Validation of the Heuristic Model for the Mobility of Quasi-free Electrons in Dense Helium Gas

Armando Francesco Borghesani

***Abstract*— The electron mobility in dense, cold Helium gas has mainly been studied because its behavior is the macroscopic manifestation of the electron self-localization phenomenon leading to the formation of electron bubbles. The measured mobility consists of the weighted contributions of quasi-free electrons and electron bubbles. A good description of the experimental outcome requires a detailed knowledge of the density dependence of the quasi-free electron mobility. As the main interest was focused on the precipitous decrease of the mobility with increasing gas density because of the localization process dominated by the rapid growth of the electron bubbles number, the low-density range, where electron bubbles are few or even absent, was not investigated with very high accuracy. As we developed a heuristic model to successfully rationalize the electron mobility in dense Argon and Neon gases, we have carried out accurate measurements of the electron mobility in order to validate such model in the temperature range $26\,\mathrm{K} < \mathrm{T} < 300\,\mathrm{K}$ and in a density range, in which the electron bubble contribution is negligible. Here, we report the new measurements and their description by means of the heuristic model.**

***Index Terms*— dense helium gas, quasi-free electron mobility, heuristic model.**

I. Introduction

THE transport properties of electrons in dense gases , i.e., loosely speaking, gases at densities larger than that of the ideal gas at 0 °C and 1 atm, are a subject of active theoretical and experimental investigation since many years because of their relevance for basic physics and for practical applications [1]. From the point of view of basic science, the knowledge of how electrons drift in a dense gas under the action of an externally applied electric field E may give important pieces of information on the electron-atom scattering interactions and on the electron energetics and dynamics in disordered media.

The rationalization of the electron mobility μ in dense gases lacks the simplicity of either the classical kinetic theory description based on the assumption of binary collisions valid in the limit of a very dilute gas or that based on the conduction band concept applicable to solids due to the symmetry of the crystal lattice. However, it is obvious that the classical kinetic theory has always been the natural starting point to develop a consistent description of the electron mobility in dense gases. Actually, the binary collision approach is not rigorously applicable in dense gas systems because, especially at low temperature and high density, the spatial extension of the electron wave packet causes the simultaneous interaction of the electron with a large number of surrounding atoms that leads to the appearance of multiple-scattering effects [2], [3]. For this reason, the local structure of the electron environment must be taken into account, thereby leading to a dependence of the mobility on the thermodynamic state of the gas.

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As a result, the zero-field, density-normalized electron mobility $\mu_0 N$ in dense noble gases, which are the simplest realization of a disordered system, shows an unexpected density dependence. $\mu_0 N$ shows a negative density effect, i.e., it decreases with increasing gas density N, in gases whose electron-atom scattering length is positive, such as Helium [4]–[6] and Neon [7], [8]. On the contrary, $\mu_0 N$ shows a positive density effect increasing with increasing N in gases with a negative scattering length, such as Argon [9]–[11].

In particular, the negative density effect, discovered at first in Helium [4] and then in Neon [7], has been intensively investigated because of the relationship between multiple-scattering effects and the formation of electron bubbles via the self-localization mechanism induced by the intrinsic gas disorder [12]–[14].

A number of theories based on multiple scattering were developed in the past to rationalize the density dependence of the mobility of thermal electrons [14]–[16], i.e., in the limit of vanishingly small reduced electric field E/N. All of them are based on a complex shift of the electron kinetic energy in a dense medium [2], [3] and on quantum corrections on the electron-atom scattering rate that depend on the ratio of the electron mean free path ℓ to its thermal wavelength λ_T [17], [18]. These theories assumed different physical mechanism to be responsible for the mobility behavior, depending on the sign of the scattering length.

Based on the assumption that the momentum-transfer scattering cross section σ_{mt} is independent of the electron energy, these theories were quite successful at describing the behavior of the zero-field, density-normalized mobility μ_0 of electrons in Helium gas but further measurements in Neon gas [7] proved them wrong. So, to achieve a unique description of the electron mobility in a dense gas independently of the scattering length sign, we developed an heuristic model [10] that retains the simple picture of binary collisions of the classical kinetic theory [1] and introduces an effective momentum-transfer scattering cross section $\sigma^{\star}_{\mathrm{mt}}$ that incorporates in a heuristic way the multiple-scattering effects suggested by theories [2], [3]: i) the quantum, density-dependent shift $E_k(N)$ of the electron

kinetic energy arising from the exclusion of the electron wave function from the hard-core volume of the atoms [19]; ii) the correlations among the scatterers [20]; iii) the back-scattering rate enhancement along paths connected by time-reversal symmetry [21].

The heuristic model, that does not rely on any adjustable parameters, has successfully been used for describing the electron mobility in dense Neon gas [7], [8] and in dense Argon gas [10], [11], [22]. It automatically accounts for both the negative- and the positive-density effect depending on the size and energy-dependence of the electron-atom momentum-transfer scattering cross section $\sigma_{\rm mt}$ and on the thermodynamic equation of state of the gas under investigation.

A further test of its validity is to apply it to mobility measurements carried out in dense Helium gas. To achieve this goal, two requirements have to be met. First of all, as in Helium electrons self-localize in bubbles, the mobility is a weighted sum of the contributions of the fast quasi-free electrons and of the very slow electronic bubble, whose fraction increases with increasing density. Thus, it is mandatory to precisely identify the density range, in which the contribution of localized states is negligible to safely apply the heuristic model for the mobility of the quasi-free electrons. Secondly, very accurate experimental data are required.

For this reason, we have carried out accurate drift mobility measurements in the temperature range $26\,\mathrm{K} < T < 300\,\mathrm{K}$ and in broad density and electric field ranges. We report here the results obtained in the density range, in which only quasi-free electrons contribute to the overall mobility, and compare them to the prediction of the heuristic model to confirm its validity.

The paper is organized as follows. In Section II a brief description of the experimental setup is given. In Section III the heuristic model is presented and is compared with the experimental outcome. Finally, in Section IV conclusions are drawn.

II. Experimental Details

The experiment has thoroughly been described elsewhere [7]. We very briefly recall here its main features. We used the pulsed Townsend photo-injection technique. The experimental cell can be pressurized up to $\approx 10\,\mathrm{MPa}$ and its temperature regulated within $\pm 0.01\,\mathrm{K}$ in the range $25\,\mathrm{K} \leq T \leq 340\,\mathrm{K}$. A home-made purification system reduces the initial Oxygen impurity content of 10 ppm to a fraction of a ppb [23]. Pressure readings are accurate within $\pm 2\,\mathrm{kPa}$. The gas density N is computed via the equation of state [24]. A d.c. H.V. generator energizes the drift capacitor up to a maximum voltage of $\approx 3\,\mathrm{kV}$. The drift distance is $d \approx 1\,\mathrm{cm}$. Bunches of electrons, containing a charge amount $4\,\mathrm{fC} < q < 400\,\mathrm{fC}$ depending on electric field and density, are injected by means of a $4\,\mu\mathrm{s}$ short VUV Xe flashlamp pulse. The electron signal is passively integrated to improve the signal-to-noise ratio and the drift time is obtained by the analysis of the signal waveform [25], [26]. The explored range of reduced electric field is limited to $E/N \leq 150\,\mathrm{mTd}$ ($1\,\mathrm{mTd}=10^{-24}\,\mathrm{V\,m^2}$) that is broad enough to significantly heat the quasi-free electrons. However, electrons always remain well below the ionization energy. As a consequence, only either thermal- or slightly epi-thermal electrons are present in this experiment, thereby making easy the analysis of the experimental outcome.

III. Experimental Results and Discussion

A. Need for accurate measurements

Historically, the mobility measurements in Helium focused on the electron localization in bubbles, i.e., the high N range has always been investigated with better accuracy than the low N one. As an example, we show in Fig. 1 some literature data at $T = 4.2\,\mathrm{K}$ [4] and some of the present results at higher temperatures [27]. It is easy to realize that the low

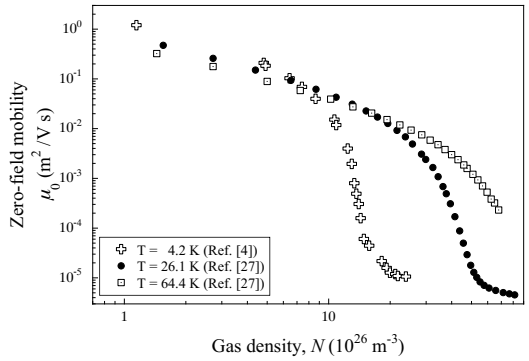

Fig. 1. Zero-field mobility μ_0 vs gas density N at several temperatures.

N side is lacking a sufficient number of data points for a comparison with any theory. Moreover, literature data in the low N region, where almost only quasi-free electrons contribute to the observed μ, are of insufficient accuracy. That is the reason to carry out new accurate measurements. To clarify this point, we compare in Fig. 2 some new data

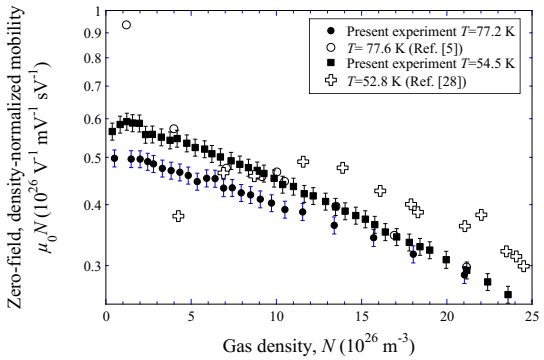

Fig. 2. Density dependence of the zero-field, density-normalized mobility $\mu_0 N$. Comparison of the present experiment accuracy with literature data.

obtained in the present experiment to literature data at nearly the same temperatures. We can note that the literature data are strongly scattered, whereas the present data show a smooth and continuous behavior. We, thus, believe that the improvement of the actual experimental accuracy over that of literature data is very evident. A similar accuracy is obtained for all presently investigated isotherms.

B. Choice of the localized-states- free density range

The observed mobility is a weighted sum of the contributions μ_F and μ_B of quasi-free electrons and electron bubbles, respectively, $\mu_F n_F + \mu_B n_B$, in which n_F and n_B are their relative fractions. The proportion of localized states increases with increasing N, as can be realized by inspecting, for instance, Fig. 1. To test the validity of the heuristic model for the mobility of quasi-free electrons, only those data have to be analyzed that lie in a density range, in which localized states are either absent or present in a negligible proportion. This range is, obviously, dependent on temperature and has to be identified experimentally.

In Fig. 3 we show how the dependence of μN on the reduced electric field E/N changes as N is increased at constant $T = 54.5\,\mathrm{K}$. Similar behavior is observed on all isotherms, in which a sufficiently high density is reached. At

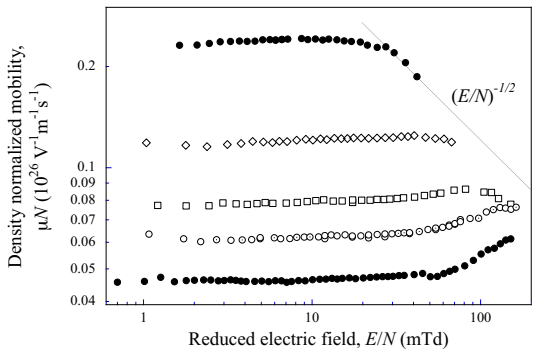

Fig. 3. μN vs E/N for several N. Note the behavior change due to the growth of the localized states fraction. The data are taken at $T = 54.5\,\mathrm{K}$ and at $N = 25.2, 37.9, 44.0, 47.2,$ and 50.8 (in units of 10^{26}) (from top). Solid line: kinetic theory prediction for electron scattering off hard-spheres at high E/N. The error bars are not shown for the sake of clarity.

low N, μN is constant at weak E/N. Then, as electrons are heated by the field, there is a crossover of the mobility field dependence to the $(E/N)^{-1/2}$ behavior, predicted by the classical kinetic theory for electron scattering off hard-spheres, as Helium atoms can be assumed to be because their scattering cross section σ_{mt} is almost independent of the electron energy, as shown in Fig. 4 [1].

By referring to Fig. 3, we note that, by increasing the density, μN starts increasing over its low-field value and reaches a maximum before merging into the $(E/N)^{-1/2}$ curve typical of epithermal electrons.

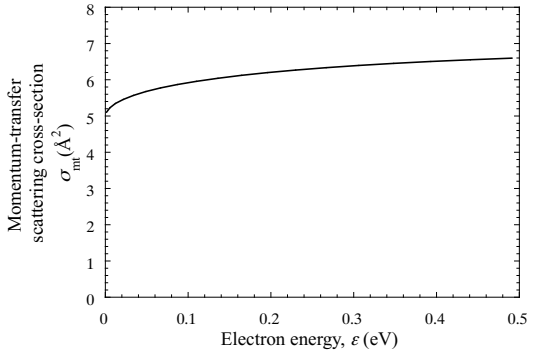

Fig. 4. Energy dependence of the electron-Helium momentum-transfer scattering cross section σ_{mt} [28].

This behavior is so rationalized. At low density, no localized states are present and the observed mobility has to be attributed to quasi-free electrons, only. At higher N, where the fraction of localized states is no longer negligible, the amount of quasi-free electrons is increased by increasing the electric field E either because epithermal electrons are more difficult to get self-localized or because some electron bubbles are broken by the field and electrons are made free to drift. As a result, the mobility increases with increasing field above its thermal value, shows a maximum, and, eventually, at high fields, it merges into the curve relative to epithermal electrons.

To identify the threshold density that separates the region, in which localized states are either absent or present in negligible proportion from the region, in which the fraction of localized states significantly contributes to the observed mobility, for each density on a single isotherm we first identify the threshold value $E^\star$ of the electric field, at which the mobility starts increasing above its thermal value. In Fig. 5 a typical example of this procedure is shown. The values of $E^\star$ determined in this way for each isotherm are plotted as a function of the density. A typical example is shown in Fig. 6. The extrapolation of $E^\star$ to zero yields the threshold density N_{th}, below which localized states do not contribute, within the experimental accuracy, to the observed mobility.

By repeating this procedure for all available isotherms, we can identify the temperature dependence of N_{th}, plotted in Fig. 7, that, for each investigated temperature, sets the upper limit of the density range, in which the localized states contribution to the measured mobility can safely be neglected and the heuristic model can be compared to the experimental outcome.

C. The physics of the heuristic model

The model originally developed for Neon [7] and Argon [10] retains the binary collision picture of the classical kinetic theory [1] but incorporates in a heuristic way three multiple scattering effects.

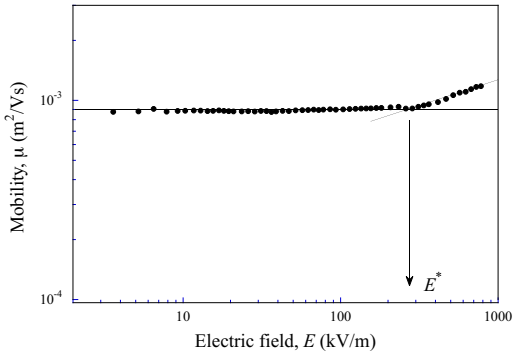

Fig. 5. Experimental determination of the threshold electric field $\boldsymbol{E^\star}$, above which the fraction of localized stated starts decreasing thereby leading to an increase of the mobility above its zero-field value. This data are recorded for $\boldsymbol{N = 50.8 \times 10^{26}\ \mathrm{m}^{-3}}$ at $\boldsymbol{T = 54.5}$ K. The lines are a guide for the eye only. The error bars are not shown for the sake of clarity.

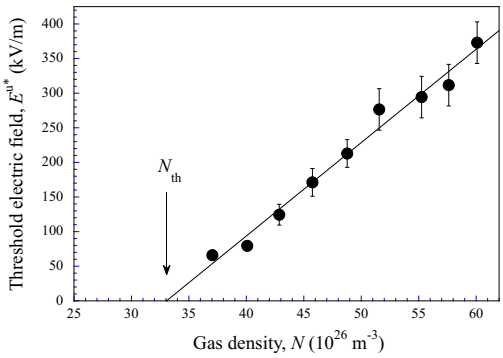

Fig. 6. Threshold electric field $\boldsymbol{E^\star}$ for the onset of the experimental detectability of localized states as a function of $\boldsymbol{N}$ at $\boldsymbol{T = 45.3}$ K. $\boldsymbol{N_{th}}$ is the threshold density. The line is a guide for the eye, only.

1) Density-dependent shift of the electron kinetic energy: The first one is the density-dependent energy shift $E_k(N)$ of the electron kinetic energy ϵ. It is a zero-point kinetic energy contribution that arises from excluding the electrons from the hard-core volume of the atoms [19]. It can explicitly be computed by locally replacing the gas structure with an ordered array of hard-spheres and by imposing that the electron wave function is translationally invariant over a distance $2r_{\mathrm{ws}}$, where the Wigner-Seitz radius is defined by the condition $(4\pi/3)r_{\mathrm{ws}}^3 N = 1$. This symmetry requirement on the wave function leads to the eigenvalue equation for the kinetic energy operator

$$\tan\left[k_0 r_{\mathrm{ws}} + \eta_0\left(k_0\right)\right] = k_0 r_{\mathrm{ws}} \tag{1}$$

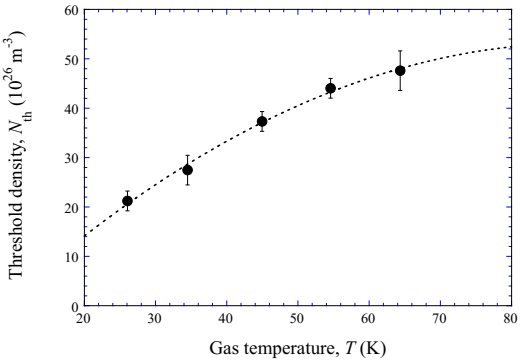

Fig. 7. Temperature dependence of the threshold density $\boldsymbol{N_{th}}$, below which the fraction of localized states is negligible.The dashed line is a guide for the eye only.

in which η_0 is the s-wave phaseshift and k_0 is the wave number eigenvalue. The kinetic energy shift is obtained as

$$E_k(N) = \frac{\hbar^2}{2m} k_0^2 \tag{2}$$

in which $\hbar$ is the reduced Planck constant and m is the electron mass. In order to account for the superposition of the tails of the atomic potentials, the scattering length $a = -\eta_0(k_0)/k_0$ is replaced by the hard-core radius of the Hartree-Fock potential $\tilde{a} = \sqrt{\sigma_t(k_0)/4\pi}$, in which σ_t is the total scattering cross section [19]. E_k produces a big effect on the density-dependence of the mobility if σ_{mt} is large but especially if it has a strong dependence on the electron kinetic energy ϵ.

2) Correlations among scatterers: The second multiple scattering effect introduced in the model is the correlation among scatterers. The electron wave packet, especially at low temperature, extends over a large spatial region of order $\lambda = h/\sqrt{2m\epsilon}$ that, at high density, contains many atoms. Thus, electrons are simultaneously scattered off them simultaneously. The total amplitude of the scattered electron wave packet is a coherent sum of all partial scattering amplitudes contributed by all involved atoms. As a result the scattering cross section is mqgnified by the gas static structure factor $S(0)(N) = N k_{\mathrm{B}} T \chi_T$, where k_{B} is the Boltzmann constant and χ_T is the isothermal gas compressibility [20].

3) Quantum self-interference of the electron wave packet: The third, and last, multiple-scattering effect is the electron back-scattering rate enhancement due to the quantum self-interference of the electron wave packet scattered off atoms located along paths connected by time-reversal symmetry [21]. The relevance of this effect on the cross section is dictated by the ratio λ/ℓ of the electron wavelength λ to its mean fre path $\ell = 1/N\sigma_{\mathrm{mt}}$. In Helium, at the temperatures of the present experiment, the electron thermal wavelength $\lambda_T = h/\sqrt{2\pi m k_{\mathrm{B}} T}$ is fairly long and the mean free path quite short because the cross section is large. Thus, the linearized treatment of this effect, valid for the case of Neon [7] and

Argon [10] because their cross sections are small at thermal and shifted energies, is no longer valid. When $\lambda/\ell \gtrsim 1$, the regime of weak localization sets in [13], [29], electrons get localized with wave function that exponentially decays in space and a mobility edge appears at finite energy ϵ_c [30]. At ϵ_c the scattering rate diverges. If $\nu_0(\epsilon) = \sqrt{2\epsilon/m}N\sigma_{\rm mt}$ is the scattering rate in the dilute-gas limit, it can be shown that the actual scattering rate $\nu(\epsilon)$ can obtained as [18], [31]

$$\nu(\epsilon) = \frac{\nu_0(\epsilon)}{1 - f\hbar\nu_0(\epsilon)/2\epsilon} = \frac{\nu_0(\epsilon)}{1 - f\lambda/\ell} \tag{3}$$

and the mobility edge energy is obtained by recursively solving the following equation

$$\epsilon_c = \frac{2}{m}\left[\frac{f}{2}\hbar N S(0)\sigma_{\rm mt}\left(\epsilon_c\right)\right]^2 \tag{4}$$

in which $f \approx 2$ [18], [32].

D. The equations of the heuristic model

It is an easy task now to implement the equations of the heuristic model without any adjustable parameters. The density-normalized mobility is formally obtained from the equations of the classical kinetic theory [1] as

$$\mu N = -\frac{e}{3}\left(\frac{2}{m}\right)^{1/2}\int_{\epsilon_c}^{\infty}\left[\frac{\epsilon}{\sigma^{\star}_{\rm mt}(\epsilon)}\right]\frac{{\rm d}g(\epsilon)}{{\rm d}\epsilon}\,{\rm d}\epsilon \tag{5}$$

in which e is the electron charge. The Davydov-Pidduck energy distribution function is given by [33]

$$g(\epsilon) = A\exp\left\{-\int_0^{\epsilon}\left[k_{\rm B}T + \frac{Me^2}{6mz}\left(\frac{E}{N\sigma^{\star}_{\rm mt}(z)}\right)^2\right]^{-1}{\rm d}z\right\} \tag{6}$$

M is the atom mass and A is the normalization constant obtained by enforcing the condition $\int_0^{\infty} z^{1/2}g(z)\,{\rm d}z = 1$.

The effective scattering cross section is obtained by incorporating the three multiple-scattering effects to obtain an effective momentum-transfer scattering cross section

$$\sigma^{\star}_{\rm mt}(\epsilon) = \mathcal{F}(w)\sigma_{\rm mt}(w)\left[1 - f\hbar\frac{N\mathcal{F}(w)\sigma_{\rm mt}(w)}{(2mw)^{1/2}}\right]^{-1} \tag{7}$$

in which $w = \epsilon + E_k(N)$. The correlating factor $\mathcal{F}$ is given by

$$\mathcal{F}(Q) = \frac{1}{(2Q)^2}\int_0^{Q} q^3 S(q)\,{\rm d}q \tag{8}$$

with $Q^2 = 2m(w - E_k)/\hbar^2$. The integral in Eq. 8 can analytically be evaluated by using the Ornstein-Zernike approximation for the structure factor $S(q) = [S(0) + (qL)^2]/[1 + (qL)^2]$ with $L^2 = 0.1l^2[S(0) - 1]$ and $l = 0.1$ nm [34]. We note that the heuristic model does not rely on any adjustable parameters. The only gas-specific features are the scattering cross section and the equation of state that are found in literature.

E. Agreement between experiment and model

By setting $E/N = 0$ in Eq. 6 the zero-field density-normalized mobility $\mu_0 N$ can be computed and compared with some experimental results in Fig. 8. The agreement between data and model is excellent. Note that the analyzed data lie within the density range, whose upper limit is set by $N_{\rm th}$, in which the contribution of localized states to the mobility is nil or negligible. Similar results are obtained for all remaining isotherms [31].

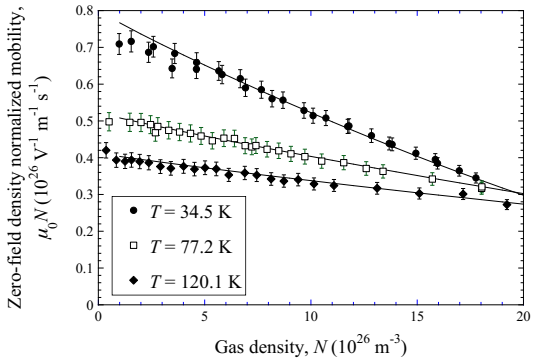

Fig. 8. Density dependence of the zero-field density-normalized mobility $\boldsymbol{\mu_0 N}$ for several temperatures. Lines: prediction of the heuristic model.

It is interesting to ascertain what is the relative influence of the energy shift and of the mobility edge energy on the observed mobility. By inspecting Fig. 9, we observe that both E_k and ϵ_c are increasing with increasing N though E_k is more than 20 times larger than ϵ_c. On one hand, as $\sigma_{\rm mt}$ does not

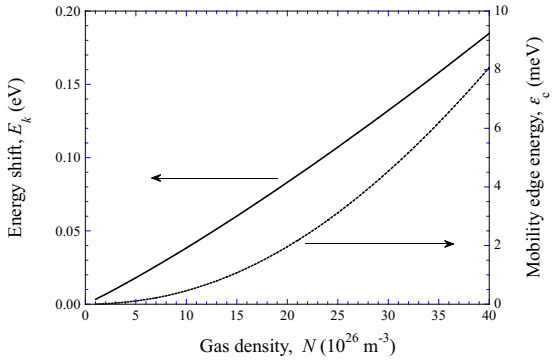

Fig. 9. Density dependence of the kinetic energy shift $\boldsymbol{E_k}$ (solid line, left scale) and of the mobility edge energy $\boldsymbol{\epsilon_c}$ (dashed line, right scale).

significantly depend on the electron energy, the energy shift induced by E_k has practically no effect on the effective cross section in contrast with the cases of Neon [7] and Argon [10] cases, whose cross sections strongly depend on the energy.

On the other hand, $\epsilon_c(N)$ is an infrared cutoff in the integral of Eq. 5 that eliminates a big fraction of the low energy states in the distribution function. This is a much more efficient mechanism to induce the strong decrease of the mobility with increasing density than the energy shift. The action of ϵ_c is also observed in Neon, though it is less important than in Helium because the cross section is smaller but still increasing with increasing energy. On the contrary, in Argon the cross section so rapidly decreases with increasing energy that makes ϵ_c negligible.

In Fig. 10 we show how the heuristic model prediction is able to even reproduce the electric field dependence of the mobility. The model well reproduces the experimental data at both weak and high electric fields. There is a slight discrepancy at intermediate fields that could be ascribed at some lack of accuracy of the scattering cross section determined from swarm experiments [28]. Actually, the experimental accuracy on the cross section is estimated to be around 10 to 15 % [35]. We note that, around a reduced electric field value $E/N \approx 10$ mTd, where the electron energy starts to rapidly increase with the field above its zero-field value, a 5 % uncertainty in the cross section produces an uncertainty on the computed density-normalized mobility as large as $|\Delta\mu N/\mu N| \approx 15\,\%$.

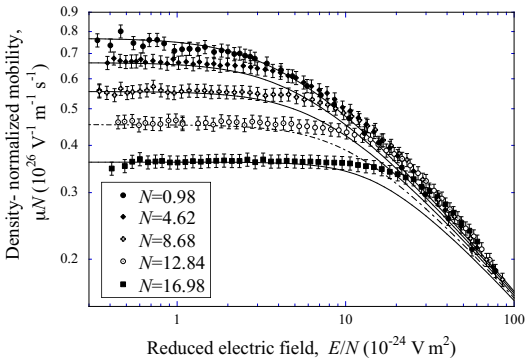

Fig. 10. Electron density normalized mobility $\boldsymbol{\mu N}$ vs reduced electric field $\boldsymbol{E/N}$ at $\boldsymbol{T = 34.5}$ K for some densities $\boldsymbol{N}$ in units of $\boldsymbol{10^{26}}$ m^{-3}Lines: heuristic model predictions.

In any case, on one hand the nice description of the electric field dependence of the mobility is not quite expected as E_k is computed from the low-energy, s-wave eigenvalue of the kinetic energy operator, Eq. 1, whereas the average electron energy significantly increases with increasing electric field. However, as the scattering cross section very weakly depends on the electron energy, we believe that this way of computing E_k should not significantly affect the computed mobility.

On the other hand, the previous multiple-scattering theories only dealt with the zero-field mobility and made no predictions at all on its electric field dependence. From this vantage point, it is clear that the heuristic model gives the researchers one further tool to help the investigation.

As a final consistency check of both measurements and model, we show in Fig. 11 the temperature dependence of the zero-density, zero-field density-normalized mobility $(\mu_0 N)_0$. In the limit of $N \to 0$, both $\epsilon_c \to 0$ and $E_k \to 0$. In this case, the equations of the heuristic model simply reduce to those of the classical kinetic theory. The excellent agreement of the experimental $(\mu_0 N)_0$ data with the theoretical prediction is a sign of the good accuracy of the experiment.

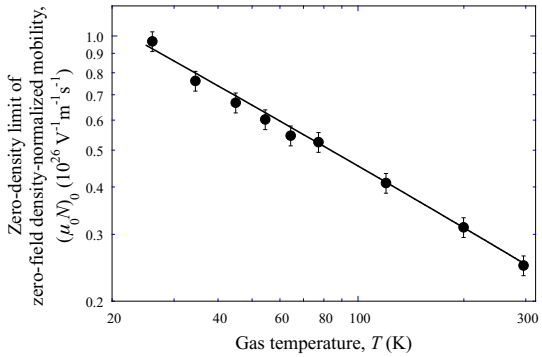

Fig. 11. Temperature dependence of the zero-field, zero-density, density normalized mobility $(\boldsymbol{\mu_0 N})_0$. Solid line: prediction of the heuristic model [31].

IV. Conclusion

In the past we have developed a simple heuristic model to describe the electron mobility in dense noble gases. It retains the classical kinetic picture of binary collisions but heuristically introduces three major scattering effects that suitably modify the electron-atom scattering cross section. The first one is the density-dependent shift of the electron energy and it is very effective when the cross section strongly depends on the electron energy. The second one, correlation among scatterers, is important when the gas is relatively near the critical point where its isothermal compressibility is very large. Finally, the third one, the quantum self-interference of the electron wave packet, strongly influences the mobility when the electron mean free path is comparable or shorter to the electron wavelength.

The model had previously proven successful is Neon and Argon. In Neon the cross section is relatively small but strongly increases with increasing electron energy. Additionally, we carried out measurements at high densities at quite low temperature and also fairly near to the critical point. By so doing, we investigated a regime, in which the electron wavelength is not much smaller than the electron mean free path and where correlations among scatterers are strong. Thus, Neon could prove the efficacy of the inclusion of all of the multiple-scattering effects in the model.

In Argon, whose cross section very rapidly decreases with increasing electron energy, the measurements were carried at higher temperatures than in Neon but also close to the critical point. In this case, the mean free path was always longer than the electron wavelength so as to make the mobility edge energy

negligible. Thus, the success of the model in Argon gas was the result of only the first two multiple-scattering effects.

Thus, to investigate the effect of the mobility edge energy, we decided to carry out accurate mobility measurements in Helium gas in a density range, in which the presence of slow localized electron bubble states is negligible, and at temperatures much higher than the critical temperature. This choice has got rid of the correlation among scatterers, because the static structure factor is always $S(0) \approx 1$. Moreover, the electron-atom scattering cross section is large but nearly energy independent. This property rules out the effect of the first multiple-scattering effect, i.e., the energy shift. At the same time, the high temperatures and densities of the experiment, together with the large value of the electron-Helium cross section, make the quantum self-interference the only relevant multiple-scattering effect. Actually, the efficacy of the model matches the accuracy of the experimental data.

The relevance of the new findings reported in this article manifold. On one hand, they confirm the validity of the heuristic model for the mobility in Helium gas in a wider temperature range than previously investigated [31]. Now, we can affirm that the heuristic model is valid in Helium whenever localized states are present in negligible proportion with respect to quasi-free electrons. On the other hand, the measurements in Helium complete the paradigmatic possible behavior of the energy dependence of the scattering cross section found in noble gases and of its influence on the multiple-scattering effects. The cross section in Helium is large and weakly energy dependent, in Neon it is small but increases rapidly with increasing electron energy, and, finally, in Argon the cross section is very large but strongly decreases by increasing electron energy and also shows the very well known Ramsauer minimum. In this way, all possible energy dependence of the cross sections are investigated. Finally, we observe that the model puts together in a heuristic way, but still within the framework of the classical kinetic theory, profound and interesting theoretical ideas developed by several authors [14], [15], [17], [32], [33], thereby giving a unified picture of the electron-atom scattering in dense, disordered media.

We can conclude that accurate mobility measurements in three noble gases have validated the heuristic model. The efficacy and simplicity of the model is due to the fact that it retains the picture of binary collisions valid in the dilute-gas limit even at high densities by suitably modifying the scattering cross section according to three main multiple-scattering effects.

Further work could be to extend the measurements and the analysis to heavier noble gases and to see if the model can be applied also to other gases, whose scattering cross section is reasonably spherically symmetric.

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Biography Section

Armando Francesco Borghesani Born in Verona (Italy), 1956. MS in Physics in 1979 with a thesis on “Anomalous behavior of the CO2 viscosity at the critical point”. From 5/1980 to 11/1981 he joined ST Microelectronics in Milan (Italy) as junior researcher in the plasma etching research group. From 11/1981 to 7/1983 he joined ENI-Ricerche in Milan as a researcher in the department for coal slurries pretreatment and conditioning. From 9/1983 to 10/1992 he was appointed as Assistant Professor in Physics of Matter at the Physics Department of the University of Padua (Italy). Since 11/1992 he is Associate Professor of Experimental Physics in the Physics & Astronomy Department of the same University. He taught courses in Mechanics, Electromagnetism and Optics, Quantum and Solid State Physics, Electronics. He investigated critical point phenomena, rheology of coal-based slurries, plasma etching of semiconductors, energetics and dynamics of electrons and ions in dense noble gases and liquids, infrared luminescence of noble gas excimers in high density gaseous environment, cathodoluminescence of rare-earth doped nonlinear crystals for developing dark-matter particle detectors, and thermal radiation of hot metal bodies. He established collaborations with groups at the Max-Planck Institute (Munich, Germany), at the Vrije Universiteit (Amsterdam, The Netherlands), and at CNRS (Grenoble, France). He is author of more than 100 peer-reviewed papers and of one monography for Oxford University Press entitled “Ions and Electrons in Liquid Helium” in addition to textbooks in Quantum Physics (in Italian). He has contributed, several times as invited speaker, to more than 40 international conferences and schools. He is actively contributing to all International Conferences on Dielectric Liquids (ICDL) since 1990 and was awarded the Hans Tropper Award at ICDL 2022.