



# Corrigendum to “Periodic rhomboidal cells for symmetry-preserving homogenization and isotropic metamaterials” [Mech. Res. Commun. 126 (2022) 104001]

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## ABSTRACT

We correct a mistake in the coefficients of a transformation matrix and accordingly update the subsequent calculations and the conclusions of our paper (Giusteri and Penta, 2022). We conclude that arrangements of spherical inclusions of isotropic materials in an isotropic matrix based on a rhomboidal cell that generates the Face-Centered Cubic lattice produce effectively isotropic composites if and only if an additional condition is satisfied. This condition entails the vanishing of a single component of the effective elasticity matrix. In spite of numerical evidence, we could not prove that this condition is always satisfied.

## 1. Correction

The matrices  $Q_{\text{sum}}$  and  $\hat{Q}_{\text{sum}}$  that appear at the end of Section 4 of the original article [1] and the subsequent calculations must be corrected as follows.

The last set of constraints that we consider comes from the  $2\pi/3$  rotation about the axis identified by  $a_1 + a_2 + a_3$ , a diagonal of the rhomboidal cell. The rotation matrix is

$$Q_{\text{sum}} = \begin{pmatrix} 1/2 & 1/(2\sqrt{3}) & \sqrt{2/3} \\ \sqrt{3}/2 & -1/6 & -\sqrt{2}/3 \\ 0 & 2\sqrt{2}/3 & -1/3 \end{pmatrix}.$$

The linear transformation associated with this rotation is

$$\hat{Q}_{\text{sum}} = \frac{1}{36} \begin{pmatrix} 9 & 3 & 24 & 12 & 12\sqrt{3} & 3\sqrt{6} \\ 27 & 1 & 8 & 4 & -12\sqrt{3} & -3\sqrt{6} \\ 0 & 32 & 4 & -16 & 0 & 0 \\ 0 & -8 & 8 & -14 & -6\sqrt{3} & 12\sqrt{6} \\ 0 & 8\sqrt{3} & -8\sqrt{3} & 14\sqrt{3} & -6 & 12\sqrt{2} \\ 9\sqrt{6} & -\sqrt{6} & -8\sqrt{6} & -4\sqrt{6} & 12\sqrt{2} & 6 \end{pmatrix}.$$

From the commutation relation  $C\hat{Q}_{\text{sum}} = \hat{Q}_{\text{sum}}C$ , with the elasticity matrix  $C$  as given in equation (11) of [1], we obtain for  $C$  the constraints

$$C_{14} = C_{41}, \quad C_{66} = C_{44} + C_{14},$$

$$C_{33} = C_{11} + \frac{1}{2}C_{14}, \quad C_{13} = C_{31} = C_{11} - C_{66} - \frac{1}{2}C_{14}.$$

The elasticity matrix thus depends only on three independent material constants,  $a = C_{11}$ ,  $b = C_{44}$ , and  $c = C_{14}$ , and assumes the general form

$$C_{\text{fcc}} = \begin{pmatrix} a & a-b & a-b-\frac{3}{2}c & c & 0 & 0 \\ a-b & a & a-b-\frac{3}{2}c & -c & 0 & 0 \\ a-b-\frac{3}{2}c & a-b-\frac{3}{2}c & a+\frac{c}{2} & 0 & 0 & 0 \\ c & -c & 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 & b & \sqrt{2}c \\ 0 & 0 & 0 & 0 & \sqrt{2}c & b+c \end{pmatrix}.$$

This becomes an isotropic elasticity matrix if and only if  $c = C_{14}$  is zero.

## 2. Amended conclusions

We have provided a necessary and sufficient condition for a periodic arrangement of spherical inclusions of an isotropic solid within an isotropic matrix to give rise to a large-scale isotropic response. Provided that such a condition is satisfied, a rhomboidal computational cell that generates the FCC lattice can be used to design composites in which the material symmetry is not affected by the periodicity of the construction. In spite of numerical evidence of the smallness of  $C_{14}$ , we could not prove analytically that this condition is satisfied exactly.

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To discuss material symmetries in a way that is independent of the reference frame, we have introduced a normalized Voigt representation, based on material directors rather than on a coordinate basis. Within this concise setup, symmetries of the inclusion lattice induce linear constraints on the entries of the 6 by 6 matrix of material coefficients that represents the linear elasticity tensor.

## References

- [1] Giulio G. Giusteri, Raimondo Penta, Periodic rhomboidal cells for symmetry-preserving homogenization and isotropic metamaterials, *Mech. Res. Commun.* 126 (2022) 104001, <http://dx.doi.org/10.1016/j.mechrescom.2022.104001>.