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Essays in Financial Econometrics

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To Alessandra

Introduction

The present doctoral thesis covers different aspects in the financial econometrics area. In particular, the research focuses on the heterogeneous agents in the market (rational and behavioural), the performance measures related to this type of agents and, more generally, the asset evaluation within a portfolio selection framework. Further, the time varying dependence among the financial markets is also considered.

In general, the financial markets represent one of the main indicators for the dynamics of the business cycle as noted by Siegel (1991). Viceversa, Hamilton and Lin (1998), for example, found that economic recessions are the main factor that leads the fluctuations in the volatility of stock returns. Therefore, there is evidence for an interdependence relationship among the economic cycle and the financial markets.

In this context, it is interesting to analyze the markets by looking at investors as decision makers in the asset selection process. Moreover, the time varying dependence among the financial markets could imply a change in the portfolio in term of diversification, with effects on investors' portfolios.

The first chapter presents a rational learning model which considers the information coming to a HARA investor from a behavioural counterpart. The main goal is to investigate this component's effect, in terms of utility function, on asset evaluation during the allocation process. This heterogeneous framework has two types of agents with two different utility functions, a rational agent with a hyperbolic absolute risk aversion (HARA) utility function and the second with a general behavioural utility function. To compare the assets, each agent uses the concept of performance measure related to utility functions. The higher the measure, the higher the expected utility of a given asset. The HARA agent is a rational learner agent. The rational learning is defined as the process undertaken through Bayesian updating of the prior beliefs. The prior beliefs derive by the utility function of the rational agent and the updating process of beliefs takes place through the presence of the behavioural counterpart. The choice is conditioned by adopting an Herding behaviour, which is the tendency for an investor to abandon her own information to imitate the behaviour of other investors. Therefore, the rational investor is conditioning her choice towards behavioural investors to give

rise to the positive feedback effect. This effect has been documented by Scharfstein and Stein (1990) on fund manager, Grinblatt et al. (1995) in mutual fund behaviour and by Devenow and Welch (1996) on forecasts made by financial analysts. The rational learner agent adopts a positive feedback strategy through herding behaviour to improve her investment.

In this regard, the two components are blended in a Bayesian manner. The model is built analogously to Black-Litterman model to obtain the aggregated measure adjusted by a weighting factor. The goal in the application of the model is to check if the positive feedback effect exists. The work shows that conditioning the choice of the HARA investor towards a behavioural direction improves the selection amongst the assets. The empirical analysis is performed on all the assets present in the NASDAQ stock exchange from December 1989 to February 2012. This chapter is a solo paper.¹

The second chapter declines with a different purpose the model developed in the first chapter. In this context, two categories of agents are considered, one rational with a risk adverse utility function and one with an S-shaped loss averse value function similar to Kahneman and Tversky (1979). Agents take investment decisions in the same way by ranking the alternative assets according to their performance measures.

We assume that a type of agent is endowed with an S-shaped loss averse value function. This produces the intuitive and empirically validated prediction that the attitude of undertaking risky investments changes according to the fluctuations of the financial market. According to this assumption, in periods of (financial and economic) recession, financial agents are attracted by more risky investments that might generate, with some positive probability, returns that compensate previous (observed) losses. On the other hand, in periods of expansion, financial agents are more reluctant to undertake a risky investment that might reduce, with some positive probability, previous (observed) capital gains. In this chapter the model estimates the relative weight of the behavioural component in the financial market. The empirical analysis is based on monthly data on the components of the S&P 500 index from January 1962 to April 2012. The relative weight of the behavioural category over the rational's one has an intuitive explanation: the higher the value of the weighting factor, the higher is the weight of the behavioural component in the aggregated measure. The estimated value of the weighting factor is obtained by maximizing the cumulated return of the one hundred most performing assets of the mixture ranking. Intuitively, the weighting factor captures the extent to which the financial market should have moved from the ordering of the rational category to the ranking of the behavioural agents to maximize the return of the "best" one hundred assets.

¹I gratefully acknowledge the comments provided by the participants at the Computational Statistics & Data Analysis Conference (2010), the Association of Southern European Economic Theorists Conference (2012) and the Italian Congress of Econometrics and Empirical Economics Conference (2013).

By choosing a selection of one hundred assets, we capture the systemic dimension of the financial market. The results confirm the existence of a significant behavioural component, which is more likely to emerge during recessions. A strong correlation emerges between the estimated relative weight series and the VIX index, which implies that the estimates substantially explain financial expectations. This is a joint paper with Professors Massimiliano Caporin and Luca Corazzini (University of Padova).²

The third chapter introduces a novel criterion for performance measure combination designed to be used as an equity screening algorithm. The combination criterion follows the general idea of linearly combining existing performance measures with positive weights. These weights are determined by means of an optimisation problem. The underlying criterion function explicitly takes into account the risk-return trade-off potentially associated with the equity screens, evaluated on a historical and rolling basis. By construction, and due to the rolling window evaluation approach, the methodology provides performance combination weights that can vary over time, thus allowing for changes in preferences across performance measures. The proposed approach is implicitly robust to the dynamic features of the returns densities, as these will affect the evaluation of performance measures that are the inputs of our screening algorithm. The final product of the linear combination of performance measure is a composite performance index, which can then be used to create asset screens. We present an empirical application that illustrates the use of the screening algorithm in a simplified portfolio allocation. This is a joint paper with Professors Monica Billio (Ca' Foscari University of Venice) and Massimiliano Caporin.³

The fourth chapter examines the financial contagion using a regime switching approach with vine copulas. Vine Copulas allows us to model easily a multivariate framework with the use of the pair-copula decomposition introduced by Aas et al. (2009). The marginals are modelled by the GARCH process with long memory volatility—in mean as introduced by Christensen et al. (2010). In particular, this model well captures the long-range dependence characterizing financial time series, allowing for asymmetric effects in the GARCH equation and for the news impact in the mean. Moreover, we decided to use Copula functions to model the dependence structure across variables. The final purpose is to use a long memory GARCH process to filter the marginal series and then to use a regime switching approach among different copula families to model the dependence structure. Diebold and Inoue (2001) highlight that these two approaches can lead to misleading results. In fact, long memory can easily be confused with structural

²The paper won the International Finance and Banking Society 2013 (IFABS) Best Poster Award. The authors gratefully acknowledge the useful comments provided by the participants at IFABS 2013, Professor Angelo Ranaldo (University of St. Gallen) and Professor Harrison Hong (Princeton University). Part of the research idea underlying the paper was developed when I was visiting the Center for Research in Econometric Analysis of Time Series (CREATES) at Aarhus University (Denmark), whose hospitality is gratefully acknowledged.

³The paper has been revised and resubmitted to the European Journal of Operations Research.

changes and viceversa. In our case, we are looking at long memory and regime switching in a complementary way, since we use them on different dimensions. Vine Copula families are considered to build the multivariate dependence structure with Pair-Copula construction methodology (Aas et al., 2009). In the empirical analysis, we focus on the main European countries (Germany, France, Italy, Spain and Netherlands) to detect contagion (and financial integration). In the thesis, a preliminary version of the paper is included in which we filtered the series using the exponential GARCH process. This is a joint paper with Professor Bent Jesper Christensen (CREATES - Aarhus University).

Introduzione

La presente tesi di dottorato verte su alcuni aspetti di econometria finanziaria. In particolare, il lavoro si concentra sui diversi tipi di agente presenti nel mercato (razionale e comportamentale), sulle misure di performance legate a questo tipo di agenti e, più in generale, alla valutazione delle attività finanziarie per la selezione dei titoli di portafoglio. Siegel (1991) ha osservato come possono variare le dipendenze tra i diversi mercati finanziari nel corso del tempo .

Si é visto che, generalmente, i mercati finanziari rappresentano uno degli indicatori principali nell'individuare la dinamica del ciclo economico. Viceversa, Hamilton e Lin (1998) hanno evidenziato come le recessioni economiche rappresentino il fattore principale nel determinare la volatilità dei rendimenti nei mercati azionari. Pertanto, esiste chiaramente un rapporto di interdipendenza tra ciclo economico e mercati finanziari.

In questo contesto, risulta quindi interessante analizzare i mercati finanziari, dalla prospettiva dei diversi tipi di investitore, durante il processo di valutazione e selezione dell'attività finanziarie di portafoglio. Inoltre, l'analisi della dipendenza tra i vari mercati finanziari lungo la dimensione temporale permette di monitorare il rischio di portafoglio dell'investitore in termini di diversificazione.

Il primo capitolo presenta un modello di apprendimento (*rational learning*) per l'investitore razionale che considera nella selezione dei titoli finanziari le informazioni provenienti da un investitore comportamentale. I due tipi di agente hanno due differenti funzioni di utilità: l'agente razionale é dotato di una funzione di utilità con avversione al rischio iperbolico (HARA) mentre l'altro agente di una funzione di utilità comportamentale generale introdotta da Zakamouline (2011). L'obiettivo principale del lavoro é quello di studiare l'effetto della componente comportamentale, espressa in termini di utilità, sulla valutazione delle attività finanziarie durante il processo di allocazione. Per valutare e ordinare queste attività, ogni agente utilizza il concetto di misura di performance legato alla propria funzione di utilità. Maggiore é il valore della misura, maggiore é l'utilità attesa di tale attività. L'agente HARA é un agente che effettua un apprendimento razionale (*rational learning process*), definito come il processo intrapreso da un'investitore razionale attraverso l'aggiornamento delle proprie *belief* (convinzioni)

iniziali. Nel caso in esame, le *belief* a priori derivano dalla valutazione dei titoli finanziari attraverso la misura di performance definita dalla funzione di utilità dell'agente razionale. Il processo di aggiornamento di queste belief iniziali scaturisce dalla presenza della controparte comportamentale. L'investitore razionale influenza la propria scelta adottando un *herding behavior* (comportamento imitativo), che rappresenta la tendenza di un investitore nel trascurare volontariamente le proprie informazioni per imitare il comportamento di altri investitori. Pertanto, l'investitore razionale condiziona la sua scelta verso gli investitori comportamentali al fine di dar luogo ad un effetto di feedback positivo (*positive feedback*). Questo effetto è stato documentato da Scharfstein e Stein (1990) sulla gestione dei fondi, Grinblatt et al. (1995) nel comportamento dei fondi comuni e da Devenow e Welch (1996) sulle previsioni degli analisti finanziari. Quindi, al fine di migliorare il proprio investimento, l'agente razionale adotta la strategia di feedback positivo attraverso l'*herding behavior*. Infatti, tenendo conto della presenza di altre tipologie di investitori, l'agente razionale agisce in modo più sofisticato rispetto alla propria condizione iniziale.

Il meccanismo di apprendimento, ovvero come la componente razionale viene condizionata verso la componente comportamentale, è di tipo Bayesiano ed il modello è costruito in modo analogo al modello di Black-Litterman. La misura aggregata è ottenuta specificando un valore di ponderazione che definisce implicitamente il peso della componente comportamentale. L'analisi empirica mostra che il condizionamento dell'investitore razionale verso una direzione comportamentale fornisce un miglioramento nella scelta delle attività finanziarie in termini di rendimenti cumulati. Il campione considerato nell'analisi riguarda tutte i titoli azionari presenti nel mercato NASDAQ da dicembre 1989 a febbraio 2012. Questo capitolo è un lavoro a firma singola.⁴

Il secondo capitolo declina in modalità diversa il modello sviluppato nel primo capitolo. In questo contesto, vengono considerate due categorie di agenti: la prima categoria, razionale con una funzione di utilità avversa al rischio e la seconda, con una funzione di utilità a S (convessa nel dominio delle perdite e concava nel dominio dei guadagni) simile a Kahneman e Tversky (1979). Gli agenti prendono decisioni di investimento allo stesso modo, ordinando in termini di utilità le attività finanziarie in base alle loro misure di performance.

Assumere che un tipo di un agente sia dotato di una funzione di utilità a S, mostra intuitivamente (ed empiricamente) che l'attitudine nell'intraprendere investimenti rischiosi cambia in base alle fluttuazioni del mercato azionario. Secondo questa ipotesi, in periodi di recessione (finanziaria ed economica), gli agenti finanziari sono attratti da investimenti più rischiosi, che possono generare, con una certa probabilità positiva,

⁴Ringrazio per gli utili commenti ricevuti dai partecipanti della conferenza Computational Statistics & Data Analysis - CFE 2010, Association of Southern European Economic Theorist - ASSET 2012 e al Congresso Italiano di Econometria e di Economia Empirica (ICEEE) 2013.

rendimenti che compensano le precedenti perdite osservate. Viceversa, in periodi di espansione, gli agenti finanziari risultano maggiormente riluttanti nel prendere posizione in investimenti rischiosi che potrebbero ridurre i guadagni precedentemente osservati.

Il modello si propone di stimare il peso relativo della componente comportamentale nel mercato finanziario. L'analisi empirica si basa su dati mensili delle componenti dello S&P 500 da gennaio 1962 ad aprile 2012. Il peso della componente comportamentale rispetto a quella razionale indica che maggiore è il valore di tale fattore di ponderazione, maggiore è il peso che assume la componente comportamentale nella misura aggregata. La stima del fattore di ponderazione è ottenuta massimizzando il rendimento cumulato di cento titoli derivanti dalla misura aggregata. Intuitivamente, il fattore di ponderazione cattura la misura in cui il mercato finanziario dovrebbe essersi spostato dall'ordinamento ottenuto dalla funzione di utilità dell'agente razionale verso l'ordinamento ottenuto dalla funzione di utilità comportamentale, al fine di massimizzare il rendimento dei "migliori" cento titoli.

La dimensione scelta per la selezione permette di catturare la componente sistemica del mercato azionario.

I risultati confermano l'esistenza di una componente comportamentale significativa che risulta emergere durante le fasi di turbolenza del mercato. Infine, l'evidenza di una correlazione tra la serie del fattore di ponderazione e l'indice VIX, implica che il fattore stimato spiega sostanzialmente le aspettative finanziarie del mercato. Questo capitolo è a firma congiunta con i professori Massimiliano Caporin e Luca Corazzini (Università di Padova).⁵

Il terzo capitolo introduce un nuovo criterio per la combinazione delle misure di performance, costruito per essere utilizzato come algoritmo di screening su titoli finanziari. Il criterio di combinazione segue l'idea generale di combinare linearmente misure di performance esistenti in letteratura. Questi pesi vengono determinati attraverso un problema di ottimizzazione di combinazione convessa dei pesi di tali misure. La funzione del criterio di ottimizzazione tiene esplicitamente conto del trade-off rischio-rendimento. Gli asset vengono valutati su una finestra temporale costruita su base storica. Per costruzione, e per l'approccio di valutazione effettuato su una finestra temporale fissa, i pesi della combinazione delle misure stimate possono variare nel tempo, consentendo quindi cambiamenti nelle preferenze nelle misure di performance. L'approccio proposto è implicitamente robusto per le caratteristiche dinamiche della funzione di densità dei

⁵Il paper ha vinto il Best Poster Award IFABS 2013 (International Finance and Banking Society). Gli autori ringraziano per gli utili commenti forniti dai partecipanti alla conferenza IFABS 2013, dal professor Angelo Ranaldo (Università San Gallo - Svizzera) ed il professore Harrison Hong (Università di Princeton - USA). Parte dell'idea di ricerca alla base del lavoro è stata sviluppata quando ero in visita al Center for Research in Econometric Analysis of Time Series (CREATES) presso l'Università di Aarhus (Danimarca), di cui ringrazio con gratitudine per l'ospitalità.

rendimenti e di come queste possono influenzare la valutazione delle misure di performance (che rappresentano i valori di input dell'algoritmo di screening). Il risultato finale della combinazione lineare delle misure di performance è un indice composito, che può essere quindi essere utilizzato per creare screening sui titoli finanziari. Un'applicazione empirica illustra l'utilizzo dell'algoritmo di screening in un schema semplificato di allocazione di portafoglio. Il capitolo è a firma congiunta con professori Monica Billio (Università Ca' Foscari di Venezia) e Massimiliano Caporin.⁶

Il quarto capitolo esamina il contagio finanziario utilizzando un approccio a cambio di regime (*regime switching*) basato sulle vine copula. Le vine copula permettono di operare facilmente in un contesto multivariato attraverso l'uso della decomposizione *pair-copula* (a copula bivariate), introdotto da Aas et al. (2009). Le serie degli indici finanziari (dette le marginali delle copula) sono modellate da processi GARCH a memoria lunga con la volatilità che entra nell'equazione delle media, Christensen et al. (2010). In particolare, questi modelli ben catturano la dipendenza lunga che caratterizza le serie finanziarie, consentendo inoltre effetti asimmetrici nell'equazione GARCH ed includendo l'impatto delle innovazioni nella media. Nel lavoro, le funzioni copula vengono utilizzate per modellare la struttura delle dipendenze tra i mercati finanziari. L'obiettivo del capitolo è quello di utilizzare il processo GARCH a memoria lunga per filtrare le serie marginali e successivamente utilizzare l'approccio a cambio di regime. Le diverse famiglie di copula utilizzate in ciascun regime, permettono di avere diverse strutture di dipendenza tra gli indici azionari nei regimi considerati.

Diebold e Inoue (2001) hanno evidenziato come i processi a memoria lunga e a cambio di regime possano portare a risultati fuorvianti. Infatti, la memoria lunga può venire facilmente scambiata per dei cambiamenti strutturali nelle serie e viceversa. Nel nostro caso, la memoria lunga e il cambio di regime vengono utilizzati in modo complementare, dal momento che vengono applicate lungo diverse dimensioni; rispettivamente, univariata e multivariata. L'analisi empirica si concentra sui principali paesi europei (Germania, Francia, Italia, Spagna e Paesi Bassi), al fine di individuare contagio finanziario o integrazione finanziaria. Il capitolo rappresenta una versione preliminare del lavoro, dove gli indici azionari sono stati modellati mediante il processo esponenziale GARCH a memoria lunga (FIEGARCH). Lo studio è a firma congiunta con il professor Bent Jesper Christensen (CREATES - Università di Aarhus).

⁶Il paper è stato revisionato e sottomesso nuovamente a l'European Journal of Operations Research.

Chapter 1

Rational learning for risk-averse investors by conditioning of behavioural agents

1.1 Introduction

The main goal of decision theory is to determine how individuals should decide and to explain how they actually decide. In particular, while the prescriptive approach indicates how a *rational* choice should be made, the descriptive one models how the decisions are effectively made. By focusing on the latter approach, it is possible to observe individuals that systematically deviate from what the prescriptive method defines as rational: this approach is called *behavioural*.

According to the efficient market hypothesis, if agents are rational and there are no frictions in the market, the security's price will reflect all the available information, and it will be equal to its fundamental value without allowing for arbitrage activities. In other words, if the market is efficient, a profitable trading strategy would not exist that would allow obtaining risk-adjusted excess returns above the market returns. On the basis of this assumption, classical financial economists such as Friedman (1953) assert that these anomalies cannot exist,¹ because if there would be some mispricing, this would imply a *de facto* arbitrage that rational investors would immediately grasp. Consequently, the mismatch would disappear at the moment.

¹A well-know story on the market efficiency tells about a professor and her student walking on the street, where at some point they find \$20 on the ground. The professor stops the student from picking up the bill by telling him that if it was really a \$20 note, it wouldn't be there anymore because someone else would already picked it up by somebody else.

However, Lamont and Thaler (2003a), amongst many others, have shown several empirical violations of the law of one price, proving the existence of arbitrage opportunities in the stock market. On the other hand, Malkiel (2003) argues the possibility that some investors are less rational than others, and thus, pricing irregularities and predictable patterns could occur in the market. Nevertheless, these patterns of irrationalities in the pricing are unlikely to continue and at the end, they will not reward a significant risk-adjusted excess return.² Moreover, as reported by Hommes (2006), in an efficient market, assuming that all agents are rational and have a perfect common knowledge of all the available information, there should be no trade.

Summing up, the classical theory asserts that the absence of an arbitrage opportunity ensures that the prices are correct, and then the market is efficient. Conversely, according to the behavioural approach, deviations from the fundamental value are due to the presence of some agents that do not act in a fully rational way. From a different point of view, we might think that a mispricing could be present in the market, but its search could be too complicated for a rational investor and unattractive because of implementation costs.³

Other heterogeneous agents have been treated in the economic literature.⁴ The different types of agents are usually distinguished on the basis of their expectations about the future asset returns. De Long et al. (1990) differentiate noise traders from sophisticated traders. The first ones (i.e., as technical analysts and stock brokers) incorrectly rely on their information. Sophisticated traders, instead, exploit these false perceptions by adopting a herding or contrarian behaviour.

Zeman (1974) introduces a fundamentalist versus a chartist model. Fundamentalists trade on the basis of the market fundamentals and economic factors, while chartists base their trades on observed historical patterns in past prices. In Grossman and Stiglitz (1980a), agents are divided into informed and uninformed. Since information is costly, prices cannot perfectly reflect all the available information in the market.

The purpose of the heterogeneous agent models is to explain stylized facts observed in financial markets, such as random walk of asset prices, no autocorrelations of asset prices, fat tails distribution of returns, and long-range volatility clustering (i.e., slow decay of autocorrelation of squared returns). In this paper, we consider a framework for heterogeneous agents: a risk-averse agent equipped with a hyperbolic absolute risk aversion (HARA) and a behavioural counterpart who is endowed with a piece-wise linear plus power utility function. The aim is to propose a rational learning model where the HARA investor considers the information coming from the behavioural counterpart.

²Market bubbles have also been considered by many economists as proof of some market irrationality, e.g Shiller(2008), while other, such as Garber (1990) analyzed the market bubbles, providing a fundamental explanation.

³See D'Avolio (2002).

⁴See Hommes (2006) for a complete survey.

We define rational learning as the process undertaken through Bayesian updating of the prior beliefs provided by agent's utility function given the presence of the behavioural counterpart.

The main goal is to investigate this component's effect in terms of utility function on asset evaluation during the selection process. We use the concept of performance measure related to utility function, where the higher the measure, the higher the expected utility provided by a given asset. In order to maintain a coherence between the two perspectives, we consider the generalized Sharpe ratio (Zakamouline and Koekebakker, 2009b), as the benchmark for a rational investor, while for the behavioural agent, we use the *Z*-ratio developed by Zakamouline (2011), starting from a general behavioural utility function. The measures proposed in the mentioned papers have been obtained by following and exploiting the *maximum principle* approach introduced by Pedersen and Satchell (2002). In that latter paper, the authors define the optimal allocation between a risky and a risk-free asset in a single-period horizon. The solution of this allocation, which provides the maximum expected utility, is an increasing function of a quantity that can be viewed as a performance measure.

Following the Bayesian approach, the model used by the rational investor to blend the two different evaluations (described by the performance measures) is analogue to the approach followed by Black and Litterman (1992a). From the the rational investor's perspective, the prior evaluation represents his or her view while the conditional part represents the *behavioral* component. Finally, the posterior provides the aggregated expectation according to the weight given to the behavioural information. In our model's application, the rational learner adopts a herding behaviour and test if conditioning his or her choice towards a behavioural direction improves the selection amongst the assets in terms of cumulative returns.⁵ The strategy aims to give rise to the positive feedback effect, which consists of buying and selling in the market based on historical prices.

In other words, if the rational investor's choice is influenced to a certain degree, assuming a behavioural component in the market, he or she act in a more sophisticated way. In fact, this agent implicitly considers the aggregated evaluation coming from the different utility functions as the best way to select amongst the assets for the next period.

Our empirical analysis is based on monthly data of all the stocks present in the NASDAQ stock exchange from December 1989 to February 2012. In terms of cumulative returns, we find an improvement on the selection of the portfolio constituents when the investor's choice is influenced to a certain degree towards the behavioural counterpart. In our opinion, this effect is produced by the interception of the behavioural component. The paper is organized as follows. In Section 1.2, we illustrate the two heterogeneous agents. The first is the investor with the HARA utility function in the expected utility framework, with the generalized Sharpe ratio as the performance measure. Then we

⁵ In some sense, the herding behaviour can be seen in the same way as the bandwagon effect.

discuss the generalized behavioural utility function, with the Z-ratio as its performance measure, developed by Zakamouline (2011). In Section 1.3, we present the rational learning model following the Bayesian approach similar to that of Black-Litterman. Finally, in Section 1.4 we perform the empirical analysis.

1.2 Heterogeneous Agent Models

In our framework, we consider two agents with different utility functions. The first decision maker is equipped with a HARA utility function, and the second with a behavioural utility function. Generally, recalling Zakamouline and Koekebakker (2009a), we define a behavioural agent as a decision maker who discriminates between an outcome above (gain) and below (loss) a reference point. Consequently, the investor's utility function behaves differently in the domain of gains and in the one of losses with a kink at the reference point.

The main difference between the two agents can be explained by their different risk attitudes: the rational investor is risk-averse in all the domains of the utility functions while the behavioural investor might show different risk preferences. Examples are risk aversion in the gains and risk-seeking in the losses, as in the S-shaped utility function by Kahneman and Tversky (1979).

Moreover, we assume no iteration amongst the decision makers. In the framework, each agent takes a decision solely according to his or her utility function.

1.2.1 The HARA utility function

The expected utility theory is considered the rational investor's reference for the optimal decision making.⁶ In this setting, an agent's risk-aversion is given by the concavity property of her wealth function.

Let's consider a general class of utility functions, concave and everywhere differentiable with a HARA,

$$U(W) = \frac{\rho}{1-\rho} \left(\frac{\lambda W}{\rho} + b \right)^{1-\rho} \quad (1.1)$$

where the absolute risk aversion is

$$ARA(W) = r(W) = -\frac{u''(W)}{u'(W)} = \lambda \left(\frac{\lambda W}{\rho} + b \right)^{1-\rho}, \text{ where } b > 0. \quad (1.2)$$

⁶A decision maker is defined rational according to the Von-Neumann-Morgenstern utility theorem, which defines a set of four axioms: *completeness*, *transitivity*, *independence* and *continuity*. The expected utility theory always satisfies this theorem.

The utility function reduces to the quadratic utility when $\rho = -1$, negative exponential utility function (CARA) when $b = 1$ and $\rho \rightarrow \infty$, and logarithmic (CRRA) when $b = 0$ and $\rho > 0$.

As reported in Zakamouline and Koekebakker (2009b), the CRRA utility function provides a performance measure consistent with a market equilibrium. The utility function is defined as,

$$U(W) = \begin{cases} \frac{1}{\rho} W^{1-\rho}, & \text{if } \rho > 0, \quad \rho \neq 1 \\ \ln W, & \text{if } \rho = 1 \end{cases} \quad (1.3)$$

where ρ measures the degree of relative risk aversion.

Mehra and Prescott (1985) indicate a ρ around 30 to be consistent with the observed equity premium in the financial market. As shown in Zakamouline and Koekebakker (2009b), the relative preferences for the moments of the distributions are similar to those of the CARA utility function when ρ is pretty high. Following the authors and for computational convenience, we consider the CARA instead of the CRRA utility function,

$$U(W) = -e^{-\lambda W}, \quad (1.4)$$

where λ represents the coefficient of risk aversion and W the investor's wealth. The utility function is reported in Figure 1.1.

The separation theorem states that all the investors with the same prior beliefs, independently from their risk aversion, will invest in the same fund of a risky asset. Sharpe (1964) and Lintner (1965) show that this market portfolio is the efficient one which represents the core of the formulation of the capital asset pricing model in a mean-variance world.⁷ Cass and Stiglitz (1970) demonstrate that if all investors in the market have a HARA utility function with the same exponent, the two-fund separation principle still holds.

Shefrin and Statman (2000) developed a behavioural portfolio theory (BPT) consistent with the Friedman and Savage (1948) puzzle, and show that generally, the mean variance frontier and the BPT do not coincide. Moreover, the two-fund separation theorem does not hold in their developed portfolio theory.

In the model, we evaluate each asset in terms of the utility function, and hence we refer to a *single* risky asset instead of a portfolio composed of risky assets. We need to do that in terms of the total order to be able to compare the different evaluations of the assets between the two type of investors. That is, we rank the assets according to their provided utility.

Generally, the expected utility of an asset i is treated as the convex combination of the utilities from the various outcomes x_i in the alternative future states of the world,

⁷To obtain the market portfolio on the efficient frontier, computational problems could arise, particularly when we consider higher moments. See Appendix A.

weighted by their respective probabilities p ,

$$\int u(x)dp. \quad (1.5)$$

The investor's wealth is considered as the amount invested in a *risky* and in a risk-free asset.

According to the maximum principle, the performance measure relates to the level of maximum expected utility provided by a given asset;⁸ the higher the performance measure, the higher the maximum expected utility to the investor.

The mean–variance proposed by Markowitz (1952) can be considered a particular case of expected utility theory when the financial returns are normally distributed. In fact, the Sharpe ratio provides the optimal solution for the maximization of the expected, utility since the distribution of returns is completely described by the first two moments.

In this regard, let's consider an investor endowed by a wealth W at the beginning of a period t_0 , where a is the amount of wealth allocated in a risky asset x and the remaining $w - a$, the part allocated in the riskless asset r_f .

At the end of the period t_1 , the investor's wealth will be

$$\tilde{w} = a \times (1 + x) + (w - a) \times (1 + r_f) = a \times (x - r_f) + w \times (1 + r_f). \quad (1.6)$$

The investor's objective is to maximize the wealth according to the choice of a ,

$$\max_a E[U(\tilde{w})] \quad (1.7)$$

and therefore the maximized expected utility will be

$$E[U^*(\tilde{w})] = E[-e^{-\lambda[a(x-r_f)+w(1+r_f)]}] = E[-e^{-\lambda[a(x-r_f)]} \times \underbrace{e^{-\lambda w(1+r_f)}}_{\bar{q}}]. \quad (1.8)$$

It is worth noting that a^* is independent from the investor's initial wealth, and we can treat $\bar{q}$ as a fixed quantity.

By setting $x_0 = w(1 + r_f)$ as in Zakamouline (2011), we can approximate the expected utility using Taylor's series,

$$E[U(\tilde{w})] = -1 + a\lambda E(x - r_f) - \frac{\lambda^2}{2}a^2 E(x - r_f)^2 + O(\tilde{w}) \quad (1.9)$$

and by the first order condition (FOC),

$$\frac{\partial E[U(\tilde{w})]}{\partial a} = \lambda E(x - r_f) - \lambda^2 E(x - r_f)^2 a = 0. \quad (1.10)$$

⁸An alternative method to define a performance measure is the axiomatization approach. See De Giorgi (2005) and Cherny and Madan (2009).

we obtain the *Sharpe ratio* as the quantity that maximizes the expected utility function,

$$a^* = \frac{1}{\lambda} \frac{\mu - r_f}{\sigma^2} = \frac{1}{\lambda} \frac{\text{SR}}{\sigma}. \quad (1.11)$$

As shown by Gatfaoui (2009), when there is a departure from Gaussianity in the financial returns, the ratio begins to be biased, both in the measurement and in the ranking amongst the assets. Therefore, several authors started to consider alternative measures. Cherny (2003) and Zakamouline and Koekebakker (2009b), amongst others, propose an improvement of the ratio with the inclusion of higher moments. In particular, the authors propose a parametric Sharpe ratio adjusted for skewness and kurtosis, assuming the *normal inverse Gaussian* (NIG) as the underlying probability distributions of the financial returns. This probability density function (pdf) is particularly suitable for distributions with fat tails.

Alternatively, using a non-parametric methodology, both authors followed the Hodges (1998) conjecture by deriving a generalized Sharpe ratio (GSR).⁹

Recalling the maximization of the expected utility,

$$E[U(\tilde{w})] = E[-e^{-\lambda(x-r_f)}] = \max_a \int_{-\infty}^{\infty} -e^{-\lambda a(x-r_f)} \hat{f}_h(x) dx, \quad (1.12)$$

where $\hat{f}_h(x)$ is the estimated kernel density function, the *GSR* is obtained by the numerical optimization of the expected utility, which considers all the empirical moments of the probability distributions:

$$GSR = \sqrt{-2 \log(-E[U^*(\tilde{w})])}. \quad (1.13)$$

We consider this ratio as the performance measure for the rational investor.

It worth noting that the *GSR* approaches the standard Sharpe ratio when the underlying distribution of the risky asset is close to the Gaussian distribution.

1.2.2 The behavioural utility function

As mentioned above, a *rational* investor should behave as described in the expected utility theory. Nevertheless, the presence of people who systematically deviate from this behaviour can be seen.¹⁰ The stock market has shown that prices present *excess volatility* much more, compared to the dynamics of their economic fundamentals.¹¹ Another important stylized fact is *clustered volatility*, where asset price movements are driven by

⁹See Zakamouline and Koekebakker (2009b) for a detailed explanation.

¹⁰The paradox of Allais (1953) was the first and the best empirical example of a systematic violation of rationality in the expected utility, particularly, the independence axiom.

¹¹See for example Cutler et al. (1989).

periods of high volatility spaced by periods of low volatility.¹² These examples enforce the hypothesis of the presence of non-rational and heterogeneous agents.

In a behavioural framework, the investor's utility function behaves differently in the domain of gains and in the one of losses, with a kink at the reference point,

$$U(W) \begin{cases} U_+(W) & \text{if } W \geq W_0, \\ U_-(W) & \text{if } W < W_0, \end{cases} \quad (1.14)$$

Different types of risks arise from the behavioural utility function; the agent equipped with this utility discriminates between expected loss and uncertainty in losses and in gains.

Zakamouline (2011) has proposed a generalized *behavioural utility function* characterized by a piece-wise linear plus power utility function,

$$U(W) = \begin{cases} 1_+(W - W_0) - (\gamma_+/\alpha)(W - W_0)^\alpha, & \text{if } W \geq W_0, \\ -\lambda(1_-(W_0 - W) + (\gamma_-/\beta)(W_0 - W)^\beta), & \text{if } W < W_0. \end{cases} \quad (1.15)$$

where W_0 is the reference point, 1_+ and 1_- are the indicator functions in $\{0, 1\}$ which define the linear part of the utility, γ_+ and γ_- are real numbers that model for the shapes of the utility, and the parameters $\lambda > 0$, $\alpha > 0$ and $\beta > 0$ are real numbers. This utility function is continuous and increasing in wealth with the existence of the first and second derivatives, with respect to the investor's wealth.

Under some conditions and by using the maximum principle, Zakamouline (2011) derives in a close-form solution the Z -ratio, the performance measure which maximizes the utility function. The expected generalized behavioural utility function can be approximated by a function of mean and partial moments of the returns,

$$Z_{\gamma_-, \gamma_+, \lambda, \beta, 1_-} = \frac{E(x) - r - (1 - \lambda - 1)LPM_1(x, r)}{\sqrt[\beta]{\gamma_+UPM_\beta(x, r) + \lambda\gamma_-LPM_\beta(x, r)}}, \quad (1.16)$$

where x is the returns series of the asset and r is set to the risk-free rate, LPM and UPM are respectively the lower and upper partial moments as defined by Fishburn(1977),

$$\begin{aligned} LPM_n(x, r) &= \int_{-\infty}^r (r - x)^n dF_x(x), \\ UPM_n(x, r) &= \int_r^{\infty} (x - r)^n dF_x(x), \end{aligned} \quad (1.17)$$

where n is the order of the partial moment of x at a given threshold r , usually the risk-free asset r_f , and $F_x(\cdot)$ is the cumulative distribution function of x .

¹²Generalized autoregressive heteroscedastic models introduced by Engle (1982a) and Bollerslev (1986) explain these type of phenomena.

This behavioural utility function allows modelling to several different preferences of a behavioural decision maker. In fact, we can obtain several behavioural types of utility through the calibration of the parameters. Consequently, we can shape the concavity and convexity in the domain of gains or losses with a given risk aversion level.

In particular, we consider here four different cases which recall some well-known behavioural utility functions. We label them in the same way as that of Zakamouline and Koekebakker (2009a).

- **Behavioural I** is Fishburn's utility function.

The agent equipped with this utility is risk averse in the domain of losses and risk neutral in the domain of gains.

The parameters are set as follows:

$$\left\{ \begin{array}{l} \gamma_+ = 0 \\ \gamma_- = .1 \\ 1_+ = 1 \\ 1_- = 1 \end{array} \right\} \quad \left\{ \begin{array}{l} \lambda = 1.5 \\ \beta = \alpha = 2. \end{array} \right.$$

The Sortino ratio is the measure that maximizes the identical utility function when the risk aversion λ is equal to 1. See Sortino and Price (1994).

- **Behavioural II** is analogous to the the utility function used in the prospect and cumulative prospect theories. In this utility function, the decision maker exhibits loss aversion point by defining it in a local sense around the reference point (Köbberling and Wakker, 2005),¹³

$$\lambda = \frac{U'(W_0-)}{U'(W_0+)},$$

where we have the left derivative in the numerator and the right derivative in the denominator .

If λ is greater than 1 the individual exhibits loss aversion.

¹³In contrast, Kahneman and Tversky (1979) define the loss aversion in a global sense,

$$-U(W_0 - \Delta W) > U(W_0 + \Delta W), \quad \forall \Delta W > 0.$$

The parameters are set as follows:

$$\left\{ \begin{array}{l} \gamma_+ = .1 \\ \gamma_- = -.1 \\ 1_+ = 1 \\ 1_- = 1 \end{array} \right\} \left\{ \begin{array}{l} \lambda = 1.5 \\ \beta = \alpha = 2. \end{array} \right.$$

- **Behavioural III** can be related to the disappointment theory (DT) introduced by Bell (1985). The decision maker experiences disappointment when an outcome is worse than expected (the reference point). Conversely, when an outcome is better than the expected one, a magnification is generated. The utility function is concave below the reference point and it can be convex above.

The parameters are set to:

$$\left\{ \begin{array}{l} \gamma_+ = .1 \\ \gamma_- = .2 \\ 1_+ = 1 \\ 1_- = 1 \end{array} \right\} \left\{ \begin{array}{l} \lambda = 1.5 \\ \beta = \alpha = 2. \end{array} \right.$$

- **Behavioural IV** is the utility where the decision maker is equipped with piece-wise power utility function with non-linear parts.

The parameters are set to:

$$\left\{ \begin{array}{l} \gamma_+ = -\alpha \\ \gamma_- = \beta \\ 1_+ = 0 \\ 1_- = 0 \end{array} \right\} \left\{ \begin{array}{l} \lambda = 1.5 \\ \alpha = 1.5 \\ \beta = 2. \end{array} \right.$$

The existence of a solution to the optimal capital allocation requires that $\beta > \alpha$; therefore, the investor does not show loss aversion. Zakamouline (2011) shows that the performance measure that maximizes their utility function is given by Tibiletti and Farinelli (2003).

It is worth noting that the general behavioural utility reduces to a quadratic utility when $\lambda = 1$, $\alpha = \beta = 2$ and $\gamma_+ = \gamma_- > 0$. If returns are normally distributed, the *CRRA*, the *CARA* and the quadratic utility are maximized in the function of the Sharpe ratio measure. Therefore, when returns converge to normality, we can relate the rational investor as a particular case of the general behavioural utility function. This

fact confirms the appropriateness of the two types of utilities. The utilities are reported graphically in Figure 1.2.

1.3 A Rational Learning Model

We assume for simplicity that the rational investor is myopic by ignoring that other agents are also engaged in a dynamic learning process. Our agent adopts a positive feedback strategy through a herding behaviour to improve his or her investment. Herding behaviour is the tendency of an investor to abandon his or her own information in order to mimic the behaviour of other investors. Therefore, the rational investor conditions her choice towards behavioural investors to give rise to the positive feedback effect. This effect has been documented by Scharfstein and Stein (1990) on fund manager, Grinblatt et al. (1995) on mutual fund behaviour and Devenow and Welch (1996) on forecasts made by financial analysts. Cont (2001) shows that it provokes fat tails returns.

Positive feedback strategy concerns trading on the basis of historical prices, that is, buy stocks when the market is improving and sell stocks when the market is declining. According to Long et al. (1989), this kind of behaviour explains correlation of asset returns, overreaction of prices to news, and price bubbles.

The purpose of our approach is to blend the assets selection by a rational investor with the one made by a behavioural investor. This combination is done by conditioning the rational choice on a behavioural ordering. The evaluation is performed in terms of expected utility.

If the rational investor modifies his or her choice by taking into account a behavioural selection in the market, then to a certain degree, he or she acts in a more sophisticated way. This agent considers implicitly the aggregated evaluation coming from the different utility functions as the best way to perform the optimal selection amongst the assets, “one step ahead” of the next period in the market.

In practice, the more weight is given to the behavioural component, the greater is that component’s relevance assumed in the market of that component by the rational investor. The two extremes are the limiting cases where the mixed selection collapses into one of the components.

As reported in Forbes (2009), if the investors are not irrational and they are learning to invest better, their learning process takes place in accordance with Bayes’ rule,

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}. \quad (1.18)$$

Therefore, the most appropriated way to obtain the aggregated measure is using the Bayesian approach. We conjugate these two components analogous to the model proposed by Black and Litterman (1992a).

In the approach, we place our perspective on an investor with the HARA utility function, which is considered the benchmark for the rational investor in the expected utility framework. Hence, the generalized Sharpe ratio is the measure used to evaluate the assets in terms of this utility function. This is our prior distribution.

The general behavioural utility function represents the additional information used along the prior distribution to infer the posterior one. The Z-ratio from Zakamouline (2011) is the measure coming from this utility function. We structure the model in a similar way as that of He and Litterman (2002). Consequently, the aggregated measure is defined by the posterior distribution.

1.3.1 The model

Generally, we consider the performance measure of an asset as a random variable independently and identically distributed,

$$PM_i \sim iid(\mu, \sigma^2). \quad (1.19)$$

We are interested in identifying the assets i with the highest values of the expected PM_i as those representing the best opportunity for the investor in terms of utility function. In performing this choice, we start from a prior distribution for μ , which is assumed to be normally distributed when centred to the generalized Sharpe ratio obtained from the optimized expected utility function,

$$\mu_{GSR} = GSR(E(U^*(a_i))) + \epsilon, \quad (1.20)$$

where ϵ is a normal distributed error with the mean equal to zero and the variance matrix, $\tau\sigma^2$. As in the Black–Litterman model, τ represents the uncertainty on the prior density. The higher τ is, the higher is the uncertainty given to the prior density. Conversely, the closer τ is to zero, the lower is the variance of the prior density, and therefore the lower the relevance given to the conditioning information represented by the behavioural component. The parameter τ is defined in $[0, \infty]$.

If we assume a coexistence between a behavioural and a rational component in the market, it is reasonable to expect an improvement in a selection that merges the points of view.

The conditioning information coming from the general behavioural utility function can be from a different source, according to the shape given to the utility. If we consider the

behavioural types of utility as described above, we have four different types of investors. Generally, we can have k number of behavioural views (as in the Black-Litterman model), where P is a k -dimensional vector on the ones (the selection vector in Black-Litterman) that combine all behavioural measures with the rational one and $\mathbb{Z}_{\gamma_-, \gamma_+, \lambda, \beta, 1_-}$ is a k -vector of the behavioural measures that declined from the Z-ratio.

$$\begin{aligned} P' &= \vec{1}, \\ \mathbb{Z}' &= (Z_1, Z_2, \dots, Z_k). \end{aligned} \tag{1.21}$$

The mean of the behavioural measure is centred to the Z-ratio plus an error term normally distributed with zero mean and variance matrix Ω :

$$\mu_{\mathbb{Z}} = \mathbb{Z}_{\gamma_-, \gamma_+, \lambda, \beta, 1_-} (E(U^*(a_i))) + \eta \tag{1.22}$$

We assume that ϵ and η are independent:

$$\begin{pmatrix} \epsilon \\ \eta \end{pmatrix} \sim N \left(0, \begin{bmatrix} \tau\sigma^2 & 0 \\ 0 & \Omega \end{bmatrix} \right) \tag{1.23}$$

with an application of the Bayes theorem, and similarly to Black and Litterman (1992a), the aggregated expectations (behavioural and rational) are distributed as a normal distribution with mean μ_a and covariance matrix M_a ,

$$\mu_a = [(\tau\sigma^2)^{-1} + P'\Omega^{-1}P]^{-1} [(\tau\sigma^2)^{-1}GSR + P'\Omega^{-1}\mathbb{Z}_{\gamma_-, \gamma_+, \lambda, \beta, 1_-}] \tag{1.24}$$

and

$$M_a = [(\tau\sigma^2)^{-1} + P'\Omega^{-1}P]^{-1},$$

where, μ_a represents the aggregated expected performance measure coming from a mixture of the two components: the first from the HARA type utility function and the second from the behavioural utility functions I to IV.

1.4 The Empirical Analysis

Our purpose is to apply the model to an investment universe composed of n assets. In fact, we want to compare the different strategies by building portfolios based on the $k < n_t$ selected constituents.¹⁴ The portfolios are based on the ranking provided by the performance measure in terms of the utility of the rational investor (the generalized Sharpe ratio), the one based on the behavioural agent (the Z-ratio) and finally, the

¹⁴We use n_t because the set number of market stocks varies with time.

one from the rational investor conditional to the behavioural component. The built portfolios are equally weighted, their components' effects and avoid a possible corner solution obtained from the utility maximization.

1.4.1 The dataset

Our investment universe is based on the full list of quoted stocks on the NASDAQ from December 1989 to February 2011, a total of 5343 assets.

Dead series are also included in the dataset. We handle the delisting and merger and acquisition of a stock by assuming that if the investor has selected this stock, he or she would have disinvested the asset during the last period of its quotation in the market. The series have been downloaded from Datastream at a monthly frequency. We also recover a proxy of the risk-free asset, the JP Morgan 1 Month Cash bond index. In a first evaluation, we refine the investment universe *a priori* by excluding the asset with a lower market value to mitigate the liquidity risk. The level of exclusion is fixed at 50%; Figure 1.3 shows the final number of assets for each period. We build the benchmark for market comparison as a value-weighted index based on the investment universe considered at a given time t .

1.4.2 Model settings and estimation

The model has been applied on rolling windows of 60 monthly returns to take into account the time-varying structure of the series. Hence, an asset enters the valuation process when its time series is longer than the dimension of the bandwidth. Bange (2000) shows that small investors reflect positive feedback trading and changes in portfolios reflect past market movements. When the market is bullish, they increase equity holdings; when the market is bearish, they decrease equity holdings. Therefore, we select the number of assets which might represent a small investor's portfolio. In this regard, we set k equal to 10; τ is scaled by the ratio of the ranges of the rational and behavioural measures.

1.4.3 Results

The descriptive analyses of the portfolios are included in Table 1.1, while Figure 1.5 reports the cumulative returns from December 1994 to February 2012. Clearly, the portfolio improves when the HARA investor selection is conditioned to the behavioural counterpart. This is what we should expect if a coexistence of the two types of investors is assumed in the market. Indeed, a mixture of the selection at a given degree of the

two components could represent a suitable proxy of the overall market expectations. Figure 1.6 shows an inverse U-shaped effect of the behavioural component on the rational investor's portfolio selection. As the weighting factor increases, the cumulative returns of the conditioned portfolio improve up to a maximum level. Then the cumulative returns begin to decrease and collapse to the behavioural component as τ tends to infinity. In particular, Figure 1.5 shows the portfolio's performance according to different values of τ , when τ is close to zero, the conditioned portfolio merely reflects rational investor's choice; conversely, when τ tends to infinity, the conditioned portfolio collapses to the behavioural selection.

It is worth stressing that we are not interested in the magnitude of τ *per se*; in fact, this will strongly depend on the range of the measures' variances. What we are interested in is the inverse U-shaped relationship of the conditioned portfolio according to τ .

If an investor with a rational utility function conditions his or her choice on the selection towards the behavioural counterpart in a certain manner, he or she obtains an improvement on the selection. The aggregate measure could be interpreted as a proxy of the *market* evaluation if we assumed the presence of these two types of investor. The inverse U-shape in Figure 1.6 clearly shows the effect of the two components.

The value-weighted benchmark has an almost identical behaviour with that of the NASDAQ composite index, which reflects more the performance of technology and growth companies. Thus, as seen in Figure 1.4, the so-called dotcom crisis in the early 2000s had a greater impact with respect to the sub-prime debt crises in the late 2000s. On the contrary, if we consider the conditioned portfolio, in each of its cases, the subprime debt crisis had a deep impact that almost dissolved the portfolio's gains, while the previous crisis had a considerably smaller impact. It could mean that the selection from the conditioned choice could reflect and explain a more general *market* sentiment. Particularly, the plot (c) in the Figure 1.5 first captures the market euphoria starting in 2005, and then the collapse in the second mid-2007.

Figure 1.7 shows the variation on the selection of the assets with respect to the previous period. Given the lower value of k , a high turnover is expected. Nevertheless, if we look at the turnover's average, as reported in the last column of Table 1.1, we notice a relatively high turnover when the selection moves from the choice given by the rational towards the behavioural component. It confirms, first of all, the coherence of the results from the model due to the same direction of the variation in the turnover, and then, more generally, the asymmetric impact of new information on the gains and losses of the behavioural investor. This investor is more willing to revise his or her asset evaluation frequently. As shown in Table 1.1, the benchmark represents the best diversified portfolio. This is a quite expected result; moreover, because we did not take into account the dependence structure amongst the selected assets. The conditioned portfolios are equally weighted, designed to detect if a change in the evaluation could change and

improve the selection. At least, they should be optimized according to the investor's utility function.

1.5 Conclusion

In this paper, we present a heterogeneous agent model which considers two decision makers; the classical risk-averse agent equipped with a HARA utility function and the agent with a general behavioural utility function. The HARA agent adopts a learning process by updating his or her beliefs according to the presence of a behavioural counterpart. The learning process takes place in a Bayesian manner, and to our knowledge, no previous literature has tried to model the two components in this way.

In practice, our model conjugates the choice of a HARA investor towards a behavioural counterpart to a certain degree, according to an exogenous weighing factor. In our investigation, the rational investor adopts a herding behaviour to give rise to a positive feedback effect in the selection of the portfolio constituents. This effect has been checked by varying the exogenous weighting factor τ .

The empirical analysis has been performed amongst all the assets listed in the NASDAQ from December 1989 to February 2012. The results show an improvement for the rational investor who adopts a learning process, modifying his or her choice in the evaluation of the assets with the behavioural counterpart. We found that this improvement has an inverse U-shaped relationship. We assume a time invariant τ , introducing a dynamic on the factor goes beyond the purpose of this paper. However, it represents the natural extension of the model.

Other applications are possible. A natural step forward should introduce a time-varying weighting factor to investigate the dynamic of the rational and behavioural components in the market across time. Another consideration for further analysis is the dependence amongst the assets or the different sectors. A deeper analysis should also focus on a particular behavioural function at time. For example, by considering the utility function with loss aversion as the behavioural component, the weighting function probability for the returns could also be introduced. In our empirical analysis, the Bayesian learner investor with a HARA utility function has adopted a *herding* strategy; further research should also analyze a *contrarian* strategy or a switching strategy.

1.6 Tables and Figures

	<i>Cum Ret</i>	<i>Ret Ann</i>	<i>Vol Ann</i>	<i>Min Ret</i>	<i>Max Ret</i>	<i>Skew</i>	<i>Kurt</i>	<i>Var 5%</i>	<i>ES</i>	<i>Sharpe r.</i>	<i>Avg-TO</i>
HARA	2.5704	0.0767	0.3936	-0.5816	0.4345	-0.3153	7.0385	0.1807	0.2474	0.0544	0.9083
$\tau = 5$	5.9530	0.1190	0.4144	-0.5564	0.3739	-0.2585	5.6512	0.1874	0.2578	0.0787	0.9132
$\tau = 20$	8.1249	0.1367	0.4200	-0.5564	0.4540	-0.1104	6.1946	0.1887	0.2554	0.0885	0.9268
$\tau = 50$	4.0550	0.0986	0.4468	-0.6795	0.4922	-0.4740	8.1064	0.2043	0.2929	0.0610	0.9288
behavioural	0.3116	0.0159	0.3955	-0.6090	0.4133	-0.6090	7.7135	0.1865	0.2678	0.0115	0.9371
Benchmark	7.0138	0.1282	0.2449	-0.2047	0.1940	-0.4453	3.5115	0.1062	0.1554	0.1429	-

TABLE 1.1: Descriptive analysis of realized portfolio returns. The columns report the cumulated returns obtained in the time range December 1994 to February 2012, the annualized average monthly return and annualized variance, the minimum and maximum monthly returns, the skewness and kurtosis indices computed on monthly returns, the 5% value-at-risk and expected shortfall, the Sharpe ratio, and the average monthly turnover.

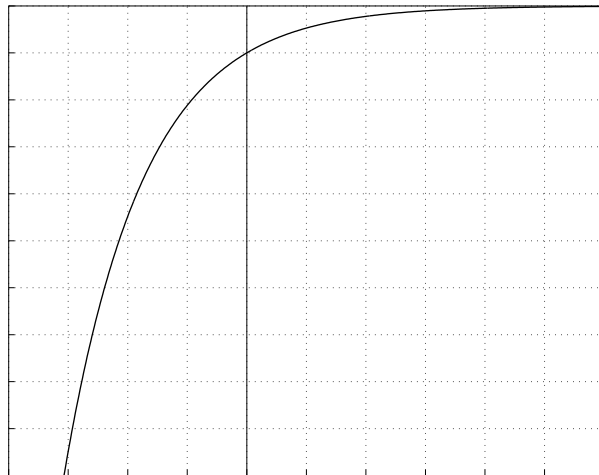


FIGURE 1.1: Negative exponential utility function with constant absolute risk aversion (CARA). λ is set equal to 1.5.

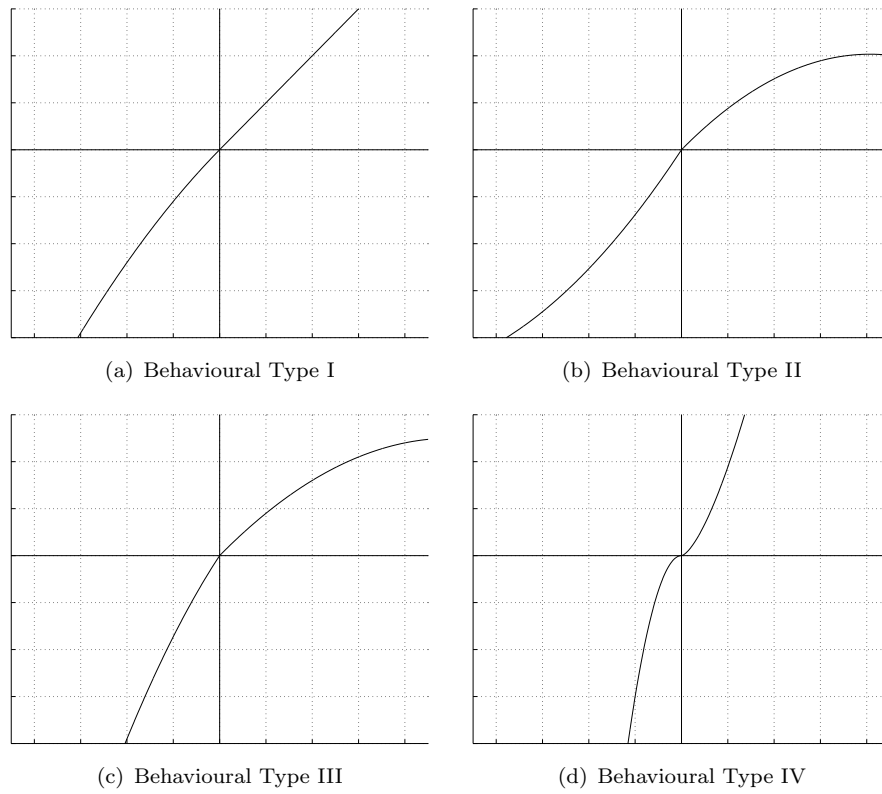


FIGURE 1.2: The four specified utility types from Zakamouline's (2011) general behavioural utility function.

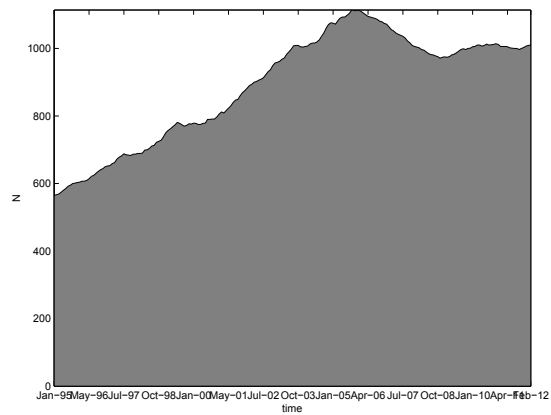
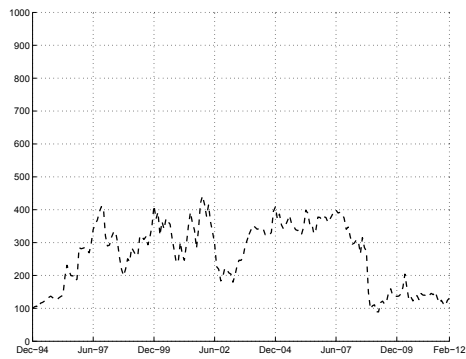


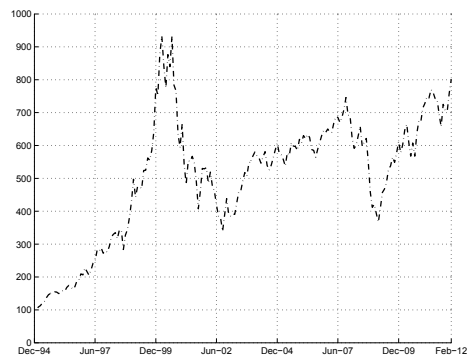
FIGURE 1.3: The number of assets in the investment universe across time.



(a) Portfolio from the HARA utility function.



(b) Portfolio from the Behavioral utility function.



(c) The benchmark of the investment universe.

FIGURE 1.4: The equally weighted portfolios of the cumulative returns on the selected assets, according to the HARA utility and the behavioural utility function. We also included the value-weighted benchmark.

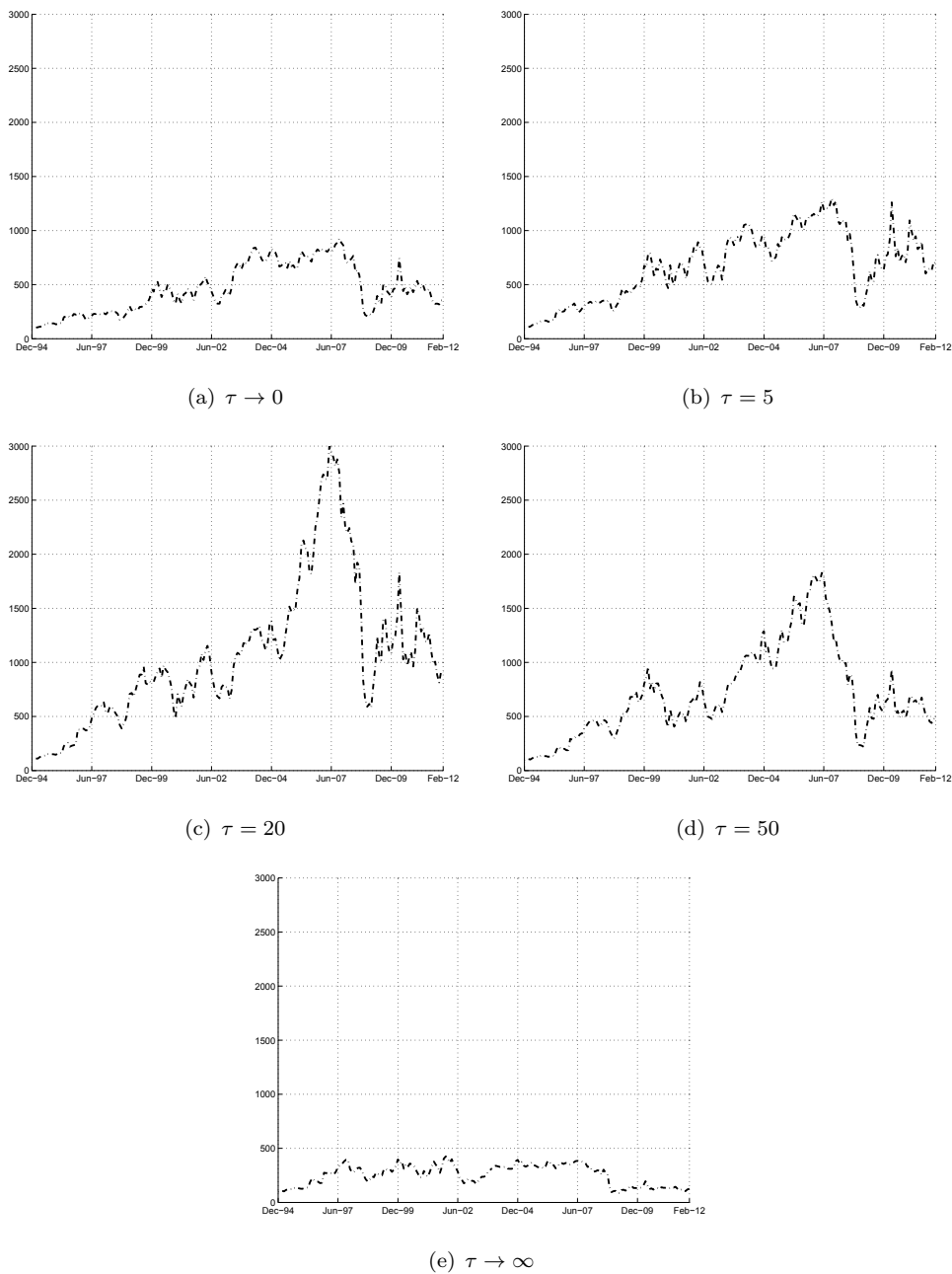


FIGURE 1.5: The effect of the behavioural component on the equally weighted portfolio of the rational's investor cumulative return. When $\tau \rightarrow 0$, the weight of the selection by the HARA utility function tends to infinity; conversely, when $\tau \rightarrow \infty$, the weight of the component tends to zero.

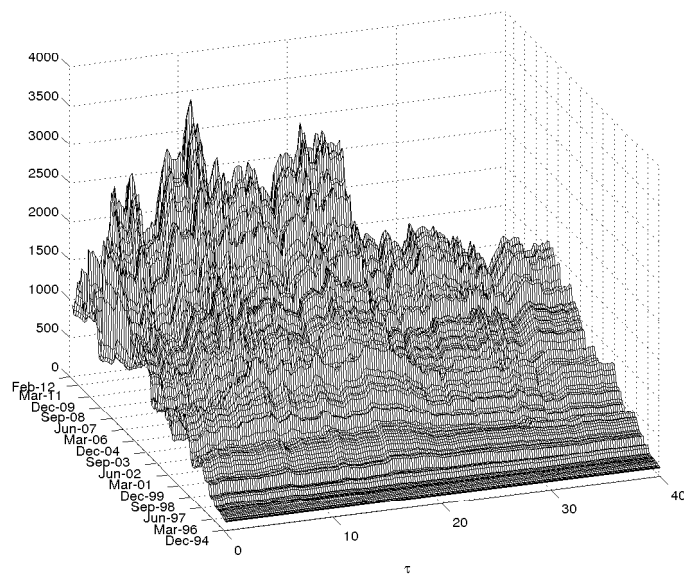
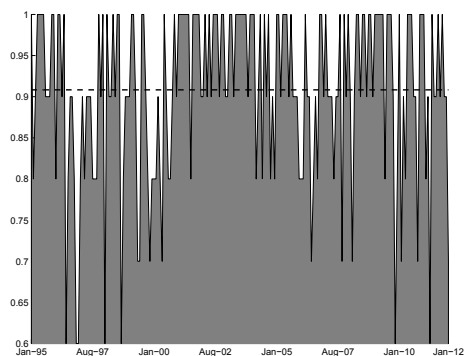
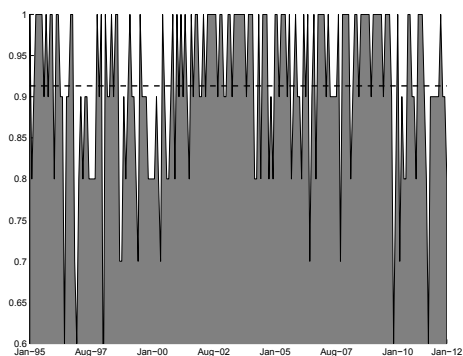
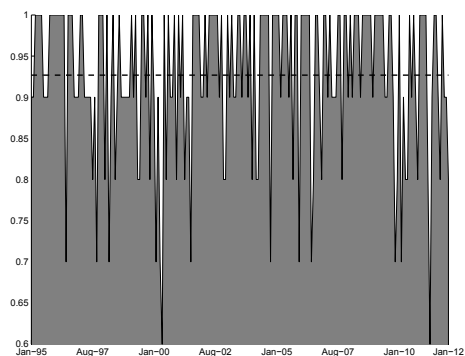
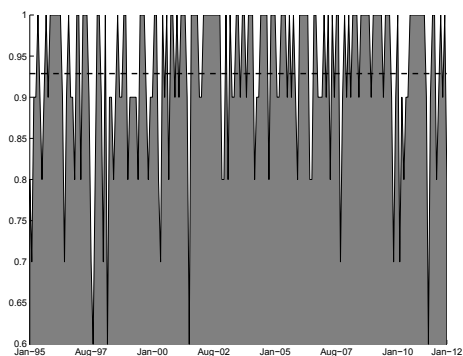
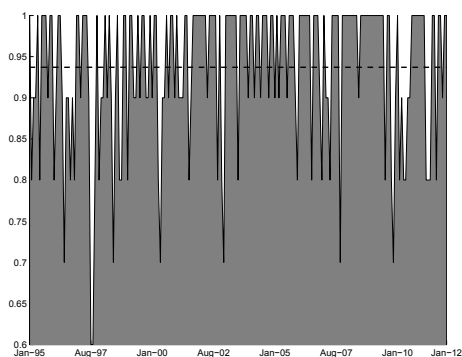


FIGURE 1.6: The change in the cumulative return of the equally weighted portfolio with τ has an inverse U-shaped relationship. For a small value of τ , the conditional portfolio is identical to the HARA utility; as τ increases, the conditional portfolio improves, and finally, when $\tau \rightarrow \infty$, the conditional portfolio collapses into the behavioural portfolio.



(a) CARA utility function portfolio.

(b) Conditional portfolio with $\tau = 5$.(c) Conditional portfolio with $\tau = 20$.(d) Conditional portfolio with $\tau = 50$.

(e) behavioural Utility function portfolio.

FIGURE 1.7: The variation on the selection of the assets with respect to the previous period. Even if there is a naturally high turnover due to the lower value of k , there is a relatively lower variation on the portfolio selected by the HARA utility function, compared to the one conditional portfolio by the behavioural component.

Chapter 2

Measuring the behavioural component of financial fluctuations: an analysis based on the S&P 500.

2.1 Introduction

The main assumption behind the traditional theory of finance (LeRoy and Werner, 2000) is that, in taking their financial investment decisions, agents are rational. Rationality refers to two aspects of agents behaviour. First, they maximize a well-conformed utility function that satisfies the (demanding) requirements of the Expected Utility Theory (EUT).

Second, financial agents use all the information available in the market and are able to optimally update beliefs according to the Bayes rule. As well known, the validity of the hypothesis of rational agent has been strongly questioned for its incapacity to account for systematic empirical puzzles, such as persistent mispricing of assets and the existence of arbitrage opportunities in the financial market (Barberis and Thaler, 2003, Lamont and Thaler, 2003b).

In order to react to the empirical impasse, financial economists have started to enrich their models with behavioural and psychological assumptions on agents decision process. In particular, according to the recent developments in behavioural finance, financial phenomena can be explained using models where agents are not fully rational (Hommes, 2006). First, it has been argued that, due to over/underconfidence and optimism, financial agents are not able to update beliefs correctly. Second and more relevant for

the purpose of this paper, behavioural economists have recognized that the requirements behind the SEUT are rather than being innocuous.

Given the huge experimental literature documenting systematic violations in risky gamble decisions, several and challenging non-Expected Utility theories have been proposed (for an extensive and comprehensive survey, see Starmer 2000). Of all the new theoretical advancements in this respect, Prospect Theory (Kahneman and Tversky, 1979, Tversky and Kahneman, 1992) has turned to be one of the most successful and intriguing approach. In their original formulation, the authors assume that (risk) preferences of agents are described by a value function that presents three main innovative features. First, agents perceive a monetary outcome as a gain or a loss relative to a reference level. Second, the agents risk attitude changes over the monetary domain according to the reference.

In particular, agents are risk-seeker and their value function is convex in the domain of losses, while they are risk averse and their value function is concave in the domain of gains. Finally, agents are loss averse in the sense that, for given change in the monetary status relative to reference, the value function is more sensitive to losses than to gains. Nowadays, there is a consistent number of studies that introduce the main assumptions of the Prospect Theory in models of financial investment decisions (among others, see Barberis et al. (2001), Benartzi and Thaler (2001)). There is no doubt about the importance of these contributions: they provide intuitive psychological explanations to important empirical inconsistencies in financial markets, such as the equity premium puzzle: although stocks on average exhibit attractive risk-return performances, investors appear to demand a substantial risk premium in order to prefer this asset to other risk-less investment opportunities.

Assuming that financial agents are endowed with an S-shaped loss averse value function produces the intuitive and empirically validated prediction that the attitude of undertaking risky investments changes according to the fluctuations of the financial market. According to this assumption, in periods of (financial and economic) recession, financial agents are attracted by more risky investments which might generate, with some positive probability, returns that compensate previous (observed) losses. On the other hand, in periods of expansion, financial agents are more reluctant to undertake a risky investment which might reduce, with some positive probability, previous (observed) capital gains.

Despite of their relevance, existing advancements of the Prospect Theory in models of financial decisions leave several empirical issues unexplored. First, it is not clear how to isolate and measure the behavioural component of the financial market. Rather than assuming that all the agents are endowed with an S-shaped loss averse value function, one could consider a more reasonable setting in which both rational (endowed with a risk averse utility function that is coherent with the standard EUT assumptions) and behavioural investors coexist in the financial market and investigate into the relative

weights of the two categories. Second, does the behavioural component of the financial market change over time? Does its evolution reflect some systematic aspects of economic/financial fluctuations? Intuitively, under the assumption of coexistence between rational and behavioural agents, the financial decisions of the two categories are more likely to diverge in periods of (economic and financial) recessions in which the latter category will take more risky investment decisions than the former.

In this paper, we propose a Bayesian mixture approach to estimate the relative weight of the behavioural component in the financial market. Our empirical analysis is based on monthly data on the five hundred components of the S&P 500 index from January 1962 to April 2012. The underlying model assumes that, at any period, the market is populated by two categories of non-strategic financial agents: one rational endowed with a risk averse utility function and the other behavioural with an S-shaped, loss aversion value function. Thus, in line with heterogeneous agents models (De Long et al., 1990, Grossman and Stiglitz, 1980b, Zeeman, 2007), the evolution of the financial market reflects the interplay between the choices made by two different types of agents. However, unlike previous contributions, we introduce heterogeneity in preferences rather than in the way in which agents process information or form their sophisticated beliefs.

In order to make their investments, at any time, each of the two categories first defines a performance measure for each of the five hundred constituents of the S&P 500 and then builds a ranking going from the most to the least performing asset. Performance measures have several advantages from an empirical point of view. First, they summarize into a single parameter the interplay between risk and return of the corresponding asset. Second, performance measures can be ordered in such a way that assets with higher measures are more performing. Third, in order to define the performance measure of an asset, the financial agent chooses that specific partition of the wealth between a riskless activity and the risky asset that maximizes her utility function (Pedersen and Satchell, 2002). Thus, the rankings defined by the two categories of agents depend on the specific feature of their utility functions: for the risk averse agents, the ranking is obtained by ordering the Generalized Sharpe Ratios of the assets (Zakamouline and Koekebakker, 2009b), while for the behavioural agents, it is obtained through the Z-ratios (Zakamouline and Koekebakker, 2009a).

Given this framework, the financial market produces a mixture ranking that is built by conditioning the prior ordering of the rational, risk-averse agents on that produced by the behavioural category. The mixture depends on a weighting factor that expresses the relative weight of the behavioural category over the rational one: the higher the value of the weighting factor, the higher is the weight of the behavioural component in the aggregated measure. In particular, in every period, the estimated value of the weighting factor is obtained by maximizing the cumulated return of the one hundred most performing assets of the mixture ranking. Intuitively, the weighting factor captures

the extent to which the financial market should have moved from the ordering of the rational category to the ranking of the behavioural agents to maximize the return of the best one hundred assets.

By choosing a selection of one hundred assets, we capture the systemic dimension of the financial market. We provide empirical arguments in favor of this conjecture. Indeed, we parametrically compare the average return of the selection produced by our methodology with that associated with two different sets of assets: the S&P 500 as a proxy of the dynamics of the financial market as a whole and the S&P 100 which focuses on the one hundred most capitalized companies. In both cases, we detect strong similarity in size and evolution over time between the estimated series and each of the two benchmarks. Our results confirm the existence of a substantial behavioural component in the financial market. The weighting factor is significantly greater than zero and, coherently with the intuitive prediction discussed above, reaches its peaks in proximity of periods of financial and economic crises. Moreover, compared to a standard model in which all agents are assumed to be rational, we find that the average return of the best (one hundred) assets of the mixture specification is more correlated to the average return of both the benchmark selections, S&P 500 and S&P 100.

To make optimal investments, financial agents build the rankings by using a substantial quantity of information on the past returns of the assets. In particular, we assume that the performance measures of an asset is defined by considering the distribution of its past returns in the previous 60 months.

In a sense, this is compatible with the idea that the performance measure defined by an agent in a period represents her best adaptive expectation on the performance of the corresponding asset in the next period. Thus, it is natural to ask whether our methodology provides any insight to explain (some proxy of) the real expectations in the financial market. We study the relationship between the estimated weighting factor (time) series and the VIX (CBOE, 2003). We find a significant, high correlation between the two time series, suggesting that the behavioural component is able to explain a substantial portion of financial expectations.

Our methodology is sufficiently flexible to apply to alternative behavioural utility functions. Thus, as robust check, we replicate our analysis by considering the utility function underlying the performance measure proposed by Tibiletti and Farinelli (2003), which is concave in the loss domain and convex in the gain domain. Again, the estimated weighting factor is significantly greater than zero, confirming the presence of a behavioural component in the market. However, relative to the first behavioural specification, the correlation with the VIX index changes substantially.

2.2 Different agents in the market

In our framework there exist two types of agents which differ on the base of their utility function. These decision makers must choose their optimal allocation and do their evaluation in terms of performance measures at the single asset level. As we will discuss later, performance measures are related to the level of maximum expected utility provided by a given single asset, and, generally speaking¹, are functions of the moments of the risky assets returns distribution. The higher the performance measure, the higher the maximum expected utility provided to the investor. Given performance measures at the single asset level, the allocation choice of the agent is made by investing in a subset of the assets (a fraction of the investment universe), those with higher scores in the performance measures.

The first type of agent which we consider might be equipped with the classical utility function coming from the expected utility theory. We thus refer in this case to the optimal choices of a *rational* agent. The chosen utility function, the power utility, belongs to the class of Constant Relative Risk Aversion (CRRA) utility functions. Notably, as shown in Zakamouline and Koekebakker (2009b), the CRRA utility functions lead to the identification of a performance measure which is coherent with market equilibrium. The utility function of the *rational* agent might thus defined as follows:

$$U(W) = \begin{cases} \frac{1}{\rho} W^{1-\rho}, & \text{if } \rho > 0, \quad \rho \neq 1 \\ \ln W & \text{if } \rho = 1 \end{cases} \quad (2.1)$$

where W is the agent's wealth and ρ measures the degree of relative risk aversion.

The power utility function has been extensively used in empirical studies, some of those aiming at identifying the value of ρ . The results of Mehra and Prescott (1985) indicates a value around 30 to ensure consistency with the observed market equity premium. As reported in Zakamouline and Koekebakker (2009b), for high values of ρ , the relative preferences across the moments of the distributions are similar to those of Constant Absolute Risk Aversion (CARA) utility functions.

In this regards, for computational convenience, we consider the a CARA instead of a CRRA utility function. Namely, we associate the *rational* agents with a negative exponential utility,

$$U(W) = -e^{-\lambda W} \quad (2.2)$$

where λ represents the coefficient of risk aversion. Such a coefficient affects the concavity property of the utility function, which is also influenced by the wealth of the investor

¹This concept is related to the maximum principle introduced by Pedersen and Satchell (2002)

W . An example of the utility function is reported in Figure 2.1.

The second type of agent we consider is characterized by a behavioural utility function. In general, we define a behavioural investor as a decision maker that discriminates an outcome above and below a reference point, i.e. gains versus losses. Consequently, the investor's utility function behaves differently in the domain of gains and in the one of losses with a kink at the reference point,

$$U(W) \begin{cases} U_+(W) & \text{if } W \geq W_0, \\ U_-(W) & \text{if } W < W_0. \end{cases} \quad (2.3)$$

where W_0 is the reference point while $U_+(W)$ and $U_-(W)$ are two functions associated with the domains of gains and losses, respectively. According to the domain considered, gains or losses, different type of risks might arise from this behavioural utility function. Recently, Zakamouline (2011) has proposed a generalized *behavioural utility function* characterized by a piecewise linear plus power utility function,

$$U(W) = \begin{cases} 1_+(W - W_0) \times (W - W_0) - (\gamma_+/\alpha)(W - W_0)^\alpha, & \text{if } W \geq W_0, \\ -\lambda(1_-(W_0 - W) \times (W - W_0) + (\gamma_-/\beta)(W_0 - W)^\beta), & \text{if } W < W_0, \end{cases} \quad (2.4)$$

where $1_+(\cdot)$ and $1_-(\cdot)$ are the indicator functions in $\{0, 1\}$ which define the linear part of the utility, and assume unit value for positive or negative arguments, respectively, and zero otherwise. Moreover, γ_+ and γ_- are real numbers that affect the shape of the utility and, finally, the additional parameters $\lambda > 0$, $\alpha > 0$ and $\beta > 0$ are real numbers. The utility function is continuous and increasing in wealth, and with proved existence of the first and second derivatives with respect to the wealth of the investor W .

The two utility functions previously described are generally considered for the evaluation of optimal investment decisions, or for the construction of optimal allocations between the risky asset and the risk-free investment, or within a set of risky assets. In our framework, the agents have to allocate their wealth across a set of risky investments. However, the allocation choices made by the decision makers is performed in term of the expected utility provided by each single asset. Then, given those single asset expected utility, the agents rank assets and invest on the top performers. Consequently, we refer to a *single* risky asset choice instead of a portfolio decision/allocation where many different risky activities are jointly considered. In this way, we are allowed to compare the different

evaluation of the two type of investors across the assets (the investment universe). In practice, we are interested in the rankings provided by the *rational* and *behavioural* utility functions. We now describe how we derive asset ranks starting from the expected utility.

The expected utility of investment i is given as the convex combination of the utilities associated with a collection of different and alternative outcomes x_i , each corresponding to the realization of a given state of the world. Each realization is weighted by its respective probability, leading to following characterization of expected utility

$$\mathbb{E}[U(X)] = \int u(x)f(x)dx, \quad (2.5)$$

where $f(x)$ is the probability density function associating to each state of the world a given probability. In this case the utility is expressed as a function of the risky asset X , to highlight their relations. However, the wealth of the investor, not explicitly appearing, is also playing a role. In fact, the wealth W is always allocated between a *risky* and a risk free asset.

According to the maximum principle, the performance measure is strictly related to the level of maximum expected utility originated by a given financial activity.² In fact, the higher is the value of the performance measure, the higher is the maximum expected utility provided to the investor.

The Mean–Variance by Markowitz (1952) is a particular case of the expected utility theory when the returns are normally distributed. In this case, the Sharpe Ratio is the optimal solution for the maximization of the expected utility (the CARA negative exponential utility function).

Let's consider a decision maker with wealth W at the begin of a period t_0 . Moreover, a denotes the amount of wealth allocated in a risky asset, while $W - a$ is the wealth allocated in the riskfree asset r_f . At the end of the period t_1 the wealth of the investor will be,

$$\tilde{W} = a \times (1 + x) + (W - a) \times (1 + r_f) = a \times (x - r_f) + w \times (1 + r_f) \quad (2.6)$$

where x is the return provided by the risky asset. In this framework, the aim of the investor is to maximize the expected utility with respect to the amount invested in the

²The axiomatization approach is an alternative method for defining the performance measure. See De Giorgi (2005) and Cherny and Madan (2009).

risky asset, a . Hence, the optimal problem corresponds to a utility maximization with respect to a ,

$$\max_a E[U(\tilde{w})]. \quad (2.7)$$

Given the CARA function and the Gaussianity assumption, the maximized expected utility will be

$$E[U^*(\tilde{W})] = E[-e^{-\lambda[a(x-r_f)+W(1+r_f)]}] = E[-e^{-\lambda[a(x-r_f)]} \times e^{-\lambda W(1+r_f)}] \quad (2.8)$$

where the last term within parentheses is a deterministic quantity.

By setting $x_0 = W(1+r_f)$ as in Zakamouline (2011), we can approximate the expected utility using Taylor's series,

$$E[U(\tilde{W})] = -1 + a\lambda E(x-r_f) - \frac{\lambda^2}{2} a^2 E(x-r_f)^2 + O(\tilde{W}). \quad (2.9)$$

From the first order condition (FOC),

$$\frac{\partial E[U(\tilde{w})]}{\partial a} = \lambda E(x-r_f) - \lambda^2 E(x-r_f)^2 a = 0 \quad (2.10)$$

we obtain the *Sharpe Ratio* as the quantity that maximize the expected utility function,

$$a^* = \frac{1}{\lambda} \frac{\mu - r_f}{\sigma^2} = \frac{1}{\lambda} \frac{\text{SR}}{\sigma}. \quad (2.11)$$

However, the Sharpe ratio begins to be biased both in the measurement of optimal allocations and in the ranking across a collection of assets when there is a departure from the normal distribution assumption for the risky asset returns. This has been empirically demonstrated in Gatfaoui (2009), among others. To overcome this issue and still remaining within the expected utility maximization framework, Zakamouline and Koekebakker (2009b), among others, suggested the introduction of a generalized Sharpe ratio. Such a quantity would be sensitive to higher order moments, and can be evaluated with a parametric or a non-parametric methodology. In the non-parametric estimation, by following the Hodges (1998) conjecture, Zakamouline and Koekebakker (2009b) derived what they called Generalized Sharpe Ratio, *GSR*.

Recall first the maximization of the expected utility,

$$E[U(\tilde{W})] = E[-e^{-\lambda(x-r_f)}] = \max_a \int_{-\infty}^{\infty} -e^{-\lambda a(x-r_f)} \hat{f}_h(x) dx \quad (2.12)$$

where $\hat{f}_h(x)$ is now the estimated kernel density function of the risky asset returns. We thus differ from the previous simplified framework as we do not impose a parametric form to the risky asset return density. The *GSR* is obtained by the numerical optimization

of the expected utility, see Zakamouline and Koekebakker (2009b),

$$GSR = \sqrt{-2 \log(-E[U(\tilde{w})])}, \quad (2.13)$$

where the argument of the log in (2.13) is defined in (2.12). Note that, by resorting to the GSR , all moments of the risky asset returns play a role, and we are thus not constraining ourselves to the evaluation of the mean and variance. Notably, the GSR approaches to the standard Sharpe ratio when the underlying distribution of the risky asset returns is close to the Gaussian. We consider GSR as the performance measure adopted by the rational investor to rank risky assets. The rational investor would prefer assets with higher GSR to assets with lower values of the performance measure.

We move now to the choices of the behavioural agent. In this case, the expected generalised behavioural utility function can be approximated by a function of the mean and of partial moments of distribution. In this regards, Zakamouline (2011) verifies that the optimal allocation of an agent depends from a ratio playing the same role of the GSR , that is, the performance measure which maximizes the utility function for the behavioural agent. The new ratio, called the Z -ratio, has been derived with the use of the maximum principle and under some conditions, see Zakamouline (2011) for further details. The Z -ratio is given as

$$Z_{\gamma-, \gamma+, \lambda, \beta} = \frac{E(x) - r_f - (1 - (W - W_0)\lambda - 1)LPM_1(x, r_f)}{\sqrt[\beta]{\gamma_+ UPM_\beta(x, r_f) + \lambda \gamma_- LPM_\beta(x, r_f)}}.$$

where LPM and UPM are, respectively, the lower and upper partial moments as defined by Fishburn(1977),

$$LPM_n(x, r) = \int_{-\infty}^r (r - x)^n dF_x(x),$$

$$UPM_n(x, r) = \int_r^{\infty} (x - r)^n dF_x(x),$$

where n is the order of the partial moment of x at a threshold level r , usually set at the risk free return, and $F_x(\cdot)$ is the cumulative distribution function of x . We stress we will assume that the behavioural agents rank the risky assets using the Z -ratio of each risky asset. Similarly to the GSR , higher values of the Z -ratio are preferred to lower values. There is one additional element we must consider when analysing the choices of behavioural agents. The utility function proposed by Zakamouline (2011) allows the construction of different preferences or beliefs of the agents through the calibration of the its parameters. Therefore, the concavity and convexity in the domain of gains and losses can be shaped in different ways and can give rise to different choices.

In this regards, we decline the general behavioural utility function in order to obtain an S-shaped utility similar to the the utility function used in prospect and cumulative prospect theory by Kahneman and Tversky (1979).³ The utility function we chose is reported in Figure 2.2 and corresponds to the following choices for the parameters: $\gamma_+ = 0.1$, $\gamma_- = -0.1$, $\lambda = 1.5$, and $\beta = \alpha = 2$. The differences between the use of this utility function with respect to the classical S-shaped of Kahneman and Tversky (1979) lies in the definition of loss aversion. In our version of the S-shaped utility function of the decision maker exhibits loss aversion in the sense of Köbberling and Wakker (2005). The loss aversion is defined around the reference point in a local sense,⁴ that is, if we define the ratio

$$\lambda = \frac{U'(W_0-)}{U'(W_0+)}$$

where in the numerator we have the left derivative and in the denominator the right derivative, the individual exhibits loss aversion if λ is greater than one. This implies that the utility function is steeper in the domain of losses: losses loom larger than corresponding gains, (Kahneman and Tversky, 1979). The main feature of the S-shaped utility function is the concavity in the gains and the convexity in the losses. In fact, the decision maker is risk adverse in the outcome above the reference point, and risk seeker below.⁵

Up to this point, we moved from the expected utility to the derivation of a performance measure. In turn, the last quantity is used by both the rational and behavioural agents to rank assets. One element is still missing, and refers to the construction of optimal allocations. We can here assume that agents allocate their wealth across the assets with highest ranks, that is highest values of the performance measure. If the market includes K assets, we might assume that the rational (behavioural) investor allocates his wealth across the $M \ll K$ assets with highest value of the GSR (Z -ratio). When

³ The best choice would be the classical utility function by (Kahneman and Tversky, 1979):

$$\begin{cases} (W - W_0)^\alpha \\ -\lambda(W_0 - W)^\beta \end{cases}$$

Nonetheless, Zakamouline and Koekebakker (2009a) shown that the existence of the solution and thus the Z -ratio requires $\beta > \alpha$ which implies the absence of loss aversion in the utility.

⁴While Kahneman and Tversky (1979) define the loss aversion in a global sense,

$$-U(W_0 - \Delta W) > U(W_0 + \Delta W), \quad \forall \Delta W > 0.$$

See Zakamouline and Koekebakker (2009a) for a detail explanation.

⁵One well known experiment from Kahneman and Tversky (1979) is the choice among two lotteries in two different settings with their related probabilities:

- (\$6,000, 25%), or (\$4,000, 25%; \$2,000, 25%)
- (-\$6,000, 25%), or (-\$4,000, 25%; -\$2,000, 25%)

In the first problem, most of the individuals in the experiment choose the second option while in the second they choose the first option. Clearly, the first setting represents a choice in the domain of gains while the second a choice in the domain of losses. This gives rise to the concavity (convexity) in gains (losses) of the S-shaped utility function.

doing that, agents might determine the optimal weights of those K assets, or simply use naive criteria such as resorting to an equally weighted allocation scheme. Note that the last choice would allow limiting the impact the estimation error and has been shown to be preferred over optimal weighting schemes by DeMiguel et al. (2009). In this work we assume that agents allocate their portfolio using equal weights across a (relatively) small number of assets.

2.3 The Market Model

As we discussed in the previous section, we assume that two types of agents are present in the market. However, we do not know which type is prevailing, neither, irrespectively of their number, which type of agent is affecting more the market fluctuations. Our objective is to determine the relevance or the impact of behavioural choices in the movements of risky asset returns. We propose to recover such a measure in an indirect fashion by starting from the presence of two types of agents. Under this assumption, the observed market behaviour is a blend of choices made by rational and behavioural agents. As a consequence, one intuitive way to recover the impact of behavioural elements is to blend the choices of rational and behavioural agents and estimate the blending parameter(s) in such a way that the combination of choices is as closer as possible to the observed market fluctuations. In the following, starting from this intuition, we present our approach for recovering the impact and relevance of behavioural beliefs in a financial market.

An investor equipped with the expected utility theory is usually considered the benchmark for the rational investor. Therefore, according to Zakamouline and Koekebakker (2009b), the generalized Sharpe Ratio may represent the measure used to evaluate the assets in terms of this utility function. For the behavioural investor, we consider the S-shaped utility function introduced by Kahneman and Tversky (1979). In this case, the Zakamouline and Koekebakker (2009a) Z -ratio drives assets evaluation.

One way of blending the choices of the two agents types is to resort to a Bayesian framework where one of the two agent's beliefs is considered a prior, while the other agent choices assume the role of additional conditioning information. As a result, the posterior will represent a composite of rational and behavioural elements. From a Bayesian perspective, we define the prior as the rational investor. Such a choice is purely subjective, but allows, in a limiting case, to obtain the rational choices as the market outcome. The conditioning component is thus represented by the behavioural investor. As the choices of the two types of agents are driven by performance measures, GSR and Z -ratio, the blending of choices is made at the performance measure level.

We thus start by assuming that both performance measures are normally distributed centred on their mean. For a generic performance measure PM we have

$$PM \sim N(\mu_{PM}, \sigma_{PM}^2). \quad (2.14)$$

Therefore, for the prior it holds that

$$\mu_{GSR} = GSR(E(U^*(\tilde{W}))) + \epsilon, \quad \epsilon \sim N(0, \sigma^2), \quad (2.15)$$

while for the conditional we have

$$\mu_Z = \mathbb{Z}_{\gamma_-, \gamma_+, \lambda, \beta}(E(U^*(\tilde{W}))) + \eta, \quad \eta \sim N(0, \omega^2). \quad (2.16)$$

Note that both distributions have mean set to the optimal choice for the agent, that is the Generalized Sharpe Ratio and the Z -ratio derived from market data. Moreover, the distributions refer to the performance measures of a single asset, that is, we have a collection of distributions, two for each risky asset present in the market. Finally, to simplify the treatment, we also assume that innovations, ϵ and η , are independent. Note that, by introducing innovations in (2.15) and (2.16) we are allowing for the presence of estimation error in the two measures. Differently, the distributional hypothesis in (2.14) takes into account the fact that agents aim at evaluating the expected value of a performance measure.

In order to determine the relevance of behavioural and rational choices, we modify the density in (2.15) by adding a multiplicative factor τ to the dispersion, leading to

$$\mu_{GSR} = GSR(E(U^*(\tilde{W}))) + \epsilon, \quad \epsilon \sim N(0, \tau\sigma^2), \quad (2.17)$$

The coefficient τ can be interpreted as the reliability or uncertainty of rational (prior) expectations. The higher the τ the less reliable (more uncertain) are the rational choices, and thus higher weight might be given to behavioural elements. Conversely, the closer τ is to zero, the lower is the uncertainty. By construction, and given the τ affects a variance, this parameter can assume values in the domain $[0, \infty]$.

The aggregation of rational and behavioural performance measures in a Bayesian framework gives rise to a composite performance measure consistent with (2.14) where mean and variance have the following expressions:

$$\mu_p = [(\tau\sigma^2)^{-1} + \omega^{-2}]^{-1} [(\tau\sigma^2)^{-1}GSR + \omega^{-2}\mathbb{Z}_{\gamma_-, \gamma_+, \lambda, \beta}] \quad (2.18)$$

and

$$\sigma_p^2 = [(\tau\sigma^2)^{-1} + \omega^{-2}]^{-1}. \quad (2.19)$$

Now, the aggregate expected measure, namely μ_p might be considered as the quantity used, at the market level, to order or rank assets. As a consequence, we might determine the role of behavioural choices through the composite measure, by looking at the optimal allocation made by an agent which is deciding where to invest his wealth across a set of risky assets ordered according to (2.18). In this case, the allocations might be evaluated in terms of past performances, while the impact of behavioural beliefs is determined by estimating the optimal τ level within a specified criterion function.

As we already noticed, we take a simplified allocation choice and consider an equally weighted investment strategy. Therefore, past performances can be evaluated as the cumulated returns of an equally weighted portfolio in a given time window, that is

$$r_p = \frac{1}{m} \sum_{l=t-m+1}^t r_{p,l} \quad (2.20)$$

where $r_{p,l}$ is the time l return of the equally weighted portfolio and m represents the time range for the portfolio evaluation (from time $t - m + 1$ to time t). The portfolio is formed by the best performing equities according to (2.18). Let us collect in the set $\mathcal{A}_t(\tau)$ the M best assets across the K included in the market. This index is a function of the parameter τ because, by changing τ the asset ranks will be affected. Moreover, the set is also a function of time, given that the impact of behavioural choices might change over time.⁶ Therefore, portfolio returns are represented as

$$r_{p,l} = \frac{1}{M} \sum_{j \in \mathcal{A}_t(\tau)} r_{j,l} \quad (2.21)$$

where $r_{j,l}$ is the return of asset j at time l ; we stress that the index j vary from 1 to K but only M values are included in the set $\mathcal{A}_t(\tau)$. Given the dependence on τ of the best performing asset set, the portfolio cumulated return in (2.20) is also a function of τ . The optimal choice of τ is determined by maximizing the portfolio returns, that is

⁶Note that, in order to to simplify the notation, we avoid adding a time subscripts to the parameter τ .

$$\begin{aligned}
\max_{\tau} f(\tau) &= \frac{1}{m} \sum_{l=t-m+1}^t r_{p,l} \\
s.t. \quad r_{p,l} &= \frac{1}{k} \sum_{j \in \mathcal{A}_l(\tau)} r_{j,l}
\end{aligned} \tag{2.22}$$

The optimal value τ^* provides the maximum cumulated return obtained by an agent investing on a subset of the risky assets traded in the market and taking decisions blending rational and behavioural choices. As a consequence, the estimated τ^* represents the relevance of behavioural choices, or, conversely, the reliability on the rational beliefs.

In fact, a high value of τ^* would imply that the rational investor should have correct her action towards a behavioural direction. On the opposite, a low value of τ^* would imply that the investor should have remained on her prior rational beliefs. The criterion function allows detecting which component, rational versus behavioural, had a larger influence on the market.

The proposed approach is intimately linked to the investment decisions taken following the model introduced by Black and Litterman (1992b).⁷ In fact, our Bayesian combination is exactly equivalent to the Black and Litterman model where the rational choices are the prior expectations on asset returns (the equilibrium returns) and the behavioural choices plays the same role of the analysts views. In our implementation, both the prior and the views are univariate. Moreover, the methodology for the evaluation of optimal choices when a subset of risky assets is selected from an investment universe, is similar to the one adopted in Billio et al. (2012), in the framework of determining a composite performance measure by weighted linear combination of standard performance indices.

2.4 Empirical Analysis

2.4.1 The S&P 500 in 1962-2012

The reference market considered is the S&P 500 from the period January 1962 to April 2012. The S&P 500 is a stock market index by Standard & Poor's based on the market capitalization of 500 leading companies traded in the US.

Generally, the stock market represents one of the most sensitive indicator for the business cycle as pointed out by Siegel (1991). Hamilton and Lin (1998), using a bivariate model

⁷See He and Litterman (2002) for a detailed explanation of the model.

with two regimes, have found that economic recessions are the main factor which leads the fluctuations in the volatility of the stock returns.

Consequently, we focus the analysis on the five hundred components that form the market index in each period.

The series have been downloaded from CRSP/COMPUSTAT at monthly frequency. The proxy used for the risk free rate is the US 3-Month Treasury Bill.

Figure 2.4 shows the log-level of the S&P500 for the considered period, the bands in the plot represents the financial crisis according Kindleberger and Aliber (2005). Figure 2.5 reports the bands of economic recessions according the National Bureau of Economic Research (NBER).⁸

Looking at the plots, it is natural to observe a match between the local minima in the estimated factor and the bands for the financial crisis. There is also a correspondence in the economic recessions. For instance, during the recession in the 1969–70 (the post-Vietnam era) a lower peak is clearly observable in Figure 2.5. This confirms the financial market as a reliable indicator for the state of the economy.

Table 2.3 contains the descriptive statistics grouped by decades. The period 1991-2000 has known a great expansion phase as it can be seen on the average returns. On the contrary, the last period from 2000-2012 instead has been the lowest in term of average returns.

2.4.2 The Model specification and Empirical Results

The model has been applied on rolling windows of 60 monthly returns to take into account the time-varying structure of the series. Thus, at a given t , we selected the assets which at least 60 observations from the S&P500's constituents. The variance of the behavioural measures is obtained using a block bootstrap procedure with a block of dimension 4 for time dependence among the returns.⁹

We filtered the optimized τ^* using a local level model in state space representation to extract the level's signal component which is allowed to vary overtime:¹⁰

$$\begin{cases} \tau_t^* = \mu_t + \epsilon_t, & \epsilon_t \sim NID(0, \sigma_\epsilon^2) \\ \mu_{t+1} = \mu_t + \xi_t, & \xi_t \sim NID(0, \sigma_\xi^2) \end{cases} \quad (2.23)$$

where μ_t is the unobserved level, ϵ_t is the observation disturbance and $\xi_{i,t}$ is the level disturbance a time t . The estimated results for the model, using the filtered τ_t^* , from the S-shaped utility function are $\hat{\epsilon}_{1,t} \sim NID(0, 0.4547)$ and $\hat{\xi}_{1,t} \sim NID(0, 0.0017)$.

⁸Table 2.1 and Table 2.2 report the list with dates of the financial crisis and respectively.

⁹The bootstrap procedure has been applied to the returns. Then the measures have been computed in each iteration and the variances have been obtained.

¹⁰See Koopman et al. (2012)

Figure 2.6 shows μ_t (henceforth, the filtered τ_t^*) including the economic recession bands according NBER.

We perform a TOBIT regression on the filtered τ_t^* by specifying the censored dependent variable in the model. We set the lower bound equal to zero to check if the constant is significantly different from zero,

$$\mu_t = c + \epsilon. \quad (2.24)$$

Table 2.4 reports the results for the regressions in decades and for all the sample. The filtered τ_t^* is statistically different from zero in all the sub-samples and in all the sample. Other descriptive statistics are also included in the table. Looking at the dynamic of the filtered τ_t^* , it is clearly observable that we have three local maxima which coincide with the three longest economic recessions. The first is the oil crisis which corresponds to the highest value of the filtered τ_t^* in the series. The second is the energy crisis which began with the Iranian revolution. According to Labonte and Makinen (2002), one of the main reason for this crisis was due to the FED's monetary policy for the inflation control. This energy crisis is often considered a "Double Dip" recession with the previous one (January 1980 - July 1980); we found an inflection point in correspondence of this crisis in the series. In this regard, we found a similar result with the crisis from December 1969 to November 1970.

The last two shortest recessions are very similar to each other: both at the beginning of a decade (early-80s and early-90s) and both of the same length of eight months. In these cases, our estimated factor does not provide any particular evidence.

Finally, the third largest recession in the considered period is the sub-prime crisis (2007-2009). It is worth noting that the level of the filtered τ_t^* after the recession starts to decay very slowly. Then it remains substantially high at the begin of the European sovereign debt crisis.

As we might expect, we find the local minima in correspondence of booming periods in our estimated factor. For instance, the first minimum is located just before the early-80s crisis (in the 1978) and the other is located just before the subprime crisis.

In the 1991-2000 decade, the economy has experienced a period of a solid economic growth; we found a relative low dynamic of the filtered τ_t^* .

Figure 2.7 represents the estimated factor including the bands for the financial crisis according to Kindleberger and Aliber (2005). Naturally, financial and economics crisis are highly interrelated and interdependent. Except for the 1987 stock market crash, in most of the cases, they just anticipate or follow each other. Looking at the crisis, it is clearly observable a local minimum in the estimated factor before the begin of the crisis and then a local maxima during the crisis.

As reported in Table (2.4), the period 1971-1980 and the period 1981-1990 contain on average the highest value and the highest standard deviation for the filtered τ_t^* .

Probably, this is due for the two recessions in each decade.

The following analysis is performed to detect if the asset selection provided by τ_t^* is related to the financial market's systemic component. If the mixed selection coming from the two types of investors (weighted by the estimated factor) reflects the systematic component in the market, it is reliable to assume the presence of these two types of decision makers. Therefore, the market returns should be explained by the portfolio returns of this selection.

In this regards, we estimate the following model,

$$r_m = c + \beta r_\tau + e, \quad (2.25)$$

where r_m is the S&P 500's return and r_τ is the return of the aggregated selection according to the τ_t^* . Opposite to the CAPM model, the market return represents the dependent variable in this model. That is, if we assume a rational and behavioural investor in the market, the selection coming from the mixture of these two agents should largely explain the market returns.

Hence, according to our assumption, the model should return an high value for β and a constant close to zero. In the estimation, we use the equally-weighted returns for the S&P 500 (the dependent variable) since the returns from the selection are defined by the equally-weighted method.

Table 2.5 reports the estimated regression. The constant represents the risk premium which is slightly positive but close to zero. Economically, the result is coherent to what we might expect. Moreover, a positive sign is consistent with the efficiency of the market portfolio as shown in Sharpe (1966) and Fama (1998), since the selection is a subset of the available assets in the market each period. The β is significant at 1% confidence level with a correspondent value of 0.90.

We perform a comparison also with the S&P 100 which includes the one hundred most capitalized companies in the US market. In this case, we use the value-weighted return series for the S&P 100 because of the short length of the equally-weighted series. The series for the index has been downloaded on Datastream and it is available from January 1973. The results are reported in Table 2.6. The β is significant at 5% confidence interval with a correspondent value of 0.78. The constant is not significant. A lower beta in this case is quite reasonable for the different underlying market. However, the risk premium is not statistically different from zero and the β captures an high level of the systematic risk.

The analysis is also performed considering the rational agent's selection provided by the generalized Sharpe ratio. If we expect a coexistence between the two agents, the GSR-selection should capture a lower systematic component of the market. That is, a lower beta in the estimated model (2.25). Table 2.7 reports the results for the regression

with the S&P 500 equally-weighted returns and Table 2.8 reports the results for the S&P100. The β coefficients are 0.83 and 0.64 respectively. These results confirm that the selection provided by the aggregated measure reflects an higher systematic part of the market with respect to the selection resulting by rational agent's utility function. Therefore, given these results, it reasonable to assume the two types of agents in the market.

2.5 The behavioural component and the VIX

At this point, we want to test if the filtered τ_t^* explains part of the market expectations. Consequently, we use our estimated variable as an explanatory variable of the market sentiment.

In this regard, we consider the CBOE Volatility Index (VIX). VIX is a stock market volatility index introduced in the 1993 on the Chicago Board Options Exchange (CBOE).¹¹ It is also called the investor's fear gauge since it is considered a measure of market expectations in the short-term period on the S&P 500's market (Whaley, 2000).¹² Thus, we consider the VIX the most appropriate choice as dependent variable to test if the filtered τ_t^* explains part of the market expectations. In Figure 2.8 we plot the filtered τ^* and the VIX.

In the regression, we use the estimated volatility of the S&P 500 as a control variable for the contemporaneous volatility in the market.

The model,

$$VIX_t = c + \beta_1 \tau_t^* + \beta_2 h_t^{1/2} \eta_t, \quad (2.26)$$

for $t = 1, \dots, n$.

VIX_t is the volatility index and $h_t^{1/2}$ is the volatility from an $APARCH(P, O, Q)$ model for the S&P500 returns. In that model, we assume that the errors are distributed with a generalized error distribution (GED). Hence, the estimated APARCH(1,1,1) model by Ding et al. (1993) is used as a control variable for the estimated market volatility.

$$\begin{cases} x_t = \underset{(.0051)}{.0019} + h_t^{1/2} \epsilon_t, \\ h_t^\delta = \underset{(.0250)}{.0256} + \underset{(.0206)}{.1035} \left(|\epsilon_{t-1}| - \underset{(.1815)}{.8519} \epsilon_{t-1} \right)^\delta + \underset{(.0473)}{.8015} \sigma_{t-1}^\delta \end{cases} \quad (2.27)$$

where $\hat{\delta} = \underset{(.2900)}{.5246}$ and $\hat{\kappa} = \underset{(.1960)}{1.6149}$.

The regression's results for the equation (2.26) are reported in Figure 2.9. Both the

¹¹See CBOE (2003).

¹²In the 2003, a new methodology for the volatility index has been proposed. It has been calculated on the S&P500 index instead of the S&P 100 index. The Black and Scholes (1973) model has been replaced by fair value of future variance.¹³

explanatory variables are significant at 1% level of confidence. The filtered τ_t^* coefficient is positive and equal to 0.3106.

Consequently, the filtered τ_t^* explains part of the VIX which is not related to the pure market volatility. We are capturing a dynamic which is related to the market expectations.

Looking at the utility functions, the difference on preferences among the agents arises in the domain of losses: it is concave in the CRRA utility and convex in the S-shaped utility function. This suggests that we should expect to observe a different behaviour of the agents during the period of crisis. Thus, when there is high volatility in the market. In this perspective, τ captures the divergence in behaviour between the two agents which is likely to emerge during turbulent financial periods.

2.6 Robustness Check

To check the consistency of our framework, we consider a different behavioural utility function: in particular, an utility function which behaves in the opposite way of the behavioural S-shaped utility. Therefore, we consider an inverse-S-shaped utility function with no loss-aversion that is concave in the domain of losses (risk adverse) and convex in the gains (risk seeking). The correspondent utility function is reported in Figure 2.3. Thus, as robustness check, we replicate our analysis considering the performance measure underlying this inverse-S-shaped utility function: the ratio proposed by Tibiletti and Farinelli (2003). In the model, we define this utility function following Zakamouline and Koekebakker (2009b).¹⁴ The estimation for the local level model in equation (2.23) for the filtered τ_t^* are $\hat{\epsilon}_{2,t} \sim NID(0, .1080)$ and $\hat{\xi}_{2,t} \sim NID(0, .0445)$.

Table 2.10 reports the estimates in decades and for all the sample. Also in this case, the filtered τ_t^* for this utility function is statistically significant from zero in all the subsamples and in the entire sample. The descriptive statistics for the estimated factor are reported in Table 2.10.

We check also if the selection captures the systematic part of the market. The model (2.25) is analysed with the S&P500 and the S&P100. The results are very similar to the S-shaped utility function case and confirms that also in this case, the selection reflect a systematic component. The estimated models are reported in Table 2.11 and in Table 2.12.

¹⁴ In order to obtain this utility function, the parameters are set to:

$$\left\{ \begin{array}{l} \gamma_+ = -\alpha, \\ \gamma_- = \beta \\ 1_+ = 0 \\ 1_- = 0 \end{array} \right\} \quad \left\{ \begin{array}{l} \lambda = 1.5 \\ \alpha = 1.5 \\ \beta = 2. \end{array} \right.$$

The most interesting part is the analysis of the relationship with the VIX in model (2.26). The results of the estimation are reported in Table 2.13. In this case, we have a negative relationship with the filtered τ_t^* which is consistent to what we should expect looking at the results of the S-shaped utility function.

2.7 Conclusions

In this paper, we considered two decision makers. A rational agent equipped with an utility function consistent with the expected utility theory and another agent equipped with a behavioural S-shaped utility function introduced by Kahneman and Tversky (1979). Using a Bayesian approach, we build a model which conjugates the two components according an optimal weighting factor. The underlying criterion function of the weighting factor has been expressed in terms of optimal cumulative returns coming from the portfolio obtained as the mixture of the two components. In the empirical analysis, we examined the S&P 500 market. This market is considered the reference for the financial market (not only) in the US. We considered this market also for the time length of the sample. Other types of market can be considered. The weighting factor is time varying and it has been estimated in each period from January 1962 to April 2012. The results confirm the existence of a substantial behavioural component in the financial market. In fact, the dynamic of the factor (in the local maxima and minima) is strictly related with the financial and economic crises. Consequently, we detect if this factor can be considered in some sense a proxy of the expectations in the market. In this regard, we analyzed the relationship between the estimated weighting factor and the VIX index. Results show a significant high correlation between the two time series, suggesting that the behavioural component is able to explain a substantial portion of financial expectations.

2.8 Tables and Figures

	Crisis	Start date	End Date
	The 1973 Oil Crisis	29-Oct-73	03-Oct-74
	The 1987 Stock Market Crash	19-Oct-87	30-Dec-88
	The 2000 Dotcom Bubble Burst	10-Mar-00	16-Apr-01
	The 2001-9-11 Terrorist Attack	11-Sep-01	09-Oct-02
	The Subprime Crisis	03-Dec-07	09-Mar-09

TABLE 2.1: The table provides the crisis list for the U.S. market according Kindleberger and Aliber (2005)

Economic Recessions		
<i>Quarterly dates are in parentheses</i>		<i>DURATION IN MONTHS</i>
December 1969(IV)	November 1970 (IV)	11
November 1973(IV)	March 1975 (I)	16
January 1980(I)	July 1980 (III)	6
July 1981(III)	November 1982 (IV)	16
July 1990(III)	March 1991(I)	8
March 2001(I)	November 2001 (IV)	8
December 2007 (IV)	June 2009 (II)	18

TABLE 2.2: The table provides the crisis list for the U.S. economic recessions according NBER available at <http://www.nber.org/cycles.html>.

<i>Period</i>	<i>1962-1970</i>	<i>1971-1980</i>	<i>1981-1990</i>	<i>1991-2000</i>	<i>2001-2012</i>	<i>All-Sample</i>
<i>Mean</i>	0.0035	0.0043	0.0086	0.0124	0.0015	0.0060
<i>Std</i>	0.0384	0.0457	0.0474	0.0385	0.0466	0.0437
<i>Skewness</i>	-0.2874	0.1588	-0.6839	-0.5130	-0.5711	-0.4108
<i>Kurtosis</i>	2.9520	4.2453	6.5393	4.4303	3.7890	4.7155
<i>Min</i>	-0.0905	-0.1193	-0.2176	-0.1458	-0.1694	-0.2176
<i>Max</i>	0.1016	0.1630	0.1318	0.1116	0.1077	0.1630

TABLE 2.3: Descriptive statistics for the S&P500 index returns for the period January 1962 - April 2012.

<i>year</i>	<i>1962-1970</i>	<i>1971-1980</i>	<i>1981-1990</i>	<i>1991-2000</i>	<i>2001-2012</i>	<i>All-Sample</i>
<i>c</i>	1.0038	1.3851	1.1955	1.0109	1.0388	1.1408
<i>s.e</i>	0.0665	0.1543	0.1236	0.0266	0.0762	0.0303
<i>p Value</i>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
<i>Skewness</i>	-0.0696	0.0503	0.5803	0.0002	0.0308	1.0816
<i>Kurtosis</i>	2.1024	1.8382	1.7605	1.9997	1.8031	3.2658
<i>Min</i>	0.8869	1.1180	1.0632	0.9636	0.9127	0.8869
<i>Max</i>	1.1178	1.6505	1.4258	1.0632	1.1619	1.6505

TABLE 2.4: Results for the TOBIT regression and the descriptive statistics for the filtered τ for the S-shaped utility function in different periods.

	Estimate	SE	tStat	pValue
(Intercept)	0.0037	0.0009	4.0066	0.0001
r_τ	0.9049	0.0175	51.7944	0.0000
R^2	0.8322			
$\bar{R}^2$	0.8319	F-test	2682.6638	0.0000

TABLE 2.5: Regression where the dependent variable is the S&P500 equally-weighted return and the explicative variable is return from the selection of the aggregated measure according τ^* for each period.

	Estimate	SE	tStat	pValue
(Intercept)	-0.0001	0.0009	-0.1177	0.9063
r_τ	0.7830	0.0172	45.4440	0.0000
R^2	0.8146			
$\bar{R}^2$	0.8142	F-test	2065.1593	0.0000

TABLE 2.6: Regression where the dependent variable is the S&P100 value-weighted return and the explicative variable is return from the selection of the aggregated measure according τ^* for each period.

	Estimate	SE	tStat	pValue
(Intercept)	0.0025	0.0007	3.4410	0.0006
r_{GSR}	0.8339	0.0121	68.9244	0.0000
R^2	0.8978			
$\bar{R}^2$	0.8976	F-test	4750.5689	0.0000

TABLE 2.7: Regression where the dependent variable is the S&P500 value-weighted return and the explicative variable is return from the selection of the Generalized Sharpe Ratio for each period.

	Estimate	SE	tStat	pValue
(Intercept)	-0.0004	0.0011	-0.3958	0.6924
r_{GSR}	0.6476	0.0182	35.6680	0.0000
R^2	0.8108			
$\bar{R}^2$	0.8104	F-test	1272.2073	0.0000

TABLE 2.8: Regression where the dependent variable is the S&P100 value-weighted return and the explicative variable is return from the selection of the Generalized Sharpe Ratio for each period.

	Estimated	Robust s.e	tStat	pValue	R_p^2
(Intercept)	-0.3022	0.0589	-5.1339	0.0000	
τ_t^*	0.3106	0.0621	4.9979	0.0000	0.0908
$h_t^{1/2}$	1.2459	0.0954	13.0639	0.0000	0.4070
R^2	0.5944				
$\bar{R}^2$	0.5913	F-test	194.15	0.0000	

TABLE 2.9: The filtered τ^* is from the behavioral utility function Type 1.

<i>year</i>	<i>1962-1970</i>	<i>1971-1980</i>	<i>1981-1990</i>	<i>1991-2000</i>	<i>2001-2012</i>	<i>All-Sample</i>
<i>Mean</i>	0.7781	1.7586	0.5576	0.3068	0.5613	0.7876
<i>s.e</i>	0.1039	0.1036	0.0304	0.0211	0.0270	0.0352
<i>p Value</i>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
<i>Skewness</i>	1.1849	0.2916	1.9085	0.8835	1.4800	2.0073
<i>Kurtosis</i>	2.8534	1.7187	6.4377	2.4280	5.2108	6.4150
<i>Min</i>	0.1879	0.1810	0.1422	0.0476	0.2026	0.0476
<i>Max</i>	2.4935	4.0520	1.6790	0.8266	1.6969	4.0520

TABLE 2.10: Results for the TOBIT regression and the descriptive statistics for the filtered τ for the inverse S-shaped utility function in different periods.

	Estimate	SE	tStat	pValue
(Intercept)	0.0035	0.0009	3.8538	0.0001
r_τ	0.9213	0.0174	52.8160	0.0000
R^2	0.8376			
$\bar{R}^2$	0.8373	F-test	2789.5345	0.0000

TABLE 2.11: Regression where the dependent variable is the S&P500 equally-weighted return and the explicative variable is return from the selection of the aggregated measure according τ^* in type 2 utility function for each period.

	Estimate	SE	tStat	pValue
(Intercept)	-0.0002	0.0009	-0.2621	0.7933
r_τ	0.7951	0.0177	44.8735	0.0000
R^2	0.8108			
$\bar{R}^2$	0.8104	F-test	2013.6313	0.0000

TABLE 2.12: Regression where the dependent variable is the S&P500 equally-weighted return and the explicative variable is return from the selection of the aggregated measure according τ^* in type 2 utility function for each period.

	Estimated	Robust s.e	tStat	pValue	R_p^2
(Intercept)	-0.0247	0.0134	-1.8486	0.0725	
τ_t^*	-0.0263	0.0121	-2.1693	0.0384	0.0164
$h_t^{1/2}$	1.6015	0.1040	15.3994	0.0000	0.5119
R^2	0.5612				
$\bar{R}^2$	0.5579	F-test	169.45	0.0000	

TABLE 2.13: The filtered τ^* is from the behavioural utility function Type 2.

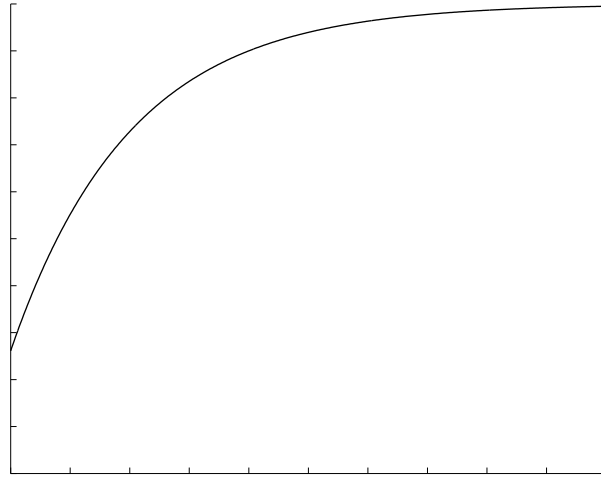


FIGURE 2.1: Negative exponential utility function with constant absolute risk aversion (CARA). λ is set equal to 1.5.

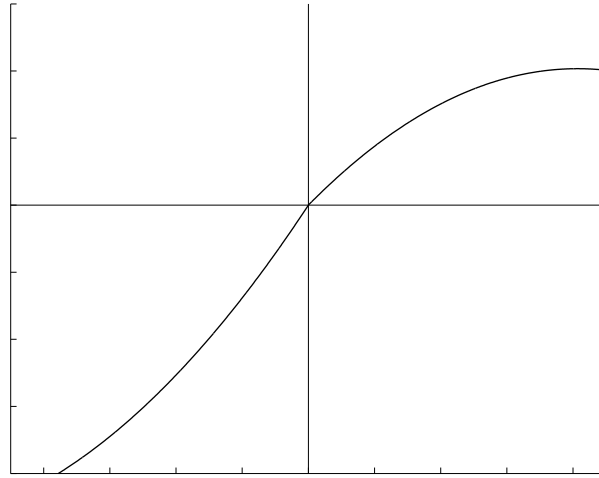


FIGURE 2.2: Behavioural S-shaped utility function similar to Kahneman and Tversky (1979).

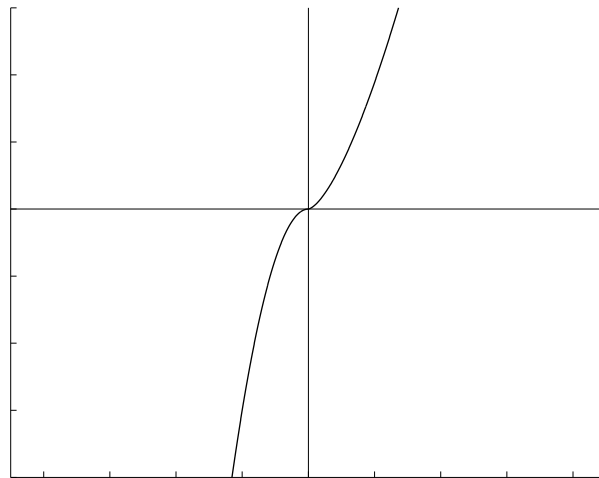


FIGURE 2.3: Behavioural inverse S-shaped utility function concave on the domain of losses and convex in the domain of gains.

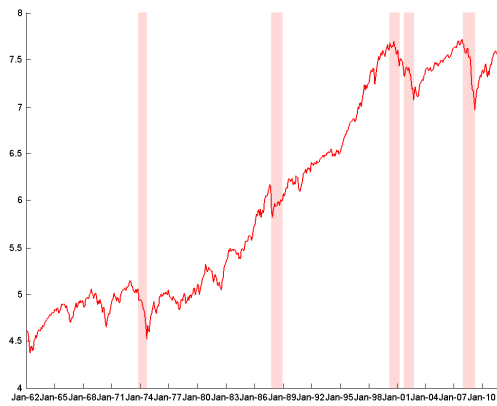


FIGURE 2.4: Log-level of the S&P500 index from January 1962 to April 2012 with bands for financial crisis. Source: Kindleberger and Aliber (2005).

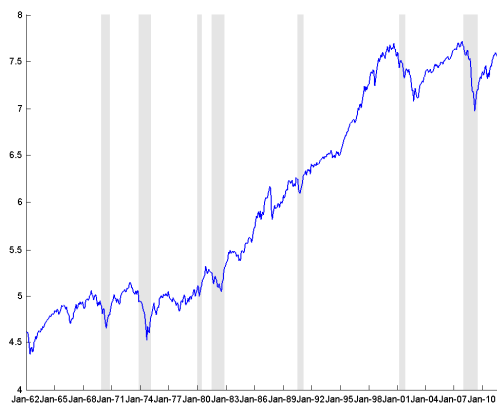


FIGURE 2.5: Log-level of the S&P500 index from January 1962 to April 2012 with bands for Economic Recessions. Source: NBER.

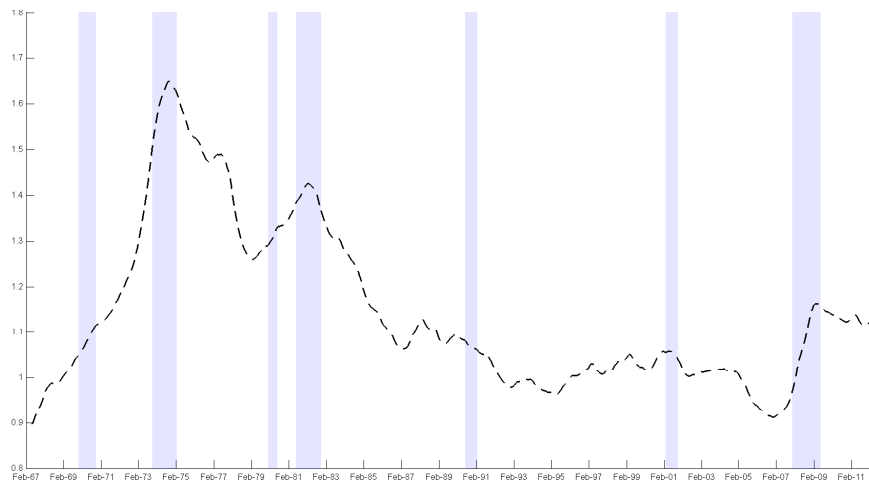


FIGURE 2.6: The filtered τ^* . The bands represent the Economic Recessions according NBER.



FIGURE 2.7: The filtered τ^* . The bands represent the Financial Crisis in the US based on Kindleberger and Aliber (2005).

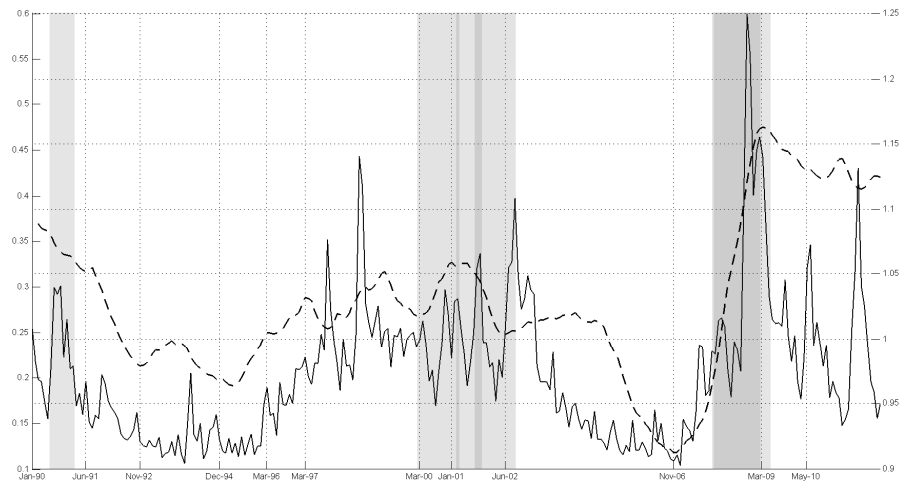
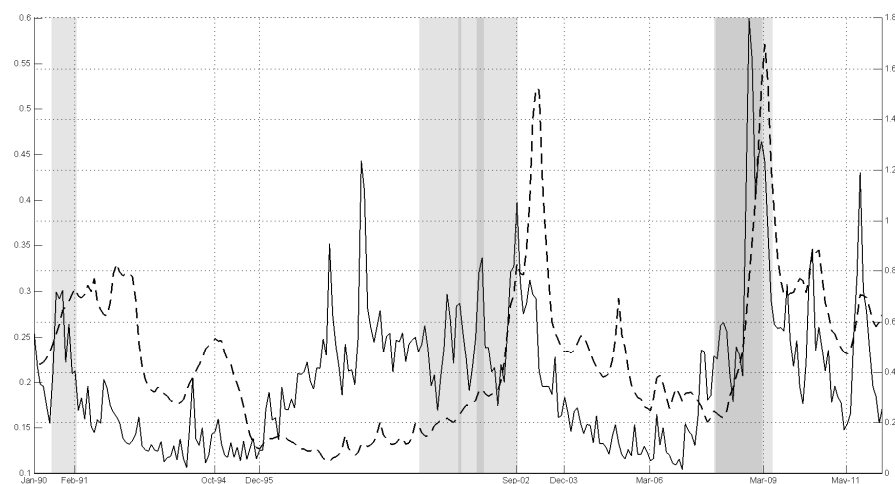
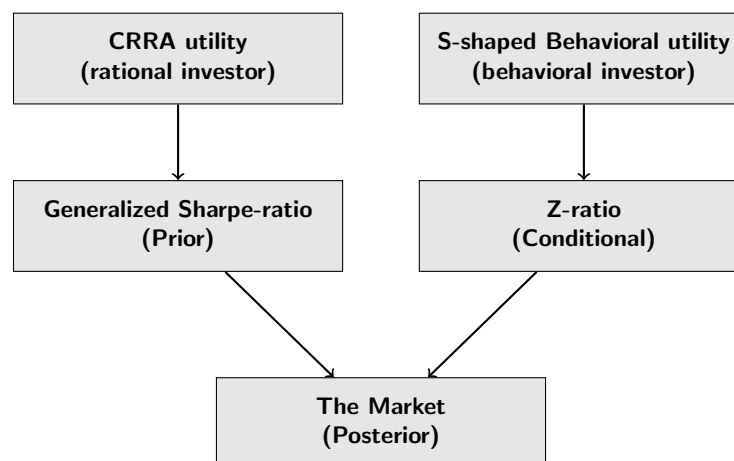
FIGURE 2.8: Vix (solid) and the filtered τ^* (dotted) from the utility function Type 1.FIGURE 2.9: Vix (solid) and the filtered τ^* (dotted) from the utility function Type 2.

FIGURE 2.10: The Bayesian model which combines the two perspectives of the agents.

Chapter 3

Backward/forward optimal combination of performance measures for equity screening

3.1 Introduction

The investment process is described by the complete set of actions taken by a portfolio manager, including the definition of the investment objectives and the associated strategic allocation, the construction of tactical asset allocation and security selection choices, and general rules for portfolio monitoring (see for example Grinold and Kahn (2000)). The security selection step focuses on identifying the most promising investment opportunities, represented by specific assets. Different approaches might be employed at this stage, inspired by technical analysis or based on a more fundamental analysis point of view. In general, security selection methodologies can be classified as qualitative or quantitative. The latter presumes the existence and the use of some quantitative tools. The broad class of quantitative security selection instruments includes the so-called equity screening rules, methodologies whose purpose is to rank a large set of asset in order to focus attention on the best ones or to exclude the worst ones. Screening rules can be used directly as security selection tools or might represent a first step in a security selection procedure; in fact, they permit to restrict the investment universe to a reasonably limited set of assets, to be analysed in greater detail by analysts. However, screening rules should not be used directly as asset allocation tools (for instance by directly investing in the best assets), since they do not control for the correlation across assets. Relevant and relatively simple examples of screening rules are given by performance measures; these are quantities that, in most cases, represent a remuneration per unit of

risk, or risk adjusted returns. In the last decade, the financial economics literature has discussed a large number of alternative performance measures; see the surveys by Aftalion et al. (2003), Le Sourd (2007), Bacon (2011), (Cogneau and Hübner, 2009, Cogneau and Hubner, 2009) and Caporin et al. (2011). The available performance indices can be classified into large families, as suggested by Caporin et al. (2011), to highlight their differences: relative performance measures (rewards per unit of risk), absolute performance measures (risk-adjusted measures referred to a benchmark or to a set of risk factors), measures derived from utility functions and measures expressed as functions of return distribution features. Note also that performance measures belonging to the same class are heterogeneous since they can be based on different quantities (such as utility functions, moments, partial moments or quantiles) or different information sets (different choices of risk factors). Furthermore, if performance measures are used to order assets (as equity screening rules), the ranks they produce for a common set of assets might be sensibly different; see Caporin and Lisi (2009). The last finding confirms that alternative measures have different views over assets, and the construction of an 'optimal' equity screening tool should take those different viewpoints into account.

A possible solution is the construction of a composite performance index to be used within an equity screening program. Few authors have considered this approach and, to our best knowledge, the only published reference is the work of Hwang and Salmon (2001). In that paper, the authors propose a combination of performance measures based on the copula function. We contribute to this strand of the quantitative finance literature by introducing a new approach for the construction of a composite performance index. Our proposal lies in between security selection and asset allocation, since our composite index is determined within a pre-specified equally weighted asset allocation scheme. Such a choice, despite being restrictive and dominated by rebalancing-based strategies (Constantinides, 1979), is motivated by the recent contributions of DeMiguel et al. (2009), who show evidence of the (statistical) equivalence between the performances of equally weighted portfolios and optimised ones (in a Markowitz sense) portfolios. Anyhow, the limitation of the calibrated portfolio weights might be removed, even if such an extension is not empirically considered in the current paper. The fixing of portfolio weights also simplifies the identification of the composite performance index, which is given by an optimal linear combination of a set of performance measures.

Within a performance evaluation framework, several authors have considered the problem of determining the optimal portfolio weights by maximizing different performance measures. They aimed at finding the 'best' performance measure; see for example Farinelli et al. (2008, 2009), among others. The outcomes of these studies were not completely conclusive, since different performance measures provide superior results over different samples and different assets. This further motivates the need for a combination

of performance measures, similar to what happens in the forecast combination literature. We thus relax the overwhelming restrictive assumption that a single performance measure provides superior results over all time periods.

In this paper we introduce a novel criterion for performance measure combination designed to be used as an equity screening algorithm. The combination criterion follows the general idea of linearly combining existing performance measures with positive weights. These weights are determined by means of an optimisation problem. The underlying criterion function explicitly takes into account the risk-return trade-off potentially associated with the equity screens, evaluated on a historical and rolling basis. By construction, and due to the rolling window evaluation approach, our method provides performance combination weights that can vary over time, thus allowing for changes in preferences across performance measures. The proposed approach is implicitly robust to the dynamic features of the returns densities, as these will affect the evaluation of performance measures that are the inputs of our screening algorithm. The final product of the linear combination of performance measure will be a composite performance index, which can then be used to create asset screens.

Apart from introducing our composite index, we discuss several implementation issues that further detail and clarify the methodology. These include the selection of performance measures, their evaluation and the optimisation of the objective function with respect to the combination weights. Those elements have a relevant role in the evaluation of the composite index and illustrate the flexibility and the features of the proposed approach.

Finally, we present an empirical application that illustrates the use of our screening algorithm in a simplified portfolio allocation. We show how combined performance indices might be used for equity asset screening.

The remainder of the paper proceeds as follows. In Section 2 we define the investment objective and introduce our screening algorithm. In Section 3, we discuss several implementation aspects. Section 4 contains an empirical example, and Section 5 concludes the paper.

3.2 The investment problem and the objective function

Our main purpose is to focus on the security selection problem faced by an investor (a portfolio manager). The investor is willing to allocate his portfolio over a subset of the assets included in his investment universe, and wants to select the asset subset using a combination of several performance measures. We presume the investor follows

one-step allocation rule: he chooses at time t the assets to form a portfolio with an investment horizon of one period, ending at $t + 1$. To be consistent, the equity screening is based on a criterion function depending on a set of performance measures evaluated using the information set available at time t . Therefore, given the information set at time t , the investor first determines the performance measures and then computes the composite performance index. This index is used as an equity screening tool, and helps the investor to identify the most interesting assets - those with higher composite performance index value. Finally, the selected assets are introduced in the portfolio, with weights to be determined by the investor. We underline how our procedure does not consider the allocation problem, but focuses on the screening of assets. The final purpose is the construction of a ranking of the assets included in the investment universe, with the identification of a subset of them which are considered the optimal assets for the construction of the portfolio.

We thus assume that the investor includes in the portfolio M assets chosen from a larger group containing N assets. Note that $M \ll N$ in order to avoid excessive transaction and rebalancing costs, while M should not be too small, otherwise diversification benefits will tend to vanish. In this study, we fix $M = 25, 50$, or 100 . Those values are reasonable in small and medium-sized managed portfolios, and will allow us to verify if changes in the number of assets will provide relevant variations in the portfolio turnover and, as a consequence, on rebalancing costs.

For simplicity, and to focus on the advantages of using a combination of performance measures, we assume the investor adopts an equally weighted portfolio composition. This implies that all the M assets included in the portfolio will have weight $1/M$. At first glance, this might seem a restrictive assumption, but equally weighted portfolios have been shown to have performances comparable to, if not better than, optimised portfolios; see DeMiguel et al. (2009).

Given the choice of M and the investment universe of N assets, the objective of the investor is to select the M assets to be included in his portfolio following an optimality criterion based on a combination of performance measures. The main contribution we provide in this paper is the peculiar screening rule we propose, which is based on an optimized linear convex combination of performance measures.

In general terms, a simple screening rule orders assets using a single performance measure. For instance, we could order the N assets by computing the Sharpe ratios of all assets, order them, and invest using an equally weighted strategy in the M assets with the highest Sharpe ratios. However, when asset return densities deviate from normality, higher order moments, partial moments or quantiles may have additional informative

content, see for instance Farinelli and Tibiletti (2008). As a result, more general and flexible performance measures could provide different asset rankings.

Our aim is to propose a more efficient screening rule by combining a set of performance measures. The combination will take advantage of different views on the assets, or, similarly, of different information, including the asset returns density, the relationship between asset returns and risk factors and the use of alternative utility functions.

Let us first introduce some notations. We define for each asset j at time t a composite performance index, $CI_{j,t}$, which is a function of Q performance indices $p_{i,j,t}$, where $i = 1, 2, \dots, Q$, and $j = 1, 2, \dots, N$. Note that this index is computed using the information set up to time t , I^t , and is used to allocate the portfolio in time t with investment horizon in $t + 1$. We impose the simplifying assumption that the set of performance measures is fixed and known a-priori (thus, the value of Q is fixed over time, and the Q performance measures used in the combination do not change over time).

We suggest the following composite index for asset j at time t :

$$CI_{j,t}(w_1, w_2, \dots, w_Q) = \sum_{i=1}^Q w_i p_{i,j,t}, \quad j = 1, 2, \dots, N, \quad (3.1)$$

$$w_i \geq 0, \quad i = 1, 2, \dots, Q, \quad \sum_{i=1}^Q w_i = 1,$$

where the weights are imposed to be positive and to sum to one. Note the weights are the same for all assets and are time-invariant. Let us assume for a while the weights are given on the composite indices for each asset, we determine the portfolio composition by investing $1/M$ of the available wealth in the M assets with the highest score of the composite index $CI_{j,t}$. However, the performance combination weights $\mathbf{w} = \{w_1, w_2, \dots, w_Q\}$ have to be estimated. We propose to determine the weights by maximizing the following criterion function:

$$\max_{\mathbf{w}} f(\mathbf{w}) = \frac{1}{m} \sum_{l=t-m+1}^t r_{p,l} - \lambda \frac{1}{m} \sum_{l=t-m+1}^t (r_{p,l} - \mu_p)^2, \quad (3.2)$$

$$r_{p,l} = \frac{1}{M} \sum_{j \in \mathcal{A}_t(\mathbf{w})} r_{j,l}, \quad (3.3)$$

where $\mathcal{A}_t(\mathbf{w})$ is the set of the M assets with the highest score of the $CI_{j,t}(\mathbf{w})$ index (note this set depends on the choice of the weights' vector); $r_{p,l}$ is the time l return of

the equally weighted portfolio over the assets included in $\mathcal{A}_t(\mathbf{w})$; the first term of the criterion function is the average return of the portfolio over the last m observations; the second term is similar to a risk measure, weighted by a risk aversion coefficient λ ; the risk measure depends on the choice of μ_p , which we set either equal to the average portfolio return, thus making the second term equivalent to the portfolio variance; alternatively, we fix μ_p equal to the average return of a benchmark over the last m observations, making the second term equivalent to a variance tracking error; finally, we suggest to set the risk aversion coefficient between 2 and 50, mimicking the standard choices in the mean-variance framework.

The overall criterion function is similar to a mean-variance utility function. However, (3.2) is not optimized with respect to the portfolio weights, which are fixed, but with respect to the performance measure weights. The intuition behind this criterion function is that we are determining the weights which, using up-to-date information, would have maximized the difference between the return and the risk of the allocated portfolio (where the risk is weighed by a risk aversion coefficient). The risk is either monitored in absolute terms, by using the portfolio variance, or in relative terms, by comparing the portfolio returns to those of a benchmark index. This second option is a reasonable choice if the investor is an investment manager. The criterion function is thus a backward evaluated mean-variance function which is used to forward allocate the portfolio with $1/M$ weights.

The proposed approach for the evaluation of the composite performance index entails a number of implicit assumptions. From a statistical point of view, the construction of performance measures at the single asset level implies a focus on the marginal distributions of each asset included in the analysis. According, the composite index in (3.1) is an equity screening tool since it does not provide optimal asset allocation. In fact, a relevant aspect is not taken directly into account, i.e. the correlation across assets. Dependence among assets has only an implicit role in the portfolio return and risk in (3.2) and (3.3), but it is not an element considered explicitly in the criterion function. Anyhow given that (3.2) penalises excessive risks and that the portfolio is equally weighted, the effect of asset correlation is partially sterilised. It might be possible that assets highly correlated with relatively good performances are included in the equally weighted portfolio, thus reducing the diversification benefits, and for this reason, we suggest the use of (3.1) within an investment process, but not directly as an asset allocation tool.

A second element not directly covered by our composite index is the dependence between performance measures. To reduce the possible negative impact of highly correlated performance measures (which would limit the benefits of the composite index), we suggest

to include performance measures with low rank correlation in (3.1), following (Eling and Schuhmacher, 2007), Eling (2008), Eling et al. (2011) and Caporin et al. (2011).

A further empirical finding is associated with the previous evidence. The function $f(\mathbf{w})$ has fixed performance weights, but when we estimate the weights over different samples (possibly partially overlapping) the performance combination weights change. In fact, they update with respect to the changing relevance of the underlying performance measures. We first relate such a feature to the evidence provided in Caporin et al. (2011) that shows time-variation in the rank correlation across performance measures, suggesting their informative content is not stable over time. The change in the performance measure relevance is also associated with a change over time of the asset return densities, or equivalently, of their moments and quantiles. In fact, if these elements vary over time, performance measures vary over time and their views over competing assets change over time.

A robust approach to capture changes in the asset return densities would be based on the construction of a proper (conditional) parametric model specifying a distributional assumption and the law of motion of the density parameters. However, such a choice would expose the analysis to specification problems and potential changes over time in the density family (even if that could be accommodated by means of mixture models). It is important to underline that we aim at introducing a model-free equity screening approach which does not depend on any distributional assumptions; it is based only on empirical quantities; and becomes thus partially non-parametric. An alternative method to capture potential changes in the asset returns densities, and in particular on their moments or quantiles, is to consider a rolling evaluation of the performance measures, as in Biglova et al. (2004). In that way we can capture potential time-varying features of the asset return moments and quantiles, which are the constituents of most performance measures. If the conditional values of the cited quantities would be time-varying, a rolling methodology will take that into account. On the contrary, if they would be time-invariant, we will only have an effect coming from sampling errors, which could be controlled by changing the size of the rolling window. We are thus implicitly assuming that the sample estimators of moments, quantiles, and quantities obtained as transformations of sample data (such as utility functions) are consistent and unbiased estimators of the corresponding conditional quantities.

3.3 Implementation issues

In the following, we discuss a number of issues that should be considered in the implementation of the composite performance index evaluation proposed in the previous section.

Those elements clarify first the definition of two elements which are pre-requisites of the equity screening methodology: the investment universe and the benchmark. Later, we highlight the flexibility of the equity screening approach, widening the concept of performance measures, which might include other indicators such as the market value of the assets. We then move to the evaluation of performance measures and to the possibility of standardizing their values. Finally, we deal with the optimisation of the criterion function $f(\mathbf{w})$, suggesting the use of genetic algorithms.

Investment universe. When the allocation is performed over time for different t , our approach does not require the portfolio cardinality M and the number of assets N to be fixed. These two could be changed over time, thus allowing for changes in the universe of available assets (companies may die, or might be involved in mergers and acquisitions, or new companies can be included in the investment universe) as well as for changes in the portfolio strategies (increasing/decreasing the diversification).

Benchmark. If a benchmark is used in the criterion function, its choice also has to be carefully considered. In fact, the benchmark has to be chosen such that it is representative of the N assets included in the analysis. This is required to evaluate an appropriate tracking error. The benchmark and the assets should thus include the effect of dead companies. In fact, the use of a specific equity market index as benchmark, together with N currently traded assets exposes the equity screening to a survivorship bias. An alternative approach that overcomes the bias and excludes dead companies is to create a synthetic benchmark using a set of N selected assets and their market values. We follow this approach for simplicity.

Definition of performance measure. Our approach is flexible, and the term "performance measures" could be interpreted in a wider sense. In fact, we could optimally combine a set of indicators we associate with listed companies. These indicators could be performance measures, but could also be liquidity measures, technical analysis indicators or company-specific variables (revenues, employees, balance sheet ratios). From a different viewpoint, the composite index we propose might be separately evaluated for a set of risk measures, as well as for a set of reward measures. From this different point of view, our criterion is close to a multi-criteria methodology, similar in some respects, to Ballester et al. (2007).

Companies' market value. Liquidity is one of the possible market constraints that could affect our modelling strategy. In fact, the selected assets can differ in terms of market value and thus liquidity, making the allocation of the optimal portfolio problematic. In extreme cases, our optimally created composite index could suggest investing

in companies with small market value, whose shares might be characterised by limited liquidity. As a result, the implementation of the portfolio could be characterised by large costs (transaction costs as well as large deviations in the price due to the limited liquidity or the impossibility of creating the portfolio because some trades could not be executed in the market due to the absence of a counterpart). In order to mitigate this aspect, and thus force the optimal portfolio to invest in small caps only if their performances are really relevant, we suggest introducing market value as a further performance measure. This would capture the liquidity effect, higher the market value and higher the liquidity. Clearly, other measures of stock liquidity can be considered.

Evaluation of performance measures. The performance indicators chosen to build the composite index are generally computed on a given sample. In order to follow the evolution over time of the asset return densities, we suggest evaluating the performance measures over a rolling window of m observations. The value of m depends on the time frequency of observations and on the total sample length; some examples could be 60 or more months, 25 or more weeks or 40 or more days. In general terms, we suggest using between 40 and 60 observations to avoid excessive volatility in performance measure values that might induce relevant changes in the construction of the composite index, in the assets included in $\mathcal{A}_t(\mathbf{w})$ and consequently, a large turnover in the portfolio. On the contrary, longer samples could significantly smooth performance measures sequences, leading to a very low turnover, but would not capture local (medium period) changes in performance measure relative rankings. With respect to the data frequency, we suggest the use of monthly data, thus adopting the equity screening approach as a tool within the investment process. Higher frequencies will induce relevant and frequent changes on the portfolio combination weights and on the asset rankings.

Standardisation. Given a list of Q performance measures, our final purpose is the construction of a composite index. However, we must recognise that different performance measures could have different ranges, thus making their combination dependent on the scale of the chosen performance measures. For this reason, we suggest considering the standardised performance measures as inputs of the composite index. Let $p_{i,j,t}$ be a given performance measure; we suggest computing the composite index $CI_{j,t}$ using the following quantities as inputs:

$$\bar{p}_{i,j,t} = \frac{p_{i,j,t} - \min \{p_{i,j,t}\}_{j=1}^N}{\max \{p_{i,j,t}\}_{j=1}^N - \min \{p_{i,j,t}\}_{j=1}^N}. \quad (3.4)$$

Such a standardisation makes the performance indices vary between 0 and 1, thus avoiding the scale effect, and ideally putting all performance measures on the same playing field.

Estimation of performance weights. The determination of the composite index requires the solution of a non-trivial optimisation problem. For each point in time, the evaluation of $f(\mathbf{w})$ in (3.2) conditional to a vector of weights $\mathbf{w}$ requires the following steps:

- Evaluate the performance measures $p_{i,j,t}$;
- Compute the standardized performance measures $\bar{p}_{i,j,t}$;
- Determine for each asset the composite index $CI_{j,t}(\mathbf{w}) = \sum_{i=1}^Q w_i \bar{p}_{i,j,t}$;
- Identify the set $\mathcal{A}_t(\mathbf{w})$;
- Obtain the ex-post return of the allocation $r_{p,l} = \frac{1}{M} \sum_{j \in \mathcal{A}_t(\mathbf{w})} r_{j,l}$ and the objective function $f(\mathbf{w})$.

The criterion function $f(\mathbf{w})$ is, however, a non-linear and non-differentiable function of the performance measure weights $\mathbf{w}$. In fact, these enter only in the construction of the set $\mathcal{A}_t(\mathbf{w})$ that contains the assets with the highest values of the index $CI_{j,t}(\mathbf{w})$. Furthermore, different values of the weights could provide the same set of 'best' assets, thus making the optimisation of $f(\mathbf{w})$ computationally demanding. We provide a graphical example to clarify this aspect. Let us assume we have three performance measures and thus two weights to be estimated (the third one is obtained through the constraint). We report in Figure (3.1) the value of the criterion function for all possible weight combinations. Notably, the surface has many flat areas and local maxima. On the basis of the previous comments, we conclude that optimisation methods based on derivatives of the function $f(\mathbf{w})$ are not appropriate.

We thus suggest the use of genetic algorithms, in particular the Differential Evolution algorithm (DE) developed by Storn and Price (1997), which is a population-based optimizer. One particular feature of DE algorithm is that it encodes every type of parameter as floating-point numbers. As reported in Price et al. (2005), this provides different advantages with respect to the bit-flipping algorithm of traditional Genetic Algorithms implementations. For instance, it induces better scales on large problems and a faster convergence. In turn, this implies a reduced computational effort.

In the DE, the starting point is determined by sampling the objective function at different random initial points. Each parameter in our objective function is bounded in

[0,1] since our parameters represent the coefficients of a convex combination, that is, the weights for performance combination. In our case, initial points are then sampled from a p -dimensional domain. In the empirical application we used the Differential Evolution optimization package for Matlab developed by Markus Buehren and available in MatlabCentral. This package is based on the code of Storn and Price (1997). Note that a single evaluation of the objective function took on average 15 seconds using an Intel 3.4 GHZ Intel Core 7 processor machine. However, the execution time can be reduced using the parallel processing on multiple cores. We set the number of population members as suggested by the author equal to 10 by the number of parameters. The algorithm stops when one of the following conditions is met: the maximum number of iterations is reached (we set the maximum at 100); the function evaluation lasts for a maximum of 60 seconds and all possible combination of parameters have been tested. For a detailed description of the algorithm, see Storn and Price (1997), Maringer (2005) and Price et al. (2005). For applications of the Differential Evolution in finance, see Maringer (2005), Gilli and Schumann (2012), Hagströmer and Binner (2009), Krink et al. (2009), Krink and Paterlini (2011) and Gilli and Schumann (2012), among others. For other applications of genetic algorithms in finance see Mohr et al. (2013), among others.

3.4 Equity screening with composite indices on the US market

We consider an empirical application of the composite index previously introduced within an asset allocation framework. The composite index might be seen here as an equity screening rule, and our purpose is to verify its advantages in terms of portfolio returns. We first list the performance measures we take into account, and later describe the data we consider. Moreover, we describe two alternative naive equity screening rules that are compared to our proposal. The empirical results are reported in a fourth subsection.

3.4.1 Selected performance measures

The results of our approach clearly depend on the choice of performance measures combined in the index $CI_{j,t}(\mathbf{w})$. The following surveys might be used to select among the large set of performance measures proposed in the financial economics literature: Aftalion et al. (2003), Le Sourd (2007), Bacon (2011), Cogneau and Hübner (2009), Cogneau and Hubner (2009) and Caporin et al. (2011). In addition, Eling and Schuhmacher (2007), Eling (2008), Eling et al. (2011) and Caporin et al. (2011) report comparisons

among alternative performance measures. In the following we list the performance measures for asset i . Those measures have been selected in order to include well-known measures, like the Sharpe, Sortino, Treynor, and Appraisal ratio, as well as measures based on partial moments, quantiles and drawdown which are not much common. Our purpose is to provide an empirical example showing the benefits associated with the combination of several measures and we do not aim at determining the optimal selection of performance measures. In the example, we evaluate the performance measures using returns data for the range $t - m$ to $t - 1$, to be used for time t equity screening. Our selection includes the following traditional performance measures:

- Sharpe ratio (Sharpe, 1966 and 1994)

$$Sh(i, t - 1, m) = \frac{\mu(r_{i,t-1} - rf_{t-1}, m)}{\sigma(r_{i,t-1} - rf_{t-1}, m)}, \quad (3.5)$$

where rf_t is the risk-free rate, $\mu(x_{t-1}, m) = \frac{1}{m} \sum_{j=1}^m x_{t-j}$ and $\sigma^2(x_{t-1}, m) = \frac{1}{m} \sum_{j=1}^m (x_{t-j} - \mu(x_{t-1}, m))^2$

- The expected return over the Mean Absolute Deviation (MAD) introduced by Konno (1990, 1991)

$$ERMAD(i, t - 1, m) = \frac{\mu(r_{i,t-1} - rf_{t-1}, m)}{MAD(r_{i,t-1} - rf_{t-1}, m)}, \quad (3.6)$$

where $MAD(x_{t-1}, m) = \frac{1}{m} \sum_{j=1}^m |r_{t-j} - \mu(x_{t-1}, m)|$;

- The Appraisal ratio, defined as

$$AR(i, t - 1, m) = \frac{\alpha_i}{\sigma[\epsilon_{i,t}]}, \quad (3.7)$$

where α_i is the intercept of the CAPM regression $r_{i,t} - rf_t = \alpha_i + \beta_i (r_t^M - rf_t) + \epsilon_{i,t}$, with r_t^M being the market return; $\sigma[\epsilon_{i,t}]$ is the volatility of the CAPM regression residuals (the volatility of the idiosyncratic shocks) and the regression parameters are estimated over the range $t - 1$ to $t - m$.

- The Treynor index (Treynor, 1964), or Risk Adjusted Return,

$$RaR(i, t - 1, m) = \frac{\mu(r_{i,t-1} - rf_{t-1}, m)}{\beta_i}, \quad (3.8)$$

where the risk adjustment is made using the systemic risk exposition as computed from the CAPM regression reported in the Appraisal ratio description;

- The M2 index by Modigliani and Modigliani (1997),

$$M2(r_{i,t}) = \mu(r_{i,t-1} - r_{B,t-1}, m) \times \frac{\sigma(r_{B,t})}{\sigma(r_{i,t})} + \sigma(r_{f,t}) - \sigma(r_{B,t}), \quad (3.9)$$

where $r_{B,t}$ identifies the return of a benchmark investment.

We also include measures based on the Drawdown, defined as the maximum loss an investor may suffer in the period $t - m$ to $t - 1$:

$$D_t(i) = \min(D_{t-j} + r_{i,t}, 0) \text{ with } D_{t-m-1} = 0.$$

The Drawdowns obtained in the range $t - m$ to $t - 1$ can be ordered from the smallest (generally negative) to the largest (generally a zero), resulting in the ordered sequence $\bar{D}_1(r_{i,t-1}), \bar{D}_2(r_{i,t-1}), \dots, \bar{D}_m(r_{i,t-1})$. In our analysis, we use the following indicators:

- the Calmar ratio of Young (1991),

$$CR(i, t - 1, m) = \frac{\mu(r_{i,t-1}, m)}{-\bar{D}_1(i)}; \quad (3.10)$$

- the Sterling ratio of Kestner (1996),

$$SR(i, t - 1, m, w) = \frac{\mu(r_{i,t-1}, m)}{-\frac{1}{w} \sum_{j=1}^w \bar{D}_j(X_{i,t})}, \quad (3.11)$$

where w identifies the number of values used for the drawdown measure;

- the Burke (1994) ratio,

$$BR(i, t - 1, m, w) = \frac{\mu(r_{i,t-1}, m)}{\left(-\frac{1}{w} \sum_{j=1}^w [\bar{D}_j(X_{i,t})]^2\right)^{\frac{1}{2}}}. \quad (3.12)$$

We also consider other measures based on Partial Moments:

- the Sortino ratio by Sortino and Van Der Meer (1991),

$$Sr(i, t - 1, m) = \frac{\mu(r_{i,t-1}, m)}{LPM(r_{i,t-1}, m, 2)}, \quad (3.13)$$

where $LPM(x_{t-1}, m, p) = \left(\frac{1}{m} \sum_{j=1}^m (-\min(x_{i,t-j}, 0))^p\right)^{\frac{1}{p}}$;

- the Kappa 3 measures by Kaplan and Knowles (2004),

$$K3(i, t - 1, m) = \frac{\mu(r_{i,t-1}, m)}{LPM(r_{i,t-1}, m, 3)}. \quad (3.14)$$

Finally, we consider a measure based on quantiles, the expected return over absolute Value-at-Risk of Dowd (2000)

$$VR(i, t-1, m, \alpha) = \frac{\mu(r_{i,t-1}, m)}{|VaR(r_{i,t-1}; \alpha)|}, \quad (3.15)$$

where $VaR(r_{i,t-1}; \alpha)$ is the α -quantile of asset i returns in the period $t-1$ to $t-m$.

In the following empirical application, we consider a rolling evaluation of the performance measures over a window of $m = 60$ days. Moreover, we compute the Sterling and Burke ratios employing the five largest Drawdowns. Finally, in the VR index, we set $\alpha = 0.05$.

As mentioned in the previous section, the MV of each company will be included as an additional performance measure to penalise smaller companies.

3.4.2 Dataset description and benchmark construction

Our dataset is based on the constituents of the S&P Composite 1500 (the 22nd of February, 2012). The time series were downloaded from Datastream at a monthly frequency from the 31st of January 1990, to the 31st of January 2012, for a total of 265 observations. We also recovered a proxy of the risk free asset, the JP Morgan 1 Month Cash bond index. To cope with survivorship bias, we restricted the dataset to a collection of assets constantly available in the analysed sample. Following this criterion, we restricted our attention to 695 assets.¹

Given that we exclude a relevant part of the assets included in the S&P500, and given that the index composition changes over time,² the S&P500 index cannot be used as a benchmark or market index to evaluate the performances of our equity screening approach. Therefore, we build a benchmark that is coherent with the selected assets. The index we construct corresponds to the value-weighted index composed of the 695 selected assets. Table (3.1) provides some information about the MV of the assets, while Figure (3.2) is the plot of the benchmark total returns since the end of January 1990. We note the well-known decreases in the equity benchmark in 2000-2001 and 2008. Moreover, we point out the increase in the selected equity average market value from 1990 to the present. The upward movement is matched with a somewhat stable coefficient of variation, with the exception of the value reported in 1999, just before the technology market bubble burst.

¹The list of assets is available upon request.

²The time series of the S&P constituents is not available to us through Datastream.

3.4.3 Portfolio allocation and naive equity screening rules

We apply our equity screening approach to the selected assets, estimating performance measures on rolling windows of 60 months. Starting from the end of January 1995 (the first month where 60 monthly returns are available), we identify, across the 695 assets, the 50 assets that maximise the criterion function in (3.2). We then create an equally weighted portfolio where each asset has weight equal to 2%, and we compute the monthly realised returns of the portfolio. The portfolio composition is modified on a monthly basis, where the criterion function (3.2) is optimised each month. At the end of this procedure, we have a total of 205 portfolio returns.

We apply the screening rule discussed in Section 2, combining two different specifications of the risk component in (3.2); we consider the portfolio variance (VO) and the tracking error volatility (TE) cases.³ Moreover, we make use of two different values for the risk aversion parameter, 1 and 20. The first corresponds to a mild penalisation of the risk, while the second mimics the choices of a more risk-averse agent.

To evaluate the performances of our equity screening algorithm, we compare it to a naive equity screening rule based on the Sharpe ratio. Therefore, with a rolling procedure similar to that outlined above, we selected the 50 assets that have higher Sharpe ratios, and use those assets to create a second equally weighted portfolio. Performances will also be compared to those of the benchmark, computed as described in the previous subsection.

The portfolio returns are compared by means of the following approaches: standard descriptive analyses of returns, including the computation of some risk measures; a horse-race over the range February 1995 to January 2012; the weights associated with the different performance measures; the turnover of the portfolios based on the screening algorithms.

3.4.4 Performance results

Table (3.2) includes the descriptive analysis of the portfolios, while Figure (3.3) shows the cumulated returns from 1995 to 2012. Compared to the benchmark, all equity screening-based portfolios provide higher cumulated returns. If we consider an investor with an initial wealth equal to 1, the portfolio with the highest cumulative return (8.53) is given by the criterion function which considers the tracking error volatility with the risk aversion coefficient set to 1. The second highest portfolio in terms of cumulative

³We set the term μ_p equal to the average portfolio return in the first case, while μ_p is equal to the benchmark return in the second case.

returns (6.69) is based on the criterion function which depends on the portfolio variance. However, the risk-aversion coefficient has a relevant impact; in fact, the TE and VO portfolios with risk aversion set to 20 are less profitable than the Sharpe-based portfolio. Respectively, (3.26) and (3.68) compared with the Sharpe-based (5.36). The result is even stronger if we analyse the turnover, which is sensibly higher for higher values of the risk aversion. In the TE strategy, it changes from 0.1822 to 0.2457. Similar, it increases from 0.1862 to 0.2208 in the VO strategy. Comparing the Sharpe index of the portfolio returns, the Sharpe-based portfolio seems to be the preferred choice (0.2288), excluding of the VO case with risk aversion set to 20 which provides a Sharpe ratio equal to 0.2314. Such a result is a consequence of the criterion used for portfolio construction, and might be expected. In terms of risk measures, we observe that all portfolios based on screening rules are more risky than the benchmark, which gives an annualized volatility equal to 0.1538. However, we stress that all screening-based portfolios have not been optimised to reduce the risk, but are simply based on an equally weighted allocation scheme. As a result, risk reductions might be achieved by optimising portfolio weights. Finally, we emphasise that risk measures decrease (in absolute terms) with increasing risk aversion, as expected, and for a risk aversion coefficient equal to 20 they are better than the benchmark and also preferred to those of the Sharpe-based portfolio. The VaR at 5% in the TE strategy changes from 0.1405 to 0.0984 while in the VO strategy from 0.0872 to 0.0573.

Overall, the introduction of screening rules provides higher returns than the benchmark, with a preference for our proposed algorithm compared to simpler screening based only on the Sharpe ratio. Risk measures are different across screening strategies, but this is a consequence of the portfolio construction that is not optimised. The turnover induced by screening rules is influenced by the degree of risk aversion, and becomes higher and more volatile with increasing risk aversion; see Figure (3.4). The Figure shows that the turnover induced by a Sharpe-based screening is oscillating between 10% and 30% on a monthly basis. Similar values are provided by the TE screening with low risk aversion in a large part of the sample. Deviations are observed during periods of high volatility (from 2007) and with higher values of risk aversion. In those two cases, the turnover induced by TE screening is higher than the turnover of Sharpe based screening, and reach values close to 50% with low risk aversion and up to 90% with high risk aversion.

In our selection procedure, the weights assigned to the different performance measures have a relevant role; moreover, they change over time, and react to the different features of the returns time series. Figure (3.5) shows an example, while Table (3.3) includes the descriptive statistics for the weights in the tracking error- and volatility-based screening. We first point out that the screening algorithm we propose generally assigns a very small weight - which is close to zero - to the Sharpe ratio, independent of form of the criterion

function and of the risk aversion coefficient level. Both tracking error and portfolio volatility objective functions provide similar performance measure weights when the risk aversion coefficient is equal to 1; in particular, we observe that the Modigliani-Modigliani index receives the largest weight. Respectively, 0.56 in the TE strategy and 0.44 in the VO strategy. Other performance measures receiving a high weight are the Appraisal Ratio and the Excess Return over Mean Absolute Deviation. The other measures are characterised by very small average weights and limited standard deviations, signalling they receive a relevant weight only occasionally as shown in Table (3.3). When the risk aversion coefficient is increased to 20, the difference between the two forms of the criterion function leads to different average weights being assigned to the performance measures. When we focus on the tracking error-based function, the MV, the Appraisal Ratio and the Excess Return over mean absolute deviation receive a weight larger than 10%. In contrast, in the second implementation based on portfolio volatility, the Burke and RAR measures increase over 10%, while the market value falls below 10%. Even for a large risk aversion level, the performance measures with small average weight have a large standard deviation, thus confirming their limited relevance. The results support our expectation of variability in the informative content of performance measures, and might also be seen as a confirmation of the potential interest in measures going beyond the Sharpe ratio.

A relevant element to emphasise is the limited weight assigned to the market value. As a consequence, the selected assets might be characterised by small market value and thus small liquidity, possibly creating difficulties in the implementation of portfolios based on those assets. This is confirmed by Figure (3.6) in which we see a sharp decrease in the market value of the selected companies in the second half of the sample. Such a behavior is common across the different implementations of the screening algorithm. To force the impact of market value in the screening algorithm, we run a second set of evaluations where we constrain the weight assigned to the market value, imposing a lower bound set to 10%.

Table (3.2) includes the portfolio return descriptive results, showing the impact of the market value bound in terms of cumulated returns, risk measures, and Sharpe ratios. Overall, imposing minimum relevance to the companies' market value leads to a slight risk reduction, as Value-at-Risk, Expected Shortfall, returns volatility and range all improve. However, the total and average returns, and the Sharpe ratio decrease, except in the case with the tracking error objective function and a high level of risk aversion. In addition, the turnover shows a decrease, which is larger for the cases where the risk aversion is set to 20. It decreases from 0.2457 to 0.1972 in the TE and from 0.2208 to 0.2013 in the VO strategy.

Comparing the market value of selected companies, we note an increase in the second part of the sample compared to the previous cases; see Figure (3.6). As a consequence, the introduction of a lower bound to the market value leads to the selection of equities the average market value of which is generally higher than the average market value of the benchmark. Weights assigned to the performance measures are partially affected by the constraint imposed on MV; see Table (3.4). However, the sets of the most influential performance measures are unchanged.

As mentioned in the previous section, the number of assets identified by our screening algorithm might be easily modified. Tables (3.5) and (3.6) contain descriptive analyses of the realised portfolio returns for 25 and 100 assets, as does Table (3.2) for the 50 assets case. By comparing the results across different values of M , we note that screening algorithms always beat the benchmark in terms of cumulated returns but not with respect to risk measures. Nevertheless, we observe a general reduction in the risk measures for increasing M , and an improvement in the Sharpe ratios for $M = 100$. With a risk coefficient aversion equal to 1, the Sharpe ratio increases to 0.2231 from 0.2111 in the TE and to 0.2284 from 0.2067 in the VO strategy. Such a finding depends on the possibility of identifying profitable investment opportunities (that is, single assets) the performance of which might be variable over time, leading to assets being "above average quality" but not necessarily "top performers". If a small number of assets is used, the selected equities are subject to more frequent changes, as shown by the average turnover (decreasing for increasing M) in Table (3.5). As a result, when the number of selected assets increases, the performances improve.

Moreover, we observe that the Sharpe ratios of the portfolios based on our screening algorithm are better than the naive approach in a few cases only, but are associated with different objective functions: when $M = 25$ the use of the tracking error-based objective function, with a large risk aversion and bounded weight of the market value, provide the best results with a Sharpe ratio equal to 0.2232. In contrast, when $M = 100$, the results are slightly better by providing a Sharpe ratio equal to 0.2649 for the objective function using the variance of the selected portfolio and a large risk aversion level, dependent from the presence of a bound on the weight of MV. The case where $M = 50$ is in the middle, with the Sharpe ratios of the naive strategy and our screening rule being very close to each other. Overall, this empirical application shows that the proposed screening algorithm is able to identify profitable investment opportunities.

3.5 Conclusions

We introduced a new screening algorithm that selects within an investment universe a subset of assets based on a composite index of performance measures. Such an index linearly combines different performance measures where the combination weights are derived from an optimisation problem that takes into account past performances associated with the "optimal" weights and the subsequent asset ranking. Accordingly, past performances lead to the asset selection for future allocations, suggesting the backward/forward equity screening name. We discuss several implementation issues of our screening algorithm and then present an empirical application based on US equities. The results show the advantage of our composite performance index in a simplified asset allocation framework. In fact, by comparing simulated equally weighed portfolio strategies, the proposed composite performance index provides superior results in terms of realised profits. Several aspects of our analysis might be further extended. Within the proposed framework we did not address the estimation of the optimal number of assets M which might be expressed as a fraction of the total number of asserts N . Moreover, asset weights might be estimated, instead of calibrating them or set them equal. Furthermore, the combined performance index could be designed to take into account the dependence structure across performance measures, or might be based on a combination of returns and risk measures rather than performance measures. In addition, several constraints can be added to the criterion function, such as limits on the risk (maximum variances, VaR constraints), maximum transaction costs, turnover constraints, just to cite some possibilities. From a practical point of view, our empirical study might be improved by the introduction of "dead" companies or by the construction of contrarian (investing in assets with the lowest composite index) or long-short (long on the best assets, short on the worst ones) strategies. Those extensions will be left to future research.

3.6 Tables and Figures

TABLE 3.1: Market value of the selected equities (in millions of USD). CV denotes the coefficient of variation.

	Mean	Stdv	Min	Max	CV
Jan-90	1976	5304.59	0.79	58827	2.685
Dec-94	2505	6911.52	2.40	87193	2.759
Dec-99	8448	34849.45	1.04	602432	4.125
Dec-04	7872	22502.83	50.52	323717	2.859
Jan-12	9229	26383.81	48.97	425608	2.859

TABLE 3.2: Descriptive analysis of realized portfolio returns and of the benchmark with 50 assets. TE denotes portfolios where the criterion function considers the tracking error volatility, while VO represents portfolio where the criterion function depends on the portfolio variance. Moreover, 1 and 20 identify the risk aversion coefficient value. The columns report the cumulated returns obtained in the range February 1995 to January 2012, the annualized average monthly return and annualized variance, the minimum and maximum monthly returns, the skewness and kurtosis indices computed on monthly returns, the 5% Value-at-Risk and Expected Shortfall, the Sharpe ratio, and the average monthly turnover.

	Cum. Ret.	Ret. Ann.	Vol. Ann.	Min	Max	Skew	Kurt	Var(5%)	ES(5%)	Sharpe	Avg-TO
Bench	2.2030	0.1011	0.1538	-0.1601	0.1044	-0.5778	3.8469	0.0650	0.0963	0.1814	-
SH	5.3647	0.1404	0.1667	-0.1876	0.1121	-0.8082	4.4216	0.0681	0.1042	0.2288	0.1624
TE1	8.5338	0.1783	0.2259	-0.2477	0.1991	-0.5968	4.3472	0.0935	0.1405	0.2111	0.1822
TE20	3.2629	0.1169	0.1601	-0.1853	0.1180	-0.6989	4.2987	0.0668	0.0984	0.2002	0.2457
VO1	6.6904	0.1611	0.2100	-0.2477	0.1463	-0.7312	4.5592	0.0872	0.1319	0.2067	0.1862
VO20	3.6833	0.1186	0.1405	-0.1646	0.1360	-0.8225	5.4076	0.0573	0.0888	0.2314	0.2208
Portfolios with a lower bound on Market Value weight in the criterion function (weight less or equal to 0.1)											
TE1	7.3774	0.1686	0.2194	-0.2383	0.1662	-0.6080	4.1221	0.0911	0.1359	0.2063	0.1723
TE20	3.8260	0.1222	0.1524	-0.1777	0.1127	-0.6593	4.2221	0.0627	0.0921	0.2194	0.1972
VO1	5.8385	0.1530	0.2054	-0.2383	0.1442	-0.7387	4.3963	0.0856	0.1297	0.2013	0.1773
VO20	3.1327	0.1113	0.1361	-0.1598	0.0943	-0.9143	5.1529	0.0558	0.0861	0.2248	0.2013

TABLE 3.3: Descriptive analysis of performance measure weights in the range February 1995 to January 2012. The minimum is not included since equal to zero for all measures. RA denotes the levels of the risk aversion. TE identifies tracking-error-based screening while VO refers to screening with portfolio variance in the objective function.

	Mean	Max	St.Dev.	Mean	Max	St.Dev.
TE	RA = 1			RA = 20		
MV	0.0041	0.0911	0.0126	0.2450	0.6544	0.1758
ERMAD	0.1198	0.9987	0.2100	0.3353	0.9340	0.3191
AR	0.2904	0.9386	0.1839	0.2623	0.9588	0.2601
BR	0.0091	0.2149	0.0222	0.0335	0.4865	0.0784
SR	0.0030	0.0727	0.0095	0.0614	0.4955	0.1231
CR	0.0024	0.1074	0.0098	0.0035	0.1238	0.0162
RaR	0.0040	0.2078	0.0193	0.0140	0.3064	0.0470
M2	0.5559	0.9342	0.2175	0.0173	0.4235	0.0647
Sr	0.0018	0.1219	0.0105	0.0024	0.2780	0.0200
K3	0.0053	0.2345	0.0258	0.0195	0.6066	0.0994
VR	0.0017	0.1226	0.0100	0.0048	0.2878	0.0299
Sh	0.0026	0.3199	0.0253	0.0009	0.1770	0.0123
VO	RA = 1			RA = 20		
MV	0.0033	0.0668	0.0092	0.0100	0.1988	0.0261
ERMAD	0.1869	0.9987	0.2904	0.1634	0.9666	0.2652
AR	0.3204	0.9817	0.2213	0.3917	0.9999	0.3631
BR	0.0140	0.3627	0.0364	0.1306	0.9108	0.1814
SR	0.0081	0.1227	0.0221	0.0563	0.4404	0.1091
CR	0.0032	0.1064	0.0121	0.0204	0.3385	0.0560
RaR	0.0088	0.2840	0.0357	0.1955	0.9066	0.3111
M2	0.4390	0.9814	0.2285	0.0114	0.1933	0.0373
Sr	0.0012	0.0389	0.0042	0.0188	0.7849	0.1005
K3	0.0073	0.2518	0.0291	0.0007	0.0882	0.0064
VR	0.0075	0.4965	0.0528	0.0010	0.0673	0.0074
Sh	0.0001	0.0064	0.0007	0.0001	0.0180	0.0013

TABLE 3.4: Descriptive analysis of performance measure weights in the range February 1995 to January 2012 with MV weight with a lower bound at 10%. The minimum is not included since equal to zero for all measures. RA denotes the levels of the risk aversion. TE identifies tracking-error-based screening while VO refers to screening with portfolio variance in the objective function.

	Mean	Max	St.Dev.	Mean	Max	St.Dev.
TE	RA = 1			RA = 20		
MV	0.1035	0.1820	0.0109	0.3236	0.6889	0.1572
ERMAD	0.1065	0.8988	0.1879	0.3025	0.8406	0.2880
AR	0.2636	0.8447	0.1652	0.2358	0.8629	0.2344
BR	0.0082	0.1585	0.0179	0.0301	0.4378	0.0706
SR	0.0025	0.0851	0.0086	0.0520	0.4459	0.1060
CR	0.0021	0.0966	0.0088	0.0029	0.1114	0.0143
RaR	0.0035	0.1870	0.0173	0.0126	0.2758	0.0423
M2	0.5027	0.8408	0.1950	0.0156	0.3812	0.0582
Sr	0.0013	0.1097	0.0081	0.0021	0.2502	0.0180
K3	0.0029	0.1734	0.0132	0.0176	0.5459	0.0895
VR	0.0010	0.0559	0.0046	0.0044	0.2590	0.0269
Sh	0.0023	0.2879	0.0228	0.0008	0.1593	0.0111
VO	RA = 1			RA = 20		
MV	0.1029	0.1601	0.0082	0.1086	0.2789	0.0229
ERMAD	0.1653	0.8988	0.2604	0.1484	0.8699	0.2396
AR	0.2890	0.8835	0.1989	0.3519	0.8999	0.3280
BR	0.0126	0.3265	0.0327	0.1187	0.8289	0.1641
SR	0.0069	0.1104	0.0190	0.0516	0.3964	0.0990
CR	0.0027	0.0958	0.0104	0.0162	0.3046	0.0468
RaR	0.0080	0.2556	0.0321	0.1765	0.8160	0.2813
M2	0.3979	0.8833	0.2074	0.0096	0.1682	0.0316
SR	0.0012	0.0350	0.0043	0.0169	0.7955	0.0925
K3	0.0066	0.2267	0.0262	0.0007	0.0794	0.0060
VR	0.0067	0.4468	0.0475	0.0008	0.0583	0.0055
Sh	0.0001	0.0058	0.0006	0.0001	0.0162	0.0011

TABLE 3.5: Descriptive analysis of realized portfolio returns and of the benchmark with 25 assets. TE denotes portfolios where the criterion function considers the tracking error volatility, while VO represents portfolio where the criterion function depends on the portfolio variance. Moreover, 1 and 20 identify the risk aversion coefficient value. The columns report the cumulated returns obtained in the range February 1995 to January 2012, the annualized average monthly return and annualized variance, the minimum and maximum monthly returns, the skewness and kurtosis indices computed on monthly returns, the 5% Value-at-Risk and Expected Shortfall, the Sharpe ratio, and the average monthly turnover.

	Cum. Ret.	Ret. Ann.	Vol. Ann.	Min	Max	Skew	Kurt	Var(5%)	ES(5%)	Sharpe	Avg-TO
Bench	2.2030	0.1011	0.1538	-0.1601	0.1044	-0.5778	3.8469	0.0650	0.0963	0.1814	-
SH	4.6209	0.1354	0.1765	-0.1958	0.1182	-0.9211	4.8543	0.0732	0.1154	0.2087	0.1845
TE1	7.4082	0.1729	0.2366	-0.1937	0.2369	-0.2312	3.7493	0.0990	0.1415	0.1959	0.2180
TE20	4.9388	0.1348	0.1586	-0.1642	0.1180	-0.5272	3.8236	0.0647	0.0939	0.2313	0.3400
VO1	8.0752	0.1756	0.2294	-0.1807	0.2369	-0.1526	3.6282	0.0954	0.1328	0.2049	0.2406
VO20	3.6774	0.1197	0.1469	-0.1842	0.1120	-1.0308	6.0695	0.0603	0.0955	0.2232	0.2843
Portfolios with a lower bound on Market Value weight in the criterion function (weight less or equal to 0.1)											
TE1	7.6419	0.1723	0.2281	-0.1831	0.2032	-0.2574	3.4093	0.0950	0.1332	0.2025	0.2159
TE20	5.4369	0.1382	0.1519	-0.1573	0.1093	-0.5565	3.8731	0.0613	0.0892	0.2474	0.2475
VO1	5.6512	0.1552	0.2237	-0.1698	0.2337	-0.0969	3.5733	0.0941	0.1267	0.1873	0.2357
VO20	3.0101	0.1110	0.1439	-0.1764	0.0999	-0.9735	5.6622	0.0595	0.0962	0.2120	0.2725

TABLE 3.6: Descriptive analysis of realized portfolio returns and of the benchmark with 100 assets. TE denotes portfolios where the criterion function considers the tracking error volatility, while VO represents portfolio where the criterion function depends on the portfolio variance. Moreover, 1 and 20 identify the risk aversion coefficient value. The columns report the cumulated returns obtained in the range February 1995 to January 2012, the annualized average monthly return and annualized variance, the minimum and maximum monthly returns, the skewness and kurtosis indices computed on monthly returns, the 5% Value-at-Risk and Expected Shortfall, the Sharpe ratio, and the average monthly turnover.

	Cum. Ret.	Ret. Ann.	Vol. Ann.	Min	Max	Skew	Kurt	Var(5%)	ES(5%)	Sharpe	Avg-TO
Bench	2.2030	0.1011	0.1538	-0.1601	0.1044	-0.5778	3.8469	0.0650	0.0963	0.1814	-
SH	6.7535	0.1508	0.1599	-0.1898	0.1242	-0.8493	4.9815	0.0642	0.1005	0.2550	0.1337
TE1	7.5915	0.1652	0.1991	-0.2307	0.1262	-0.7565	4.3627	0.0817	0.1257	0.2231	0.1423
TE20	5.1371	0.1360	0.1541	-0.1635	0.1206	-0.7079	4.4306	0.0625	0.0964	0.2400	0.1761
VO1	7.1007	0.1591	0.1878	-0.2291	0.1298	-0.8484	4.7995	0.0768	0.1187	0.2284	0.1430
VO20	4.7616	0.1292	0.1346	-0.1623	0.0960	-0.9998	5.7702	0.0537	0.0873	0.2619	0.1776
Portfolios with a lower bound on Market Value weight in the criterion function (weight less or equal to 0.1)											
TE1	6.8952	0.1592	0.1950	-0.2334	0.1259	-0.8254	4.5355	0.0802	0.1236	0.2201	0.1362
TE20	5.1409	0.1356	0.1520	-0.1621	0.1146	-0.7105	4.3957	0.0615	0.0935	0.2428	0.1551
VO1	5.8876	0.1490	0.1854	-0.2334	0.1270	-0.9107	5.0157	0.0764	0.1185	0.2175	0.1408
VO20	4.7109	0.1284	0.1323	-0.1592	0.0944	-0.9877	5.5716	0.0527	0.0864	0.2649	0.1680

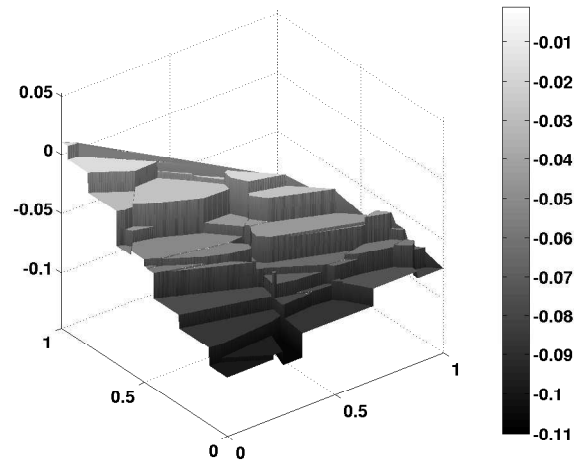


FIGURE 3.1: Surface of the objective function for the determination of the composite index weights with three performance measures. The vertical axis refers to the objective function while the other two axis report the weights of two performance measures (the third being obtained through the constraint on combination weights).

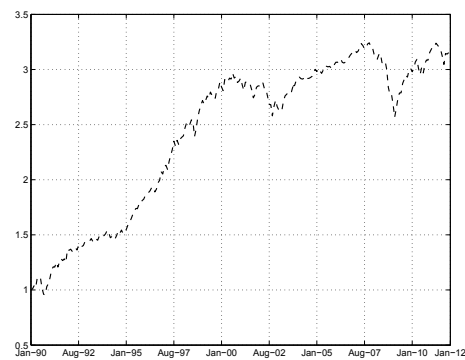


FIGURE 3.2: Cumulated returns of the benchmark in the period 1990-2012

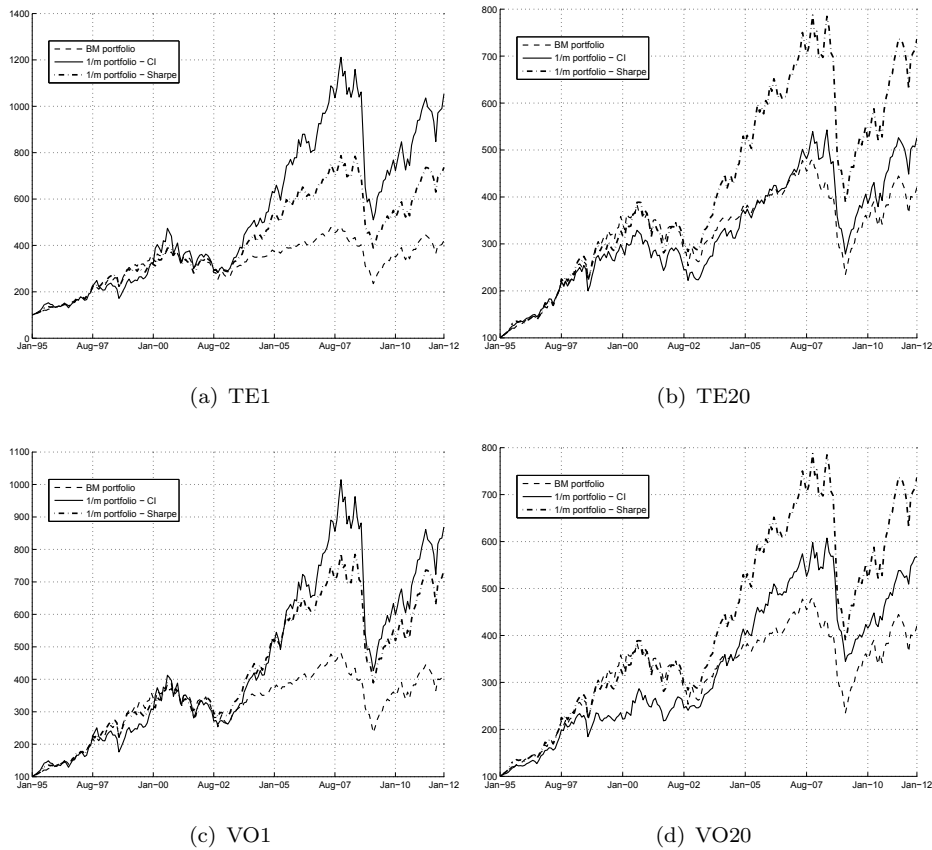


FIGURE 3.3: Cumulated returns of the strategies and of the benchmark in the range 1995-2012

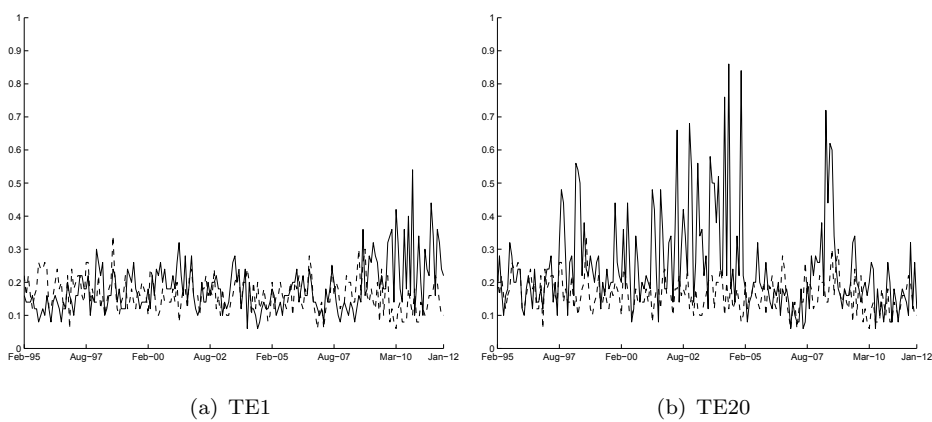


FIGURE 3.4: Turnover induced by Sharpe-based screening (dotted line) and by the TE screening with risk aversion coefficient equal to 1 (bold line - left picture) and equal to 20 (bold line - right picture)

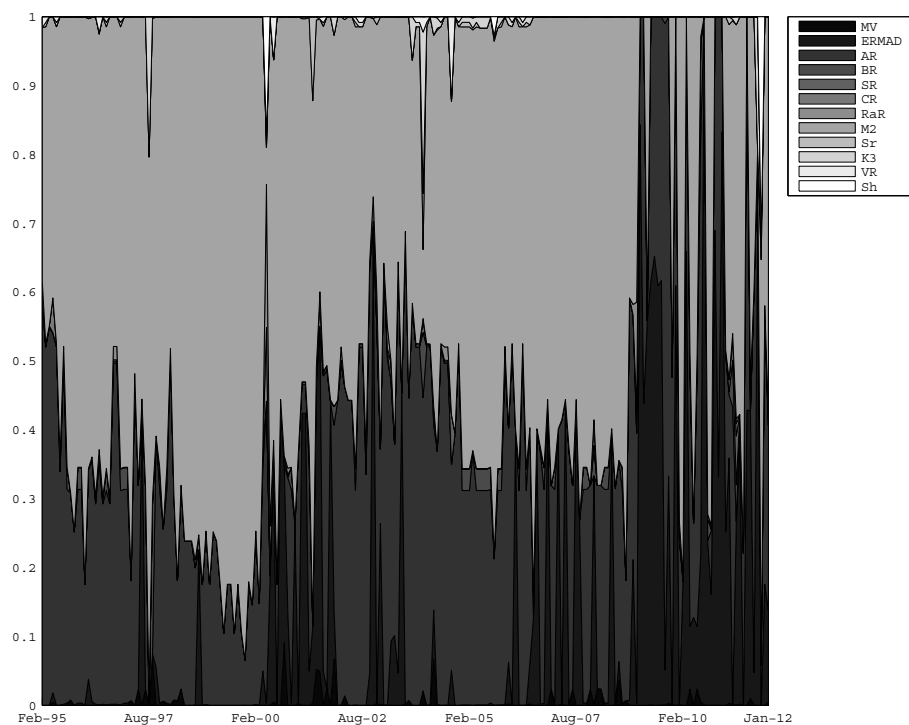


FIGURE 3.5: Performance measures weights in the TE screening with risk aversion coefficient equal to 1

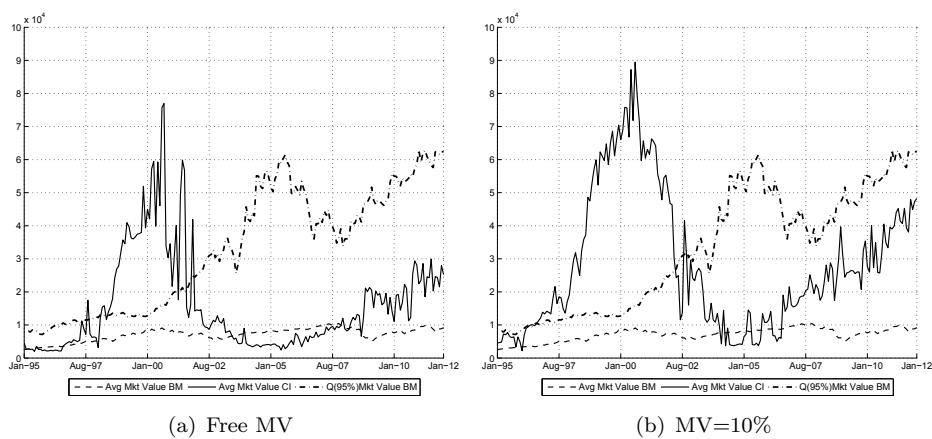


FIGURE 3.6: Average market value of the assets selected by the TE screening with risk aversion equal to 1 with unconstrained (bold line - left figure) or constrained market value weight (market value fixed at 10% - bold line - right figure). The figures also report the average market value of the companies included in the benchmark (dashed line), and the 95% quantile of the market value of the companies included in the benchmark (dotted line).

Chapter 4

Capturing European equity markets dependence with the Markov Switching Copula FIEGARCH-M model

4.1 Introduction

The recent financial crisis has again drawn attention of the financial literature in the interdependence among the financial markets during turbulent periods. *Contagion* defined in the sense of Forbes and Rigobon (2002) means a significant increase in cross-market co-movements during a crisis. If these co-movements remains also in the steady periods, it does not imply contagion but a more general interdependence. In this regard, this definition is crucial because it allows to detect whereas or not there is contagion.

During the years extensive research has been devoted to the analysis of financial contagion using the most various econometric techniques. Research on contagion based on correlation breakout has been conducted providing different findings. For instance, early studies as Kaplanis (1988) and Ratner (1992) have shown a stable correlation between markets. Other studies as Bertero and Mayer (1990), Longin and Solnik (1995), Calvo and Reinhart (1996) and Baig and Goldfajn (2000) found positive correlation during the market crisis. Boyer et al. (1997) and Loretan and English (2000) argued that changes in market volatility can lead to biased correlations¹. To overcome this issue, the aforementioned authors proposed a correction which allows for heteroskedasticity. In particular,

¹The authors provide an example by splitting a sample with a constant correlation which returned changes in correlations.

Forbes and Rigobon (2002) focused on the 1987 Market Crash, the Mexican devaluation and the Asian Crisis using a VAR estimation. They found no evidence of contagion but solely a high degree of dependence. Nevertheless, Billio and Pelizzon (2003) has shown that, even if adjusted for heteroskedasticity, the correlation coefficients can be misleading depending highly on the window used for the in-crisis and out-crisis sample. Thus, contagion was refused.

At this point literature extended towards different approaches. Non-linearity was embraced with the use of tail correlation in the extreme value theory framework (Hartmann et al., 2004). An interesting approach has been the introduction of Markov switching models which allows for structural breaks in the process on different regimes without requiring a predefined specification of the crisis periods.

Billio and Caporin (2005) presented a DCC model allowing regime switching both on the unconditional correlation and in the parameters.

Recent research focused on the copula literature which became very popular for the flexibility in the construction of the multivariate dependence between financial assets. See Patton (2012) for a literature survey about copula on time series.

In the contagion framework, a variety of studies have been done. Lo and Wilke (2010) makes use of mixed copula to model the cross markets dependence which are defined as a convex combination of different copulas. Other studies detected contagion allowing time varying copula function.

More in general, copulas have been used to capture asymmetric dependence in the international markets. Patton (2006a,b) has been the first to introduce the concept of conditional copula on the studies of the asymmetric dependence in the exchange rates. Jondeau and Rockinger (2006a) used the copula functions with Garch marginals allowing time varying skewed-t disturbances.

In this regard, Manner and Reznikova (2012) provided a good survey for different type of time varying copula used in the financial literature.

Rodriguez (2007) detected contagion in the sense of Forbes and Rigobon (2002) by modeling dependence with switching parameter in the Copula and Swarch model in the marginals. In that paper, contagion is described by structural breaks in tail dependence. Peng and Ng (2012) suggested a dynamic mixed copula approach in order to capture the time-varying tail dependence between market and volatility indices. Instead, Weiß (2011) used a mixed copula to detect bank contagion and bailout effects by estimating a mixed copula with event study methodology.

Chollete et al. (2009) proposed a Vine Copula methodology in a multivariate regime-switching showing that this types of dependence structures provide an higher performance in likelihood terms with respect to other models. They also showed the limit of the mixed copula in capture the tail dependence and the importance of the regime switching copula in the risk management.

In this paper we follow a similar procedure. We adopt a Regime-Switching approach using a D-Vine copula in order to detect asymmetric dependence among the financial markets of the Euro area. We analyze the main Countries of the area: Germany, France, Italy, Spain and Netherlands. In this regard, we use two types of Copula families: the Gaussian copula which does not allow for upper and lower tail dependence and the Rotated-Gumbel which admits lower tail dependence. We model the marginals in a parametric way using the FIEGARCH with volatility in mean introduced by Christensen et al. (2010) that includes the exponential asymmetry and the in-mean effect. This model well captures the features of financial time series as the negative premia (Ang et al., 2006), financial leverage effect (Black, 1976) and volatility feedback (Campbell and Hentschel, 1992).

We follow the approach of Chollete et al. (2009) which proposed a Vine Copula methodology in a multivariate regime-switching. The authors imposed the regime switching only in the dependence structure and not in the marginals. This is particular suitable in our framework since we filter the marginals using a long memory Garch model while we use the Markov Switching approach in the dependence structure.

Diebold and Inoue (2001) shown that Markov switching model and long memory are strictly related and often are easily confused. In our case, they are complementary since we use them on different dimensions.

The paper is organized as follows. In Section 4.2, we briefly illustrate the Copula and the D-Vine Copula. In Section 4.2.2, we review the Garch processes which are used to model the marginals. In Section 4.2.3, we describe the filter of Hamilton (1989) which is also the procedure used also by Chollete et al. (2009). Finally, in Section 4.4, we present the empirical analysis of the Eurozone.

4.2 Copulas

In this section we briefly review the theory of Copula. For a complete introduction on the theory behind copulas see Roger (2006).

Copulas are functions which links marginal distribution functions to a multivariate distribution function. The fundamental concept in Copula theory is the theorem of Sklar(1959) which defines the copula.

Theorem 1. (Sklar,1959). Let H be a joint distribution functions with margins $F_1, \dots, F_n$. Then there exists a Copula C such that for all $x \in R^n$,

$$H(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n)). \quad (4.1)$$

If the marginals $F_1, \dots, F_n$ are continuous then C is unique on $\text{Ran}(F_1) \times \dots \times \text{Ran}(F_n)$. Conversely, if C is a copula and $F_1, \dots, F_n$ are distribution functions, then the function H defined above is a joint distribution function with margins $F_1, \dots, F_n$. See Roger (2006) for the proof.

Therefore, Copula is the function which provides the dependence structure in the marginals. If the F_i 's are differentiable and C and H are n -differentiables, we can separate the marginals from the dependence structure by deriving both sides of (4.1). Hence, we obtain,

$$\frac{\partial^n D(x_1, \dots, x_n)}{\partial x_1, \dots, x_n} = \frac{\partial^n C(F(x_1), \dots, F(x_n))}{\partial x_1, \dots, x_n} \prod_{i=1}^n f_i(x_i). \quad (4.2)$$

The copula contains the information of the dependence among the marginals. In fact, the multivariate density is expressed as the product of the copula and the marginals.

We assume that our copula belongs to a parametric family $C_\theta, \theta \in \Theta \subset R^k$ and the marginals $F_1, \dots, F_n$ are also modeled parametrically with the Garch family processes where the residuals are *i.i.d* random variables.

Then, the probability integral transform (PIT) is applied by $U_i = F(x_i, \phi_i)$ where ϕ_i is the vector of the parameters. Thus, the residuals with PIT are distributed as an uniform random variable which allows to model the Copula.

As reported in Manner and Reznikova (2012), the parameters of the copula are separated from the parameters of the marginals because we are dealing with instantaneous causality where each variable depends only on its past and not on the past of the other variables.

4.2.1 D-Vine Copula

Vine Copula allows to model multivariate data using a bivariate copula decomposition as building blocks for the dependence structure. This family have been introduced in statistics by Bedford and Cooke (2002). Many different financial applications followed using vine-copula to detect asymmetric dependencies (Brechmann and Czado, 2012, Brechmann et al., 2012, de Melo Mendes et al., 2010, Dissmann et al., 2012, Fischer et al., 2009, Joe et al., 2010). Following Aas et al. (2009), whose firstly presented this pair-copula decomposition in finance, we briefly describe the process for the Drawable-Vine Copula (D-Vine).

Given a vector of n random variables $x_1, \dots, x_n$, we can decompose the joint density function as,

$$f(x_1, \dots, x_n) = f(x_1) \cdot f(x_2|x_1) \cdot f(x_3|x_1, x_2) \dots f(x_n|x_1, \dots, x_{n-1}). \quad (4.3)$$

The joint distribution contains all the information about the marginals and their dependence structure. Moreover, we can decompose equation (4.3) into a cascade of bivariate copula,

$$f(x_2|x_1) = c_{12}(F_1(x_1), F_2(x_2)) f(x_3|x_1), \quad (4.4)$$

where $F_i(\cdot)$ represents the cdf of each random variable x_i . The second term can be decomposed into a cascade of bivariate copulas,

$$\begin{aligned} f(x_3|x_1, x_2) &= c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) f(x_3|x_1) \\ &= c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) c_{13}(F_1(x_1), F_2(x_3)) f_3(x_3). \end{aligned} \quad (4.5)$$

Following the same procedure, we can decompose each terms in (4.3) until we obtain the joint density as a product of pair-copulas, using the different conditional probability distributions.

For example, a three-variate density can be expressed as,

$$\begin{aligned} f(x_1, x_2, x_3) &= c_{12}(F_1(x_1), F_2(x_2)) c_{13}(F_1(x_1), F_2(x_3)) \\ &\quad c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) f_1(x_1) f_2(x_2) f_3(x_3), \end{aligned} \quad (4.6)$$

where the copula density is,

$$\begin{aligned} c(x_1, x_2, x_3) &= c_{12}(F_1(x_1), F_2(x_2)) c_{13}(F_1(x_1), F_2(x_3)) \\ &\quad c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)). \end{aligned} \quad (4.7)$$

The pair-copula construction makes use of conditional distribution as in Joe(1996),

$$F(x|v) = \frac{\partial C_{x, v_j|v_{-j}}(F(x|v_{-j}), F(v_j|v_{-j}))}{\partial F(v_j|v_{-j})}, \quad (4.8)$$

where v_{-j} denotes the vector v excluding the v_j component. The choice for v is completely arbitrary.

In high-dimensional multivariate data we can model the dependence structure with an high number of possible combination of pair-copula constructions. Bedford and Cooke (2001, 2002) organized them defining regular-vine as a general class.

In particular, canonical-vine and d-vine are two special cases of the R-vine copula. The main difference among the two types of vine lies in the role of the marginals. In the canonical-vine a predefined variable plays a pivotal role which governs the interaction of the data. Therefore, the pivotal variable represents a key variable at the root of the dependence structure where all the other variables are conditioned. In the d-vine copula, instead, all the variables are *inter pares* in the graph structure. Figure 4.1 and Figure 4.2 illustrate an example of C-vine and D-vine with 5 variables respectively.

In our paper, we consider the D-Vine copula type, since we do not perform any inference

about the possible pivotal role about one of the stock indexes in the analyzed financial market.²

The density of a D-vine with n variables is defined as,

$$\prod_{k=1}^n f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|i+1,\dots,i+j-1} \left(F(x_i|x_{i+1}, \dots, x_{i+j-1}), F(x_{i+j}|x_{i+1}, \dots, x_{i+j-1}) \right), \quad (4.9)$$

where j identifies the trees and i iterates over the edges in each tree. In the D-Vine copula, the whole decomposition is defined by $n(n-1)/2$ bivariate copulas.

4.2.2 Marginals Model

The marginal distributions are modeled with univariate GARCH models. Autoregressive conditionally heteroskedastic models have been introduced by Engle (1982b) and their generalized ARCH extension can be found in Bollerslev (1986). These types of processes consider the variance conditional on the past as a linear function of the squared past values of the financial time series.

The process or mean equation x_i is expressed as,

$$x_t = \mu + h^{\frac{1}{2}} z_t, \quad (4.10)$$

where z_t is a *i.i.d* random variable and $\epsilon_t \equiv h^{\frac{1}{2}} z_t$.

The GARCH(p,q) is defined as,

$$h_t = \omega + \Psi(L)\epsilon_{t-1}^2, \quad (4.11)$$

where $\omega > 0$, $\Psi(L) = \frac{\alpha(L)}{\beta(L)}$.

$\alpha(L)$ and $\beta(L)$ are polynomials in L of degrees $q \geq 0$ and $p \geq 0$, respectively, which have no roots in common,

$$\alpha(L) = 1 + \sum_{i=1}^q \alpha_i L^i, \alpha_i \neq 0, |z| \leq 1,$$

$$\beta(L) = 1 - \sum_{i=1}^p \beta_i L^i, \beta_i \neq 0, |z| \leq 1.$$

In order to ensure the positivity and the stationarity of the conditional variance h_t , there must be some restrictions on the parameters: $\alpha_i \geq 0$, $\beta_i \geq 0$, and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$. Moreover, there is a symmetric response to the positive and negative shocks. That is,

²Otherwise, a natural pivotal role could be played by Germany.

negative returns have the same impact of the positive returns in the volatility in the model.

Nelson (1991) propose an Exponential-GARCH model (EGARCH) in order to overcome the aforementioned weaknesses,

$$\log(h_t) = \omega + \Psi(L)g(z_{t-1}), \quad (4.12)$$

where $g(z_t)$ function provides an asymmetric response to the shocks,

$$g(z_t) = \theta z_t + \gamma (|z_t| - E(z)). \quad (4.13)$$

In the empirical application of the GARCH(1,1), often happens that α_1 and β_1 are close to unity. In practice, there is a persistence in the estimated conditional variance. This is due to a well observed phenomenon in the financial markets: the hyperbolic decay of the autocorrelation of the squared (and absolute) returns. Instead, this shocks die out at an exponential rate in the $I(0)$ process as GARCH types.

In this regard, Engle and Bollerslev (1986) introduced the Integrated-GARCH (IGARCH) imposing the restriction $\alpha_1 + \beta_1 = 1$, in order to capture the persistence effect.

The IGARCH is an $I(1)$ process where there is no mean reversion. Nelson (1991) showed that the unconditional variance of the process increases linearly with time when the constant of the process is positive.

A class of process which allows a flexibility between this two model's types is the fractional integrated process $I(d)$ with $0 < d < 1$.

In fact, the fractional orders of integration well captures the hyperbolic decay of the autocorrelation of the squared returns. For this reason, the fractionally integrated process is also called long-memory process.

Baillie et al. (1996) proposed the fractionally integrated GARCH,

$$h_t = \omega + \Psi(L)(1 - L)^{-d}\epsilon_{t-1}^2, \quad (4.14)$$

where $(1 - L)^{-d}$ is the fractional operator defined as,

$$(1 - L)^{-d} = \sum_{j=0}^{\infty} \frac{\Gamma(j + d)}{\Gamma(j + 1)\Gamma(d)} L^j, \quad (4.15)$$

with $-1/2 < d < 1/2$, in order to guarantee the process to be stationary and invertible. If $d = 0$, the FIGARCH process reduces to the standard GARCH model and if $d = 1$, it reduces to the IGARCH model.

Later, Bollerslev and Ole Mikkelsen (1996) suggested the fractionally integrated EGARCH,

in order to take into account for asymmetry in the volatility,

$$\log(h_t) = \omega + \Psi(L)(1 - L)^{-d}g(z_{t-1}), \quad (4.16)$$

with in this case $d < 1/2$.

Recalling the mean equation (4.10), Engle et al. (1987) introduced the GARCH-in-mean model (GARCH-m) in order to use the conditional variance as a representation of the time-varying risk premium,

$$x_t = \mu + f(h_{t-1}), \quad (4.17)$$

where $f(h_t) = \delta h_t$. Ang et al. (2006), Christensen and Nielsen (2007) considered instead the volatility innovations, $g(z_t)$, in the mean.

Christensen et al. (2010), introduced the FIEGARCH with volatility in-mean with an autoregressive dynamic in the mean.

$$x_t = \mu_0 + \sum_{i=1}^k \mu_i x_{t-i} + \lambda g(z_t) + \epsilon_t. \quad (4.18)$$

4.2.3 Regime-Switching and Copula families

The dependence in our dataset are modeled using a Regime-Switching model. In this regard, we use the same approach of Chollete et al. (2009) allowing the latent process varying in two regimes.

In particular, we want to consider one regime with lower tail dependence for the *contagion* period and the other regime period with no tail dependence. Since we use pair-copula decomposition to model the multivariate data, we consider henceforth, only the bivariate case.

In general, two random vectors X_1, X_2 are upper (lower) tail-dependent,

$$\begin{aligned} \lambda_U &\equiv \lim_{v \rightarrow 1-} P \{X_1 > F_1^{-1}(v) | X_2 > F_2^{-1}(v)\} > 0, \\ \lambda_L &\equiv \lim_{v \rightarrow 1-} P \{X_1 \leq F_1^{-1}(v) | X_2 \leq F_2^{-1}(v)\} > 0, \end{aligned} \quad (4.19)$$

and if the limit exists. F_i^{-1} denotes the generalized inverse distribution function of X_i . If λ_U is equal to zero, X_1 and X_2 are upper tail-independent. The same applies for λ_L . In our case, we consider the Gaussian copula³ for the regime with no tail dependence and the rotated-Gumbel copula for the regime with lower tail dependence.

In the Gaussian copula, we define the distribution function as,

$$C(x_1, x_2, \Sigma) = \Phi_{\Sigma}(\Phi^{-1}(x_1), \Phi^{-1}(x_2)), \quad (4.20)$$

³The Gaussian copula has zero upper and lower tail dependence when there is not perfect correlation.

where Φ^{-1} is the cdf of the standard normal distribution, $(\Phi^{-1}(x_1), \Phi^{-1}(x_2), \Sigma)$ is the normal cumulative distribution and Σ is the correlation matrix.

The density function of the Gaussian bivariate copula is,

$$c_\rho(u_1, u_2) = \frac{1}{\sqrt{1-\rho^2}} \exp \left[\frac{-[\Phi^{-1}(x_1)^2 + \Phi^{-1}(x_2)^2 - 2\rho\Phi^{-1}(x_2)]}{2(1-\rho^2)} + \frac{\Phi^{-1}(x_1)^2 + \Phi^{-1}(x_2)^2}{2} \right], \quad (4.21)$$

where ρ is the correlation coefficient which lies in $[-1, 1]$.

As described above, there is no upper and lower tail dependence in the Gaussian Copula, $\lambda_U = \lambda_L = 0$.

The relationship between the Pearson's ρ and the Kendall's τ is defined by

$$\tau = 2 \arcsin(\rho)/\pi. \quad (4.22)$$

Let's consider the rotated-Gumbel for the second regime. The distribution function of the Gumbel copula is described by

$$C_G(x_1, x_2, \theta) = \exp \left[- \left((-\log x_1)^\theta + (-\log x_2)^\theta \right)^{\frac{1}{\theta}} \right], \quad (4.23)$$

with the following the density function,

$$c_G(x_1, x_2, \theta) = \frac{C_G(x_1, x_2, \theta)(\log x_1 \cdot \log x_2)^{\theta-1}}{x_1 x_2 \left((-\log x_1)^\theta + (-\log x_2)^\theta \right)^{2-\frac{1}{\theta}}} \times \left[\left((-\log x_1)^\theta + (-\log x_2)^\theta \right)^{\frac{1}{\theta}} + \theta - 1 \right], \quad (4.24)$$

where $\theta \in [1, \infty)$. The Gumbel-Copula allows for upper tail dependence.

Since we are interested to detect the lower tail dependence we use the rotated version of the Gumbel-Copula defined as,

$$C_{RG}(x_1, x_2, \theta) = x_1 + x_2 - 1 + C_G(1 - x_1, 1 - x_2, \theta) \quad (4.25)$$

where the density is given by

$$c_{RG}(x_1, x_2, \theta) = c_G(1 - x_1, 1 - x_2, \theta). \quad (4.26)$$

Therefore, we have $\lambda_U = 0$ and $\lambda_L = 2 - 2^{\frac{1}{\theta}}$ in the rotated-Gumbel.

The relationship between the parameter θ and the Kendal's τ is defined by

$$\tau = 1 - \frac{1}{\theta}. \quad (4.27)$$

4.2.3.1 Markov Switching

In the regime switching estimation, we follow the same procedure of Chollete et al. (2009), in the sense that the marginal distributions do not depend on the regime. Only the dependence structure is regime-dependent.

Diebold and Inoue (2001) show using Monte Carlo simulation that long memory and Markov Switching models often are confused. When sometimes long memory is detected, there is instead a simply structural change in the process and viceversa. In our case, we are looking at long memory and regime switching in a complementary way, since we use them on different dimensions.

Using the filter of Hamilton (1989) procedure, we assume that the multivariate process X_t depends on a latent variable (binary) which describes the current regime for the dependence structure. The regime switching describes two states on the D-Vine copulas, where each vine is build using pair-copula construction.⁴ The regimes are described by $j = 1, 2$ and the density function of the data, conditional of being in regime j , is defined by:

$$f(X_t|X_{t-1}, s_t = j) = c^{(j)}\left(F_1(x_{1,t}, \dots, F_n(x_{n,t}); \theta_c^{(j)}\right) \prod_{i=1}^n f_i(x_{i,t}; \theta_{m,1}), \quad (4.28)$$

where s_t is the binary latent variable, $c^{(j)}(\cdot)$ is the D-Vine copula in regime j , $f_i(\cdot)$ is the density of the marginal distribution x_i and F_i is the distribution function. θ_c^j and θ_M^i describe the parameters of the copula in each regime and each marginal distribution respectively.

The transition probability matrix for the Markov chain process which defines the latent state is,

$$P = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \quad (4.29)$$

where $p_{i,j}$ represents the transition probability from state i , at time t , to state j , at time $t + i$.

4.3 Estimation

Joint Maximum Likelihood estimation can be difficult to implement given the large number of the unknown parameters that could rise in the estimation of the Copula and its marginals:

$$(\hat{\theta}_m, \hat{\theta}_c) = \arg \max_{\theta_m, \theta_c} L(\theta_m, \theta_c), \quad (4.30)$$

⁴In Appendix B we report the simulations for the bivariate regime-switching copula case.

where θ_m and θ_c are the parameters of the marginals and the RS copula respectively. $L(\theta_m, \theta_c)$ is the Log-Likelihood function,

$$\sum_{t=1}^T \log f(X_t; \theta_m, \theta_c).$$

A one-step maximization approach is clearly not feasible when there is an increase in dimensionality. Moreover, in our application we have also to estimate the regime switching using the EM algorithm.

Shih and Louis (1995) introduced a two-step ML procedure called inference from marginals (IFM). Patton (2006a) has shown the asymptotic efficiency of this methodology.

Therefore, we can adopt a two-step procedure,

$$L(\theta_m, \theta_c) = L_m(\theta_m) + L_c(\hat{\theta}_m, \theta_c). \quad (4.31)$$

First the marginals are estimated and then, the dependence parameters are estimated from the copula likelihood,

$$\begin{aligned} \hat{\theta}_m &= \arg \max_{\theta_m} L_m(\theta_m), \\ \hat{\theta}_c &= \arg \max_{\theta_c} L_c(\hat{\theta}_m, \theta_c), \end{aligned} \quad (4.32)$$

where,

$$\begin{aligned} L_m(\theta_m) &= \sum_{t=1}^T \sum_{i=1}^n \log f_i(\theta_{m,i}), \\ L_c(\hat{\theta}_m, \theta_c) &= \sum_{t=1}^T \log c\left(F_1(\hat{\theta}_{m,1}), \dots, F_1(\hat{\theta}_{m,n}); \theta_c\right). \end{aligned}$$

It is worth to pointing out that the parameter vector θ_c contains both the parameters of the d-vine copulas in the two regimes and the parameters from the transition matrix. Finally, we stimate the markov chain s_t using the filter of Hamilton (1989),

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \otimes \eta_t}{\mathbf{1}'(\hat{\xi}_{t|t-1} \otimes \eta_t)}, \quad (4.33)$$

$$\hat{\xi}_{t+1|t} = P' \hat{\xi}_{t|t}, \quad (4.34)$$

$$\eta_t = \begin{pmatrix} c^{(1)}\left(F_1(x_{1,t}), \dots, F_n(x_{n,t}); \theta_c^1\right) \\ c^{(2)}\left(F_1(x_{1,t}), \dots, F_n(x_{n,t}); \theta_c^2\right) \end{pmatrix}, \quad (4.35)$$

where η_t contains the copula density at time t conditional to being in each regime, P is the transition matrix, $\mathbf{1}$ is a vector of ones (2×1) and $\otimes$ is the Hadamard product.

Given a starting values $\hat{\xi}_{1|0}$ and the parameter vectors $\theta_c^{(1)}$ and $\theta_c^{(2)}$, by iterating equation (4.33) and (4.34) for $i = 1, \dots, T$ is possible to calculate the values of $\hat{\xi}_{t|t}$ and $\hat{\xi}_{t+1|t}$ and the values of the log likelihood function,

$$L_c(\theta_m, \theta_c, X) = \sum_{t=1}^T \log \left(\mathbf{1}'(\hat{\xi}_{t|t-1} \otimes \eta_t) \right). \quad (4.36)$$

Standard errors of the estimates are computed following the theorems of Newey and McFadden (1994), where under regularity conditions,

$$\sqrt{T}(\hat{\theta} - \theta_0) \overset{A}{\rightsquigarrow} N(0, \hat{V}^{-1} \hat{M} \hat{V}'^{-1}), \quad (4.37)$$

with

$$\hat{V}^{-1} = \begin{bmatrix} \nabla_{\theta_m \theta_m} L_m(\hat{\theta}_m, X) & 0 \\ \nabla_{\theta_m \theta_c} L_c(\hat{\theta}_m, \theta_c, X) & \nabla_{\theta_c \theta_c} L_c(\hat{\theta}_m, \theta_c, X) \end{bmatrix}$$

and

$$\hat{M} = \text{var} \left[\sum_{t=1}^T \left(\frac{1}{\sqrt{T}} \nabla'_{\theta_m} L_m(\hat{\theta}_m, X_t), \frac{1}{\sqrt{T}} \nabla'_{\theta_c} L_c(\hat{\theta}_m, \theta_c, X_t) \right) \right].$$

4.4 Empirical Analysis

In this section we report the estimation results. In our analysis we consider the main country of the Eurozone: Germany, France, Italy, Spain and Netherlands.⁵

4.4.1 Results for the Marginal Model

The price index data have been downloaded from Datastream at daily frequency from January 1st, 1998 to May 2nd, 2013. The total number of observations is 4001.

We refine the series by excluding the trading days from the sample where one or more markets were closed for holidays. The final sample is constituted by 3883 observations. The descriptive statistics of the log-returns are reported in Table 4.1. The average daily log-returns are very close to zero while the minimum and the maximum daily log-returns are around 10% in magnitude. Clearly, the data show evidence of non-normality with presence of negative skewness (except for Spain) and excess of kurtosis.

Table 4.2 reports the Pearson's correlation matrix. The correlations are quite strong, as we expect in a common market, and all significantly different from zero. In particular, France and Netherlands exhibit an high correlation of 0.91.

We estimated the univariate models presented in Section 4.2.2. The results are presented

⁵Respectively, the stock indexes considered are Dax 30, Cac 40, Ftse Mib, Ibex 35, Ibex 35 and AEX index.

in Table 4.3 for the GARCH(1,1) models, Table 4.4 for the EGARCH(1,1,1) models, Table 4.5 for the FIGARCH(1,1) models, and Table 4.3 for the FIEGARCH(1,1) model. In all the models the coefficient are significant at 5% confidence interval except for the α in the FIGARCH and FIEGARCH models. Also Chollete et al. (2008) found a similar results for France (the unique country in common with our dataset) using a GARCH with skewed-t innovations.

According to AIC and BIC criteria the models to be preferred are the FIEGARCH(1,d,1) and the EGARCH(1,1,1). In fact, these models well captures the asymmetric effects between positive and negative returns. The AIC and BIC are reported in Table 4.6 and Table 4.5.

We performed a Ljung-Box test in the residuals of the models with different lags as shown in Table (4.7-4.8-4.9-4.10). The null hypothesis of no autocorrelation is not rejected. The test has been performed also in the squared of the residuals. In this case, we rejected the null hypothesis of no autocorrelation in all the models except for the FIEGARCH(1,d,1). This suggests that other dynamics may enter in the series. The univariate modeling needs further consideration. For example, allowing autoregressive effect in the mean, volatility in the mean and other distributional hypothesis for the disturbances.

Finally, in order to avoid biased results in the estimated parameter of the copula, we also applied the Kolmogorov-Smirnov to test the null hypothesis that the cdf of the data are not different from an uniform $[0, 1]$. The results strongly reject the alternative hypothesis.

4.4.2 Estimation of Bivariate Regime Switching Copula

In this section, we consider the regime switching on the bivariate copula in order to analyze the interdependence among two countries. Consequently, we explore 10 relationships between the stock indexes.

In order to detect the asymmetry in the lower tail for a regime and tail independence in the other regime, we consider the Gaussian and Rotated-Gumbel copula. We consider also the case where there is no tail dependence in regimes but a simply a correlation breakout. Therefore, we analyze the case Gaussian-Gaussian copula, where we assume there is a regime with low correlation and the other one with high correlation.

We estimated the regime-switching using the marginals modeled with the FIEGARCH(1,d,1) process, since it is preferable to the other models according the AIC and BIC criteria.⁶ Table 4.11 reports the results for the Gaussian-Rotated-Gumbel case and Table 4.12 for the Gaussian-Gaussian case. Smoothed probabilities are reported in Figure 4.3 and in

⁶We report also the estimations with different models for the marginals in the Appendix C.

Figure 4.4 respectively.

The results show an high Kendall's τ in both the regimes. On average, there is a variation of 0.20 from one regime to the other in the τ .

Probably, a reason for this strongly dependence in both regimes is due to the higher interdependence in the Euro area.

At this point, it could be interested to structure the Markov switching in terms of financial integration in the Euro area. It is clearly observable in both Figure 4.3 and Figure 4.4 a change in regime from the beginning of 2000s. This is consistent with Hardouvelis et al. (2006) which found, an increased integration of the European stock markets at the beginning of the 1990s.

In fact, if we assume a financial integration among the markets, it could be interesting to specify a markov switching copula, allowing a regime with lower tail dependence and the other with upper tail dependence. In case of financial integration, we should expect the latent state to be a proxy of the financial cycle.

An other technical reason lies in the marginals. The mean of the considered GARCH model is described only by the constant, no ARMA effects are included and also no garch-in-mean effects. We should expect an improvement on the estimation once we capture more dynamic in the marginals.

Nevertheless, some consideration at this stage of analysis can be done. It is clearly observable a strong relationship between France and Netherlands. This is consistent with the high correlation observed in Table 4.2. In the case of the Gaussian–Rotated–Gumbel case, the Kendall's τ changes in regime from 0.60 to 0.78 and from 0.58 to 0.78 in the Gaussian-Gaussian copula case.

An other case clearly defined is between France and Italy and Spain and Netherlands; there is a variation in the Kendall's τ from 0.56 to 0.74 in the Gaussian–Rotated–Gumbel case and from 0.50 to 0.68 in the Gaussian-Gaussian case.

The AIC and BIC for the estimated regime-switching bivariate copula show better result in the Gaussian-Gaussian case. This confirms again a general interdependence in the European markets.

4.4.3 Estimation of D-Vine Regime Switching Copula

TO BE DISCUSSED

4.5 Conclusion

In this paper we analyzed the dependence structure among the european financial markets. In this regard, we use two types of Copula families: the Gaussian copula which

does not allow for upper and lower tail dependence and the Rotated-Gumbel which admits lower tail dependence.

We model the marginals in a parametric way using the FIEGARCH with volatility in mean approach introduced by Christensen et al. (2010), which includes both the exponential asymmetry and the volatility in-mean effect. This model well captures the features of financial time series compared to other GARCH processes, as the negative premia (Ang et al., 2006), financial leverage effect (Black, 1976) and volatility feedback (Campbell and Hentschel, 1992). We followed the approach of (Chollete et al., 2009) which proposed a Vine Copula methodology in a multivariate regime-switching, imposing the change in regime only in the dependence structure and not in the marginals.

4.6 Tables and Figures

	Mean	Std	Skewness	Kurtosis	Min	Max
Germany	0.0001	0.0162	-0.0640	6.7499	-0.0887	0.1080
France	0.0000	0.0155	-0.0082	7.2112	-0.0947	0.1059
Italy	-0.0002	0.0159	-0.0901	7.0143	-0.0860	0.1088
Spain	0.0000	0.0159	0.0314	7.5070	-0.0959	0.1348
Netherland	-0.0001	0.0155	-0.1239	8.3018	-0.0959	0.1003

TABLE 4.1: Descriptive statistics of daily index log-returns for the Eurozone countries.

	Germany	France	Italy	Spain	Netherlands
Germany	1	-	-	-	-
France	0.88	1	-	-	-
Italy	0.80	0.88	1	-	-
Spain	0.78	0.86	0.86	1	-
Netherlands	0.85	0.91	0.83	0.81	1

TABLE 4.2: Pearson's correlation matrix for the Eurozone countries. Hypothesis of no correlation is rejected at $\alpha = 0.01$.

GARCH	Germany	France	Italy	Spain	Netherlands
ω	$2.477 \cdot 10^{-6}$ ($6.767 \cdot 10^{-7}$)	$2.009 \cdot 10^{-6}$ ($7.109 \cdot 10^{-6}$)	$1.529 \cdot 10^{-6}$ ($4.815 \cdot 10^{-6}$)	$2.130 \cdot 10^{-6}$ ($6.822 \cdot 10^{-6}$)	$2.049 \cdot 10^{-6}$ ($5.643 \cdot 10^{-6}$)
α	0.0895 (0.0107)	0.0840 (0.0112)	0.0976 (0.0125)	0.0994 (0.0142)	0.1016 (0.0122)
β	0.9014 (0.0108)	0.9091 (0.0118)	0.8996 (0.0116)	0.8955 (0.0134)	0.8908 (0.0121)
AIC	-22035.79	-22233.14	-22331.12	-22105.78	-22735.15
BIC	-22054.54	-22251.90	-22349.87	-22124.53	-22753.90

TABLE 4.3: Estimates of univariate GARCH(1,1) models. The estimated parameter is reported in bold when we do not reject the null hypothesis that it is equal to zero at 5%.

EGARCH	Germany	France	Italy	Spain	Netherlands
ω	-0.1922 (0.0342)	-0.1607 (0.0310)	-0.1389 (0.0291)	-0.1737 (0.0326)	-0.1416 (0.0275)
α	0.1548 (0.0162)	0.1333 (0.0151)	0.1709 (0.0182)	0.1555 (0.0178)	0.1501 (0.0212)
β	0.9774 (0.0040)	0.9812 (0.0036)	0.9836 (0.0034)	0.9795 (0.0038)	0.9836 (0.0032)
γ	-0.1001 (0.0151)	-0.1008 (0.0129)	-0.0804 (0.0119)	-0.0991 (0.0124)	-0.0920 (0.0156)
AIC	-22157.50	-22367.80	-22413.12	-22238.79	-22855.56
BIC	-22182.51	-22392.80	-22438.12	-22263.80	-22880.57

TABLE 4.4: Estimates of univariate EGARCH(1,1) models. In the estimation the alternative representation was used: $\log(h_t) = \omega \sum_{i=1}^q \alpha_i \left[\frac{z_{t-1}}{-} |z_{t-1}| - E(z_{t-1}) \right] + \beta_i \log(h_{t-1})$. The estimated parameter is reported in bold when we do not reject the null hypothesis that it is equal to zero at 5%.

FIGARCH	Germany	France	Italy	Spain	Netherlands
ω	$1.305 \cdot 10^{-5}$ ($4.791 \cdot 10^{-5}$)	$1.124 \cdot 10^{-5}$ ($3.713 \cdot 10^{-5}$)	$9.341 \cdot 10^{-5}$ ($2.932 \cdot 10^{-5}$)	$1.169 \cdot 10^{-5}$ ($4.281 \cdot 10^{-5}$)	$9.934 \cdot 10^{-5}$ ($5.295 \cdot 10^{-5}$)
α	0.0329 (0.0344)	0.0953 (0.0531)	0.0375 (0.1041)	0.0799 (0.0355)	0.0341 (0.1977)
d	0.6275 (0.0868)	0.5754 (0.0768)	0.5924 (0.0562)	0.5796 (0.0722)	0.6347 (0.0915)
β	0.6384 (0.0782)	0.6299 (0.0744)	0.6011 (0.0919)	0.6122 (0.0619)	0.6244 (0.1167)
AIC	-22047.43	-22238.19	-22365.93	-22126.23	-22747.88
BIC	-22072.44	-22263.20	-22390.93	-22151.23	-22772.88

TABLE 4.5: Estimates of univariate FIGARCH(1,d,1) models. The estimated parameter is reported in bold when we do not reject the null hypothesis that it is equal to zero at 5%.

FIEGARCH	Germany	France	Italy	Spain	Netherlands
ω	-8.3926 (0.1699)	-8.3701 (0.1735)	-8.2194 (0.2132)	-8.3572 0(.1780)	-8.4744 (0.2246)
α	-0.2371 (0.2186)	-0.3653 (0.1346)	-0.0049 (0.3078)	-0.3812 (0.1336)	-0.1807 (0.2231)
d	0.4140 (0.0972)	0.3785 (0.0701)	0.4900 (0.0582)	0.4021 (0.0719)	0.5088 (0.0556)
β	0.8347 (0.0971)	0.8938 (0.0352)	0.7490 (0.1058)	0.8644 (0.0487)	0.7848 (0.0778)
θ	-0.1046 (0.0196)	-0.1111 (0.0191)	-0.0783 (0.0162)	-0.1157 (0.0196)	-0.0934 (0.0169)
γ	0.1534 (0.0235)	0.1355 (0.0214)	0.1516 (0.0249)	0.1715 (0.0242)	0.1434 (0.0232)
AIC	-22176.70	-22390.35	-22453.29	-22265.43	-22880.78
BIC	-22132.94	-22346.59	-22409.53	-22221.67	-22837.02

TABLE 4.6: Estimates of univariate FIEGARCH(1,d,1) models. Classical representation was used. The estimated parameter is reported in bold when we do not reject the null hypothesis that it is equal to zero at 5%.

GARCH	Germany	France	Italy	Spain	Netherlands
1	0.80	0.25	0.90	0.27	0.21
2	0.96	0.22	0.81	0.36	0.32
4	0.84	0.11	0.54	0.50	0.22
8	0.81	0.15	0.29	0.53	0.11
12	0.71	0.29	0.45	0.66	0.17
16	0.84	0.43	0.64	0.50	0.23
32	0.95	0.75	0.96	0.95	0.43
100	0.62	0.35	0.41	0.46	0.22

TABLE 4.7: The table reports the p -values of the Ljung-Box statistics for test of no autocorrelation of residuals from GARCH(1,1) model.

EGARCH	Germany	France	Italy	Spain	Netherlands
1	0.74	0.43	0.72	0.13	0.33
2	0.83	0.13	0.80	0.18	0.59
4	0.68	0.03	0.42	0.35	0.19
8	0.61	0.05	0.14	0.36	0.09
12	0.59	0.15	0.24	0.54	0.14
16	0.67	0.26	0.38	0.42	0.24
32	0.89	0.63	0.81	0.90	0.47
100	0.60	0.36	0.29	0.46	0.28

TABLE 4.8: The table reports the p -values of the Ljung-Box statistics for test of no autocorrelation of residuals from EGARCH(1,1) model.

FIGARCH	Germany	France	Italy	Spain	Netherlands
1	0.31	0.10	0.93	0.08	0.01
2	0.58	0.13	0.79	0.20	0.01
4	0.85	0.13	0.42	0.18	0.04
8	0.94	0.25	0.56	0.34	0.13
12	0.77	0.24	0.69	0.22	0.03
16	0.84	0.39	0.87	0.15	0.06
32	0.87	0.89	0.87	0.40	0.10
100	0.37	0.12	0.03	0.01	0.02

TABLE 4.9: The table reports the p -values of the Ljung-Box statistics for test of no autocorrelation of residuals from FIGARCH(1,d,1) model.

FIEGARCH	Germany	France	Italy	Spain	Netherlands
1	0.74	0.40	0.92	0.17	0.35
2	0.80	0.11	0.91	0.18	0.63
4	0.65	0.03	0.47	0.38	0.25
8	0.60	0.05	0.18	0.46	0.10
12	0.61	0.15	0.31	0.64	0.12
16	0.73	0.24	0.43	0.59	0.20
32	0.92	0.63	0.84	0.94	0.44
100	0.65	0.36	0.35	0.54	0.34

TABLE 4.10: The table reports the p -values of the Ljung-Box statistics for test of no autocorrelation of residuals from FIEGARCH(1,d,1) model.

FIEGARCH	Gaussian Regime			Rot-Gumbel Regime			AIC	BIC	$\hat{\rho}_{11}$	t-stat	$\hat{\rho}_{22}$	t-stat
	$\hat{\rho}$	t-stat	$\hat{\tau}_1$	$\hat{\theta}$	t-stat	$\hat{\tau}_2$						
Ger,Fra	0.76	3.60	0.55	4.22	72.17	0.76	-6276.49	-6257.73	0.97	7.23	0.99	113.47
Ger,Ita	0.60	36.94	0.41	3.13	193.93	0.68	-4529.41	-4510.66	0.95	58.74	0.98	60.82
Ger,Spa	0.69	45.28	0.49	3.31	204.60	0.70	-4053.84	-4035.09	0.97	153.85	0.97	1007.29
Ger,NL	0.80	49.58	0.59	4.27	264.51	0.77	-5397.28	-5378.53	0.99	60.98	0.98	60.59
Fra,Ita	0.77	47.57	0.56	3.84	237.82	0.74	-5430.78	-5378.53	0.97	60.98	0.98	60.59
Fra,Spa	0.78	63.07	0.57	3.90	237.44	0.74	-4976.85	-4958.10	0.97	61.09	0.96	68.78
Fra,NL	0.81	50.14	0.60	4.47	276.38	0.78	-6454.35	-6435.60	0.98	60.94	0.99	61.24
Ita,Spa	0.76	43.22	0.55	3.59	222.75	0.72	-4555.64	-4536.89	0.97	93.78	0.96	69.34
Ita,NL	0.74	1535.93	0.53	3.67	1642.85	0.73	-4445.55	-4426.80	0.97	2447.94	0.96	1609.23
Spa,NL	0.74	46.06	0.53	3.29	203.86	0.70	-4036.79	-4018.04	0.99	61.31	0.98	60.85

TABLE 4.11: The table reports parameter estimates for the bivariate dependence structure in the regime switching Gaussian-Rotated-Gumbel copula with marginals modeled with FIEGARCH(1,d,1). We also include the Kendall's τ .

FIEGARCH	Gaussian Regime 1			Gaussian Regime 2			AIC	BIC	$\hat{\rho}_{11}$	t-stat	$\hat{\rho}_{22}$	t-stat
	$\hat{\rho}_1$	t-stat	$\hat{\tau}_1$	$\hat{\rho}_2$	t-stat	$\hat{\tau}_2$						
Ger,Fra	0.94	830.97	0.79	0.75	40.77	0.54	-6435.83	-6417.08	0.98	398.10	0.97	1448.51
Ger,Ita	0.89	13459.73	0.70	0.54	925.71	0.36	-4677.92	-4659.17	0.98	11911.69	0.93	4465.54
Ger,Spa	0.90	50.00	0.71	0.65	12.79	0.45	-4158.36	-4139.61	0.97	73.47	0.96	27.37
Ger,NL	0.94	2071.72	0.77	0.78	48.92	0.57	-5475.54	-5456.79	0.98	785.71	0.98	404.85
Fra,Ita	0.92	8023.28	0.74	0.71	655.97	0.50	-5607.05	-5456.79	0.99	404.85	0.97	785.71
Fra,Spa	0.92	7838.34	0.75	0.73	3497.23	0.53	-5092.07	-5073.32	0.97	3612.16	0.96	2773.99
Fra,NL	0.94	21926.15	0.78	0.79	3502.25	0.58	-6618.80	-6600.05	0.99	18437.10	0.99	8613.73
Ita,Spa	0.90	8326.98	0.71	0.68	2384.91	0.48	-4665.16	-4646.41	0.97	8694.08	0.95	4267.67
Ita,NL	0.89	55.03	0.70	0.65	40.08	0.45	-4535.66	-4516.91	0.98	60.57	0.95	58.87
Spa,NL	0.88	140.24	0.69	0.70	68.30	0.49	-4113.60	-4094.85	0.98	136.40	0.98	117.53

TABLE 4.12: The table reports parameter estimates for the bivariate dependence structure in the regime switching Gaussian-Gaussian copula with marginals modeled with FIEGARCH(1,1d,1). We also include the Kendall's τ . The Regime 1 identifies the highest dependence structure.

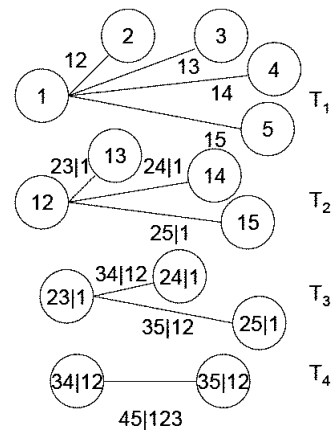


FIGURE 4.1: Example of a C-Vine copula with 5 variables.

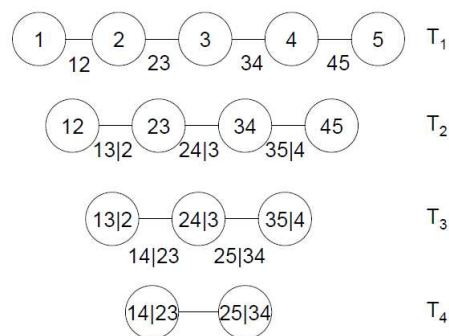


FIGURE 4.2: Example of a D-Vine copula with 5 variables.

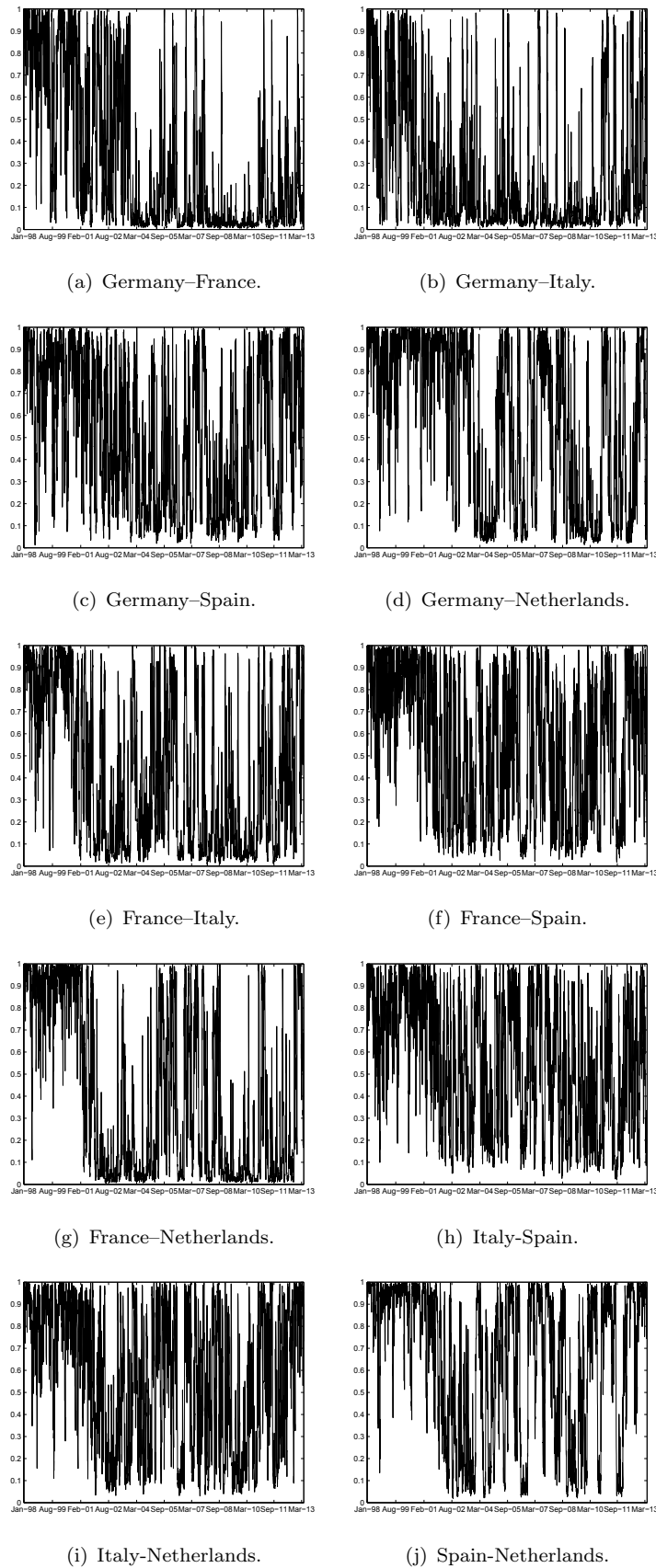


FIGURE 4.3: Smoothed probability with marginals modeled with FIEGARCH(1,d,1) model. The two regime: the bivariate Gaussian copula and the rotated-Gumbel copula.

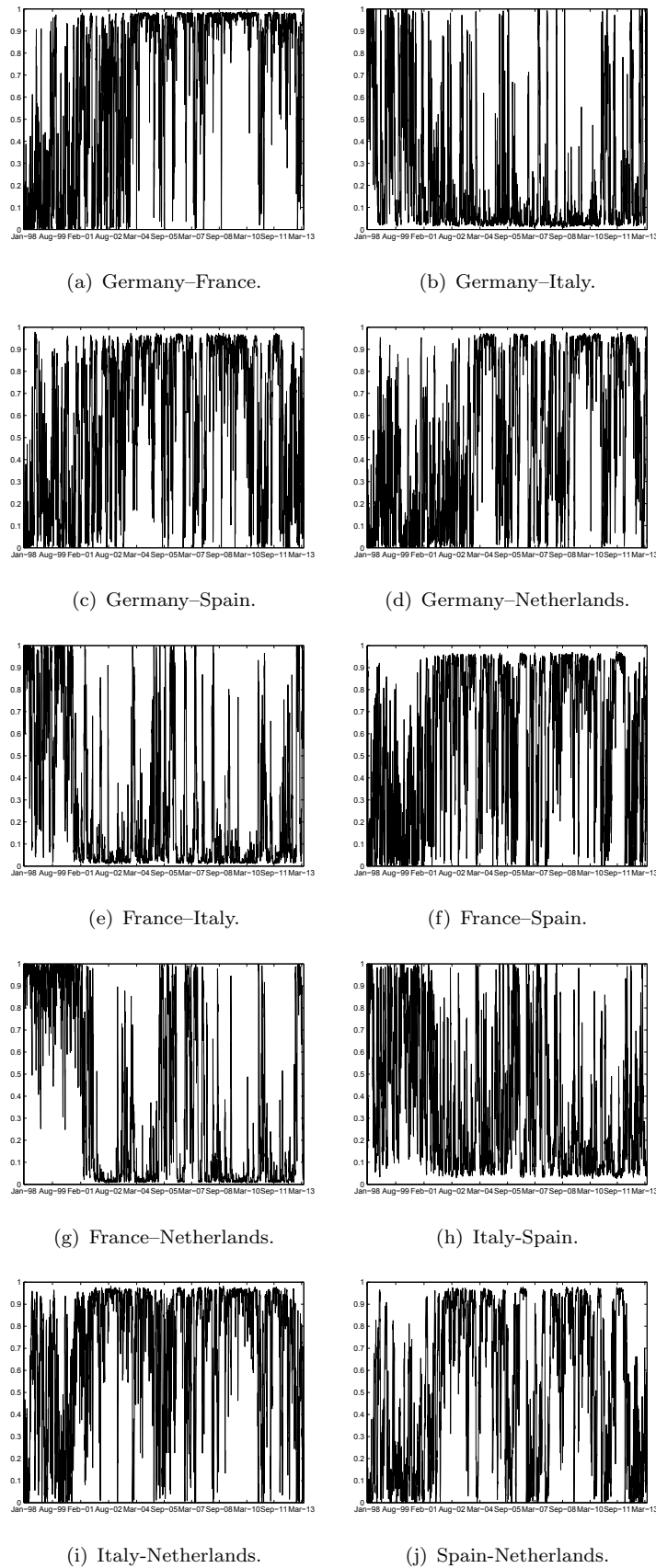


FIGURE 4.4: Smoothed probability with marginals modeled with FIEGARCH(1,d,1) model. Both the regimes are described a bivariate Gaussian copula.

Appendix A

Portfolio's Optimization

Stylized facts reveals the non-normality of returns by showing fat-tailed distributions;¹ many authors deal with higher moments in the asset allocation by using of Taylor's series;²,

$$\begin{aligned} U(W) &= - \sum_{k=0}^{\infty} \frac{(-\lambda W)^k}{k!} = \\ &= -1 + \lambda W - \frac{\lambda^2}{2} W^2 + \frac{\lambda^3}{3!} W^3 - \frac{\lambda^4}{4!} W^4 + O(W^5). \end{aligned} \quad (\text{A.1})$$

It is possible to express the CARA expected utility function maximized by the investor in terms of the portfolio's moments with n assets,

$$E[U(W)] \approx \lambda \mu_p - \frac{\lambda^2}{2} \sigma_p^2 + \frac{\lambda^3}{3!} s_p^3 - \frac{\lambda^4}{4!} \kappa_p^4 \quad (\text{A.2})$$

$$\begin{aligned} \mu_p &= \alpha' \mu \\ \sigma_p^2 &= \alpha' M_2 \alpha \\ s_p^3 &= \alpha' M_3 (\alpha \otimes \alpha) \\ \kappa_p^4 &= \alpha' M_4 (\alpha \otimes \alpha \otimes \alpha) \end{aligned} \quad (\text{A.3})$$

and

$$\begin{aligned} M_2 &= E[(R - \mu)(R - \mu)'] \\ M_3 &= E[(R - \mu)(R - \mu)' \otimes (R - \mu)'] \\ M_4 &= E[(R - \mu)(R - \mu)' \otimes (R - \mu)' \otimes (R - \mu)'], \end{aligned} \quad (\text{A.4})$$

where M_2 is the co-variance matrix, M_3 and M_4 are the co-skewness and co-kurtosis and $\otimes$ is the Kroenecker product. This technique, introduced in Athayde (2001), makes

¹See Cont (2001) for a survey on the stylized facts and statistical issues.

²For major references, see Harvey et al. (2010) and Guidolin and Timmerman (2005), as reported in Jondeau and Rockinger (2006b).

the tensors easy to handle by allowing their expression as matrices. It is straightforward to see that a computational problem arises when the number of assets n grows in dimensionality,

n	μ	M_2	M_3	M_4	k
10	10	55	220	715	1000
50	50	1275	22100	292825	316250
100	100	5050	171700	4421275	4598125

Particularly, at the low frequency of the financial data, given the relatively “small” sample with respect to the number of k parameters needed, it clearly becomes infeasible in the computation to consider all the investment universe (or a large subset) in the allocation process. Hence, it is necessary to perform a selection amongst the universe or use other techniques such as principal component analysis or factor analysis.

Appendix B

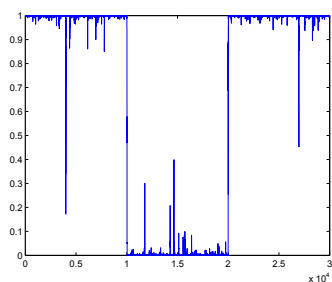
Simulation of Bivariate RS-Copula

In this Appendix we perform some simulations on the Bivariate RS-copula. The values for the copula are expressed with Kendall's τ . We simulate 30000 observations for each trial. The purpose is to create a latent variable from a steady state that switch at some point in the turbulent state and then return in the steady. The latent state is represented by the Gaussian copula regime where there is no tail dependence among the two series and the turbulent state with the rotated-Gumbel copula where there is lower tail dependence.

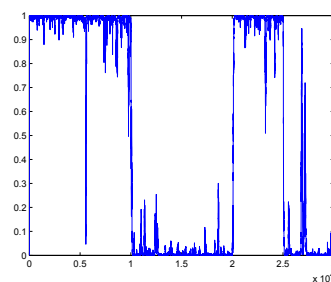
The results are reported in Table B.1 The parameter Kendall's τ is set to different values for each trials and Figure (B.1) shows the smoothed probabilities, respectively.

	Steady Gaussian	Turbulent Gumbel	Transition probabilities		Change in regime
			p_{11}	p_{22}	
τ	0.5	0.75			2
$\hat{\tau}$	0.5026	0.7446	0.9999	0.9999	
t-stat	71.00	391.56	99.99	99.99	
T	20000	10000			
τ	0.2	0.3			3
$\hat{\tau}$	0.1985	0.3042	0.9999	0.9999	
t-stat	4227.75	15147.77	1003114.80	1298921.85	
T	15000	15000			
τ	0.8	0.8			4
$\hat{\tau}$	0.8004	0.8002	0.9999	0.9998	
t-stat	95.13	500.48	99.99	99.98	
T	17500	12500			
τ	0.1	0.9			2
$\hat{\tau}$	0.0845	0.8909	0.9991	1.0000	
t-stat	98.53	109.23	99.90	99.99	
T	29000	1000			

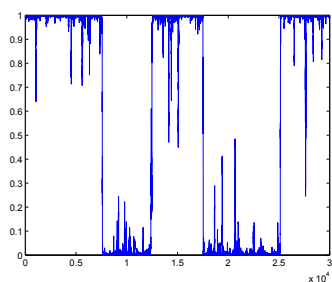
TABLE B.1: Results for the four simulations on the Gaussian-Rotated-Gumbel regime switching copula.



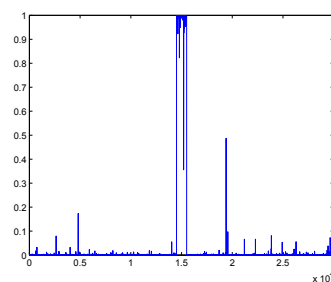
(a) Simulation 1



(b) Simulation 2



(c) Simulation 3



(d) Simulation 4

FIGURE B.1: Smoothed probability simulated in the steady and turbulent regime with bivariate Gaussian and Rotated-Gumbel copulas.

Appendix C

Estimation Results for all the Models

In this section we include all the estimation results of the Regime-Switching Copulas and the Smoothed probabilities for all the models used to filter the marginals.

GARCH	Gaussian Regime			Rot-Gumbel Regime			AIC	BIC	$\hat{\rho}_{11}$	t-stat	$\hat{\rho}_{22}$	t-stat
	$\hat{\rho}$	t-stat	$\hat{\tau}_1$	$\hat{\theta}$	t-stat	$\hat{\tau}_2$						
Ger,Fra	0.79	48.89	0.58	4.42	273.80	0.77	-6430.74	-6411.99	0.99	61.18	0.99	61.48
Ger,Ita	0.66	41.09	0.46	3.39	209.96	0.71	-4606.70	-4587.95	0.96	59.44	0.98	60.55
Ger,Spa	0.70	43.38	0.49	3.56	220.31	0.72	-4164.08	-4145.33	0.97	60.01	0.97	59.73
Ger,NL	0.80	49.72	0.59	4.23	261.74	0.76	-5532.69	-5513.94	0.99	61.20	0.99	61.02
Fra,Ita	0.81	49.83	0.60	4.28	264.92	0.77	-5633.04	-5513.94	0.98	61.20	0.98	61.02
Fra,Spa	0.79	48.77	0.58	4.17	257.98	0.76	-5101.39	-5082.64	0.96	59.46	0.95	58.56
Fra,NL	0.82	50.88	0.61	4.78	295.67	0.79	-6614.29	-6595.54	0.97	60.20	0.98	60.60
Ita,Spa	0.76	33.47	0.55	3.75	239.20	0.73	-4674.25	-4655.49	0.97	88.93	0.96	80.14
Ita,NL	0.76	90.15	0.55	3.89	86.94	0.74	-4564.20	-4545.45	0.97	2716.54	0.96	492.01
Spa,NL	0.75	46.65	0.54	3.56	220.49	0.72	-4153.84	-4135.09	0.99	61.05	0.97	60.20

TABLE C.1: The table reports parameter estimates for the bivariate dependence structure in the regime switching Gaussian-Rotated-Gumbel copula with marginals modeled with GARCH(1,1). We also include the Kendall's τ .

EGARCH	Gaussian Regime			Rot-Gumbel Regime			AIC	BIC	$\hat{\rho}_{11}$	t-stat	$\hat{\rho}_{22}$	t-stat
	$\hat{\rho}$	t-stat	$\hat{\tau}_1$	$\hat{\theta}$	t-stat	$\hat{\tau}_2$						
Ger,Fra	0.78	57.72	0.57	4.37	268.63	0.77	-6298.02	-6279.27	0.98	120.26	0.99	238.74
Ger,Ita	0.63	482.04	0.43	3.26	534.41	0.69	-4488.61	-4469.86	0.95	1463.22	0.98	2727.47
Ger,Spa	0.70	2713.83	0.49	3.46	939.43	0.71	-4031.29	-4012.54	0.97	2977.82	0.97	2107.88
Ger,NL	0.80	7391.95	0.59	4.22	3243.46	0.76	-5379.30	-5360.54	0.98	12009.43	0.98	9708.37
Fra,Ita	0.77	1742.95	0.56	3.92	2241.02	0.74	-5433.08	-5360.54	0.97	12009.43	0.98	9708.37
Fra,Spa	0.77	98.47	0.56	3.92	240.85	0.74	-4953.13	-4934.37	0.96	272.17	0.96	44.85
Fra,NL	0.79	211.39	0.58	4.33	276.20	0.77	-6443.59	-6424.84	0.98	740.65	0.99	2036.59
Ita,Spa	0.75	41.86	0.54	3.70	228.53	0.73	-4551.20	-4532.45	0.96	109.27	0.95	64.62
Ita,NL	0.73	3968.77	0.52	3.68	2140.36	0.73	-4422.71	-4403.96	0.96	6931.62	0.95	6339.21
Spa,NL	0.75	46.14	0.54	3.42	211.90	0.71	-4005.09	-3986.33	0.98	60.89	0.97	59.88

TABLE C.2: The table reports parameter estimates for the bivariate dependence structure in the regime switching Gaussian-Rotated-Gumbel copula with marginals modeled with EGARCH(1,1,1). We also include the Kendall's τ .

FIGARCH	Gaussian Regime			Rot-Gumbel Regime			AIC	BIC	$\hat{\rho}_{11}$	t-stat	$\hat{\rho}_{22}$	t-stat
	$\hat{\rho}$	t-stat	$\hat{\tau}_1$	$\hat{\theta}$	t-stat	$\hat{\tau}_2$						
Ger,Fra	0.83	51.24	0.62	5.57	344.60	0.82	-6355.01	-6336.26	0.98	60.95	0.98	60.52
Ger,Ita	0.71	13.82	0.50	4.28	264.54	0.77	-4513.06	-4494.31	0.98	64.71	0.97	41.49
Ger,Spa	0.70	73.99	0.49	4.27	263.60	0.77	-4146.76	-4128.00	0.98	58.80	0.96	67.31
Ger,NL	0.77	17.84	0.56	4.62	279.80	0.78	-5521.40	-5502.65	0.98	72.88	0.98	32.65
Fra,Ita	0.77	113.23	0.55	5.07	313.85	0.80	-5517.91	-5502.65	0.98	72.88	0.98	32.65
Fra,Spa	0.76	53.24	0.55	4.99	295.80	0.80	-5041.54	-5022.79	0.98	174.88	0.97	105.08
Fra,NL	0.80	42.88	0.59	5.81	359.98	0.83	-6409.27	-6390.52	0.98	57.30	0.98	132.73
Ita,Spa	0.69	42.95	0.49	4.23	261.78	0.76	-4592.27	-4573.52	0.98	60.50	0.97	60.34
Ita,NL	0.70	37.82	0.49	4.49	278.04	0.78	-4350.02	-4331.27	0.98	104.29	0.97	167.58
Spa,NL	0.68	208.94	0.48	3.87	239.61	0.74	-3918.64	-3899.88	0.99	137.01	0.98	68.60

TABLE C.3: The table reports parameter estimates for the bivariate dependence structure in the regime switching Gaussian-Rotated-Gumbel copula with marginals modeled with FIGARCH(1,d,1). We also include the Kendall's τ .

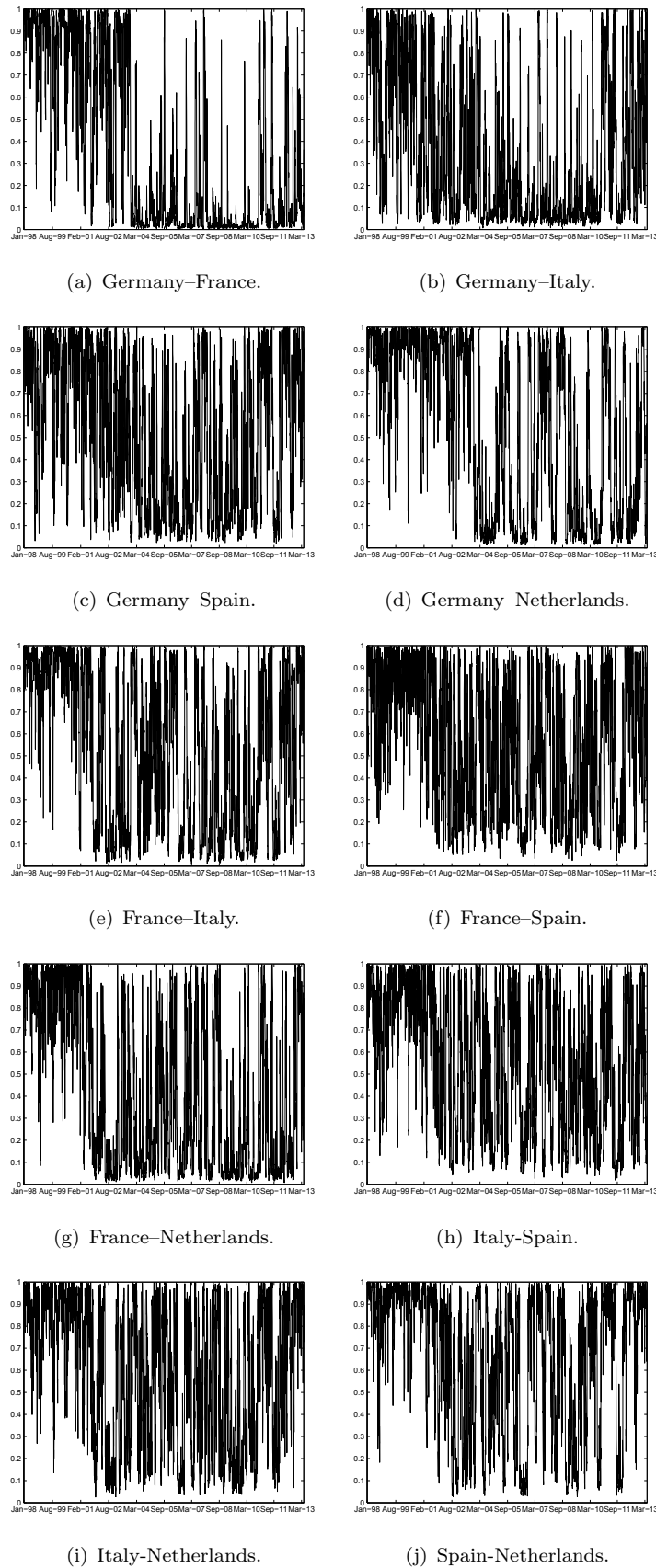


FIGURE C.1: Smoothed probability with marginals modelled with GARCH(1,1) model.
The two regimes: a bivariate Gaussian copula and a rotated-Gumbel copula.

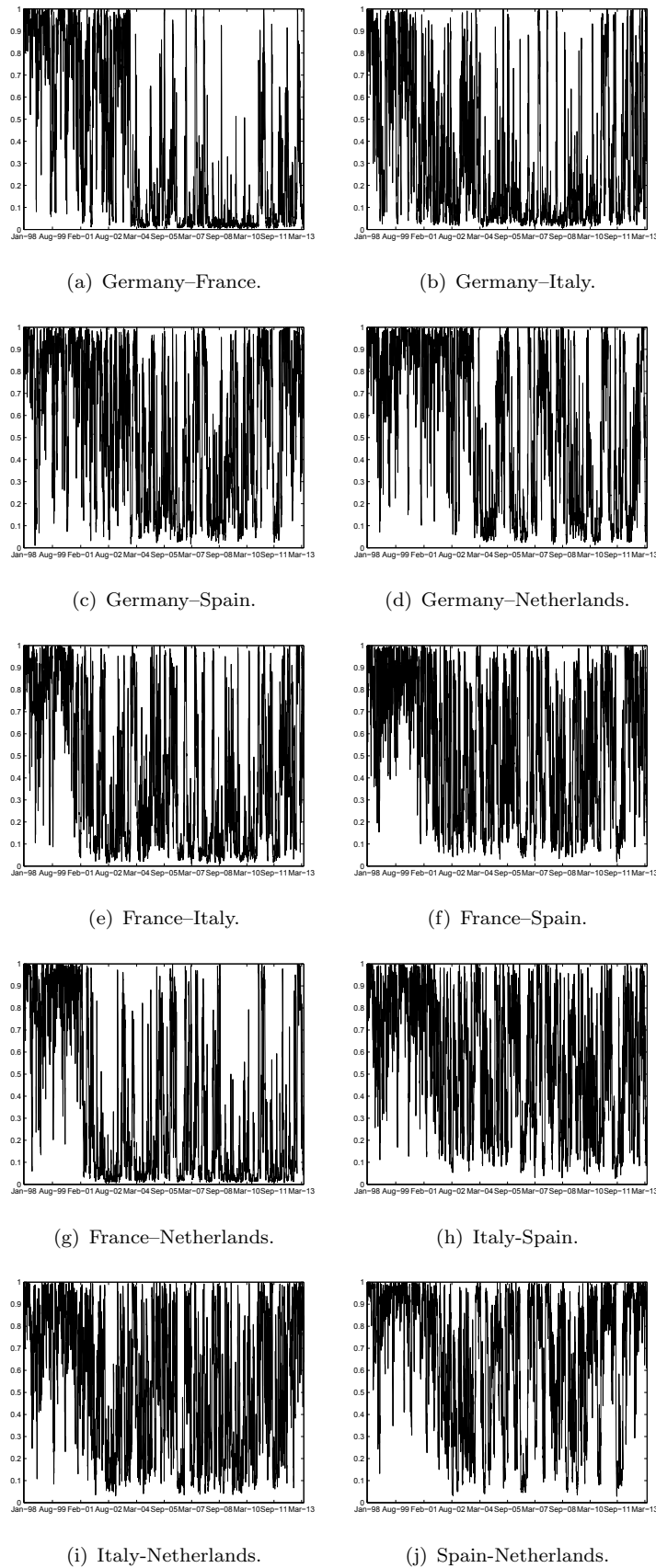


FIGURE C.2: Smoothed probability with marginals modelled with EGARCH(1,1,1) model. The two regimes: a bivariate Gaussian copula and a rotated-Gumbel copula.

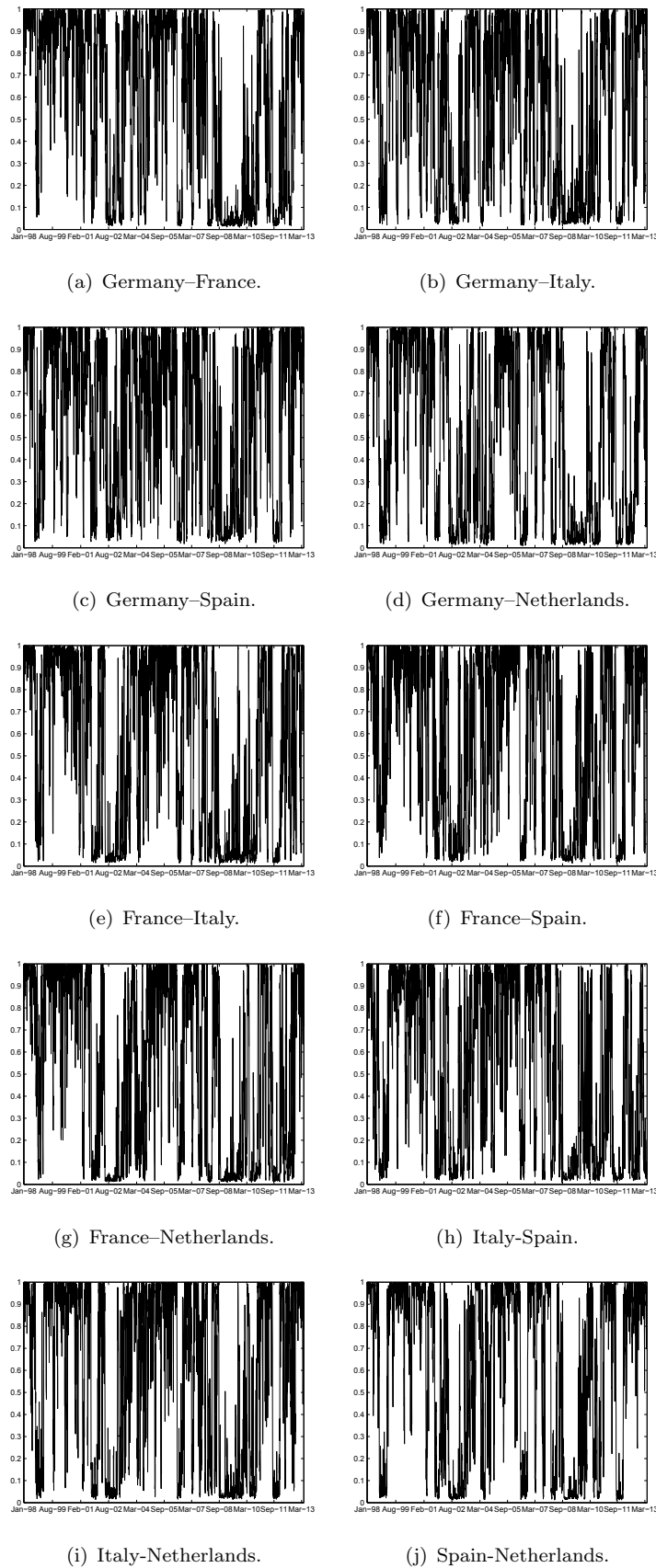


FIGURE C.3: Smoothed probability with marginals modelled with FIGARCH(1,d,1) model. The two regimes: a bivariate Gaussian copula and a rotated-Gumbel copula.

GARCH	Gaussian Regime 1			Gaussian Regime 2			AIC	BIC	$\hat{\rho}_{11}$	t-stat	$\hat{\rho}_{22}$	t-stat
	$\hat{\rho}_1$	t-stat	$\hat{\tau}_1$	$\hat{\rho}_2$	t-stat	$\hat{\tau}_2$						
Ger,Fra	0.95	24354.76	0.80	0.78	4109.00	0.57	-6581.78	-6563.03	0.99	16124.81	0.98	8608.85
Ger,Ita	0.90	522.32	0.72	0.61	36.03	0.42	-4749.24	-4730.48	0.98	130.33	0.95	60.44
Ger,Spa	0.89	6405.41	0.70	0.63	1583.76	0.43	-4237.56	-4218.81	0.98	6104.65	0.97	4289.53
Ger,NL	0.94	6377.71	0.77	0.79	1543.79	0.58	-5648.25	-5629.50	0.99	6188.01	0.99	5702.79
Fra,Ita	0.93	9463.34	0.76	0.77	3136.52	0.56	-5714.87	-5629.50	0.98	5702.79	0.97	6188.01
Fra,Spa	0.93	268.40	0.76	0.75	88.34	0.54	-5180.02	-5161.27	0.94	94.41	0.95	105.98
Fra,NL	0.93	27724.99	0.77	0.77	4315.96	0.56	-6749.57	-6730.81	0.998	52672.99	0.99	16851.70
Ita,Spa	0.90	73.60	0.72	0.70	57.66	0.49	-4756.79	-4738.04	0.97	122.07	0.95	55.08
Ita,NL	0.91	9380.28	0.73	0.71	4384.73	0.50	-4632.45	-4613.70	0.97	8917.11	0.96	8784.34
Spa,NL	0.91	7117.92	0.73	0.73	3490.38	0.52	-4197.51	-4178.76	0.97	5520.29	0.98	7861.28

TABLE C.4: The table reports parameter estimates for the bivariate dependence structure in the regime switching Gaussian-Gaussian copula with marginals modeled with GARCH(1,1). We also include the Kendall's τ . The Regime 1 identifies the highest dependence structure.

EGARCH	Gaussian Regime 1			Gaussian Regime 2			AIC	BIC	$\hat{\rho}_{11}$	t-stat	$\hat{\rho}_{22}$	t-stat
	$\hat{\rho}_1$	t-stat	$\hat{\tau}_1$	$\hat{\rho}_2$	t-stat	$\hat{\tau}_2$						
Ger,Fra	0.95	293.44	0.79	0.76	103.11	0.55	-6443.38	-6424.63	0.99	413.09	0.97	69.66
Ger,Ita	0.90	1189.79	0.71	0.57	28.86	0.39	-4651.16	-4632.41	0.98	417.79	0.94	122.56
Ger,Spa	0.90	6726.64	0.71	0.64	1348.39	0.45	-4132.91	-4114.16	0.97	3641.28	0.96	3116.16
Ger,NL	0.93	57.02	0.76	0.77	44.53	0.56	-5488.75	-5470.00	0.98	83.02	0.98	85.04
Fra,Ita	0.92	12286.11	0.74	0.70	1315.47	0.49	-5605.60	-5470.00	0.99	83.02	0.97	85.04
Fra,Spa	0.92	13201.28	0.74	0.73	7868.71	0.52	-5077.66	-5058.91	0.98	7279.52	0.97	5537.87
Fra,NL	0.94	6640.93	0.77	0.76	2314.28	0.55	-6619.49	-6600.74	0.99	13912.24	0.98	4781.89
Ita,Spa	0.90	117.30	0.71	0.67	72.22	0.47	-4660.19	-4641.43	0.97	62.06	0.95	63.08
Ita,NL	0.88	1.75	0.69	0.61	0.13	0.42	-4526.93	-4508.18	0.98	6.91	0.95	777.71
Spa,NL	0.88	2883.00	0.69	0.68	2389.66	0.48	-4095.70	-4076.95	0.98	5635.39	0.97	4652.05

TABLE C.5: The table reports parameter estimates for the bivariate dependence structure in the regime switching Gaussian-Gaussian copula with marginals modeled with EGARCH(1,1,1). We also include the Kendall's τ . The Regime 1 identifies the highest dependence structure.

FIGARCH	Gaussian Regime 1			Gaussian Regime 2			AIC	BIC	$\hat{p}_{11}$	t-stat	$\hat{p}_{22}$	t-stat
	$\hat{\rho}_1$	t-stat	$\hat{\tau}_1$	$\hat{\rho}_2$	t-stat	$\hat{\tau}_2$						
Ger,Fra	0.96	26490.83	0.82	0.81	2346.30	0.60	-6441.47	-6422.72	0.97	2737.21	0.97	2539.54
Ger,Ita	0.94	13387.18	0.78	0.70	3434.28	0.50	-4521.13	-4502.38	0.97	8221.38	0.98	10543.13
Ger,Spa	0.93	9277.04	0.76	0.68	2476.22	0.48	-4153.82	-4135.07	0.96	7216.65	0.97	9969.71
Ger,NL	0.95	63.53	0.80	0.76	47.76	0.55	-5618.31	-5599.56	0.98	133.47	0.98	56.56
Fra,Ita	0.96	10642.06	0.81	0.76	1690.13	0.55	-5544.32	-5599.56	0.97	56.56	0.98	133.47
Fra,Spa	0.95	13689.59	0.80	0.74	2565.67	0.53	-5073.40	-5054.65	0.96	9514.48	0.97	9067.47
Fra,NL	0.97	15281.12	0.83	0.79	3682.38	0.58	-6498.53	-6479.78	0.98	3571.80	0.98	3339.35
Ita,Spa	0.94	57.97	0.77	0.68	41.93	0.47	-4672.30	-4653.54	0.97	60.17	0.97	60.13
Ita,NL	0.95	18366.42	0.79	0.70	2511.03	0.49	-4398.44	-4379.69	0.97	8704.60	0.98	10333.91
Spa,NL	0.93	13450.56	0.76	0.68	5049.04	0.48	-3975.48	-3956.73	0.98	9522.40	0.99	15899.76

TABLE C.6: The table reports parameter estimates for the bivariate dependence structure in the regime switching Gaussian-Gaussian copula with marginals modeled with FIGARCH(1,d,1). We also include the Kendall's τ . The Regime 1 identifies the highest dependence structure.

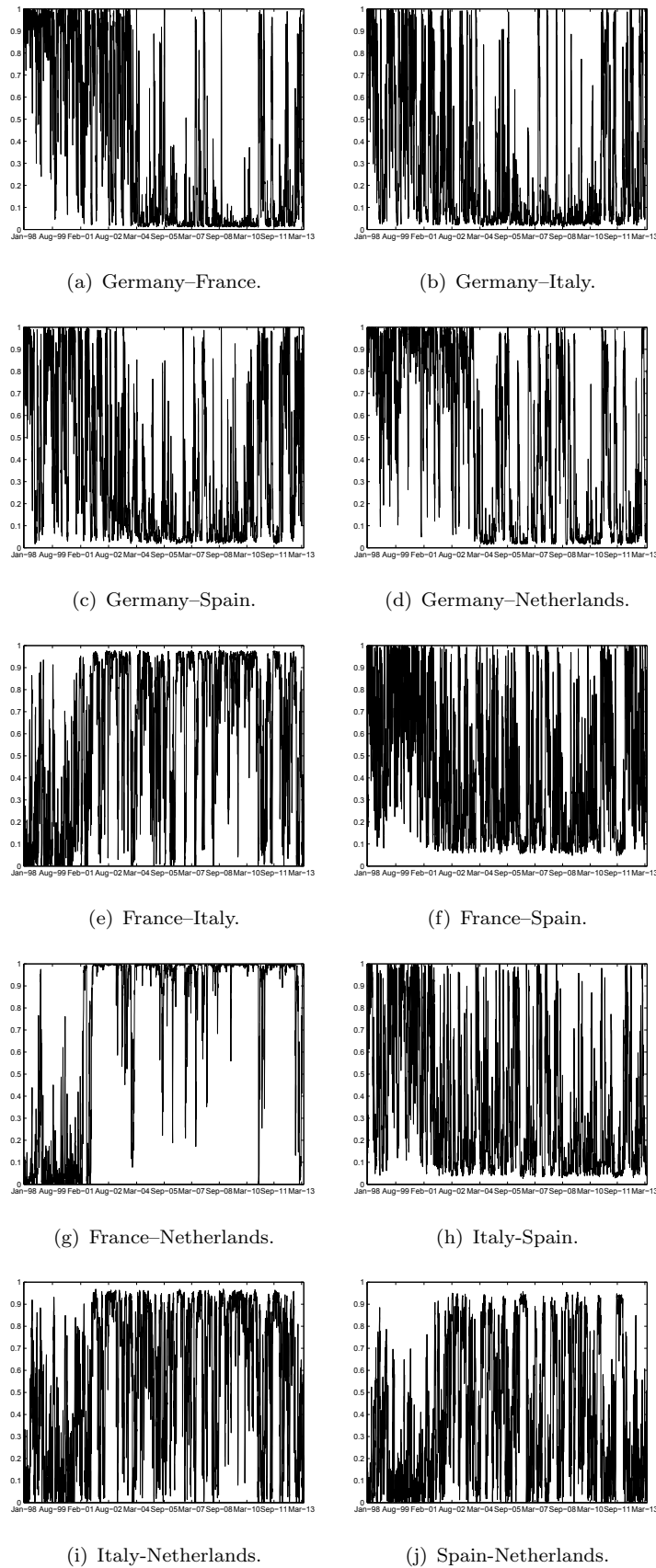


FIGURE C.4: Smoothed probability with marginals modeled with GARCH(1,1) model. Both the regimes are described a bivariate Gaussian copula.

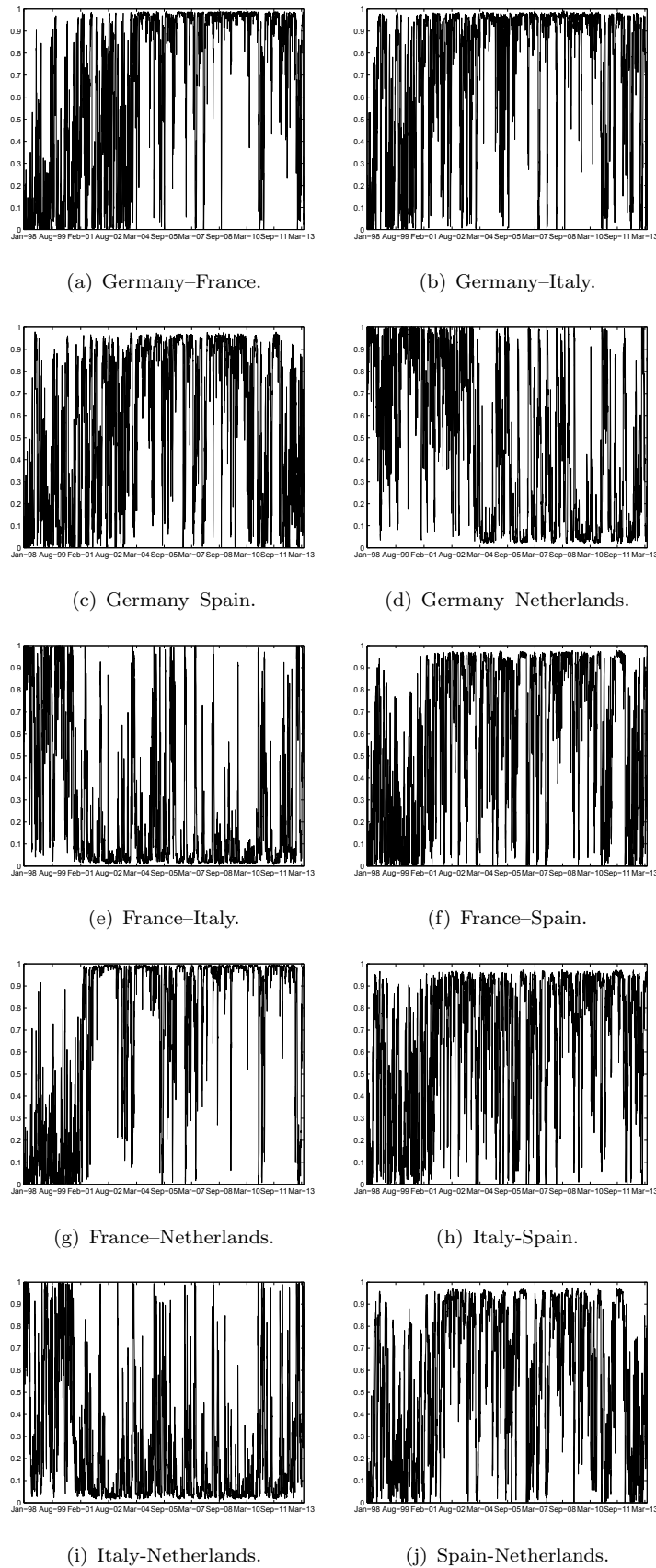


FIGURE C.5: Smoothed probability with marginals modeled with EGARCH(1,1,1) model. Both the regimes are described a bivariate Gaussian copula.

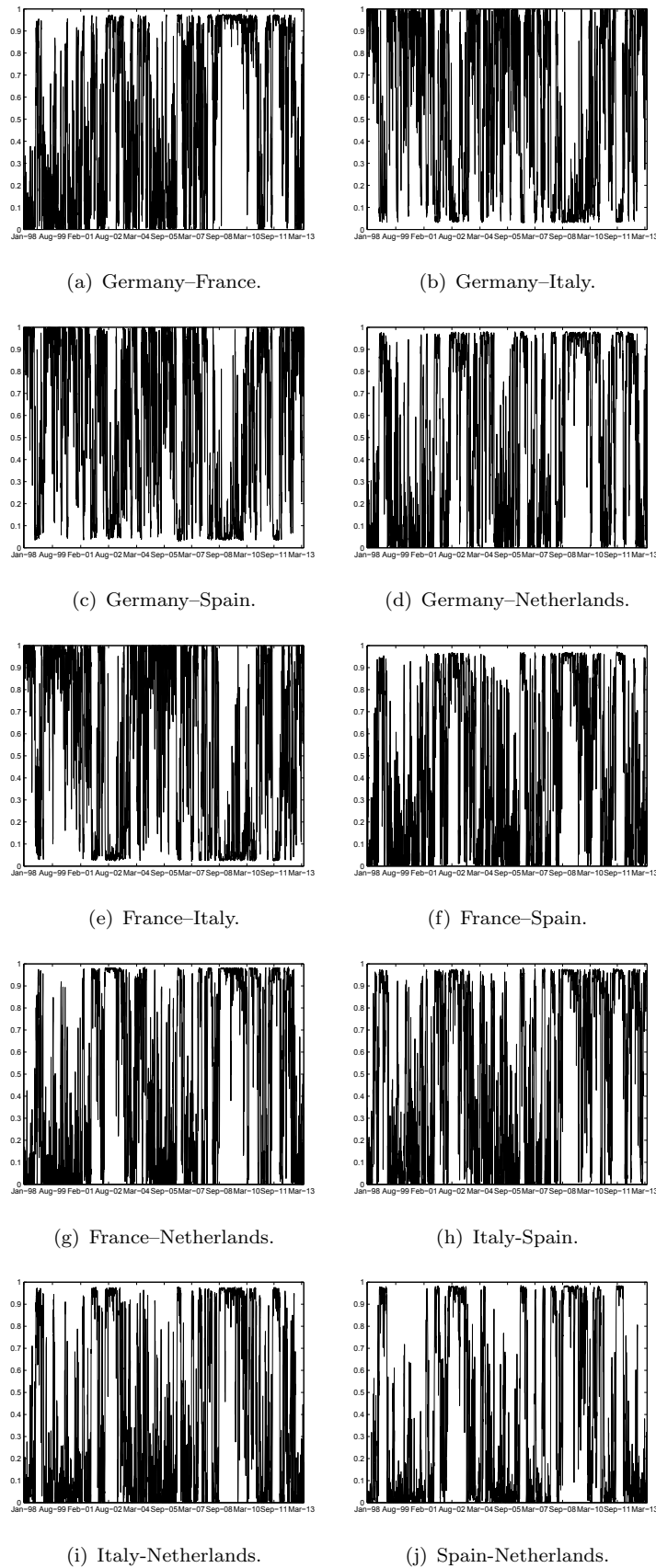


FIGURE C.6: Smoothed probability with marginals modeled with FIGARCH(1,d,1) model. Both the regimes are described a bivariate Gaussian copula.

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