal point there. Anyway, the real optimal solution should take into account the expected/required thrust profile with time.

It is important to highlight that the relation between thrust and mixture ratio variations during time has the opposite trend respect to the case of throttling, so compensating the thrust variation with time trough throttling (like in [23, 24]) has the consequence to exacerbate the mixture ratio shift with time.

In case of a pressure dependency m of the fuel regression rate (for example trough the addition of energetic additives to the fuel) the mixture ratio shift with time follows the same trend as before (with n=0.5 as neutral point) but the results get slightly worse as the reduction (increment) of fuel induces a reduction (increment) of pressure that in turn decreases (increases) further the regression rate (see an example in Fig. 2).

On the contrary, the mixture ratio shift with throttling is mitigated by the pressure dependency as the new neutral point is now defined by the sum of the two exponents n+m being equal to 1 [2] (see an example in Fig. 3 and 4).

In Fig. 3 and 4 the characteristic velocity curve obtained from a thermochemical code [45] refers to

90% hydrogen peroxide and **polyethylene** (as in the rest of the paper) and the starting point is set at the optimum mixture ratio. Slightly different results are obtained with other propellant combinations for the same regression rate laws.

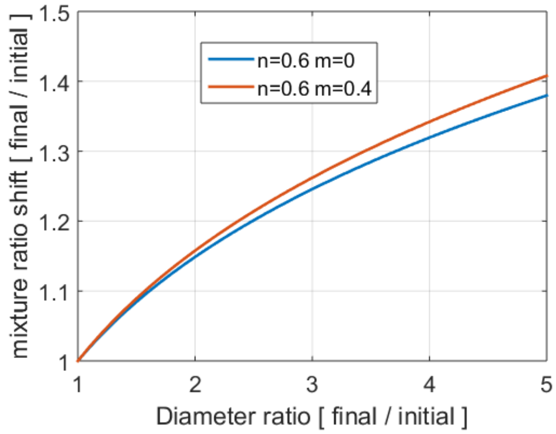

Fig. 2. Mixture ratio shift with time.

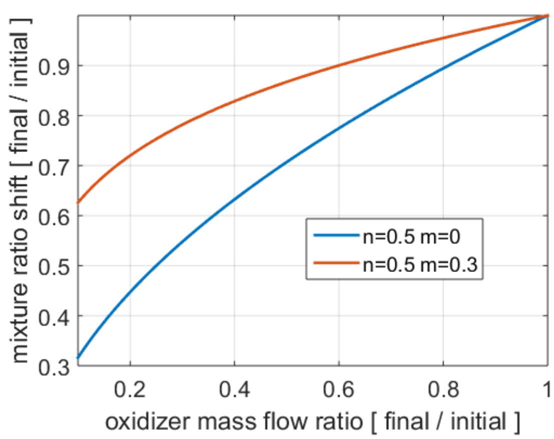

Fig. 3. Mixture ratio shift with throttling.

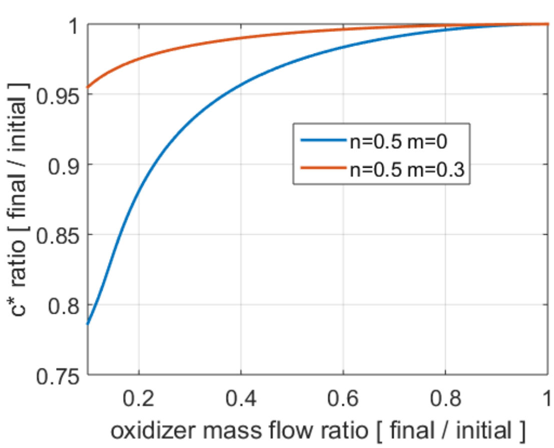

Fig. 4. Characteristic velocity with throttling. Initial chamber pressure: 50 bar.

However, it is important to note that any propellant combination will have not only its own *c** curve but also regression rate exponents *n* and *m*, so the real results has to be calculated case by case. Often, adding energetic additives has the effect to reduce *n* and increase *m*.

In theory, the perfect regression rate law (mixture ratio insensitive to time and throttling) will require both exponents equal to 0.5, this has not been obtained in practice yet.

In general, adding metals in the hybrid fuel has a twofold benefit (other than on the energetic content): a reduction of the mixture ratio shift with throttling because of the pressure dependency and a lower sensitivity of *c** to the mixture ratio [46], both keeping the specific impulse nearer to its optimum value.

However, as a drawback, with the pressure dependency the mixture ratio becomes dependent on nozzle erosion. Moreover, in general, adding energetic additives to the fuel has the effect to decrease the optimum mixture ratio, exacerbating the issue of fuel residuals.

When throttling down a conventional motor (i.e. without a special device like a pintle on the nozzle throat) there is a reduction in chamber pressure. Chamber pressure has a second order effect on *c** (except for very low values) but could have a significant impact on the thrust coefficient during atmospheric flight. This penalty is shared also by liquid rockets and by hybrid rockets with mixture ratio control features.

It is interesting to determine the total specific impulse losses during throttling in atmospheric flight. As an example, a non-throttleable motor against a motor with a mass flow throttling ratio of 5 is considered. The nozzle is adapted to an altitude around 7.8 km to avoid separation at sea level. In case of the throttleable motor the adaptation considers the minimum thrust, thus the nozzle expansion ratio is slightly smaller. Two cases are considered: one with a classical hybrid with n=0.5 and the other without mixture ratio shift. For the classical hybrid the throttling ratio is referred to the oxidizer mass flow. The optimum mixture ratio is set at maximum thrust. The results are presented in Fig. 5-10.

In Fig. 5 and the followings *up* refers to the ratio of the specific impulse of the throttleable case at maximum thrust to the non-throttleable case.

The *up-down* refers to the ratio of the specific impulse of the throttleable case at maximum thrust to that at minimum thrust.

The *down* curve refers to the ratio of the specific impulse of the throttleable case at minimum thrust to the non-throttleable case.

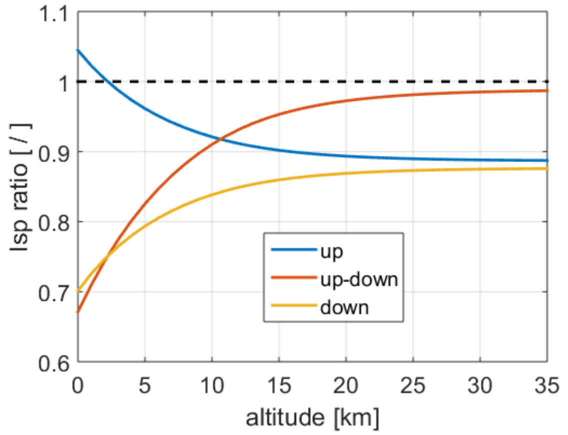

Fig. 5. Specific impulse penalty of a throttleable motor with altitude without mixture ratio shift. Max chamber pressure: 25 bar.

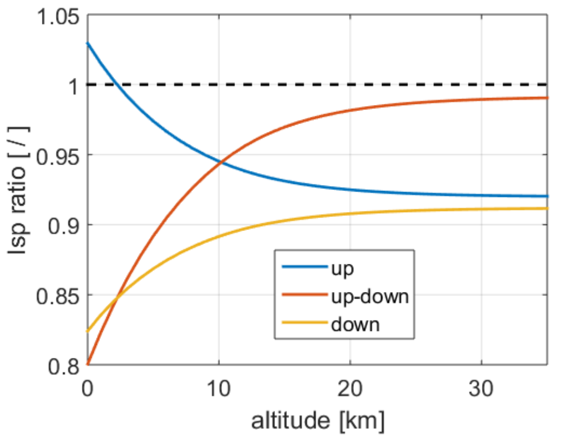

Fig. 6. Specific impulse penalty of a throttleable motor with altitude without mixture ratio shift. Max chamber pressure: 100 bar.

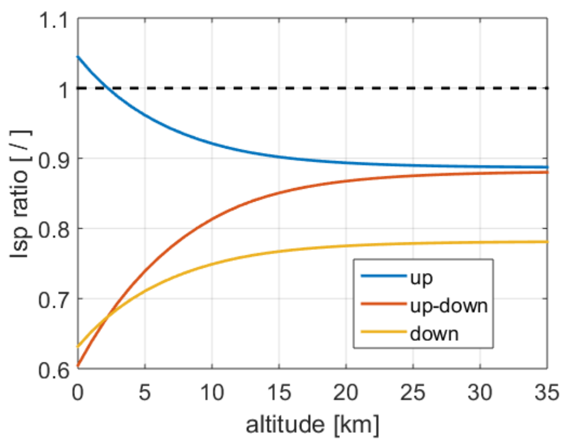

Fig. 7. Specific impulse penalty of a throttleable motor with altitude with mixture ratio shift, *n*=0.5. Max chamber pressure: 25 bar.

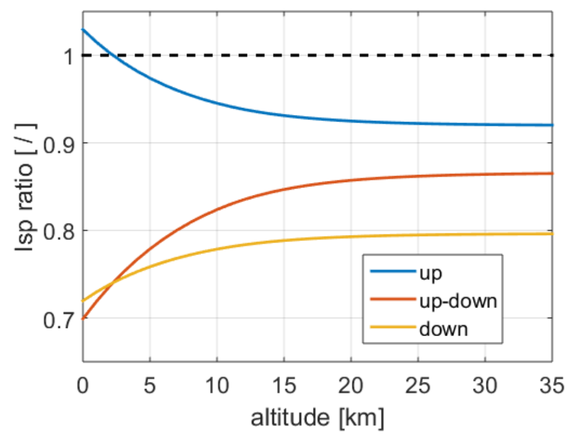

Fig. 8. Specific impulse penalty of a throttleable motor with altitude with mixture ratio shift, *n*=0.5. Max chamber pressure: 100 bar.

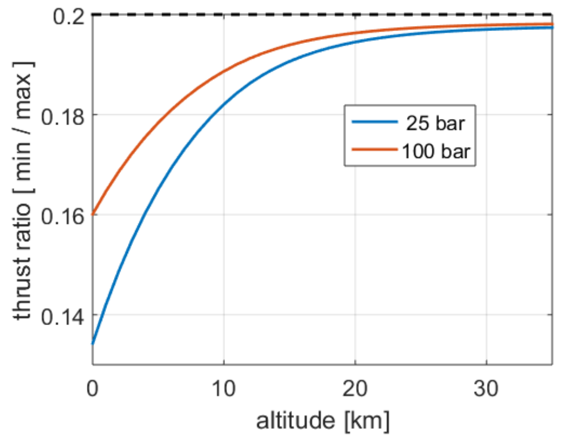

Fig. 9. Thrust ratio of a throttleable motor (5:1) with altitude without mixture ratio shift.

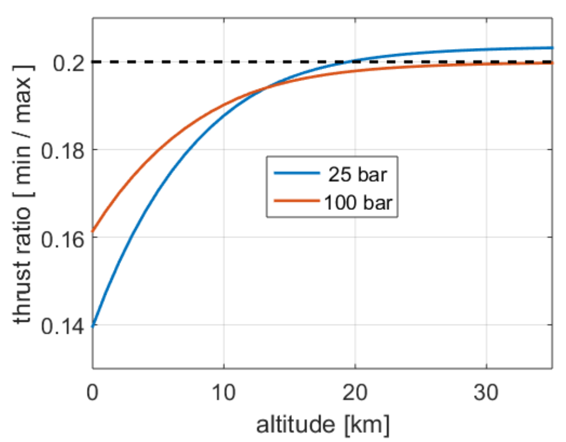

Fig. 10. Thrust ratio of a throttleable motor (5:1) with altitude with mixture ratio shift, *n*=0.5.

As the maximum thrust is selected at the optimum mixture ratio, the specific impulse losses (*up*) between the throttleable and non-throttleable cases are related only to the lower expansion ratio (sized for the minimum thrust). The lower expansion ratio provides a slight advantage at very low altitudes where the larger nozzle is over-expanded.

The ratio (*up-down*) between the specific impulse at maximum thrust and minimum thrust is determined by the variation of the thrust coefficient with pressure for both cases plus the c^* penalty mainly for the case with mixture ratio shift. As expected, the ratio tends almost to unity at high altitudes where the back pressure is negligible for the case with mixture ratio control. A slight penalty is present due to the variation of c^* with pressure. In the other case the mixture ratio becomes fuel rich and the specific impulse at minimum thrust is reduced.

The total penalty during throttling (*down*) is due to the lower nozzle expansion ratio (penalty *up*) combined with the lower pressure ratio and eventual mixture ratio shift (penalty *up-down*).

As maximum chamber pressure is increased the penalty related to the thrust coefficient is reduced. Therefore, the relative impact of the mixture ratio shift is lower at lower pressures. Considering that in general a motor operating at low pressures is simpler and cheaper, mitigating the mixture ratio shift with added complexity for throttleable motors is more indicated for high performing motors operating at high pressures. Moreover, because high throttling ratios mean lower minimum pressures and higher mixture ratio shifts, again the combination high pressure - mixture ratio control is more interesting for very high throttling ratios. In several ways, a mixture ratio-controlled hybrid rocket is in-between a classical hybrid and a liquid.

The ratio between the maximum and minimum thrust is higher than the variation of the oxidizer mass flow, mainly due to the effect of the thrust coefficient (see Fig. 9-10). As the back pressure is reduced with altitude the thrust ratio approaches the oxidizer mass flow ratio except for the effect of the mixture ratio shift (and a smaller effect of chamber pressure variation). The global effect is more pronounced for low initial chamber pressures.

The previous analyses could vaguely represent a rocket motor that has to perform a sort of propulsive re-entry/landing.

In case of a booster during ascent it is interesting to calculate the specific impulse and the maximum throttling ratio of a throttleable motor using the same nozzle of a non-throttleable motor. In this case the throttling ratio is determined by separation of the flow on the over-expanded nozzle. The results for the case with constant mixture ratio are shown in Fig. 11-13, while those for the case of a classical hybrid with n=0.5 are shown in Fig. 14-16.

With the constraint of a nozzle adapted for maximum thrust no throttling is possible at sea level.

As the altitude increases and the back pressure decreases the minimum chamber pressure and consequently the thrust can be safely reduced (see Fig. 11 and 14).

In Fig. 12 and 15 the thrust with altitude for different oxidizer throttling levels is plotted. The intersection of each curve with the envelope curve determines the minimum altitude where the corresponding level of throttling can occur.

In Fig. 13 and 16 the specific impulse with altitude for different oxidizer throttling levels is plotted. Again, the intersection of each curve with the envelope curve determines the minimum altitude where the corresponding level of throttling can occur.

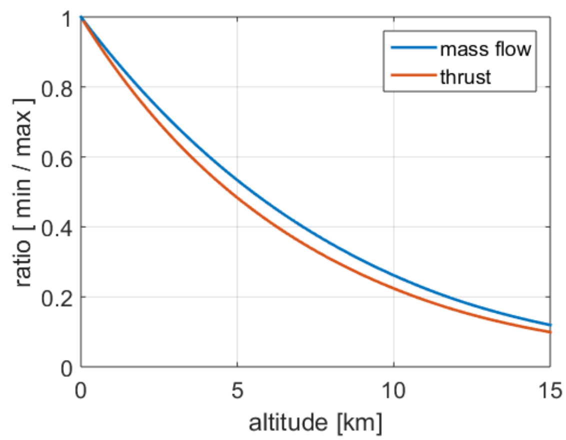

Fig. 11. Minimum thrust with altitude of a throttleable motor with the same nozzle as a non-throttleable one. Constant mixture ratio. Initial chamber pressure: 50 bar.

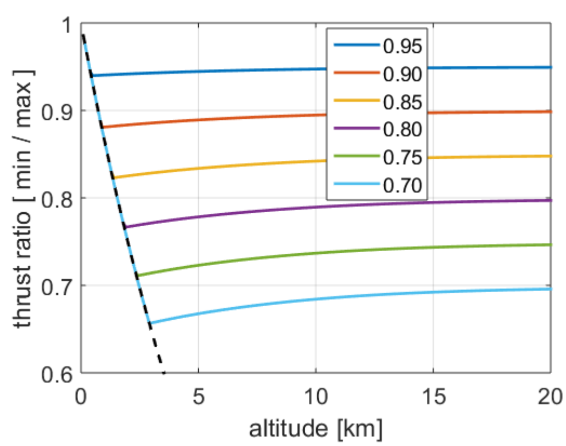

Fig. 12. Thrust with altitude of a throttleable motor. Constant mixture ratio. Parametric with throttling ratio. Initial chamber pressure: 50 bar.

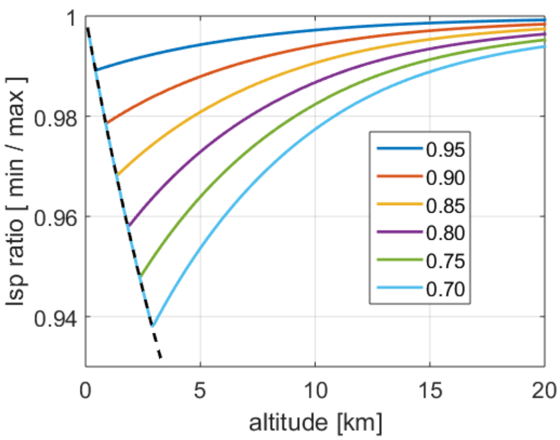

Fig. 13. Specific impulse with altitude of a throttleable motor. Constant mixture ratio. Parametric with throttling ratio. Initial chamber pressure: 50 bar.

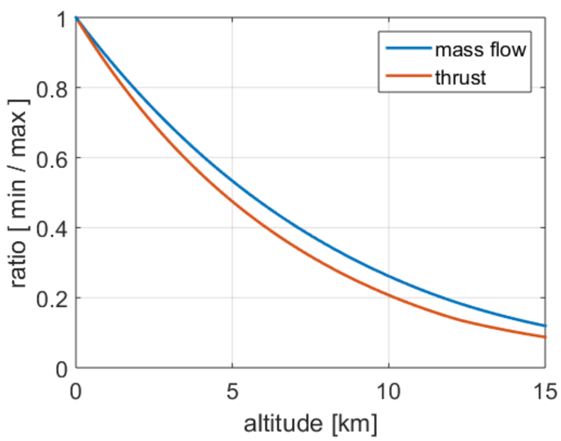

Fig. 14. Minimum thrust with altitude of a throttleable motor with the same nozzle as a non-throttleable one. Hybrid with n=0.5. Initial chamber pressure: 50 bar.

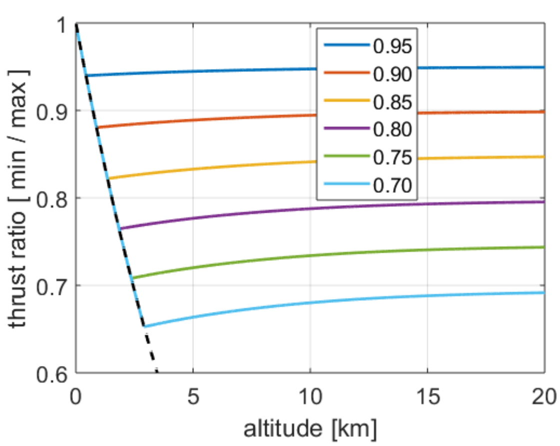

Fig. 15. Thrust with altitude of a throttleable motor. Hybrid with n=0.5. Parametric with throttling ratio. Initial chamber pressure: 50 bar.

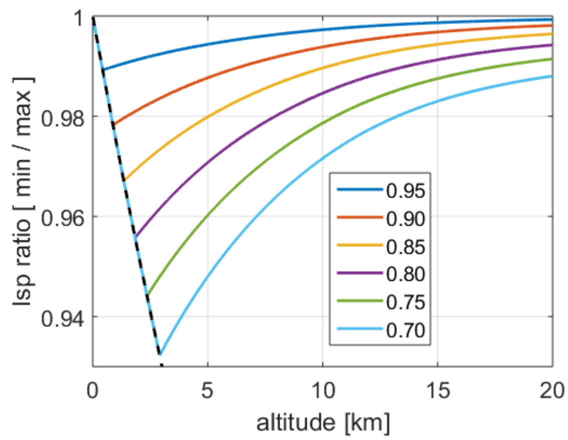

Fig. 16. Specific impulse with altitude of a throttleable motor. Hybrid with n=0.5. Parametric with throttling ratio. Initial chamber pressure: 50 bar.

It is possible to see that, unless the initial chamber pressure is too low, a typical throttling down at altitude (10-15 km for common launchers) to limit the maximum dynamic pressure induces only a small penalty on the thrust coefficient. In this case of limited throttling, also the penalty related to the mixture ratio shift is relatively small, so the usefulness of the added complexity of a mixture ratio control is debatable.

The mixture ratio shift has a significant impact on several aspects: not only a penalty on the specific impulse but also a change in the exhaust characteristics, affecting nozzle erosion and the plume behaviour (heat transfer, signature, spacecraft contamination, air pollution…).

Moreover, if the required thrust profile is not known in advance, a non-predicted mixture ratio shift will leave propellant residuals. The total impulse (and ΔV) penalty from residuals generation can become much larger than the one related to the specific impulse losses if the foreseen thrust profile could have unexpected large variations. The worst case for a defined throttling level capability is to consider that the motor could possibly operate the whole burning time both at maximum or minimum thrust. For the typical high optimal mixture ratio occurring in hybrid propulsion the best residuals mitigation is to design the system to always consume all the oxidizer and to have a fuel margin for any situation [47]. Therefore, the design mixture ratio should be the minimum one, which is reached at minimum thrust. Thus, no residuals are obtained at minimum thrust, while the maximum amount of (fuel) residuals is generated at maximum thrust and its (relative) value is:

$$res_f = 1 - \left(\frac{\varphi_{min}}{\varphi_{max}}\right) = 1 - \left(\frac{\dot{m}_{ox\,min}}{\dot{m}_{ox\,max}}\right)^{1-n} \quad (3)$$

where the second part is valid in case of no pressure dependency of the regression rate. From the previous discussions, for a pressure dependency, the amount of residuals is lower. The relative value of residuals referred to the total propellant is:

$$res_{tot} = \frac{res_f}{\varphi_{min}+1} \quad (4)$$

Since the maximum residuals penalty is generated at maximum thrust, as a compensation the optimal mixture ratio can be fixed at this point. In this way the maximum thrust has only the (maximum) residuals penalty while the minimum thrust has only the (maximum) specific impulse penalty. The result is:

$$res_{tot} = \frac{1-\left(\frac{\dot{m}_{ox\,min}}{\dot{m}_{ox\,max}}\right)^{1-n}}{\left(1+\varphi_{max}\left(\frac{\dot{m}_{ox\,min}}{\dot{m}_{ox\,max}}\right)^{1-n}\right)} \quad (5)$$

For all the intermediate levels the total impulse penalty is the combination of the specific impulse penalty and the residuals penalty.

The residuals (at maximum thrust) for different throttling ratios and *n* exponents at an optimal mixture ratio of 7.5 are plotted in Fig 17 and 18.

The figures show the significant penalty incurred if high throttling ratios are required but their relative duration on the total burning time is completely unknown.

However, this is an extreme situation as in most applications there is a certain degree of knowledge of the expected thrust profile. For very complex missions a statistical analysis (e.g. Monte Carlo) of the possible scenarios could be performed in order to find the optimal solution that minimize the average or maximum (depending on requirements) total impulse (or ΔV) losses in a certain confidence interval.

Anyway, a mixture ratio control becomes attractive for random high throttling ratios.

Another source of residuals is related to unexpected mixture ratio shift due to manufacturing tolerances, parameters uncertainties and motor performance variations from burn to burn [47].

However, in this case a good design (including residual management) and a consistent manufacturing/quality control could limit the penalty to very low values [47] and so a mixture ratio control seems not worthy.

In order to control the mixture ratio in real-time possible solutions include a combination of direct and indirect measurements, like oxidizer mass flows, chamber pressure, regression rate [48, 49] and mixture ratio [50].

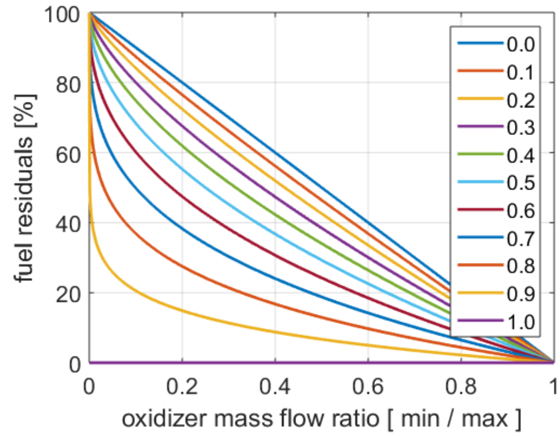

Fig. 17. Fuel residuals for different (unpredicted) throttling levels. Parametric with *n*.

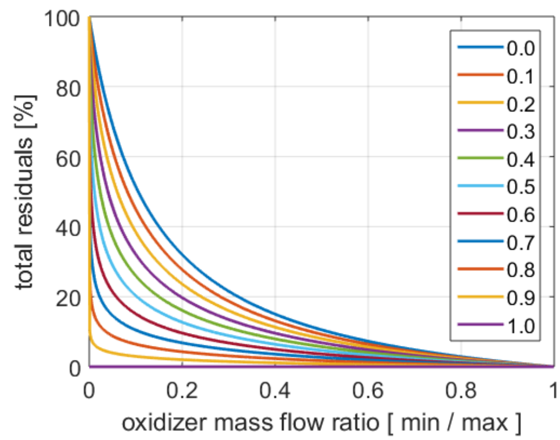

Fig. 18. Total residuals for different (unpredicted) throttling levels. Parametric with *n*.

3. AOIM

The first proposed way to control the mixture ratio in a hybrid rocket motor is the injection of part of the oxidizer in the aft-chamber. This solution requires adding a further (throttleable) feedline and injection system to the aft-chamber:

$$\dot{m}_{ox\,tot} = \dot{m}_{ox\,p} + \dot{m}_{ox\,aft} \quad (6)$$

Only the port oxidizer mass flow affects the fuel regression rate, while the other can be used for the mixture ratio compensation.

The injection in the aft-chamber presents some peculiarities respect to the one at the head-end of the motor. In fact, from one (positive) side the oxidizer is injected in a very hot environment, enhancing vaporization and reactivity. On the other (negative) side there is less time/space for combustion to occur. The final result has to be evaluated case by case depending on design solutions, propellant combination and so on.

Compared to a classical hybrid the number of feed lines is doubled, however, the system retains some of the hybrid advantages compared to a liquid. Again, it can be thought as a sort of an intermediate step between the two.

A related but different technology that has also attracted some limited attention is the aft injected gas generator hybrid. In this case the oxidizer is injected on the exhausts of a (throttleable) solid propellant fuel-rich gas generator [51].

It is interesting to calculate the amount of oxidizer to be injected both in front and to the rear of the motor for different throttling levels (see Fig. 19-20).

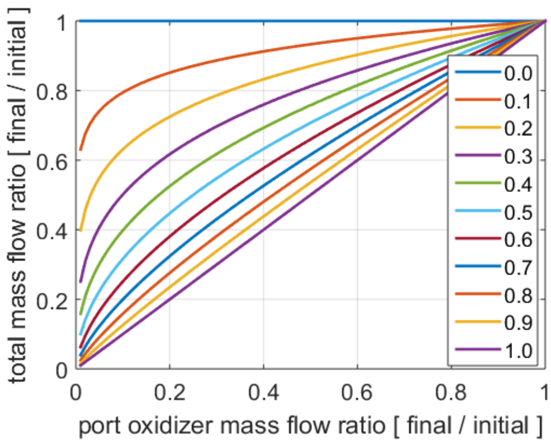

Fig. 19. AOIM total throttling ratio for different port throttling levels. Parametric with *n*.

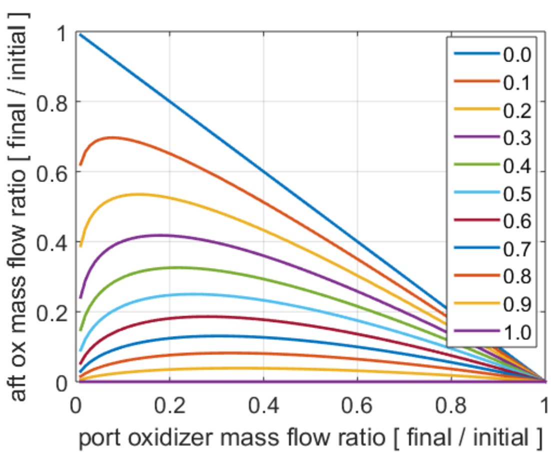

Fig. 20. AOIM aft injection for different port throttling levels. Parametric with *n*.

As the port oxidizer mass flow is reduced, the fuel mass flow remains slightly higher due to the *1-n* shift in the port mixture ratio. In order to keep the global mixture ratio at the optimal value, some oxidizer has to be injected in the aft-chamber. The final result is that the total mass flow (and thrust) decrease is lower than the port one. In particular, for *n*=0.5 (i.e. 1-*n*=0.5), the thrust variation is only the square root of the oxidizer mass flow variation in the port. Consequently, in order to obtain a certain throttling ratio, the oxidizer mass flow in the port has to be throttled by the second power of the target throttling ratio (e.g. 1:25 for a 1:5 global result).

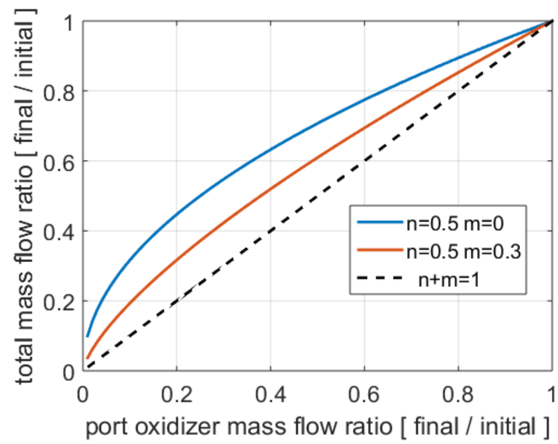

Fig. 21. AOIM total throttling ratio for different pressure dependencies.

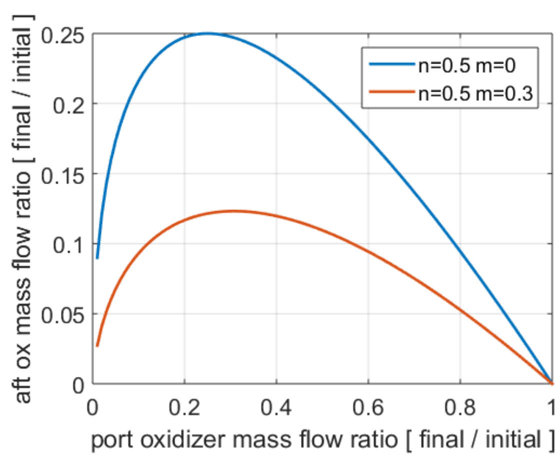

Fig. 22. AOIM aft injection for different pressure dependencies.

It is also worth noting that, as the motor is throttled down, the mass flow in the aft chamber has to be initially increased, then reaches a maximum value and finally starts to drop as the aft-injected oxidizer mass flow approaches the total one. For *n*=0.5 the maximum aft-chamber oxidizer mass flow for throttling is 25% of the total one. This means that while a classical hybrid needs an injector plate sized for 100% of the mass flow, in the full-throttleable AOIM case there are two injection plates, one sized for 100% of the mass flow and the other for 25%.

This confirms again that the positive features of the AOIM solution do not come for free in terms of weight and costs. As the value of n approaches zero the technique becomes unfeasible.

In case of a pressure dependency the results improve because the mixture ratio shift in the port is reduced (see Fig. 21-22).

The AOIM can be used also to compensate the mixture ratio shift with time (see Fig. 23-28).

For $n<0.5$ the port oxidizer flow has to decrease in order to reduce the upshift of fuel mass flow. Meanwhile, for compensation, further oxidizer needs to be injected in the post chamber.

On the opposite, for $n>0.5$ the port oxidizer flow has to increase in order to reduce the downshift of fuel mass flow. Meanwhile, for compensation, the oxidizer injected in the post chamber needs to be reduced.

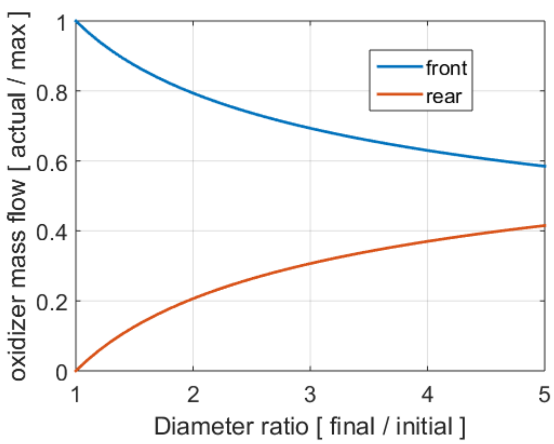

Fig. 23. AOIM oxidizer mass flows for constant mixture ratio with time, n=0.4.

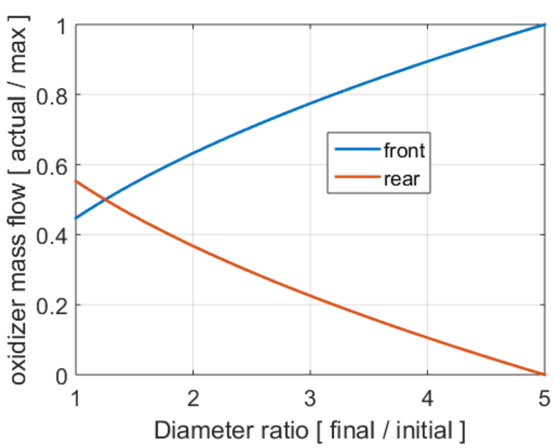

Fig. 24. AOIM oxidizer mass flows for constant mixture ratio with time, n=0.6.

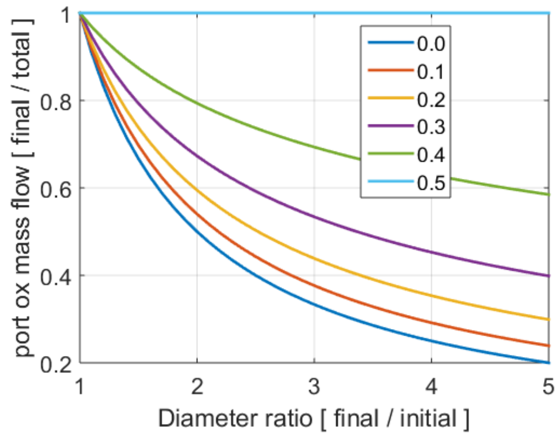

Fig. 25. AOIM forward injection for constant mixture ratio with time, parametric with $n \leq 0.5$.

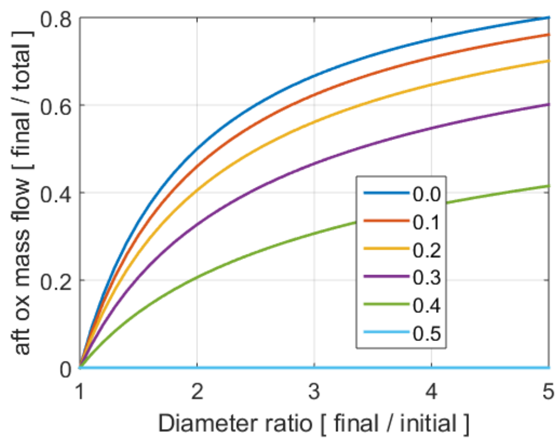

Fig. 26. AOIM aft injection for constant mixture ratio with time, parametric with $n \leq 0.5$.

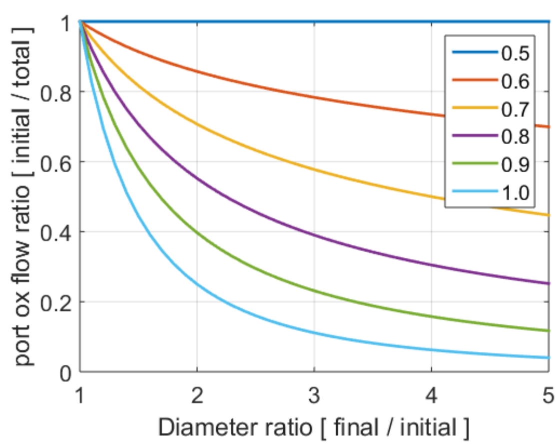

Fig. 27. AOIM forward injection for constant mixture ratio with time, parametric with $n \geq 0.5$.

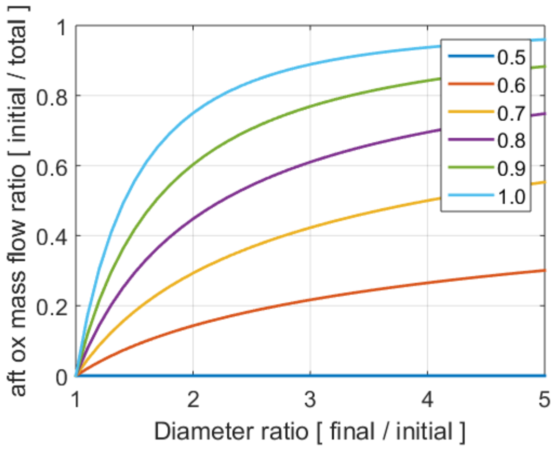

Fig. 28. AOIM aft injection for constant mixture ratio with time, parametric with $n \geq 0.5$.

As the diameter ratio increases and/or *n* depart from 0.5, both feed-systems tend to require to be sized for the full mass flow.

The port mixture ratio varies always in the same way as in the classical hybrid with an amplification due to the required counter-throttling. For this reason, while the mathematical modelling could work, it is always important to keep in mind that too large variations of the port mixture ratio could affect combustion efficiency and grain consumption uniformity.

Moreover, both injection systems (rear and forward) need to work properly in a wider and wider range as the diameter ratio increases and/or *n* depart from 0.5.

Finally, it is important to remember that the port oxidizer mass flow must be bounded between specific limits [44].

Concluding, while the AOIM has the capability to avoid the troublesome mixture ratio shift, it requires a more complex plumbing and design and cannot be stretched to the limits (i.e. too high throttling ratios at low *n* or too large diameter ratios for *n* far from 0.5).

It is interesting also to investigate what happen if, in order to simplify the system, the AOIM is employed with only one throttleable feedline (the other providing a constant mass flow).

If the controlled valve is the rear one, it is easy to determine that the behaviour with throttling is the same as a classical hybrid with n=0 (no matter the *n* of the fuel, as the front oxidizer flow is constant), so it is not useful.

On the contrary if the controlled valve is the forward one, it is possible to demonstrate that the mixture ratio shift is reduced, and as the front oxidizer mass flow reaches a lower threshold a reversal occurs (see Fig. 29).

In fact, as the port oxidizer flow is reduced, also the fuel mass flow decreases but, because the fuel reduction is sublinear, in a classical hybrid the mixture ratio goes down. In this case, on the contrary, at a certain point the mixture ratio is dominated by the ratio of the constant rear oxidizer flow divided by the decreasing fuel mass flow, and so the mixture ratio starts to go up. So, for moderate throttling levels this could also considered as an option in-between a classical hybrid and a full AOIM. As in the full AOIM also in this case there is a reduction of sensitivity (or gain) between the input (frontal oxidizer throttling) and the output (thrust variation).

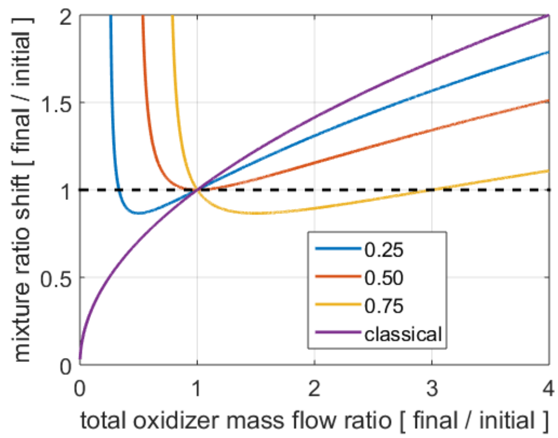

Fig. 29. AOIM with only forward injection throttling. Parametric with initial ratio between rear and front oxidizer mass flow; n=0.5.

Regarding the mixture ratio shift with time, it is interesting to calculate what is the required thrust variation in order to maintain a constant mixture ratio (i.e. the mixture ratio shift with throttling is used to compensate the shift with time). The results for a classical hybrid and the AOIM with only aft-throttling are shown in Fig. 30.

In a classical hybrid for n<0.5 the fuel mass flow increases with time. To keep the mixture ratio constant, it is necessary to throttle up. This behaviour has little practical application. On the contrary for n>0.5 the fuel mass flow decreases with time and so, to keep the mixture ratio constant, it is necessary to throttle down. For *n* moderately above 0.5, a decreasing thrust profile could have some practical application, for example in sounding rockets [52]. Obviously, for *n* approaching 1 (throttling insensitivity) the solution collapses.

The AOIM with only aft-throttling has a similar behaviour but is able to keep the mixture ratio constant with a lower thrust variation (or higher *n* and/or higher diameter ratio).

In both cases a pressure dependency (e.g. metal addition) will worsen the results (as the throttling sensitivity is reduced as $n+m$ approaches 1).

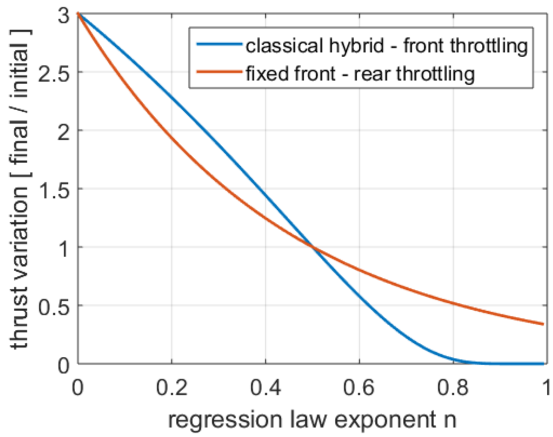

Fig. 30. Thrust variations for constant mixture ratio: classical hybrid vs AOIM with only rear injection throttling. Diameter ratio = 3.

The AOIM with only forward-throttling has a very complex nonlinear behaviour with regions of multiple solutions and regions of no-solutions. Even if the results are fascinating mathematically, they are of little practical value, so, for the sake of brevity, they are not reported here except for one case to mention. For $n<0.5$ but near to it and an important initial aft-injected oxidizer mass fraction, the mixture ratio can be kept constant with a moderately regressive thrust profile.

4. A-SOFT

Another alternative way to control the mixture ratio in a hybrid rocket motor has been proposed by Shimada et al. [38-43] and is called A-SOFT. The basic idea is that the mixture ratio shift can be compensated acting in real time on the *a* value of the fuel regression rate. It is now well known that swirl injection can significantly increases hybrid regression rate [53-59]. So, if the injection swirl number can be changed on demand, the regression rate can be adjusted in order to maintain a constant mixture ratio with time and/or throttling.

To accomplish this, again, two (throttleable) feed systems are required, this time both placed in the forward end of the motor, one axial and the other swirled:

$$\dot{m}_{ox\ tot} = \dot{m}_{ox\ p} = \dot{m}_{ox\ ax} + \dot{m}_{ox\ tan} \quad (7)$$

Balancing the amount of axial vs swirl injected oxidizer mass flow is potentially possible to calibrate the required value of swirl number and, consequently, regression rate.

The fuel regression rate in the A-SOFT configuration is described by the following equation [43]:

$$\dot{r} = a' G_{ox}^n = a(1 + S_e^2)^m G_{ox}^n \quad (8)$$

The effective swirl number is related to geometric swirl number of the tangential injector trough the ratio between the axial and tangential oxidizer mass flows:

$$S_e = S_g / (1 + \dot{m}_{ox\ ax} / \dot{m}_{ox\ tan})^2 \quad (9)$$

If only the axial injection is activated, the effective swirl number is equal to zero, on the contrary, if only the tangential injection is activated, the effective swirl number is equal to the geometric swirl number of the tangential injector. However, it is not always possible to close completely a feedline, so in general:

$$0 \le S_{min} \le S_e \le S_{max} \le S_g \quad (10)$$

In particular, a minimum amount of swirl number could be necessary in order to guarantee sufficient level of combustion stability and efficiency.

For the swirl injection to be effective, at least the tangential oxidizer needs to be gasified. This requires something like a catalyst, a gas generator or a regenerative heat exchanger. In practice, once such device is necessary, it is generally more convenient to gasify the whole oxidizer mass flow.

The exponent *m* (not to be confused with the previous pressure dependency) determines the sensitivity of the regression rate to a change in the effective swirl number. For large swirl number the dependency tends to become proportional to *2m*.

In [54] it was demonstrated that a good correlation based on physical basis was obtained considering $2m \approx n$, which gave a final dependency around 0.555. In [53] and [60] a nearly linear dependency was found (i.e. $2m \approx 1$), at least in the range of swirl numbers explored.

However, Ozawa [61] determined a much weaker dependency $2m \approx 0.2$ ($m \approx 0.1$).

It is worth noting that classical swirled experiments like in [53-60] used different swirl injectors from test to test, while Ozawa [61] experiments used a fixed two-feeds mixed axial/tangential injection with different relative mass flows that is more representative of a real A-SOFT.

Anyway, since the value of *m* has a deep impact on the A-SOFT behaviour (as it will be shown in the following), it is necessary to perform further research on this aspect.

It is now worth investigating the behaviour of the A-SOFT during throttling (see Fig. 31-34).

In Fig 31-34 a minimum Swirl number of 2 has been fixed in order to guarantee good motor performance at the lowest thrust.

In Fig. 31-33 the same regression rate law as Shimada has been used (n=0.65 m=0.1).

In Fig. 34 the value of *m* has been increased to 0.5 for comparison.

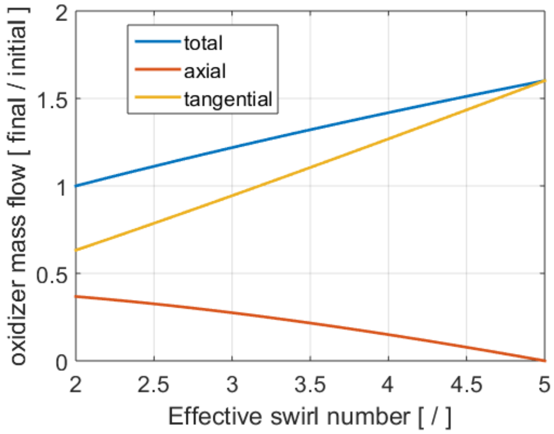

Fig. 31. A-SOFT throttling behaviour. n=0.65 m=0.1. S_{min} = 2. S_{max} = S_g = 5.

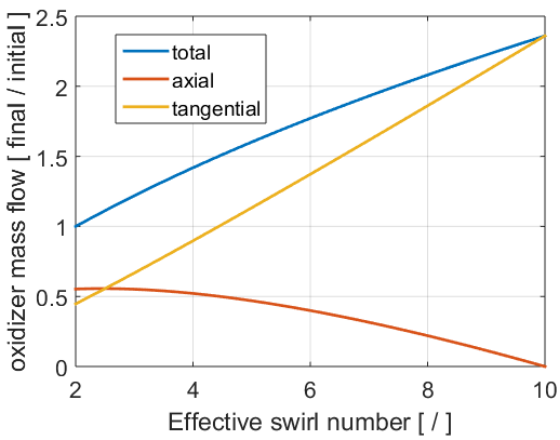

Fig. 32. A-SOFT throttling behaviour. n=0.65 m=0.1. S_{min} = 2. S_{max} = S_g = 10.

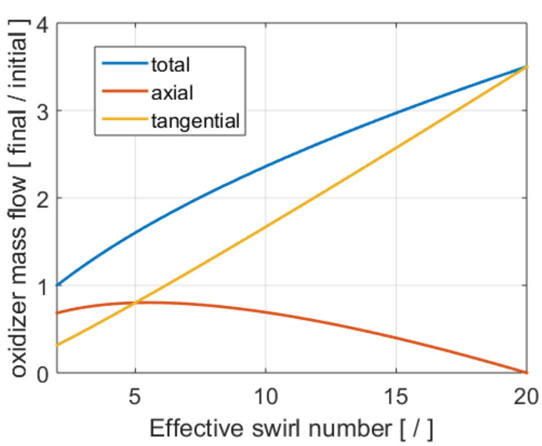

Fig. 33. A-SOFT throttling behaviour. n=0.65 m=0.1. S_{min} = 2. S_{max} = S_g = 20.

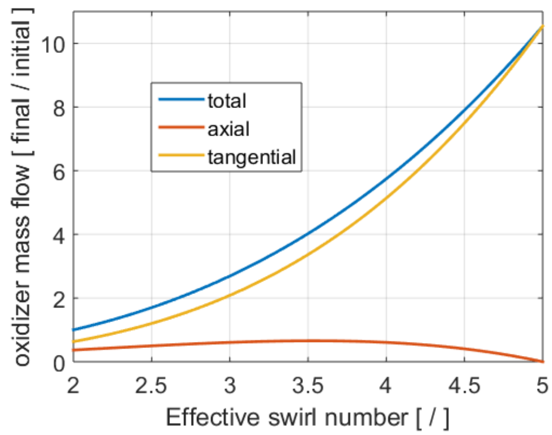

Fig. 34. A-SOFT throttling behaviour. n=0.65 m=0.5. S_{min} = 2. S_{max} = S_g = 5.

Since in a classical hybrid throttling up will increase the mixture ratio, in an A-SOFT it is necessary to increase the swirl number in order to further boost the regression rate. Consequently, throttling up is performed increasing the tangential oxidizer mass flow. At the beginning, the resulting effect is even excessive, so also the axial oxidizer mass flow has to be increased. After a certain effective swirl number, the effect weakens and so the axial mass flow has to be decreased in order to further boost the effective swirl number.

The point of maximum axial mass flow shifts to the left as the value of S_{max} and/or m is reduced.

The maximum swirl number has been set equal to the geometric swirl number of the tangential injector, thus at maximum thrust the axial flow ceases and all the oxidizer is injected tangentially. On the opposite, at minimum thrust there is still some tangential flow (as S_{min}>0).

Increasing S_{max} allows to obtain higher throttling ratios. The required maximum axial mass flow is always lower than the minimum total oxidizer mass flow (it will be equal for S_{min}=0).

The effect of m is to increase the sensitivity to the tangential throttling, therefore with a much lower variation of swirl number it is possible to obtain significantly higher throttling ratios.

The maximum value of S_{max} is limited by several reasons. For example, increasing the swirl number has a strong influence on the nozzle discharge coefficient (decrease) and a detrimental effect on the specific impulse, the latter particularly at lower nozzle expansion ratios [62-65] and higher back pressures.

Moreover, everything else kept equal, increasing the swirl number will increase the injection velocity and therefore Mach number, requiring a higher injection pressure drop.

Both effects are significantly reduced with an increase of chamber pressure (i.e. combustion chamber contraction ratio). Therefore, motors operating at higher chamber pressures are allowed to reach higher maximum swirl numbers.

Another aspect to take into account is the increase of nozzle heat transfer/erosion with the swirl number.

It is now interesting to evaluate the behaviour of the A-SOFT with time (see Fig. 35-36).

Even with a weak swirl dependency m=0.1 and a small maximum swirl number of 5 the A-SOFT is able to smoothly hold constant the mixture ratio with only relatively small adjustments of the two oxidizer mass flows.

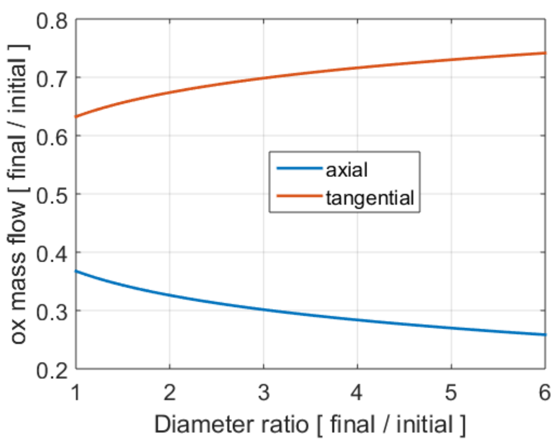

Fig. 35. A-SOFT behaviour for constant mixture ratio with time. n=0.65 m=0.1. S_{min} = 2. S_{max} = S_g = 5.

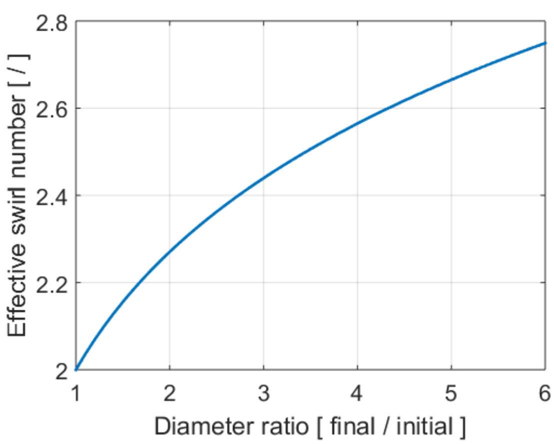

Fig. 36. A-SOFT effective swirl number for constant mixture ratio with time. n=0.65 m=0.1. S_{min} = 2. S_{max} = S_g = 5.

It is now interesting, again, to briefly discuss what happens if, for simplicity, only a single feedline has a throttleable valve. For throttling purposes, the feedline to control is the tangential one. In fact, in a classical hybrid, throttling up the oxidizer flow will increase the mixture ratio. Throttling up the tangential oxidizer flow will increase the effective swirl number, boosting up the regression rate. The final results are plotted in Fig. 37.

The single valve A-SOFT is able to mitigate the mixture ratio shift during throttling and, for large S_{max} and/or m and/or n, to initially obtain the opposite trend as the increase in swirl number prevails on the standard mixture ratio shift. For specific combinations of n, m and S_{max} a relatively flat behaviour can be obtained for a certain range of throttling ratios.

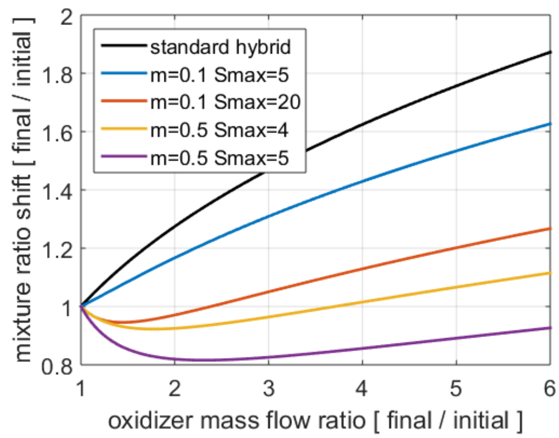

Fig. 37. A-SOFT with only tangential throttling. n=0.65, S_{min} = 2.

On the contrary, throttling only the axial feedline will behave in the opposite way, enhancing the original mixture ratio shift of the standard hybrid.

As in the general case, the opposite trend occurs considering the mixture ratio shift with time. In fact, the relatively throttling-insensitive single tangential valve A-SOFT has no possibility to effectively compensate the mixture ratio shift with time.

On the contrary, if the fuel mass flow decreases (increases) with time, a relatively small decrease (increase) in the axial oxidizer flow will have two synergistic mechanisms (swirl number and throttling) for mixture ratio compensation. The final result is qualitatively similar to the standard hybrid of Fig. 30 but quantitatively better.

5. Comparison between AOIM and A-SOFT

The AOIM and A-SOFT have both the potential ability to keep constant the mixture ratio during throttling and with time. Both also require (for optimum performance) two controlled feedlines and injection systems. However, as highlighted by the previous analyses, several differences are also present.

From an architectural point of view the AOIM requires the injection of oxidizer in the (relatively far) post-chamber with all its foreseen complexity and relative lack of experience in the literature. On the contrary, the two feedlines and injector plates of the A-SOFT could be better integrated in an almost single item in the front head of the hybrid motor in a similar way as in liquid engines (perhaps also exploiting additive manufacturing), with less departure from a well-tested conventional swirled hybrid configuration.

The A-SOFT requires gaseous injection while the AOIM not necessarily, however, for high throttling ratios it is easy to work with gas injection even for the AOIM (or any classical hybrid) due to concerns over atomization/vaporization.

Regarding the behaviour, previous simulations showed that the AOIM tend to be more attractive for large throttling ratios than the A-SOFT, particularly if the A-SOFT is limited in terms of m and S_{max}. This is a reason for the need to better investigate these two parameters in A-SOFT motors.

On the contrary the A-SOFT can deals much more easily with the mixture ratio shift with time; this is due to the fact that the port oxidizer mass flow is kept constant, while in the AOIM it is changed in the wrong direction (in terms of port mixture ratio) to control the fuel mass flow and needs a higher (aft-injected) counter-compensation.

In any case it is important to remember that the previous calculations considered the behavior with time and throttling independently. As a throttleable motor needs still to work for a specific burning time, in practice the two effects result combined, each one using part of the allowable oxidizer feed throttling range. Therefore, in any situation, a high degree of compensation with time will reduce the margins for throttling and vice versa. That's why, again, n=0.5 can be in general somehow preferable also for throttling because it keeps the maximum and minimum possible thrusts constant with time.

In [43], Ozawa has drawn a graph showing the regions of applicability of the two technologies. However, the graph considered the same grain dimensions, i.e. port diameter and length. For this reason, the A-SOFT, with its possible higher regression rates operates for larger thrusts and the axial AOIM for lower thrusts. Unfortunately, in practice there is a limit in the maximum port oxidizer flux. Considering in first approximation the same port diameter and the same maximum oxidizer mass flux, the two motors can deliver the same maximum thrust, with the swirled one requiring a much shorter grain according to the following equation:

$$L_{f\,A-SOFT} = L_{f\,AOIM} / (1 + S_{max}^2)^m \qquad (11)$$

Modifying Ozawa figure for different grain lengths and same maximum oxidizer flux (and flow) the following Fig. 38-39 are obtained.

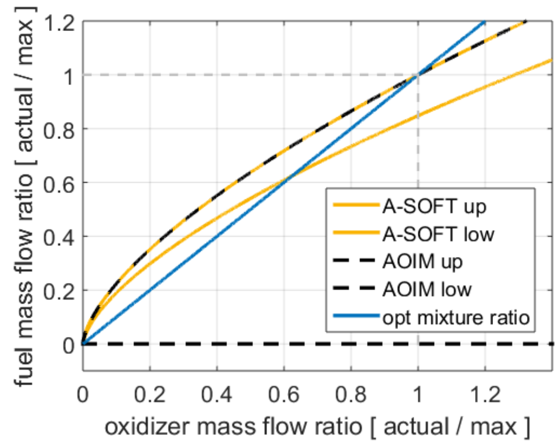

Fig. 38. A-SOFT vs AOIM. Same max oxidizer flux, different fuel grain lengths. n=0.65, m=0.1. S_{min} = 2. S_{max} = S_g = 5.

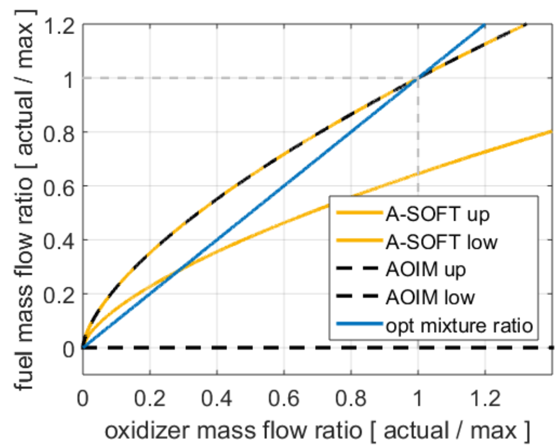

Fig. 38. A-SOFT vs AOIM. Same max oxidizer flux, different fuel grain lengths. n=0.65, m=0.1. S_{min} = 2. S_{max} = S_g = 20.

The result is the same upper bound for both technologies and an extended lower bound (theoretically up to zero) for the AOIM. Obviously, as S_{max} and/or m increase, the A-SOFT lower bound also approaches zero.

In [43] it is claimed that a swirled AOIM will combine both regions of operation; however, with the current hypothesis of same maximum oxidizer flux limit the swirled AOIM will behave exactly as an axial AOIM (i.e. better throttling and worse time behavior compared to A-SOFT) except for a shorter grain length (and potentially better baseline combustion efficiency and stability).

6. Other options

One alternative way to deal with the issues of motor throttling is to avoid the need of throttling in the first instance. For example, instead of throttling the motor, a similar effect can be achieved by a series of constant thrust pulses, a technique called Pulse Width Modulation (PWM). Compared to throttling the motor, in this case there is no mixture ratio shift, no chamber pressure and thrust coefficient reduction, no changes in the exhaust conditions, no need of complex plumbing and injection designs able to operate in a wide range. What is required is a multiple ignition capability, another possible feature of hybrid rocket motors.

One drawback is a slight specific impulse penalty related to the phases of thrust rise and decay during ignition and shutdown, respectively. Base drag increase during coast phases has also to be taken into account, together with the inability to use a thrust vector control for manoeuvres during periods of motor off. Moreover, for few pulses, a non-optimized flight profile could induce trajectory losses, while for a high number of pulses the impact of the thrust ramps/tails becomes non-negligible.

Hybrids tend to have slower transients compared to liquids of the same thrust because of the larger combustion chamber size, the thermal lag in the fuel grain and, for ignition, the generally lower reactivity of the solid fuel, thus they are not best suited for high frequency pulses.

In a hybrid rocket, throttling up and down is generally faster and precise than the complete start and stop sequence, so a throttleable motor is more favourable for precise control (e.g. soft-landing).

Another option is to use a cluster of smaller motors or combustion chambers and operate only a certain number of them to achieve different thrust levels. Again, no issues related to motor throttling are present, even if only certain finite thrust levels can be achieved depending on the number of motors and their relative thrusts. If the thrust levels are only descending and not separated in time, only thrust termination is required, on the contrary, if they are also upward and/or spaced in time the re-ignition capability is again necessary.

Issues of thrust misalignment constrain the arrangement and the possible combinations of thrusters' activation in order to keep symmetry respect to the centre of mass.

For equal motors, clustering generally brings development and production advantages respect to an equivalent single larger unit at the expense of a penalty in the inert mass fraction. If the motors have different sizes more combinations of thrust levels can potentially occur for the same total number of motors at the expense of a higher system complexity.

It is important to remember that hybrid propulsion is less flexible than liquid propulsion in this regard. In fact, equal liquid motors of the same thrust can deliver different total impulses as they are fed for less or more time. On the contrary, hybrid motors are designed to deliver a certain total impulse determined by the amount of fuel placed in the combustion chamber.

A technique used in solid propulsion called off-loading could allow for a slight variation in the amount of fuel casted, but its range is limited.

Moreover, in case of unpredicted required thrust profiles, a cluster of liquid engines can always deplete all its propellants, while in a hybrid rocket the unspent fuel in a combustion chamber cannot be used by the others, limiting the flexibility of this approach.

A combination of the techniques exposed in this chapter and conventional throttling can also be foreseen in order to mitigate the motor throttling requirements. A typical example is the SpaceX Falcon 9 booster, which has 9 slightly throttleable Merlin engines and only one is used on landing (and 3 during re-entry). Moreover, the booster lands thanks to a manoeuvre called hover slam, where the thrust of the rocket is superior to the weight of the system and so, after slowdown, only careful timing of motor shutdown allows for safe touchdown. This is in contrast with Blue Origin New Shepard suborbital vehicle that requires a highly throttleable engine to perform controlled hovering before touchdown.

7. Deep throttling

Other than the mixture ratio shift, hybrid throttling comes with issues in common with liquid engines, like the need to keep a high combustion efficiency and stability in a wide range of mass flows.

In particular, properly controlling the mass flow and the injection characteristics are two key elements for a throttleable system.

Regarding the first point, if the mass flow is controlled with a simple valve, a reduction of the mass flow is obtained with a significant increase of pressure losses in the feedline.

On the opposite, a proven successful technique to control the mass flow in a large range with limited pressure drop is the variable are cavitating venturi or cavitating pintle [28, 29, 66-68]. A cavitating venturi is used to decouple the mass flow from chamber pressure, with a benefit in combustion stability and ease of control. Varying the throat area through a movable pintle allows a wide variation of mass flow for the same upstream pressure.

Regarding the issue of injection performance, a decrease of the mass flow through a fixed injector will cause a reduction in the injection pressure drop, negatively affecting atomization and injector stiffness.

The main solutions for hybrid throttling that have been considered are briefly presented in the following subsections. All of them are somewhat borrowed from liquid propulsion [69].

7.1 Simple fixed geometry injector

The easiest way to throttle a hybrid motor is to simply vary the mass flow through a valve without any other changes. However, this option, even if attractive for its simplicity, as highlighted before, brings a reduction in the injector pressure drop with a potential loss of injection performance that could affect motor stability and efficiency, unless the initial pressure drop is very high.

Whitmore [23, 24] showed a remarkable 66:1 turndown ratio on a lab-scale motor using nitrous oxide as oxidizer. However, the scale of the motor was small and the injection was certainly improved by the two-phase flow of nitrous oxide and the very high pressure drop. In the general case, based also on the liquid literature, this technique is not best suited for large throttling ranges of high performing systems.

7.2 Dual manifold

This technique foresees the use of multiple (generally two) injector regions that are controlled independently in order to partialize the oxidizer flow through the closure of one or more lines.

This solution has been tested successfully in several occasions, for example by ONERA [70, 71], UTC [10-12] and in the JIRAD program [20], reaching throttling ratio as high as 10:1.

An analogous technology has been developed at Stanford University for the Peregrine sounding rocket [72, 73]. A custom-made throttling plate that mates to the injector face inside the injector manifold is rotated to control the oxidizer mass flow rate between 50-100% of the nominal value.

This kind of solutions are not best suited for continuous throttling but they are particularly attractive for boost-sustain thrust profiles. In this case, in order to compensate the mixture ratio shift, an interesting idea is to use a dual concentric grain composed by a high regressing fuel (e.g. paraffin) in the inner core and a lower regressing fuel (e.g. a classical plastic) in the outer periphery in order to match the mixture ratio during both phases [74, 75].

7.3 Gas injection

This technique foresees the injection of a small quantity of gas in order to increase the pressure drop and improve atomization at reduced liquid mass flows.

An evident drawback is the added complexity and weight. Moreover, if the gas is inert, it can negatively affect propulsion performance. Better results could be achieved when the system is already pressure-fed with a gaseous oxidizer (e.g. gaseous oxygen [76]) that can be spilled for this purpose.

This technique lost the competition with the pintle injector for the Lunar Module during the Apollo program. In the hybrid field it was tested by Waidmann [77]. It has been claimed that helium-injection has been also used on SpaceShipTwo RocketMotorTwo for stability reasons [78].

7.4 Variable area injector

This technique foresees the use of a movable pintle injector in order to guarantee good atomization in a wide range of mass flows (even down to shut-off). This solution has been tremendously successful in liquid propulsion as demonstrated by the Moon Landings [79]. On the opposite, in the hybrid field it has been proposed sometimes [80] but little activity has been performed, particularly in regard to throttling behaviour.

7.5 Gaseous injection

This technique foresees the injection of a gaseous oxidizer. In this way the problem is by-passed in advance. In fact, gaseous injection is much more robust and insensitive than liquid injection, as it naturally skips the complex steps of atomization/vaporization. Anyway, as the oxidizer is stored more efficiently in liquid form, this solution requires a way to gasify the oxidizer. As before, possible means to achieve this are the use of a catalyst, a gas generator or a regenerative heat exchanger. However, for the last two, the problem tends to shift upstream with the need to guarantee proper control and vaporization at different thrust levels. Moreover, they require added hardware complexity.

The catalyst represents the simplest, most robust and versatile option. In fact, impressive results with throttling ratios up to 10:1 and complex thrust profiles, sometimes on demand in real time, have been obtained by Purdue [25], NAMMO [26, 27] and UNIPD [28-32] in simple hybrid rocket configurations with catalytically decomposed hydrogen peroxide.

8. Conclusions

Hybrid rockets have several possible positive features; among them, the potential ability to be deeply throttled on demand. Unfortunately, classical hybrids suffer a mixture ratio shift with both throttling and time.

Several ideas have been considered along the years to overcome these issues, playing with the propellants and/or the architecture.

The addition of energetic additives often introduces a pressure dependency in the fuel regression rate law that mitigates the shift with throttling but exacerbate the one with time.

Two main techniques have been proposed to directly control the mixture ratio in hybrid rockets, the aft-injection of oxidizer (AOIM) and the altering-intensity swirled injection (A-SOFT).

In the first case, further oxidizer is added to compensate the unbalance in the mixture ratio, while in the second, the injection swirl number is controlled to directly adjust the fuel regression rate to the required value.

Both methodologies increase the complexity of the hybrid architecture trough the need for two independent controllable feed lines and injections.

The AOIM has the largest throttling range but is more troublesome to compensate with time, while, on the contrary, the A-SOFT is more limited in throttling but is very comfortable in the compensation with time.

The A-SOFT throttling range is strongly dependent on the maximum achievable swirl number and the regression rate sensitivity to the swirl variation.

Possible alternatives to avoid or reduce the throttling needs are the use of multiple chambers or pulse width modulation.

Several possibilities are available to achieve large throttling ratios keeping at the same time high levels of combustion efficiency and stability; among them, the use of a variable area cavitating venturi to control the oxidizer mass flow coupled with a catalyst bed to decompose the oxidizer into hot gas has shown the most promising results.

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