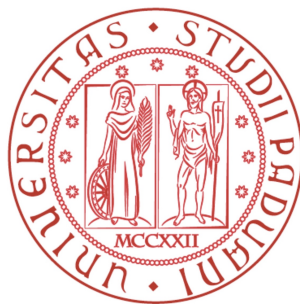




UNIVERSITÀ DEGLI STUDI DI PADOVA
DIPARTIMENTO DI INGEGNERIA INDUSTRIALE

SCUOLA DI DOTTORATO IN INGEGNERIA INDUSTRIALE
INDIRIZZO INGEGNERIA DELL'ENERGIA
XXVIII CICLO

Multiphysical Finite Element Simulation of Contour Induction Hardening of Gears

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01 Febbraio 2016

To my mother

Abstract

Induction heating has been widely applied in the field of heat treatment of components for the automotive and aerospace sectors, in particular for the hardening of a huge variety of applications. The main advantages of using this technology counts the high level of repeatability achievable in the treated product, together with the high velocity and automation of treatment, factors both able to ensure production efficiency and reduced environmental impact.

Nowadays, numerical methods are becoming more and more important as a reference method of analysis, in order to optimize the main parameters of the process, also thanks to the possibility of coupling different physical between them, a result which until a few years ago would not have been possible.

The aim of this work is the analysis and numerical modeling of the process of induction hardening of gear wheels for the aerospace industry.

In this thesis it will be shown how, starting from the electromagnetic and thermal coupled models, already extensively uased in the last years by both the research and the industrial sectors, it is possible to calculate the phase transformations that occur in the steel during the heating and cooling stages.

The algorithm developed will be firstly applied on the case of a simple 2D geometry, and then the complexity level will be gradually increased (both from computational and process point of view), applying the algorithm to an induction hardening process of a gear. The numerical results thus obtained will be verified experimentally.

Sommario

Il riscaldamento a induzione è stato diffusamente impiegato nel settore dei trattamenti termici di componenti per i settori automobilistico ed aerospaziale, in particolare per la tempra di una vasta varietà di applicazioni. I principali vantaggi legati all'utilizzo di questa tecnologia risiedono nell'elevato grado di ripetibilità ottenibile nel prodotto trattato, unitamente alla elevata velocità ed automazione di trattamento, fattori entrambi in grado di garantire una produzione efficiente e dal ridotto impatto ambientale.

Oggigiorno, i metodi numerici si stanno affermando sempre più come principale metodo di analisi, in modo da ottimizzare i principali parametri di processo, anche grazie alla possibilità di accoppiare diverse fisiche tra di loro, risultato che fino a pochi anni fa non sarebbe stato possibile.

Scopo di questo lavoro è l'analisi e la modellazione numerica del processo di tempra ad induzione su ruote dentate per l'industria aerospaziale.

Nel corso della tesi si mostrerà come, partendo dai modelli elettromagnetici e termici accoppiati, già diffusamente sviluppati negli anni sia a livello di ricerca che a livello industriale, è possibile anche calcolare le trasformazioni di fase che avvengono nell'acciaio nel corso del riscaldamento e del raffreddamento. L'algoritmo sviluppato verrà poi applicato dapprima su di un caso semplice 2D, per poi incrementare la difficoltà (sia computazionale che di processo), applicandolo ad un trattamento di tempra ad induzione di una ruota dentata. I risultati numerici così ottenuti verranno verificati sperimentalmente.

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Chapter 1

Introduction

The use of heat to affect steel properties, the so-called heat treatment, is one of the oldest of all technologies, dating back to the Iron Age. Though, it was not until the end of the 19th and the begin of the 20th century that this process came to be systematically investigated and its scientific backgrounds fully understood [1].

The formulation and application of modeling and simulation methods for metallic materials and manufacturing process design and development is rapidly increasing. Classic models that predict the behavior of metals under processing conditions are continuing to be used and enhanced with greater understanding of the mechanisms that control the evolution of microstructure, texture, and defects. New theories and associated mathematical models are being developed and applied to metallic alloy systems for existing and new processing methods. To complement empirical descriptions of material behavior during processing, so-called first-principles approaches, such as those based on atomistic and molecular dynamics calculations, are now being developed to provide fundamental understanding of the mechanisms that control observed behaviors of existing and emerging alloys, such as those for unique or highly demanding applications [2]-[3].

In particular, the induction heating processes has been widely applied starting from the beginning of the 20th century. When the induction surface hardening age began in early 1930s, there were attempts to develop analytical methods for calculation of induction surface hardening processes. The first advanced analytical methods for the calculation of one-dimensional (1D) and even 2D electromagnetic (EM) and thermal fields in induction hardening systems have been developed by Razorenov and Vologdin [4], followed by Curtis, Stansel, Rodigin, and others [5]-[6]-[7]-[8]-[9]-[10]. The analytical solutions were based on such concepts as series expansion, separation of variables, Bessel and other special functions applications, Fourier transformation, and so on. These methods could be applied for domains of a simple geometry and mainly linear materials. Even with these restrictions the calculations were rather lengthy and cumbersome until the introduction of computers. Then the era of numerical simulation began. Nowadays, numerical methods allow us to calculate coupled EM, thermal, and other fields in systems of any complex geometry with linear or nonlinear materials. The first numerical method based on the solution of nonlinear differential equations for closely coupled EM and thermal processes during a steel rod hardening was published by Kogan in 1966 [11]. Actually, it was the first

computer simulation of 1D induction heating process of steel in time domain (TD), i.e., with calculation of the EM field waveform.

Despite the long history of induction system modeling with analytical and numerical methods, it is only since the end of the 1980s that numerical modeling (computer simulation) became a widely used tool for the study and design of 2D induction systems. Simulation of three-dimensional (3D) systems is in the process of intensive development and may be considered as an emerging technology.

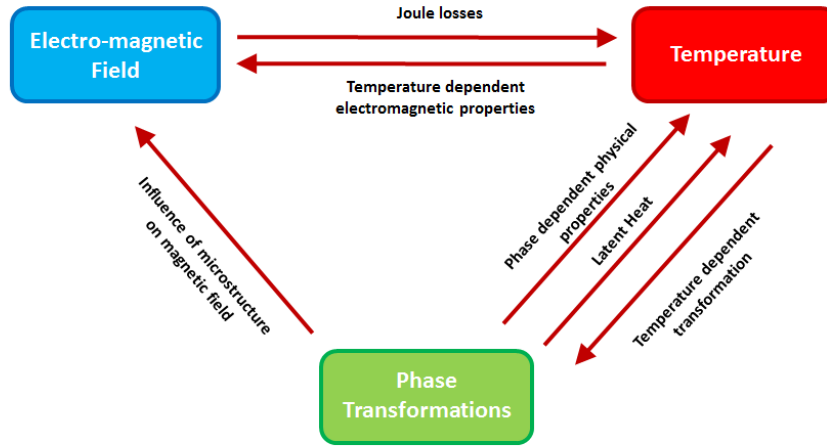


Figure 1.1: Multiphysical approach on induction hardening problem

Nowadays the Finite Element Method is the most used technique in order to design components and processes. Moreover, the recent advancements in parallel (e.g., graphics processing units, GPUs) and distributed (e.g., clusters) computing have made it now possible to break the wall of one trillion floating-point operations per second (teraflop) on a single node, thus enabling users to perform very demanding 3-D multi-physics simulations.

The new challenge, today, is called multiphysical simulations, in which different physics are solved and coupled in the same numerical solution, taking into account the mutual interaction between each of them.

In this thesis, the work done during the last three year of doctoral studies at the Laboratory of Electroheat of University of Padova is summarized. The research can be divided in three main branches:

- The multiphysical simulation of the contour induction hardening process
- The implementation of a FEM procedure for the calculation of Simultaneous Double Frequencies (SDF) induction heat treatments

- The investigation of the combined effect of microstructure and heating rate on the austenitization kinetics of steel.

The multiphysical simulation of contour induction hardening process has been developed at Laboratory of Electroheat of University of Padova and in collaboration with the Ecole de Technologie Supérieure of Montréal (Canada) for the experimental tests. The simulation of SDF induction heat treatments has been completely developed at the Laboratory of Electroheat by me and Antonio Marconi, which gave me a great help during his master's thesis. The last task, the investigation of austenitization kinetic, has been carried out in collaboration of two other groups of my department, in particular with Alessio Settimi and Luca Pezzato of the Metallurgy Group and Michele Novella of the Precision Manufacturing Engineering Group for the dilatometric tests.

Following, a brief summary of the thesis is discussed, presenting the content of each chapter:

In Chapter 2, a general discussion about gears manufacturing will be presented, focusing the attention, in the last part, on the induction heating technologies used in gears heat treatment.

Then in Chapter 3 the numerical modeling of electromagnetic phenomena involved in induction hardening simulation will be presented, starting from the field theory to the FEM modelization.

In Chapter 4, the numerical formulation of the thermal problem applied to quenching process is described, showing how to develop a robust procedure in order to solve this kind of problem, even in case of critical conditions.

In Chapter 5 the metallurgical solid solid phase transformations that occur in steels during heating and quenching are described.

In Chapter 6 an algorithm for the solution of magneto-thermo-metallurgical model will be developed and applied to simple test-case like a 2D-axisymmetric model.

In Chapter 7 the algorithm developed in chapter 6 will be applied to a complex 3D geometry, considering magnetic and thermal non-linearities and phase-dependent material's properties. At the end the model developed will be verified through experimental tests with an industrial scale machine.

In Chapter 8 an algorithm has been developed in order to solve the electromagnetic and thermal coupled simulation in the time domain, without exceeding with the computational time. In this way a simultaneous double frequency induction hardening process can be modeled, considering also the strong non-linearities introduced by the magnetic behavior of the steel.

In Chapter 9 the influence of the previous microstructure and the heating rate on the austenitization kinetics of 39NiCrMo3 steel. The analysis has been carried out through dilatometric tests.

Finally in Chapter 10 the final remarks will be presented. The obtained results also have a scientific relevance that have allowed their presentation to numerous national and international conferences and publication in scientific journals as reported at the end of the thesis.

Chapter 2

Induction Hardening of Gears

2.1 Introduction

Gears are machine elements that transmit rotary motion and power by the successive engagements of teeth on their periphery. Gears have been in use for more than three thousand years and they are an important element, in all manner, of machinery used in current times. Gears are used in a very wide set of applications, and just to name few of them: automotive, aerospace, transportation, oil and gas industries, marine, machine tool and large mills [12].

2.2 Gear Nomenclature

In this section a brief overview of the most used terms in gear nomenclature will be proposed to the reader. For a better comprehension refer to Figure 2.1.

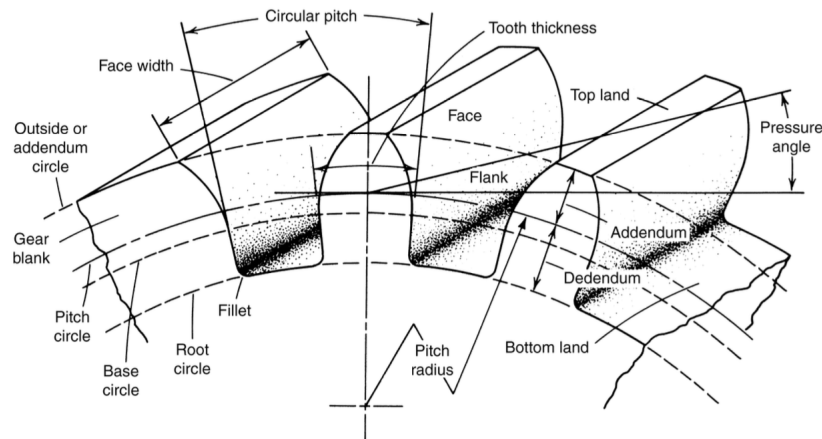


Figure 2.1: Scheme of a typical gear tooth nomenclature [12]

- **active profile** The part of the gear tooth profile that actually comes in contact with the profile of the mating gear while in mesh.
- **addendum** The height of the tooth above the pitch circle.
- **backlash** The amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the operating pitch circle.
- **base circle** The circle from which the involute tooth profiles are generated.
- **bottom land** or **root** The surface at the bottom of a tooth space adjoining the fillet.
- **center distance** The distance between the axes of rotation between two mating gears.
- **circular pitch** Length of arc of the pitch circle between corresponding points on adjacent teeth.
- **circular thickness** The length of arc between the two sides of a gear tooth at the pitch circle.
- **dedendum** The depth of the tooth below the pitch circle.
- **diametral pitch** A measure of tooth size in the English system. In units, it is the number of teeth per inch of pitch diameter. As the tooth size increases, the diametral pitch decreases. Diametral pitches range from 0.5 to 200. Coarse pitch gears are those with $DP < 20$. Fine pitch gears are those with a $DP > 20$.
- **face width** The length of the gear teeth in an axial plane.
- **fillet radius** The radius of the fillet curve at the base of the gear tooth.
- **helix angle** The angle between any helix and an element of its cylinder. In helical gears and worms, it is at the standard pitch circle unless otherwise specified.
- **normal section** A section through a gear that is perpendicular to the tooth at the pitch circle.
- **pitch circle** The circumference of a gear measured at the point of contact with the mating gear.
- **pitch diameter** The diameter of the pitch circle.
- **pitch line** In a cross section of a rack, the pitch line corresponds to the pitch circle in the cross section of the gear.
- **pitch point** The tangency point of the pitch circles of two mating gears.

- **pressure angle** The angle between a tooth profile and a radial line at its pitch point. The pressure angle of an involute gear tooth is determined by the size ratio between the base circle and the pitch circle. Common pressure angles used by the gear industry are 14.5, 20, and 25 .
- **top land** The surface of the top of a tooth.

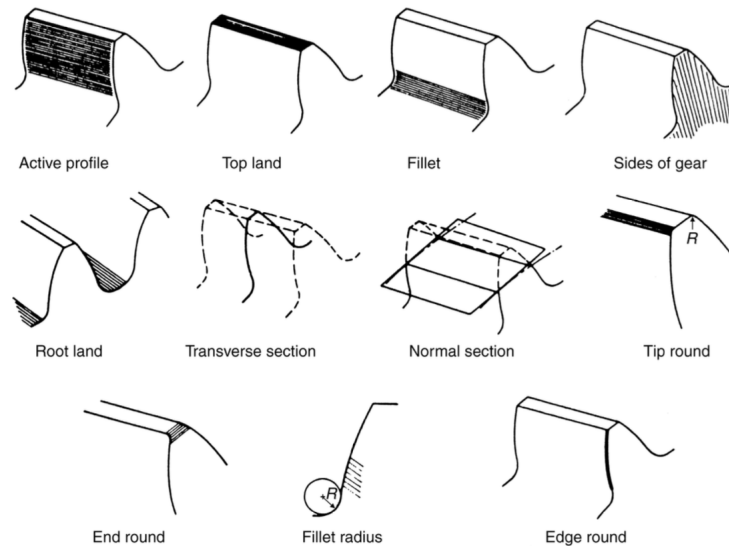


Figure 2.2: Scheme of the principal tooth nomenclature [12]

2.3 Materials Selection

Steel selection depends upon features of the gear working conditions, required hardness, and cost. Low-alloy and medium-carbon steels with 0.4–0.55% carbon content (i.e., AISI 4140, 4340, 1045, 4150, 1552, 5150) are quite commonly used for gear heat treatment by induction.

When discussing induction hardening it is imperative to mention the importance of having favorable metal conditions before gear hardening. Hardness repeatability and the stability of the hardness pattern are grossly affected by the consistency of the microstructure before heat treatment (referred to as microstructure of a green gear) and the steel's chemical composition [13]–[14].

Favorable initial microstructure, including a homogeneous fine-grained quenched and tempered martensitic structure with hardness of 30–34 HRC leads to fast and consistent metal response to heat treating with the smallest shape and size distortion and a minimum amount of grain growth. This type of initial microstructure results in higher hardness and deeper hardened case depth compared to the ferritic-pearlitic initial microstructure. If the initial microstructure of the gear has a significant amount of coarse pearlite and most

importantly coarse ferrites or clusters of ferrites, then these microstructures cannot be considered favorable because gears with such structures will require longer austenization time and higher austenizing temperatures to make sure that diffusion-type processes are completed and homogeneous austenite is obtained.

In opposition to quenched and tempered prior microstructure, steels with large carbides (i.e., spheroidized microstructures) have poor response to induction hardening and also require in the need for prolonged heating and higher temperatures for austenization. Longer heat time leads to grain growth, the appearance of coarse martensite, data scatter, an extended transition zone, and essential gear shape distortion. Coarse martensite has a negative effect on tooth toughness and creates favorable conditions for crack development.

The surface condition of the gear is another factor that can have a distinct effect on gear heat-treating practice. Voids, microcracks, notches, and other surface and subsurface discontinuities as well as other stress concentrators can initiate crack development during induction hardening when the metal goes through the expansion-contraction cycle; thermal gradients and stresses can reach critical values and open notches and microcracks. Conversely, a homogeneous metal structure with a smooth surface free of voids, cracks, notches, and the like improves the heat-treating conditions, and positively affects important gear characteristics such as bending fatigue strength, endurance limit, gear durability, and gear life.

Medium and high frequencies have a tendency to overheat sharp corners; therefore, gear teeth should be generously chamfered, if possible, for optimum results in the heating process. Because gears provide transmission of motion and force, they belong to a group of the most geometrically accurate power transmission components. A gear's geometrical accuracy and ability to provide a required fit to its mate greatly affect gear performance characteristics. Typical required gear tolerances are measured in microns, therefore the ability to control such undesirable phenomena as gear warpage, ovality, conicality, out-of-flatness, tooth crowning, bending, growth, shrinkage, and the like plays a dominant role in providing quality gears.

2.4 Manufacturing Processes of Gears

Gear manufacturing is one of the most complicated of the metal cutting processes. Metal gears, for industrial sectors are commonly produced by metal removal processes, which can be grouped into two general categories [12]

Rough machining (or gear cutting) operations

The methods of gear cutting can be divided broadly into form cutting and generating processes. The tooth profile in the former is obtained by using a form cutting tool. This may be a multiple-point cutter used in a milling machine or a broaching machine, or a single-point tool for use in a shaper. A variation of the form cutter method is based on transferring the profile from a templet. This method is used in the shaperlike cutting of large tooth forms and in cutting bevel gear teeth on a bevel gear planer. In all these

processes, the workpiece is held stationary until a tooth is finished; the piece is then indexed for successive teeth.

In the generating process, the tooth profile is obtained by a tool that simulates one or more teeth of an imaginary generating gear. A relative rolling motion of the tool with the workpiece generates the tooth surface. This method is used in the hobbing, shaping, and milling processes for manufacturing spur and helical gears and in the face milling and face hobbing of bevel gears.

A wide variety of machines are used to cut gear teeth. First, the cutting tool can be threaded and gashed. If so, it is a hob, and the method of cutting is called hobbing. Second, when the cutting tool is shaped like a pinion or a section of a rack, it will be used in a cutting method called shaping. Third, in the milling process, the cutting tool is a toothed disk with a gear tooth contour ground into the sides of the teeth. The fourth general method uses a tool (or a series of tools) that wraps around the gear and cuts all teeth at the same time. Methods of this type are termed broaching, punching, or shear cutting.

Finishing (or high precision machining) operations

Mainly consists on shaving, grinding, honing and lapping and are applied to gears in order to improve the accuracy and surface finish of previously prepared teeth. There are several methods available to improve the quality of gears, following the standard rough machining operation. Gear shaving is a chip forming finishing operation that removes small amounts of metal from the working surfaces of gear teeth. Its purpose is to correct errors in index, helix angle, tooth profile, and eccentricity. Shaving is carried out prior to gear heat treatment. Grinding is a technique of finish machining utilizing a rotating abrasive wheel. Grinding is an effective means of finishing heat-treated high-hardness steels (40 HRC and above). Gear honing is a particularly effective method of removing nicks and burrs from the active profiles of the teeth after heat treatment. Lapping is a low-speed low-pressure abrading operation used to refine the tooth surface and to reduce noise levels in gear sets. The mating gear and pinion are run together under a controlled light load while a mixture of abrasive compound and suitable vehicle are pumped onto the pair. As with grinding and honing, lapping is also carried out after gear heat treatment.

2.5 Basics of Induction Heating

Induction heating is one of the most used processes in order to heat up electrically conductive materials. The first induction heat treating processes were developed in the early 1930s, nearly simultaneously in the United States and in Russia, for hardening of crankshaft pins and journals [41]. Nowadays, this technology is extensively used in many sectors such as induction hardening, hot working of metals (rolling, forging and stamping), welding, brazing or induction melting. The induction heating has several specific features that give many advantages respect other methods:

- Very fast heating rates without surface overheating due to high specific power densities in combination with internal power sources

- High production rates and small footprints of installations due to short heating times
- Contactless energy transfer without surface damage
- Possibility to heat parts in any atmosphere and vacuum
- High selectivity of heating in depth and surface
- Very good controllability and repeatability of the process
- Higher quality of the product (no or very low surface oxidation or decarburization, higher mechanical properties, favorable distribution of residual stresses, low distortions)

In contrast to carburizing and nitriding, induction hardening does not require heating the whole gear. With induction the heating can be localized only to the areas where metallurgical changes are desired (i.e. flank, root, and gear tip can be selectively hardened) and the heating effect on adjacent areas is minimum. Depending upon the application a tooth hardness typically ranges from 42 to 60 HRC.

One of the goals of induction gear hardening is to provide a fine-grained fully martensitic layer on specific areas of the gear to increase hardness and wear resistance while allowing the remainder of the part to be unaffected by the process. The increase in hardness improves contact fatigue strength as well. A combination of increased hardness, wear resistance, and the ability to provide a fine martensite structure, often allows the substitution of inexpensive medium- or high-carbon steel or low-alloyed steel for more expensive highly alloyed steels [16]-[17]-[15]. Another goal of induction gear hardening deals with the ability to provide significant compressive residual stresses at the surface. This is an important feature, since it helps to inhibit crack development as well as resist tensile bending fatigue.

2.5.1 Physical Description of the Phenomenon

In this process, a time-varying magnetic field is produced by a time-varying current that flows in an induction coil. When a workpiece is placed into the coil an electromotive force is induced, according to the Farady's law:

$$e = -N \frac{d\Phi}{dt} \quad (2.1)$$

where e is the induced voltage (V), N the number of turns and $d\Phi/dt$ is the rate of change of magnetic flux (Wb/s).

At any moment the direction of the induced current in the workpiece is approximately opposite to that in the inductor coil, and in general its flow pattern will describe a kind of "shadow image" of the coil conductors. The induced currents also generate their own magnetic fields, which are in opposition to the field generated by the coil and thereby prevent the field from penetrating to the center of the object.

The currents induced into the workpiece heat up the material by Joule effect. To this primary heating mechanism a second one occurs in ferromagnetic materials when energy is dissipated during the reversal of magnetic domains. However, this effect is usually ignored in induction heating, because its contribution is much smaller than Joule losses.

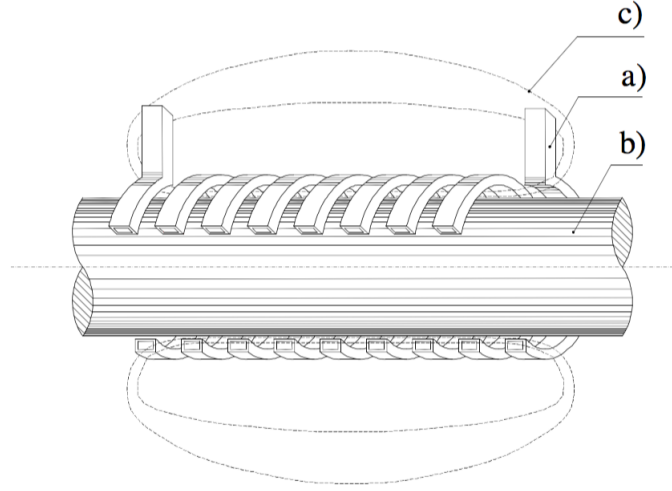


Figure 2.3: Induction Heating set-up. a) coil b) workpiece c) equiflux lines [18]

Skin Effect

As one may know from the basis of electricity, when a direct current flows through a conductor that stands alone (bus-bar or cable), the current distribution within the conductor's cross section is uniform. However, when an alternating current flows through the same conductor, the current distribution is not uniform, but the maximum value of the current density is located on the surface of the conductor, and the current density decreases exponentially towards the center. This phenomenon is called *skin effect*. The skin effect is of great practical importance in electrical engineering, since the 86% (approximately) of power density distributions are located in a specific superficial layer called *penetration depth* δ . The penetration depth is defined as:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_r\mu_0}} = \sqrt{\frac{\rho}{\pi\mu_r\mu_0 f}} \quad (2.2)$$

where ρ is the resistivity of the material ($\Omega \cdot m$), μ_0 and μ_r the vacuum magnetic permeability ($\mu_0 = 4\pi \cdot 10^{-7} H/m$) and the relative magnetic permeability of the material, while ω is the angular frequency of the current (s^{-1}) and f the frequency (Hz). Considering a current in a large planar object (with a thickness much bigger then the penetration depth and not affected by end effect), distribution of current and power densities from the surface can be easily evaluated as:

$$J(x) = J_0 \exp(-x/\delta) \quad (2.3)$$

$$W(x) = W_0 \exp(-x/\delta) \quad (2.4)$$

From what has been said since this point, it is easily deducible that the higher the frequency of the current the smaller will be the skin depth.

It is important to note how the penetration depth is of primary importance in the induction heating, especially when applied to ferromagnetic materials. In this latter case the penetration depth is not constant during the heating process but tends to vary for the

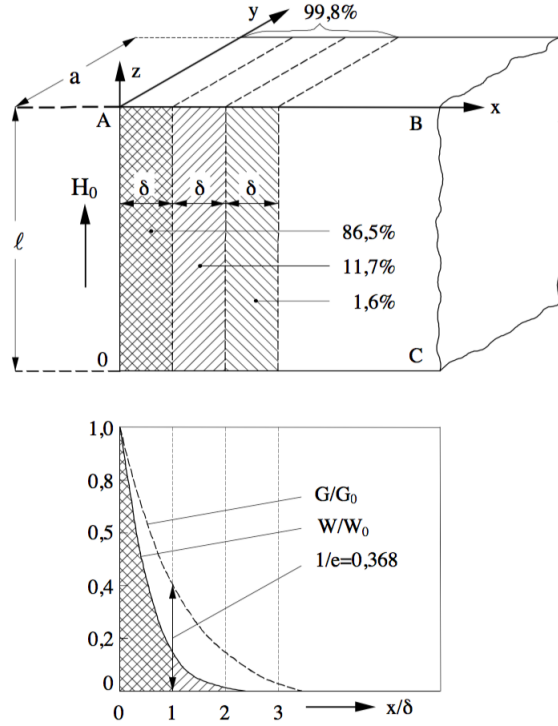


Figure 2.4: Distribution of current and power densities in a undefined half-plane [18]

effect of temperature and magnetic field on the electromagnetic properties of the material. In particular the resistivity tends to increase with the temperature and the global behavior of the material can be approximated with the general rule:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \quad (2.5)$$

where ρ_0 is the material resistivity at 0°C and α an empirical parameter fitted by experimental data.

A little more complicated is the discussion regarding the magnetic permeability which is affected by both the temperature and the magnetic field strength. The material shows a demagnetization effect when its temperature overcome the Curie temperature. The magnetization curve describes the nonlinear relationship between magnetic flux density $B(T)$ and magnetic field intensity $H(A/m)$. The relative magnetic permeability of the steel can be approximated as a constant value until the knee of the B-H curve; after this point the material shows the typical saturation effect and the $\mu_r \rightarrow 1$ when $H \rightarrow \infty$. The composition of these two effects leads to a very complicated behavior of the magnetic permeability of the material during the heating process.

Proximity effect

Previously, when the skin depth has been discussed, the current density in a stand-alone conductor has been considered. In most practical applications this is not the case. Most often there are many other conductors in close proximity. These conductors have their own magnetic field, which interact with nearby fields, and as a result, the current and power density distributions will be distorted.

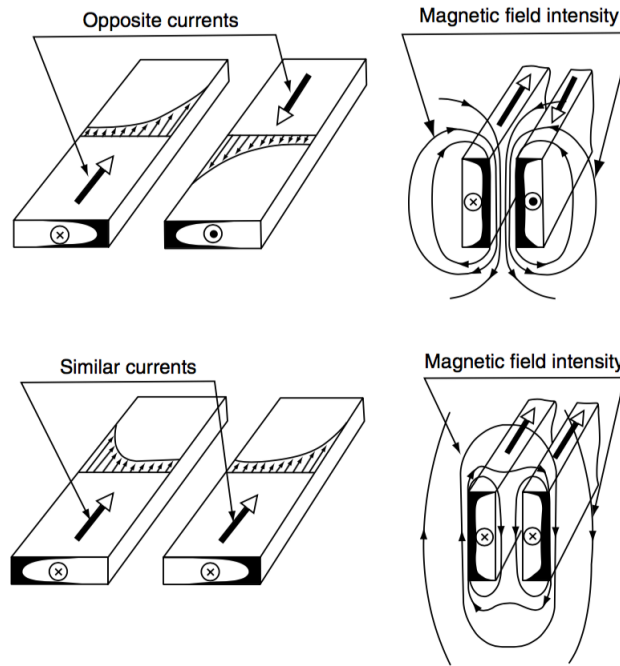


Figure 2.5: Proximity effect cases [15]

Figure 2.5 shows the skin effect and magnetic field distribution in a conductor that stand alone. When another current carrying conductor is placed near the first one, the current in both conductors will be redistributed. If the currents flowing in the conductors have opposite directions, then both currents will be concentrated in the areas facing each other. However, if the currents have the same direction, then these currents will be concentrated on opposite sides of the cross sections.

Ring Effect

Up to now the current density distribution in a straight conductor has been discussed. If that current-carrying bar is bent to shape it into a ring, then the current density will be redistributed. Magnetic flux lines will be concentrated inside the ring, and therefore the density of the magnetic field will be higher inside the ring (Figure 2.6).

Outside the ring, the magnetic flux lines will be disseminated. As a result, most of the current will flow within the thin inside surface layer of the ring. The ring effect plays a negative role in the induction heating of internal surfaces (so called I.D. heating), where

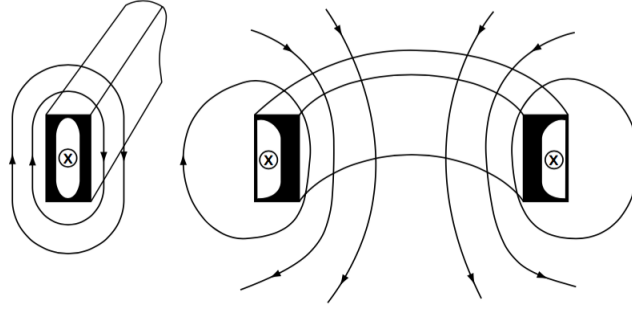


Figure 2.6: Ring effect [15]

the induction coil is located inside the workpiece. In this case, this effect leads to coil current concentration on the inside diameter of the coil. This makes the coil-workpiece coupling poor and, therefore, decreases the coil efficiency.

2.5.2 Induction Heat Treating Equipment

An induction heating system typically consists of a power supply, a workstation, an inductor (heating) coil, controls, and work-piece handling units. When a steel or cast iron is being hardened, the system may also include a quenching system, depending on the hardenability of the particular grade. Some annealing practices may also be enhanced by accelerated cooling. The basic architecture of an induction heating system includes a workstation (or heat station), which contains load matching components such as output transformers and capacitors, plus high-frequency contactors, protective devices, cooling-water manifolds, and quench valves. The principal function of the workstation is to provide the proper electrical impedance match between the output of the power supply and the inductor (induction heating coil) for optimum power transfer into the heated load. The coil is normally mounted on the front of and close to the workstation. Workstation components may sometimes be installed in the power supply enclosure, as is often the case in small solid-state, medium-frequency heaters, and in many of the vacuum-tube oscillator units.

Besides the induction coil and workpiece, the power supply is probably the most important component of an overall induction heating system. The function of the power supply is to accept power from a line at (usually 50 or 60 Hz) and to deliver AC electrical energy through the load matching circuit to the inductor coil circuit at a useful voltage and frequency. Over the years, three basic frequency ranges have evolved for the classification of AC induction heating systems:

- low-frequency systems (50 or 60 Hz line-frequency systems);
- medium-frequency systems (converters in the range 5-30 kHz);
- high-frequency systems (100-450 kHz).

Medium- and high-frequency power supplies suggest the need for conversion of line frequency power, preferably three phase, to a single phase of higher frequency. A very basic block

diagram that applies to nearly all induction heating power supplies is shown in Figure 2.7.

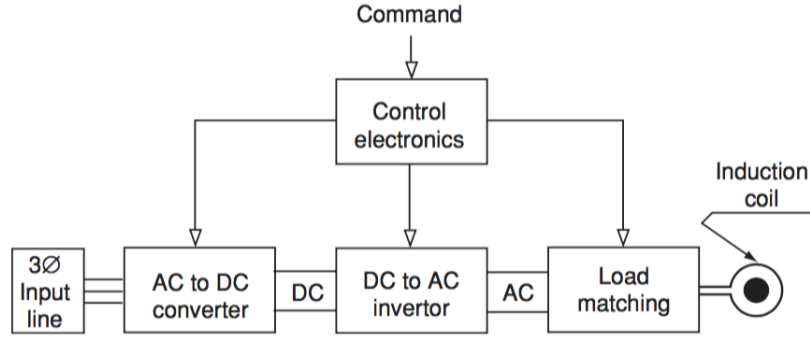


Figure 2.7: Scheme of an induction heating system [17]

The first block represents the AC to DC converter or rectifier. This section may provide a fixed DC voltage, a variable DC voltage, or a variable DC current. The second block represents the inverter or oscillator section, which switches the DC to produce a single-phase AC output. The third block represents the load-matching components, which adapt the output of the inverter to the level required by the induction coil. The control section compares the output of the system to the command signal and adjusts the DC output of the converter, the phase or frequency of the inverter, or both to provide the desired heating. The choice of a power source depends primarily on the desired frequency, although there is often considerable overlap of application capability for any given unit in its frequency range. The frequency required for efficient induction heating is determined by the material properties (i.e., resistivity and relative magnetic permeability) and the work-piece cross-sectional size and shape. In a great variety of cases, a suitable frequency may also be impacted by the application requirements such as case depth in surface hardening of steels or the allowable temperature gradient for through hardening (see Figure 2.33).

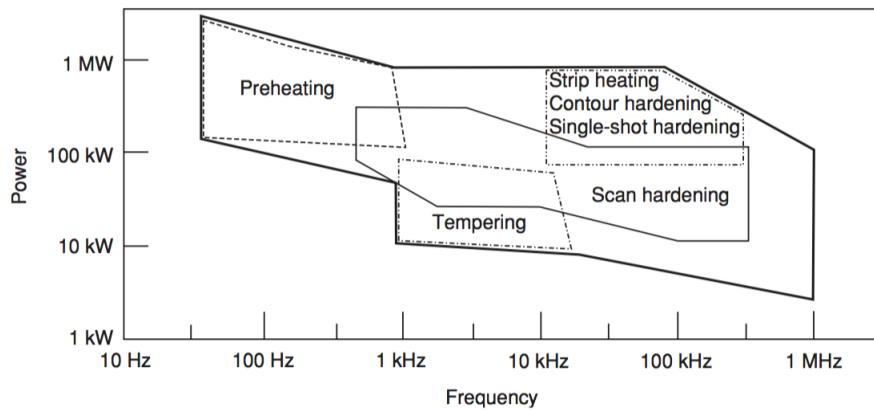


Figure 2.8: Typical induction heat treatment applications [17]

In general, however, equipment cost per kilowatt increases with frequency, so a power

supply of the lowest possible frequency that will accomplish the job efficiently is usually the best place to start when considering equipment. Power requirements for induction heating are also dependent on the specific application. For through heating applications, the power needed is generally based upon the amount of material processed per unit of time, the peak average temperature, and the material's heat capacity at this temperature. A power determination for localized heating and other operations, such as surface hardening of steel, is not as simple because of the effects of heat conduction to the adjacent metal and/or unhardened core. Medium-frequency power supplies using solid-state frequency conversions make up the vast majority of induction heating and melting power supplies in use today. The inverter circuits that convert DC to AC use solid-state switching devices such as thyristors and transistors. For high power and lower frequencies, large thyristors are commonly used. These systems, which utilize high-power silicon-controlled rectifiers (SCRs) having rapid turn-on/turn-off times, have largely replaced motor-generator sets for medium frequency conversion. The basic advantages of solid-state power supplies are their improved efficiency, low initial cost and maintenance, and availability in a multitude of sizes and frequencies. With the exception of an AC-AC solid-state converter, which does not have wide usage in heat-treating applications, all of the commercial medium-frequency solid-state inverters for induction heating use a two-step conversion in which 50 Hz AC is changed to DC and the DC changed back to AC at the desired frequency. Diodes or thyristors first rectify three-phase line frequency to produce direct current, and solid-state logic then converts DC into AC by either a swept-frequency or load-resonant inverter. In both types of solid-state inverters, the system frequency is tuned to the load, whereas in motor-generator circuits the load is tuned to the fixed frequency of the generator. This greatly simplifies load matching when the electrical characteristics of the load change during a heating cycle.

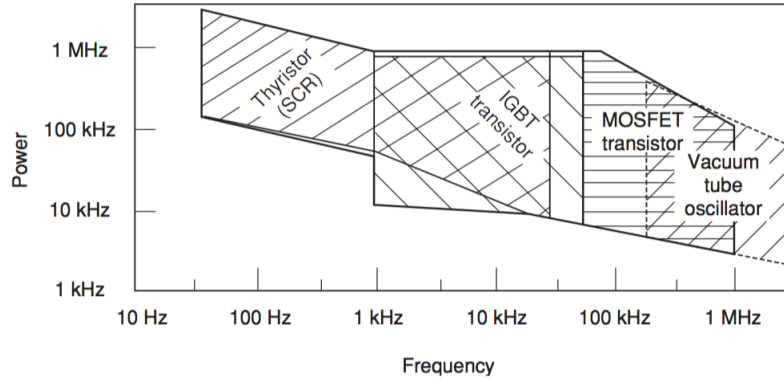


Figure 2.9: Modern inverter types for induction heat treatment [17]

For frequencies above 150 kHz, frequency conversion can be performed by either vacuum-tube or transistor systems (see Figure 2.9). Vacuum-tube systems have been widely used in the past, thanks to their flexibility and capability of attaining frequencies up to about 2 MHz for induction heating. However, the low efficiency (50-70% depending on vacuum tube type), limited life time (4000 - 6000 operating hours) and high voltages required

(10-12 kV), made the vacuum tube a rather inefficient equipment [19]. Recently, transistor systems are more and more used because of their ability to be turned on and off very fast with low-switching losses. Presently, transistor systems like the so-called Insulated-Gate Bipolar Transistor (IGBT), produce RF heating energy in the frequency span 20 to 100 kHz and a wide range of power outputs. Metal-Oxide Semiconductor Field-Effect Transistor (MOSFET) allow to extend the frequency range up to 500 kHz with acceptable power levels (>100 kW). The major difference of the two switches can be found within their switching capability and robustness. The MOSFET is capable of reaching higher frequencies but is quite susceptible to high voltages. On the contrary, the IGBT is rather slower but very robust.

2.6 Induction Hardening of Gears

One of the goals of induction gear hardening is to provide a fine-grained fully martensitic layer on specific areas of the gear to increase hardness and wear resistance, while allowing the remainder of the part to be unaffected by the process. The increase in hardness also improves contact fatigue strength.

The first step in designing an induction gear heat treatment is to specify the required surface hardness and hardness profile. Figure 2.10 shows typical locations of gear profile measurements. In some cases, depending upon the type of gear and its application, some operators create specific procedures for gear profile measurements. Insufficient hardness, as well as an interrupted (broken) hardness profile at tooth contact areas, will shorten gear life, due to poor load-carrying capacity, premature wear, tooth bending fatigue, rolling contact fatigue, pitting and spalling. They can even result in some plastic deformation of the teeth. A through-hardened gear tooth, with a hardness exceeding 60HRC, is too brittle and will often experience a premature brittle fracture. Hardened case depth should be adequate (not too large and not too small) to provide the required gear tooth properties [13].

There is a common misconception that a uniform contour profile is always the best pattern for all gear hardening applications. It is not. In many cases, a certain hardness gradient profile can provide a gear with better performance. Operating load condition (whether there are occasional, intermittent, or continuous loads) has a pronounced effect on the type of gear, tooth geometry and hardness profile. Loads lasting up to 30 minutes per day are considered occasional loads. Loads lasting several minutes per hour are considered to be intermittent-type loads. Continuous loads last from 10 to 24 hours [22]-[23].

Let's briefly evaluate a variety of hardening patterns (Figure 2.11) and their effect on gear load-carrying capacity and life [20]-[21]

- Pattern A is a flank hardening pattern that has been used since the late 1940s for hardening large gears (with teeth modulus of eight and larger). The hardened pattern occupies the tooth flank area and ends prior to the tooth fillet. This provides the required wear resistance, but the typical failure mode of gears with this type of pattern is bending fatigue caused by repeated loading. A crack typically initiates in the tooth root/fillet area. In the hardened/non-hardened transition region, the

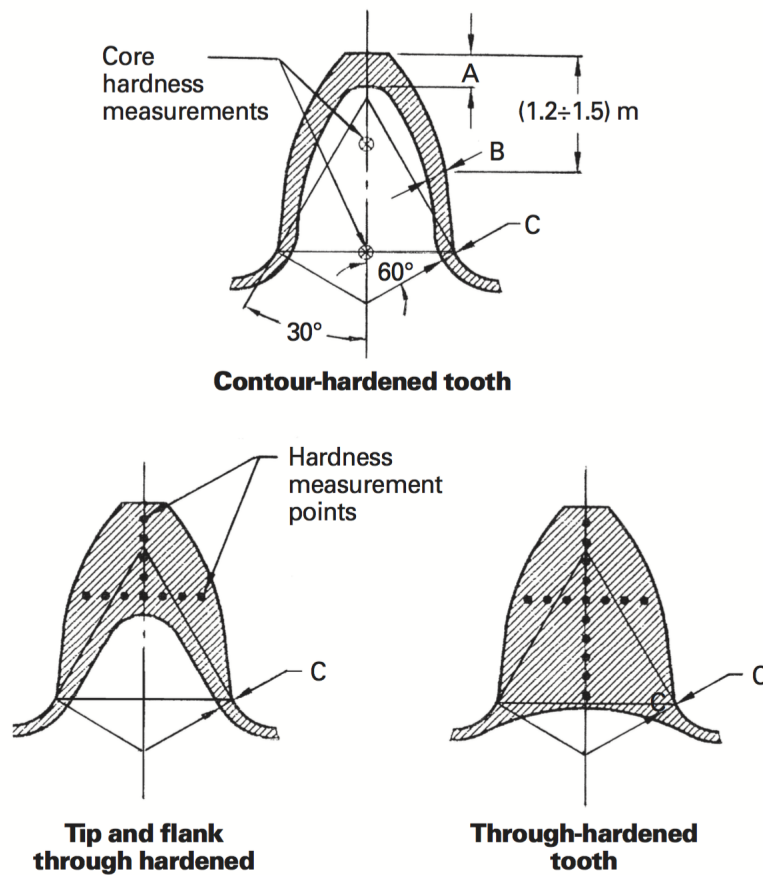


Figure 2.10: Typical locations of gear profile measurements [13]

residual stresses change from compressive in the hardened area to tensile in the non-hardened area. The maximum tensile residual stresses are located just below the end of the hardened pattern. A combination of applied tensile stresses with tensile residual stresses creates a favourable condition for early crack development in the root/fillet area, particularly for heavily-loaded gears. Therefore when pattern A is imparted, mechanical hardening (i.e., roll or ball hardening) is typically required to harden the fillet area and develop useful compressive residual stresses there that will resist bending fatigue. When mechanical hardening is not employed, use of a pattern that hardens the root areas as well, such as that pictured in pattern I, is typically recommended.

- Pattern B is a flank and tooth hardening pattern. This pattern has a shortcoming similar to the previous one – a poor load-carrying capacity – yet can be used in cases where wear resistance is of prime concern. Patterns E, F and G provide better results when a combination of wear, tear and bending fatigue resistance is required.
- Pattern C is a tooth-tip hardening pattern. In this case, the gear has minimum

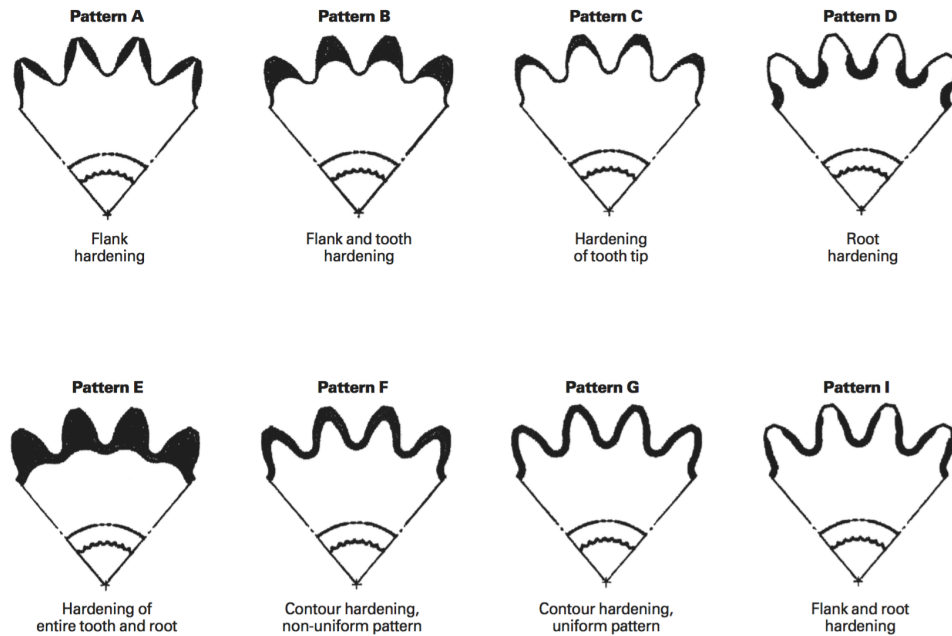


Figure 2.11: Induction hardening patterns of gears. [13]

shape distortion. In addition, the application of gears with this pattern is extremely limited because the two most important tooth areas (flank and root) are not hardened. Indeed, due to unfavourable residual stress distribution, the bending fatigue strength of a gear with this pattern, as well as patterns A and B, can even be 25% lower than that of the gear prior to hardening (“green” gear) [24]. In most cases, patterns F and G would be better choices.

- Pattern D is a root hardening pattern. The maximum bending stresses are located in the tooth fillet area; therefore, this pattern provides good fillet/root strengthening, with a combination of hard surface, sufficient case depth and compressive stresses. The root is essentially reinforced; thus the maximum tensile residual stresses are shifted far away from the root/fillet surface to a depth where tensile residual stresses will not complement applied tensile stresses during service and create bending fatigue fracture. However, application of this pattern is quite limited. Since the tooth flank is not hardened, it provides poor wear resistance that may result in removal or displacement of metal particles from the gear surface. Theoretically, it is possible to imagine the necessity of using this pattern as well as the previous one; however, it is more practical to use another, such as pattern I.
- Pattern E is one of the most common induction hardening patterns, particularly for small-size gears and sprockets. Because the body of the tooth is through-hardened, some quench cracking may occur. In addition, there is danger of brittle fracture in

gears with through-hardened teeth, particularly those subjected to shock loads. The core should be able to withstand impact loads and prevent plastic deformation of the gear teeth. It should also have some ductility. This is why low-temperature tempering is often applied. The core strength is measured by its hardness. Low-temperature tempering lowers the final hardness down to 52 to 58HRC and provides some ductility and toughness to the gear teeth. This pattern offers good resistance to wear and pitting.

- Patterns F and G are popular patterns for medium-size gears in many applications. According to pattern F, the case depth in the tooth root area is typically 30 to 40% of that at the tooth tip. A slightly larger hardened depth at the tooth pitch line, compared with at the root, is beneficial in preventing spalling and pitting. It is very important to harden the entire gear perimeter, including the flank and root area. An uninterrupted hardened pattern on all contact areas of the tooth indicates good wear properties of the gear. It also ensures the existence of uninterrupted distribution of desirable compressive stresses at the gear surface. Because gear teeth are not hardened through, a relatively ductile tooth core (30 to 44HRC) and a hard surface (56 to 62HRC) provide a good combination of important gear properties, such as exceptional wear resistance, toughness and bending strength, and promote superior gear durability.
- Pattern I is one of the most popular choices for induction hardening of large gears and pinions (i.e., gears with OD of 300mm and larger) with coarse teeth (modules greater than eight). This pattern provides an exceptional combination of fatigue and wear strength, as well as resistance to shock loading and scuffing (severe adhesion wear where metal particles transfer from one tooth to another). This is very important for heavily-loaded gears and pinions experiencing severe shock loads. It is recommended that, for these applications, surface hardness should not be very high (typically in the range of 55 to 59HRC). If surface hardness exceeds 61 to 62HRC, the gear might be too brittle and could experience some tooth bending failures.

2.6.1 Induction Hardening Methods

Induction hardening can meet production needs in processing a range of gears designs, from small sprockets to very large gears used in transmissions. Two main concepts are to harden only the gear tooth or part of it (only where the tooth makes contact, for example) and to harden the entire gear circumference including the tip and root of each tooth. Other considerations such as gear size, module, production rate, power availability influence the choice of the process [25].

Tooth-by-Tooth Hardening

This technique is commonly used to harden flanks and/roots; for example, large sprockets such as those found in off-road equipment, or ring gears found in transmissions. In the process, the coil comes onto the tooth to harden root and flanks, or flanks and tip. Possible approaches are single shot and scanning operations. Some processing, such as the so-called

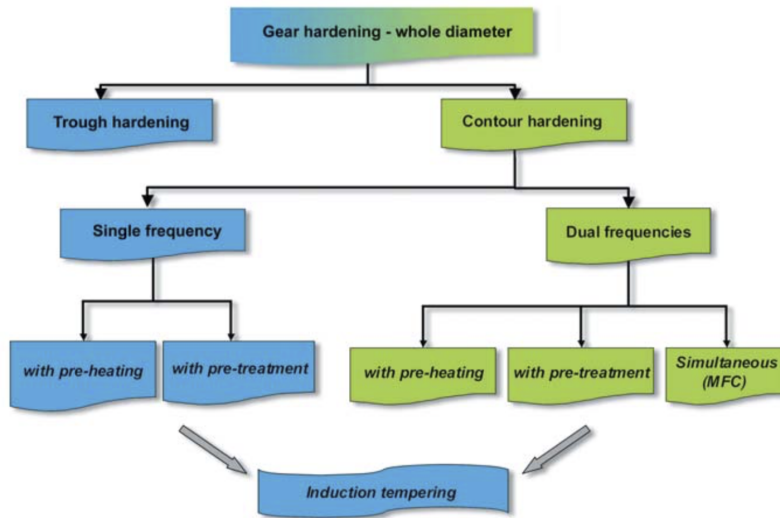


Figure 2.12: Contour induction hardening processes [25]

NATCO approach, performs the hardening operation under water, which minimizes the distortion resulting from the transformation. A current method used for large gears (e.g., slewing rings) uses more measuring devices, which make it possible to guarantee a perfect location of the coil vs. the part even when part is not well centered. This also provides the necessary process repeatability. The air gap between coil and tooth must be constant during the heating process, but more important is the repeatability of positioning from one tooth to another. To achieve regular control using measuring probe, a CNC axis gives the best result. Other solutions used to ensure a constant gap between coil and tooth flanks and/or root include mechanical when using a contact probe in neighboring teeth and ground-fault detection in connection with the control system (CNC or PLC) giving a set point when approaching the root of the tooth.

Gear Spin Hardening

In spin hardening, the entire gear is brought up to the austenitizing temperature via an encircling coil and subsequently quenched, which allows either through hardening of the gear down to the tooth root (similar to case hardening) or hardening the outer surface at a uniform or irregular distance from the face. Figure 2.12 shows different methods for contour hardening corresponding to different heating techniques for inductive gear spin hardening.

Through Hardening on Gear Teeth

For gears that are predominantly subject to wear (e.g., sprockets), the tooth is through hardened using a relatively low specific power [26]. If the frequency is too low, there is the risk that beyond the Curie temperature (point where the material changes from magnetic to nonmagnetic) the induced eddy current largely flows only in the root circle, and the

tem- perature of the tooth lags behind. Quenching for through hardening can be either submersion or spraying depending on the material [27]. De- pending on factors from the module to the frequency, it is recommended to delay the quenching sequence to achieve a uniform temperature between the tooth and the root circle. Tempering after through hardening is essential for crack prevention.

Gear Hardening on Tooth Perimeter (Contour or No Contour)

When hardening at an irregular distance from the tooth face, or (ideally) at a uniform distance from the tooth face, one can choose between the single frequency concept (SFC), in which an single frequency power supply is used, and the dual fre- quency method (including multiple frequency concept, or MFC) using two frequency levels (either simultaneously or not).

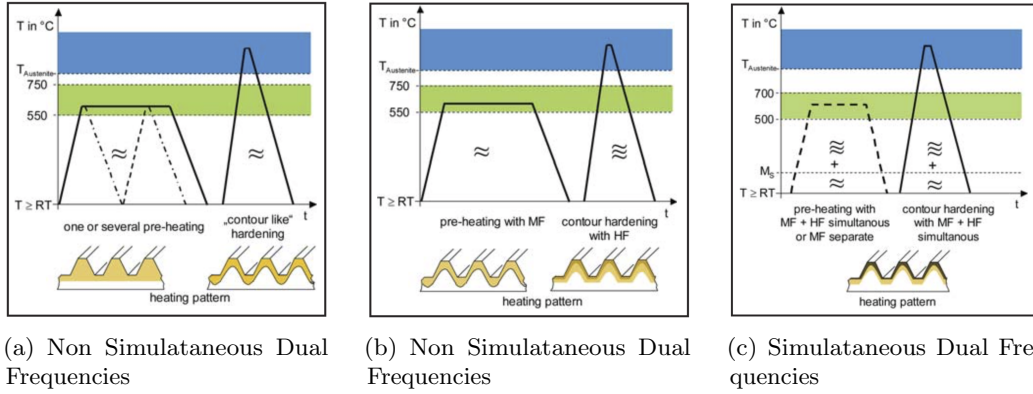


Figure 2.13: Induction hardening processes [25]

Single Frequency Concept

In the process, the inductor is fed from a single power supply using a frequency selected according to the module. Heating (austenitizing) is done either via single shot in one step or with preheating using reduced power to reach a temperature between 550 to 750°C and subsequent final heating using a higher specific power to the hardening temperature. Preheating can also consist of several pulsed cycles. The purpose of preheating is to reach an adapted high austenitizing temperature in the root circle during final heating, without overheating the tooth tip. The schematic principle can be seen in Figure 2.13a. To achieve hardening profiles at an irregular distance from the tooth face using a single frequency, short heating times combined with a high specific power is usually required. Both parameters (time and power) depend on the hardened gear module. It is not always certain that the user has the optimum power supply with respect to available frequency, or whether it can be correspondingly adjusted. In this case, better results can be achieved for contour hardening using a generator in which the working frequency is higher rather than lower for the given module. If the frequency is higher than in the ideal case, it is possible to heat the root

circle by means of thermo-conduction during the preheating phase to subsequently achieve hardening results at an irregular distance from the face with a final heating impulse using a high specific power. On the other hand, if the generator frequency is too low for a given module, through hardening of the tooth or an inadequate austenitizing temperature of the tooth tip is unavoidable for physical reasons. In addition, if the initial microstructure is not suitable for gear hardening, further improvement in the hardening result may be achieved via induction initial hardening and tempering operations before the actual hardening itself. For this, heating of the entire gear area to an austenitizing temperature corresponding to the material is carried out. Cooling by self-quenching (mass quenching) is followed by a short inductive tempering of the entire gear. The actual hardening zone at an irregular distance from the face is then generated by a short final heating.

Dual-Frequency Methods

Dual-frequency hardening using separate frequencies allows achieving hardening profiles similar to those achieved using case hardening. Two different frequencies, one after the other, are applied to the gear. The tooth part is preheated (depending on the material) using a low frequency and limited power to a temperature between 550 and 750°C. The frequency should be such that the preheating preferably takes place in the root circle region. After a short delay, austenitizing using a higher frequency and higher power density is then provided. Heating times are measured in tenths of seconds or seconds (depending on the module and gear thickness/length) in this final heating phase, and requires then a very accurate, fast monitoring system [ref Marquis 5]. Figure 2.13b Due to the requirements of material hardenability in the contour hardening process, prequenching and tempering by induction of the tooth area prior final hardening may be considered as an alternative process (just as in the case of the single frequency contour hardening process).

Simultaneous Frequency Hardening

In this method, a lower medium frequency (MF) and a higher (HF) frequency are fed in the inductor simultaneously. This is accomplished using two inverters and feeding a single coil using proper filtering devices, avoiding rejection of frequencies and perturbations within the electrical circuits. Such system envisioned in the mid 1920s was patented by EFD Induction in 1992 (ELVA patent) using the advantages of transistor power supplies. Figure 13 shows a gear being processed using MFC. Preheating, which is necessary in the normal dual frequency method using separate frequencies, is not used [28]. Dual-frequency contour hardening using simultaneous frequencies is done by heating in the region of the root circle by means of the MF, and in the tooth tip region using the HF (Figure 2.13c). However, the short heating times required in this process place higher demands on the generator and machine engineering. The MFC process is suitable when the use of simultaneous frequencies is necessary, such as in the bevel gears having small to medium size module. It is also very suitable when material and final metallurgical structure required by the end-user allows using a short heating time.

Chapter 3

Electromagnetic Modeling

3.1 Introduction

The electrical and magnetic fields are all governed by a set of equations, defining the curl and the divergence of the field quantities and known as *Maxwell's equations*, together with equations defining the continuity of the field quantities. Depending on particular physical conditions, the nature of the field can be classified into three types: In the first, when the fields are wholly invariant with time, or when the rate of change with the time is sufficiently low as to be treated as invariant for practical purposes they are described respectively as *static* or '*quasi*'-static. In the second, when the rates of change are low, induced or '*eddy*' currents exist; and in the third, rates of change are sufficiently high that the fields are influenced by displacement current and wave behavior. In numerical simulation of induction heat treatments, our attention can be focused on the second case in which Maxwell's equations can be reduced to diffusion equation. For steady-state harmonic conditions this yields an implicit (phasor) solution for the field variation. The formulations here presented have been taken from the following references: [29]-[30]-[31]-[32]-[33]-[34].

3.2 Field Theory

Maxwell's equations are a set of fundamental equations that govern all macroscopic electromagnetic phenomena. The equations can be written in both differential and integral forms.

3.2.1 The General Integral Form

For general time-varying fields, Maxwell's equations in integral form are given by:

$$\oint_C \mathbf{E} \, dl = -\frac{d}{dt} \iint_S \mathbf{B} \, ds \quad (\text{Faraday's Law}) \quad (3.1)$$

$$\oint_C \mathbf{H} \, dl = \frac{d}{dt} \iint_S \mathbf{D} \, ds + \iint_S \mathbf{J} \, ds \quad (\text{Maxwell-Ampère Law}) \quad (3.2)$$

$$\oint_S \mathbf{D} \, ds = \iiint_V \rho \, dv \quad (\text{Gauss's Law}) \quad (3.3)$$

$$\oint_S \mathbf{B} \, ds = 0 \quad (\text{Gauss' Law - magnetic}) \quad (3.4)$$

where $\mathbf{E}$ is the electric field intensity, $\mathbf{D}$ the flux density, $\mathbf{H}$ the magnetic field intensity, $\mathbf{B}$ magnetic flux density, $\mathbf{J}$ is the electric current density and ρ_e is the free electric charge density.

In 3.1 and 3.2, S is an arbitrary open surface bounded by contour C , whereas in 3.3 and 3.4, S is a closed surface enclosing volume V . Another fundamental equation, known as the *equation of continuity*, is given by:

$$\oint_S \mathbf{J} \, ds = -\frac{d}{dt} \iiint_V \rho \, dv \quad (3.5)$$

This equation, which can be derived from 3.2 and 3.3, is the mathematical form of the law of conservation of charge.

Equations 3.1 to 3.5 are valid in all circumstances regardless of the medium and the shape of the integration volume, surface and contour. They can be considered as the fundamental equations governing the behavior of electromagnetic fields.

3.2.2 The General Differential Form

Maxwell's equations in differential form can be derived from 3.1 to 3.5 by using Gauss's and Stokes's theorems. Consider a point in space where all the field quantities and their derivatives are continuous. Application of Gauss's and Stokes's theorems to 3.1 - 3.5 yields:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (\text{Faraday's Law}) \quad (3.6)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \quad (\text{Maxwell-Ampère Law}) \quad (3.7)$$

$$\nabla \cdot \mathbf{D} = \rho_e \quad (\text{Gauss's Law}) \quad (3.8)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{Gauss's Law - magnetic}) \quad (3.9)$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad (\text{equation of continuity}) \quad (3.10)$$

Among the five equations above, only three are independent for the case of time varying fields and thus are called *independent equations*. Either the first three equations, 3.6 to 3.8, or the first two equations 3.6 and 3.7, with 3.10 can be chosen as such independent equations. The other two equations, 3.9 and 3.10 or 3.8 and 3.9, can be derived from the independent equations and thus are called *auxiliary* or *dependent equations*.

3.2.3 Electro- and Magnetostatic Fields

When the field quantities do not vary with time, the field is called static. In this case, 3.6, 3.7 and 3.10 can be written as:

$$\nabla \times \mathbf{E} = 0 \quad (3.11)$$

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (3.12)$$

$$\nabla \cdot \mathbf{J} = 0 \quad (3.13)$$

whereas equations 3.8 and 3.9 remain the same. It is evident that in this case there is no interaction between electric and magnetic fields; therefore, we can have separately either an *electrostatic* case described by equations 3.8 and 3.11 or a *magnetostatic* case described by equations 3.9 and 3.12, with equation 3.13 as natural consequence of equation 3.12.

3.2.4 Time-Harmonic Fields

When field quantities in Maxwell's equations are harmonically oscillating functions with a single frequency, the field is referred to as *time-harmonic*. Using the complex phasor notation, equations 3.6, 3.7 and 3.10 can be written in a simplified form as:

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B} \quad (3.14)$$

$$\nabla \times \mathbf{E} = j\omega\mathbf{D} + \mathbf{J} \quad (3.15)$$

$$\nabla \cdot \mathbf{J} = -j\omega\rho \quad (3.16)$$

where the time convention $e^{j\omega t}$ is used and suppressed and ω is angular frequency. It is evident that in this case, the electric and magnetic fields must exist simultaneously and they interact with each other, and it is also evident that the static case is the limiting case of the harmonic fields as the frequency ω approaches zero.

The use of time-harmonic fields is not as restrictive as it first appears. Using Fourier analysis, any time-varying field can be expressed in terms of time-harmonic components via the Fourier transforms

$$\mathbf{E}(t) = \int_{-\infty}^{\infty} \mathbf{E}(\omega) e^{j\omega t} d\omega \quad (3.17)$$

$$\mathbf{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{E}(t) e^{j\omega t} dt \quad (3.18)$$

Therefore if a time-harmonic field is known for any ω , its counterpart in the time domain can be obtained by evaluating equation 3.17.

3.2.5 Constitutive Relations

The three independent equations among the five Maxwell's equations described above are in *indefinite* form since the number of equations is less than the number of unknowns. Maxwell's equations become *definite* when *constitutive relations* between the fields quantities are specified. The constitutive relations describe the macroscopic properties of the medium being considered. For a simple medium, they are:

$$\mathbf{D} = \epsilon \mathbf{E} \quad (3.19)$$

$$\mathbf{B} = \mu \mathbf{H} \quad (3.20)$$

$$\mathbf{J} = \sigma \mathbf{E} \quad (3.21)$$

where the constitutive parameters ϵ , μ , σ denote, respectively, the permittivity, permeability and electrical conductivity of the medium. These parameters are tensors for anisotropic media and scalars for isotropic media. For inhomogeneous they are functions of the position, whereas for homogenous media they are not.

3.2.6 Quasi-Static Electromagnetic Field Equations

Since this point the general form of Maxwell's equations has been presented, but considering the premises discussed in the introduction the model can be simplified.

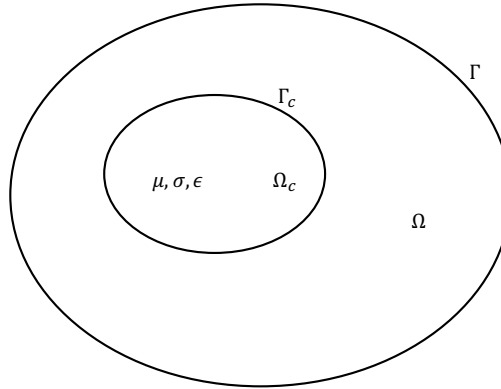


Figure 3.1: Simple model configuration of arbitrary shape

For this purpose an elementary model (shown in figure 3.1) can be introduced, in which a volume of conducting material Ω_c , with magnetic permeability μ , electric permittivity ϵ and electrical conductivity σ , bounded by a surface Γ_c is contained within a volume of free space Ω bounded by a surface Γ which may be extended to infinity.

If the wavelengths of any time-varying fields are large compared with the physical dimensions of the problem, the displacement current term in 3.7 can be neglected compared with the free current density $\mathbf{J}$ and there is no radiation. This regime means that the pre-Maxwell field equations, the so called *quasi-static case* where Ampere's law is a good

approximation, are applicable. In this situation the field equations can be approximated by:

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon} \quad (3.22)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3.23)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3.24)$$

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (3.25)$$

The current density in a conductor moving with relative velocity $\mathbf{v}$ is generated by the Lorentz force and is given by:

$$\mathbf{J} = \sigma\{\mathbf{E} + \mathbf{v} \times \mathbf{B}\} \quad (3.26)$$

which incorporates Ohm's law. The current continuity condition is:

$$\nabla \cdot \mathbf{J} = 0 \quad (3.27)$$

The four basic field vectors must satisfy the following conditions at the interfaces between regions by subscripts 1 and 2 of different material properties:

$$(\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0 \quad (3.28)$$

$$(\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n} = \omega \quad (3.29)$$

$$(\mathbf{H}_2 - \mathbf{H}_1) \times \mathbf{n} = \mathbf{K} \quad (3.30)$$

$$(\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n} = 0 \quad (3.31)$$

where $\mathbf{K}$ and ω are the surface current and charge density respectively. These conditions are referred to as *boundary or interface conditions*. Equation 3.28 express the continuity of magnetic flux across a boundary and equation 3.29 is the equivalent form in terms of electric flux. Equation 3.30 results from Ampere's law and equation 3.31 expresses the continuity of the tangent component of electric field strength.

Magnetic Vector Potential

Since the field vector $\mathbf{B}$ satisfies a zero divergence condition (equation 3.23), it can be expressed in terms of a vector potential $\mathbf{A}$ as follows:

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (3.32)$$

and then, from equations 3.24 and 3.32, it follows that:

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \quad (3.33)$$

Hence, by integration, one obtains:

$$\mathbf{E} = -\left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V\right) = \mathbf{J} \quad (3.34)$$

where V is a scalar potential. Neither $\mathbf{A}$ nor V are completely defined since the gradient of an arbitrary scalar function can be added to $\mathbf{A}$ and the time derivative of the same function can be subtracted from V without affecting the physical quantities $\mathbf{E}$ and $\mathbf{B}$. These changes to $\mathbf{A}$ and V are the so-called gauge transformations, and the uniqueness of the solution is usually ensured by specifying the divergence (or gauge) of $\mathbf{A}$ together with the necessary boundary conditions. Thus in the region Ω_c the field equations in terms of $\mathbf{A}$ and V are as follows:

$$\nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} + \sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right) = \mathbf{J} \quad (3.35)$$

Hence,

$$\nabla \cdot \sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right) = 0 \quad (3.36)$$

and in the global region where $\sigma = 0$ and $\nabla \times \mathbf{H} = \mathbf{J}_s$, equation 3.23 reduces to

$$\nabla \cdot \mu \nabla \phi = 0 \quad (3.37)$$

where ϕ is the reduced magnetic scalar potential with $\mathbf{H} = \mathbf{H}_s - \nabla \phi$ for a source field $\mathbf{H}_s$. If μ is a constant then equation 3.37 is known as Laplace's equation.

At points just inside the conductor, equations 3.29 and 3.31 imply:

$$\mathbf{J}_n = -\sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right) \cdot \mathbf{n} = 0 \quad (3.38)$$

Hence,

$$\frac{\partial \mathbf{A}_n}{\partial t} + \frac{\partial V}{\partial \mathbf{n}} = 0 \quad (3.39)$$

at conductor surfaces, and at interfaces, across which the conductivity changes from σ_1 to σ_2 , equation 3.29 implies that:

$$\sigma_1 \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V_1 \right) \cdot \mathbf{n} = \sigma_2 \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V_2 \right) \cdot \mathbf{n} \quad (3.40)$$

Magnetic Scalar Potential

As has already noted the free space region Ω , shown in figure 3.1 can be modelled using a scalar potential (equation 3.37) and furthermore, static magnetic fields can likewise be modelled in all regions. For example, the magnetic field $\mathbf{H}$ can be partitioned into two

fields namely, the field generated by the prescribed sources $\mathbf{H}_s$ and the field arising from induced magnetism in ferromagnetic materials $\mathbf{H}_m$. Thus,

$$\mathbf{H} = \mathbf{H}_m + \mathbf{H}_s \quad (3.41)$$

and, since from equation 3.25 $\nabla \times \mathbf{H}_m = 0$, it follows that:

$$\mathbf{H} = -\nabla\phi + \mathbf{H}_s \quad (3.42)$$

where ϕ is called the *reduced scalar potential*, and by definition for conductor source regions with current density $\mathbf{J}_s$ the source field is given by

$$\mathbf{H}_s = \frac{1}{4\pi} \int_{\Omega} \mathbf{J}_s \times \nabla \left(\frac{1}{R} \right) d\Omega \quad (3.43)$$

where $R = |r' - r|$ is the distance from the source point r' to the field point r , equation 3.43 is known as the Biot-Savart law.

In many cases, this can be integrated to give an analytical expression for $\mathbf{H}_s$;

3.3 Finite Element Modelization

The finite element method is a numerical tool for determining approximate solutions to a large class of engineering problems. Among the various numerical methods that have evolved over the years, the most commonly used techniques are the finite difference, finite volume and finite element methods.

The finite difference is a well-established and conceptually simple method that requires a point-wise approximation to the governing equations. The model, formed by writing the difference equations for an array of grid points, can be improved by increasing the number of points. Although many heat transfer problems may be solved using the finite difference methods, as soon as irregular geometries or an unusual specification of boundary conditions are encountered, the finite difference technique becomes difficult to use.

The finite volume method is a further refined version of the finite difference method and has become popular in computational fluid dynamics. The vertex-centred finite volume technique is very similar to the linear finite element method.

The finite element method considers that the solution region comprises many small, interconnected, sub-regions or elements and gives a piece-wise approximation to the governing equations, that is, the complex partial differential equations are reduced to either linear or nonlinear simultaneous equations. Thus, the finite element discretization (i.e., dividing the region into a number of smaller regions) procedure reduces the continuum problem, which has an infinite number of unknowns, to one with a finite number of unknowns at specified points referred to as *nodes*. Since the finite element method allows to form the elements, or sub-regions, in an arbitrary sense, a close representation of the boundaries of complicated domains is possible.

Most of the finite difference schemes used in fluid dynamics and heat transfer problems can be viewed as special cases within a weighted residual framework. For weighted residual

procedures, the error in the approximate solution of the conservation equations is not set to zero, but instead its integral, with respect to selected ‘weights’, is required to vanish. Within this family, the collocation method reproduces the classical finite difference equations, whereas the finite volume algorithm is obtained by using constant weights.

The solution of a continuum problem by the finite element method is approximated by the following step-by-step process:

1. *Discretize the continuum*

Divide the solution region into non-overlapping elements or sub-regions. The finite element discretization allows a variety of element shapes, for example, triangles, quadrilaterals. Each element is formed by the connection of a certain number of nodes. The number of nodes employed to form an element depends on the type of element (or interpolation function).

2. *Select interpolation of the shape function*

The next step is to choose the type of interpolation function that represents the variation of the field variable over an element. The number of nodes form an element; the nature and number of unknowns at each node decide the variation of a field variable within the element.

3. *Form element equations*

Next, we have to determine the matrix equations that express the properties of the individual elements by forming an element Left Hand Side (LHS) matrix and load vector.

4. *Assemble the element equations to obtain a system of simultaneous equations*

To find the properties of the overall system, we must assemble all the individual element equations, that is, to combine the matrix equations of each element in an appropriate way such that the resulting matrix represents the behaviour of the entire solution region of the problem. The boundary conditions must be incorporated after the assemblage of the individual element contributions

5. *Solve the system of equations*

The resulting set of algebraic equations, may now be solved to obtain the nodal values of the field variable, for example, temperature.

6. *Calculate the secondary quantities*

From the nodal values of the field variable, for example, temperatures, we can then calculate the secondary quantities, for example, space heat fluxes.

3.3.1 Elements and Shape Functions

By dividing the solution region into a number of small regions, called elements, and approximating the solution over these regions by a suitable known function, a relation between the differential equations and the elements is established. The functions employed to represent the nature of the solution within each element are called *shape functions*, or

interpolating functions, or basis functions. They are called interpolating functions as they are used to determine the value of the field variable within an element by interpolating the nodal values.

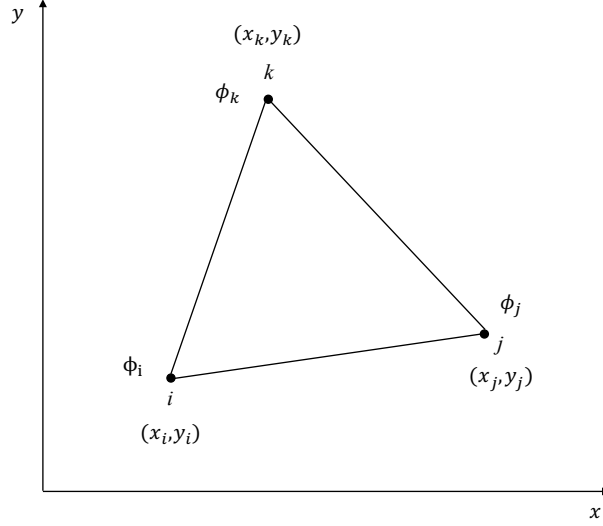


Figure 3.2: A linear triangular element

The simplest geometric shape that can be employed to approximate irregular surfaces is the triangle and it is one of the popular elements currently used in finite element calculations. The two-dimensional linear triangular element, also known as a simplex element, is represented by:

$$\phi(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y \quad (3.44)$$

where the polynomial is linear in x and y and contains three coefficients. Since a linear triangle has three nodes (Figure 3.2), the values of α_1 , α_2 and α_3 are determined from:

$$\begin{aligned} \phi_i &= \alpha_1 + \alpha_2 x_i + \alpha_3 y_i \\ \phi_j &= \alpha_1 + \alpha_2 x_j + \alpha_3 y_j \\ \phi_k &= \alpha_1 + \alpha_2 x_k + \alpha_3 y_k \end{aligned} \quad (3.45)$$

which result in the following:

$$\begin{aligned} \alpha_1 &= \frac{1}{2A} [(x_j y_k - x_k y_j) \phi_i + (x_k y_i - x_i y_k) \phi_j + (x_i y_j - x_j y_i) \phi_k] \\ \alpha_2 &= \frac{1}{2A} [(y_j - y_k) \phi_i + (y_k - y_i) \phi_j + (y_i - y_j) \phi_k] \\ \alpha_3 &= \frac{1}{2A} [(x_k - x_j) \phi_i + (x_i - x_k) \phi_j + (x_j - x_i) \phi_k] \end{aligned} \quad (3.46)$$

where A is the area of the triangle given by:

$$2A = \det \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} = (x_i y_j - x_j y_i) + (x_k y_i - x_i y_k) + (x_j y_k - x_k y_j) \quad (3.47)$$

Substituting the values of α_1 , α_2 and α_3 into equation 3.45 and collating the coefficients of ϕ_i , ϕ_j and ϕ_k , we get

$$\phi = N_i \phi_i + N_j \phi_j + N_k \phi_k = N_i \begin{bmatrix} N_i & N_j & N_k \end{bmatrix} \begin{Bmatrix} \phi_i \\ \phi_j \\ \phi_k \end{Bmatrix} \quad (3.48)$$

where

$$\begin{aligned} N_i &= \frac{1}{2A}(a_i + b_i x + c_i y) \\ N_j &= \frac{1}{2A}(a_j + b_j x + c_j y) \\ N_k &= \frac{1}{2A}(a_k + b_k x + c_k y) \end{aligned} \quad (3.49)$$

and

$$\begin{aligned} a_i &= x_j y_k - x_k y_j & b_i &= y_j - y_k & c_i &= x_k - x_j \\ a_j &= x_k y_i - x_i y_k & b_j &= y_k - y_i & c_j &= x_i - x_k \\ a_k &= x_i y_j - x_j y_i & b_k &= y_i - y_j & c_k &= x_j - x_i \end{aligned} \quad (3.50)$$

If we evaluate N_i at node i , where the coordinates are (x_i, y_i) , then we obtain

$$(N_i)_i = \frac{1}{2A} = [(x_j y_k - x_k y_j) + (y_j - y_k)x_i + (x_k - x_j)y_i] = \frac{2A}{2A} = 1 \quad (3.51)$$

Similarly, it can readily be verified that $(N_j)_i = (N_k)_i = 0$. Thus, we see that the shape functions have a value of unity at the designated vertex and zero at all other vertices. It is possible to show that

$$N_i + N_j + N_k = 1 \quad (3.52)$$

everywhere in the element, including the boundaries.

3.3.2 Formulations

After briefly describing the various elements used in the context of finite element analysis, we shall now focus our attention on determining the element characteristics, that is, the relation between the nodal unknowns and the corresponding loads or forces in the form of the following matrix equation, namely,

$$[\mathbf{K}]\{\phi\} = \{\mathbf{f}\} \quad (3.53)$$

where $[\mathbf{K}]$ is the thermal stiffness matrix, $\{\phi\}$ is the vector of unknown field variable and $\{\mathbf{f}\}$ is the load, or forcing vector. Several methods are available for the determination of the approximate solution to a given problem. We shall consider three methods in the first instance.

1. Ritz method
2. Rayleigh Ritz method (Variational)
3. Weighted residual methods

Variational Method

In order to establish the global behavior of the problem the local element description, characterized by equation 3.48, must be integrated over the entire domain in some way in order to take account of the boundary and interface conditions. One approach is to apply a variational principle corresponding to the defining equation for instance a necessary condition for the potential energy in a electrostatic system to be a minimum is that the electrostatic potential must satisfy the Laplace equation. For the special case of the Laplace equation in electrostatics this energy can be expressed as:

$$W = \int_{\Omega} \frac{1}{2} \mathbf{E} \cdot \mathbf{D} d\Omega = \frac{1}{2} \int_{\Omega} \epsilon E^2 d\Omega = \frac{1}{2} \int_{\Omega} \epsilon \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right] d\Omega \quad (3.54)$$

Substituting equation 3.48 into 3.54 after summing over all the n elements leads to:

$$W = \sum_{n=1}^n \frac{1}{2} \epsilon \int_{elem} \left[\left(\sum_1^3 \frac{\partial N}{\partial x} \phi_i \right)^2 + \left(\sum_1^3 \frac{\partial N}{\partial y} \phi_i \right)^2 \right] d\Omega \quad (3.55)$$

in order to minimize the total energy expression, equation 3.55 must be differentiated with respect to a typical value of ϕ_i and equated to a zero in the usual way. Thus,

$$\frac{\partial W}{\partial \phi_j} = \sum_{n=1}^n \frac{1}{2} \epsilon \int_{elem} \left[\left(\sum_1^3 \frac{\partial N}{\partial x} \phi_i \right) \frac{\partial N_j}{\partial x} + \left(\sum_1^3 \frac{\partial N}{\partial y} \phi_i \right) \frac{\partial N_j}{\partial y} \right] d\Omega = 0 \quad (3.56)$$

taking care to merge contributions for elements to shared nodes, equation 3.56 is seen to be of the form,

$$K\phi = 0 \quad (3.57)$$

with

$$K_{ij} = K_{ji} = \int_{elem} \epsilon \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) d\Omega \quad (3.58)$$

The above formulation as it stands will only apply to the Dirichlet problem where assigned potentials are specified at the boundaries, however the energy functional can be easily

extended to include the Neumann problem case and indeed also to the Poisson problem with sources. The system set of equations (equation 3.57) will become non-homogeneous and well posed when the boundary values are assigned.

Method of Weighted Residuals

A very direct approach that leads to the numerical solution of a partial differential equation of the Poisson type is to use a local element approximation like equation 3.48 containing unknown parameters and substitute into the defining equation. Suppose that locally,

$$u = \sum N_i a_i \quad (3.59)$$

where N_i are suitable chosen shape functions and that a_i are a set of parameters (field values) to be determined. Substitute equation 3.59 into Poisson equation to obtain

$$R = (\nabla \cdot \kappa \nabla \sum N_i a_i + Q) = 0 \quad (3.60)$$

where, in general R will only vanish if a_i is the exact solution. Equation 3.60 is the residue equation and is a measure of the error introduced by using the approximate solution equation 3.59. Direct application of equation 3.60 will produce a rudimentary solution, provided the N_i satisfy the boundary conditions on Poisson equation. This method is known as point matching. The solution is matched to a number of points within the domain of the problem equal to the number of parameters. These points are sometimes called collocation points and the method the collocation method. A far better approximation is obtained if the residue error is forced to zero in an average sense over the domain of the problem rather than a series of unrelated local operations. To this end the equation 3.60 is written

$$\int_{\Omega} \omega R d\Omega = 0 \quad (3.61)$$

where ω_i is a specially chosen function called a weighting function. Equation 3.60 is known as the weighted residual approximation to the solution. It should be clear that if equation 3.60 is satisfied for any ω_i then also Poisson equation must be satisfied at all points in the domain, otherwise if $\nabla \cdot \kappa \nabla u \neq -Q$ at one more points then it would be possible to chose a function ω_i which would violate equation 3.60. If the number of weighting functions is chosen equal to the number of trial parameters a set of linear equations is obtained,

$$\sum \left(\int_{\Omega} \omega_i \nabla \cdot \kappa \nabla N_j d\Omega \right) a_j = - \int_{\Omega} \omega_i Q d\Omega \quad (3.62)$$

which is of the form

$$K_{ij} a_j = g_i \quad (3.63)$$

This approach enables nearly all of the commonly used methods for determining numerical solutions to be unified; particular techniques are available for equation 3.62 by particular

choices of weighting functions. There are an infinite number of choices for w_i but four particular functions are most often used. Depending on the choice of the weighting functions, different names are given

- *Collocation*: $w_i = \delta(x - x_i)$

$$\int_{\Omega} R \delta(x - x_i) dx = R_{x=x_i} = 0 \quad (3.64)$$

- *Sub-Domain*: $w_i = 1$, note the sub-domain Ω_i in the integration

$$\int_{\Omega_i} R dx = 0 \quad \text{with } i = 1, 2, \dots, n. \quad (3.65)$$

- *Galerkin*: $w_i = N_i$ that is, the same trial functions as used in T

$$\int_{\Omega} N_i R dx = 0 \quad \text{with } i = 1, 2, \dots, n. \quad (3.66)$$

- *Least Squares*: $w_i = \partial R / \partial a_i$

$$\int_{\Omega} \frac{\partial R}{\partial a_i} dx = 0 \quad \text{with } i = 1, 2, \dots, n. \quad (3.67)$$

Chapter 4

Thermal Modeling of Quenching

4.1 Thermal Problem

Heat transfer is that section of engineering science that studies the energy transport between material bodies due to a temperature difference and is based on three principal phenomena:

1. Conduction
2. Convection
3. Radiation

The conduction mode of heat transport occurs either because of an exchange of energy from one molecule to another, without the actual motion of the molecules, or because of the motion of the free electrons if they are present. Therefore, this form of heat transport depends heavily on the properties of the medium and takes place in solids, liquids and gases, if a difference in temperature exists.

Molecules present in liquids and gases have freedom of motion, and by moving from a hot to a cold region, they carry energy with them. The transfer of heat from one region to another, due to such macroscopic motion in a liquid or gas, added to the energy transfer by conduction within the fluid, is called heat transfer by convection. Convection may be free, forced or mixed. When fluid motion occurs because of a density variation caused by temperature differences, the situation is said to be a free, or natural, convection. When the fluid motion is caused by an external force, such as pumping or blowing, the state is defined as being one of forced convection. A mixed convection state is one in which both natural and forced convections are present. Convection heat transfer also occurs in boiling and condensation processes.

All bodies emit thermal radiation at all temperatures. This is the only mode that does not require a material medium for heat transfer to occur. The nature of thermal radiation is such that a propagation of energy, carried by electromagnetic waves, is emitted from the surface of the body. When these electromagnetic waves strike other body surfaces, a part is reflected, a part is transmitted and the remaining part is absorbed.

All modes of heat transfer are generally present in varying degrees in a real physical problem. The important aspects in solving heat transfer problems are identifying the significant modes and deciding whether the heat transferred by other modes can be neglected [34]-[35]-[36]-[37]-[38].

4.1.1 The Laws of Heat Transfer

Conduction

It is important to quantify the amount of energy being transferred per unit time and for that we require the use of rate equations. For heat conduction, the rate equation is known as *Fourier's law*, which is expressed for one dimension as:

$$q_x = -k \frac{dT}{dx} \quad (4.1)$$

where q_x is the heat flux in the x direction (W/m^2); k is the thermal conductivity (W/mK) and dT/dx is the temperature gradient (K/m).

Convection

For convective heat transfer, the rate equation is given by *Newton's law* of cooling as

$$q = h(T_s - T_a) \quad (4.2)$$

where q is the convective heat flux (W/m^2), $(T_s - T_a)$ is the temperature difference between the surface and the fluid (K) and h is the convection heat transfer coefficient (W/m^2K). The convection heat transfer coefficient frequently appears as a boundary condition in the solution of heat conduction through solids. We assume h to be known in many such problems.

Radiation

The maximum flux that can be emitted by radiation from a black surface is given by the *Stefan-Boltzmann Law*, that is,

$$q = \sigma T_s^4 \quad (4.3)$$

where q is the radiative heat flux, (W/m^2); σ is the Stefan-Boltzmann constant, in (W/m^2K^4) and T_s is the surface temperature, (K). The heat flux emitted by a real surface is less than that of a black surface and is given by:

$$q = \epsilon \sigma T_s^4 \quad (4.4)$$

where ϵ is the radiative property of the surface and is referred to as the *emissivity*. The net radiant energy exchange between any two surfaces 1 and 2 is given by

$$q = F_\epsilon F_G \sigma A_1 (T_1^4 - T_2^4) \quad (4.5)$$

where F_e is a factor that takes into account the nature of the two radiating surfaces; F_G is a factor that takes into account the geometric orientation of the two radiating surfaces and A_1 is the area of surface 1.

4.1.2 Heat Balance Equation

The determination of temperature distribution in a medium (solid, liquid, gas or combination of phases) is the main objective of a conduction analysis, that is, to know the temperature in the medium as a function of space at steady state and as a function of time during the transient state. Once this temperature distribution is known, the heat flux at any point within the medium, or on its surface, may be computed from Fourier's law. A Taylor series expansion results in:

$$\begin{aligned} q_{x+dx} &= q_x + \frac{\partial q_x}{\partial x} \Delta x \\ q_{y+dy} &= q_y + \frac{\partial q_y}{\partial y} \Delta y \\ q_{z+dz} &= q_z + \frac{\partial q_z}{\partial z} \Delta z \end{aligned} \quad (4.6)$$

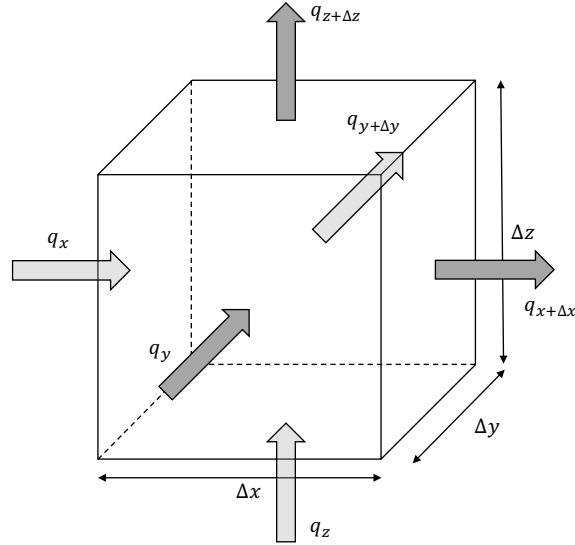


Figure 4.1: A differential control volume for heat conduction analysis

Note that the second- and higher-order terms are neglected in the above equation. The heat generated in the control volume is $Q\Delta x\Delta y\Delta z$ and the rate of change in energy storage is given as

$$\gamma\Delta x\Delta y\Delta zC_p\frac{\partial T}{\partial t} \quad (4.7)$$

Now, with reference to figure 5.1, the energy balance can be written as:

$$Q\Delta x\Delta y\Delta z + q_x + q_y + q_z = \gamma\Delta x\Delta y\Delta z C_p \frac{\partial T}{\partial t} + q_{x+\Delta x} + q_{y+\Delta y} + q_{z+\Delta z} \quad (4.8)$$

Substituting equation 4.6 into equation 4.8 and rearranging results in

$$-\frac{\partial q_x}{\partial x}\Delta x - \frac{\partial q_y}{\partial y}\Delta y - \frac{\partial q_z}{\partial z}\Delta z + Q\Delta x\Delta y\Delta z = \gamma\Delta x\Delta y\Delta z \frac{\partial T}{\partial t} \quad (4.9)$$

The total heat transfer Q in each direction can be expressed as

$$\begin{aligned} Q_x &= \Delta y\Delta z q_x = -k_x \Delta y\Delta z \frac{\partial T}{\partial x} \\ Q_y &= \Delta x\Delta z q_y = -k_y \Delta x\Delta z \frac{\partial T}{\partial y} \\ Q_z &= \Delta x\Delta y q_z = -k_z \Delta x\Delta y \frac{\partial T}{\partial z} \end{aligned} \quad (4.10)$$

Substituting equation 4.10 into equation 4.9 and dividing by the volume $\Delta x\Delta y\Delta z$, we get

$$\frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] + Q = \gamma C_p \frac{\partial T}{\partial t} \quad (4.11)$$

Equation 4.11 is the transient heat conduction equation for a stationary system expressed in Cartesian coordinates. The thermal conductivity, k , in the above equation is a vector. In its most general form, the thermal conductivity can be expressed as a tensor, that is,

$$k = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \quad (4.12)$$

All the material properties, considered since this point have been considered as a constant or just function of the the direction of the heat flux (anisotropy). In the case of numerical simulation of heat treatments the material properties are strongly affected by the temperature and by phase transformations.

The thermal properties of a phase mixture can be approximated by a linear rule of mixture

$$P(T, \xi_k) = \sum_{k=1}^N P_k \xi_k \quad (4.13)$$

in which P is the overall temperature-dependent thermal property of the phase mixture, P_k the thermal property of the k – *th* constituent of the phase mixture and ξ_k the volume fraction of k – *th* constituent.

The heat balance equation, as is, cannot be used in heat treatments simulations because it doesn't take into account the latent heat absorbed or released during phase

transformations. For the sake of simplicity, heat released due to phase transformation is usually assumed as the change in the enthalpy per unit volume (ΔH_k).

So, the latent heat release rate per unit volume can be calculated as:

$$\frac{dQ_{LH}}{dt} = \sum_{k=1}^N c_k \xi_k + \sum_{k=1}^N \Delta H_k \frac{d\xi_k}{dt} \quad (4.14)$$

From the numerical point of view there are two possible ways to consider the effect of the latent heat:

- *Inclusion of the latent heat effect in a fictitious specific heat:*

$$c^* = \sum_{k=1}^N c_k \xi_k + \sum_{k=1}^N \Delta H_k \frac{d\xi_k}{dt} \quad (4.15)$$

- *Considering the latent heat as an "internal power density source"*

4.1.3 Boundary Conditions

The heat conduction equations, discussed in the previous section, will be complete for any problem only if the appropriate boundary and initial conditions are stated. With the necessary boundary and initial conditions, a solution to the heat conduction equations is possible. The boundary conditions for the conduction equation can be of two types or a combination of these: the Dirichlet condition, in which the temperature on the boundaries is known and/or the Neumann condition, in which the heat flux is imposed:

- *Dirichlet condition*

$$T = T_0, \quad \text{with } \Gamma_T. \quad (4.16)$$

- *Neumann condition*

$$q = -k \frac{\partial T}{\partial n} = C, \quad \text{with } \Gamma_{qf}. \quad (4.17)$$

In Equations 4.16 and 4.17, T_0 is the prescribed temperature, Γ the boundary surface, n is the outward direction normal to the surface and C is the constant flux given. The insulated, or adiabatic, condition can be obtained by substituting $C = 0$. The convective heat transfer boundary condition also falls into the Neumann category and can be expressed as

$$q = -k \frac{\partial T}{\partial n} = h(T_s - T_a), \quad \text{with } \Gamma_{qc}. \quad (4.18)$$

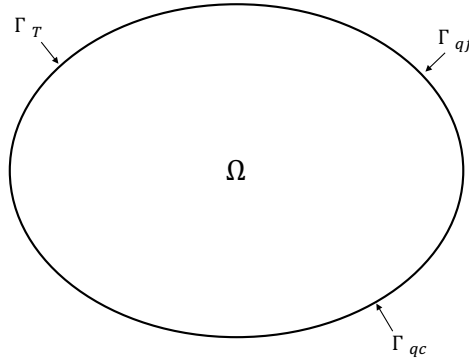


Figure 4.2: Boundary conditions on thermal domain

4.1.4 Heat Transfer Modelization During Quenching

In the previous section the heat exchange boundary conditions have been discussed in a general way, but the description of quenching process requires a special attention. During quenching the hot workpiece is rapidly cooled down through the contact with a colder fluid, and its cooling rate is not constant during the process, but it is strongly affected by the superficial temperature of the body.

Three different stages can be identified in immersion quenching processes, each one with different characteristics (Figure 4.3)[19]:

- **Water blanket stage:** The first stage of cooling is characterized by a quenchant vapour blanket around the part. This is also referred to as the vapour blanket cooling stage and is characterized by the Leidenfrost effect, a phenomenon in which a liquid, in near contact with a mass significantly hotter than the liquid's boiling point, produces an insulating vapour layer which keeps that liquid from boiling rapidly [39]. The vapour blanket develops and is maintained while the supply of heat from the interior of the part to the surface exceeds the amount of heat needed to evaporate the quenchant and maintain the vapour phase itself. Relatively slow cooling occurs during this period because the vapour envelope acts as an insulator, and cooling occurs mainly by radiation through the vapour film. The temperature above which a total vapour blanket is maintained is called the characteristic temperature of the liquid and is also known as the Leidenfrost temperature. This stage of cooling is not usually present in parts quenched in aqueous solutions containing more than about 5 wt% of an ionic material such as potassium chloride, lithium chloride, sodium hydroxide, or sulfuric acid. Cooling curves for these solutions start immediately with the boiling stage. The presence of the salts at the hot metal quenchant interface initiates nucleate boiling almost immediately. The water blanket stage cooling is not observed when quenching in non-volatile quenchant media such as molten salt baths. Conversely, heat transfer in gas quenchants such as air and inert gases occurs exclusively by a vapour blanket mechanism.

- **Boiling stage:** The highest cooling rates occur in the so-called nucleate boiling stage. During this period, the vapour envelope collapses, and high heat extraction rates are achieved that are associated with nucleate boiling of the quenchant on the metal surface. Heat is rapidly removed from the surface as liquid quenchant contacts the metal surface and is vaporized.
- **Convective cooling stage:** This stage is called the liquid cooling stage and begins when the temperature of the metal surface is reduced below the boiling point of the quenching liquid. Below this temperature, boiling stops and cooling takes place by conduction and convection into the quenchant. Convective stage cooling rates are dependent on the viscosity of the quenchant: all other factors being equal, cooling rates decrease with increasing viscosity.

The initial temperature of the component is generally well above the boiling point of the quenchant. During quenching, successive stages of heat transfer characterize the cooling process: vapor blanket stage, nucleate boiling, and convective stage each associated with a distinct cooling regime [42].

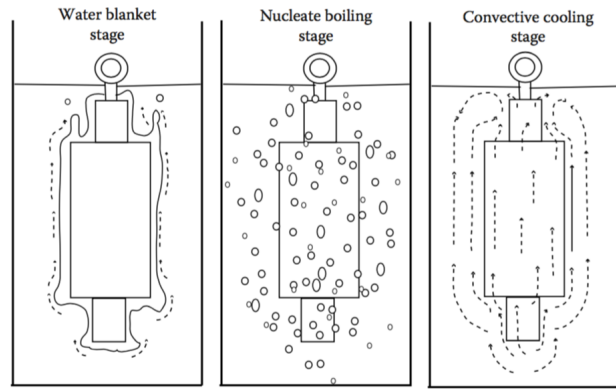


Figure 4.3: Schematic representation of critical stages of quenching [40]

During the initial transient boiling stage, liquid is in contact with the hot surface, causing intense boiling. This stage is rapidly terminated as soon as sufficient vapor has been generated to completely cover the whole surface. The rate of heat removal is very low in the vapor blanket stage due to the low thermal conductivity of the vapor layer. The contribution of radiation to the overall heat transfer in this stage is remarkable. Below the burnout point, the heat flux decreases until the surface temperature reaches the boiling point of the quenchant. The change from the vapor blanket stage to nucleate boiling is not abrupt; a transition occurs when the surface heat transfer coefficient and the amount of liquid=solid contact increases. This increase and decrease in the transition regions is achieved by the movement of the “wetting front” that separates the vapor blanket and nucleate boiling, on the surface of the immersion quenched component. In most cases, the wetting front ascends the cooling surface with a significant velocity during nucleate boiling, whereas it descends in the fluid direction during film boiling [43].

The final stage of the cooling of the quenched component is reached as the temperature of the component approaches the boiling point of the liquid; the rate of vaporization decreases and the heat transfer is reduced rapidly to the value associated with convective flow. This stage is termed as the convective cooling stage. Convective cooling rates are highly dependent on the fluid flow velocity, the viscosity, and the heat capacity of the quenchant. Hence, agitation and the use of a low-viscosity quenchant are expected to increase the cooling rate. For example, the convective cooling rate in water is high due to its low viscosity and high heat capacity. Thus, the convective cooling stage makes a remarkable contribution to the overall cooling rate. On the other hand, convective cooling in oil is less intensive due to relatively high viscosity and lower heat capacity.

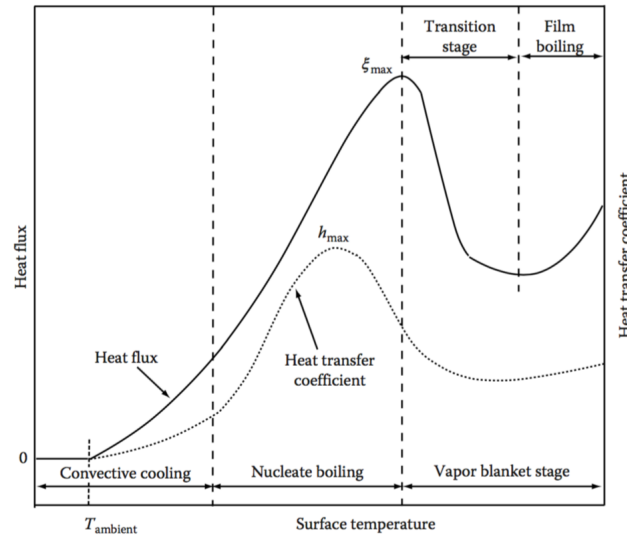


Figure 4.4: Variation of heat flux and heat transfer coefficient with temperature during critical stages of quenching [40]

Different fluids can be chosen as quenching medium, each one with different properties:

- **Water:** It is the most used quenchant for through hardening of carbon and low-alloyed steels. The convective cooling rate in water is high due to its low viscosity and high heat capacity. Thus, the convective cooling stage makes a remarkable contribution to the overall cooling rate.
- **Oil:** Quench oils are traditionally preferred quenchants when lower cooling rates are required. A variety of different quenching oils tend to show a prolonged first stage, a short second stage with a much lower cooling rate, and finally a prolonged convective cooling stage with a very modest cooling rate
- **Polymer solution:** Aqueous polymer solutions such as poly(alkylene glycol) and poly(vinyl pyrrolidone) are other alternative quenchants. They differ from quench oils with their low wetting times leading to explosive-like (Newtonian) wetting

behavior, which may improve the uniformity of the cooling. Theoretically, their cooling characteristics may be improved by additives similar to the quench oils. However, no such additive has been identified for aqueous polymer quenchants until today

4.2 Finite Element Modelization

The finite element method is a numerical tool for determining approximate solutions to a large class of engineering problems. The method was originally developed to study the stresses in complex air-frame structures [44] and was later extended to the general field of continuum mechanics [46]. There have been many articles on the history of finite elements written by numerous authors with conflicting opinions on the origins of the technique ([45]-[47]-[48]-[49]). The finite element method is receiving considerable attention in engineering education and in industry because of its diversity and flexibility as an analysis tool.

4.2.1 FE Method for Two-Dimensional Unsteady Heat Conduction

The method of weighted residuals, especially the embodiment of the Galerkin FEM, is a powerful mathematical tool that provides a technique for formulating a finite-element solution approach to practically any problem for which the governing differential equation and boundary conditions can be written.

Here, two-dimensional unsteady heat conduction is taken as example to derive the FEM formulations by Galerkin's weighted residual method.

The governing PDE for unsteady, two dimensional heat conduction with an internal heat source is [41]

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q - \gamma C_p \frac{\partial T}{\partial t} = 0 \quad (4.19)$$

The initial condition, supposing the temperature field is known, can be written as:

$$t = 0 \quad : \quad T = T_0 \quad (4.20)$$

The right-hand side of equation 4.19 equals zero when the column vector T , the exact solution of the temperature field, is substituted. On the contrary, the approximate solution T makes the residual error:

$$R = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q - \gamma C_p \frac{\partial T}{\partial t} \quad (4.21)$$

The basic idea of the weighted residual method is to construct a suitable weight function so that the integration of products by residual error and weight function equals zero; the approximate solution on the whole domain can thus be obtained. Therefore, we have:

$$\iint_D W_i \left(\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q - \gamma C_p \frac{\partial T}{\partial t} \right) dx dy \quad (4.22)$$

where W_i is the weight function.

Several variations of the weighted residual method exist and the techniques vary primarily in how the weight functions are determined or selected. The most common techniques are point collocation, subdomain collocation, least squares and Galerkin's Method [50].

The second order differential item in equation 4.22 can be transformed into a first-order item by integration in parts. Therefore, the second-order differential item can be expressed as:

$$\begin{aligned} \iint_D W_i \left(\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \right) dx dy = \\ - \iint_D \lambda \left(\frac{\partial W_i}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial W_i}{\partial y} \frac{\partial T}{\partial y} \right) dx dy + \int_S W_i \lambda \frac{\partial T}{\partial n} ds \end{aligned} \quad (4.23)$$

and

$$\int_S W_i \lambda \frac{\partial T}{\partial n} ds = \int_{S_1} \lambda \frac{\partial T}{\partial n} ds + \int_{S_2} \lambda \frac{\partial T}{\partial n} ds + \int_{S_3} \lambda \frac{\partial T}{\partial n} ds \quad (4.24)$$

Since the temperature on boundary S_1 is known, we have $\int_{S_1} W_i \lambda (\partial T / \partial n) ds = 0$. Substituting equations 4.23 and 4.24 into equation 4.22, we can get:

$$\iint_D \lambda \left(\frac{\partial W_i}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial W_i}{\partial y} \frac{\partial T}{\partial y} \right) dx dy + W_i \left(\gamma C_p \frac{\partial T}{\partial t} - Q \right) dx dy = \int_{S_2} \lambda \frac{\partial T}{\partial n} ds + \int_{S_3} \lambda \frac{\partial T}{\partial n} ds \quad (4.25)$$

The element analysis is to establish the formulations by which the continuous function in the subdomain ΔD can be expressed by the node value, i.e., the function of the node values. Supposing there are n elements in the solution domain, and there are m nodes in each element with the temperature T_i ($i=1,2,\dots,m$), the unknown temperature $T_{(x,y,z)}^e$ defined in the element can be expressed as the interpolation function of each node. Therefore, we have:

$$T_{(x,y,z)}^e = N_1 T_1 + N_2 T_2 + \dots + N_m T_m = [N_i] \{T_i\} \quad (4.26)$$

where N_i is the shape function, which is a function of the components of each element nodes (x_i, y_i, z_i) and those of the location (x, y, z) .

For example, the shape functions of a triangle element with three nodes can be simply derived and expressed as:

$$\begin{aligned} N_i(x, y) &= \frac{1}{2A}(a_i + b_i x + c_i y) \\ N_j(x, y) &= \frac{1}{2A}(a_j + b_j x + c_j y) \\ N_m(x, y) &= \frac{1}{2A}(a_m + b_m x + c_m y) \end{aligned} \quad (4.27)$$

Although the shape functions for different types of elements have different forms, they have common characteristics and can be obtained by the geometrical method. Especially the coefficients in the shape functions are the functions of the coordinates of each node, that is:

$$N_i(x, y, z) = F(x_i, y_i, z_i, x, y, z) \quad i = 1, 2, \dots, m \quad (4.28)$$

Obviously, the shape functions are determined only by the coordinates of each node and the type of element. Hence, the temperature on certain points in a element can be expressed as a function of node temperature, namely, the column vector T_i .

Equation 4.25 is applicable in the whole solution domain D , so for its subdomain, so for its subdomains, i.e., each element ΔD , we have:

$$\begin{aligned} \iint_{\Delta D} \lambda \left(\frac{\partial W_i}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial W_i}{\partial y} \frac{\partial T}{\partial y} \right) dx dy + W_i \left(\gamma C_p \frac{\partial T}{\partial t} - Q \right) dx dy \\ = \int_{\Delta S_2} \lambda \frac{\partial T}{\partial n} ds + \int_{\Delta S_3} \lambda \frac{\partial T}{\partial n} ds \end{aligned} \quad (4.29)$$

In Galerkin's weighted residual method, the weight functions are chosen to be identical to the trial function. Here, the shape functions are taken as the weight functions, for example $W_i = N_i(x, y)$, and substitute into 4.29. Hence, we have:

$$\begin{aligned} \iint_{\Delta D} \lambda \left(\frac{\partial N_i}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial T}{\partial y} \right) dx dy + N_i \left(\gamma C_p \frac{\partial T}{\partial t} - Q \right) dx dy \\ = \int_{\Delta S_2} N_i \lambda \frac{\partial T}{\partial n} ds + \int_{\Delta S_3} N_i \lambda \frac{\partial T}{\partial n} ds \end{aligned} \quad (4.30)$$

Substituting equation 4.26 into 4.30 and introducing the boundary conditions:

$$\begin{aligned} \iint_{\Delta D} \left[\lambda \frac{\partial N_i}{\partial x} \left(\frac{\partial N_i}{\partial x} T_i + \frac{\partial N_j}{\partial x} T_j + \frac{\partial N_m}{\partial x} T_m \right) + \lambda \frac{\partial N_i}{\partial y} \left(\frac{\partial N_i}{\partial y} T_i + \frac{\partial N_j}{\partial y} T_j + \frac{\partial N_m}{\partial y} T_m \right) \right. \\ \left. + N_i \gamma C_p \left(N_i \frac{\partial T_i}{\partial t} + N_j \frac{\partial T_j}{\partial t} + N_m \frac{\partial T_m}{\partial t} \right) - N_i Q \right] dx dy \\ = \int_{\Delta S_2} q N_i ds - \int_{\Delta S_3} h N_i (N_i T_i + N_j T_j - T_a) ds \end{aligned} \quad (4.31)$$

Taking $W_j = N_j(x, y)$ and $W_m = N_m(x, y)$, and deriving in exactly the same way, can be obtained as follows:

$$\begin{aligned}
 \iint_{\Delta D} & \left[\lambda \frac{\partial N_j}{\partial x} \left(\frac{\partial N_i}{\partial x} T_i + \frac{\partial N_j}{\partial x} T_j + \frac{\partial N_m}{\partial x} T_m \right) + \lambda \frac{\partial N_j}{\partial y} \left(\frac{\partial N_i}{\partial y} T_i + \frac{\partial N_j}{\partial y} T_j + \frac{\partial N_m}{\partial y} T_m \right) \right. \\
 & \left. + N_j \gamma C_p \left(N_i \frac{\partial T_i}{\partial t} + N_j \frac{\partial T_j}{\partial t} + N_m \frac{\partial T_m}{\partial t} \right) - N_i Q_i \right] dx dy \\
 & = \int_{\Delta S_2} q N_j ds - \int_{\Delta S_3} h N_j (N_i T_i + N_j T_j - T_a) ds \quad (4.32)
 \end{aligned}$$

$$\begin{aligned}
 \iint_{\Delta D} & \left[\lambda \frac{\partial N_m}{\partial x} \left(\frac{\partial N_i}{\partial x} T_i + \frac{\partial N_j}{\partial x} T_j + \frac{\partial N_m}{\partial x} T_m \right) + \lambda \frac{\partial N_m}{\partial y} \left(\frac{\partial N_i}{\partial y} T_i + \frac{\partial N_j}{\partial y} T_j + \frac{\partial N_m}{\partial y} T_m \right) \right. \\
 & \left. + N_m \gamma C_p \left(N_i \frac{\partial T_i}{\partial t} + N_j \frac{\partial T_j}{\partial t} + N_m \frac{\partial T_m}{\partial t} \right) - N_i Q \right] dx dy \\
 & = \int_{\Delta S_2} q N_m ds - \int_{\Delta S_3} h N_m (N_i T_i + N_j T_j - T_a) ds \quad (4.33)
 \end{aligned}$$

For the interior elements, the right-hand side of equation 4.31 through 4.33 equals zero respectively; while for the boundary element, only the right-hand side of equation 4.33 equals zero with the assumption that only node i and j are on the boundary.

An equation group constructed by equations 4.31 through 4.33 contains only the three unknowns, T_i , T_j , T_m . After rearrangement, it can be written in matrix form as follows:

$$[K]^e \{T\}^e + [C]^e \frac{\partial}{\partial t} \{T\}^e = \{p\}^e \quad (4.34)$$

where

$[K]^e$ is the element stiffness matrix

$\{T\}^e$ is the column vector of temperature on element nodes (unknown)

$[C]^e$ is the element heat capacity matrix

$\{p\}^e$ is the element constant vector item

The element stiffness matrix assembled from equations 4.31 through 4.33 is:

$$\begin{aligned}
 [K]^e = \iint_{\Delta D} & \lambda \begin{bmatrix} \frac{\partial N_i}{\partial x} \frac{\partial N_i}{\partial x} + \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} & \frac{\partial N_j}{\partial x} \frac{\partial N_i}{\partial x} + \frac{\partial N_j}{\partial x} \frac{\partial N_j}{\partial x} & \frac{\partial N_m}{\partial x} \frac{\partial N_i}{\partial x} + \frac{\partial N_m}{\partial x} \frac{\partial N_j}{\partial x} \\ \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial x} \frac{\partial N_m}{\partial x} & \frac{\partial N_j}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_j}{\partial x} \frac{\partial N_m}{\partial x} & \frac{\partial N_m}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_m}{\partial x} \frac{\partial N_m}{\partial x} \\ \frac{\partial N_i}{\partial x} \frac{\partial N_m}{\partial x} + \frac{\partial N_i}{\partial x} \frac{\partial N_m}{\partial x} & \frac{\partial N_j}{\partial x} \frac{\partial N_m}{\partial x} + \frac{\partial N_j}{\partial x} \frac{\partial N_m}{\partial x} & \frac{\partial N_m}{\partial x} \frac{\partial N_m}{\partial x} + \frac{\partial N_m}{\partial x} \frac{\partial N_m}{\partial x} \end{bmatrix} dx dy \\
 & + \int_{\Delta S_2} q \begin{bmatrix} N_i \\ N_j \\ 0 \end{bmatrix} ds + \int_{\Delta S_3} h \begin{bmatrix} N_i N_i & N_j N_i & 0 \\ N_j N_i & N_j N_j & 0 \\ 0 & 0 & 0 \end{bmatrix} ds \quad (4.35)
 \end{aligned}$$

Substituting equation 4.27 into 4.35 and integrating, we obtain:

$$[K]^e = \frac{\lambda}{4A} \begin{bmatrix} b_i^2 + c_i^2 & b_i b_j + c_i c_j & b_i b_m + c_i c_m \\ b_j b_i + c_j c_i & b_j^2 + c_j^2 & b_j b_m + c_j c_m \\ b_m b_i + c_m c_i & b_m b_j + c_m c_j & b_m^2 + c_m^2 \end{bmatrix} + \frac{h l_{ij}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.36)$$

where:

A is the area of element ΔD

l_{ij} is the length of the exterior boundary; $l_{ij} = 0$ for interior element

The coefficients $b_i, b_j, b_m, c_i, c_j, c_m$ are determined by the coordinates of nodes. Obviously, every element in $[K]^e$ is determined.

For the element heat capacity matrix, we have:

$$[C]^e = \iint_{\Delta D} \gamma C_p \begin{bmatrix} N_i N_i & N_j N_i & N_m N_i \\ N_i N_j & N_j N_j & N_m N_j \\ N_i N_m & N_j N_m & N_m N_m \end{bmatrix} dx dy \quad (4.37)$$

Substituting equation 4.27 into equation 4.37 and integrating, we obtain:

$$[C]^e = \frac{\gamma C_p A}{12} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad (4.38)$$

The element constant vector item $\{p\}^e$ in equation 4.34 can be explained as:

$$\{p\}^e = \{p_Q\}^e + \{p_q\}^e + \{p_h\}^e \quad (4.39)$$

where $\{p_Q\}^e, \{p_q\}^e, \{p_h\}^e$ originate from the internal heat source, heat flux in the second kind of boundary condition, and the heat transfer coefficient in the third kind of boundary condition, respectively.

The item $\{p_Q\}^e$ can be obtained as:

$$\{p_Q\}^e = \int_{\Delta D} Q N_i dx dy + \int_{\Delta D} Q N_j dx dy + \int_{\Delta D} Q N_m dx dy \quad (4.40)$$

When the internal heat source intensity Q is a constant, we have:

$$\{p_Q\}^e = \frac{AQ}{3} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \quad (4.41)$$

When the internal power source is a linear function we have:

$$\{p_Q\}^e = \frac{A}{12} \begin{bmatrix} 2Q_i & Q_j & Q_m \\ Q_i & 2Q_j & Q_m \\ Q_i & Q_j & 2Q_m \end{bmatrix} \quad (4.42)$$

Where Q_i, Q_j, Q_m are the internal sources on nodes i, j and m , respectively.

The item $\{p_q\}^e$ due to the heat flux on the boundary can be obtained by:

$$\{p_q\}^e = \int_{\Delta S_2} q N_i ds + \int_{\Delta S_2} q N_j ds + \int_{\Delta S_2} q N_m ds \quad (4.43)$$

When the heat flux through the boundary q is a linear function, we have:

$$\{p_q\}^e = \frac{l_{ij}}{2} q \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \quad (4.44)$$

When the heat flux through the boundary q is a linear function, we have:

$$\{p_q\}^e = \frac{l_{ij}}{6} q \begin{Bmatrix} 2q_i + q_j \\ q_i + 2q_j \\ 0 \end{Bmatrix} \quad (4.45)$$

The item $\{p_h\}^e$ due to the heat transfer coefficient on the boundary can be obtained by:

$$\{p_h\}^e = \int_{\Delta C_3} h T_a N_i ds + \int_{\Delta C_3} h T_a N_j ds + \int_{\Delta C_3} h T_a N_m ds = \frac{l_{ij}}{2} h T_a \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \quad (4.46)$$

Therefore, the continuous function $T_{(x,y,z)}^e$ on the subdomains, i.e., the element ΔD , has been expressed by the node temperature, and the algebraic equation group with the unknown variables of node temperature has been established.

For the whole solution domain, the algebraic equation groups on each element are assembled together so that a large-scale equation group can be obtained as follows:

$$[K]\{T\} + [C] \frac{\partial}{\partial t} \{T\} - \{P\} = 0 \quad (4.47)$$

where

$$\begin{aligned} \text{General stiffness matrix: } [K] &= \sum [K]^e \\ \text{Heat capacity matrix: } [C] &= \sum [C]^e \\ \text{Heat flux matrix: } [P] &= \sum [P]^e \end{aligned}$$

The solving of equation 4.47 brings the solution of the temperature field for the unsteady two-dimensional heat conduction. The evolution of temperature can be observed recording the node temperature at each time step, and the heating (or cooling) curves at special points can also be extracted from the results files.

4.2.2 FE Method for Three-Dimensional Unsteady Heat Conduction

The principles and procedure in the FEM for three-dimensional unsteady heat conduction are identical to those in the two-dimensional one, as presented in the previous section. The derivation of its FEM formulation is more complex and can be found in related monographies [51]-[52]-[53]. Here, only the derived results are briefly outlined.

The governing PDE of the three-dimensional unsteady heat conduction with an internal heat source is:

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q = \gamma C_p \frac{\partial T}{\partial t} \quad (4.48)$$

Supposing that there are total n elements (m nodes in each element) and p nodes after the discretization of the whole solution domain, equation 4.48 can be transformed into an algebraic equation group as follows:

$$[K]\{T\} + [C] \frac{\partial}{\partial t} \{T\} - \{P\} = 0 \quad (4.49)$$

$$[K] = \sum [K]^e \quad (4.50)$$

$$[C] = \sum [C]^e \quad (4.51)$$

$$[P] = \sum [P]^e \quad (4.52)$$

$$[K]^e = \iiint_{V^e} [B]^T [T] [B] dv + \iint_{S^e} h [N]^T [N] ds \quad (4.53)$$

$$[C]^e = \iiint_{V^e} \gamma C_p [N]^T [N] dv \quad (4.54)$$

$$[P]^e = \iiint_{V^e} Q [B]^T dv + \iint_{S^e} h T_a [N]^T ds \quad (4.55)$$

$$[D] = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \quad (4.56)$$

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \dots & \frac{\partial N_P}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \dots & \frac{\partial N_P}{\partial y} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \dots & \frac{\partial N_P}{\partial z} \end{bmatrix} \quad (4.57)$$

$$[N] = [N_1 N_2 \dots N_P] \quad (4.58)$$

where

V^e is the element volume

S^e are the exterior boundaries

In the unsteady temperature field, the variable temperature $T_{(x,y,z,t)}$ is a function of a spatial location and time. Equation 4.48 is the spatial discretized formulation with the time differential item $\partial T / \partial t$, which can be discretized by the difference method in a uniform format as follows:

$$\theta \left(\frac{\partial T}{\partial t} \right)_t + (1 - \theta) \left(\frac{\partial T}{\partial t} \right)_{t-\Delta t} = \frac{1}{\Delta t} (T_t - T_{t-\Delta t}) \quad (4.59)$$

If $\theta = 1$, the backward difference format of time discretization is:

$$\left(\frac{\partial T}{\partial t} \right)_t = \frac{1}{\Delta t} (T_t - T_{t-\Delta t}) \quad (4.60)$$

Substituting equation 4.60 into equation 4.49, we get:

$$\left([K] + \frac{1}{\Delta t} [C] \right) \{T_t\} = \frac{1}{\Delta t} [C] \{T_{t-\Delta t}\} + \{P\} \quad (4.61)$$

If $\theta = 1/2$, the Crank-Nicolson difference format (also named central difference format) of time discretization can be obtained as:

$$\frac{1}{2} \left[\left(\frac{\partial T}{\partial t} \right)_t + \left(\frac{\partial T}{\partial t} \right)_{t-\Delta t} \right] = \frac{1}{\Delta t} (T_t - T_{t-\Delta t}) \quad (4.62)$$

Substituting equation 4.62 into equation 4.49, the corresponding FEM algebraic equation can be obtained as follows:

$$\left([K] + \frac{2}{\Delta t} [C] \right) \{T_t\} = \left(\frac{2}{\Delta t} [C] - [K] \right) \{T_{t-\Delta t}\} + \frac{1}{\Delta t} (\{P\}_t + \{P\}_{t-\Delta t}) \quad (4.63)$$

The Crank-Nicolson difference format is, in general, more accurate than the backward difference method, in that it does not give preference to either temperature at time step $t - \Delta t$ or time step t , but rather, gives equal credence to both.

In Finite Difference Method (FDM) of time, the key parameter governing solution accuracy is the selected time step Δt . In a fashion similar to the FEM, in which the smaller the elements are, physically, the better is the solution, the FDM converges more rapidly to the true solution as the time step is decreased.

When the temperature field at time step $t - \Delta t$ is known, and input into equation 4.61 or equation 4.63, the temperature $\{T_t\}$ on all nodes at time step t can be thus solved, and the marching solution can progress in time until a steady state is reached. The nodal temperatures, recorded step by step at different time, constitute the evolution history of the three-dimensional temperature field.

Chapter 5

Theory and Modelization of Metallurgical Phase Transformations

5.1 Introduction

The purpose of heat treatment is to cause desired changes in the metallurgical structure and thus in the properties of metal parts. Heat treatment can affect the properties of most metals and alloys, but ferrous alloys, principally steels, undergo the most dramatic increases in properties, and therefore structural changes in iron-carbon alloys are considered in this chapter.

In general, the most stable steel structures are produced when a steel is heated to the high-temperature austenitic state (to be defined later) and slowly cooled under near-equilibrium conditions.

5.2 The Fe-C Diagram

The complex phenomena that occur during heat treatments of steels are due to the different solubility of Carbon and other alloying elements in the cristalline lattice of the iron. The problem can be gradually approached, firstly considering the transformations that appen in equilibrium conditions, when the time do not affect the transformations.

The structure of iron-carbon alloys can contain either pure carbon (graphite) or a chemical compound (cementite) as the carbon-enriched component. Cementite is present even in relatively slowly cooled alloys: a long holding at elevated temperatures is required to decompose cementite to iron and graphite. For this reason the iron-carbon diagram is usually treated as the iron-iron carbide diagram. The former is stable, whereas the latter is metastable.

This kind of problem can be studied, for carbon steels, using the so-called Fe-C diagrams (Figure 5.1), that are the phase diagrams of the iron and carbon solution.

The pure iron presents some characteristic points. In particular:

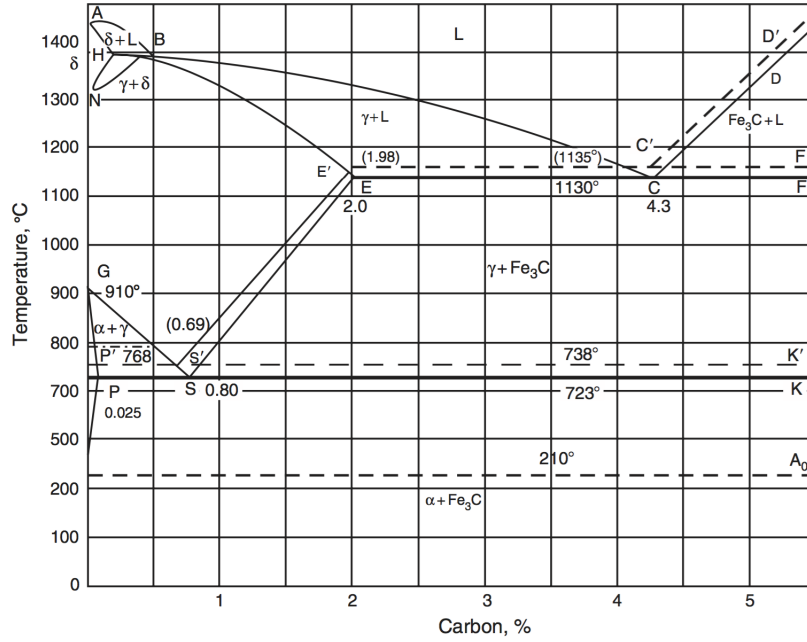


Figure 5.1: The Fe-C diagram

- The solidification temperature (1536 °C), in which the molten iron solidifies to the crystalline structure bcc named δ -Fe
- The transformation of δ -Fe (bcc) (1392°C) into a fcc stable lattice called γ -Fe
- The transformation of γ -Fe (fcc) (911 °C) in the stable lattice called α -Fe.

A maximum solubility of carbon in α -Fe is observed at 721°C and is equal to 0.018% C. The structure of the steel containing 0–0.02% C comprises ferrite and tertiary cementite. A further increase in the carbon content leads to the appearance of a new structural component, a eutectoid of ferrite and cementite (pearlite). Pearlite appears first as separate inclusions between ferrite grains and then, at 0.8% C, occupies the entire volume. Pearlite represents a two-phase mixture, which usually has a lamellar structure. As the carbon content of steel is raised to over 0.8%, secondary cementite is formed along with pearlite. The secondary cementite is shaped as needles. The amount of cementite increases as the carbon content is increased.

The temperature that identifies the change of the iron properties during the heating is conventionally indicated with the letter *A* followed by a number, as follows:

- $A_1=723$ °C. This point does not exist in pure iron, but only for Fe-C alloys and correspond to the eutectoidic transformation $\alpha \Leftrightarrow \gamma$.
- $A_3=911$ °C. Allotropic transformation temperature $\alpha \Leftrightarrow \gamma$. This temperature changes in function of the carbon content (from 911 °C for pure iron, to 723 °C when the carbon content is around 0.8%)

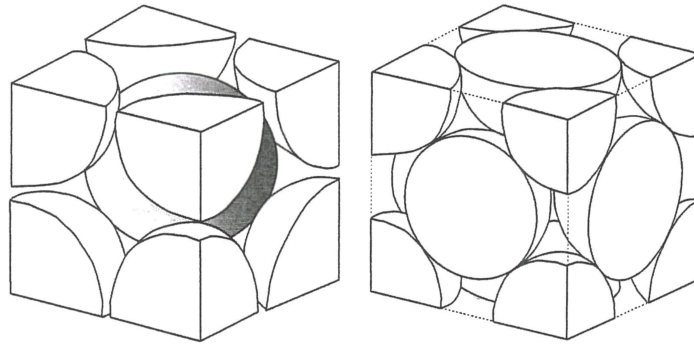


Figure 5.2: Body Centered Cubic (bcc) microstructure and Face Centered Cubic (fcc) [54]

In equilibrium conditions, the transformations are reversible and should occur at the same temperature. Practically this behavior does not occur, since the transformations take place in non-equilibrium conditions, in a wide range of temperatures which depend on the heating or cooling rates that affect the hysteresis magnitude (overheating or undercooling necessary for the activation of the transformation). In order to distinguish the transformation points (also called *critical points*) the letter "c" (chauffage) may be added to the heating ones, and the letter "r" (refroidissement) to these determined during the cooling stage.

5.3 Austenite Formation Kinetics

The way austenite is formed when a certain steel is heated depends very much on the steel's starting microstructure.

Let us take as an example an unalloyed eutectoid steel with 0.8% C. At room temperature the cementite (Fe_3C) plates of the pearlite are in direct contact with ferrite. The carbon atoms from cementite have a tendency to diffuse into the ferrite lattice. The higher the temperature, the greater this tendency is. Upon heating, on reaching the A_{C1} temperature ($723^{\circ}C$), the transformation of ferrite into austenite starts immediately adjacent to the cementite plates. After that the cementite plates start to dissolve within the newly formed austenite, becoming thinner and thinner. So two processes take place at the same time: the formation of austenite grains from ferrite and the dissolution of cementite plates in the austenite lattice.

Experiments have shown that the process of ferrite-to-austenite transformation ends before all the cementite has been dissolved. This means that after all the ferrite has transformed into austenite, small particles of cementite will remain within the austenite grains.

The duration of austenitizing process depends on the austenitizing temperature and the steel composition. The influence of time at austenitization can best be explained by the diagrammatic illustrations shown in Figure 5.3.

The holding time at austenitizing (hardening) temperature depends on the desired

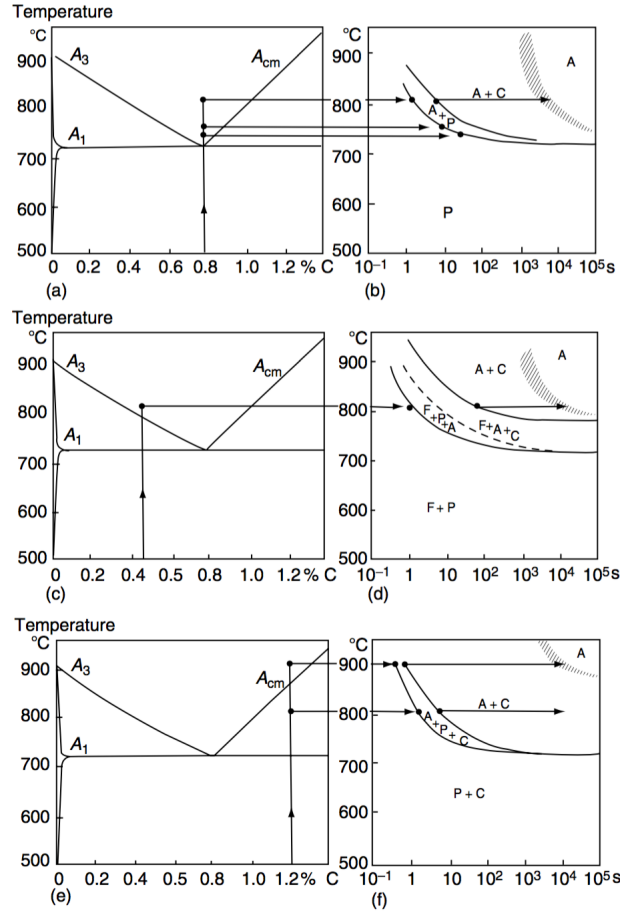


Figure 5.3: Austenitization process for steels with different carbon content

degree of carbide dissolution and acceptable grain size, taking into account that the grain growth increases with higher austenitizing temperatures and longer holding times. Since the amount of carbide is different for different types of steel, the holding time (from the metallurgical point of view) depends on the grade of steel. However, carbide dissolution and the holding time are dependent not only on the austenitizing temperature but also the rate of heating to this temperature. Varying the rate of heating to this temperature will have an effect on the rate of transformation and dissolution of the constituents. The influence of the role of heating (and correspondingly of the holding time) on carbide dissolution, grain growth, and hardness after hardening for various grades of steel has been studied in detail and published in literature.

These time-temperature-austenitizing diagrams (Zeit-Temperatur-Austenitisierung Schaubilder in German) have been produced either as isothermal diagrams (the steel specimens were heated rapidly at the rate of 130°C/s to the temperature in question and held there for a certain predetermined time) or as continuous heating diagrams (the steel specimens were heated continuously at different heating rates). In practice, the continuous

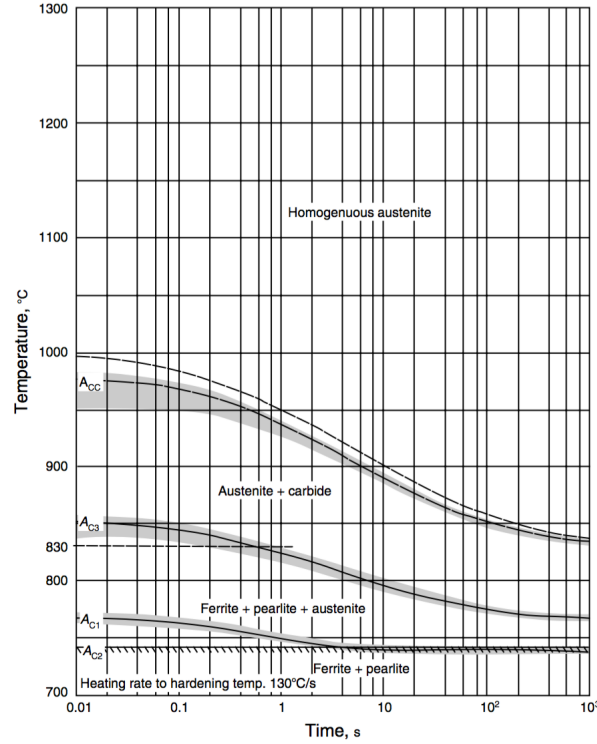


Figure 5.4: TTA diagram for 50CrV4 steel

heating diagrams are much more important because every austenitizing process is carried out at a specified heating rate.

Induction hardening is characterized by internal power densities which can generate a high heating rate (in the range of $100 \div 10000^\circ\text{C/s}$). The classical approach, based on TTA diagrams can not be used since it does not take into account the effects, which play a fundamental role in these treatments, such as the effect of the heating rate and initial microstructure of the steel.

The transformation of pearlite to austenite takes place when the workpiece is heated above the conventional temperature A_{C1} . Cementite lamellae in the pearlite colonies first dissolve and carbon diffuses outward into the surrounding ferrite. To achieve such an extent of carbon diffusion, at a high heating rate, superheating is required. Additionally, part or all of the ferrite originally surrounding the pearlite colonies may transform to austenite, depending on the value of the temperature relative to the A_{C1} , where considerable superheating of the ferrite might also occur. The pearlite becomes austenite containing 0.8% carbon, and the ferrite becomes austenite with negligible carbon content. Thereafter, carbon diffuses from a high concentration to a low concentration region, homogenizing the carbon distribution. On subsequent cooling, all austenite with carbon content above a critical value displacively transforms to martensite, and the rest reverts to ferrite. However, due to the finite rate of carbon diffusion, at a high heating rate a superheating is required to enhance the diffusion rate in order to achieve austenite transformation.

It is clear how austenitization process is strongly controlled by diffusion phenomena, where time, temperature and cementite dispersion play a fundamental role. Finer initial microstructures, with small and dispersed carbides, are more suitable for high-heating rate processes since carbides are easily solvable in the austenitic matrix.

Starting from CHT diagrams, the total ammount of transformed phase can be expressed, in first approximation as:

$$\xi_\gamma = \begin{cases} 0, & \text{if } T < A_{C1}(\dot{T}). \\ \frac{T - A_{C3}(\dot{T})}{A_{C3}(\dot{T}) - A_{C1}(\dot{T})}, & \text{if } A_{C1}(\dot{T}) \leq T \leq A_{C3}(\dot{T}). \\ 1, & \text{if } T > A_{C3}(\dot{T}). \end{cases} \quad (5.1)$$

where ξ_γ is the volume fraction of austenite, $A_{C3}(\dot{T})$ and $A_{C1}(\dot{T})$ the transformation temperatures in function of the heating rate and T the temperature.

5.4 Phase Transformations During Quenching

In this section the possible microstructures achievable during the austenite decomposition will be presented. In order to simplify the dissertation, an eutectoidic steel - in which A_1 and A_3 coincide - can be considered.

Pearlite

After the austenitization, cooling down very slowly the steel, at the temperature A_{R1} should take place the following phenomena:

- transformation of $\text{Fe}\gamma$ (fcc) into $\text{Fe}\alpha$ (bcc)
- formation of cementite due to the reduced solubility of carbon in ferrite
- coalescence of cementite

If the cooling is slow enough, the transformation will occur in quasi-equilibrium conditions and A_{R1} will not be very different from A_{C1} . The three mechanisms discussed above will occur and the final microstructure obtained is named *globular pearlite*, that is a ferritic matrix with cementite homogeneously dispersed.

Increasing the cooling rate, the austenite transformation begins at a lower temperature, accentuating the hysteresis phenomena. When A_{R1} decreases between 700 and 600 °C:

- the diffusion of carbon in austenitic matrix is still high, but decreases with the temperature
- slow transformation kinetics $\gamma \Rightarrow \alpha$ due to the small undercooling respect A_1
- coalescence of cementite stops 10-15 °C below A_1 , so the perlitic microstructure presents a lamellar shape.

With the decreasing the transformation temperature of austenite (or increasing the cooling rate), the microstructure becomes more fine and the hardness of the steel is enhanced. This effect depends upon the dispersion of the harder phase (cementite) in the softer phase (ferrite).

When A_{R1} decreases around 500-600°C:

- carbon diffusion decreases
- α phase nucleation strongly increases for the effect of the undercooling
- the transformation $\gamma \Rightarrow \alpha$ occurs rapidly because both the governing phenomena (nucleation and carbon diffusion) are optimized.
- no coalescence

The pearlite lamellae are so fine to be not observable with optical microscope.

Bainite

When A_{R1} decreases between 500°C and 350 °C:

- low carbon diffusion
- high nucleation rate due to the high undercooling
- $\gamma \Rightarrow \alpha$ transformation rate is controlled by the carbon diffusion

The microstructure obtained is called *upper bainite*. The bainite microstructure consists of aggregates of ferrite plates separated by thin films of austenite, martensite, or cementite. These aggregates of plates are called sheaves. Upper bainites are harder than pearlites, but more brittle, because of the higher dispersion of cementite and huge ferritic plates in which the fracture can easily propagate.

When A_{R1} falls between 350 and 220 °C

- carbon diffusion is almost negligible, but still exists
- high undercooling and high nucleation tendency
- $\gamma \Rightarrow \alpha$ transformation rate is controlled by the carbon diffusion, and so it is very slow.

The microstructure obtained through this mechanism is extraordinarily fine, justified by the limited mobility of the atoms and carbon diffusion. This microstructure, named *lower bainite*, has the maximum dispersion of cementite in ferritic matrix, giving high hardness and high toughness to the steel.

Martensite

Under 200°C the carbon diffusion is completely cancelled, so the austenite crystals (fcc) can not transform into ferrite (bcc), separating cementite. Carbon atoms stay trapped in the crystalline lattice, that transforms in a tetragonal lattice through a rotation of some atomic planes. This new metastable phase is called *martensite*.

The most recent theories suggest that martensite generates through the inclination of of a common plane of the austenite and the martensite.

The martensite transformation occurs athermally below M_s , i.e., the extent of transformation is proportional to undercooling below the M_s and does not depend on the time spent below the M_s . The transformation from austenite to martensite proceeds as the temperature is reduced below M_s until the martensite finish temperature (M_f) is reached, at which point 100% martensite is expected. However, if the M_f temperature is below room temperature, then some austenite may be retained if only cooled to room temperature. The martensite transformation is also sensitive to external and internal stresses.

5.5 TTT and CCT Diagrams

Time-temperature-transformation diagrams for isothermal transformation (IT diagrams) and for continuous cooling transformation (CCT diagrams) are used to predict the microstructure and hardness after a heat treatment process or to specify the heat treatment process that will yield the desired microstructure and hardness. The use of the either type of diagram requires that the user be acquainted with its specific features, possibilities, and limitations.

TTT Diagrams

Figure 5.5 shows an IT diagram of the low-alloy steel DIN 50CrV4. The regions of transformation of the structural phases ferrite (F), pearlite (P), and bainite (B) as positioned in the time-temperature diagram (the abscissa of which is always in logarithmic scale) are valid only under conditions of fast quenching from the austenitizing temperature to the chosen transformation temperature and subsequent holding at that temperature. This is the way the diagram itself was developed. Therefore the IT diagram may be read only along the isotherms.

Therefore, an agreement is reached, according to which the curve marking the beginning of transformation denotes 1% of relevant phase originated, and the curve marking the end of transformation denotes 99% of the austenite transformed. Only the formation of martensite takes place without diffusion, instantly, depending only on the temperature below the M_s point. Some IT diagrams, when read along the isotherms, enable the user to determine the percentages of phases transformed and the hardness achieved.

CCT Diagrams

Figure 5.6 shows the CCT diagram of the low-alloy DIN 50CrV4 steel. When comparing the curves for the start of transformation in CCT and IT diagrams for the same heat and steel

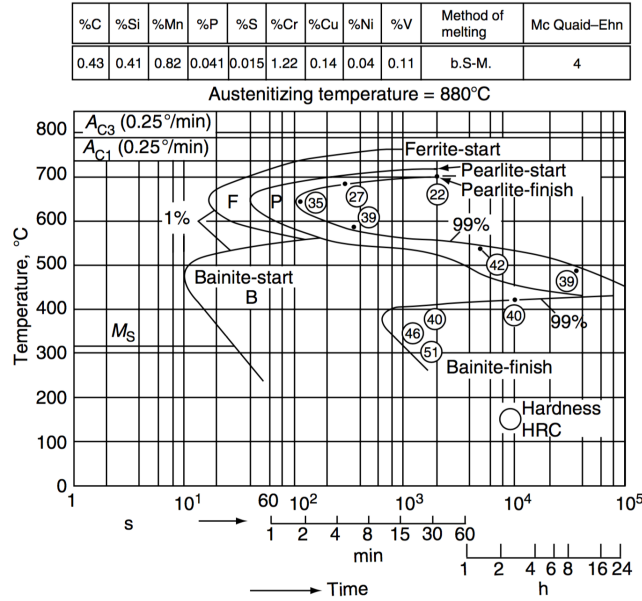


Figure 5.5: TTT diagram of 50CrV4 steel

grade, we found that in the CCT diagram the curves are slightly shifted to longer times and lower temperatures. This indicates that in CCT processes the transformation starts later than in IT processes. The basis of this phenomenon is related to the incubation time and can be found in a hypothesis of Scheil. It should also be noted that with higher austenitizing temperature the curves denoting the start of transformation of a particular phase can be shifted to longer times. It is necessary, therefore, when using a CCT diagram, to ascertain that the austenitizing temperature used to develop the CCT diagram corresponds to the austenitizing temperature of the parts treated.

A CCT diagram is developed in the following way. Many small specimens austenitized and cooled within a dilatometer with different cooling rates. Start and finish of transformation of relevant phases with each cooling curve are recorded and these points are connected to obtain the regions of transformation for the relevant phases. Therefore, a CCT diagram can be read only in the way in which it was developed, i.e., along the cooling curves.

5.6 Numerical Modeling of Transformation Diagrams

The Time-Temperature Transformation (TTT) and Continuous Cooling Transformation (CCT) diagrams are useful tools in thermomechanical processing of steels. Such diagrams depend on a so great number of variables that it is impossible to produce enough experimental diagrams for generalised use. For this reason, significant work has been undertaken to develop models that can calculate TTT and CCT diagrams for steels. An efficient method for the numerical prediction of transformation diagrams has been proposed by Kirkaldy and Venugopalan [56] and reviewed by Victor Li [55]. The general form of isothermal diagrams

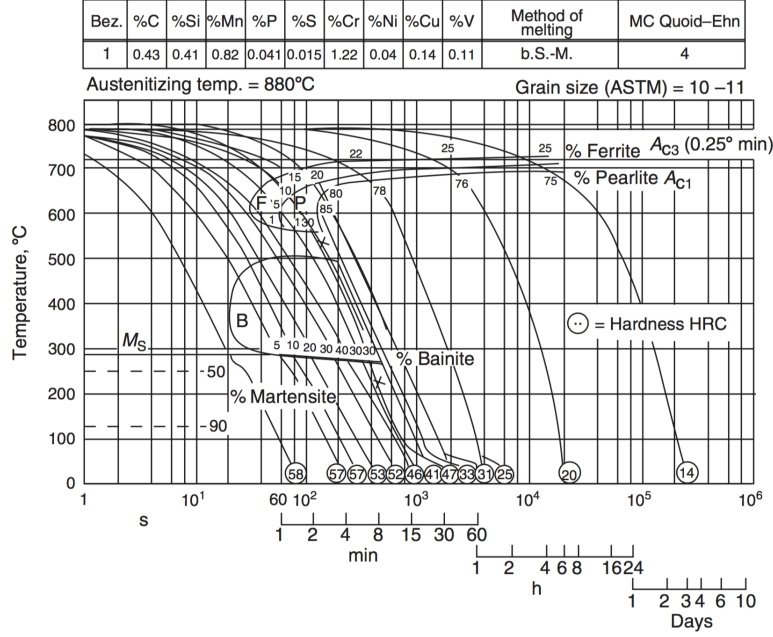


Figure 5.6: Continuous Cooling Diagram (CHT) of 50CrV4 steel

can be described by Zener [57] and Hillert [58] type formula:

$$\tau(X, T) = \frac{F(C, Mn, Si, Ni, Cr, Mo, G)}{\Delta T^n \exp(-Q_{act}/RT)} S(X) \quad (5.2)$$

where τ is the time needed to transform X volume fraction of austenite at the temperature T , F is a function of steel composition (expressed in wt%) and austenite grain size, ΔT is the undercooling, Q_{act} the activation energy for diffusion, n an empirical constant determined by the effective diffusion mechanism ($n = 2$ for volume and $n = 3$ for boundary diffusion) and $S(X)$ is the reaction rate term, proposed by Victor Li, which approximate the sigmoidal effect of transformation kinetics.

The reaction rate term has been described in [55] as:

$$S(X) = \int_0^X \frac{dX}{X^{0.4}(1-X)(1-X)^{0.4X}} \quad (5.3)$$

The location of the 'nose' in each C-curve in isothermal diagrams is jointly determined by the values of n and Q_{act} . From Equation 5.2, at the nose temperature, it is possible to derive:

$$\frac{d}{dT} [\Delta T^n \exp(-Q_{act}/RT)] = 0 \quad (5.4)$$

which lead the relationship:

$$Q_{act} = \frac{nRT_{nose}^2}{\Delta T} \quad (5.5)$$

where T_{nose} is the temperature of the nose. Equation 5.5 can be calibrated by using experimental data taken from literature and getting good estimations of the optimal value of Q_{act} . It was found in Victor et al. [55] that the Q_{act} values have a median value of 27500 kcal/mol°C. For simplicity, this value is used for each diffusion-controlled phase transformation.

Under isothermal condition, the time required for ferritic transformation is:

$$\tau_F = \frac{\exp(1.00 + 6.31C + 1.78Mn + 0.31Si + 1.12Ni + 2.70Cr + 4.06Mo)}{2^{0.41G}(A_{C3} - T)^3 \exp(-27500/RT)} S(X) \quad (5.6)$$

for the pearlitic transformation:

$$\tau_P = \frac{\exp(-4.25 + 4.12C + 4.36Mn + 0.44Si + 1.71Ni + 3.33Cr + 5.19\sqrt{Mo})}{2^{0.32G}(A_{C1} - T)^3 \exp(-27500/RT)} S(X) \quad (5.7)$$

and for the bainitic transformation:

$$\tau_B = \frac{\exp(-10.23 + 10.18C + 0.85Mn + 0.55Ni + 0.90Cr + 0.36Mo)}{2^{0.29G}(B_S - T)^2 \exp(-27500/RT)} S(X) \quad (5.8)$$

where the critical temperatures can be approximated with the following formulas, based on thermodynamical calculations or regression analysis.

Lusk et al. [62] derived a purely empirical formula based on regression analysis on approximately 4000 steel grades, for the determination of A_{C3} :

$$A_{C3} = 883.49 - 275.89C + 90.91C^2 - 12.26Cr + 16.45CCr - 29.96CMn + 8.49Mo \\ - 10.80CMo - 25.56Ni + 1.45MnNi + 0.76Ni^2 + 13.53Si - 3.47MnSi \quad (5.9)$$

A_{C1} temperature can be estimated through a relation proposed by Trzaska and Dobrzanski [64]-[63]

$$A_{C1} = 739 - 22.8C - 6.8Mn + 18.2Si + 11.7Cr - 15Ni - 6.4Mo - 5V - 28Cu \quad (5.10)$$

B_S can be calculated using the equation proposed by Steven and Haynes [61]

$$B_S = 656 - 58C - 35Mn - 75Si - 15Ni - 34Cr - 41Mo \quad (5.11)$$

The M_S formula proposed by Kung and Raymond [59] based on modification of the original linear formula by Andrews [60] has been adopted:

$$M_S = 539 - 423C - 30.4Mn - 17.7Ni - 12.1Cr - 7.5Mo + 10Co - 7.5Si \quad (5.12)$$

Kung and Raymond reported that this formula has better accuracy than the M_S formula of Steven and Haynes.

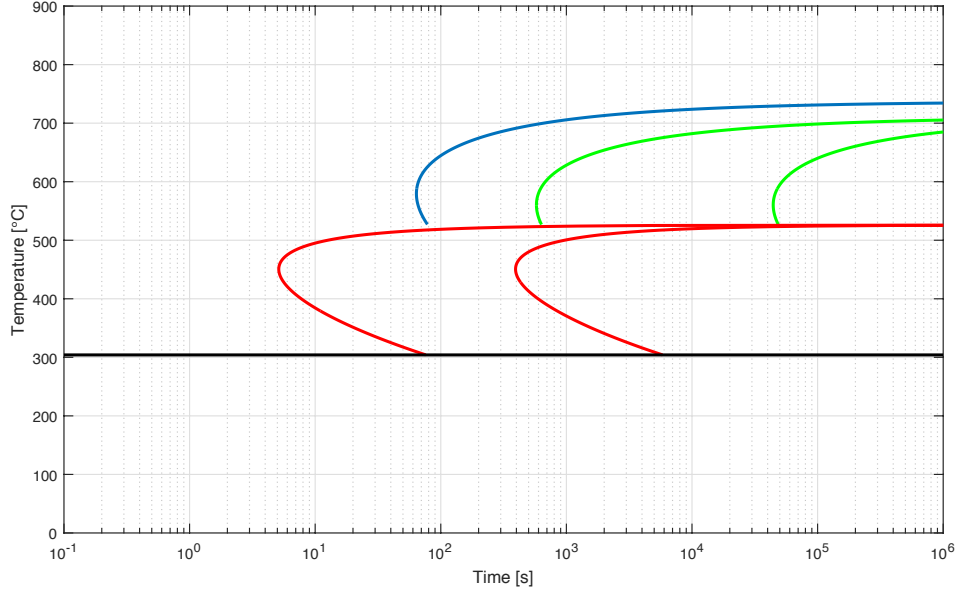


Figure 5.7: TTT diagram of AISI 4340, modeled through numerical approach

5.7 Modeling of Diffusion-Controlled Transformations

In order to describe correctly the transformations that occur during quenching process, it is fundamental to consider that the transformations are influenced by two concurrent phenomena such as the *undercooling* and the *carbon diffusion*. The transformation kinetics can be divided in three main stages: initial nucleation, growth of initial nuclei with steady nucleation and finally site saturation and impingement of the grains.

5.7.1 Isothermal Transformation Model

Several mathematical models have been proposed for the description of isothermal kinetics of solid-solid phase transformations, most of which are based on the same principle with minor modifications. The initial transformed amount of phase can be described as:

$$\xi_k = b_k t^{n_k} \quad (5.13)$$

where b_k and n_k are two coefficients directly extracted from the isothermal diagrams and can be defined as follows:

$$n_k = \frac{\log\left(\frac{\ln(1-\xi_k^f)}{\ln(1-\xi_k^s)}\right)}{\log\left(\frac{t_s}{t_f}\right)} \quad (5.14)$$

$$b_k = -\frac{\ln(1-\xi_k^f)}{(t_s)^{n_k}} \quad (5.15)$$

where t_s and t_f are the time needed, at the actual temperature, to transform respectively the amount ξ_k^s and ξ_k^f of k-th phase (generally 0.01 and 0.99). As transformation proceeds, the available volume for nucleation becomes exhausted and later, growing phase boundaries impinge each others, both leading to a decrease in transformation rate. A more general equation can be written as:

$$\dot{\xi}_k = (1 - \xi_k)^{r_k} n_k b_k (b_k t)^{n_k - 1} \quad (5.16)$$

where r is the saturation parameter which depends upon the growth mode. Different choices of r results in different kinetics of transformations [65]. Integrating this reaction rate relation:

$$\xi_k = \begin{cases} 1 - \exp(-b_k t^{n_k}), & r = 1 \quad (\text{Avrami}) \\ 1 - (1 + b_k t^{n_k})^{-1}, & r = 2 \quad (\text{Austin-Rickett}) \\ 1 - (1 + (r_k - 1)b_k t)^{\frac{r_k - 1}{n_k}}, & r \neq 1 \end{cases} \quad (5.17)$$

The most used formulation used for the description of solid-solid phase transformations during quenching in steel is the Avrami formula and since this point it will be used as reference formulation.

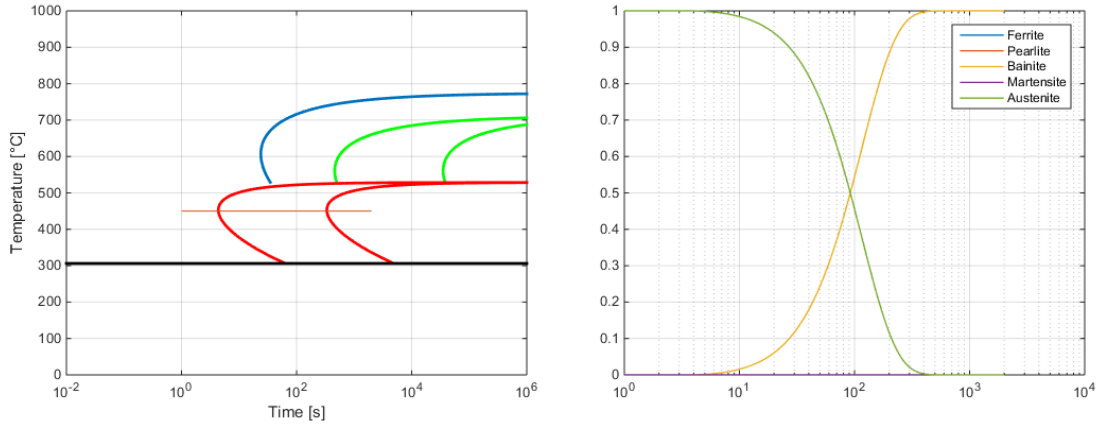


Figure 5.8: Example of isothermal transformation: temperature trend (left) and transformation kinetics (right)

This expression may be corrected to account for phase transformations that start from a phase mixture and do not saturate completely:

$$\xi_k = \xi_k^0 + (\xi_k^e - \xi_k^0)(1 - \exp(-b_k t^{n_k})) \quad (5.18)$$

where ξ_k^0 is the initial amount of k-th phase and ξ_k^e the equilibrium concentration.

5.7.2 Anisothermal Transformation Model

Isothermal phase transformation kinetics cannot be used to describe correctly quenching phenomena, because based on constant temperature observations. Each temperature corresponds to a different transformation kinetics, so it is impossible to sum, in a easy way the different contribution to phase transformation during the cooling stage. On the other hand, numerical procedures, based on isothermal diagrams, have been developed also for non-isothermal treatments, dividing the thermal history in a sum of isothermal steps and introducing an additivity rule.

The additivity principle, firstly proposed by Scheil [66], permits to sum the contribution of different kinetics in order to evaluate the total amount of transformed phase during an anisothermal transformation process. Additivity principle has long been discussed, reviewed and adopted by many authors [67]-[68]-[69]-[70]-[71]-[72]-[73]-[74]. According to Scheil's additivity rule, if $\tau(\xi_k, T)$ is the isothermal time required to reach certain transformed amount ξ_k , the same transformation amount will be reached under anisothermal conditions when the following Scheil's sum (S) equals unity:

$$S = \int_0^t \frac{dt}{\tau(\xi_k, T)} = 1 \quad (5.19)$$

or, for the numerical purpose the Scheil's sum can be expressed as:

$$S = \sum_{i=1}^N \frac{\Delta t}{\tau(\xi_k, T_i)} \approx 1 \quad (5.20)$$

where Δt and τ_i are the time step size and isothermal time to reach ξ_k at the current time step. This rule can be exploited in the calculation of both the incubation time and the anisothermal kinetics of transformation.

After the completion of incubation time, growth kinetics needs to be calculated. Considering the Avrami kinetic equation, a fictitious time t^* , which is dependent on the fraction transformed up to the end of the previous time step, is calculated:

$$t^* = \left(-\frac{\ln(1 - \xi_k(t))}{b_k} \right)^{\frac{1}{n_k}} \quad (5.21)$$

Next, the fictitious time is incremented by step size (Δt) to calculate the new fictitious transformed fraction. Then, the fictitious transformed fraction is further corrected to take into account the amount of austenite available for the transformation and reactions that do not saturate to full completion.

Figure 5.10 summarizes this procedure, which yields the following equation:

$$\xi_k^t = \xi_k^{max} (\xi_\gamma^{t-1} - \xi_k^{t-1}) (1 - \exp(b_k(t^* + \Delta t)^{n_k})) \quad (5.22)$$

where ξ_k^{max} is the maximum fraction of the product phase. In the case of pearlitic or bainitic transformations, it is the fraction of austenite at the beginning of the transformation ($\xi_p^{max} = \xi_b^{max} = \xi_\gamma^{max}$).

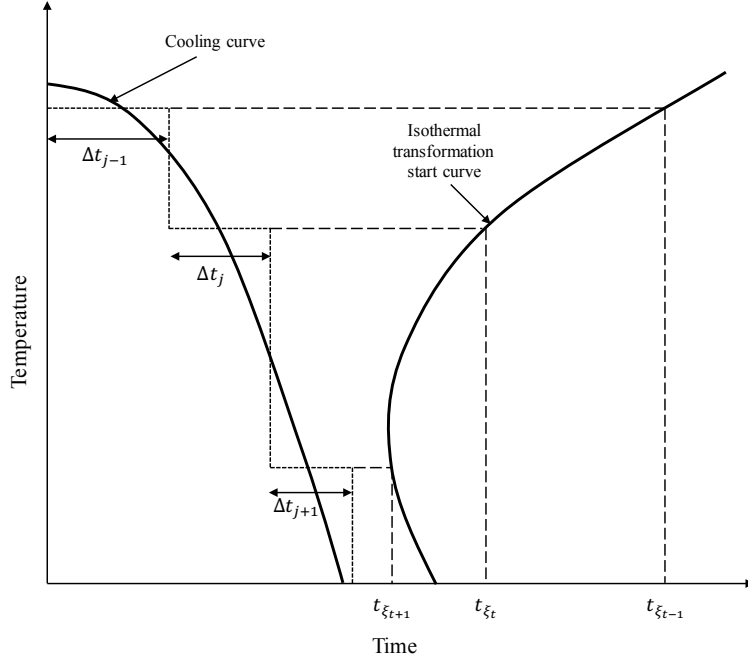


Figure 5.9: Thermal history discretization and incubation time

5.8 Modeling of Martensitic Transformation

Martensite is commonly considered to form by a time-independent transformation below M_S temperature. Physically, there exists a nucleation and growth stage, but the growth rate is so high that the rate of volume transformation is almost entirely controlled by nucleation. In fact, the austenite/martensite interface moves almost at the speed of sound in the solid. Therefore, its kinetics is essentially not influenced by the cooling rate and cannot be described by Avrami type of kinetic equations.

The amount of martensite formed is often calculated as a function of temperature using the law established by Koistinen and Marburger [75]

$$\xi_m = \xi_\gamma (1 - \exp(-\Omega(M_S - T))) \quad (5.23)$$

where Ω is considered constant for many steels and equal to 0.011.

This relation proposed by Koistinen and Marburger has been extensively used in literature, but tends to underestimate the value of the transformed phase in alloyed steels. A reviewed version of this formula has been recently proposed by Lee and Van Tyne [76]. Koistinen and Marburger model has been derived only considering the effect of undercooling respect M_S temperature, which is not, however, the only parameter that affect the martensite transformation. The model proposed by Lee and Van Tyne can be expressed as follows:

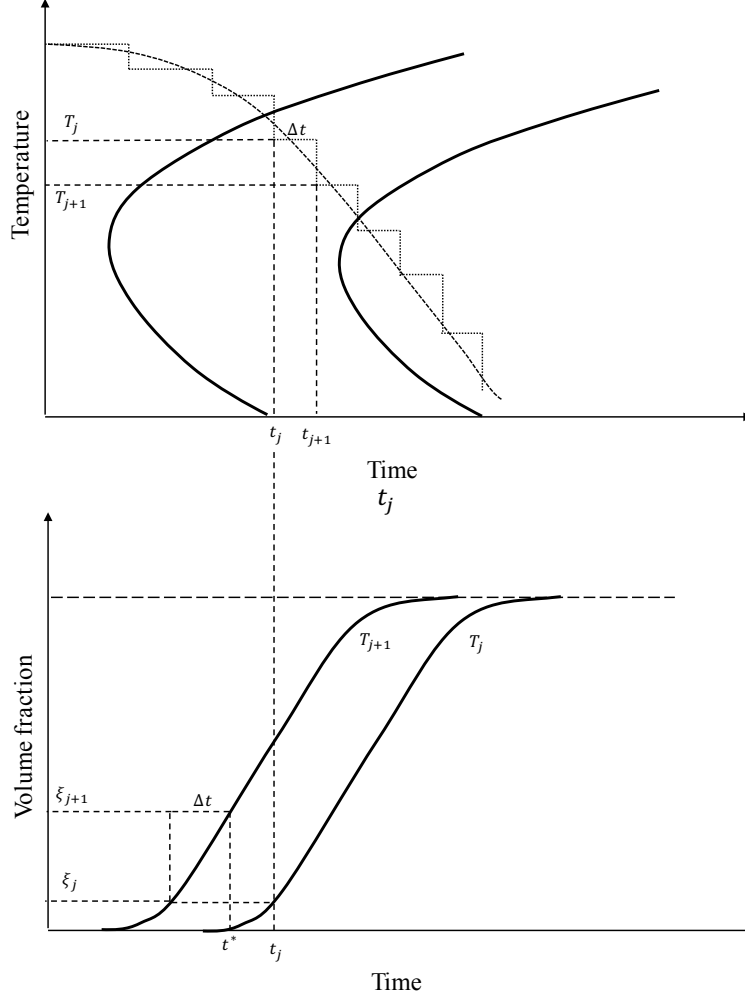


Figure 5.10: Anisothermal transformation kinetics with thermal history discretization

$$\xi_m = \xi_\gamma (1 - \exp(-K_{LV}(M_S - T)^{n_{LV}})) \quad (5.24)$$

where K_{LV} and n_{LV} are two parameters dependent by the chemical composition of the steel and equal to:

$$K_{LV} = 0.0231 - 0.0105C - 0.0017Ni + 0.0074Cr - 0.0193Mo \quad (5.25)$$

$$n_{LV} = 1.4304 - 1.1836C + 0.7527C^2 - 0.0258Ni - 0.0739Cr + 0.3108Mo \quad (5.26)$$

This model fit very well the transformation kinetics of martensite in low-alloyed steel giving also an accurate prediction of other transformation properties, such as M_{90} and retained austenite.

Chapter 6

Two Dimensional Benchmark Model

6.1 Introduction

The analysis of multiphysical problems is still considered, nowadays, a hard task, due to the several numbers of interactions between the different physics involved. The simulation of induction hardening processes require a precise description of all the phenomena, starting from the calculation of the eddy currents induced in the workpiece by the variation of the magnetic field produced by the coil, passing through the description of heat diffusion and the modelization of metallurgical phase transformations that occur, typically, in very short time.

Even if the physical formulations used in the model are appropriate, in order to obtain a reasonable result, the material needs to be characterized in a proper way.

Currently many FEM packages are able to propose valid formulations and coupling capabilities for induction heating problems (Cedrat Flux, Ansys, JMAG), where the classical electromagnetic formulations are coupled with thermal problem. On the other hand, many FEM packages have been proposed over the years, in order to help manufacturing industries which work in the field of heat treatments. Just to name some of them, Forge, Deform or Dante are able to predict the resulting metallurgical transformation at the end of a quenching process. It can be noticed that there is a lack for what concern a complete multiphysical approach to induction hardening processes.

The analysis has been divided in two main algorithms: the first one, an electromagnetic-thermal coupled simulation, has been developed with Cerdad Flux, the latter is a thermo-metallurgical simulation, implemented with Comsol and Matlab.

The electromagnetic-thermal problem has been solved by the use of a so-called indirect coupling method (Figure 6.1). This technique calls for an iterative process which consists of multiple elementary solving sequences for the electromagnetic and heat transfer problems. Practically, for each time-step of the solution, Maxwell's and Fourier's equations are solved multiple times, iteratively exchanging the results of calculations and accordingly updating material physical properties. This solving loop is repeated until a given precision of results (i.e., a sufficiently small variation of the residual) is reached [77]-[15]. The main

assumption here is that temperature variations are not significant during a certain interval of time predetermined by the user. Depending on the application, at least five iterations are typically required to make corrections in the electromagnetic field distribution via recalculation of the heat sources. For the great majority of induction heating applications, the indirect coupling is highly effective. Of course, the accuracy of the results are greatly affected by the chosen time-stepping for the solution and the required precision for the convergence of each step.

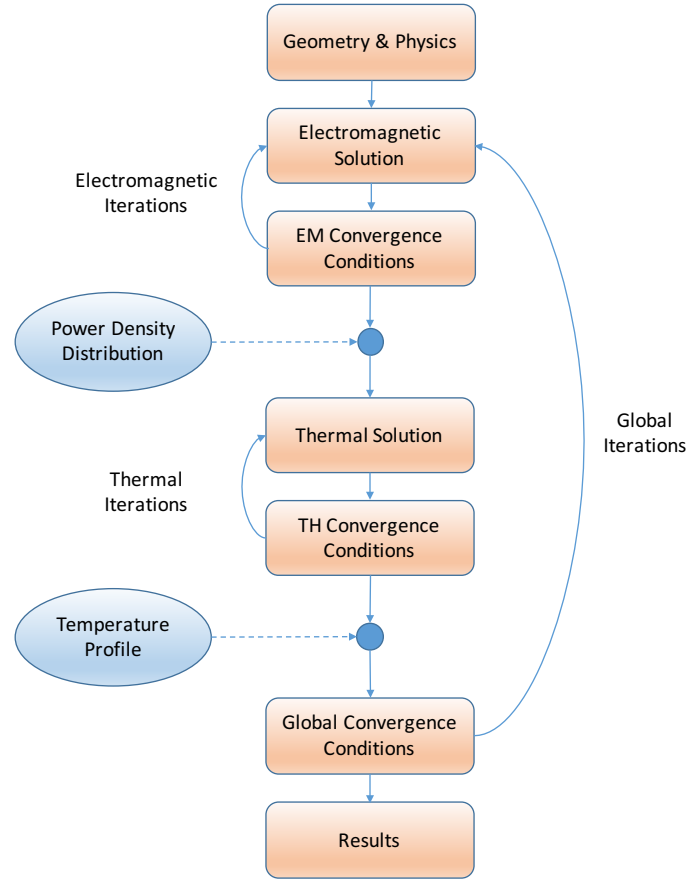


Figure 6.1: Indirect coupling scheme used in electromagnetic-thermal coupled simulation

When the electromagnetic-thermal coupled problem has been solved for each time step of the heating process, the power densities has been exported and used as internal heating source in the thermo-metallurgical model. This type of coupling is also called two-step coupling, because there are not any iteration in order to verify the precision of the results. For this reason the time step must be the smaller as possible, in order to do not introduce important variation of the temperature between each step (which could appreciably change the temperature dependent material's properties).

In Figure 6.2 the complete coupling scheme between the algorithms is shown.

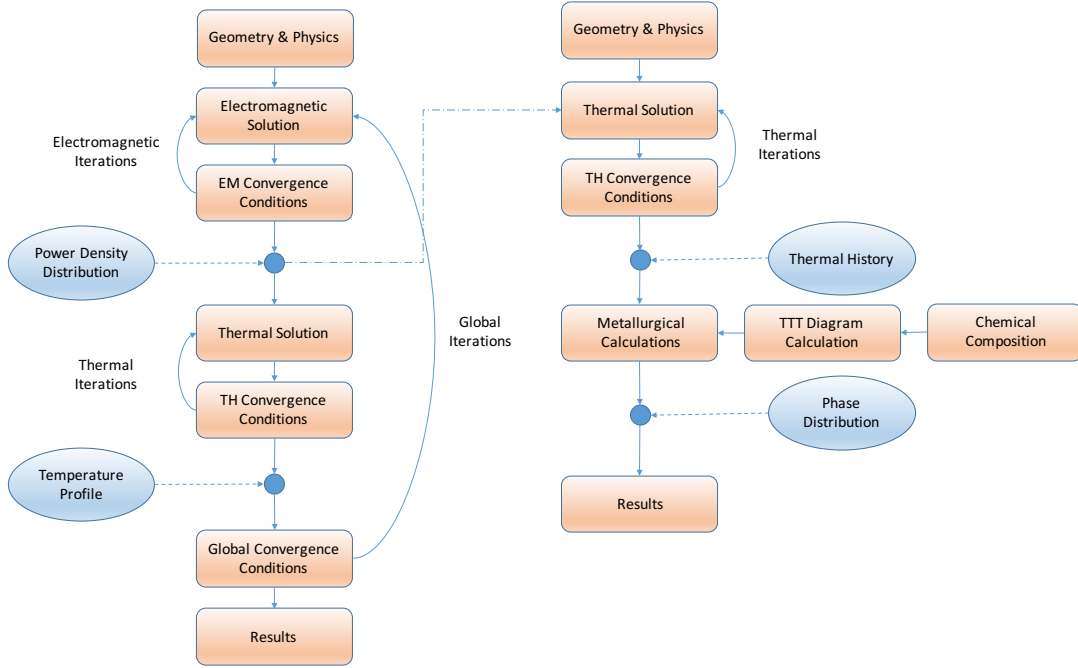


Figure 6.2: General scheme of overall simulation

In the thermo-metallurgical simulation the temperature rise, caused by the internal power densities imported from the previous simulation, is used to calculate the phase transformations during the process.

Comsol Multiphysics does not have a module for the solid-solid phase transformations in steel, so it has been developed with an external routine, exploiting the capabilities of Comsol/Matlab interfacing. At the end of each time step of the thermal-transient problem, the metallurgical phase distribution is evaluated with the procedure explained in Chapter 5 and the material's properties are updated for the next time step. At the same time, when transformations occur, also the latent heat of transformation is calculated and used as internal power density in the calculation.

During the quenching phase the workpiece is no more heated up and only thermo-metallurgical simulations are carried out. In this case special heat exchange boundary conditions (temperature dependent convection coefficient) has been imposed on the surfaces between air and workpiece.

6.2 Material's Properties

6.2.1 Chemical Composition

The AISI 4340 is a high strength steel grade commonly used for high quality mechanical components which are subjected to high stress loads, such as heavy duty shafts, gears,

axles, spindles, couplings, pins etc.

Thanks to the medium carbon content and presence of chromium, nickel, and molybdenum, it is particularly suitable for induction surface hardening applications requiring good hardenability and mechanical properties of the core. Its chemical composition is reported in Table 6.1 [78]-[79].

Table 6.1: Chemical composition in %wt of AISI 4340

Element	Weight %
C	0.38 - 0.43
Mn	0.60 - 0.80
P	0.035 (max)
S	0.04 (max)
Si	0.15 - 0.30
Cr	0.70 - 0.90
Ni	1.75 - 1.90
Mo	0.20 - 0.30
Fe	bal

The considered steel grade is used in the quenched and tempered condition, which is obtained by means of a prior heat treatment, producing a homogeneous martensitic microstructure, conferring relatively high hardness to the core (i.e., 35-40 HRC) after the tempering process in the range of 550-630°C. After this treatment, the microstructure results in tempered martensite, with small and dispersed carbides which guarantee high toughness to the base material.

By surface hardening such a kind of structure, it is possible to obtain a very hard case (i.e., 55-60 HRC) at considerable depths (i.e., 10-15 mm), without excessively increasing quenching intensity, which may result in cracking of the part upon severe cooling.

6.2.2 Electromagnetic Properties

Electrical Resistivity

The electrical resistivity is an intrinsic property that quantifies how strongly a given material opposes the flow of electric current. A low resistivity indicates a material that readily allows the flow of electric current. Resistivity is commonly expressed as a linear function of the temperature, according with the relation:

$$\rho(T) = \rho_0(1 + \alpha_\rho(T - T_0)) \quad (6.1)$$

in which T_0 is the reference temperature (K), ρ_0 is the resistivity at the reference temperature ($\Omega \cdot m$) and α_ρ is the temperature coefficient of resistivity (K^{-1}).

This model can be assumed as valid any for such types of materials or small temperature ranges of variations of resistivity in steels. As can be seen in Figure 6.3 the behavior of resistivity with the temperature is not really straight, but, after a non-linear behavior for

low temperatures, it presents a strong variation around the austenitization temperatures, due to the effect of phase transformations that occur at those temperature. A description of the polynomial function that approximate the non-linear behavior with temperature of the resistivity can be found in [?]

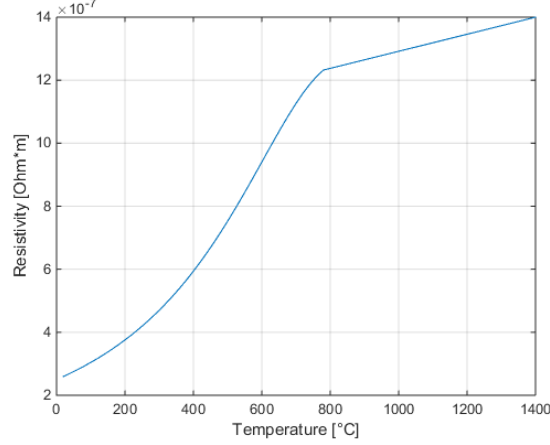


Figure 6.3: AISI 4340 resistivity behavior in fuction of temperature

Magnetic Permeability

Magnetic materials are generally characterized by a hysteresis cycle, which is represented by a closed surface in the $(\mathbf{H}, \mathbf{B})$ space. As it is di cult to correctly model hysteresis, a univocal $B(H)$ relationship is usually adopted for magnetic materials; soft magnetic materials, which are characterised by very low values of the coercive field, can be modelled by their curve of first magnetization, thus neglecting hysteresis losses. The $B(H)$ dependency used in the simulations is defined by the following relations:

$$B(H) = \mu_{r0}H + J_s \frac{H_a + 1 - \sqrt{(H_a + 1)^2 - 4H_a(1 - a)}}{2(1 - a)} \quad (6.2)$$

$$H_a = \mu_{r0}H \left(\frac{\mu_r - 1}{J_s} \right) \quad (6.3)$$

where μ_{r0} is the initial value of the relative magnetic permeability, J_s is the magnetic flux density saturation value (T), and a is an adjustment coefficient for the sharpness of the transition knee from unsaturated to saturated conditions ($0 < a < 0.5$; the smaller the coefficient, the sharper the curve transition knee). These coefficients have been tuned using a fitting technique based on experimental data taken from [80]. The obtained values are reported in Table 6.2 and the resulting curve is depicted in Figure 6.4.

The presented model can also be extended by introducing the effect of material demagnetization, when heated above the Curie temperature. This aspect can be taken into account by means of a temperature coefficient based on exponential functions, as follows:

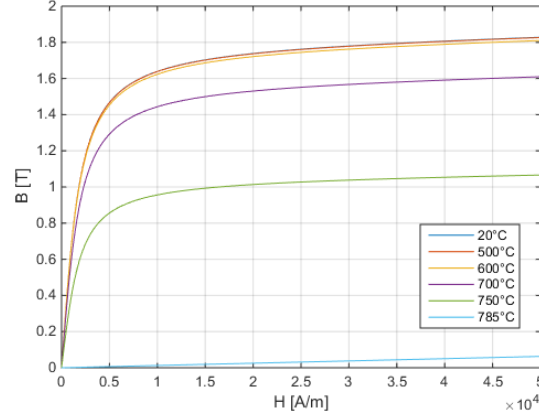


Figure 6.4: AISI 4340 relative magnetic permeability behavior in fuction of temperature and magnetic field strenght

$$B(H, T) = \mu_{r0}H + J_s \frac{H_a + 1 - \sqrt{(H_a + 1)^2 - 4H_a(1 - a)}}{2(1 - a)} COEF(T) \quad (6.4)$$

$$COEF(T) = \begin{cases} 1 - \exp\left(\frac{T - T_C}{C}\right), & \text{for } T < T_1 \Rightarrow (T_1 - T_C) = C \ln(0.9). \\ \exp\left(\frac{10(T_2 - T)}{C}\right), & \text{for } T > T_1 \Rightarrow (T_2 - T_1) = 0.1C \ln(0.1). \end{cases} \quad (6.5)$$

where T_C is the Curie temperature and C is the temperature constant. The exponential decay (Equation 6.5) is more or less pronounced depending on the value of the temperature constant: higher values of C (i.e., $C=100$) correspond to faster decays, lower values (i.e., $C=10$) to slower ones. The used values are reported in Table 6.2 and the resulting curves are depicted in Figure 6.4.

Table 6.2: Parameters used in the model for the description of megnetic properties of AISI 4340

Parameter	Unit	Value
μ_{r0}	-	600
J_s	T	1.8
a	-	0.5
T_c	°C	785
C	°C	40

6.2.3 Thermo-physical Properties

Thermo-physical properties are essential for the description of the temperature distribution in the workpiece during the heating and quenching stage. As it has been said in the previous chapters, the material during quenching process modeling can be seen as a mixture of different phases, each one with different thermo-physical properties dependent by the temperature. It is easily understandable that these material's properties need to be described in a different manner respect the electromagnetic properties (which are only small affected by the phase change). The global material's behavior can be described with the rule of mixture:

$$P(T, \xi_k) = \sum_{k=1}^N P_k \xi_k \quad (6.6)$$

in which P is the overall temperature-dependent thermal property of the phase mixture, P_k the thermal property of the k – *th* constituent of the phase mixture and ξ_k the volume fraction of k – *th* constituent. Here, the principal thermo-physical properties, used for thermo-metallurgical computation are presented, each one divided into the singular contributions of each phase. For this work the material's properties of ferrite, perlite and bainite have been considered as equal, while a big difference can be noticed between ferrite/pearlite/bainite, austenite and martensite.

Density

The density, or more precisely, the volumetric mass density, of a substance is its mass per unit volume. A strong variation in density accompanies the phase transition between BCC microstructure (typical of ferrite and pearlite) when it transforms into austenite FCC. In Figure 6.5 the density values, in function of the temperature, have been reported for the principal phases involved.

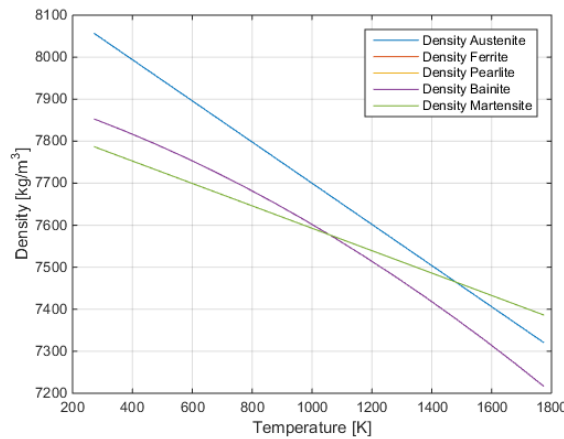


Figure 6.5: Temperature dependency of density for the main phases of AISI 4340

Thermal Conductivity

Thermal conductivity is a thermo-physical material property, which characterizes the quantity of heat transferred through a unit surface area in a unit time when the heat transfer is generated by a temperature gradient. In other words, it represents the ability of a material to transfer heat. In Figure 6.6 the thermal conductivity values, in function of the temperature, have been reported for the principal phases involved.

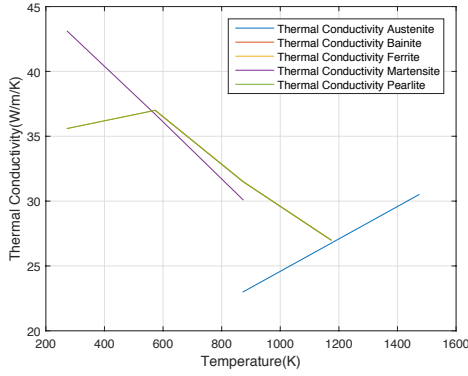


Figure 6.6: Temperature dependency of thermal conductivity for the main phases of AISI 4340

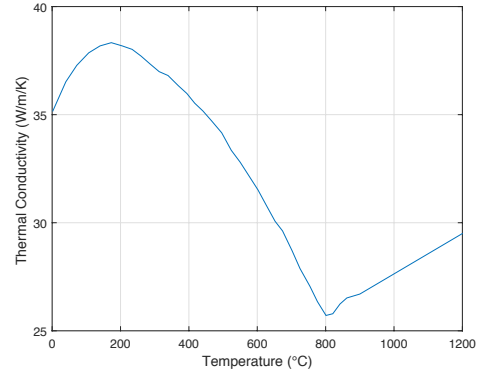


Figure 6.7: Temperature dependency of thermal conductivity of AISI 4340 steel

In the electromagnetic-thermal model developed in Flux, only the heating process has been considered and the phases involved in the process are the ferrite/pearlite and the austenite. From this point of view, the global thermal conductivity can be seen as the base material's property until the austenitization temperature and the austenite's one can be considered. The global behavior is shown in 6.7.

Heat Capacity

The heat capacity of a material is the heat quantity necessary to raise its temperature by one Celsius degree. In other words, it is a quantity that offers the possibility to evaluate the amount of heat exchanged by a body during a process characterized by a temperature variation. In Figure 6.8 the heat capacity values, in function of the temperature, have been reported for the principal phases involved.

The problems encountered for the description of the thermal conductivity in the electromagnetic-thermal model can be found also in the case of specific heat. The latent heat of phase transformation can be included in the simulation through a local increasing of the heat capacity (Flux considers the volumetric heat capacity which is the heat capacity per unit volume) in correspondence of the supposed transformation range of temperature.

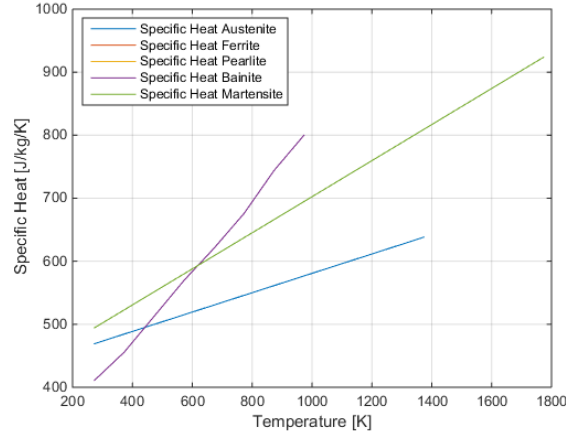


Figure 6.8: Temperature dependency of specific heat for the main phases of AISI 4340

6.3 Electromagnetic-Thermal Analysis

In this section the implementation of the electromagnetic-thermal coupled simulation will be described. A simple geometry has been chosen in order to reduce the geometrical complexity effects and to concentrate the work on theoretical and methodological issues.

6.3.1 Geometry and Mesh

The test-article is a round bar made of AISI 4340 with a total length of 100 mm and a diameter of 32 mm. The inductor used is a double-turn square-section coil made of copper and its main dimensions are shown in Table 6.3(b).

Table 6.3: Geometric dimensions of the model

(a) Billet Dimensions			(b) Coil Dimensions		
Parameter	Unit	Value	Parameter	Unit	Value
Billet Height	mm	100	Coil Height	mm	8
Billet Radius	mm	16	Coil Width	mm	8
			Coil Thickness	mm	1.5
			Inner Radius	mm	20

To obtain an acceptable solution of the electromagnetic problem, a good practice is the definition of a region, that is called infinite box, in which the magnetic field goes to zero, in other words it is imposed what happens theoretically to infinity. The infinite box, that in our case is a ring-shaped air region, is defined by an inner radius and an outer radius. The infinite box is a strong simplification and may affect the physical solution, if it has not been chosen properly. In order to avoid such kind of problems the dimension of the infinite box may be at least 10-20 times bigger than the maximum dimension of the active part.

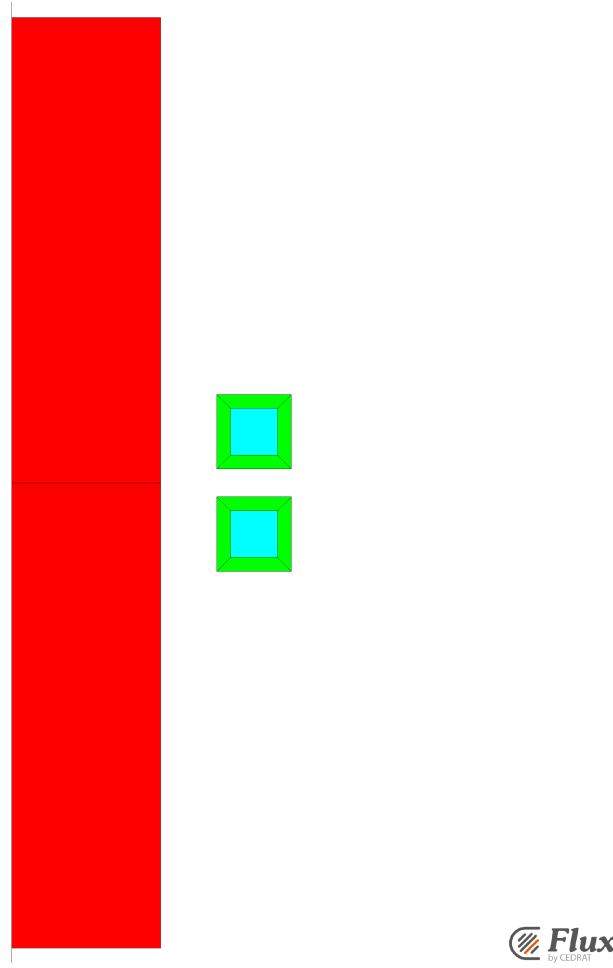
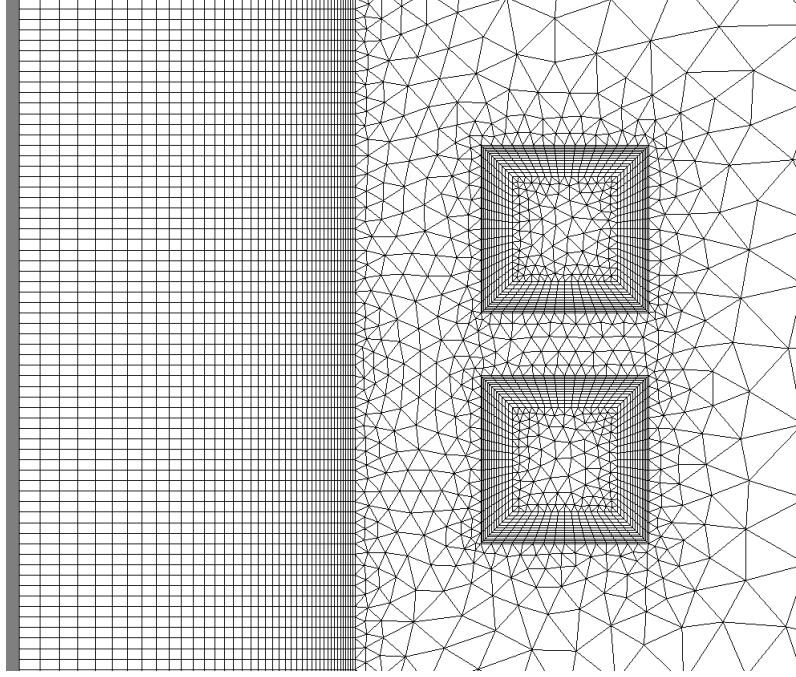


Figure 6.9: 2D geometry of the electromagnetic model

The mesh is composed by 14081 second-order elements. The mesh in the billet and in the coil is composed by quadrilateral element, while in the air and inside the coil the mesh is typically triangular in order to take advantage of the relaxing capabilities of the meshing algorithm.

In induction problems, in order to obtain a realistic description of the phenomena, the precise calculation of power densities distributions is mandatory. Induction heating is characterized by internal heating sources, located in a thin superficial layer, while the internal part of the workpiece is not affected. A good way to approach the problem is to respect the empirical rule of almost 2-3 elements in the first penetration depth. In Table 6.4 the minimum skin depths for the different materials involved in the model are shown.

During the heating process, the skin depth in the billet changes due to the effect of the temperature and the magnetic field strenght on the material's properties. Remembering the definition of skin depth ($\delta = \sqrt{(2\rho)/(\omega\mu\mu_0)}$), it can be easily understood that the higher the temperature the higher will be the skin depth. Generally, when the mesh is

**Figure 6.10:** Detail of the mesh of electromagnetic model**Table 6.4:** Skin depths for the materials in the model at 6300 Hz

Penetration Depth	μ_r	ρ ($\Omega \cdot \text{m}$)	Value (mm)
δ_{steel}	600	$2.5 \cdot 10^{-7}$	0.13
δ_{copper}	1	$2.0 \cdot 10^{-8}$	0.90

realized, the better way to do it is just to consider the worst case as possible (in this case room temperature, with the maximum magnetic permeability), in order to calculate the dimension of the superficial elements.

6.3.2 Physics Implementations

Although the strong coupling of the systems of thermal and magnetic equations is very interesting from the theoretical point of view, in many cases this is completely uninteresting from the computational point of view, due to the important difference between the values of the thermal and magnetic time constants. Indeed:

- from the electromagnetic point of view, in induction heating the range of frequencies is comprised between 50 Hz and 1 MHz; consequently, the period of the electromagnetic phenomena has values always lower than 0.02 seconds;
- from the thermal point of view, the value of the time constant of a transient thermal phenomenon is of the order of a second, often higher, practically never lower than 0.1

s. However, to treat a strong coupling, there is only one “time” variable for all the phenomena involved in the coupling. The reduced value of the time step which must be adapted to the smaller time constant, will lead to prohibitive computation times, as the time of the study must be related to the greatest time constant. Thus, in order to obtain acceptable values of the computation time in case of magnetic/thermal coupling, it is necessary to decouple the thermal and the magnetic solving processes.

In Flux the two main physics are weakly coupled. The thermal and the electromagnetic processes are solved separately, but the two systems of equations are coupled by means of some terms [81]-[82]-[83]-[84]:

- The **temperature** (resulting from thermal solution) is a quantity which is involved in the system of electromagnetic equations by means of the material physical properties such as magnetic permeability and resistivity.
- The **power losses** (resulting from the electromagnetic solving) is the term corresponding to the heating source of the system of thermal equations.

In the particular case, if we consider that the electric currents have a sinusoidal dependence, the magnetic state of the device can be considered as harmonic and the variation with respect to the temperature of the physical properties is slow and continuous, so we can assume that the model can be approximated with the Flux modules *Steady State AC Magnetic* for the electromagnetic part and *Transient Thermal* for the thermal calculation. In figure 7.5 a scheme of the Flux’s algorithm for the solution of Magneto-Thermal problem is shown.

A scheme of the principal electrical parameters that characterize the heating process are shown in Table 6.5. The time of two seconds is the time needed (deduced after some trials), with this conditions, to obtain a reasonable temperature profile on the sample, which will permit to be easily analyzed after the thermo-metallurgical simulations. There are some additional technological aspects to take into account during the heating process, such as the maximum temperature reached, that should not exceed the value of 1100°C, otherwise the austenitic grain size could considerably increase.

The losses in the coil can be strongly reduced if the thickness of the coil respects the relation $s \geq \frac{\pi}{2} \delta$ [18]. In this case the coil geometry was fixed, so the frequency has been tuned in order to verify the previous relation.

Table 6.5: Process parameters

Parameter	Unit	Value
Current	A	3700
Frequency	Hz	6300
Time	s	2

The electromagnetic problem has been solved by means of vector magnetic potential formulation, imposing the flowing current on the coil, which has been kept constant during the whole process.

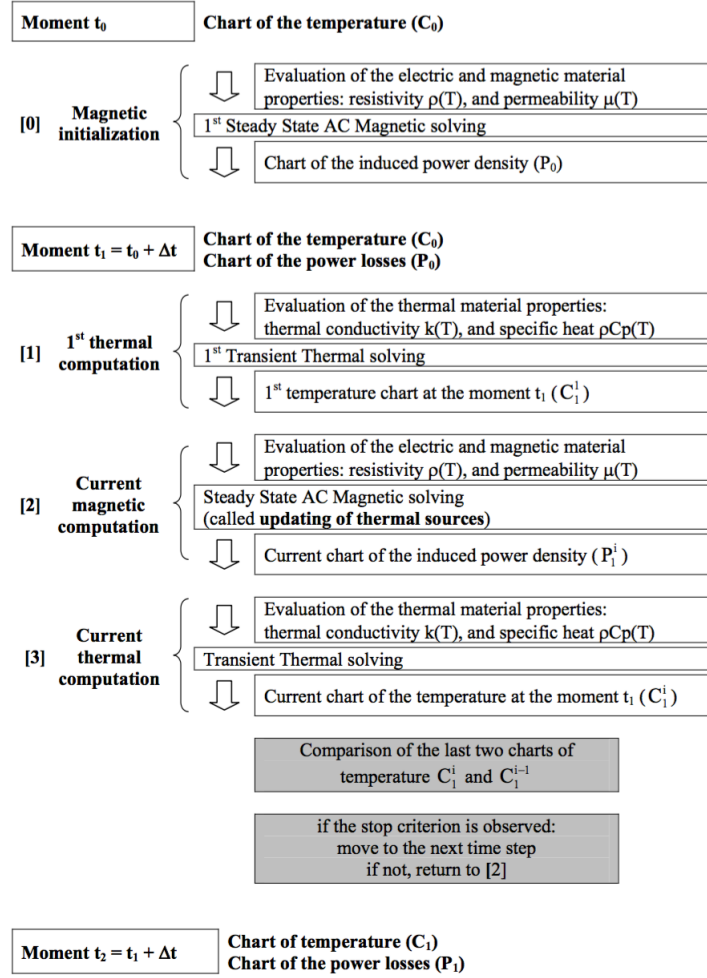


Figure 6.11: Scheme of the solution algorithm for electromagnetic-thermal coupled problem [81]-[82]-[83]-[84]

During the heating process heat losses by convection and radiation have been considered on the surfaces of contact between steel and air. Convection heat exchange parameter is generally dependent by the exchanging medium and the relative velocity between the fluid and the body, but in this case (rotation speed around 200 rpm), it can be assumed equal to 15-20. The emissivity of this kind of steel can be assumed equal to 0.8 [18].

The linear equation system has been solved with the direct solver MUMPS (Multifrontal Massively Parallel Solver) [85]-[86]. Both the thermal and the magnetic problems are characterized by non-linear problem and their solution can be obtained using the Newton-Raphson algorithm.

The time stepping for thermal transient problem is fixed and equal to 0.05 seconds each.

6.3.3 Results

In Figure 6.12 the current density distribution in the coil is shown. The current density is located on the inner side of the coil section because of the characteristic induction heating effects (skin, ring and proximity effect).

In Figure 6.14 and Figure 6.15 the power density distribution at the beginning and at the end of the process are shown. This distribution is drastically changed during the heating process for the effect of resistivity increasing and the demagnetization effect of Curie Temperature.

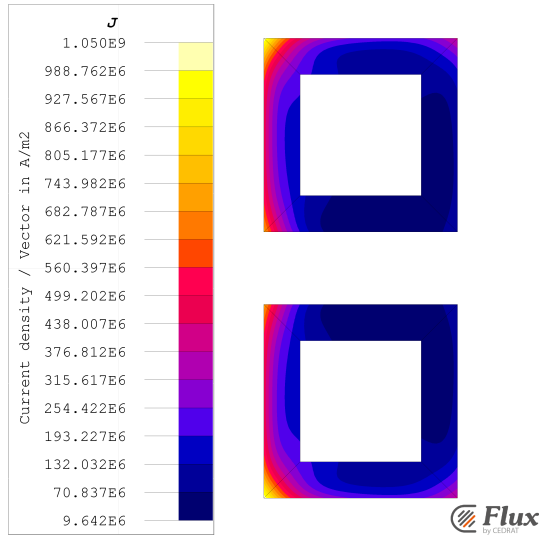


Figure 6.12: Current density distribution in the coil

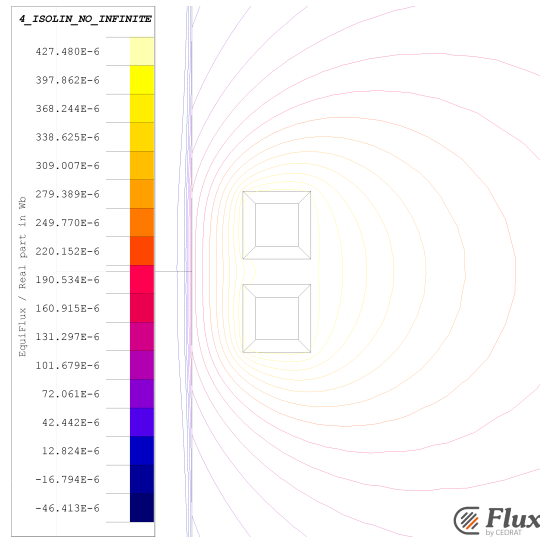


Figure 6.13: Equiflux lines

In Figure 6.16 the radial power density distribution for three different times has been shown. The measurements are related to the central section of the billet. The blue curve is the radial power density distribution at the beginning of the process. This distribution appears to be much more deep respect what was initially predicted, considering the maximum magnetic permeability. The permeability is considerably reduced considering the magnetic saturation effect (Figure 6.17.)

The second curve is related to the power density distribution after one second of treatment, just before the achievement of the Curie Temperature. The heating depth is enhanced for the effect of the increment of resistivity with the temperature. The last curve is related to the final power density distribution, when the Curie temperature has been overcome on a considerable part of the billet. The superficial value of power density is strongly reduced respect the beginning of the process, but the maximum value is located in the inner part of the treated body in correspondence of the transition zone between the demagnetized portion of material and the magnetic one.

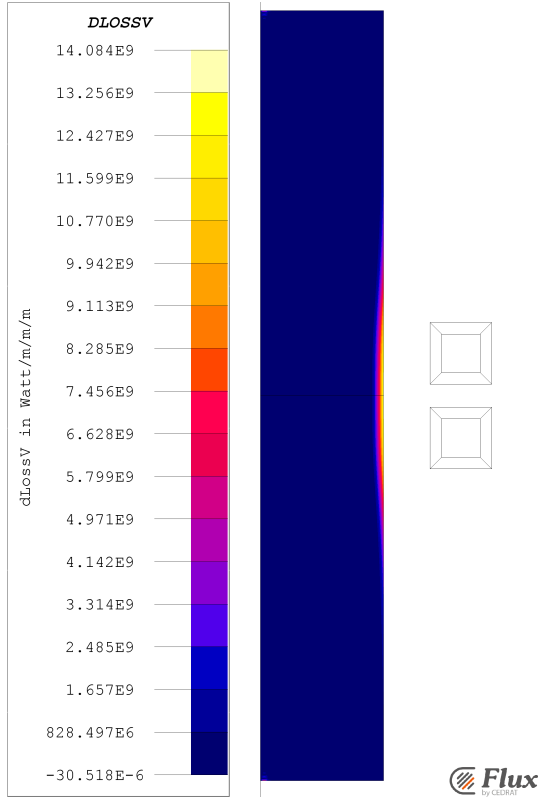


Figure 6.14: Power density distribution at the beginning of the process

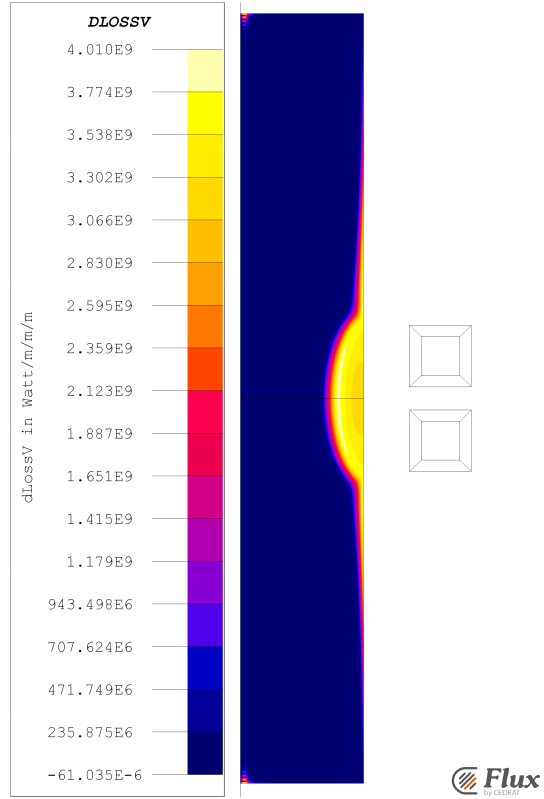


Figure 6.15: Power density distribution at the end of heating process

6.4 Thermo-Metallurgical Analysis

6.4.1 Geometry and Mesh

The geometry of the model is the same as described in the electromagnetic model but, for thermo-metallurgical computation, the coil and surrounding air are not taken into account.

A fine mesh in the thermo-metallurgical model is mandatory, not only to interpolate correctly the power densities imported from the electromagnetic computation, but also to predict properly the phase transformations. Phase transformation will occur initially on the surface, but then it moves towards the center of the billet, while the temperature is rising. For this reason the mesh has to be fine enough, not only on the superficial layer (as in electromagnetic problem), but also on the rest of the treated body. The mesh is composed by 14000 first-order quadrilateral elements. The radial dimension of the elements changes linearly from 0.15 mm on the surface to 0.3 mm in the billet's center, while the the height of each elemet is kept constant and equal to 0.5 mm. A picture of the mesh used in Comsol is shown in Figure 6.19, with a detail of the central portion of the billet.

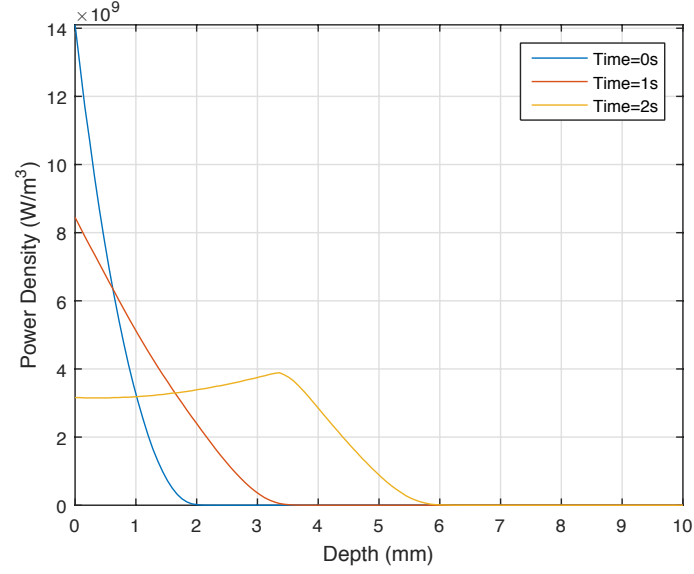


Figure 6.16: Power density distributions at the beginning of the process, after 1 seconds and after 2 seconds

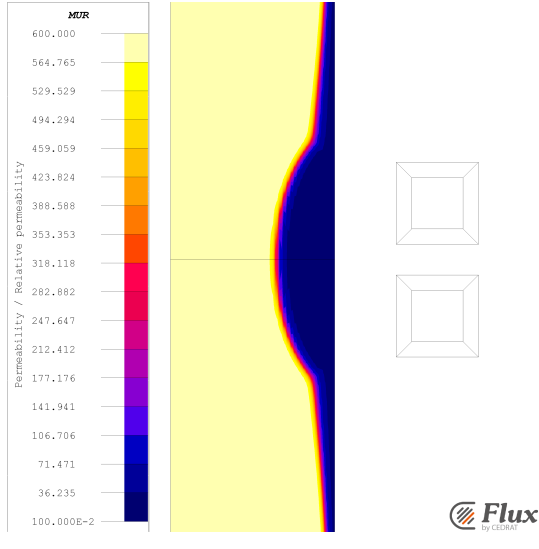


Figure 6.17: Relative magnetic permeability distribution after 2 seconds of heating

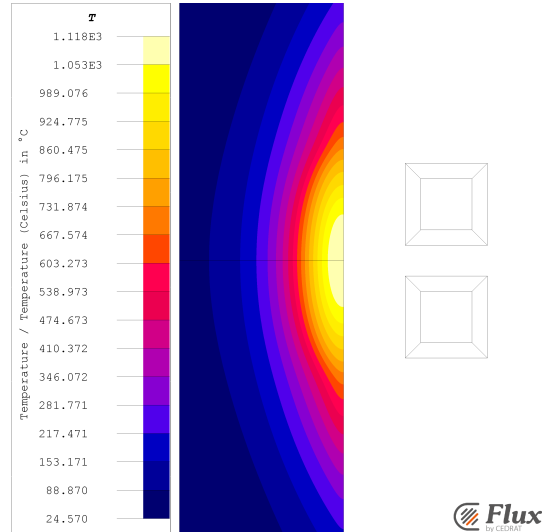


Figure 6.18: Temperature distribution after 2 seconds of heating

6.4.2 Physics Implementations

The model has been solved interfacing the FEM package COMSOL Multiphysics, for the transient thermal calculations, with MATLAB for the computation of metallurgical phase transformations. In order to use the data calculated through the electromagnetic

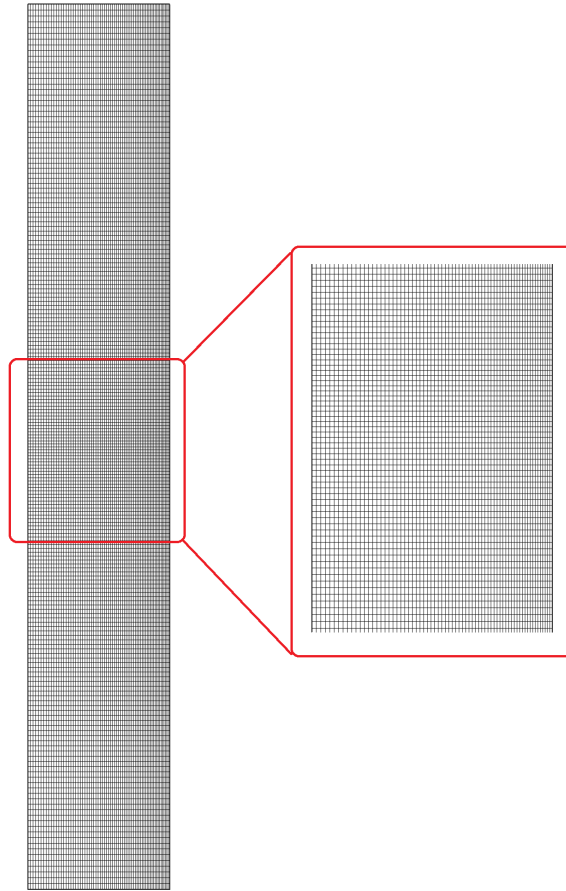


Figure 6.19: Detail of the first-order quadrilateral mesh used in Comsol for thermal calculation

simulation, the power density distribution has to be interpolated in order to adapt correctly to the new mesh. The global material's properties have been calculated at the beginning of each time step as a composition of the phases' properties multiplied for the weight fraction with an external routine developed in Matlab. These values have been evaluated on the nodal point of the mesh, so they need to be interpolated before the use in the thermal model. During the heating step the same thermal boundary conditions used in the electromagnetic-thermal model have been applied on the exchanging surfaces. After each time step the nodal thermal history and the temperature rate are used in order to predict the phase transformations which occur during heating. As shown in previous chapters the material is heated up in order to transform part of it in austenite. This kind of transformation is predicted with CHT diagrams.

When the heating process is finished the workpiece is rapidly cooled down in order to transform the austenite into martensite. The cooling process can be divided in two steps:

1. the first one in which the workpiece is moved towards the shower. The duration of

this step depends upon the movement speed of the machine spindle, but in our case it can be assumed equal to 0.5. The heat exchanges during this period are the same of the heating process (same heat convection coefficient and emissivity).

2. a second step in which the material is quenched in the shower until the sample reaches the room temperature. The expected duration is around 30 seconds and it is characterized by a temperature-dependent heat convection coefficient (Figure 7.17).

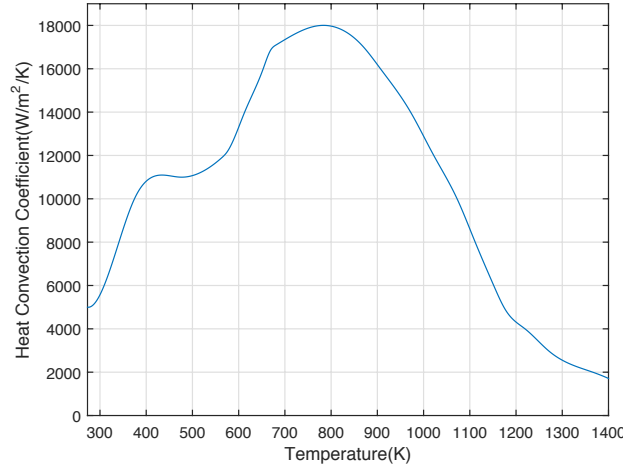


Figure 6.20: Heat convection coefficient in fuction of surface's temperature

Phase transformations during quenching can be predicted through the use of isothermal diagrams (Figure 7.18), which may be defined with the algorithm explained on previous chapters. The nodal thermal history has been discretized in isothermal steps, in order to apply the anisothermal transformation kinetic models with Scheil's additivity principle.

Both during heating and quenching the latent heat of transformation is evaluated as:

$$\frac{dQ_{LH}}{dt} = \sum_{k=1}^N c_k \xi_k + \sum_{k=1}^N \Delta H_k \frac{d\xi_k}{dt} \quad (6.7)$$

and used as a power density absorbed or released during the transformation depending if the current transformation is endothermic or exothermic.

Once the microstructure has been evaluated the material's properties for the following time step are updated in function of phase composition and temperature.

6.4.3 Results

When the whole simulation is finished, the phase distributions (austenite, bainite, ferrite, martensite and pearlite) can be verified directly in the post-processors of Comsol Multiphysics. In figure 6.22 and Figure 6.23 the temperature distribution, at the end of heating step, and the corresponding austenite distribution are shown. The differences in

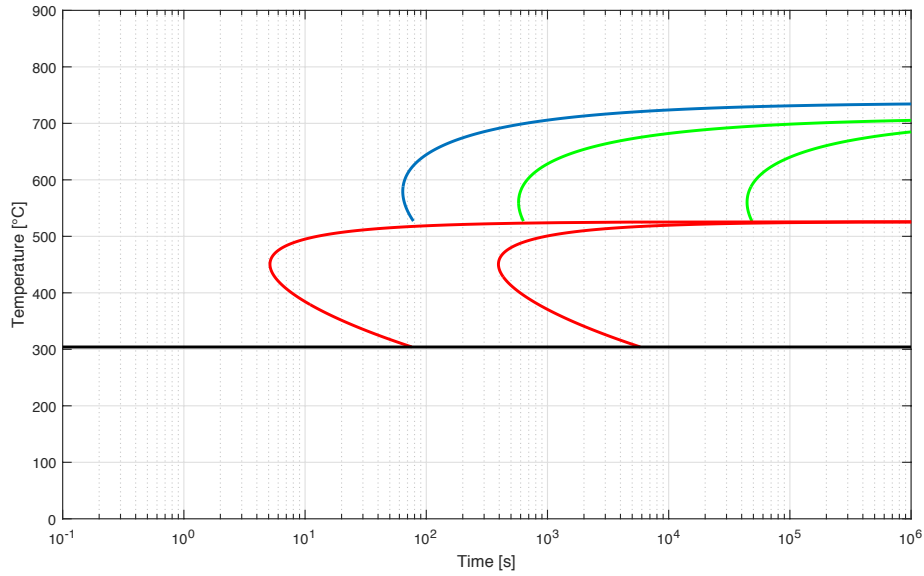


Figure 6.21: TTT diagram of AISI 4340

temperature calculation between the electromagnetic-thermal model (Figure 6.18) and thermo-metallurgical one (Figure 6.22) are limited to 8 °C (less than 1%) on the maximum temperature. This small difference can justify the simplification introduced decoupling the magneto-thermo-metallurgical model in two separated simulations.

In figure 6.24 a comparison between temperature and martensite distributions during quenching is shown. The pictures are related to different time steps of simulation and the corresponding time is calculated since the beginning of the shower. The surface temperature decreases fast at the beginning of the quenching process, but then, the heat removal from the workpiece is controlled by the thermal conductivity of the material and a pretty long time is required to completely cool down the body.

The material begins to transform to martensite on the surface, starting from the edges of the austenitized part, since these zones are surrounded by a colder part and the temperature drop is higher. After around 6.5 seconds the transformation interests more or less the entire austenitized part, with the transformation front which is located in the inner part.

After 30 second of shower quench the billet can be considered completely cooled down and as can be seen on Figure 6.25(e) the austenite completely transform into martensite. The thin layer between the previous microstructure (mixture of pearlite and ferrite) around the martensitic area is a transition zone in which martensite, ferrite and pearlite coexist at the same time.

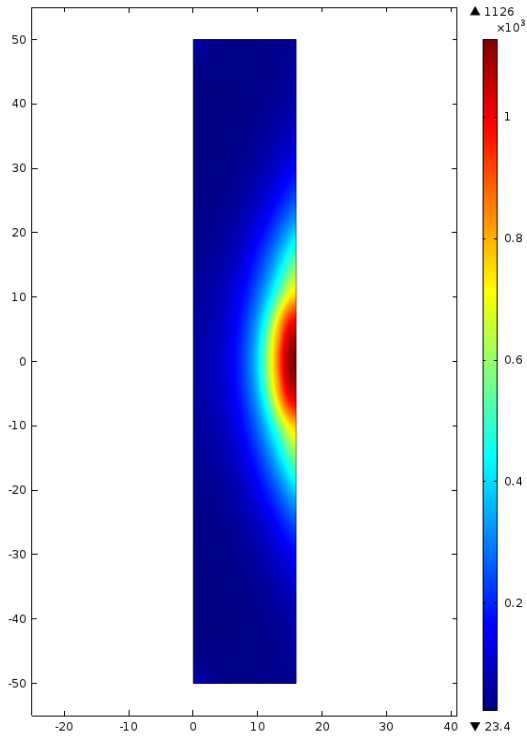


Figure 6.22: Temperature distribution at the end of the heating process

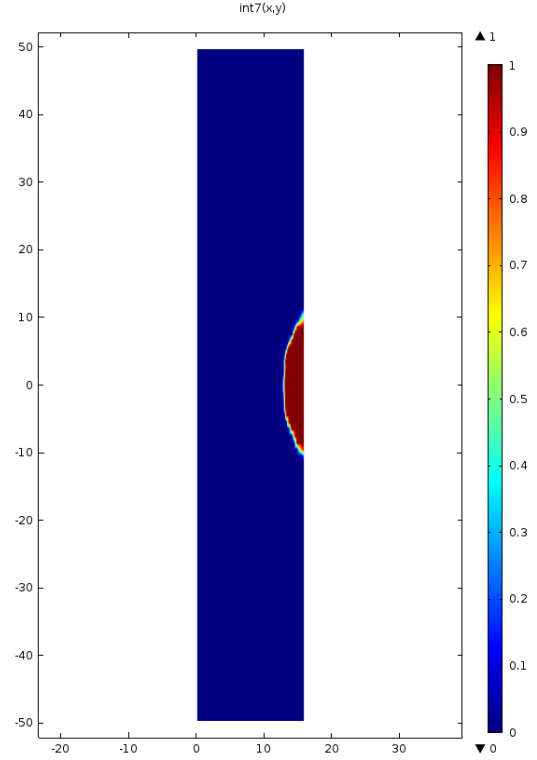


Figure 6.23: Austenite distribution at the end of the heating process

6.5 Experimental Validation

Once the simulation gives the expected results, an experimental validation has been developed in order to verify the martensite distribution at the end of of the entire process.

The tests have been carried out with an industrial induction hardening machine fed by an IGBT converter 200 kW 2-30 kHz. The resonance frequency may be set changing the capacitance of the capacitors bank. The process parameters have been verified using a Rogowsky coil for the measurement of the current and frequency in the coil.

Before the treatment the billet has been quenched in oil and tempered at 580°C in order to achieve the desired microstructure. After the heating the sample has been quenched in a shower of water-based polymer solution of Polyalkylenglicole (PAG) with a concentration of 12% for 30 seconds. The sample has been later cut down on its central section in order to compare the experimental results with the simulation. The surface has been polished and etched with Nital 3% in order to make it visible the metallurgical microstructure. Figure 6.25 shows a comparison between the experimental results (on the left) with the martensite distribution obtained through numerical simulation.

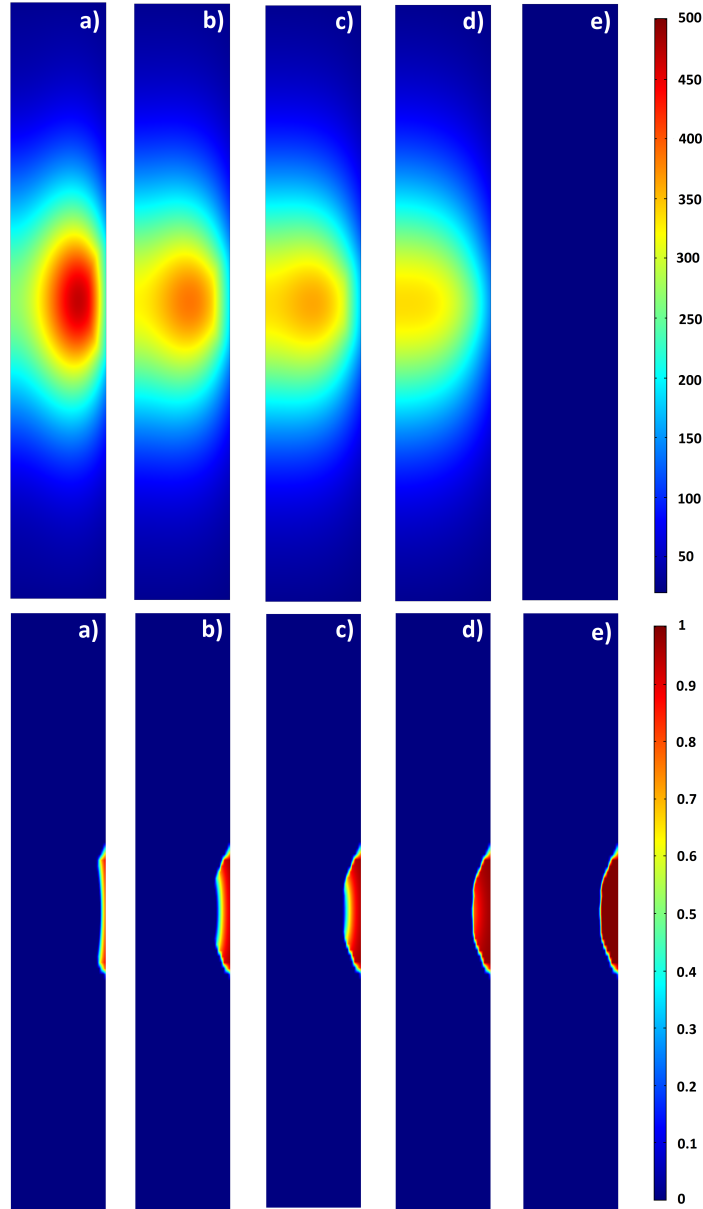


Figure 6.24: Comparison between temperature profiles and martensite distribution after 4s (a), 5s (b), 5.5s (c), 6.5s (d), 30s (e)

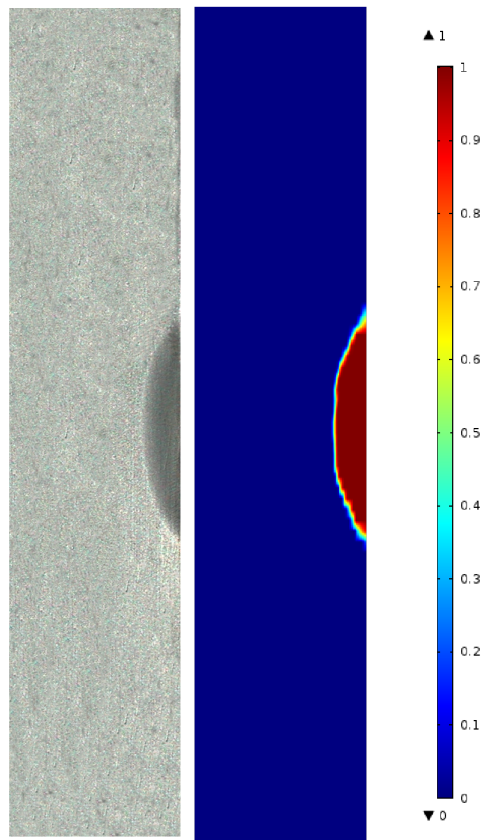


Figure 6.25: Comparison between experimental distribution of martensite (left) and numerical simulation (right)

Chapter 7

3D Model

7.1 Introduction

Induction hardening is widely applied for the heat treatment of components mainly in the aeronautical and automotive sectors because of its peculiar advantages like high quality and repeatability of process and its easy automation. Contour induction hardening is particularly used in the last years as a valid substitution of carburizing processes in order to guarantee to the gear's surface very high resistance to contact fatigue and tooth bending fatigue. The advantages of induction hardening technology are various, but the set-up of the process generally requires a particular attention. For many years, the choice of process parameters has been done through trial and error approach, reducing drastically the convenience of this technology, due to the high costs of development. In the last years Finite Element simulations have been proposed as a method for process design in order to determine the temperature profile during the process.

The new challenge can be considered the multiphysical-multiscale approach in order to describe the microstructural evolution of the steel during heating and quenching and its contribution to generation of residual stresses.

A multiscale multiphysical finite element (FE) analysis is presented in this chapter for the prediction of microstructural evolution during induction hardening processes on a complex geometry such as a gear used in aeronautical sector.

An ad hoc external routine has been developed in order to calculate the metallurgical phase changes during heating and cooling process associated with non-isothermal transformations. This routine has been coupled with commercial FEM codes able to solve the coupled electromagnetic and thermal problem that typically describes the induction heating processes.

7.2 Test Article

The reference workpiece is a 56-teeth cylindrical spur gear, characterized by a pitch diameter of 140 mm ($m=2.5$ mm). Thanks to their relatively easy design and symmetric geometry, spur gears are relatively simple to manufacture and heat treat, thus representing an ideal

candidate for the purposes of this study. The employed material is a low-alloy, medium carbon steel with 0.40% carbon content (AISI 4340), exhibiting good hardenability and high superficial hardness, both beneficial factors for the selected application. The material's properties of this steel have already been discussed in the previous chapter.

The considered steel grade is used in the quenched and tempered condition, which is obtained by means of a prior heat treatment, producing a homogeneous martensitic microstructure, conferring relatively high hardness to the core (i.e., 35-40 HRC) after the tempering process in the range of 550-630°C. After this treatment, the microstructure results in tempered martensite, with small and dispersed carbides which guarantee high toughness to the base material.

By surface hardening such a kind of structure, it is possible to obtain a very hard case (i.e., 55-60 HRC) at considerable depths (i.e., 10-15 mm), without excessively increasing quenching intensity, which may result in cracking of the part upon severe cooling.

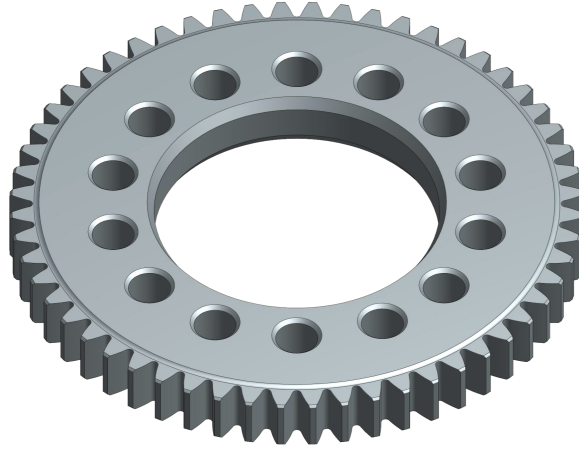


Figure 7.1: Gear geometry

7.3 Electromagnetic-Thermal Analysis

7.3.1 Geometry and Mesh

The geometry of the model is composed by the gear, a single turn coil, two flux concentrators placed on the planar faces of the gear and a stainless steel support to clamp all these parts together (Figure 7.2a). The importance of the two flux concentrators is due to the possibility to reduce the edge effect on the root of the gear, but it will be discussed in more detail later.

The process of contour induction hardening of gears is characterized by a final stage in which high frequencies (in the range of 100-400 kHz) are used in order to achieve the typical contour austenitized shape along the gear profile. The use of high frequencies requires a very fine mesh along the tooth profile and the reduction of model complexity is mandatory. The model can be reduced exploiting the symmetries present in the model, in

particular along the middle plane of the gear and the radial symmetries. In this way the model can be simplified to a quarter of a tooth, reducing the overall complexity of a factor 1/224 (Figure 7.2b).

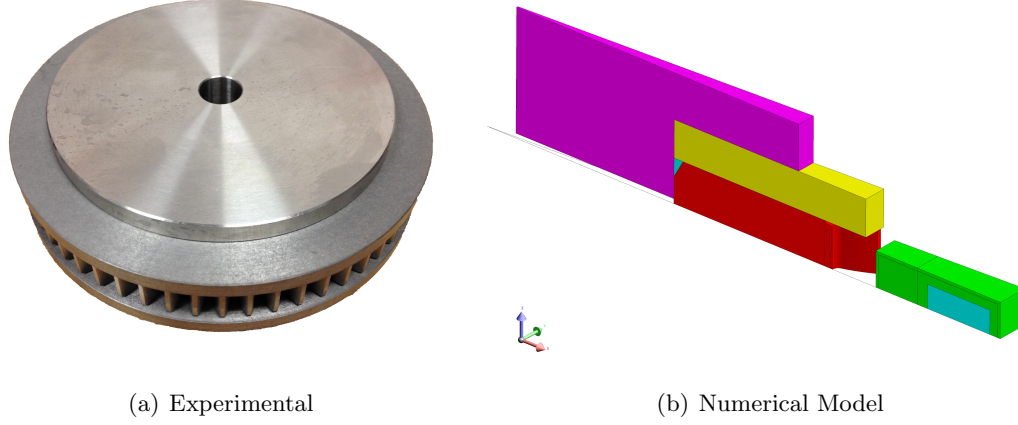


Figure 7.2: Experimental layout of the gear (left) and geometry of the electromagnetic model: Gear (red), Coil (green), Flux Concentrator (yellow), Support (magenta) (right)

The mesh is composed by 57903 nodes and 269282 first-order volumic elements. The mesh mainly consists on tetraedral elements. except on a superficial layer on the coil, in where the mesh is hexaedral. On the gear's profile surface the mesh has been mapped in order to make it the more regulare as possible.

In induction heating problems, in order to obtain a realistic description of the phenomena, the precise calculation of power densities distributions is mandatory. Induction heating is characterized by internal heating sources, located in a thin superficial layer, while the internal part of the workpiece is not affected. A good way to approach the problem is to respect the empirical rule of almost 2-3 elements in the first penetration depth. In Table 7.1 the minimum skin depths for the main conductive materials involved in the model are shown.

Table 7.1: Skin depths for the materials in the model at different frequencies

Penetration Depth	Frequency (Hz)	μ_r	ρ ($\Omega \cdot m$)	Value (mm)
δ_{steel}	10000	600	$2.5 \cdot 10^{-7}$	0.10
δ_{copper}	10000	1	$2.0 \cdot 10^{-8}$	0.71
δ_{steel}	185000	600	$2.5 \cdot 10^{-7}$	0.02
δ_{copper}	185000	1	$2.0 \cdot 10^{-8}$	0.16

The skin depths for AISI 4340, predicted in Table 7.1 must be taken with special care. They refer to the maximum value of permeability, achieved with no or low magnetic fields. In the case of induction hardening the magnetic field magnitude on the surface of the

workpiece can easily reach values of 10^5 H/m, strongly reducing the value of the magnetic permeability on the interested area. Considering, in first approximation, a value of relative magnetic permeability equal to 10 (and this assumption is not far from real values), the skin depths become equal to 0.79 mm for a frequency of 10 kHz and equal to 0.18 for the frequency of 185 kHz. With this assumption the skin depths are around eight times bigger than the worst case and can be used in order to design a proper mesh, without increasing excessively the computational time of the problem.

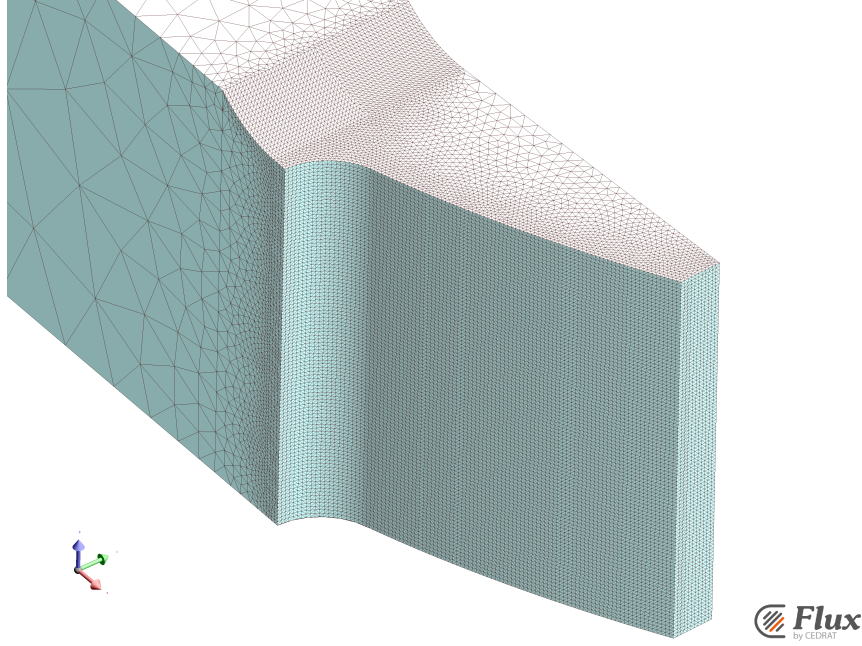


Figure 7.3: Detail of the mesh on the gear tooth

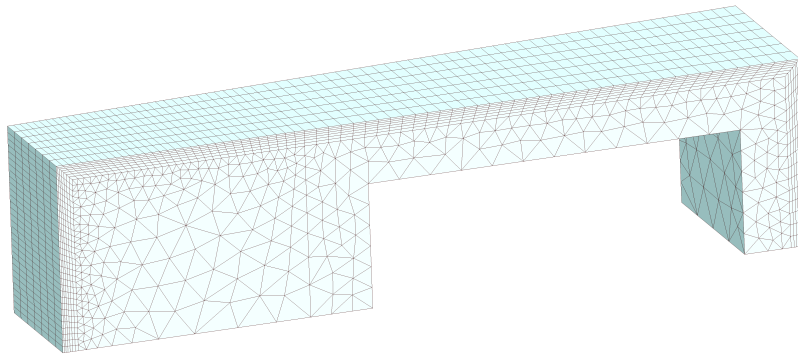


Figure 7.4: Detail of the mesh on the coil

The dimension of the elements on the gear's profile is kept constant and equal to 0.07 mm, and not any relaxation algorithm has been applied on the surface. Doing so, it is

possible to respect the empirical rule of at least two elements for penetration depth in the worst case (high frequency treatment). The volumic mesh has been slightly relaxed towards the central part of the tooth, where the effects of induced currents are supposed to be lower (Figure 7.3).

In the coil, the mesh has been designed on a radial face and extruded along the circumferential direction (Figure 7.4). A superficial layer 0.5 mm thick has been introduced in the geometry in order to define a mesh controlled volume. In this area the bigger part of the coil's elements is concentrated in order to calculate correctly the current density distributions. The dimension of the superficial elements is 0.05 mm and has been linearly relaxed to 0.1 mm after 0.5 mm.

In the rest of the model the mesh is kept free in order to exploit the relaxation capabilities of the meshing algorithm. The tetrahedral mesh of the model has been realized by means of Advancing Front (Netgen) Algorithm [87].

7.3.2 Physics Implementation

Although the strong coupling of the systems of thermal and magnetic equations is very interesting from the theoretical point of view, in many cases this is completely uninteresting from the computational point of view, due to the important difference between the values of the thermal and magnetic time constants. Indeed:

- from the electromagnetic point of view, in induction heating the range of frequencies is comprised between 50 Hz and 1 MHz; consequently, the period of the electromagnetic phenomena has values always lower than 0.02 seconds;
- from the thermal point of view, the value of the time constant of a transient thermal phenomenon is of the order of a second, often higher, practically never lower than 0.1 s. However, to treat a strong coupling, there is only one “time” variable for all the phenomena involved in the coupling. The reduced value of the time step which must be adapted to the smaller time constant, will lead to prohibitive computation times, as the time of the study must be related to the greatest time constant. Thus, in order to obtain acceptable values of the computation time in case of magnetic/thermal coupling, it is necessary to decouple the thermal and the magnetic solving processes.

In Flux the two main physics are weakly coupled. The thermal and the electromagnetic processes are solved separately, but the two systems of equations are coupled by means of some terms [81]-[82]-[83]-[84]:

- The **temperature** (resulting from thermal solution) is a quantity which is involved in the system of electromagnetic equations by means of the material physical properties such as magnetic permeability and resistivity.
- The **power losses** (resulting from the electromagnetic solving) is the term corresponding to the heating source of the system of thermal equations.

In the particular case, if we consider that the electric currents have a sinusoidal dependence, the magnetic state of the device can be considered as harmonic and the variation with

respect to the temperature of the physical properties is slow and continuous, so we can assume that the model can be approximated with the Flux modules *Steady State AC Magnetic* for the electromagnetic part and *Transient Thermal* for the thermal calculation. In figure 7.5 a scheme of the Flux's algorithm for the solution of Magneto-Thermal problem is shown. A scheme of the of the process is shown in Table 7.2.

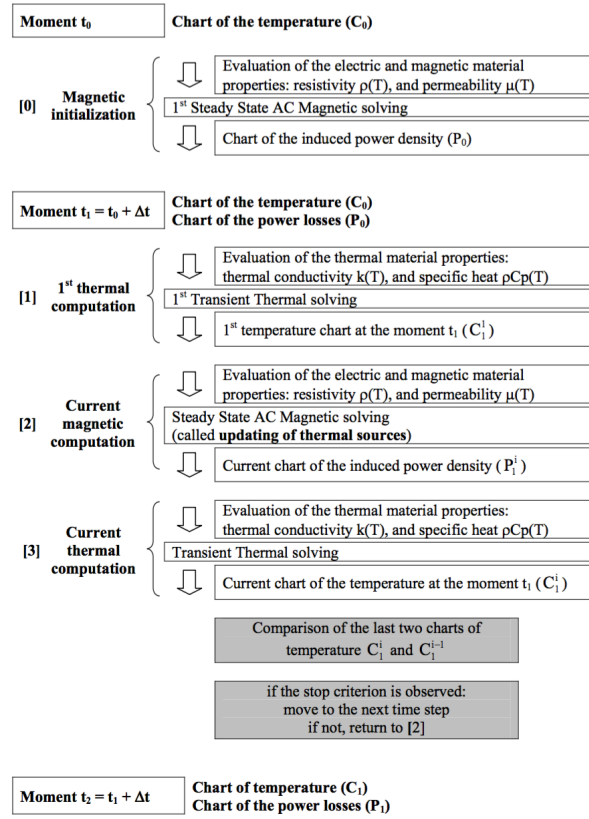


Figure 7.5: Scheme of the electromagnetic-thermal coupled simulation algorithm [81]-[82]-[83]-[84]

Generally, for gears which have modules with $m \leq 3$, the dual frequency process gives better results when MF (medium frequency) and HF (high frequency) sources are applied independently, respect to the simultaneous dual frequency method as discussed in [88].

At the beginning of the process, before the real contour hardening, a medium frequency preheating has been introduced in order to increase the initial temperature of the gear. This process may entail several advantages since the risk of cracking during the heating process is drastically reduced and, at the same time, it permits a wider hardened depth in the gear root. To do so the temperature along the tooth needs to be more constant as possible and low-power long-time power feed is mandatory. After the preheating process, a dwell time is necessary in order to let the heat to diffuse and equalize the temperature on the tooth. After that, the gear is treated with single-shot high-frequency power shot, in

Table 7.2: Process steps

Process Step	Time (s)
Preheating	10
Dwell Time	10
Heating	0.23
Shower Delay	0.5
Shower	30

Table 7.3: Process parameters during MF and HF heating

(a) Medium frequency preheating			(b) High frequency heating		
Parameter	Unit	Value	Parameter	Unit	Value
Voltage	V	0.27	Current	V	4.5
Frequency	Hz	10000	Frequency	Hz	185000
Time	s	10	Time	s	0.23

which the classical contour austenitized profile can be achieved. When the heating process is finished the gear is rapidly moved in the shower ring and quenched.

There are some additional technological aspects to take into account during the heating process, such as the maximum temperature reached, that should not exceed the value of 1100°C, otherwise the austenitic grain size could considerably increase.

Table 7.4: Material's data

Material	ρ ($\Omega \cdot m$)	μ_r	k (W/m/K)	C_p (J/Kg/K)	γ (Kg/m ³)
Copper	$2 \cdot 10^{-8}$	1	-	-	-
Fluxtrol	-	25	6	900	6100
AISI 304	$71 \cdot 10^{-8}$	1	15	500	8000

In the electromagnetic model all the components involved in the process have been taken into account. The role of the gear and the coil has been already discussed previously, but it is important to define how the flux concentrator disks are mandatory in a contour induction process like this one. Two disks of flux concentrators (Fluxtrol 50) are placed on the planar faces of the gear; this shrewdness is necessary to reduce the overheated zone, caused by the strong edge effect that commonly occurs on the root edge, during induction hardening treatment. Magneto-dielectric materials modify the induced current distribution due to the slot effect.

Also the clamping system of the gear, made of stainless steel, has been modeled. The material's properties for copper, Fluxtrol 50 and AISI 304 are shown in Table 7.4.

The electromagnetic problem has been solved by means of magnetic vector potential AV formulation with Coulomb's gauge. Since with this kind of formulation is not possible

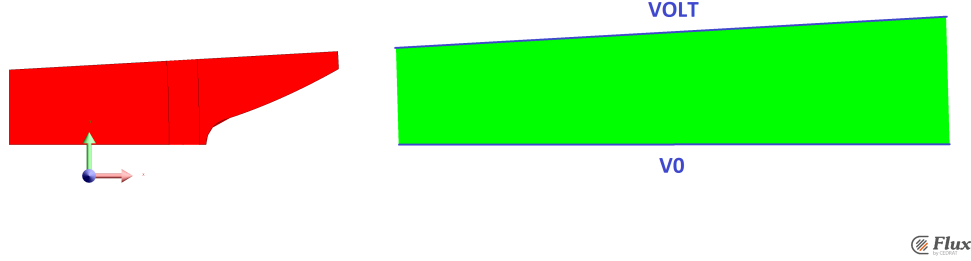


Figure 7.6: Imposed potential boundary condition

to feed the coil with an imposed current, a fixed voltage drop has been applied. In this case (Figure 7.6) on the two radial sections of the coil, two different voltages have been applied, one equal to zero and the second one is controlled by the user.

As shown in Table 7.3(a) and Table 7.3(b), two different set of process parameters have been used for the electromagnetic simulation.

During the heating process heat losses by convection and radiation have been considered on the surfaces of contact between thermal conductive bodies and air. Convection heat exchange parameter is generally dependent by the exchanging medium and the relative velocity between the fluid and the thermal conductive bodies, but in this case, it can be assumed equal to 15-20. The emissivity of this kind of steel can be assumed equal to 0.8 [18].

The linear equation system has been solved with the direct solver MUMPS (MULTifrontal Massively Parallel Solver) [85]-[86]. Both the thermal and the magnetic problems are characterized by non-linear problem and their solution can be obtained using the Newton-Raphson algorithm.

The time stepping for thermal transient problem is fixed and equal to 0.01 seconds each.

7.3.3 Results

Here some results from the electromagnetic-thermal coupled model will be presented. In Figure 7.9 the current density distribution at the beginning of the high frequency step is shown. It can be easily seen how the current is located on the profile, while the inner part of the tooth is not interested by eddy currents. Figure 7.7 shows the Joule losses distribution due to eddy currents at the beginning of the high frequency step. The Joule losses are equally distributed along the profile of the gear, with a small concentration on the edge of the root because of the edge effect. The distribution drastically change when part of the gear tooth overcome the curie temperature of the material. Figure 7.8 shows the power densities distribution at the end of the heating process. The maximum value is located all along the root of the tooth, while the tip seems to be only partially interested.

Figures 7.12 and 7.13 show the current and power trend calculated with simulations

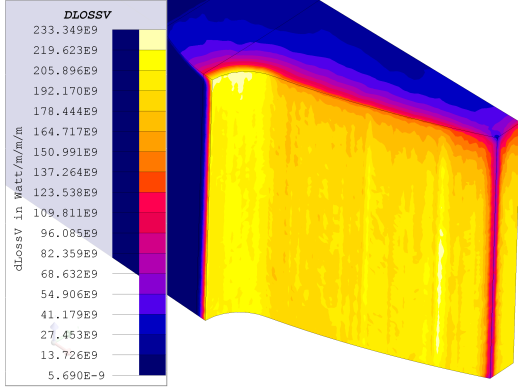


Figure 7.7: Power density distribution at the beginning of HF step

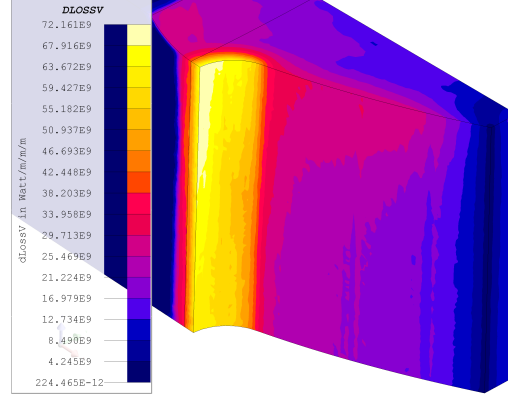


Figure 7.8: Power density distribution at the end of HF step

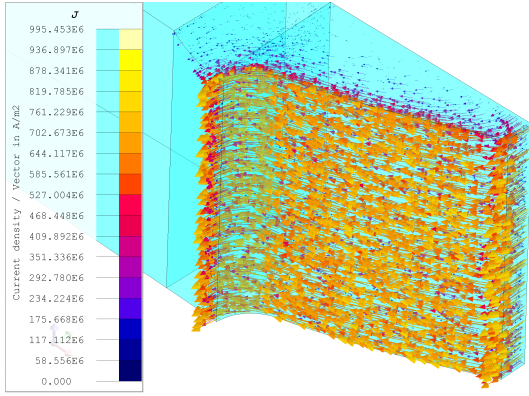


Figure 7.9: Current density distribution at the beginning of HF step

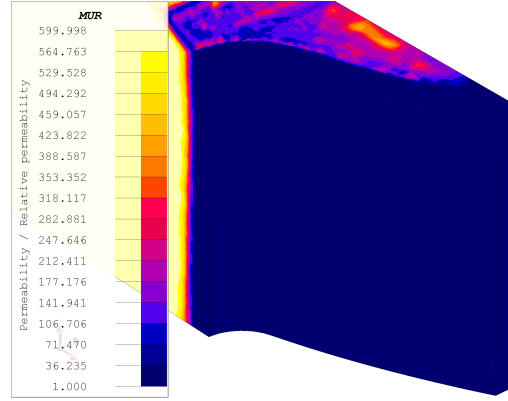


Figure 7.10: Magnetic permeability distribution at the end of HF step

during the preheating process. Since the voltage applied to the coil is kept constant, the current during the process tends to decrease, for the effect temperature on resistivity. In the same way, power absorbed by the system decreases too.

A different trend for current and power absorbed during high frequency step is visible in Figures 7.14 and 7.15. The integral value of the power is much higher in this case, due to the heating rate required. In fact, in order to guarantee a contour heated profile, the heating time has to be short enough to minimize the heat diffusion towards the inner part of the tooth. At the same time, the heating process cannot be too fast, because the extremely high power required (for the experimental validation) and also because fast processes may be interested by the effect of the coil tails. This last effect can be neglected if the gears performs at least 2-3 complete turns during the heating process; this means that, for a rotation speed of 800 rpm the minimum reasonable time to minimize the effect of the coil tails is between 0.15-0.22 seconds.

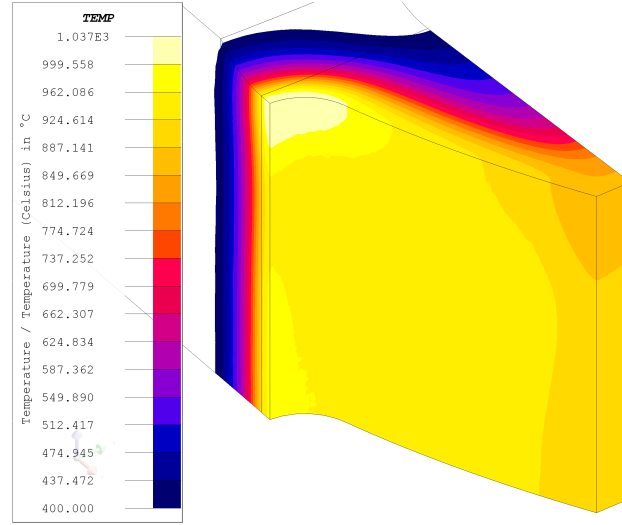


Figure 7.11: Temperature distribution at the end of HF step

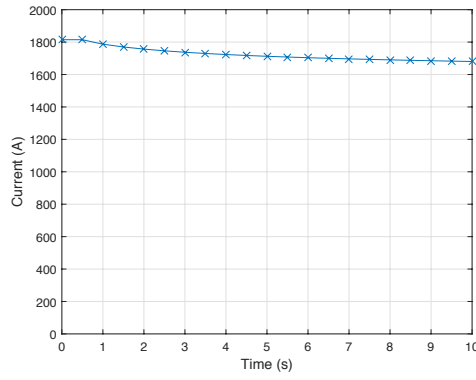


Figure 7.12: Coil current during MF stage

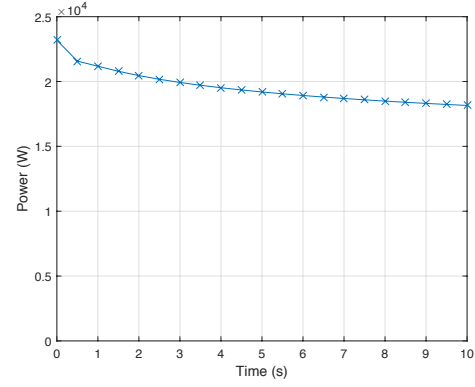


Figure 7.13: Total power during MF stage

The general trend of the power absorbed looks different, respect the preheating case, because with an impedance increment due to the change of resistivity with the temperature, the demagnetization effect due to the overcoming of Curie point, acts as a concurrent effect which tends to decrease the value of impedance. This can be easily seen in figure 7.15, where the maximum value of power absorbed in the high frequency step is located after around 0.15 seconds of treatment. Here, the major part of the gear profile overcomes the Curie's temperature making much deeper the area interested by magnetic induction.

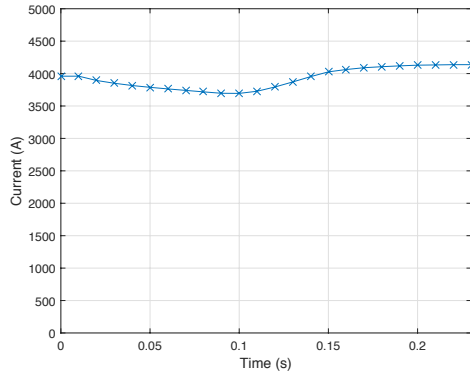


Figure 7.14: Coil current during HF stage

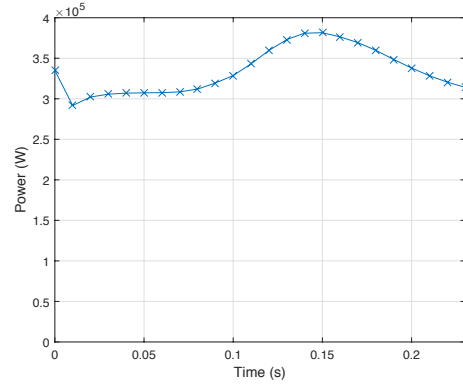


Figure 7.15: Total power during HF stage

7.4 Thermo-Metallurgical Analysis

7.4.1 Geometry and Mesh

The geometry of the model is the same as described in the electromagnetic model but, for thermo-metallurgical computation, the coil and surrounding air are not taken into account. A fine mesh in the thermo-metallurgical model is mandatory, not only to interpolate correctly the power densities imported from the electromagnetic computation, but also to predict properly the phase transformations. Phase transformation will occur initially on the tooth tip, but then it moves towards the root and the center of the tooth, while the temperature is rising. For this reason the mesh has to be fine enough, not only on the superficial layer (as in electromagnetic problem), but also on the rest of the tooth, while can be relaxed on the solid core of the gear and in the flux concentrator and stainless steel support, which are heated only by conduction. The mesh is composed by 125567 first-order elements, divided between tetrahedral, pyramid and hexahedral elements. On the gear tooth a superficial free quadrilateral mesh has been realized, with a controlled dimension of the elements equal to 0.07 mm. The superficial mesh has been, then, extruded along the tooth height, maintaining a reasonable ratio between elements' dimensions.

The same meshing approach has been used in the fitting between tooth and gear core, in order to guarantee a mesh dimension continuity in the volume linked with the tooth. In the rest of the geometry the mesh has been relaxed in order to speed up the computational time. A picture of the mesh used in Comsol is shown in Figure 7.16.

7.4.2 Physics Implementation

The model has been solved interfacing the FEM package COMSOL Multiphysics, for the transient thermal calculations, with MATLAB for the computation of metallurgical phase transformations.

In order to use the data calculated through the electromagnetic simulation, the power density distribution has to be interpolated in order to adapt correctly to the new mesh.

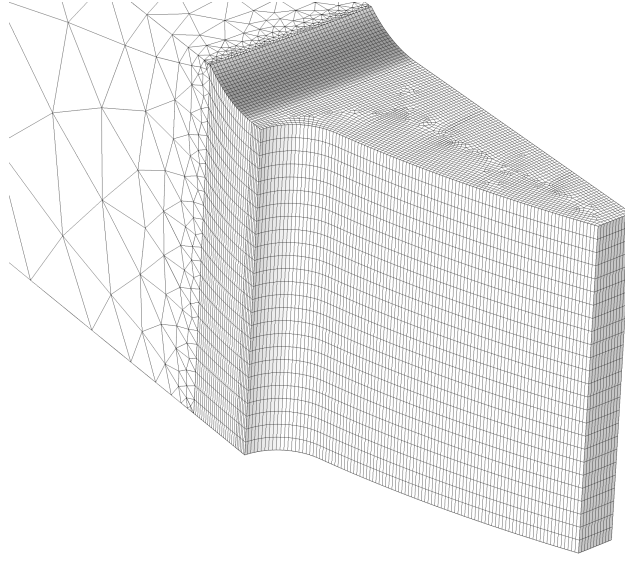


Figure 7.16: Detail of the mesh in the thermo-metallurgical model

The global material's properties have been calculated at the beginning of each time step as a composition of the phases' properties multiplied for the weight fraction with an external routine developed in Matlab. These values have been evaluated on the nodal point of the mesh, so they need to be interpolated before the use in the thermal model. During the heating step, the same thermal boundary conditions used in the electromagnetic-thermal model have been applied on the exchanging surfaces. After each time step the nodal thermal history and the temperature rate are used in order to predict the phase transformations which occur during heating. As shown in previous chapters the material is heated up in order to transform part of it in austenite. This kind of transformation is predicted with CHT diagrams.

When the heating process is finished the workpiece is rapidly cooled down in order to transform the austenite into martensite. The cooling process can be divided in two steps:

1. the first one in which the workpiece is moved towards the shower. The duration of this step depends upon the movement speed of the machine spindle, but in our case it can be assumed equal to 0.5. The heat exchanges during this period are the same of the heating process (same heat convection coefficient and emissivity).
2. a second step in which the material is quenched in the shower until the sample reaches the room temperature. The expected duration is around 30 seconds and it is characterized by a temperature-dependent heat convection coefficient (Figure 7.17).

Phase transformations during quenching can be predicted through the use of isothermal diagrams (Figure 7.18), which may be defined with the algorithm explained on previous chapters. The nodal thermal history has been discretized in isothermal steps, in order to apply the anisothermal transformation kinetic models with Scheil's additivity principle.

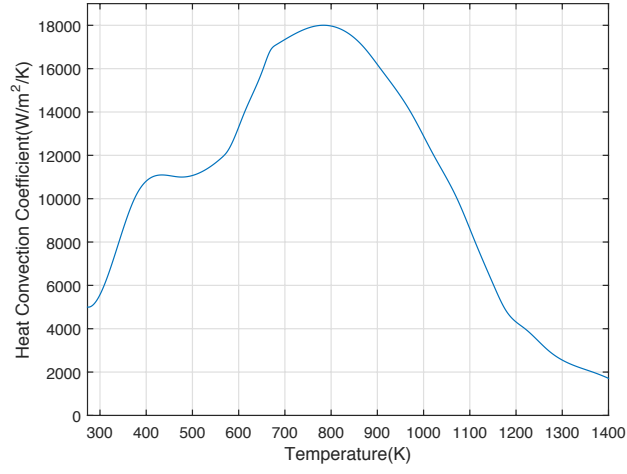


Figure 7.17: Heat convection coefficient in fuction of surface's temperature

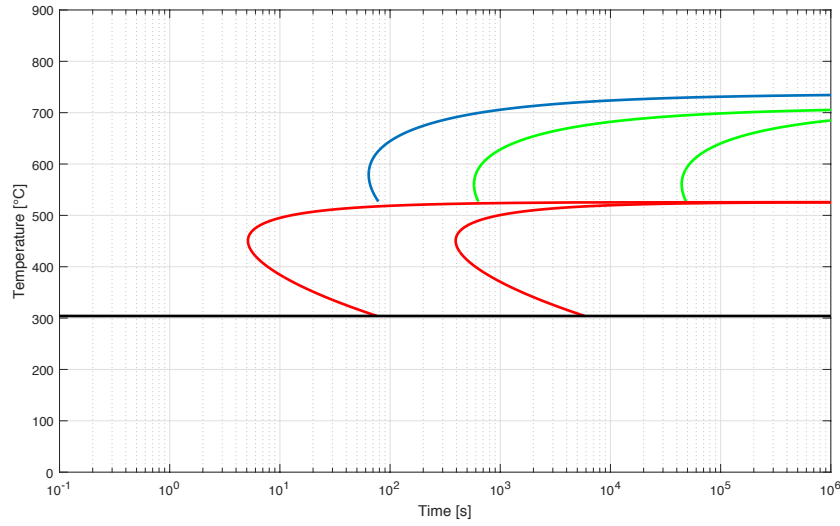


Figure 7.18: TTT diagram of AISI 4340

Both during heating and quenching the latent heat of transformation is evaluated as:

$$\frac{dQ_{LH}}{dt} = \sum_{k=1}^N c_k \xi_k + \sum_{k=1}^N \Delta H_k \frac{d\xi_k}{dt} \quad (7.1)$$

and used as a power density absorbed or released during the transformation depending if the current transformation is endothermic or exothermic.

Once the microstructure has been evaluated the material's properties for the following time step are updated in function of phase composition and temperature.

7.4.3 Results

When the whole simulation is finished, the phase distributions (austenite, bainite, ferrite, martensite and pearlite) can be verified directly in the post-processors of Comsol Multiphysics. In figure 7.19 and Figure 7.20 the temperature distribution, at the end of heating step, and the corresponding austenite distribution are shown. The differences in temperature calculation between the electromagnetic-thermal model (Figure 7.11) and thermo-metallurgical one (Figure 7.19) are limited to 30 °C (less than 3%) on the maximum temperature. This small difference can justify the simplification introduced decoupling the magneto-thermo-metallurgical model in two separated simulations.

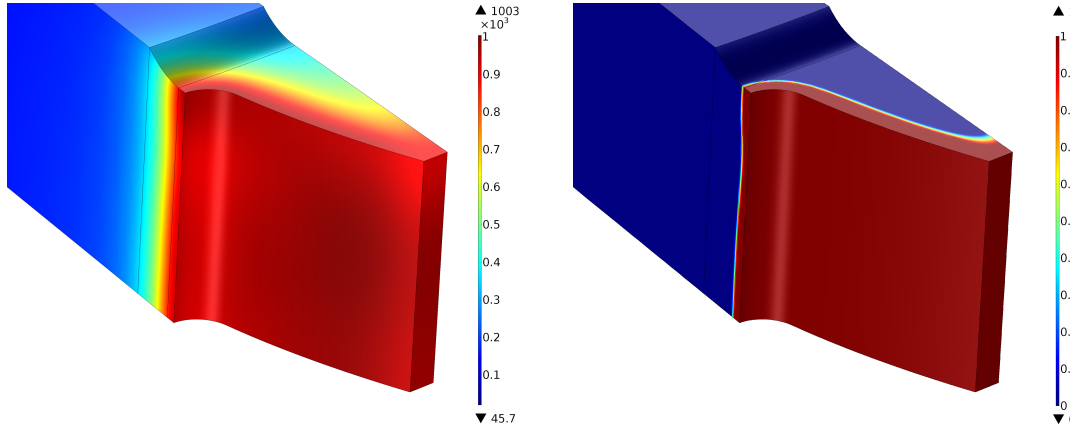


Figure 7.19: Temperature distribution at the end of heating

Figure 7.20: Austenite distribution at the end of heating

A possible difference on temperature calculation, between electromagnetic-thermal and thermo-metallurgical model may derive from the calculation of latent heat of transformation. In this latter model, the latent heat absorbed or released during the phase transformation, is strickly evaluated starting from the enthalpy change. In the electromagntetic-themal model, instead, it is just simply supposed, introducing a possible error in the calculation of temperature profile, especially when high heating rates characterize the heating process. On the other hand a small mistake may have been introduced because of the interpolation of the values of the power densities in the thermo-metallurgical model: in the two simulations the mesh generated is different, because the different physics require a different type of mesh in the tooth geometry.

In figures 7.21 and 7.22 the spatial distribution of released power due to phase transformation during quenching are shown for two different times.

In figure 7.23 the transformation kinetic of Austenite during the heating process is shown for different time steps of heating process. The austenite distribution is compared with the temperature field in the gear tooth for the corresponding time step.

The austenitic transformation begins on the tooth flank, after around 0.12 seconds of high frequency heating. The transformation, then, continues towards the tip and the root of the tooth. After 0.15 seconds the flank is almost completely transformed, while the microstructure on the edges are still unvaried. Only a small portion, on the root edge,

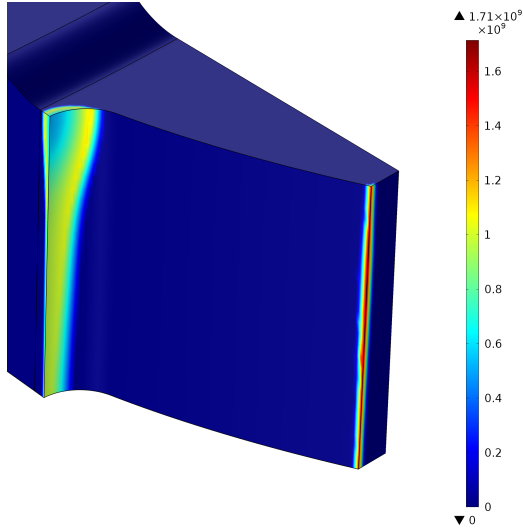


Figure 7.21: Latent heat released after 0.25 seconds of quenching

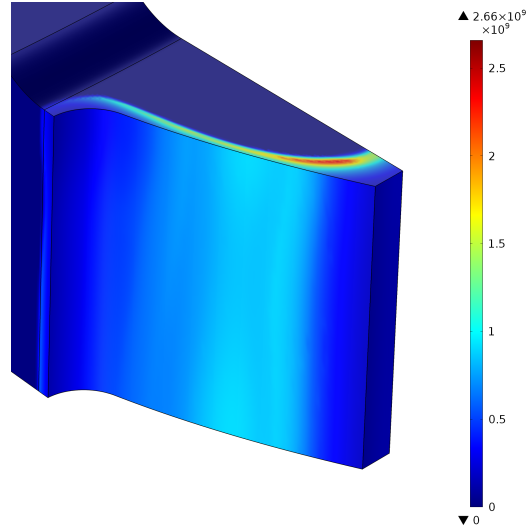


Figure 7.22: Latent heat released after 0.55 seconds of quenching

starts to transform due to the overheating induced by edge effect. After 0.19 seconds, the tooth profile is almost totally austenitized, while just only two small spot, located on the tip edge and below the root edge, are still colder. At 0.23 second the profile is totally austenitized.

During quenching process the austenitized part of material is rapidly transformed in martensite. In figure 7.24 the transformation kinetic of Martensite during the quenching process is shown for different time steps of cooling. The martensite distribution is compared with the temperature field in the gear tooth for the corresponding time step.

The transformation begins after 0.24 seconds of quenching (0.74 seconds since the end of heating process) in the root of the gear and in the corner between tip and flank. These parts are the most subjected to cooling phenomena: the first one is affected by a heat sink effect from the colder core of the gear, while in the second one the ratio between volume and exchanging surface is very small. Then, the transformation proceeds towards the center of the flank's surface and the tip is the first part to complete the martensitic transformation, while the transformation in the root seems to be slowed down because of its thermal inertia. The last part of the transformation kinetic is the slowest one, because the superficial convective exchange is limited and the heat removal from the inner part of the tooth is controlled by the thermal conduction through the steel. After 30 second of quenching the gear is completely cooled down and the martensitic transformation is completed.

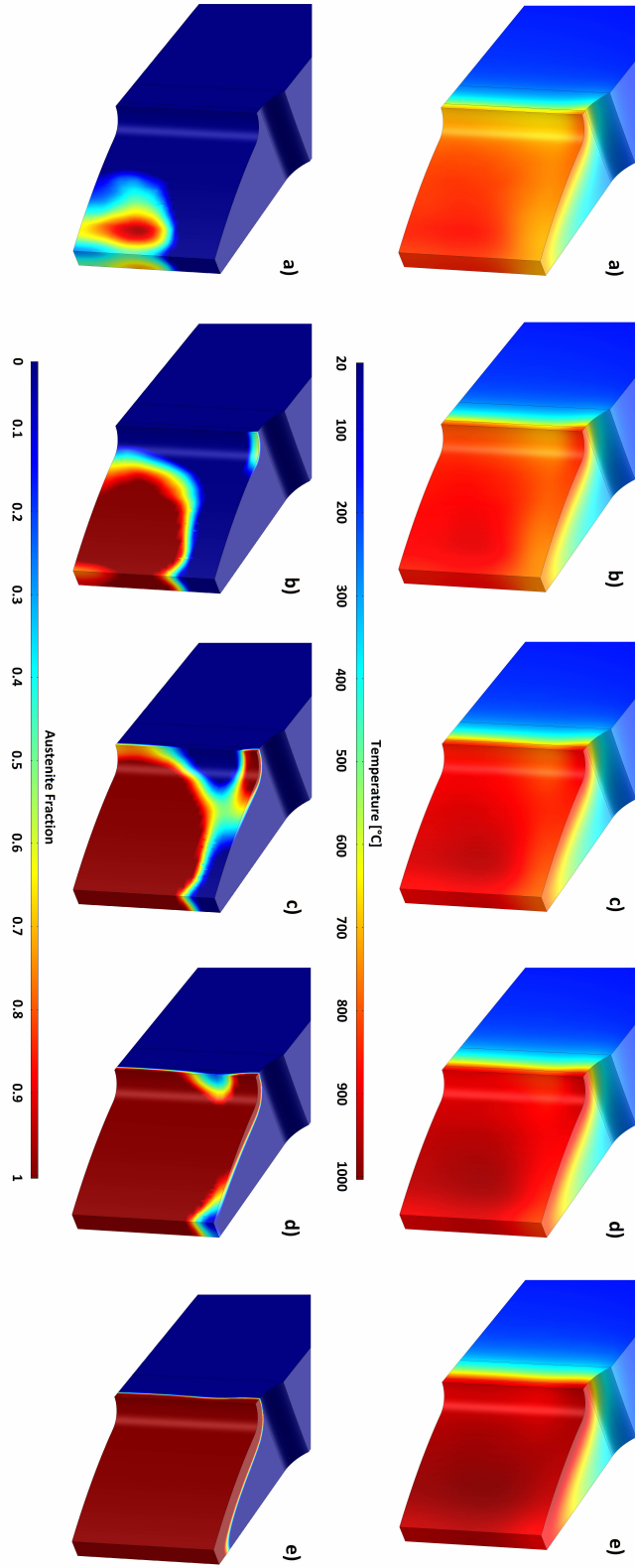


Figure 7.23: Evolution of temperature and austenite phase distribution during heating phase, respectively after a) 0.13 s, b) 0.15 s, c) 0.17 s, d) 0.19 s and e) 0.23 s of HF heating

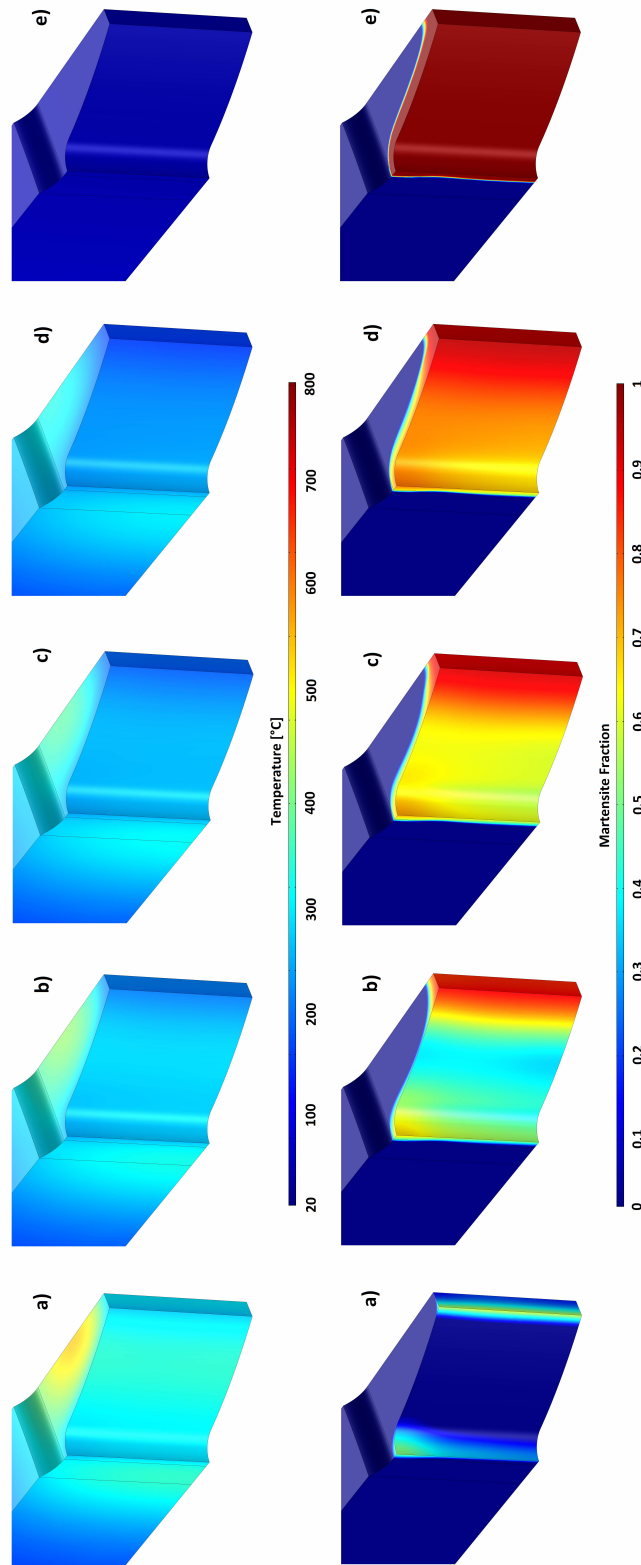


Figure 7.24: Evolution of temperature and martensite phase distribution during heating phase, respectively after a) 0.0.25 s, b) 0.0.44 s, c) 0.0.54 s, d) 0.64 s and e) 30 s of quenching

7.5 Experimental Validation

The numerical model developed has been verified experimentally through a contour induction hardening process on a vertical induction hardening machine. The tests have been carried out at the Center of Thermal Technology (CTT) of the Mechanical Engineering Department of Ecole de Technologie Supérieure (ETS) in Montréal (Canada). Here, the facilities available include a simultaneous double-frequency induction hardening machine (EFD SINAC 650).

7.5.1 The Induction Hardening Machine

The EFD induction heating machine consists of four parts:

1. The medium-frequency IGBT converter that can treat parts with a constant frequency of 10 kHz with a maximum power of 600 kW.
2. The high-frequency IGBT converter which operates with a frequency between 150 kHz and 250 kHz. This generator has the feature to adjust the frequency depending on the inductor used and the workpiece. The maximum power achievable is 350 kW
3. The active part of the machine which contains the inductor, shower cooling the workpiece and the filter that permit the simultaneously use of both the generators.
4. The control panel, that allows to select the desired power, to choose between a manual or automatic processing, to adjust the position of the workpiece and display measurements data taken during the treatment.

The operation of this machine can be summarized using the simplified circuit design shown in the figure. A diode rectifier is connected to a mains 3 x 480 V (orange on the diagram). The current thus rectified is sent to the frequency converters that provide current to the desired (green) frequency. From there the current passes through a matching transformer to reduce the voltage and increase the intensity (in yellow). Finally, a capacitor reactive power compensation (red) is used to provide the required reactive power. Indeed, inductive machines consume a lot of reactive power, which leads to a large power consumption, voltage drop and large energy losses. To minimize adverse impacts on the power grid, it is necessary to create its own reactive power through capacitors. In the figure, top left, the HF generator, bottom left is the MF generator, and right filter. This allows not only to filter the current signal, but it also prevents the RF current back in MF generator because both HF and MF outputs are connected to the same inductor.

7.5.2 The process

In order to go on with the experimental tests the coil, the stainless steel support and the magneto-dielectric disks have been designed properly. In Figures 7.26 and 7.27 the drawings of these components are shown. The inductor is made by a single turn coil with an internal diameter of 150 mm and 14 mm high. The coil tail has been shaped in order to insert a Rogowski coil for the current measurements during the test. For the quenching shower, a

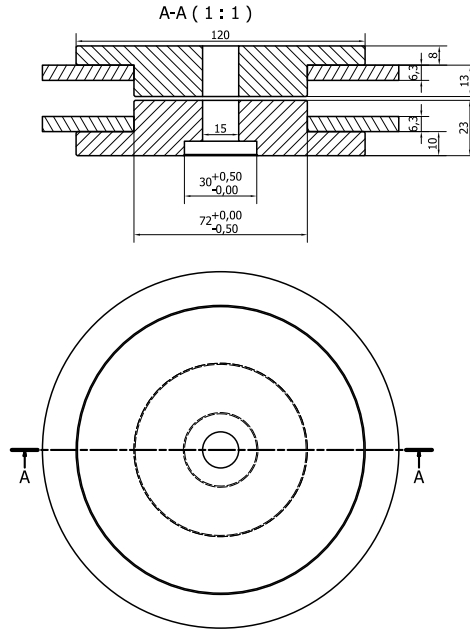


Figure 7.27: Drawing of the support of the gear and the flux concentrators

7.5.3 Results

As validation of the simulation results, the recipe developed with numerical methods has been tested experimentally. The gear, after the treatment, has been cut along vertical and horizontal sections and etched with Nital 3% in order to make visible the martensite distribution. The etched samples have been analyzed with the optical microscope and stereo microscope in order to verify the microstructure and to compare the experimental results with the numerical ones.

In Figure 7.28 a comparison between the experimental results and the martensite distribution calculated through the numerical algorithm is shown. In this case, the medium plane section of the gear and the transversal section of the gear root have been taken as comparison case. A remarkable correspondance between the two analysis is notable: in the plane section the FEM calculation predicts the transformed phase in a proper way all along the gear profile.

The same considerations can be adduced for the vertical section of the root. Here the profile seems to be very straight, exhibiting a very low edge effect on the root's edge. The model developed is precise enough to predict, also, a small defect like a little reduction of the hardened depth, just below the root's edge. This problem is strictly connected with the coil design, since the coil's height depends on the commercial dimensions of copper tubes, which exist only in a limited number of dimensions.

Table 7.5: Hardness measurements on section A

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Depth from Surface	HV	HV	HV	HV	HV	HV	HV	HV	HV	HV
0.1	687	721	705	676	696	689	689	730	676	678
0.2	698	726	723	666	707	721	668	783	705	683
0.3	687	721	726	683	685	733	637	680	719	670
0.4	663	712	719	351	702	693	351	345	691	676
0.5	668	595	705	362	691	709	348	364	691	631
0.6	400	326	689	339	365	685	345	365	719	364
0.7	356	322	687	321	349	716	343	385	700	359
0.8	349	349	674	352	327	521	349	351	373	330
0.9	359	358	674	343	371	353	336	359	380	343
1.0	351	348	696	330	364	361	338	369	348	330
1.1	359	367	716	382	356	353	346	361	362	354
1.2	330	373	696	351	343	361	362	366	364	389
1.3	348	398	705	356	362	345	353	364	359	362
1.4	357	371	691	369	362	346	366	375	328	345
1.5	341	363	643	353	345	346	361	379	354	345
1.6	-	-	536	-	-	-	-	-	-	-
1.7	-	-	376	-	-	-	-	-	-	-
1.8	-	-	357	-	-	-	-	-	-	-
1.9	-	-	359	-	-	-	-	-	-	-
2.0	-	-	351	-	-	-	-	-	-	-
2.1	-	-	342	-	-	-	-	-	-	-
2.2	-	-	349	-	-	-	-	-	-	-
2.3	-	-	364	-	-	-	-	-	-	-
2.4	-	-	364	-	-	-	-	-	-	-

Table 7.6: Hardness measurements on sections B and C

	B_{TOP}	B_{MID}	B_{BOT}	C_{TOP}	C_{MID}	C_{BOT}
Depth from Surface	HV	HV	HV	HV	HV	HV
0.1	649	676	702	676	698	655
0.2	698	680	668	651	689	624
0.3	670	672	693	663	668	655
0.4	502	320	419	659	659	635
0.5	362	333	371	676	639	628
0.6	357	361	360	631	647	591
0.7	344	342	351	598	647	561
0.8	356	371	365	491	659	463
0.9	367	342	360	418	663	442
1.0	357	355	361	345	668	349
1.1	349	357	347	322	655	342
1.2	352	342	357	325	663	333
1.3	346	350	339	328	659	326
1.4	327	336	339	322	651	336
1.5	323	339	344	323	620	321
1.6	-	-	-	327	521	331
1.7	-	-	-	339	406	342
1.8	-	-	-	335	322	344
1.9	-	-	-	350	321	328
2.0	-	-	-	340	322	341
2.1	-	-	-	337	324	332
2.2	-	-	-	330	336	354
2.3	-	-	-	340	340	341
2.4	-	-	-	338	342	337

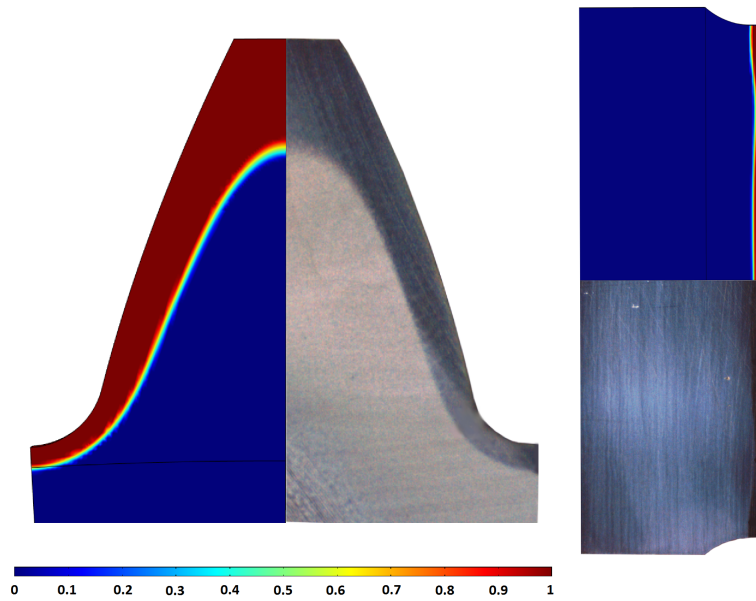


Figure 7.28: Comparison between numerical model and experimental distribution of martensite on middle section of the gear and vertical section of the root

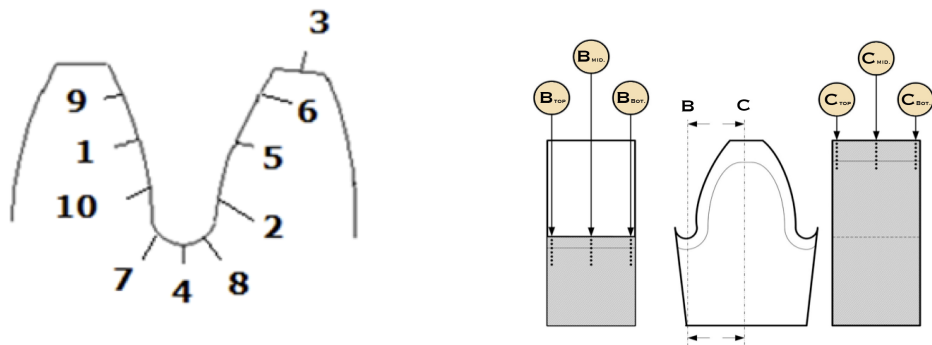


Figure 7.29: Inspection scheme on sections A, B and C

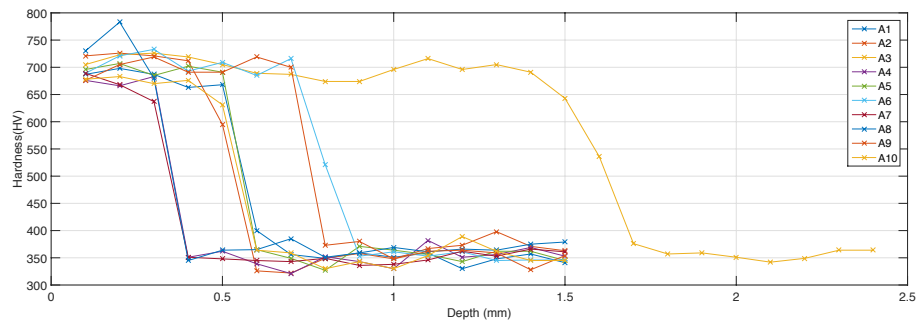


Figure 7.30: Hardness paths on section A

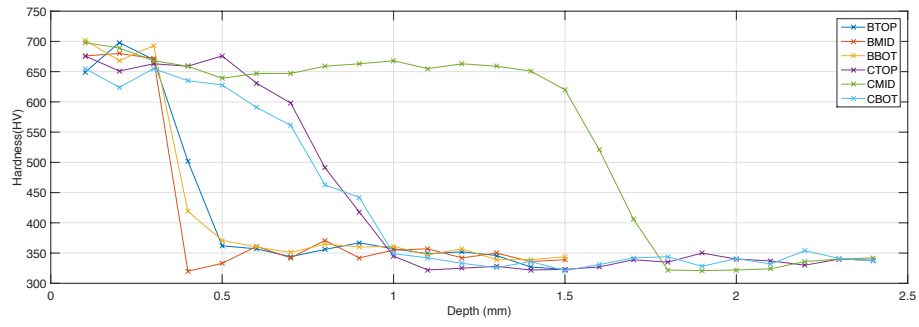


Figure 7.31: Hardness paths on sections B and C

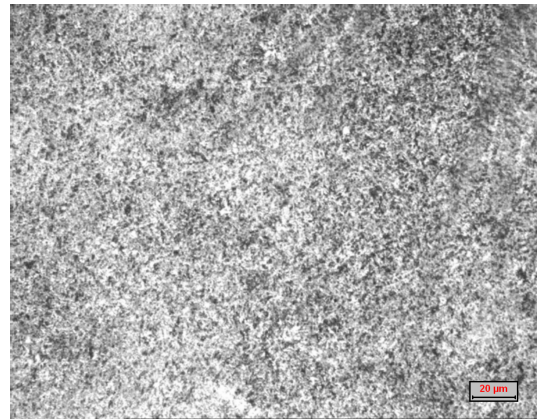
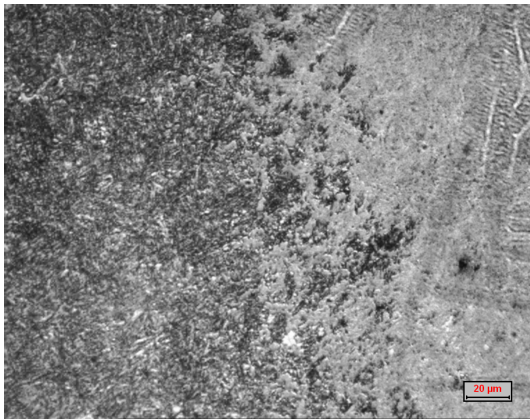


Figure 7.32: Inspection scheme on sections A, B and C

Chapter 8

Numerical Simulation of Simultaneous Double Frequency Induction Hardening Process

8.1 Introduction

It has long been known that, with medium frequency, the root area of the tooth can be hardened. However, this also resulted in through-hardening of the tooth when attempting to achieve the desired depth of root hardening. It was then discovered that, if a medium frequency is applied in advance to the part in a preheating scenario, this would compliment the effects of the high frequency later applied.

With Simultaneous Double Frequency (SDF) treatments, medium and high frequencies are simultaneously supplied to one inductor. It is a frequency blend where the medium frequency is used to provide the ground wave upon which the high frequency is superimposed. What is so unique about this process is that the amplitudes of both frequencies can be controlled independently: one inductor can be used, and one generator can produce both frequencies simultaneously. The ability to control the amplitudes of either medium or high frequency allows one to specifically tailor the energy density delivered to the part, thereby allowing one to better control the current flow to “follow” the contours of the part. To insure true-contour hardening of the tooth without through-hardening and effecting the core of the part, short heating times of less than 0.5 seconds are required [89].

In this chapter, the implementation of a finite element simulation for the solution of electromagnetic-thermal coupled problem during Simultaneous Double Frequency processes is discussed, taking into account the material’s nonlinearities that characterize the electromagnetic and the thermal problem.

As already done in previous chapters, the algorithm has been applied to a simple geometry (billet) in order to avoid geometric complexities, and to concentrate the efforts on the description of the physics. Anyway, the algorithm developed can be easily extended to a more complex case such as a different geometry or a 3D model.

This part of the work has been developed in collaboration with Antonio Marconi and a

deeper description of the work done can be found in his thesis [90].

8.2 Description of the Problem

Classically, the simultaneous double frequency problems, applied to induction hardening case, has been sometimes solved decoupling the electromagnetic simulation in two different frequency-domain problems, the first one solved evaluating the effect of the medium frequency treatment and the second one with the high frequency. The nodal power densities distribution so evaluated were, then, summed by means of superimposition method and used in the transient thermal problem as driving force.

A similar approach cannot be used in induction hardening process simulation, since the magnetic properties of the treated material are highly non-linear and a similar method does not take into account the saturation effect, committing a considerable mistake on the evaluation of the heated profile.

Typically a process of surface induction hardening is extremely fast, as long as the power induced in the workpiece is concentrated in a reduced thickness and the frequency range is in the order of $10^1 - 10^2$ [kHz]. This means that the process time to be simulated in the thermal model is in the range of:

$$T_{therm} \approx 10^{-1} [s]$$

instead the electromagnetic period to be simulated in the electromagnetic model is much smaller, for example:

$$T_{em} = \frac{1}{f} \approx 10^{-5} [s]$$

where $f[Hz]$ is the frequency of electrical sources. Shannon's theorem says that to describe properly a waveform with good accuracy, we need at least $N = 40$ samples per period. This means that a good time step to solve such a kind of problem is:

$$dt = \frac{T_{em}}{N} \approx 2.5 \cdot 10^{-7};$$

Afterwards a surface induction hardening process, with an completely coupled (electromagnetic and thermal) model might need an extremely large number of time steps to be numerically computed:

$$n_{steps} = \frac{T_{therm}}{dt} \approx 4 \cdot 10^5 [steps]$$

It is clear that this approach needs too many iterations to be solved.

Taking into account also the complications in the problem as nonlinearity of the PDEs (such as magnetic saturation and Curie's transformation), to obtain the convergence of the solution could be necessary to reduce the time step dt and increase the number of elements of the mesh. This means that also a simple geometry requires a long time to be solved, probably several weeks. A similar solution time is not acceptable for a realistic use of the algorithm for the study of the problem, so a different way must be found in order to reduce the total computational time.

In order to solve this problem the magneto-thermal problem has been decoupled solving separately the transient electromagnetic and transient thermal problem.

It can be assumed, if the temperature grows relatively of few degrees during this time period, that the electromagnetic parameters, such as magnetic relative permeability and electrical conductivity do not significantly vary with temperature. So, it is plausible to assume that in a time interval sufficiently small, the power induced in the workpiece is constant, and it is precisely the average value of the power induced.

These observations imply that it is possible to solve firstly the EM transient problem, using the material's properties calculated considering the current temperature profile. Then it is possible to derive the average heating power distribution calculated in the electromagnetic period, and assume this power as time constant source in a time step of the thermal transient. Finally the temperature distribution can be obtained, and used in order to calculate the new materials properties distribution, which will be used in the later electromagnetic simulation, and so on until the end of the process [91]-[94].

Practically it is possible to do that assuming that for a certain integer number of electromagnetic periods N_{em} , the waveform of the electromagnetic phenomena (such as magnetic field, current, power ...etc) can't vary as long as the temperature does not change. This means that if t_k and t_{k+1} are the initial and the last points in time at which the thermal transient is solved,

$$dt_{therm} = t_{k+1} - t_k \quad (8.1)$$

then the thermal time step must be an integer multiple of the electromagnetic period:

$$dt_{therm} = N_{em} \cdot T_{em} \quad (8.2)$$

where the electromagnetic period is the period defined by the medium frequency, in a simultaneous double frequency method (f_{mf} and f_{hf}), instead in a single frequency induction hardening is the period defined by the chosen frequency.

$$T_{em} = \begin{cases} \frac{1}{f} & \text{with single frequency supply} \\ \frac{1}{f_{mf}} & \text{with SDF supply} \end{cases} \quad (8.3)$$

It is important to highlight that the time step dt_{therm} has to be chosen sufficiently small in order to have limited temperature variation (max 40 °C) in a time step, because the surface will be characterized by the highest heating rate, at least for most of the process. The maximum acceptable temperature difference $\Delta\theta$ of a surface point between two magnetic periods T_{em}^k and T_{em}^{k+1} is in the order of 30-40 °C. This value is a good compromise between acceptable computation times and accuracy of the model.

The concept iterative scheme of the coupled model is sketched in Figure 8.2.

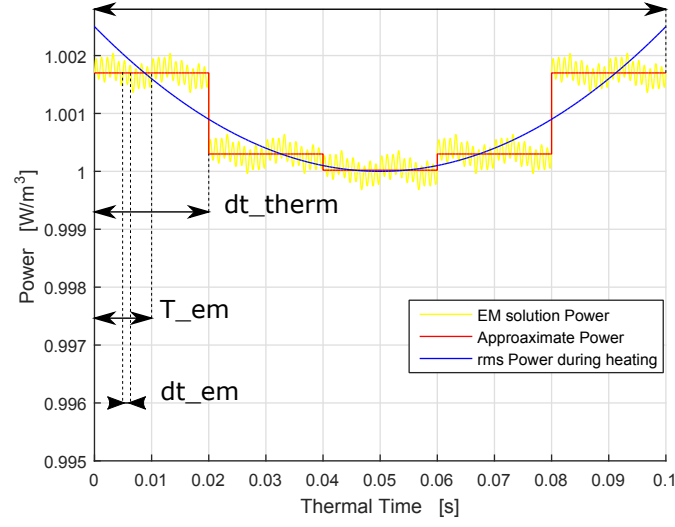


Figure 8.1: Qualitative example of power approximations in the model

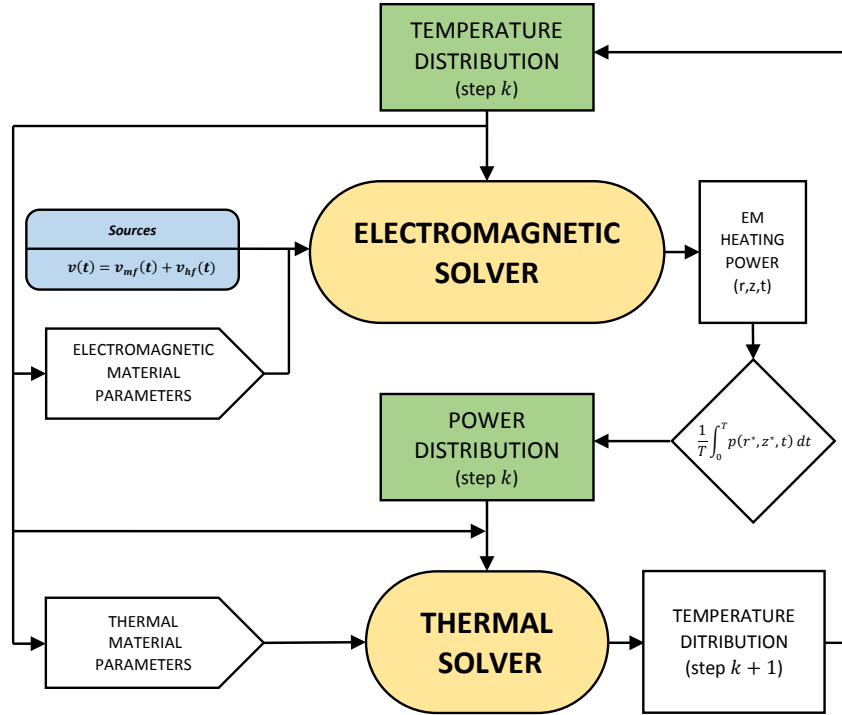


Figure 8.2: Flowchart of the FEM solution of the coupled problem.

8.3 Description of the Model

Since this work is aimed to implement a numerical way for the study of the simultaneous double frequency treatments, a simple geometry, like a billet coupled with a coil conductor, has been used. This assumption has been done to solve in the time domain both the problems, avoiding extremely long times. It is well known the effect of the temperature on the penetration of the magnetic field and on the distribution of the thermal sources. But it is extremely complicated to understand how much important might be the magnetic saturation during the process, especially its effect in a simultaneous double frequencies process.

8.3.1 The geometry

The sizes of the steel billet modeled in Comsol are:

Table 8.1: Dimensions of the billet.

Constant	Value	Unit
<i>radius</i>	10	<i>mm</i>
<i>height</i>	50	<i>mm</i>

The inductor used for this simulation is a simple geometry, since it is made by a squared section copper tube. The losses in the coil can be strongly reduced if the thickness of the coil respects the relation $s \geq \frac{\pi}{2}\delta$ [ref Lupi]. The coil thickness has been therefore chosen in order to guarantee this assumption.

Table 8.2: Dimensions of the water cooled coil.

Constant	Value	Unit
$r_{int-ind}$	12	<i>mm</i>
h_{ind}	10	<i>mm</i>
s_{ind}	1.5	<i>mm</i>

To obtain an acceptable solution of the electromagnetic problem, a good practice is the definition of a region, that is called infinite box, in which the magnetic field goes to zero, in other words it is imposed what happens theoretically to infinity. The infinite box, that in our case is a ring-shaped air region, is defined by an inner radius $r_{inf-int}$ and an outer radius $r_{inf-ext}$. The infinite box is a strong simplification and may affect the physical solution, if it has not been chosen properly. In order to avoid such kind of problems the dimension of the infinite box may be at least 10-20 times bigger than the maximum dimension of the active part.

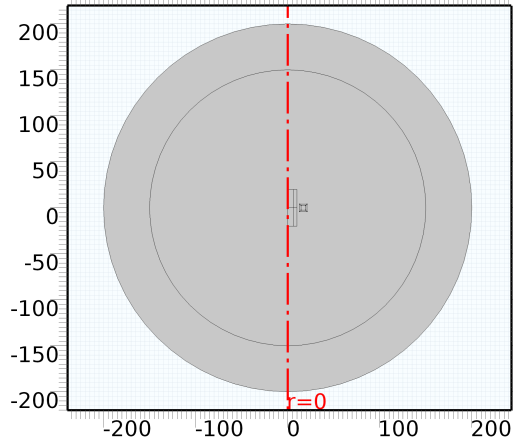


Figure 8.3: Overall view of geometry and physics of the problem

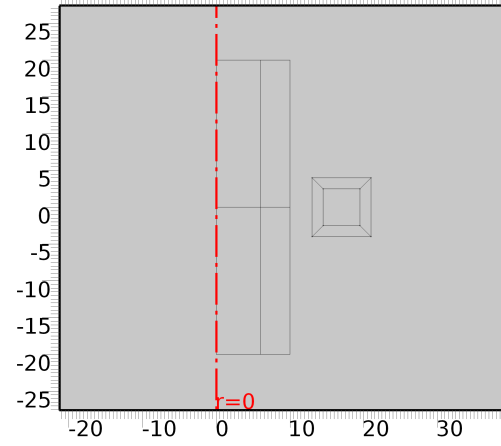


Figure 8.4: Detail of the geometry of the model

Table 8.3: Dimensions of the infinite elements.

Constant	Value	Unit
$r_{inf-int}$	150	[mm]
$h_{inf-ext}$	200	[mm]

8.3.2 The mesh

The mesh is composed by 18354 second-order elements. The mesh in the billet and in the coil is composed by quadrilateral element, while in the air and inside the coil the mesh is typically triangular in order to take advantage of the relaxing capabilities of the meshing algorithm.

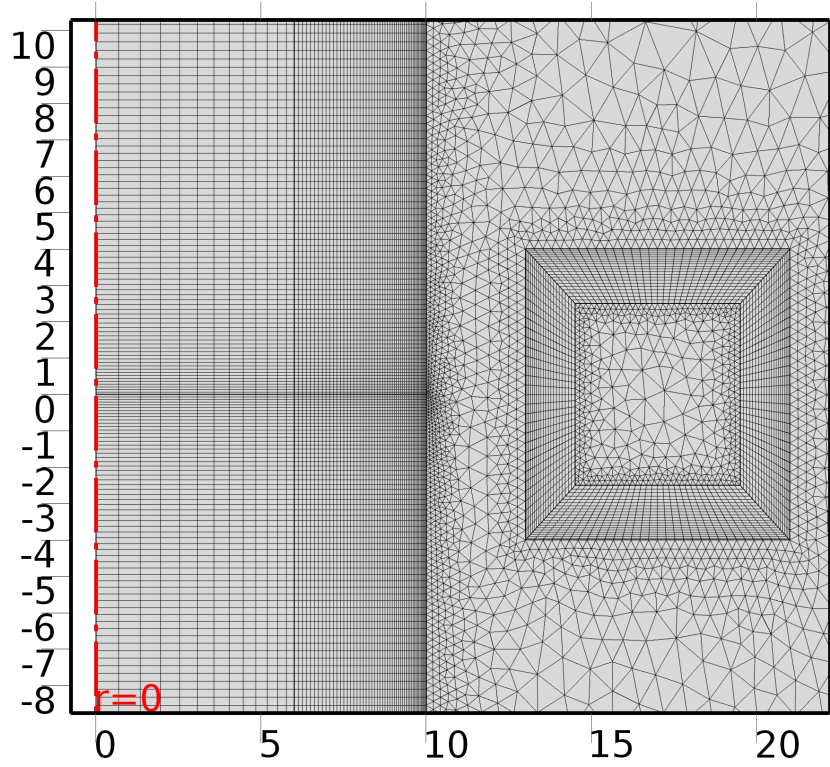
In induction problems, in order to obtain a realistic description of the phenomena, the precise calculation of power densities distributions is mandatory. Induction heating is characterized by internal heating sources, located in a thin superficial layer, while the internal part of the workpiece is not affected. A good way to approach the problem is to respect the empirical rule of almost 2-3 elements in the first penetration depth.

The billet has been divided in four rectangles in order to exploit the possibility to modify the mesh density in each region. The mesh in the superficial rectangles is the thinner in the model, in order to describe properly the electromagnetic and thermal quantities which define the heating process. In the inner regions the mesh has been always kept mapped, but with bigger elements since it is slightly affected by electromagnetic fields. The inductor has been divided into trapezoids, but with elements whose minimum width is defined by the skin depth in copper at 100 kHz. Instead, the air and the inductor cooling has been meshed with free triangular elements.

Below the different penetration depths δ are listed, they are combination of different values of the material parameters.

Table 8.4: Penetration depths in function of material

Penetration Depth	Frequency [kHz]	μ_r	ρ [Ωm]	Value [mm]
δ_{steel}	10	600	$2.5 \cdot 10^{-7}$	0.3
δ_{steel}	100	600	$2.5 \cdot 10^{-7}$	0.1
δ_{steel}	10	1	$2.5 \cdot 10^{-7}$	0.8
δ_{steel}	100	1	$2.5 \cdot 10^{-7}$	0.5
δ_{Cu}	10	1	$2.0 \cdot 10^{-8}$	1.0
δ_{Cu}	100	1	$2.0 \cdot 10^{-8}$	1.3

**Figure 8.5:** Mesh structure of the billet and of the coil.

8.3.3 Electromagnetic Model

The electromagnetic problem can be easily represented with a flowchart of the solution model, that is shown in Figure 8.7.

Power supply

The inductor is voltage supplied. In particular, a voltage equal to the superposition of two sinusoidal waveform with two different frequencies has been imposed on the coil . From the

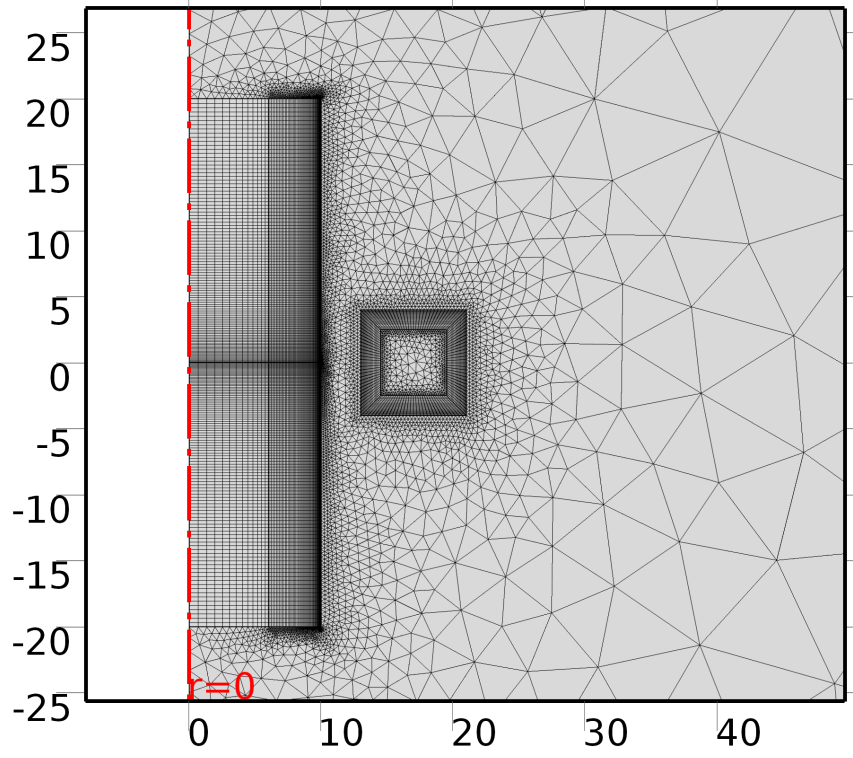


Figure 8.6: Overall mesh of the model

electrical point of view this is equivalent to put in series two voltage generators independent of each other. As already seen, to obtain a power that is the sum of different terms it is chosen an integer ratio between the two frequencies.

$$k = \frac{f_{hf}}{f_{mf}} \quad (8.4)$$

with $k \in \mathbb{N}$. Then, if V_{hf} [V] is the rms value of high frequency and V_{mf} the rms value of medium frequency:

$$v(t) = V_{mf} \cos(2\pi f_{mf} t) + V_{hf} \cos(2\pi f_{hf} t) \quad (8.5)$$

The inductor has been modeled as a solid entity in which the current is not omogeneously distributed inside the section, but it is affected by skin, ring and proximity effect. This is a proper way to model an inductor taking into account many problems as edge effects, saturation and evolution of a wave form in time, particularly if penetration depth is much smaller than the dimensions of the coil.

The voltage source is well known and imposed during all the electromagnetic period:

$$\mathbf{J}_e = \sigma \frac{v(t)}{2\pi r} \mathbf{e}_{coil} \quad (8.6)$$

This means that the current density $\mathbf{J}_e$ [A/m²], imposed normal through the inductor section (it is weel known that for a 2D-Axial Symmetric model, current desities admit only

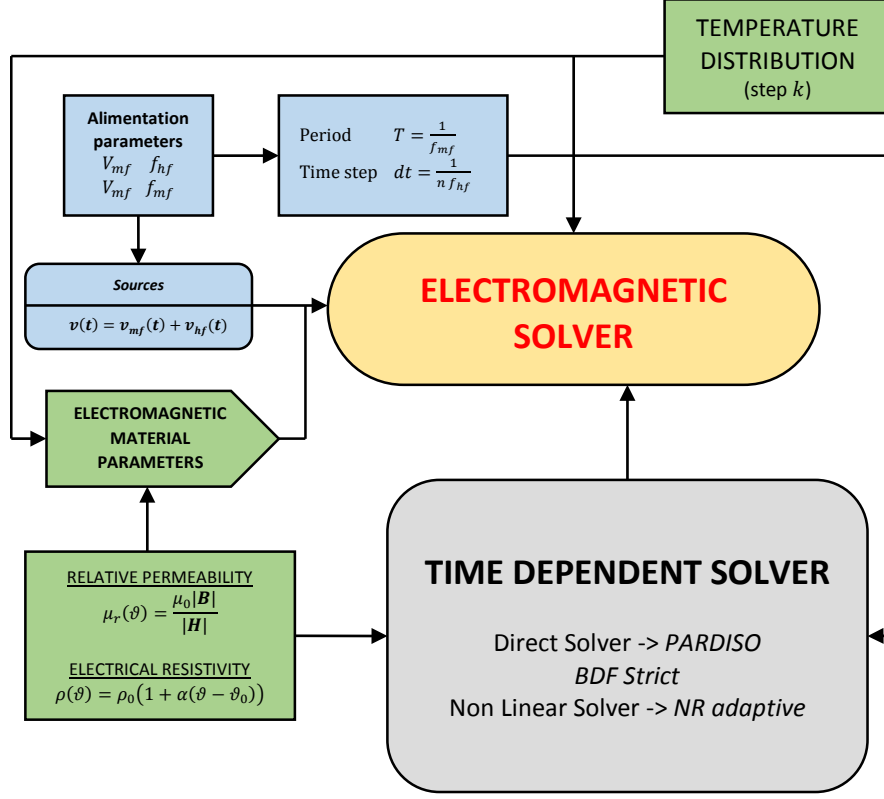


Figure 8.7: Flow chart of the electromagnetic model. The problem is solved in time domain, and the PDEs are characterized by non linear coefficients.

φ -out-of-plane-component), can be computed for each element of the mesh. It is possible to split the current density inside the coil inductor as two terms at different frequencies:

$$\mathbf{J}_e = \sigma \frac{V_{hf} \cos(2\pi f_{hf})}{2\pi r} \mathbf{e}_{coil} + \sigma \frac{V_{mf} \cos(2\pi f_{mf})}{2\pi r} \mathbf{e}_{coil} \quad (8.7)$$

so it is possible to affirm that for the single turn coil the superimposition of the effects is valid. This is even more true, if we consider that the material used for the inductor is copper. Now Maxwell's equations that have been seen in previous chapters make possible the computation of the problem. The current that flows in the coil during the process is one of the most verifiable parameter in order to understand the process, so starting from the current density distribution on the coil section the integral value can be evaluated as:

$$i_{coil} = \int_{S_{coil}} \mathbf{J} \cdot \mathbf{e}_{coil} dS \quad (8.8)$$

Steel Billet

This region is the workpiece that must be heated during the process. This physic region is characterized by initial high value of relative magnetic permeability. So high values of magnetic flux are expected inside.

An ideal, linear region, with constant μ_r typically imposes B-normal component through interface between air and workpiece. Magnetic potential countour are typically perpendicular into interface of a material with very high μ_r . Instead during the time evolution of electromagnetic problem, steel saturates and magnetic potential equiflux lines are tangential to interface between air and steel billet. The system of equations for the billet is solved by the boundary element method with iterative correction of relative magnetic permeability and electric resistance, every time increment depending of temperature in the element.

The specific Joule heat sources $w(r, z)$ have been calculated for each element, for each time step of the time-transient electromagnetic solver.

$$w(r, z) = \frac{1}{\sigma} |\mathbf{J}(r^*, z^*) \cdot \mathbf{J}(r^*, z^*)| \quad (8.9)$$

where the current density for each element of the mesh is:

$$\mathbf{J} = -\sigma \frac{\partial \mathbf{A}}{\partial t} = -\sigma \frac{\mathbf{A}(\mathbf{t}_{k+1}) - \mathbf{A}(\mathbf{t}_k)}{t_{k+1} - t_k} \quad (8.10)$$

8.3.4 Thermal Model

The thermal problem is modeled by imposing a distribution of heating power in the billet. The model takes into account the heat loss during the process, by imposing thermal exchange boundary conditions on the surface, taking account of losses by convection and radiation. The radiative loss, when the temperature exceeds 800 [° C], become much more significant. The air temperature is set to 20 [°C]

The thermal model, as already introduced, takes into account the variation of the parameters during the process. In other words, the material properties in the heat balance equation are functions of temperature. This means that the thermal model is solved through a non-linear solver. The inductor is not modeled in the thermal process.

The material properties has already thoroughly described in the previous chapters and will be no longer discussed.

8.4 Coupling

The coupling between the two physics is performed, respectively, through the temperature distribution in the billet and through the heating power. From the latter can easily be obtained the trend in time (in the magnetic period) of the induced power: where the material properties in each element of the mesh of the billet are a distribution function of the temperature distribution computed in the previous thermal step. Obviously the model must start from an initial condition of uniform distribution of temperature $\theta_0 = 20$ °C. At

this point, the coupling with the thermal model is carried out thanks to the integration of the distribution of electromagnetic power in the period. The result is then the average value of the heating power for each element that constitutes the mesh. In practice from electromagnetic solution are pulled out all the nodes constituting the mesh, and are integrated through an appropriate numerical integration method. Because as written, the electromagnetic model was solved considering linear shape functions, simple integration via method of trapezoids provides excellent results in low-cost computing. This is even more true when you consider that every electromagnetic period is sampled with 401 points, that means in our case a time step of $2.5 \cdot 10^{-7}$ s.

$$w(\mathbf{P}, t) = \begin{bmatrix} w(\mathbf{P}_1, t_1) & w(\mathbf{P}_1, t_2) & \dots & w(\mathbf{P}_1, t_{n-1}) & w(\mathbf{P}_1, t_n) \\ w(\mathbf{P}_2, t_1) & w(\mathbf{P}_2, t_2) & \dots & w(\mathbf{P}_2, t_{n-1}) & w(\mathbf{P}_2, t_n) \\ \dots & \dots & \dots & \dots & \dots \\ w(\mathbf{P}_m, t_1) & w(\mathbf{P}_m, t_2) & \dots & w(\mathbf{P}_m, t_{n-1}) & w(\mathbf{P}_m, t_n) \end{bmatrix} \quad (8.11)$$

where n is the number of samples in a period, and m the number of Lagrange points, that depends from the quality of the mesh. A Lagrange point in an axial symmetric model is univocally defined from its coordinates: $\mathbf{P} = [x, y]$. To obtain the average power in a period:

$$\langle w(\mathbf{P}) \rangle = \frac{1}{T} \int_{t_0}^{t_f} w(\mathbf{P}, t) dt = \frac{1}{T} \int_0^T w(\mathbf{P}, t) dt \quad (8.12)$$

Than for the Stevin law, it is possible to approximate the Integral 8.12, in other words the area below the power function, with the area of the trapezoid. Because of the magnetic potential is linear, and the time discretization is constant, it is also the exact integral, than:

$$\langle w(\mathbf{P}) \rangle \approx \frac{dt}{T} \left(\frac{w(\mathbf{P}, 0) + w(\mathbf{P}, T)}{2} + \sum_{i=2}^{n-1} w(\mathbf{P}, t_i) \right) \quad (8.13)$$

So you get the distribution of the average power inside the billet . This distribution is then the only heat source in the thermal problem, assumed constant over the period of the thermal solver. At the end of electromagnetic solution, it reaches a certain temperature distribution in the billet to step k . This distribution is set as input to step $k + 1$ of the electromagnetic model . The computation of the thermal problem has as output a temperature distribution, that is the input of the next electromagnetic problem.

$$\theta_k(\mathbf{P}) \rightarrow \theta_{k+1}(\mathbf{P}) \quad (8.14)$$

8.4.1 Solvers

The resolution of the electromagnetic problem is the most time consuming part of the solution of the model . In fact the non-linear magnetic behavior (i.e. the correlation between the magnetic field $\mathbf{H}$ [A/m] and the magnetic flux $\mathbf{B}$ [T] through a feature typically highly non-linear) makes it very slow and costly resolution. The commercial software Comsol makes possible to define with reasonable freedom Magnetic feature through a map [magnetic field-magnetic flux]. In practice the software after the resolution of the

rotor of the magnetic potential, obtains the magnetic flux $\mathbf{B}$, then it calculates the module at a specific element and for that value of magnetic flux goes through linear interpolation to derive the relative value of the magnetic field $|\mathbf{H}|$ [A/m]. To ensure a convergence of the method and the obtaining of a solution, it was chosen as the direct solution method: Pardiso [ref sito pardiso].

Table 8.5: Electromagnetic time dependent solver configuration

Time dependent solver	$dt = 2.5 \cdot 10^{-7}$ [s]	$t_i = 0$ [s]	$t_f = 2.0 \cdot 10^4$ [s]
Direct Solver	PARDISO		
BDF-Strict	$t_{min} = 1 \cdot 10^{-8}$ [s]	$t_{max} = 1 \cdot 10^{-8}$ [s]	
Non Linear Method	Newton-Raphson	Authomatic	

The solution of the thermal problem is much easier then the electromagnetic problem, which is the most time consuming part of the computational problem.. It is important to remember that for the thermal model, for each time step the distirbution oh the heating power is supposed constant within the computational period. The heating power $w(r, z)$ is the avarage induced power in an electromagnetic period, so it is possible to solve the Fourier's equation with a more large time step, because it is supposed constant.

Table 8.6: Thermal problem time dependent solver configuration

Time dependent solver	$dt = 1 \cdot 10^{-4}$ [s]	$t_i = 0$ [s]	$t_f = 5.0 \cdot 10^3$ [s]
Direct Solver	PARDISO		
BDF-Strict	$t_{min} = 1 \cdot 10^{-4}$ [s]	$t_{max} = 1 \cdot 10^{-4}$ [s]	
Non Linear Method	Newton-Raphson	Automatic	

8.5 Results

8.5.1 Electromagnetic Analysis

This section presents the results of the study of the electromagnetic problem, solved in the time domain. In particular, it focuses on the effects of temperature on the magnitudes of electromagnetic oscillating fields in time. Also a numerical solution, with a time-dependent solver allows to study with extreme accuracy (at least from the numerical point of view) the effect of magnetic saturation.

The solution of the problem, in the electromagnetic time domain, enables to plot trends that characterize the electrical quantities such as currents in the coil and the total power. The solution of the FEM model allows to query physical entities such as the induction coil and and derive the trends of their electrical parameters characterizing the power supply. The current measurement has been done integrating the current densities on the coil section, while the total active power is refered to the integration of the mean power density distribution of the system.

From the electrical point of view, it is interesting to analyze the variation in the thermal time of electrical quantities. Since for each period of electromagnetic simulation the temperature distribution of the billet is assumed constant, it is possible to analyze, the effects that temperature has on waveforms of the electromagnetic phenomenon. The model has been developed in order to set the voltage from an external source, so it does not change over time. Instead, as already seen, current and power are strongly affected by temperature.

In Figure 8.8 three different behaviour of current for three different temperature distributions have been plotted, respectively at the beginning of the process, when the temperature is uniform inside the billet and two different temperature conditions (in the period [0.1-0.105] s and in the period [0.195-0.22] s).

These two curves show how the average temperature of the surface electromagnetically coupled with the inductor plays a key role. In fact, although the resistivity is increased (which causes a reduction of the RMS value of the current) the value of the demagnetization effect of magnetic permeability is much more important with the heating time because the average temperature of the coupled surface is much higher than the temperature at which it begins to reduce the relative permeability, or in other words $T > 650C$.

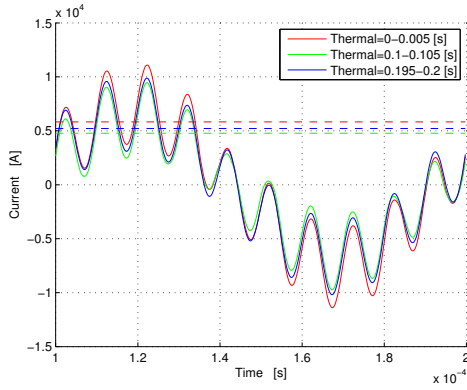


Figure 8.8: Current evolution in several electromagnetic periods.

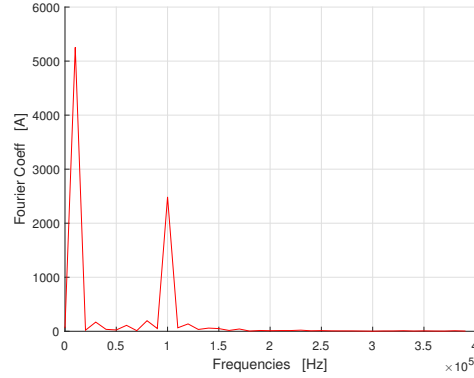


Figure 8.9: Fourier analysis of current spectrum of [0-0.005] s.

From the results it is clear that the voltage has only two frequency components. One evolves in time with the medium frequency $f_{mf} = 10kHz$ and one evolves in time with the high frequency $f_{mf} = 100kHz$.

Completely different is the behaviour of the instantaneous power and its harmonic content. From the plots it is possible to see well the four frequency components introduced in the section of the electrical model of the problem, plus the DC content. In Figure 8.10 the power trend during the electromagnetic period has been analyzed for different thermal time steps. In Figure 8.11 the Fourier analysis of power spectrum has been shown. It is important to remember that during the process, due to the change of the electromagnetic parameters, the value of the power and current consumption in the period varies in a highly non-linear way with the process time.

From the electrical point of view, it is extremely interesting to understand how evolve

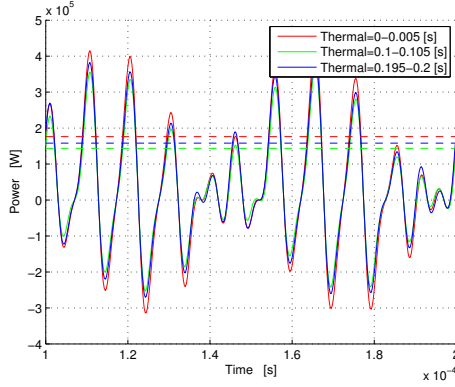


Figure 8.10: Power evolution in several electromagnetic periods.

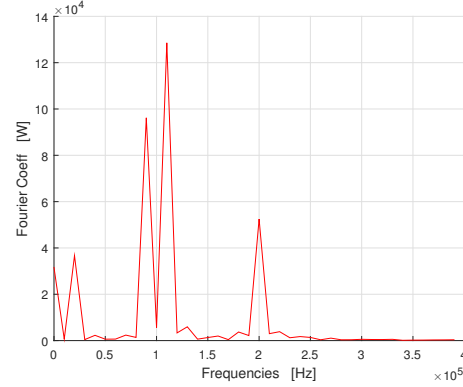


Figure 8.11: Fourier analysis of power spectrum of [0-0.005] s.

the active power absorbed by the sysyem during the process and the integral value of the current that flows in the coil. So it is possible to define, for each thermal step, the average value of the power absorbed, that is the total active power of the load. This is plotted in Figure 8.13. The same thing can be done for the current 8.12 It is clearly possible view that it varies in a non-linear way during the SDF hardening process.

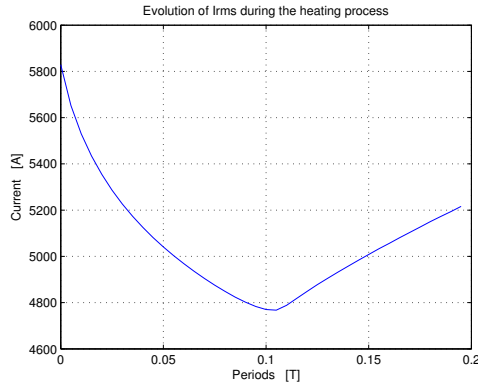


Figure 8.12: RMS Current during the process.

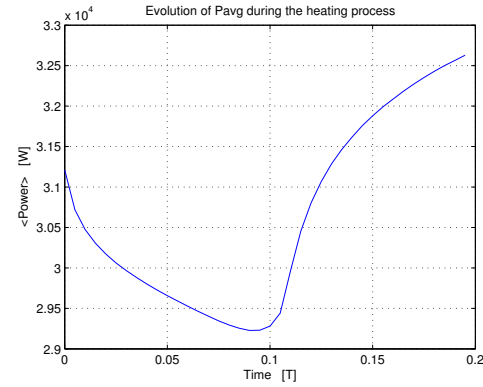


Figure 8.13: Avarage value of the power P_{AVG} during the process.

It is particularly interesting to analyze what happens, from the electromagnetic point of view, when the current passes through zero. In fact in this instant the power delivered by the inductor is zero, but the time derivative of the magnetic field is very high. The magnetic field generated by the inductor is zero. So the inductor does not generate any current in the billet. The current density on the surface of the billett cannot instantaneously go to zero, however. Thus there is a current density which sign depends on the supply current of the inductor at the previous instant. This surface current density then generates a magnetic field inside the piece of opposite sign, which generates opposite currents than those that have generated.

In the simulations, the inversion of the sign of the current density along the radius of the workpiece can be seen, since there are points where the current density goes to zero. This phenomenon from an electromagnetic point of view is extremely interesting. It is possible to understand that there are streams that do not concatenate the inductor, but whose lines are closed again inside the billet, or in the air. The phenomenon can be observed for different temperature distributions, or in other words, for different instants of the heating process.

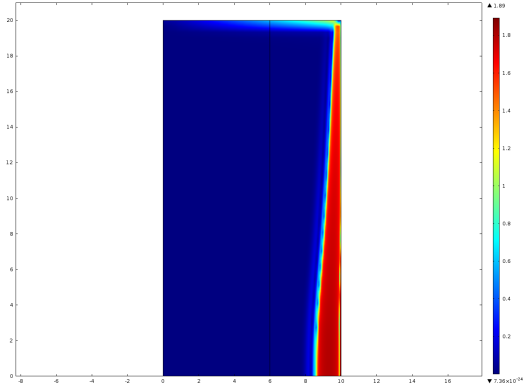


Figure 8.14: Magnetic Flux amplitude $norm\mathbf{B}$ [T] plotted at time $t_0 = 1.3664 \cdot 10^{-4}$ [s] in the first period of heating process.

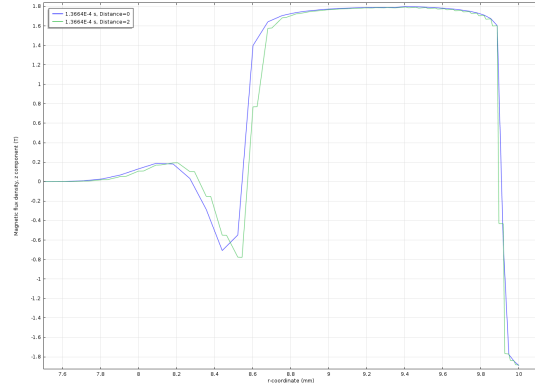


Figure 8.15: z -component of Magnetic Flux B_z [T] plotted at time $t_0 = 1.3664 \cdot 10^{-4}$ [s] in the first period of heating process

In Figure 8.14 is represented the amplitude of magnetic flux when the current goes to zero and for an uniform temperature distribution, that means the temperature is everywhere equal to 20 °C. It is possible to verify that magnetic flux inside the billet goes to zero. This phenomenon is more clear if it is considered the z -component of Magnetic Flux: B_z that for the geometry of the model is the most relevant. It is possible view that the z -component changes the direction along the radial coordinate of the workpiece. From the Figure 8.15 it is possible view that B_z plotted in a cutline parallel to r -direction and in the center of the billet B_z is extremely intense and with a negative sign on the surface of the billet, instead at 9.9 [mm] it changes sign and it maintains a very high and positive value for a thickness $\delta = 1.3$ [mm]. This effect can be explained considering that the inductor current at that instant is zero, so there is no magnetic field from the inductor.

In the same way it possible to plot the behaviour along the r -coordinate of the current density J_φ that of course must change sign in accord with what happens in Figure 8.15.

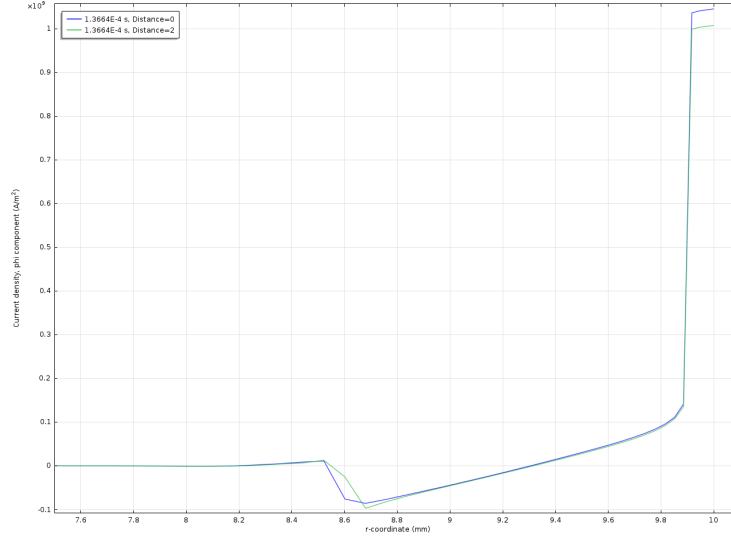


Figure 8.16: φ -component of Current density J_φ [A/m^2] plotted at time $t_0 = 1.3664 \cdot 10^{-4}$ [s] in the first period of heating process, for two different cutlines.

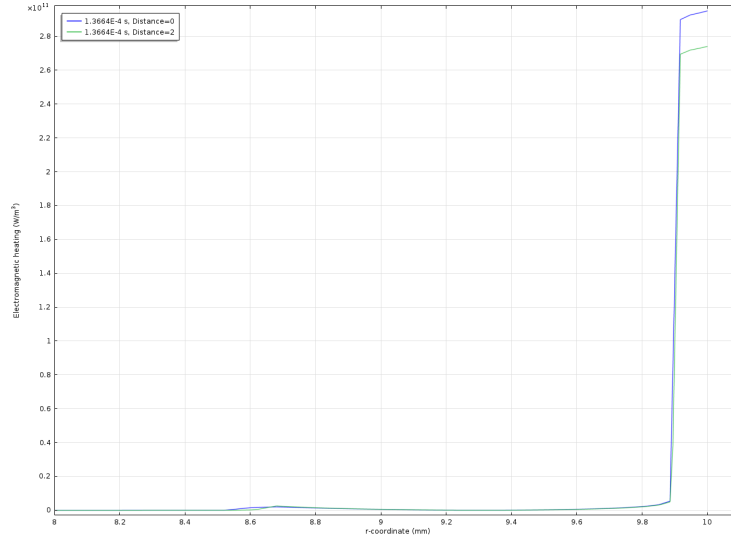


Figure 8.17: Specific heating induced power w [W/m^3] plotted at time $t_0 = 1.3664 \cdot 10^{-4}$ [s] in the first period of heating process, for two different cutlines.

For this time step, it is interesting also to analyze the induced power and relative magnetic permeability along the central cutline. These two quantities are, in fact, the two most interesting variables of the problem, the first from the thermal point of view and the last from the purely electromagnetic one. It is evident that the induced power, when the current passes through zero, remains confined in an extremely reduced thickness, when the temperature is far below the Curie point. Instead, because of the magnetic field $\mathbf{H}$ is

however very intense, a large portion of space overcomes the saturation point, that means the relative magnetic permeability is extremely small.

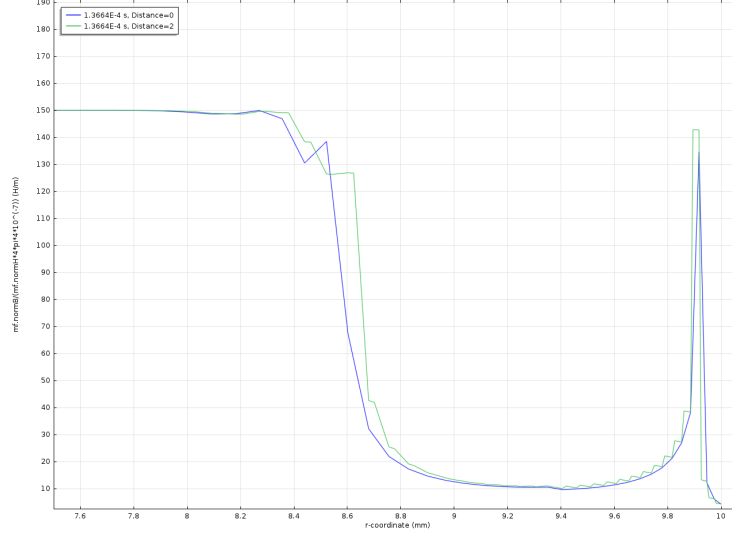


Figure 8.18: Relative magnetic permeability μ_r [-] plotted at time $t_0 = 1.3664 \cdot 10^{-4}$ [s] in the first period of heating process, for two different cutlines.

Going ahead with the treatment, the temperature on the workpiece's surface increases, and more and more points exceed the Curie transition zone. In Figures from 8.19 to 8.22 the magnetic flux densities after 0.1 seconds of heating are shown.

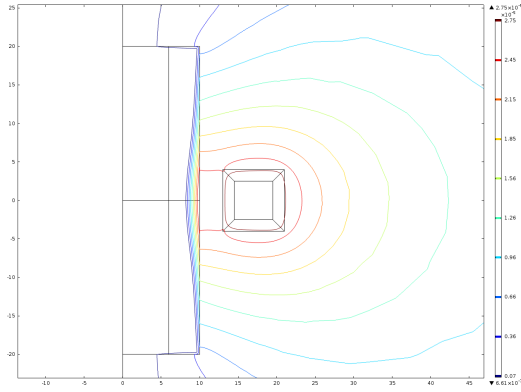


Figure 8.19: Behaviour of $2\pi A_\varphi$ at magnetic time $1.2225 \cdot 10^{-4}$ s for the temperature distribution at time 0.1 s

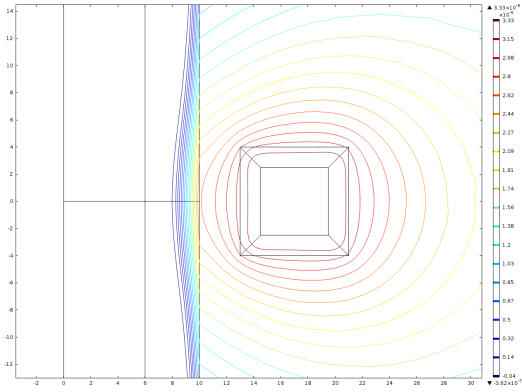


Figure 8.20: Behaviour of $2\pi A_\varphi$ at magnetic time $1.3 \cdot 10^{-4}$ s for the temperature distribution at time 0.1 s

Figure 8.19 shows that when the current assumes its maximum value, the inductor is fully coupled with the billet. Instead when the current falls to zero, Figure 8.21 shows a toroidal structure of magnetic energy in the airgap between inductor and billet. This

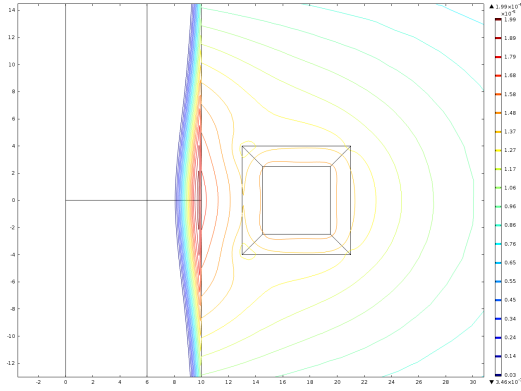


Figure 8.21: Behaviour of $2\pi A_\varphi$ at magnetic time $1.445 \cdot 10^{-4} s$ for the temperature distribution at time 0.1 s

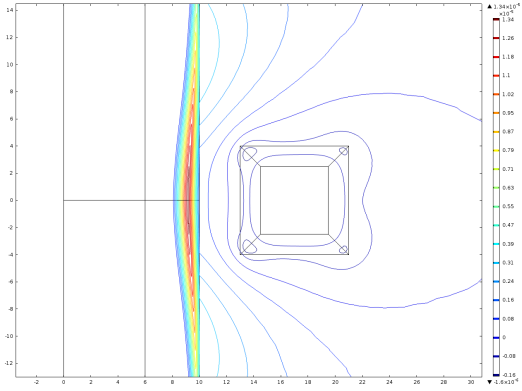


Figure 8.22: Behaviour of $2\pi A_\varphi$ at magnetic time $1.5 \cdot 10^{-4} s$ for the temperature distribution at time 0.1 s

means that the surface current density that exists on the billet generates itself a strong magnetic field that induces current inside the inductor. Than when the coil current become negative the situation is changed as shows Figure 8.22.

It is also interesting to plot the same trends for different temperature distributions. For example, with reference to Figure 8.23, it is clear, that when a very large portion of the billet is characterized by temperatures higher than Curie Temperature, the material changes its magnetic properties and becomes diamagnetic. If you combine this phenomenon with the passage by zero of the current, you get an even more complex behaviour with respect to that discussed previously.

From Figure 8.24 it is possible to notice that the magnetic field penetrates very deeply into the workpiece. when part of the billet overcomes the Curie temperature. So the overcoming of the critical temperature, causes the penetration of the magnetic flux $\mathbf{B}$ towards the axis of the billet.

This is the most interesting phenomenon of the heating process. It causes, during the last part of the process, the displacement of the thermal sources to different areas that one would expect if they did not take into account the effect of Curie transition. This means that overall electromagnetic periods, heat sources are located very deeply. This causes a heating much more intense in innermost zones. It results in a Austenite thickness wider than presumed.

In the same way it possible to plot the behaviour along the r -coordinate of the current density J_φ that of cours must changes sign in accord with what happens in Figure 8.15. It is clear that if the current density is proportional to the time derivative of the magnetic flux $\mathbf{B}$, the distribution of induced currents is closely related to the distribution inside the piece of relative permeability. It is possible to observe from the figure of the distribution of the φ -component of the current density that it penetrates very deeply into the piece.

In the same way that was done for the heating power, it is possible to derive the average distribution of the current density, in a electromagnetic period. It is interesting to consider

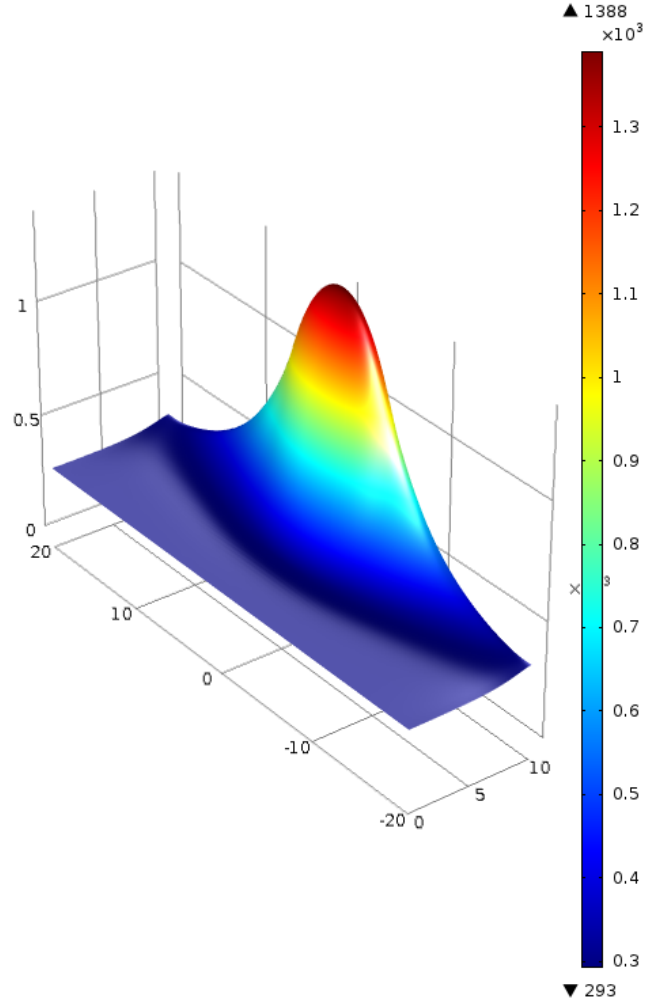


Figure 8.23: Temperature distribution [K] at heating time $t_{ht} = 2.15 \cdot 10^{-1}$ [s]

how the most important electromagnetic parameters vary with temperature distribution. It is possible to observe a similar phenomenon that occurs for average heating power. It means that don't consider a behaviour of electromagnetic phenomena as function of temperature could involve not acceptable errors, and than not acceptable heating profiles.

Figure 8.29 shows that, when many points of the billet exceeds the critical temperature of Curie, it occurs a shift of the current density towards the axis of the workpiece, not only for different instants of the period, but actually also in terms of average values. This behavior is also increased by the effect of saturation.

it is also interesting to analyze the distributions of the magnitudes elettromangetiche for example when the current assumes the maximum value within the period electromagnetic. This occurs when the derivative with respect to time of the current assumes the zero value. Because the current assumed at this instant the maximum value, then the magnetic field

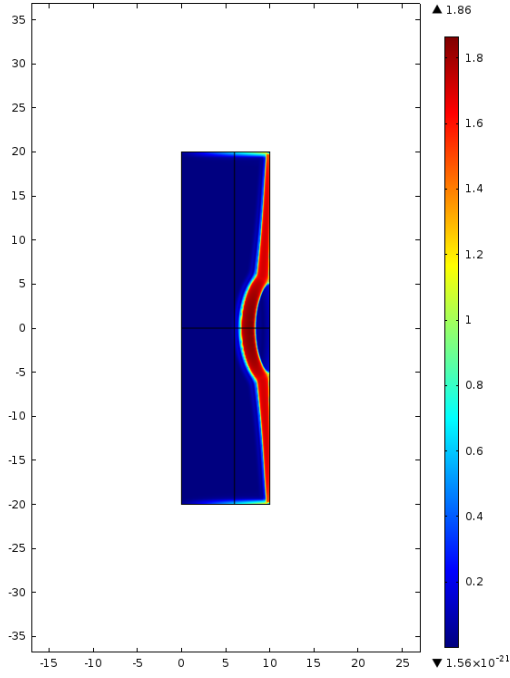


Figure 8.24: Amplitude of Magnetic Flux $norm\mathbf{B}$ [T] plotted at time $t_0 = 1.3618 \cdot 10^{-4}$ [s] for a temperature distribution with many points over T_c .

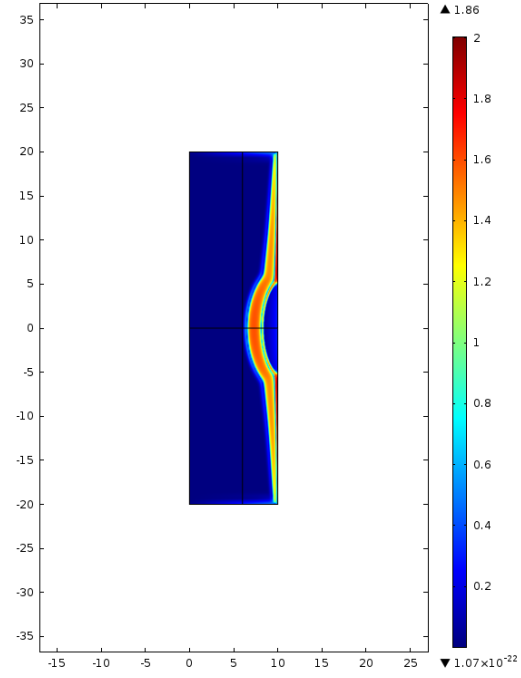


Figure 8.25: Amplitude of Magnetic Flux $norm\mathbf{B}$ [T] plotted at time $t_0 = 1 \cdot 10^{-4}$ [s] for the temperature distribution at heating time $t_{hf} = 0.215$ [s]

generated by the inductor is maximum. Magnetic flux has already saturated, so the magnetic permeability is minimum. As was done for other electromagnetic quantities it is useful and interesting plot developments for different temperature distributions.

In Figure 8.30 it is clear that a large portion of space is saturated. In fact the magnetic flux $\mathbf{B}$ remains almost constant for about $\delta = 1$ mm and then reduces in value. Also relative permeability shows this behaviour.

Extremely interesting is the distribution of heating power into the workpiece (Figure 8.33, although the temperature is well separated from the critical temperature, the saturation tends to push the distribution of the thermal power at greater depth from the surface.

Finally in this section are reported the trends for the final temperature distribution. In addition there are also the distributions of the electromagnetic quantities along the surface of the workpiece. In fact, in the final stages of the piece there was a large portion (a semi ellipse) in relative permeability unit. We see that the corners are concentrated strong current densities and thus strong heating powers that cause a localized overheating (in distributions are seen two peaks). This behavior is increased for example when the current assumes the maximum value, namely for the time instant $t_0 = 1.225 \cdot 10^{-4}$, in other words when the piece is in condition of maximum saturation. For this reason here are reported plots on the billet surface and on a cutline parallel to the first, but spaced 1 [mm].

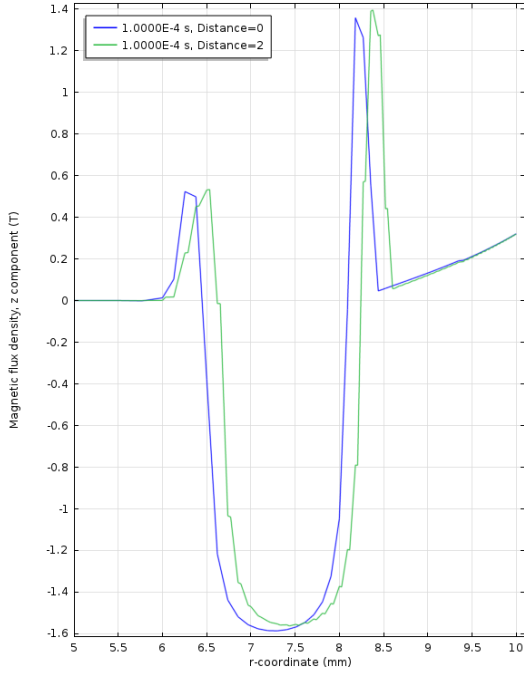


Figure 8.26: z -component of Magnetic Flux B_z [T] plotted at time $t_0 = 1 \cdot 10^{-4}$ [s], for the temperature distribution at heating time $t_{hf} = 0.215$ [s] for two different cutlines.

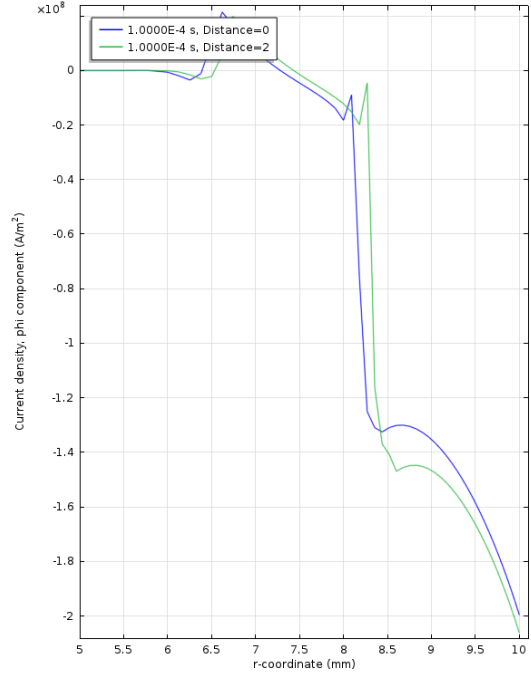


Figure 8.27: φ -component of Current density J_φ [A/m²] plotted at time $t_0 = 1 \cdot 10^{-4}$ [s], for the temperature distribution at heating time $t_{hf} = 0.215$ [s] for two different cutlines.

8.5.2 Numerical thermal results

The analysis of post-processing has been done with the help of commercial software MATLAB. First of all it is important to define a map of the points which the figures refer to. The points selected for the analysis of the data are the points of greatest interest from the point of view of the electromagnetic and thermal process. The process of induction hardening in dual frequency typically want as output data (or specific process) parameters such as temperature sistribution in the directions x,y (r, z in the model). Of considerable interest in this work there are also other parameters such as the heating rate, the distribution of the power induced and the values of the electromagnetic fields inside the billet.

In detail it is also possible to analyze what happens on the main points of interest. In Figure 8.34 the temperature evolution over the time for the interensting points along the radial direction has been plotted. points are located along the transversal section of the billet, corresponding to the middle section of the coil. The temperature on the surface reaches a value close to 1100°C, while it rapidly drops when the attention is moved towards the center of the body.

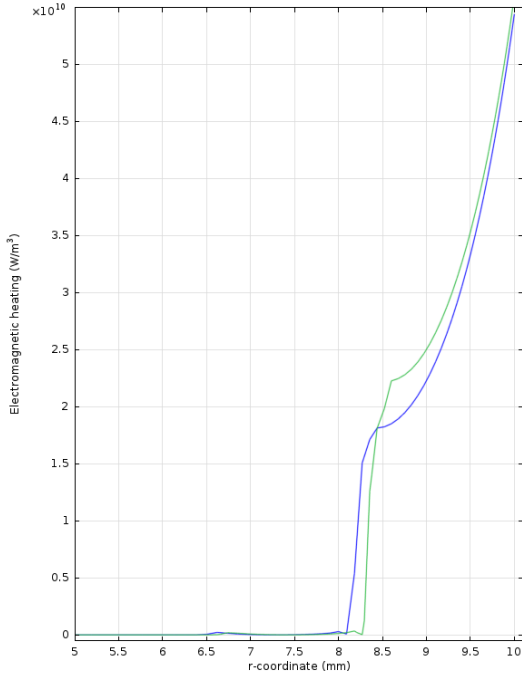


Figure 8.28: Specific heating induced power w [W/m^3] plotted at time $t_0 = 1 \cdot 10^{-4}$ [s], for the temperature distribution at heating time $t_{hf} = 0.215$ [s], for two different cut-lines.

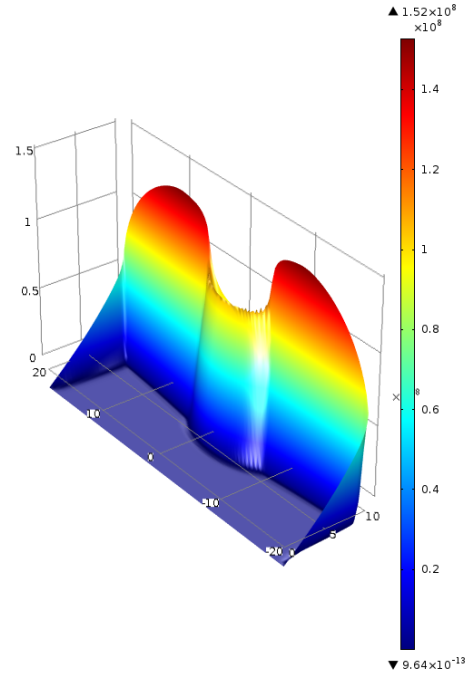


Figure 8.29: AVG distribution of amplitude of current density $normJ$ [T] computed for the temperature distribution at heating time $t_{hf} = 0.215$ [s].

Table 8.7: List of the points analyzed in the results

Point	r -coordinate [mm]	z -coordinate [mm]
A	10	0
B	0	0
C	5	0
D	7.5	0
E	10	2
F	10	5
G	10	20
H	8	0

Since the inductor is much smaller than the billet heated, only a small band of billet is well coupled with the inductor, so the distribution of the heating power is confined in a central band with respect to the inductor. During the heating in fact temperature

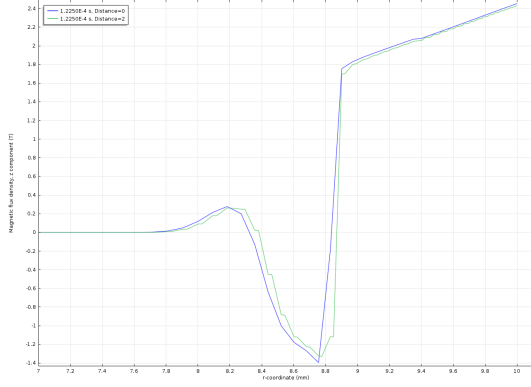


Figure 8.30: z -component of Magnetic Flux B_z [T] computed for the temperature distribution at initial condition for two different cutline along r -coordinate.

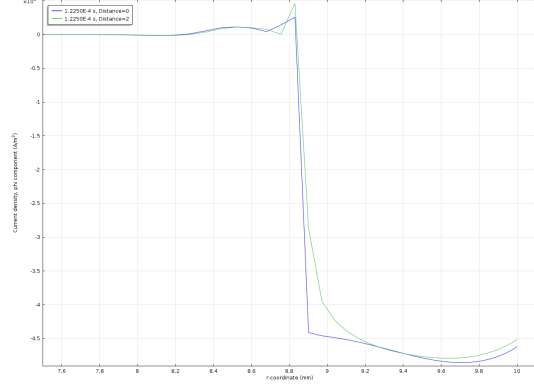


Figure 8.31: φ -component of current density J_φ [A/m^2] computed for the temperature distribution at initial condition for two different cutline along r -coordinate.

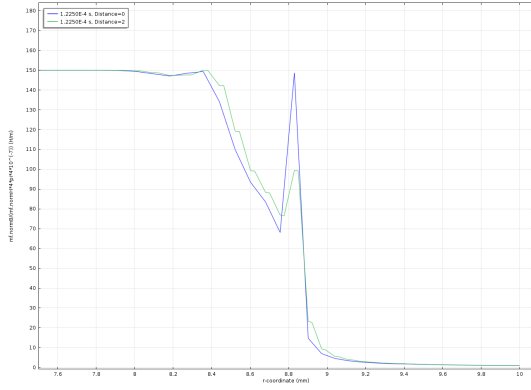


Figure 8.32: Relative permeability distribution μ_r [-] computed for the temperature distribution at initial condition for two different cutline along r -coordinate.

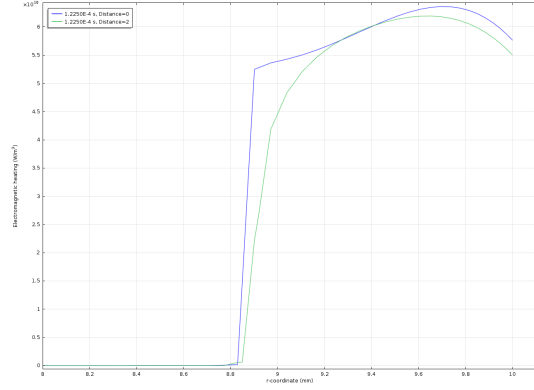


Figure 8.33: Specific heating power distribution w [W/m^3] computed for the temperature distribution at initial condition for two different cutline along r -coordinate.

gradients along the z -coordinate are very intense. From Figure 8.35 you can see the evolution on the most interesting points of the surface of the billet.

To understand the temperature trend, for example some distributions of temperature have been also plotted, in sections parallel to the height of the billet in two different time instants, before the demagnetization effect the Curie and at the time of 0.1 seconds when a good portion of the billet has passed the critical temperature.

From the Figure 8.40 it is clear how much important is the rate changes during the

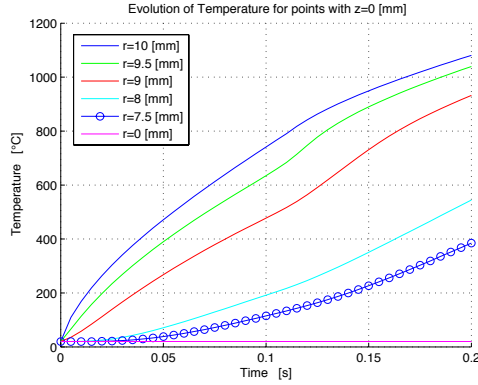


Figure 8.34: Temperature evolution in time of main points at 0 [mm] z -coordinate

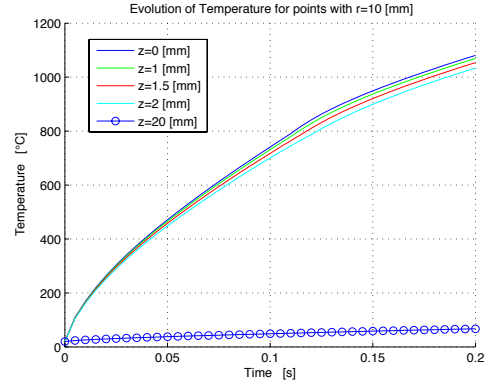


Figure 8.35: Temperature evolution in time of main points at 10 [mm] r -coordinate

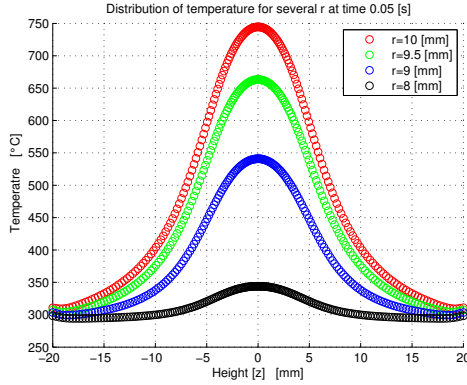


Figure 8.36: Temperature distribution for different r values at process time of 0.05 [s]

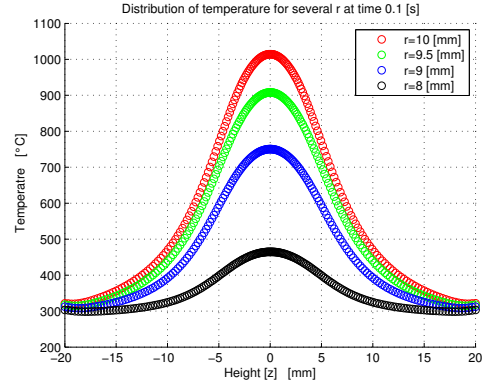


Figure 8.37: Temperature distribution for different r values at process time of 0.1 [s]

heating process. There is a strong thermal inertia of the layers of material that in the first steps of the process do not undergo by induced powers.

From the theory it is also known that the rate of heating is useful for defining the temperature A_{C3} , which is known to be a function of the heating rate. Because of induced power is not constant inside the billet, also heating rate will be not constant inside the billet. This means that the transformation to austenite begins at different temperatures, and is not simultaneous in to the workpiece. The results highlight that heating rate are extremely strong all along the process. Figure 8.40 shows a sharp drop of the heating rate of the points nearest to the inductor already from earliest moments of the process.

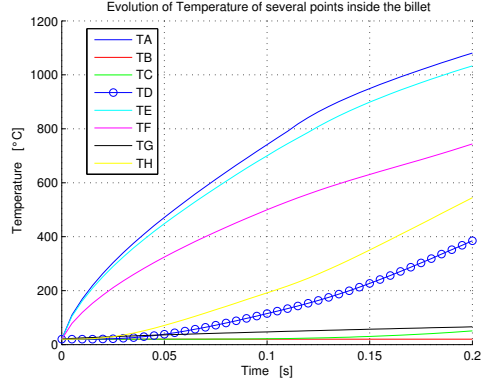


Figure 8.38: Temperature evolution in time of several points inside the billet.

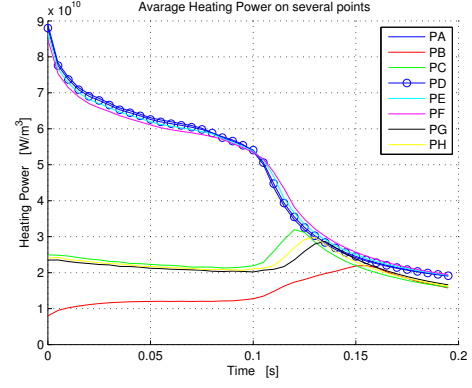


Figure 8.39: Heating rates evolution in time of several points inside the billet.

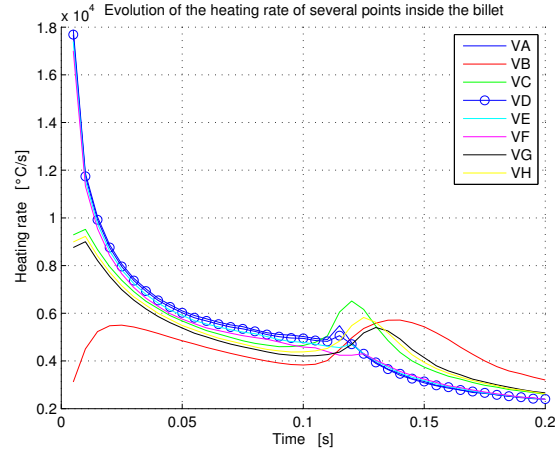


Figure 8.40: Heating rates evolution in time of several points inside the billet.

Chapter 9

Influence of Microstructure and Heating Rate on Austenitization Kinetic of 39NiCrMo3 Steel

9.1 Introduction

The formation of austenite in steel is an inevitable occurrence during most of the industrial processes. Indeed quenching, normalizing, annealing, gas carburizing and surface transformation hardening are examples of heat treatments involving austenitization as first stage of the process. For this reason, the investigation of austenite formation is very important, from both a scientific and a technological point of view. Although this consideration, there has been little work carried out on the formation of austenite as compared with the high amount of studies on its decomposition. Moreover, some of these works are quite disconnected and qualitative. The initial condition of the austenite determines the development of the final microstructure and the mechanical properties of the materials. In particular, for mechanical properties, it's important heating rate, austenite homogeneity, grain size and its distribution [93]. For example Danon et al. [95], studying the influence of the heating rate and austenitization temperature on austenitic grain growth in martensitic steel, reported that the heating rate employed had a strong influence on austenitic grain size and its distribution. Another important aspect to be considered is the prior microstructure: indeed different initial microstructures determine that of austenite obtained during heating. In 1943, Roberts and Mehl reported a study of austenite formation from ferrite-lamellar pearlite and ferrite-spheroidized pearlite, determining the nucleation and growth characteristics of the transformation [96]. More recent studies have indicated that austenite formation in low alloy steel consists of two phenomena: pearlite dissolution and proeutectoid ferrite to austenite transformation. Each of these take place by nucleation and growth [97]-[98]-[99]-[100]. Japanese researchers, instead, have recently examined the formation of austenite from a prior martensitic steel microstructure [101]-[102]-[103]-[104]. They show that acicular austenite is derived from the elongated martensitic laths and/or packets, and is favored by slow heating rate [101]-[102], low austenitizing temperature

and by acicular prior microstructure [103]-[104]. Globular austenite, instead, requires developing opposite conditions than those described above. Furthermore, it is necessary also an investigation of austenitization process under rapid heating and short austenitization time. These conditions are typical of thermal treatments by electromagnetic induction. In these processes, due to Joule effect, an induced current heats the workpiece to be treated. Usually the heating rates employed are extremely high, and often exceeds 200°C/s [15]. Oliveira et al. [97], Macedo et al. [105], but also other investigators [106] observed that an increase of heating rate increase the critical austenitization temperatures (in particular Ac3) and the rate of austenite formation. These considerations are correct only for not prior martensitic microstructure. In the latter case, critical points increase with heating rate only for slow heating rate [107]. Instead, for high heating rate critical points decrease and/or stabilize. In describing induction-heating treatment, besides prior microstructure, it is essential to determine not only the critical points, but also the heating rate employed. These parameters are extremely important in design of induction treatment. Dilatometry could be used to study austenite formation under continuous heating. Indeed, during the transformation phases of steel, there are some volume variations because of changes in the crystalline structure [108]. The present paper describes the influence of prior microstructure and the heating rate on austenite formation. The low alloy steel has been heat-treated in order to develop three microstructures: a ferritic/lamellar pearlitic, a martensitic, and a ferritic/spheroidized pearlitic. Dilatometry has been used to simulate induction heat treatment, and also to determine the fundamental parameters governing austenite formation.

9.2 Experimental Procedure

9.2.1 Material

The material used during the tests is th 39NiCrMo3 (EN10083-3:2006), which chemical composition is shown in Table 9.1

Table 9.1: Chemical composton in %wt of 39NiCrMo3

Element	Weight %
C	0.35 - 0.43
Mn	0.50 - 0.80
P	0.025 (max)
S	0.035 (max)
Si	0.40
Cr	0.60 - 1.00
Ni	0.70 - 1.00
Mo	0.15 - 0.25
Fe	bal

The 39NiCrMo3 has high characteristics of toughness, good deformability and high

hardenability. In particular, Carbon content affects the toughness, Nickel improves the hardenability, the toughness and the mechanical properties, Chromium improves the hardenability and the Molybdenum content avoids a strong embrittlement due to the formation of precipitates, during slow cooling in the range 400-450°C. This material is often used in components exposed to dynamic loads, such as gears and shafts. The material has been furnished in round bars of 12 mm diameter, from which the samples for the dilatometric tests have been obtained.

In order to verify the chemical composition of the steel used, a chemical analysis by Optical Emission Spectroscopy (OES) has been carried out, giving the results shown in Table 9.2

Table 9.2: Chemical composition in %wt of 39NiCrMo3 by OES

Element	Weight %
C	0.43
Mn	0.74
P	0.008
S	0.033
Si	0.36
Cr	0.93
Ni	0.85
Mo	0.24
Al	0.021
Cu	0.099
Pb	0.15
Fe	bal

9.2.2 Dilatometric Sample

The sample used in the dilatometric tests has been derived from the classical shape of round cross-section for tensile tests, but the length between the clampings and the shoulder has been extended in order to plug the electrical connections for the direct current heating of the sample.

Just after the manufacturing process, the samples have been normalized in order to relax the residual stresses introduced during the machining. The normalization process has been carried out in furnace at 850°C for 1 hour, followed by air cooling.

The microstructure of the base material, after normalization, has been verified through optical microscopy. From Figure 9.2 it is possible to observe that the crystalline grains are very fine, uniform and homogeneously distributed.

The material hardness has been verified before and after the normalization process through microvickers hardness measurements. The measurements have been carried out with a load of 300 g and the measurement has been repeated 15 times in order to define the mean and the standard deviation. The results are shown in Table 9.3.

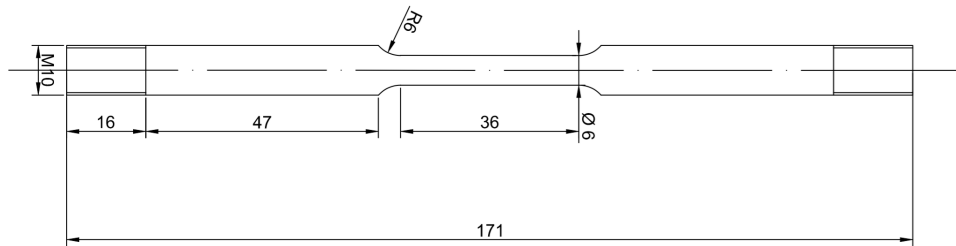


Figure 9.1: Drawing of the dilatometric test sample

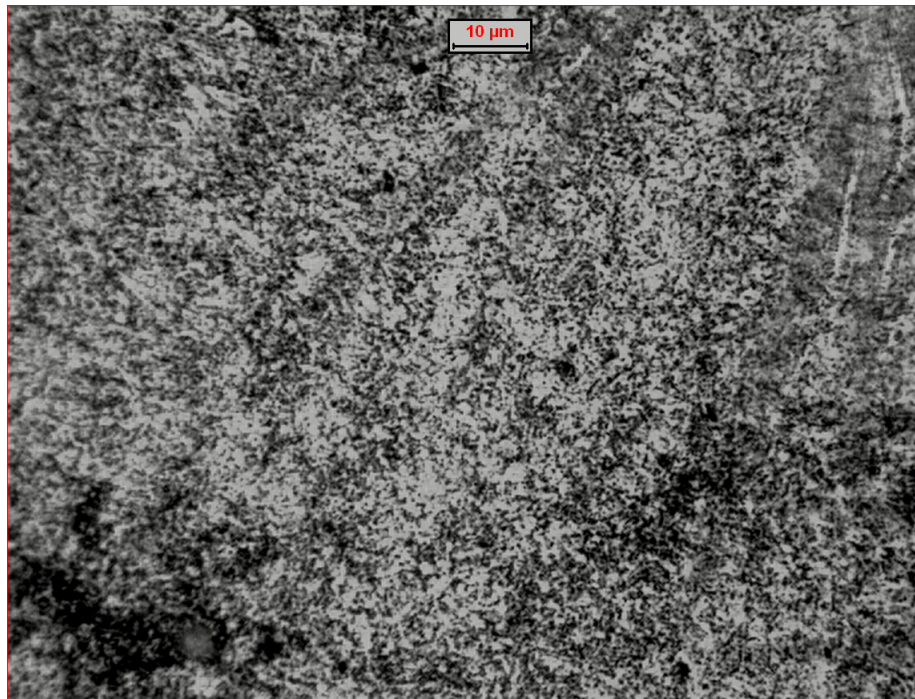


Figure 9.2: Optical micrograph at 1000x of the base material after 2% Nital etching. Transvertical section

9.2.3 Preliminary Heat Treatments

As it has been already said, the aim of the work is the study of the influence of the microstructure and the heating rate on the austenitization kinetics of this steel grade. In order to verify the influence of the microstructure the samples have to be hardened and different tempering processes have to be applied. The samples have been heated in furnace at the temperature of 850°C for 1 hour and then oil quenched, in order to do not introduce dangerous distortion which may threaten the alignment of the sample in the machine.

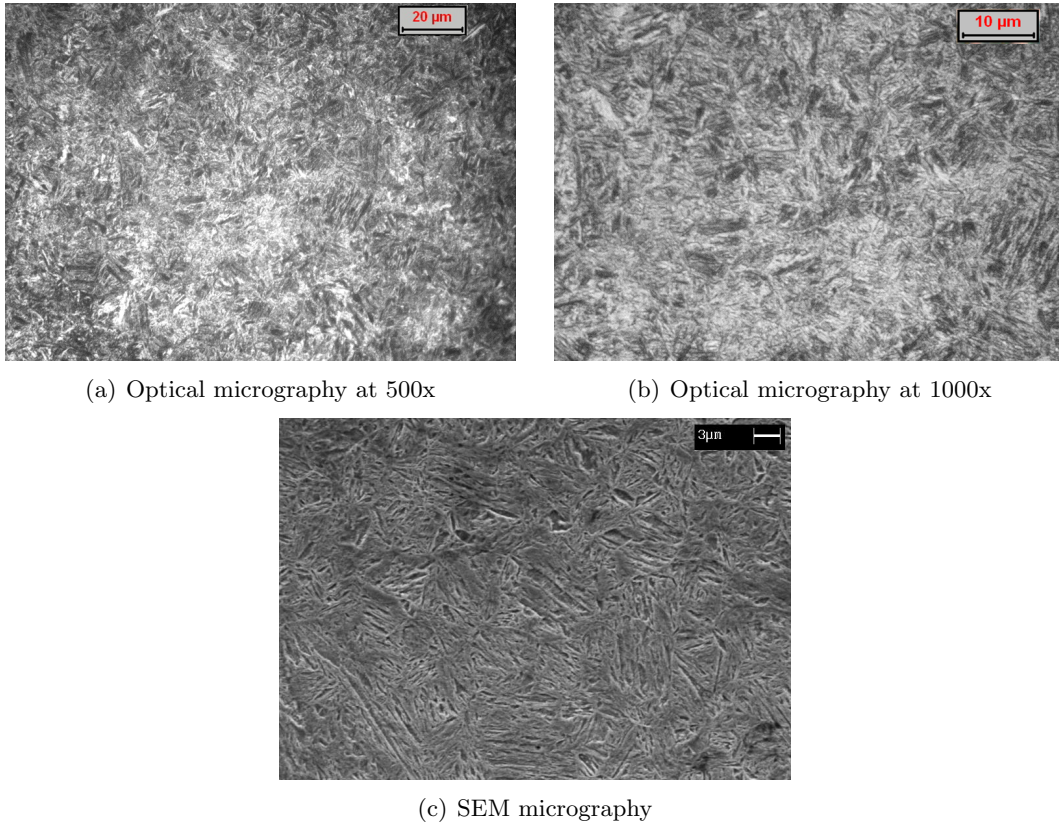
The three tempering processes that will be here discussed are:

- 180°C for 4h

Table 9.3: Hardness (HV0.3) of 39NiCrMo3 before and after normalization

Sample	Hardness (HV)	Standard Deviation
Base material	330	14.97
Normalized	320	15.57

- 580°C for 3h
- 580°C for 6h

**Figure 9.3:** Micrographies of material after tempering at 180°C for 4 hours

The first treatment produces a low temperature tempered martensite, as can be seen in Figure 9.3, where the acicular shape of this kind of microstructure is well defined in the Scanning Electron Microscope (SEM) picture. In the Optical Microscope (OM) pictures the microstructure seems to be too fine to be well observed, even if the martensitic microstructure is recognizable. Here, thanks to the tempering treatment, the martensitic structure starts to relax its crystalline lattice due to the carbon diffusion, even if the temperature is still too low to permit a significant phenomenon. The tetragonal microstructure (typical of martensite) transforms into cubic martensite (or supersaturated ferrite) and the carbon

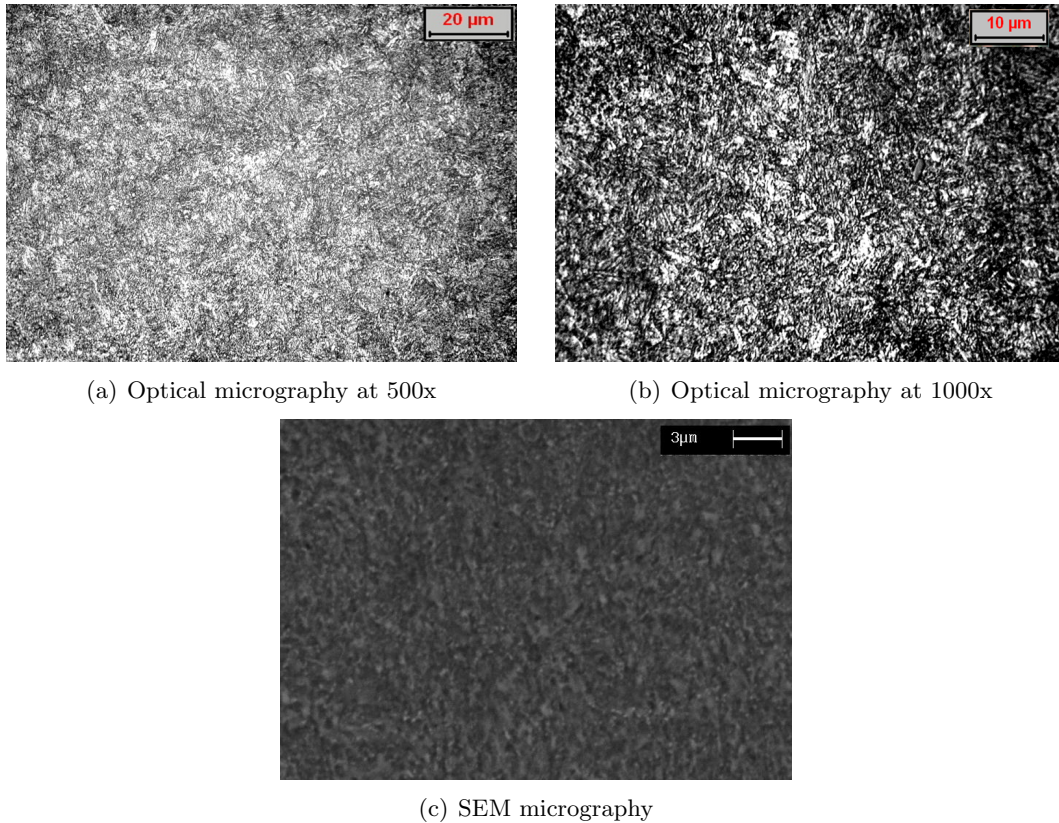


Figure 9.4: Micrographies of material after tempering at 580°C for 3 hours

reacts with iron and generating the ε' carbides which are precursors of cementite [54]. This type of treatment, maintains a very high hardness, similar to the pure martensitic one, and a strong acicularity if observed with the microscope. The ε' carbides cannot be observed with optical microscope, but only through a Transmission Electronic Microscope (TEM).

The low temperature tempering processes require long time to take place, so in order to obtain good transformations at least 8 hours are required for treatments at 150°C and around 4 hours if the treatment has been carried out at 200°C.

Heating up a steel, above the temperature of 250°C, a progressive transformation of the ε' carbides takes place, becoming cementite or more complex carbides. Increasing more and more the temperature (between 300-350°C) the carbides start to coalesce and grow, due to the increased mobility of carbon in the crystalline lattice. The coalescence of carbides softens the steel and increases the toughness, especially when the treatment is carried out above the 540°C.

The second type of microstructure studied in this chapter is derived from a process of quenching and tempering at 580°C for 3 hours. In this case the martensite is completely transformed into ferrite and the carbon migrates in order to form small globular carbides, well dispersed in the ferritic matrix. In Figure 9.8 the microstructure after this kind of treatment has been analyzed through optical and SEM microscopy. Before the analysis

the samples have been polished and etched with Nital 2%, in order to make visible the microstructure. In the pictures the microstructure appears as mixture of ferrite and very fine and well dispersed globular cementite.

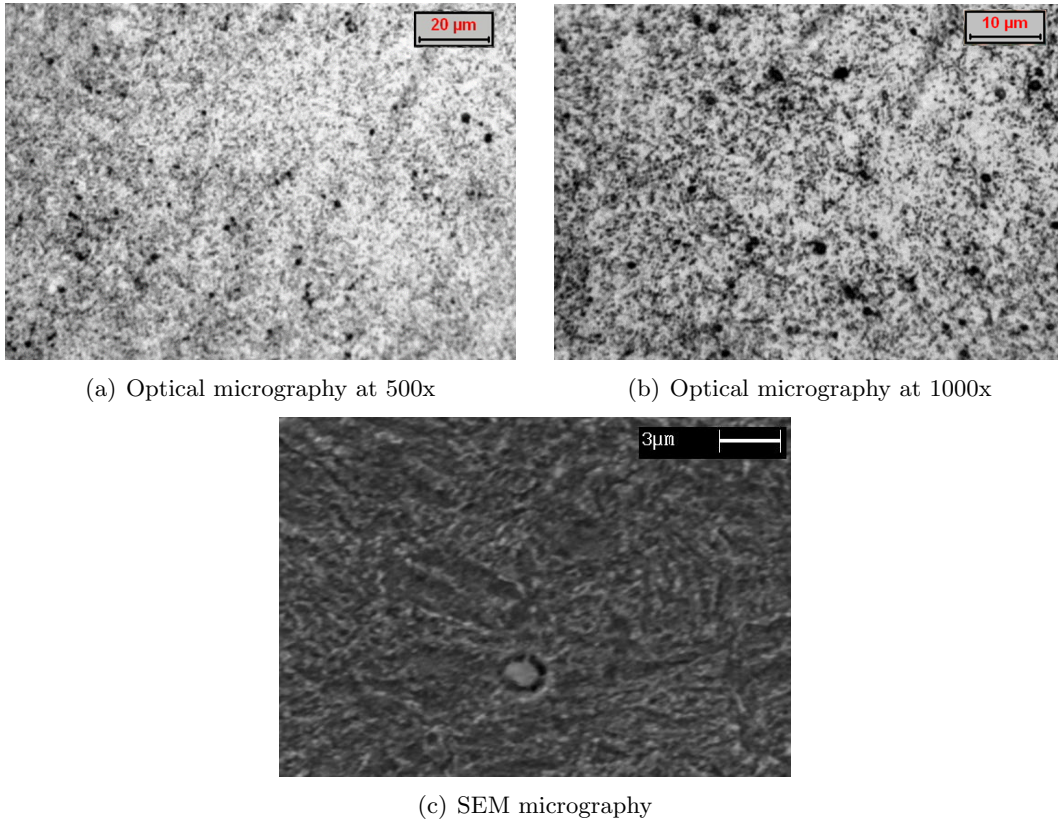


Figure 9.5: Micrographies of material after tempering at 580°C for 6 hours

The last microstructure studied is similar to the second one, but the treatment time has been increased to 6 hours. In this case the carbides have more and more time to coalesce, so bigger carbides are expected. This assumption can be easily verified in Figure 9.5, where the microstructure after this kind of treatment has been analyzed through optical and SEM microscopy. The microstructure in optical microscope pictures appears immediately much coarser than the previous case, since big black carbides are evident, even if they are well distributed. In Figure 9.5c, the picture obtained through SEM microscopy shows the detail of a single carbide.

9.2.4 Dilatometric Tests

Testing Instrumentation

The dilatometric tests has been performed using a DSI Gleeble 3800. This instrument consist of a chamber, a heating system based on resistive heating, a control unit and a measurement system. Dilatometer specimens, with dimensions of 25-mm length and

6-mm diameter, were mounted between one fixed and one movable jaw in the center of the chamber. Dimensional change due to heating was measured along a diameter at the center of the specimen length by an electromechanical transducer (LVDT). At the same time, temperature measurement has been made using 0.25-mm diameter Chromel-Alumel thermocouples, which was spot-welded on the outside surface of the specimen.

9.2.5 Determination of Critical Temperatures

During the test, the diameter of the specimen increases with temperature up to the achievement of A_{C1} . A_{C1} is the temperature at which austenite begins to form during heating process. From this point on, the elongation decreases with increasing temperature up to A_{C3} . This is the ending temperature of the austenitic transformation: subsequently the diameter of the specimen will proportionally increase with the temperature. Both the critical points can be determined by changes in the slope of the curve temperature-displacement, obtained at the end of the test dilatometer, as shown in Figure 9.6.

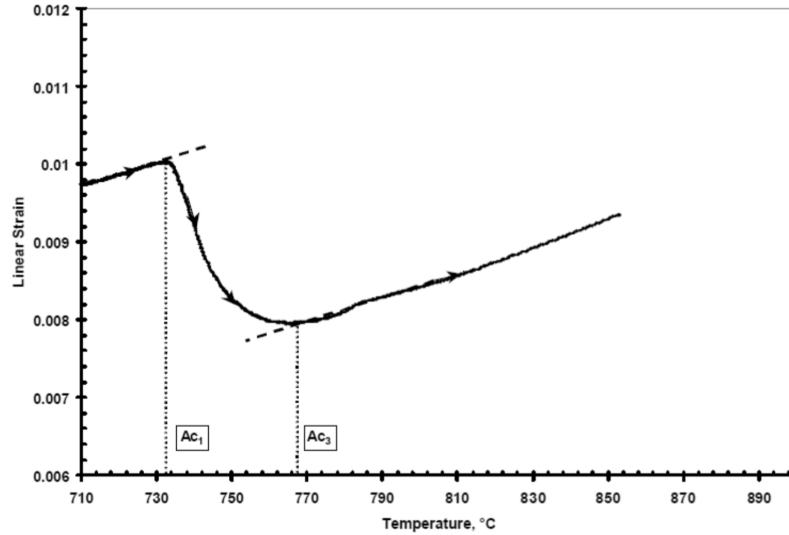


Figure 9.6: Typical shape of dilatometric curve

9.3 Results

9.3.1 580 3h

The transformation pearlite/ferrite - austenite is diffusive and involves the migration of carbon atoms to considerable distances. Its kinetics can be divided into two stages: dissolution of pearlite and transformation of the ferrite into austenite [63]. In the first stage the nuclei of austenite are formed on the boundary between the ferrite crystals cementite, that is where there is the maximum probability of formation, both for spontaneous atomic fluctuations, either because the austenite find part of the activation energy necessary for the

its nucleation (heterogeneous). This implies a high rate of formation of austenitic nuclei due to the small distances required for the diffusion of carbon. Once the first stage is finished, the austenite formed from the pearlite grows at the expense of the ferrite, until all the ferrite has been transformed into austenite; this process is the second stage of austenitization. Normally from the dilatometric curves it is not possible to determine differences between dissolution of pearlite and transformation $\alpha \rightarrow \gamma$. The possibility to split the two stages depends on the type of dilatometer used (a high resolution dilatometer allows to determine the temperature at which the transformation $\alpha \rightarrow \gamma$ begins), the material and the heating rate (only for low heating rate it is possible to assess the transformation $\alpha \rightarrow \gamma$) [108].

For what concerns the influence of the steel composition on the transformation kinetic, in the presence of austenite-stabilizing elements (Ni, N) will speed up the process of austenitizing both to decrease the critical points, both for the pearlite-generation effect, which results in an increment of the size of ferrite/carbides interface. The addition of carbide-former elements (Cr, Mo), if on the one hand accelerates the austenitization, on the other hand reduces to the thermodynamic stability of the relative carbides. Another important factor of influence is the extension of the ferrite/pearlite (or carbides) interface: for a certain steel of a given composition, the finer the initial structure (higher level of dispersion of the carbides and high ferrite interfaces/carbides), the higher the nucleation and the shorter are the diffusive paths and, consequently, the transformation times.

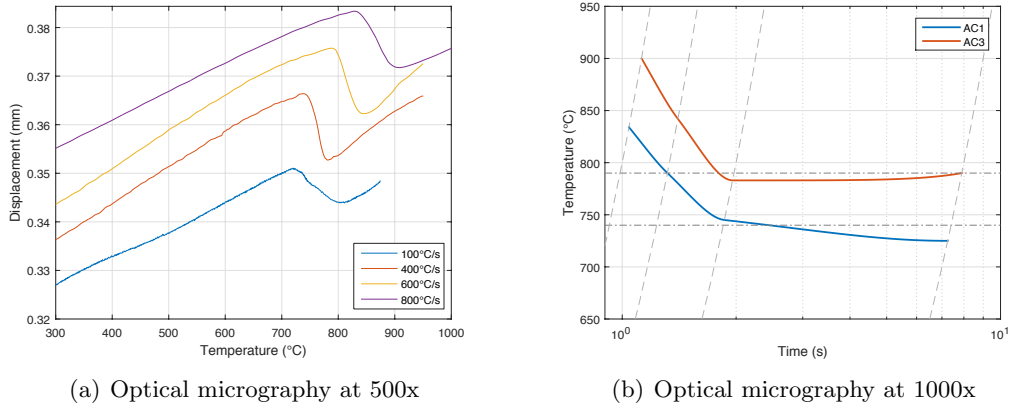


Figure 9.7: Micrographies of material after tempering at 580°C for 3 hours

Table 9.4: Critical Temperatures for 39NiCrMo tempered at 580°C for 3 hours

Heating Rate (°C/s)	A_{C1} (°C)	σ	C_v	A_{C3} (°C)	σ	C_v
100	725	8.7	1.17%	790	16.7	2.07%
400	745	9.6	1.30%	783	18.7	2.39%
600	790	5.2	1.00%	842	5.1	0.61%
800	834	2.5	0.30%	900	10.1	1.15%

Therefore, with microstructure that consists of fine and dispersed carbides will with

more rapidly, also requiring a lower austenitizing temperature. If the starting and ending times of the process are detected during the continuous heating, and this is repeated for different heating rate, the diagram that is obtained is of the type shown in Figure 9.7b. This is named CHT (Continuous Heating Transformation). From the graphs in Figures 9.7a and 9.7b it can be seen as the higher the heating rate, the higher the critical temperatures for austenitic transformation. Indeed, in addition to phenomena of thermal hysteresis, the increase of the heating rate reduces the duration of the diffusion processes that take place during the step of austenitizing. This will require, therefore, a higher energy and will be completed at higher temperatures than the equilibrium condition [106]. In particular, it results that the A_{C3} temperature of the austenitic transformation (which is determined only by the diffusion) will be more affected than the transformation starting temperature. In fact A_{C1} is mainly determined by free energy available for nucleation, a quantity relatively influenced by the heating rate [106].

Table 9.5: Hardness (HV0.3) of 39NiCrMo3 after quenching and tempering at 580°C for 3 hours

Sample	Hardness (HV)	Standard Deviation
Tempered 580°C 3h	350	15.85

9.3.2 580 6h

Also in this case, the transformation is a diffusive one, since the previous microstructure is always a mixture of ferrite and cementite. For this reason, the considerations made previously, regarding the austenitization kinetic are still valid. However, unlike the tempering at 580 °C for 3 hours, this one leads to obtain a coarser microstructure with bigger and more stable carbides. A reduced degree of dispersion of the carbides in the pearlite, due to the smaller extension of the ferrite/carbides interface, decreases the nucleation speed of austenite and increases the length of the diffusional path of carbon and alloying other elements. Accordingly, temperature and processing time required will be greater than those necessary for a structure with carbides finely dispersed. Due to the heating time it is possible to obtain an excessive enlargement of the austenitic grain, known as "overheating". In slow cooling, the ferrite or proeutectoid cementite separate in the form of meshes along the grain boundaries or of large needles oriented in different directions. This structure is said to Widmanstätten. For a quenching treatment, however, there is the formation of coarse martensite with deleterious effects on important properties such as toughness, resilience, fatigue resistance and cracks generation risk. The following figure shows the diagram CHT. The reduced degree of dispersion of the carbides in the matrix causes a further increase of the critical points with respect to the previous case (material tempered at 580 °C, 3h). This is always linked to the need to increase the heating time because of not "effective" response of the material, resulting in more and the reduced extension of the interfaces ferrite/carbides and the low rate of austenite nucleation.

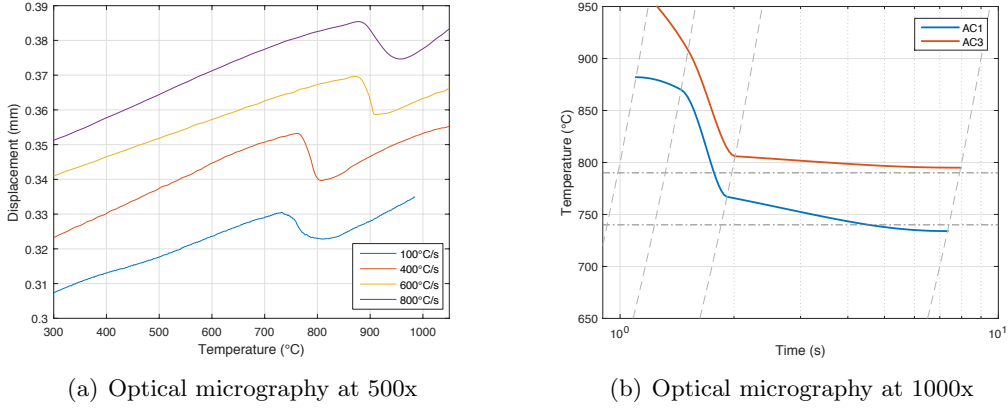


Figure 9.8: Micrographies of material after tempering at 580°C for 3 hours

Table 9.6: Critical Temperatures for 39NiCrMo tempered at 580°C for 6 hours

Heating Rate (°C/s)	A_{C1} (°C)	σ	C_v	A_{C3} (°C)	σ	C_v
100	734	21.3	2.85%	795	20.6	2.60%
400	767	13.0	1.69%	806	20.1	2.50%
600	870	1.7	0.20%	908	1.5	0.17%
800	882	1.0	0.13%	958	2.0	0.22%

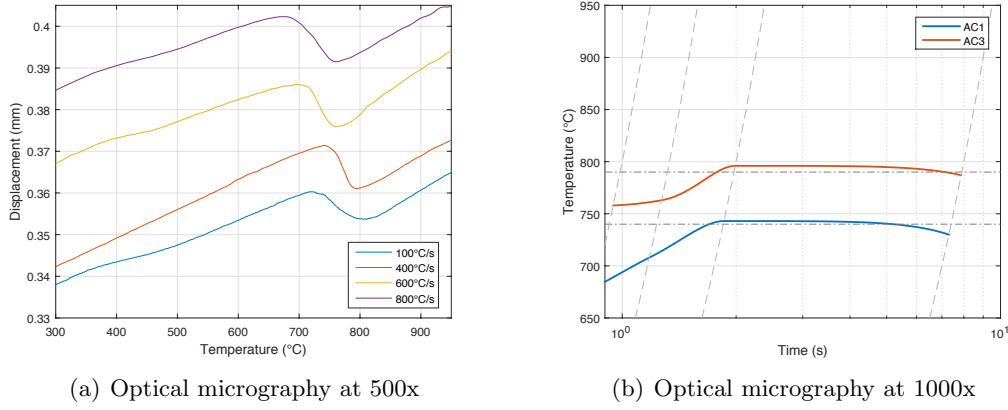
9.3.3 180 4h

From Figures 9.9 can be easily observed that the critical temperatures tend to initially increase for low heating rates, but then decrease for the higher heating rates. The initial increase of the critical temperatures suggests that the martensite-austenite transformation at low heating rates takes place by means of a diffusive mechanism. With high heating rates, however, due to inhibition of the diffusion phenomena, the transformation is dominated by a displacive mechanism (defined austenite reversion), whereby the critical points tend to decrease with the heating rate [107].

Some studies [6] show that the austenite reversion has the same crystallographic orientations of the residual austenite in the initial martensitic microstructure. This suggests that there are the same steps (Bain's distortion, "lattice invariant shear" and slip) both in the direct transformation (martensite formation) and in inverse one (re-austenitization). Moreover, it appears that the individual crystals of martensite can revert so as to obtain equivalent crystals of austenite [109]. In other cases, however, especially for alloys with high carbon content, each crystal of martensite can produce numerous regions of reversed austenite disoriented between them. The main characteristics of reversed austenite are its fine structure, the high dislocation density, the presence of small twins and stacking faults, although most of the latter is no longer present at the end of trasformazione. The high density of dislocations derives not only from the one already present in the initial martensitic microstructure, but also by the contribution of the

Table 9.7: Hardness (HV0.3) of 39NiCrMo3 after quenching and tempering at 580°C for 6 hours

Sample	Hardness (HV)	Standard Deviation
Tempered 580°C 3h	305	15.70

**Figure 9.9:** Microographies of material after tempering at 580°C for 3 hours

various deformations, associated with the austenite reversion mechanism which occur in a structure already initially imperfect [107]-[109]. It was still unclear if the stacking faults participate in the transformation mechanism, or are formed during the recovery process after the transformation and necessary to accommodate the stressed microstructure [109]. The little crystal twinings are considered the initial stage of the recovery processes of austenite reversion: their small size are connected to the high degree of deformation in the microstructure. The displavive transformation at high heating rate is also supported by two factors:

- Martensite and austenite have the same chemical composition, due to the lack of diffusion phenomena during the transformation that occur during the cooling process;
- The presence of a plan in the austenitic cell which is found unchanged (neither rotated nor distorted) also in the martensite. This plan is called the invariant plan (Habit plane).

The austenite obtained by not diffusive processes also presents a high hardness and resistance. This effect is associated with defects in the crystal structure: mainly the high dislocation density, but also the twinings [109]. Moreover, the strengthening depends in a complex way by numerous factors, such as heat treatment conditions or the structure/material type. In particular:

- The volume fraction of martensite in the initial microstructure, which determines the volume fraction of austenite produced;

Table 9.8: Critical Temperatures for 39NiCrMo tempered at 180°C for 4 hours

Heating Rate (°C/s)	A_{C1} (°C)	σ	C_v	A_{C3} (°C)	σ	C_v
100	730	16.8	2.30%	787	20.0	2.54%
400	743	9.5	1.28%	796	19.0	2.39%
600	708	9.7	1.38 %	763	18.2	2.38%
800	680	7.9	1.17%	758	12.2	1.60%

Table 9.9: Hardness (HV0.3) of 39NiCrMo3 after quenching and tempering at 180°C for 4 hours

Sample	Hardness (HV)	Standard Deviation
Quenched	705	34.93
Tempered 180°C 4 h	660	32.21

- The type of martensite initially present: this affects the density and distribution of defects at the end of the transformation;
- The structural conditions of the pre-processed retained austenite (for example, the presence of dislocations introduced in the austenite during the martensitic transformation);
- The level of volumetric shrinkage generated by the transition from FCC to BCC; this is an important factor in the generation of dislocations in the final austenitic structure.

At low heating rates, the transformation martensite-austenite takes place according to a diffusive mechanism. The main characteristics of this process are:

- Need of a certain over-heating to overcome the activation energy of transformation [107];
- Atoms diffusion through a short-range incoherent interface [107].

The nucleation of austenite takes place both within and at the edges of the martensitic crystals, with a higher speed than the previous microstructures (pearlite-ferrite) [110]. The austenite thus obtained has a morphology almost equiaxed, coarser than the reversed one, with curved edges, and less jagged [107]. Also, unlike the reversed austenite, the one obtained by diffusion has a low dislocation density and is free of twinings: its hardness is therefore less than the reversed one [107].

9.4 Conclusions

The work done was aimed at evaluate the influence of both the prior microstructure and the heating rate on the austenitizing kinetics. To do so, samples of different microstructures

of 39NiCrMo3 have been firstly characterized by optical microscope, then tested through dilatometric tests at different heating rate (100, 400, 600, 800 °C/s). The dilatometry, in fact, allows to verify in real time the evolution of the transformation in terms of dimensional variations of the heated sample. The diagrams temperature-displacement allow to determine the critical points.

The results obtained in this work can be summarized as follows:

- For the two prior microstructures, the austenitic transformation is diffusive and consists of two phenomena: dissolution of pearlite and ferrite transformation to austenite. Regarding the effect of the microstructure, the important factor is the extension of the interfaces ferrite/cementite: for a certain steel at a given temperature, the finer the initial structure, the higher the nucleation of austenite (short diffusion paths) and lower the processing time. So a microstructure with finely dispersed carbides is transformed much more quickly than one with larger carbides;
- The critical points of the ferrite/cementite microstructures increase with the heating rate, not only due to thermal hysteresis phenomena, but also for the greater energy required to activate the diffusion processes, which are hampered by the heating rate;
- For the martensitic microstructure critical points increase with the heating rate up to a maximum, and then decrease.
- The transformation martensite→austenite, at low heating rates, is of diffusive type and leads the formation of austenite almost equiaxed, with no jagged edges with low dislocation density;
- With high heating rate, however, the same transformation is done with a type displacive mechanism. The austenite thus obtained presents crystallographic orientations identical to that residual present in the initial martensitic microstructure. Furthermore single crystals of martensite will transform into equivalent crystals of austenite; In other cases, however, crystals of martensite produce numerous regions of austenite reversion disoriented between them;
- The increase in hardness of the reversed austenite is associated to the high dislocation density in the final structure and the presence of twinings and stacking faults.

Chapter 10

Final Remarks

In this thesis a robust method for the analysis of contour induction hardening processes of gears has been proposed.

The work has been divided in three main branches.

In the first one, the induction hardening process has been modeled through a multi-physical FEM simulation in which the metallurgical phase distribution can be evaluated for each moment of the simulation. The transformation kinetics have been divided in diffusive and non-diffusive, applying a different formulation for the two cases. In particular, in the case of diffusive transformations, the classical formulation based on JMAK method has been extended to the anisothermal case through the Scheil's additivity rule. Since one of the most complex task, in multiphysical simulation, can be considered the high number of material's properties, a smart method for the numerical evaluation of TTT diagrams have been implemented. The algorithm has been applied on a real test case of a 56-teeth spur gear for aeronautical industry. Through the simulation the appropriate system set-up has been developed (in terms of coil shape, type of support and use of magnetic flux concentrator), and it has been possible to choose the right process parameters in order to achieve the desired results. The simulations have been verified through experimental tests carried out at Ecole de Technologie Superieure of Montreal (Canada) in an industrial scale induction hardening machine.

The results obtained at the end of the tests have been compared with the numerical analysis, showing an impressive agreement.

The second part of the work was the numerical simulation of Simultaneous Double Frequencies (SDF) induction hardening processes. For what concern this project, different works have been proposed in the past by different authors, generally using an excessive level of simplification, for example considering the steel as a material with a linear magnetic behavior. On the other hand, an accurate solution of the problem may be obtained solving a electromagnetic transient simulation, but as explained in Chapter 8, it would require a too long time (several months), such as to not allow the application to real cases.

In the case studied in this thesis, the magneto-thermal problem has been decoupled solving separately the transient electromagnetic and transient thermal problem. It can be assumed, if the temperature does not grow excessively during this time period, that the electromagnetic parameters, such as magnetic relative permeability and electrical

conductivity do not significantly vary with temperature. So, it is plausible to assume that in a time interval sufficiently small, the power induced in the workpiece is constant, and it is precisely the average value of the power induced. These observations imply that it is possible to solve firstly the EM transient problem, using the material's properties calculated considering the current temperature profile. Then it is possible to evaluate the average heating power distribution calculated in the electromagnetic period, and assume this power as a time independent source in a time step of the thermal transient simulation. In this manner it is possible to simulate a SDF induction hardening process, considering correctly the magnetic saturation effect, drastically reducing the total computational time.

In the last part of the thesis the investigation of the effect of the previous microstructure and the heating rate on the austenitization kinetics of 39NiCrMo3 steel has been analyzed. The analysis has been carried out through dilatometric tests, in order to identify the critical temperatures (A_{C1} and A_{C3}). The samples have been realized with three different microstructures and tested with four different heating rates (100, 400, 600, 600 °C/s). The microstructure of the samples has been verified through optical microscopy (500X and 1000X) and also through Scanning Electronic Microscope (SEM) analysis. From the results, it is possible to observe that, as expected, the critical temperatures generally arise increasing the heating rate and the carbides dimension, but in the case of samples quenched and tempered at 180°C different mechanisms are activated. In this latter case, in fact, the critical temperatures increase passing from 100 to 200°C/s, but drastically decrease for higher heating rates. This effect, partially studied in the literature is also called "*Austenite Reversion*" and is well discussed in Chapter 9.

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