

Detailed studies of ^{12}C structure and reactions

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Research Article

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Posted Date: October 18th, 2023

DOI: <https://doi.org/10.21203/rs.3.rs-3442662/v1>

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Additional Declarations: No competing interests reported.

Version of Record: A version of this preprint was published at Few-Body Systems on December 5th, 2023.

See the published version at <https://doi.org/10.1007/s00601-023-01870-5>.

Detailed studies of ^{12}C structure and reactions

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Received: date / Accepted: date

Abstract We are reporting here on a series of theoretical investigations with both algebraic models and geometric cluster models of alpha clusters in ^{12}C , focusing on the structure of the ground state, the first excited 0^+ state and the second excited 2^+ state with the purpose, in particular, of establishing if the rotational bands are compatible with rigid structures or rather if they are quantum mixture of different configurations. In a first series of paper [1,2], we assume a rigid equilateral triangle shape and study in detail several properties that descend from the algebraic framework, such as the energy spectrum, electromagnetic observables and calculate the transition densities in order to extract elastic and inelastic cross-sections for various processes. In a second series of papers [3,4], we solve the three-body Schrödinger equation with orthogonality conditions using the stochastic variational method with correlated Gaussian basis functions. The two-body density distributions indicate that the main configurations of both the 0_2^+ and 2_2^+ states are acute isosceles triangle shapes coming from $^8\text{Be}(0^+)+\alpha$ configurations and find some hints that the second 2^+ state is not an ideal rigid rotational band member of the Hoyle state band.

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1 Introduction

In many branches of the physical sciences and in astrophysics, there is a great interest in having a complete understanding of the properties of the nuclide ^{12}C , the main stable form of carbon composed of 6 protons and 6 neutrons.

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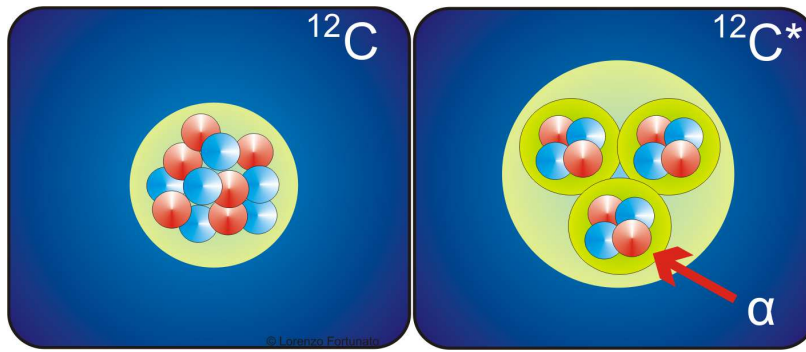


Fig. 1 A cartoon depiction of the ground states (often interpreted as a spherical shell model state) and the Hoyle excited 0^+ state, often represented as three alpha particles. New models challenge that view.

In recent years, there has been a surge of studies, using many different models and highlighting various aspects, especially the alpha-clusterization, i.e. the tendency of some of its constituents to merge into 4-particle lumps of 2 protons and 2 neutrons, making up an effective alpha particle. Some models assume or predict a spherical shell model configurations for all states in ^{12}C , others require that the Hoyle state, i.e. the excited 0^+ state at 7.65 MeV, and other states close to the alpha-separation threshold (Ikeda) might have a three alpha particles molecular structure and others yet conjecture that the rotations and vibrations of the three alphas, arranged accordingly to some geometrical shape, might explain most part of the low-lying energy spectrum (See Fig. 1).

In a long series of papers[1–4], the nuclear physics group in Padova (Italy), in strict collaboration with the groups in Catania (Italy), Seville (Spain) and Sapporo (Japan), has made a careful examination of the role of clusterization in the structure and reactions of the crucially important nuclei carbon-12 and oxygen-16. Our understanding of the life supporting ^{12}C nucleus still unfolds multiple unsolved problems and it has an impact on other branches of science. Let us list a few issues regarding this nucleus:

- single particle vs. cluster structure: only very recently refined versions of the shell model are starting to capture the emergence of clusterization [5] starting ab initio from the underlying nucleon-nucleon forces
- the role of discrete symmetries in explaining the energy spectrum and electromagnetic transitions and moments has emerged recently, allowing for a detailed comparison of the experimental energy spectrum with the prediction of the \mathcal{D}_{3h} group[6]
- the role of fermionic degrees of freedom in altering the simplified bosonic picture of cluster models and in particular the role of Pauli exclusion principle in altering some of the prediction made so far [7]
- the consistency of structure and reaction theories is very important. Too often theorists limit their speculations to the spectrum and electromagnetic transitions, but structure models should also be used in reaction calcula-

tions in order to compare with cross-section data for a number of reactions [1,2]

- the topic of Bose-Einstein condensation in this and other α -conjugate nuclei is a much debated issue
- the abundance of this particular isotope in the universe has an importance that transcends the pure interest in nuclear physics, touching on disciplines such as astrophysics, abiogenesis and anthropogenesis

Despite a large corpus of accumulated studies (See for instance Ref. [8,9]), one concludes that it is definitely worth to investigate this nucleus even more closely.

1.1 Previously presented material

In two occasions, in the two preceding editions of this conference, at the EFB23 in Aarhus (Denmark) 2016 and at the EFB24 Guildford (UK) 2019, we had presented detailed mathematical studies on the electromagnetic selection rules for ^{12}C in a 3 alpha cluster model [10,11] and on theoretical speculations on how to determine the shape of nuclear molecules with a sort of nuclear Raman scattering using beams of polarized γ -rays to determine the depolarization ratio of the outgoing scattered radiation [12,13].

2 Algebraic Cluster Model

Bijker and Iachello [6] have clearly demonstrated the successful application of the Algebraic Cluster Model, or ACM, in which the vibrational-rotational spectrum of alpha-conjugate nuclei like ^{12}C and ^{16}O is calculated by assuming spinless α particles with pure boson character, placed at the vertexes of geometric structures, such as an equilateral triangle and a tetrahedron. Each geometry brings with it a discrete group, that is a subgroup of the full spherical symmetry, namely $SO(3) \supset \mathcal{D}_{3h}$ and $SO(3) \supset T_d$, respectively. One can see in the figures of Ref.[6] examples of the spectra and the level of description that they can achieve.

Note that rotational bands *do not* conform to the usual quadrupole rotational bands that we are used to in nuclear physics, but they have a different symmetry! Taking as an example carbon-12, this implies bands composed of states with quantum numbers L^π compatible with the transformation of an equilateral triangle into itself. In other words, a resolution of the spherical states onto those allowed by the discrete dihedral subgroup \mathcal{D}_{3h} is the driving idea here.

The ACM disregards the Pauli exclusion principle, in the sense that the three alphas are not just elementary bosons, but rather are composite fermionic systems with all the nucleons in a $(1s)^4$ configuration. Thus the missing quanta of the Wildermuth condition must be attributed to the relative motion (two Jacobi vectors) [7] and this might alter the simple picture explained so far.

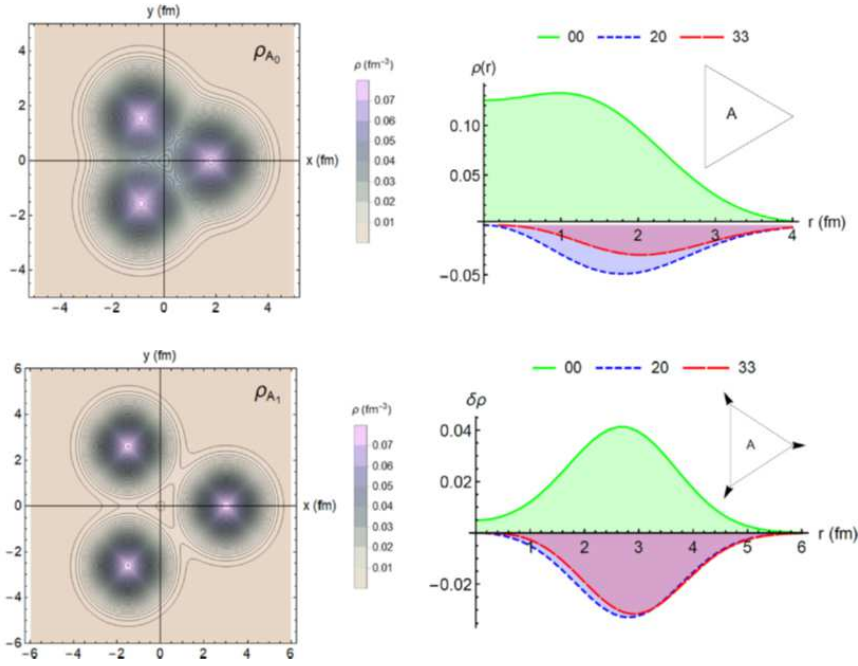


Fig. 2 Densities and decomposition in spherical harmonics components of the ground state (first line) and Hoyle state (second line) as a function of the radius. The pair of numbers indicate $\lambda\mu$, compatibly with the triangular symmetry. Adapted from Ref. [1].

2.1 Densities, transition densities and reactions observables

With the purpose of checking reactions observables against experimental data, we have studied a slightly more realistic version of the ACM in which each α particle is not point-like, but rather described by a gaussian density distribution of the form

$$\rho_{gs}(\mathbf{r}, \{\mathbf{r}_k\}) = \sum_{k=1}^3 \rho_{\alpha}(\mathbf{r} - \mathbf{r}_k) \quad , \quad \rho_{\alpha}(\mathbf{r}) = \left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha r^2} \quad (1)$$

where \mathbf{r}_k are the equilibrium positions of the alpha particles at the vertexes of an equilateral triangle and $\alpha = 0.56(2) \text{ fm}^{-2}$ is an empirical parameter to adjust the size of the individual α -particle. These are then expanded in spherical harmonics as

$$\rho_{gs}(\mathbf{r}) = \sum_{\lambda\mu} \rho_{gs}^{\lambda,\mu}(r) Y_{\lambda,\mu}(\theta, \varphi) \quad (2)$$

where only a few terms appear in the sum: those compatible with the triangular symmetry (so for instance for the ground state A-type band, $\{\lambda, \mu\} = \{0, 0\}, \{2, 0\}, \{3, 3\}, \{4, 0\}, \{4, 3\}, \{5, 3\}, \dots$).

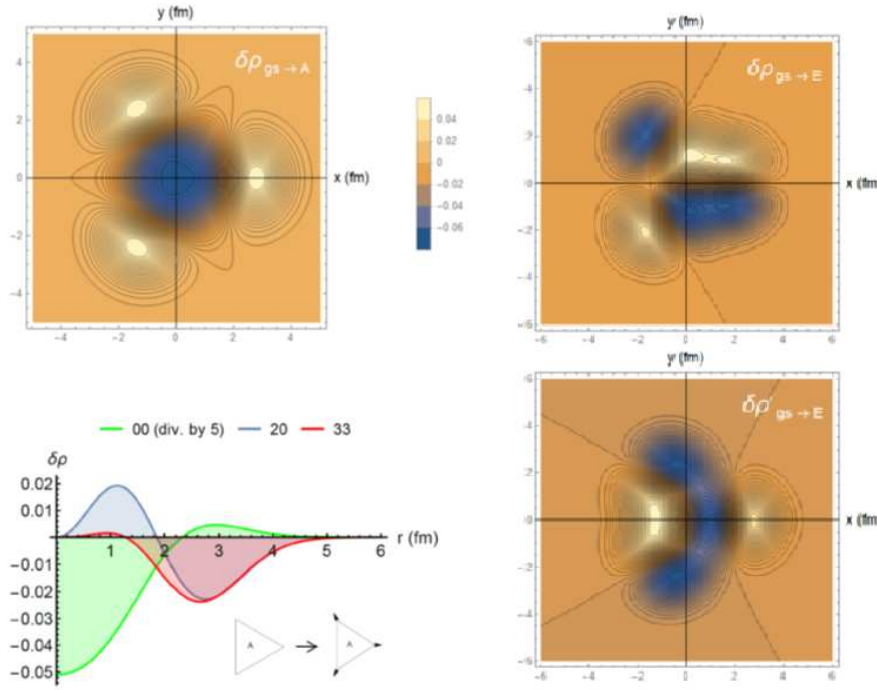


Fig. 3 Transition density of the A-type vibration (Holy state) and decomposition (left), transition densities of the two degenerate states of the E-type vibration (right). Adapted from Ref. [1].

One can see in Fig. 2 an example of contour plots as a function of coordinates in the plane containing the α 's for the density of the ground state, indicated with A_0 , because there are zero quanta of excitation, and of the Hoyle state, seen as the band-head of the first quantum of excitation of A-type, i.e. A_1 , corresponding to a breathing mode of the three α particles in and out. On the right, the expansion into the first few radial components is shown in colors, green being the spherical one.

In Fig. 3 we show the transition densities, for excitations from the ground state to the first excited A-type vibration (upper left) and to the doubly-degenerate E-type vibration (right panels). They represent (at first order) the difference between the initial and final densities. Without entering into many details, we also show the expansion of the g.s. to Hoyle state transition density in spherical harmonics (lower left), where the $\{0, 0\}$ monopole term is dominant.

The quantities shown above contain all the structure information that allows to compute form factors for inelastic processes such as, for example, the $\alpha+^{12}\text{C}$ scattering and from the form factors one can compute cross-sections. We show in Fig. 4 a few examples of form factors (matrix elements) for this process and a few examples of computed differential cross-sections as compared

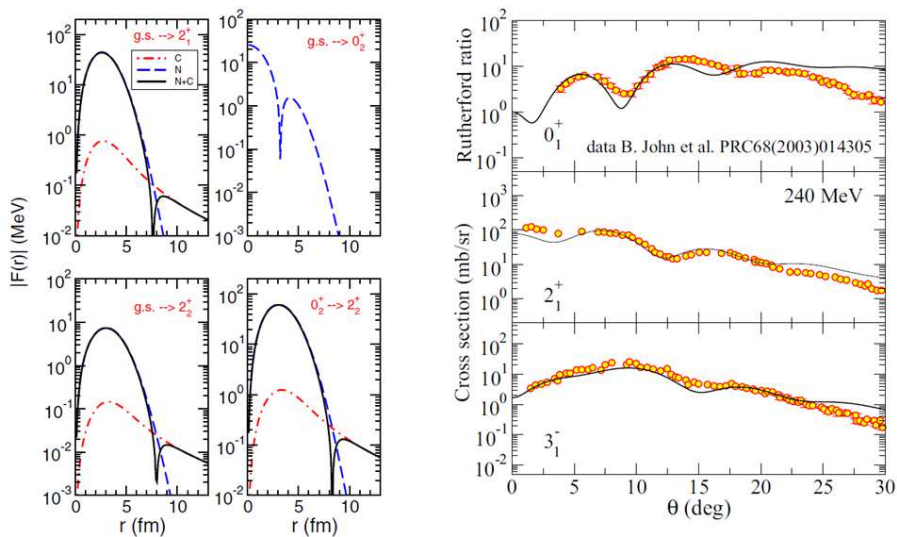


Fig. 4 Form factors for the $\alpha+^{12}\text{C}$ inelastic scattering to selected states, with beak-down into Coulomb and nuclear contributions (left) and differential cross-sections (right), compared with published data (reference on the figure) Adapted from Ref. [1]).

to experimental data (yellow dots). The results are satisfactory and indicate that a simple description of ^{12}C in terms of three α particles, not only allows for a reliable description of its spectrum end electromagnetic properties, but it gives also a good description of reaction observables.

3 Microscopic approaches lead to a different interpretation

Algebraic cluster models and molecular models have their own utility and strength, but we can approach the problem from a more microscopic few-body perspective. A three-body approach based on a stochastic variational method with correlated Gaussian basis functions, developed mainly by Japanese colleagues H. Moriya and W. Horiuchi, can be applied to the present problem [3, 4]. Two methods can be used, an orthogonality condition model (OCM) and a shallow potential model (modified Ali-Bodmer alpha-alpha potential), that use quite different strategies. This model, in either of the variants, does not suffer from the limitations linked to the Pauli exclusion principle.

We can see in Fig 5 the comparison of the resulting two-body density distribution for the ground and Hoyle state in the two methods. The distribution of the 0_2^+ state is clearly more external, compatible with a first breathing oscillation of the clusters in and out, but while the ground state is peaked around the equilateral triangle geometry (almost diagonal line upper panel), the Hoyle state has a somewhat more acute main shape, with also equilateral and obtuse components.

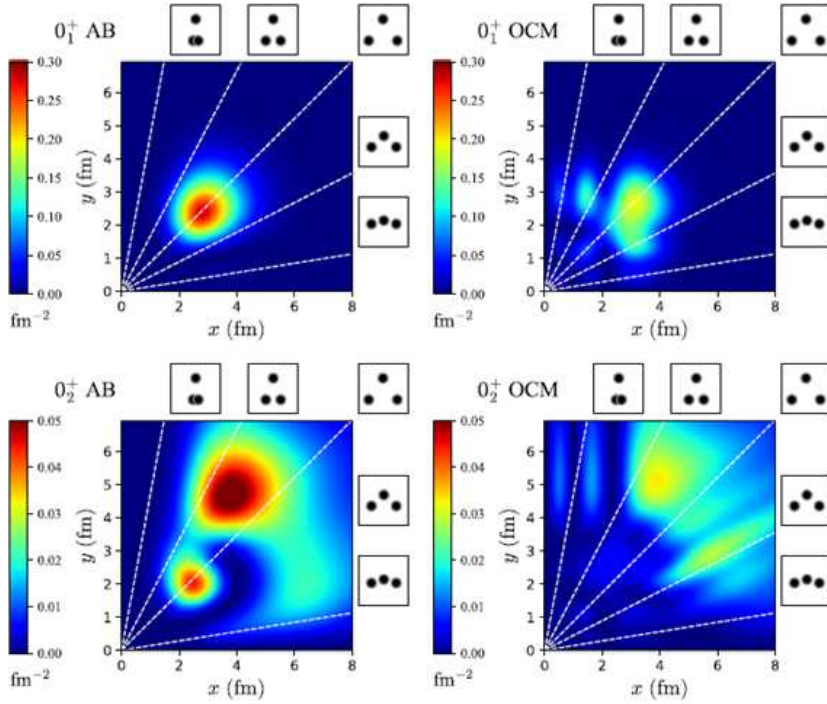


Fig. 5 Two-body density distribution for the ground and Hoyle states with two different methods (see text). Lines correspond to different geometrical arrangements. From Ref. [3].

The two-body density distributions indicate also (not shown here, see references) that the main configurations of both the second 0_2^+ and 2_2^+ states have acute isosceles triangle shapes coming mostly from $^8\text{Be}(0^+) + \alpha$ configurations and find some hints that the second 2_2^+ state is not an ideal rigid rotational band member of the Hoyle state band. Please, see Ref.[3,4] for a detailed analysis. The lower left panel of Fig. 5 is almost identical to Fig.4 in Ref. [14], that was also shown by S. Ishikawa at this conference, arriving at the same conclusions.

It is indeed to be expected that nuclear systems get distorted with raising excitation energy, due to the composite nature of the alpha particles (four correlated fermions, subject to the Pauli principle, instead of just a spin 0 boson). When you incorporate these effects, you get some distortion of the geometry in favour of an acute triangle, showing some $^8\text{Be}(0^+) + \alpha$ correlations.

4 Conclusions

We have discussed here two different aspects: 1) that the algebraic description can describe reliably a number of reaction observables and 2) that three-body

approaches hint at a more sophisticated mixing of quantum states than the simple geometrical model would.

As yet, one cannot say that one model (ab initio shell model and/or variants, phenomenological cluster models, algebraic, semi-algebraic or others) prevails over others in the description of this nucleus. It is, by now, firmly established that clusterization play a crucial role and this is not yet fully incorporated in more microscopic models. A merging of these ab initio techniques including nucleon-nucleon correlations that satisfy constraints coming from symmetry considerations is perhaps to key to solve this intricate issue.

5 Acknowledgements

These studies and the presentation at the EFB25 conference have been supported by the University of Padova and by the INFN, under the MONSTRE initiative.

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