Algebraic model studies of the structure and reactions of ^{12}C and ^{16}O

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Abstract. The role of algebraic models and geometric cluster models of alpha clusters in the structure of ground and first excited 0^+ and second excited 2^+ states in ^{12}C are evaluated with the aim of establishing if the rotational bands are compatible with rigid structures or if they are instead overlaps of different configurations. A rigid equilateral triangle shape is assumed in Ref.[2, 3]: the energy spectrum, electromagnetic observables are calculated and the transition densities are compared with elastic and inelastic cross-sections for various processes. In a different approach [4, 5], we solve the three-body quantum problem for 3 alphas using the orthogonality condition and the stochastic variational method with correlated Gaussian basis functions. The two-body density distributions indicate that the main configurations of both the second 0^+ and 2^+ states are acute iscosceles triangle shapes with a ^{8}Be(0^+)+α configuration. We also find clues that the second 2^+ state is probably not a member of the rigid rotational band built on the Hoyle state.

1 Introduction

The present contribution will summarize recent works on $\alpha-$clustering in light nuclei ^{12}C and ^{16}O, highlighting a few important aspects regarding the two-body density distribution in both the Algebraic Cluster Model (ACM) and three-body models.

Traditionally, the properties of p-shell nuclei have been understood in terms of the spherical shell model and this description typically works well for the ground state and for the description of a number of other quantities. Despite this, many reaction observables cannot be satisfactorily explained unless one stretches the model beyond its reach or introduces cluster degrees of freedom. Instead of pursuing a spherical shell model picture, pushing basis dimensions to include phonons of several $\hbar\omega$'s, one must recognize the role of α particles, i.e. lumps of two protons and two neutrons, that stick together inside a nucleus forming an equivalent α-particle. Often one just speaks of α-particle correlations to refer to this situations. This α particle *in medium*, i.e. in the bulk of nuclear matter, is probably the most accurate view, but molecular-type models in which each cluster is taken as a structureless building block, have been attempted in the past with various degree of success. This extreme "frozen" view, that forgets the internal fermionic structure of the α's and the Pauli principle, is not without merit. The α particle is indeed a tightly bound and compact spin-0 boson that justifies the molecular approaches.

Bijker and Iachello [1] have clearly demonstrated the successful application of the ACM, to the vibration-rotation spectrum of alpha-conjugate nuclei like ^{12}C and ^{16}O, seen in a geometric picture, in which 3 or 4 α-particles are placed at the vertexes of an equilateral triangle or of a tetrahedron, respectively. The energy spectrum is convincingly regrouped into rotational bands that do not conform to the usual quadrupole rotational bands that we are used to, but they have different symmetry and selection rules! Restricting ourselves to just ^{12}C, the angular momenta and parities are compatible with the transformations of an equilateral triangle into itself, which amount to the $\mathcal{D}_{3h}$ discrete symmetry. The vibrational bands, as calculated by importing group theoretical methods from molecular physics, take the character of the group representations, called A and E, and in each band a non-standard rotational sequence appears. For example, for the ground state A-type band, the allowed angular momentum and third component pairs are $\{\lambda,\mu\} = \{0,0\},\{2,0\},\{3,3\},\{4,0\},\{4,3\},\{5,3\},\cdots$, while for the E-type bands, we have $\{\lambda,\mu\} = \{1,1\},\{2,1\},\{2,2\},\{3,1\},\{3,2\},\cdots$. Many properties of these nuclei, such as electromagnetic transitions, moments and selection rules [6], are deeply affected by such an extreme molecular model.

2 Reaction observables of clusterized nuclei

It is interesting to see in some details, to what extent the geometric distribution of three cluster affects

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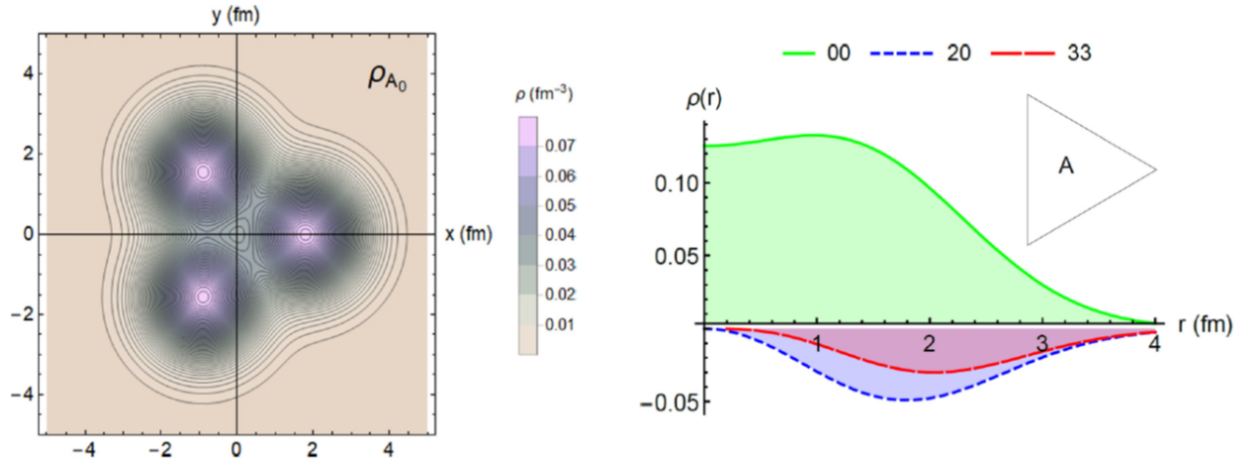

Figure 1. Section of density, ρ_{gs} in the xy plane and the first few terms of the spherical harmonic expansion of the ground state, $\rho_{gs}^{\lambda,\mu}$.

reactions observables, such as form-factors and cross-sections and how these can be used to extract useful indications.

One refinement is to introduce finite density distributions, that allow for a more precise modelisation. A gaussian form is used to represent the density of each α particle, as:

$$\rho_\alpha(\vec{r}) = \left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha r^2} \tag{1}$$

with the parameters as in Ref. [2]. The 3D integral is normalized to 1, so that the charge and mass of the particle is obtained upon multiplication by Z or A. The density of ^{12}C is obtained by summing three α particles located at the vertices of a triangle, indicated as $\vec{r}_k$, as

$$\rho_{gs}(\vec{r},\{\vec{r}_k\}) = \sum_{k=1}^{3} \rho_\alpha(\vec{r}-\vec{r}_k) = \sum_{\lambda\mu} \rho_{gs}^{\lambda,\mu}(r) Y_{\lambda,\mu}(\theta,\varphi) \tag{2}$$

Once Eq.1 is set, the whole density is dependent only on the parameter β, the distance from the center, that can be fixed to about 1.8 fm. The density can be expanded in spherical harmonics (see Fig. 1) to highlight the fact that not all of the $\{\lambda,\mu\}$ components are present: only those who conform to the rules of the $\mathcal{D}_{3h}$ symmetry given above are non-null. The same expansion can be done for the Hoyle state band (A-type vibration with one quantum of oscillation in this model) and for the doubly-degenerated E-band, obtaining analogous expressions for the components. One finds interesting results for the E-band, for example, the two generate states corresponding to the intrinsic shape of the whole band show a ^{8}Be-α structure as illustrated in Fig. 2. One wants to do the same spherical harmonic expansion not only to the intrinsic densities of each band, but also to the transition densities, that are the crucial quantities to determine connection between the various bands. By this method, one can calculate form factors that can

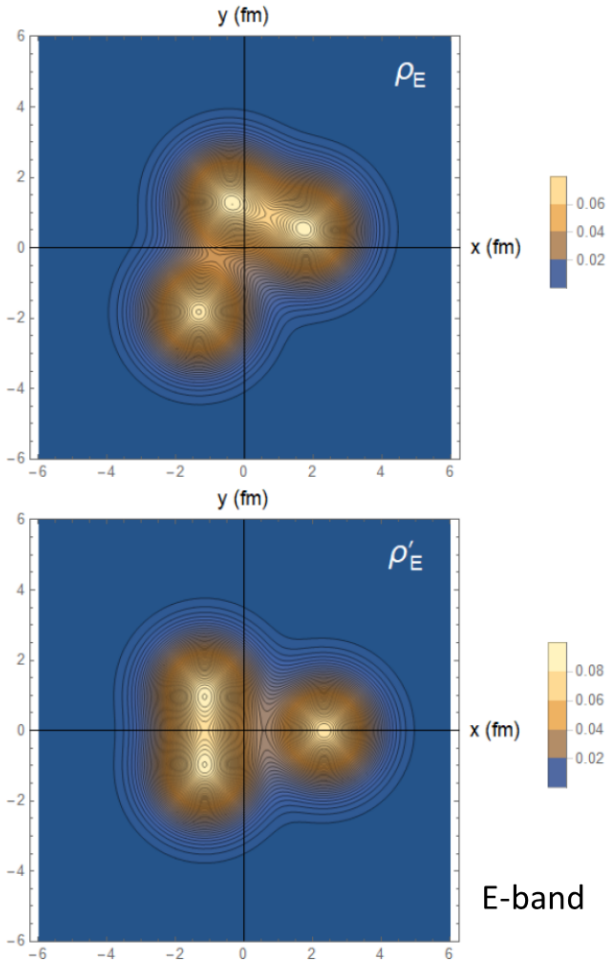

Figure 2. Section of density for the two degenerate intrinsic states of the E-band: a clear ^{8}Be-α structure emerges.

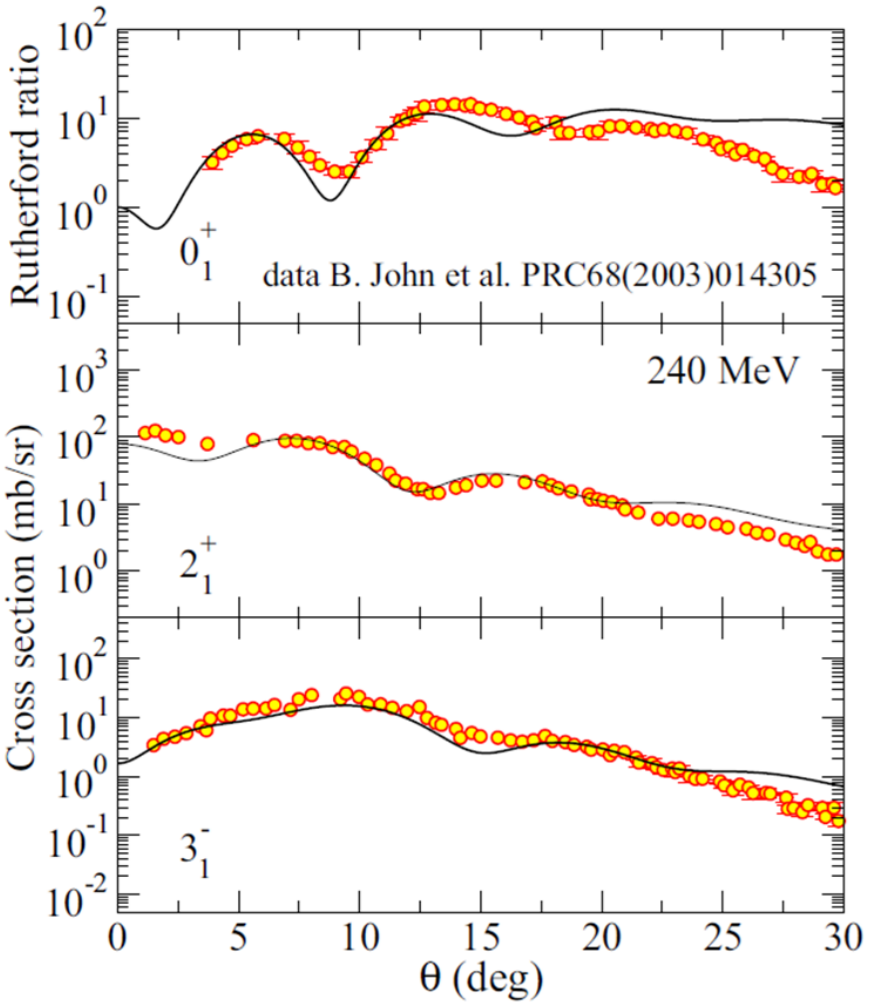

Figure 3. Elastic and inelastic DWBA cross-sections for in α+^{12}C process at 240 MeV bombarding energy, compared with data [8].

be plugged into standard DWBA codes and obtain reaction observables such as differential and total cross-sections. We refer the readers to Ref.[2, 3] for a more detailed discussion and we show only the elastic and inelastic cross-section to the first 2^+ and 3^+ states in Fig. 3, excited in α+^{12}C process at 240 MeV bombarding energy, compared with experimental data [8].

They confirm that with just a simple triangular model one catches all the gross features, not only of the nuclear structure, but also of reaction dynamics of ^{12}C.

In a similar fashion, [3] one can attack more complex reactions such as $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ using also the properties of tetrahedral symmetry of ^{16}O in the ACM.

3 Three-body models using SVM

A different approach has been followed in Ref.[4, 5] where a three-body stochastic variational method using correlated Gaussian basis functions has been applied to solve the full problem in a very accurate numerical way. Two different variants have been adopted for ^{12}C: 1) orthogonality condition model (OCM) ; 2) the use of shallow potentials (in this case a modified Ali-Bodmer alpha-alpha potential).

The ACM “forgets" the Pauli Exclusion principle, in the sense that, if the three alphas are all $(1s)^4$ configurations, then the missing quanta of the Wildermuth condition must be attributed to the relative motion (two Jacobi vectors) [7]. The present approach does not suffer from this limitation.

The aim was to study in detail the composition/structure of each state. In particular, Ref. [4] concentrates on the Hoyle state and Ref. [5] on the second $J^\pi = 2^+$ state of ^{12}C. From the results of calculations with the two methods for the two-body density distributions in Fig. 4, one can see that the 0^+_2 state is clearly more external, compatible with a first breathing oscillation of the clusters in and out.

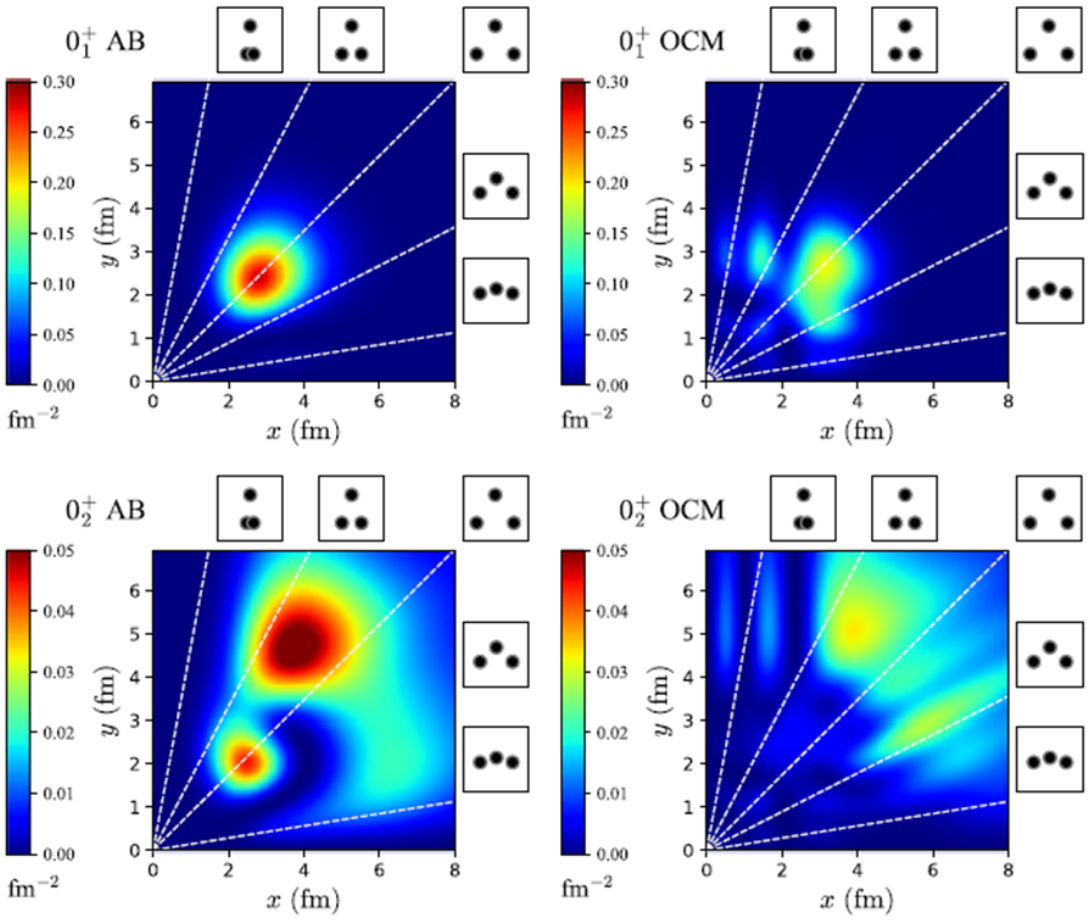

Figure 4. Two-body density distributions for the ground state and the Hoyle state within the two methods, OCM and AB, see text.

Similar calculations can be performed in the interesting case of the second 2_2^+ as it is not completely clear if this state is built on the Hoyle state of if it contains different components from other configurations.

The two-body density distributions (See Fig. 5 and Ref. [5] for pictures) indicate that the main configurations of both the second 0_2^+ and 2_2^+ states have acute iscosceles triangle shapes coming mostly from $^8\mathrm{Be}(0^+) + \alpha$ configurations and find some hints that the second 2_2^+ state is not an ideal rigid rotational band member of the Hoyle state band. Densities get distorted with raising energy and we can claim that the rotor picture is not so rigid at this level.

4 Conclusions

Clusterization is an essential ingredient to understand structure and reactions of light nuclei. The discrete geometrical symmetry group of the equilateral triangle $\mathcal{D}_{3h}$ (containing also the permutations of the three alphas) shows its importance not only in the energy spectrum of ^{12}C, but also in reaction properties, like elastic and inelastic scattering. The same can be said of the tetrahedral configurations T_d of four alphas in ^{16}O.

Even though these algebraic methods are great, because they capture the essence of the rotational-vibrational molecular type of motion in these nuclear molecules, they do not incorporate microscopic correlations, that most certainly show up in higher lying states due to the composite nature of the alpha particles (four correlated fermions, subject to the Pauli principle, instead of just a spin 0 boson). When you incorporate these effects, you get some distortion of the geometry in favour of an acute triangle, showing some ^{8}Be+α correlations. Due to the limited space available in this conference contribution, we refer the reader to the cited literature for a more comprehensive coverage.

5 Acknowledgements

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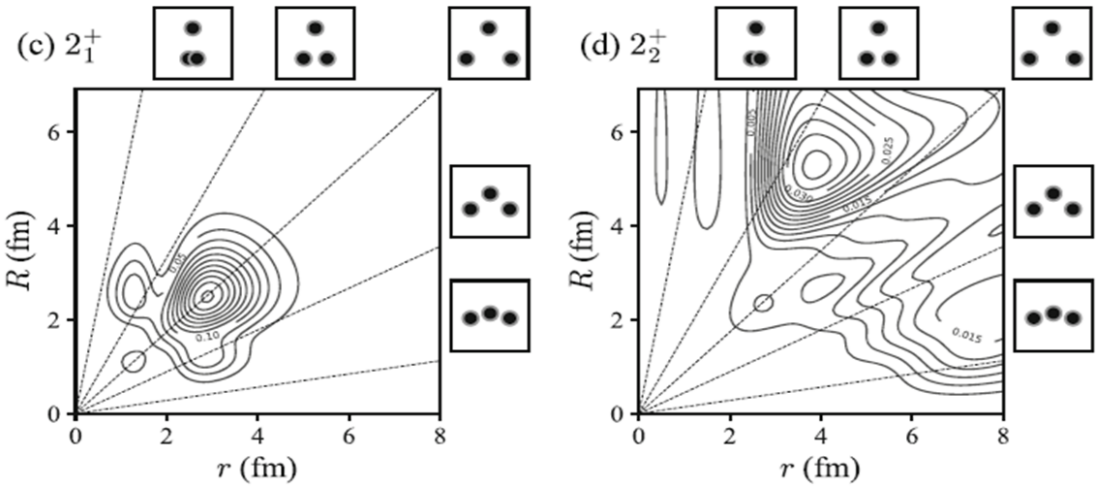

Figure 5. Comparison of the two-body density distributions for the first and second 2^+ states of ^{12}C. The right panel shows a more mixed configuration than just an equilateral triangle.

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