



Building *more mathematico* in Renaissance Venice

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Abstract

In the mid-fifteenth century, Venice inherited Cardinal Bessarione's collection of ancient manuscripts, an exceptional legacy that contributed to the renewal of mathematical studies. This essay outlines the spread of mathematics in Venice in the early Renaissance, when scholars actively discussed Euclid and universal proportions. Based on this cultural context, the author briefly analyzes the role of polyhedra and an example of the application of proportions to architecture: the church of San Francesco della Vigna in Venice. This church is an interesting and rare case of 'declared' application of proportions to architecture, since we have specific indications in Francesco Zorzi's memorandum written in 1535.

Keywords Design theory · Renaissance Venice · Francesco Zorzi · Harmonic proportions · Polyhedra

Mathematical Collections and Scholars in Renaissance Venice

In the Renaissance, Venice became a center of mathematical studies. The first and most important reason is the bequest of the Greek and Latin manuscripts of Cardinal Bessarion to Venice. But a series of other scholars born in the city, such as Bartolomeo Zamberti and Giovanni Battista Memmo, or settled in Venice, such as Luca Pacioli and Giorgio Valla, contributed to the development of mathematics and its related disciplines with their humanist debate.

We can say that Cardinal Bessarion's collection of manuscripts transmit ancient culture to Renaissance Italy. According to the inventory of 1472, the collection numbers several hundred of Greek and Latin manuscripts (Labowsky, 1979: 147), and is considered the largest such collection in Europe. In the end, the collection was bequeathed to the Venetian Republic, forming the nucleus of the *Libreria di San Marco*. He left his splendid library to the city, when, on 31 May 1468, Bessarion

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sent the Doge of Venice the note *donatio inter vivos*, when the donation was given ‘the value of a rite: Venice was consecrated heir of the miraculous Hellenic culture, and in a superior life of the spirit it is made responsible for a common heritage of the human race.’¹ Among these manuscripts there are many mathematical works. Indeed, Bessarion’s interest in mathematical problems can be seen in the notes that he adds in the margins of his manuscripts of Archimedes, Euclid, Ptolemy, Apollonius, Aristarchus, Diophantus, Proclus, Hero, Aristotle, Theodosius and Serenus (Rose 1975: 45). Thus, beside the work of Pappus, the *Libreria di San Marco* contained every major classical source for the use of Renaissance mathematicians.

At the close of fifteenth century, Venice held another great humanist collection of mathematical works: the library of Giorgio Valla of Piacenza. Valla’s mathematical manuscripts became famous among the scholars of the late Quattrocento and, for this reason, many mathematicians visited his house in Venice to copy *Codex A* of Archimedes, a work lost after its entry into the Estense collection in the 1560 s (Heiberg 1896: 108). Giorgio Valla used *Codex A*, as well as the rest of his mathematical manuscripts, to write the *De Rebus Expetendis et Fugiendis* (Venice 1501), an encyclopedic work compiled by translating and paraphrasing classical authors. For the next forty years at least, the *De Rebus Expetendis* remained almost the only printed source of reference for the works of Apollonius, Archimedes, the commentaries of Eutocius, and Hero (Rose 1975: 47).

The Venetian Bartolomeo Zamberti was thoroughly versed in Valla’s learning. In the preface of his Latin translation of Euclid’s *Elements* (Venice 1505) he discusses Plato as the figure who combines mathematics with moral and natural philosophy. Zamberti approached the Euclidean work with a completely different spirit from that of Campanus or Regiomontanus, authors who attempted to confer mathematical coherence on the *Elements*, without worrying if this meant drawing on Arabic sources, adding new propositions or changing, even radically, some proofs. Instead, Zamberti, as a Venetian humanist, aspired to pass down the original Euclidean text and to achieve this aim, he translated, as faithfully as possible, the Greek codex at his disposal. In his book Zamberti specifies that one would find the barbarisms introduced by Campanus, such as the word *helmuain*, nor a mistranslation of the fifth definition in Book V (Zamberti 1505: 6). Indeed, Zamberti’s main objective was the reconstruction of the *Elements* in its original perfect state. Zamberti’s translation of the *Elements*, which pays little attention to mathematical coherence and focuses more on fidelity to the original text, provoked the reaction of another important mathematician who lived in Venice at that time: Luca Pacioli. Pacioli published a revised version of the Campanus’s edition. The title page of Pacioli’s *Euclidis Opera* (Venice 1509) acclaims Campanus as *interpres fidissimus*, blaming the errors on the carelessness of copyists.

Another mathematician active in Venice in the early sixteenth century was the patrician Giovanni Battista Memmo, who on 8 October 1530 persuaded the Senate

¹ ...il valore di un rito: Venezia è consacrata erede della miracolosa cultura ellenica, e in una vita superiore dello spirito è fatta responsabile di un patrimonio comune del genere umano (Garin 1994: 39).

to set up a public chair of mathematics in Venice. The main work of Memmo was his Latin translation of the Greek text of Apollonius, *Apollonii Pergei, philosophi, mathematicae excellentissimi Opera* (Venice 1537).

As we have seen, the Renaissance Venetian humanist tradition demonstrated a deep interest in mathematics; indeed, this discipline occupied a central position in the curriculum of the *Scuola di Rialto*, the place where Mathematics was taught in Renaissance Venice. Since Mathematics in the *Scuola di Rialto* was directly linked to the practical applications of sciences, this pragmatic approach had profound repercussions in art and daily life.

Solids and Proportions in Renaissance Venice

Fra Luca Pacioli gave a famous lecture on proportions in 1508 at St. Bartholomew church, seat of the public mathematics chair of the *Scuola di Rialto*, with the aim of demonstrating that God organised the universe on the basis of precise mathematical rules (Black 2013: 87–104). The following year Pacioli published his book entitled *Divina proportione* (Venice 1509), expanding the contents of the lecture and discussing numbers, plane and solid geometry in greater detail. The foundations of Pacioli's proportionate universe are the Platonic solids, which his master Piero della Francesca described from a mathematical point of view in the *Libellus de quinque corporibus regularibus* and showed how to represent them in *De prospectiva pingendi*. Polyhedra embody the application of mathematics in art (Gario 1979). It is no coincidence that, in his treatise on divine proportion, Pacioli lists the masters of intarsia among the greatest experts of perspective because their figurative repertoire is famously based on the composition of complex solids. As an example, Pacioli refers to the Zoccolanti choir stalls in San Francesco della Vigna (Pacioli 1509: 23r), executed by Giovan Marco Canozzi. This work of art was demolished to build a new church that would become emblematic, since Jacopo Sansovino's new project respected specific harmonic proportions according to the *more mathematico*. The new San Francesco della Vigna is based on the science of numbers and musical proportions as tools for deciphering the mysteries of God and the universe. Sansovino's references are the physics of Plato's *Timaeus*, the harmonic-proportional speculations of Vitruvius and the geometry of solids described by Luca Pacioli (Lorenzin 2013: 23).

Therefore, this Venetian cultural context, focused on proportions in which mathematics, science, philosophy, religion, and esotericism find a common exegetical horizon, and traces back to regular and semi-regular polyhedra. According to the Renaissance zeitgeist, mathematical consonances connect the terrestrial reality ('atomically' composed by the Platonic solids) to the immutable laws of celestial harmonies, of which music is a tangible expression. Obviously, the supreme polyphonic perfection of the super-sensible world remains unattainable, but works of art must imitate it by showing a mathematical-proportional order (Field 1997: 241–289). The Platonic and Archimedean solids work very well to this purpose, so much so that their surfaces are often used to accommodate sundials for reading the time. The Museum of the History of

Fig. 1 S. Buonsignori. *Multiple-faced sundial on a dodecahedron* (1587). Image: Florence, Museum of the History of Science



Fig. 2 H. Holbein. *Multiple-faced sundial on a solid. Detail of Nicola Kratzer's portrait* (1528). Image: Paris, Musée du Louvre



Science in Florence preserves, for example, multiple sundials created on the faces of a dodecahedron; it is a refined creation by Stefano Buonsignori, dating back to 1587 (Miniati 1991: 50) (Fig. 1). But there are many other artworks that associate regular and semi-regular polyhedrons with instruments for measuring time and reading celestial motions, such as the portrait of Nikolaus Kratzer, realized by

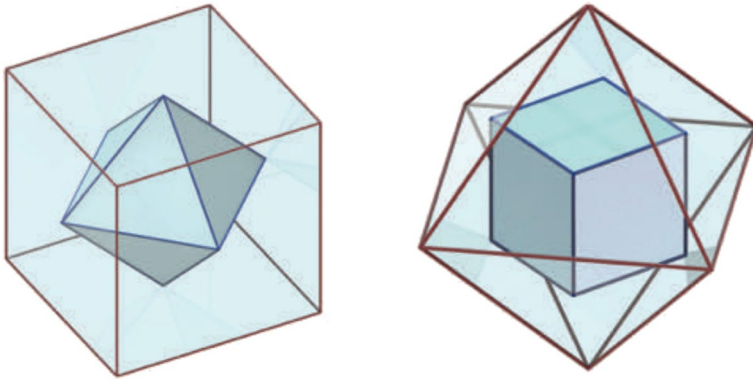


Fig. 3 Octahedron derived from a cube (left); cube derived from an octahedron (right). Image: author

Hans Holbein in 1528, which shows a sundial obtained by truncating the corners of an octahedron (Tosi 2007: 17) (Fig. 2).

Furthermore, polyhedra have numerical and geometrical features. The lateral surfaces and volumes of the dodecahedron and the icosahedron, with sides measuring one unit, can be expressed as a function of the golden section that Luca Pacioli's called divine proportion. Let's consider a segment AB . It can be divided into two unequal parts AC and CB in such a way that the whole segment stands to its larger part as this stands to the smaller one AB : $AC = AC : CB$. Greek mathematicians and philosophers were fascinated by this curious proportion, which seemed to reflect in itself the divine perfection and harmony. Hence the use of the adjective 'divine' to denote the 'proportion' that governs this singular relationship. Coming back to the features of Platonic solids, the cube and the octahedron are duals: they share the same number of edges, equal to 12; they show symmetries in the number of faces and vertices, respectively: 6 faces and 8 vertices in the first; 8 and 6 in the second. The dodecahedron and the icosahedron are similarly duals; they are characterized by 30 edges, where the dodecahedron has 12 faces and 20 vertices, while the icosahedron counts the opposite. These numerical characteristics can be translated geometrically into a process by which one solid derives from another. Thus, by joining the centres of the 6 lateral surfaces of a cube we obtain an octahedron, and vice versa (Fig. 3). One can proceed in the same way to obtain a dodecahedron from an icosahedron and, on the contrary, an icosahedron from a dodecahedron (Fig. 4), keeping in mind the ratio between the 'generating' polyhedron and the 'generated' one can always be expressed as a function of the divine proportion. Among the Platonic solids, the tetrahedron is the only one to 'generate' itself (Monteleone 2020: 79) (Fig. 5).

The investigation of the symmetrical and proportional properties of polyhedra, symbols of both the terrestrial and celestial order, is fully part of the study of mathematics, in particular regarding Euclid's Book XIII of the *Elements*, which contains their theoretical basis. In the first part of this essay I mentioned the different interpretation of Zamberti and Pacioli regarding the original text of the Greek mathematician. The reason for the dispute is philological; Paul Lawrence Rose

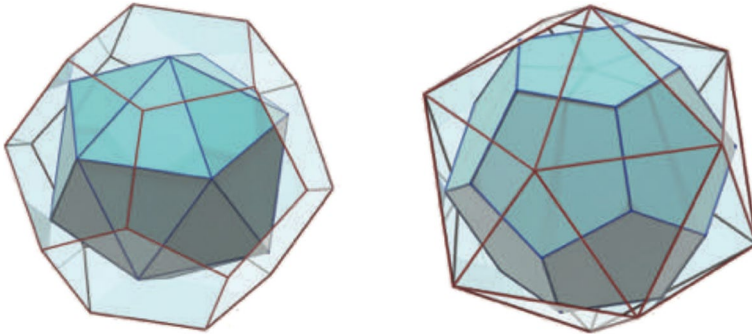
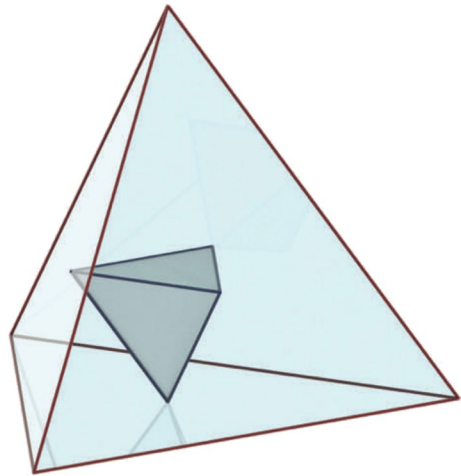


Fig. 4 Icosahedron derived from a dodecahedron (left); dodecahedron derived from an icosahedron (right). Image: author

Fig. 5 Tetrahedron derived from a tetrahedron. Image: author



explains the reasons that push Bartolomeo Zamberti and Luca Pacioli to face each other virtually on the printed page (Rose 1975: 143–149). Basing himself on the teachings of his master Giorgio Valla, Zamberti published his *Euclidis megarensis philosophi Platonici mathematicarum disciplinarum Janitoris* (Zamberti 1505), a Latin translation of Euclid's *Elements* from a Greek manuscript attributed to Theon of Smyrna, following a strictly humanist approach. Pacioli, on the contrary, defended the interests of mathematics from philological questions and responded four years later with *Divina proportione* (Pacioli 1509) a re-proposed, amended and updated version of Campanus (Giovanni Campano da Novara).

Pacioli felt the need to re-evaluate the *Elements* translated by Campanus because this work, while raising problems of textual exegesis, retains the correct mathematical rigor, thanks to the arithmetic transcriptions of many propositions. What interests us most is that the dispute includes the Greek terms *αναλογία* and *λογος*, which Zamberti translates respectively with the Latin words *proportio* and

ratio, distinguishing between proportion – i.e., the direct relationship between dimensions – and proportionality – i.e., the correspondence that homogeneous quantities establish. Pacioli translates both Greek terms with the lemma *proportio*, as does Campanus (Ciocci 2017: 245).

Renaissance scholars were aware that the discussion on the terms *proportio* and *ratio* involves architecture as well as other disciplines that base their speculative reasoning on vision: optics or *perspectiva naturalis*; painting or *perspectiva artificialis*; survey to measure distances; cartography to estimate and represent the earth; astronomy and gnomonics to calculate time and contemplate the celestial order. Vitruvius develops the link between vision and proportion in the preface to Books III and V. Kim Williams extensively studied the Venetian scholar Daniele Barbaro's comments on Vitruvius, attributing meaning to the words associated with geometric and harmonic proportions (Williams 2019b). In addition to the works by Daniele Barbaro, it is emblematic that Venice holds other specialized publications on the relationship between mathematics and arts, such as Palladio's *I quattro libri dell'architettura* (Venice 1570) and Silvio Belli's *Della proporzione et proporzionalità* (Venice 1573). To this list of treatises that deal with mathematics and art one can also add the renewed interest in Leon Battista Alberti's *concinntas*, promoted by a Venetian reprint of *De re aedificatoria*, edited by Cosimo Bartoli (Venice 1565).

Therefore, the edition of Bartolomeo Zamberti's *Elements* corroborates the link between the mathematical terms *proportio*, *ratio* and art. In practice, just as harmonic relationships delight the ear, metric relationships delight the sight. In this regard Daniele Barbaro says that: 'those proportions in sounds that delight the ear, when likewise applied to bodies, delight the eye' (Williams 2019a: 456). For this reason, good architecture must proceed by composing spaces and elements by supporting specific harmonious relationships.

Arithmetic and Geometric Proportions

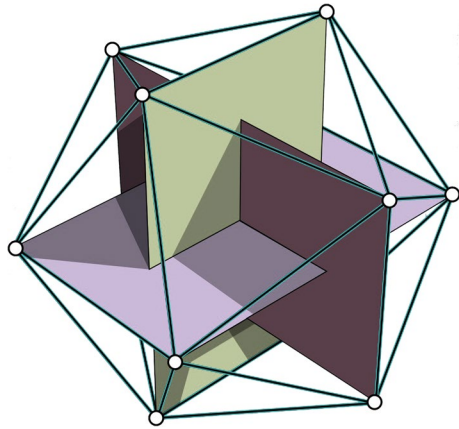
After Rudolf Wittkower published his book titled *Architectural principles in the Age of Humanism* in 1952, many scholars delved into the relationship between architecture and proportions, sharing his distinction between arithmetic and geometric proportions. In practice, Wittkower's Appendix II argues that: 'the mediaeval artist tends to project a pre-established geometrical norm into his imagery, while the Renaissance artist tends to extract a metrical norm from the natural phenomena that surround him' (Wittkower 1952: 159). Indeed, breaking with the design method of the architects of the Middle Ages, the Renaissance architects looked at metrical proportion as guiding principle to reveal harmony. This is also why Renaissance architects espoused Vitruvius' module system, that is based on arithmetic proportions. However, at one point in his analysis Wittkower seems to contradict himself, concluding that simple, arithmetic proportions may arise from geometrical procedures for instance: 'Where Francesco di Giorgio suggests using a mediaeval, incommensurable geometrical scheme for the design of a church, it actually serves to lead him to a

rational module' (Wittkower 1952: 161). Or: 'Where Alberti talks about his own (apparently geometrically determined) designs of buildings, he is sadly aware of his errors and of the necessity to adjust them by correct numbers' (Wittkower 1952: 161). Yet the coincidence between the results of geometric proportions to those of arithmetic proportions had already been noted and addressed in Daniel Barbaro's commentary on the concept of eurythmy expounded by Vitruvius. Indeed, when Vitruvius states that: 'The fine number called eurythmy is the graceful visual appearance and appropriate form in the composition of the members. This is made when the members of the work are appropriate, as the height to the width, the width to the length, and finally everything corresponds to the compartition proper to it' (Williams 2019a: 64). Daniele Barbaro comments as follow: 'Since all proportion is born of numbers, he has used the aforesaid name in everything where there is proportion. The width to the height and the length of the works must agree with the visual appearance and this is not done without proportion' (Williams 2019a: 64). So, we can say Wittkower is not contradicting himself when he states arithmetic proportions may arise from geometrical procedures because, while it is true that there is a different design approach between the architects of the Middle Ages (geometry) and the Renaissance (mathematics), it is equally true that their goal is the same, namely harmony in architecture. Indeed, eurythmy is a visual quality that manifests itself whenever architecture is designed on the basis of harmonic consonances, whether geometric or arithmetic. Proportions, then, are not only an essential tool for controlling design but also for gratifying our sight with harmony.

It is worth taking a step back at this point to explain these aspects further. Pythagoras had dealt with the interaction between music and numbers, made evident by simple ratios: 1:2—1:3—1:4—2:3—3:4. Let us consider a string of a guitar that can vibrate and establish 'n' its measure; well, the same vibration to another string that measures the half, that is, ' $\frac{1}{2}$ n,' generates a sound one octave higher (*ottava*); we have thus defined the ratio of 1:2. On the other hand, if the shorter string is ' $\frac{2}{3}$ n' the difference in sound will be a fifth (*diapente*—2:3), if it is ' $\frac{3}{4}$ n' the difference in tone will be a fourth (*diatessaron* 3:4). By applying these simple relationships in design, we link architecture to musical harmonies by transforming the pleasure that causes sound to the ear into the pleasure that generates the view to the eye.

Unlike Music, which bases intervals and chords on commensurable quantities, architecture also contemplates proportions that cannot be measured arithmetically, such as the golden section. The fact that golden section cannot be expressed by means of a fraction (that is, by rational numbers) simply means that it is impossible to find two whole numbers whose ratio exactly matches the ratio of two lengths. The golden section is an interesting amalgamation of two meanings, the quantitative and the aesthetic, because although it is defined geometrically it is able to make shapes pleasing the eye. It is in this context that regular polyhedra gain importance in the Renaissance as symbols of harmony. Indeed, Platonic solids generate in their intrinsic measures many instances of harmonic relationships. Let us take one case for all: The icosahedron is characterized by 12 vertices that can be interpreted as the vertices of 3 golden rectangles that intersect orthogonally. The ratio of the sides of each of these rectangles is equal to the golden section (Fig. 6).

Fig. 6 3 golden rectangles inside an icosahedron. Image: author



Building *More Mathematico*

The events related to the design and construction of the church of San Francesco della Vigna in Venice constitute an occasion more unique than rare to examine the question of proportions applied to architecture. I dealt with this subject in a recent essay (Monteleone 2023); here I summarize its contents and refer the reader to the earlier publication.

It does not happen very often that contemporary scholars can consult an original Renaissance document regarding the application of mathematics to architecture and the mystical and religious meanings associated with it. The memorandum written in 1535 by Francesco Zorzi for the design of the church of San Francesco della Vigna offers a unique opportunity to reconstruct the metrical harmonies of this building. The belief held by all the protagonists of this architectural story is that beauty passes through proportions to become a political manifesto, an expression of prestige and power. Therefore, the architecture of San Francesco della Vigna is an instrument to show the solidity and harmony of the Venetian Republic (Tafuri 1985: 161). Doge Andrea Gritti promoted the construction of innovative buildings following the principles of the ancients. He assigned this task to the architect Jacopo Sansovino, who settled in Venice after the sack of Rome in 1527 and, at least in the case of San Francesco della Vigna, to Francesco Zorzi, a friar of the church expert in Kabbalah and Bible interpretation. In this renovation of the *imago urbis* the ideas of Zorzi, whose success must be attributed to his ability to reconcile Christianity, Judaism, and ancient philosophy, find their place. His best-known work, *De harmonia mundi totius cantica tria* (Venice 1525), establishes a dialogue between the dogmas of the church and the mystical, hermetic, and kabalistic trends of the time. Zorzi's harmonic theories find their practical application in the reconstruction of San Francesco della Vigna. In 1534 the Doge supported the reconstruction of this earlier Gothic church, asking Zorzi to point Sansovino in the right direction so that the building can respect the *harmonia mundi*, i.e., mystical, musical and proportional harmony (Foscari and Tafuri 1983: 53). In 1535 Zorzi drafted a

Fig. 7 Spaces with proportion 2:3 (sesquialtera, or *diapente*).
Image: author

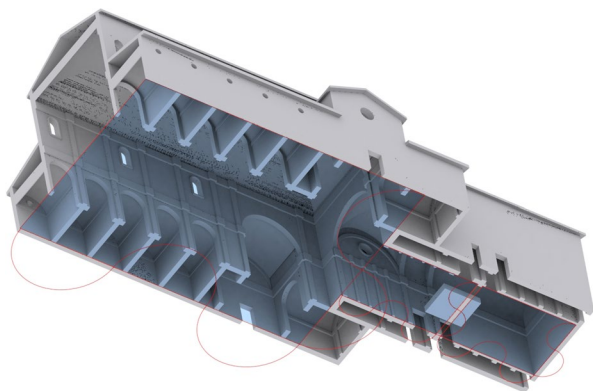
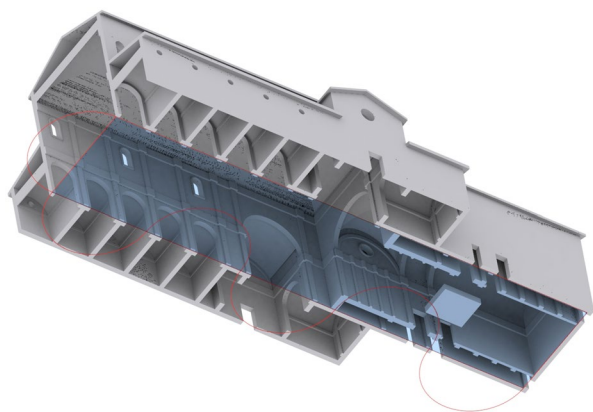


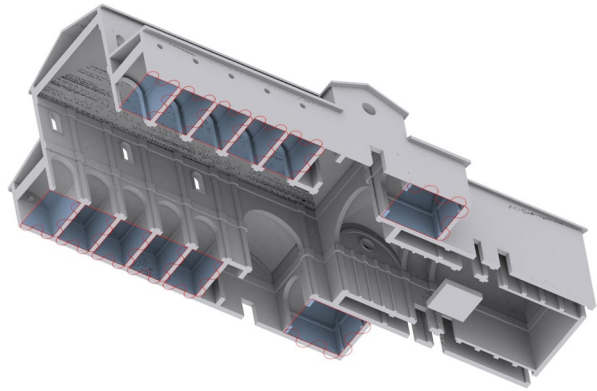
Fig. 8 Space of quintuple ratio.
Image: author



memorandum that proposed building the church according to the proportions indicated in Plato's *Timaeus* for the construction of the world. The design of San Francesco della Vigna thus follows a divine thought of harmony. Since the memorandum also recalled the precepts of Vitruvius, Sansovino adopted Zorzi's indications, and the whole building was realized according to a mystical numerology.

Zorzi's harmonic theories are based on number 3, and Sansovino too builds the whole church on the basis of the number 3, the ratios of 1:2 (double ratio, or *diapason* in music), 2:3 (sesquialtera, or *diapente*) and 3:4 (sesquitercia, or *diatessaron*). The ratio between the width and length of the nave (9:27) can be seen to form a 9:18:27 progression. The altar at the end of the nave is 9 feet long and 6 feet wide, in the ratio of 2:3. The ratio of 2:3 has also been applied to the width and length of the altar, and to the choir, behind the altar (Fig. 7). The entire length of the church is 5 times 9, a quintuple ratio with its width (Fig. 8). The chapels at the two sides of the nave are 3 feet wide, or one-third the width of the nave (3:9). The width and length of each side chapel are in the ratio 3:4 (Fig. 9). The ratio of the minor chapel's width to that of the altar is 1:2, while the ratio of

Fig. 9 Spaces with proportion 3:4 (sesquitercia, or *diatessaron*). Image: author



the width of the transept to that of the chapels is 3:4. Considering now the section of the building, we can conclude that the heights respect proportions quite similar to those of the widths. The nave has a height equal to 5 times 3, or 15 feet, and thus is again a quintuple ratio with its width. The same proportional homogeneity of width-height and width-length is also found in the side chapels in the ratio of 3:4.

Thanks to the memorandum written by Francesco Zorzi it is possible to reconstruct these proportional relationships, which follow the musical ratios. But why are the protagonists of this story so interested in proportion? The answer can be found in the religious notions (Bullinger 1895: 55). Number 3 is said to be Moses's inspiration in his construction of the Ark of the Covenant. St. Paul, in his letters to the Corinthians, writes that this number is explicit in the proportions of the human body. Number 3 is even believed to have inspired the proportions of Solomon's Temple in Jerusalem. In the end, Jacopo Sansovino's design for San Francesco della Vigna was never fully completed since the construction of the facade passed to Andrea Palladio, who adopted a proportional system inspired by Vitruvius' rules. However, inside the church, the dimensions of the rooms in plan faithfully follow the indications provided by Francesco di Giorgio with his memorandum (Monteleone 2023). However, just considering the inside of the church, from a symbolic point of view, the nave of San Francesco della Vigna represents Christ and human body perfection, the transept on the other hand symbolizes Christ on the cross, the twelve chapels the apostles, and so on. The idea was to show Venice as a new Rome: a rich and renewed city, that arose after the failures of the war against the League of Cambrai; a city that takes up the classical legacy of mathematics and applies geometric proportions to architecture to make Venice a reflection of the world's harmony.

Conclusion

A series of circumstances, including to the bequest of Cardinal Bessarion, made Venice a unique place for the development of mathematics in the Renaissance. Some of the best scholars of the time, including Giorgio Valla, Bartolomeo Zamberti,

Luca Pacioli and Giovanni Battista Memmo, had access to ancient treatises that they could study and translate in order to disseminate knowledge. However, the humanist approach to manuscripts raises problems of scientific rigor which fuelled the discussion and in-depth study of some concepts. In particular, the knot of translating the terms *proportio* and *ratio*, which involve sight and, consequently, art, remained to be resolved.

The case of San Francesco della Vigna demonstrates that in Renaissance Venice there was a widespread belief that beauty could only be achieved through appropriate harmonious proportions. This context of cultural renewal, where mathematics applied to architecture, that is, building *more mathematico*, allowed Doge Andrea Gritti to extend science and knowledge to every manifestation of Venetian power, including the urban image. So, Sansovino's design for the church of San Francesco della Vigna became part of this grandiose political project, because it reveals proportional relationships related as much to ancient philosophy as to Judaism and Christianity.

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