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THERMAL CONTENT DYNAMICS IN MAGNETICALLY CONFINED PLASMAS

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Abstract

The present thesis is devoted to the physical investigation of magnetic reconnection events in the RFX-mod device operating in the Reversed Field Pinch (RFP) configuration for the magnetic confinement of a thermonuclear plasma. The analysis is carried out through a combined numerical and experimental approach. On the numerical side, thermal content (i.e. ion) dynamics is studied through simulations via the Hamiltonian, guiding-center code ORBIT. On the experimental side, a preliminary activity, aimed at the refurbishment of the Neutral Particle Analyzer (NPA) data acquisition system, is presented. This is meant in view of the upgraded RFX-mod2 device, whose commissioning is expected to start by the end of 2026.

Experiments conducted on the Madison Symmetric Torus (MST), a device similar to RFX-mod, have demonstrated that magnetic reconnection accelerates plasma charged particles. Similarly, in RFX-mod, significant ion heating has been observed through NPA measurements. Particle energization associated with magnetic reconnection is a fundamental topic also in space plasma physics; however a proper description of the phenomenon is still far from being achieved. The main objective of the thesis is therefore to investigate plasma particle dynamics during magnetic reconnection events, with particular emphasis on the case of the discharge #29324 at time $t = 218$ ms, when a strong magnetic reconnection event induced a Plasma-Wall Interaction (PWI) that was directly observed by a CCD camera detecting the first wall.

The study considers a gradual approach of increasing complexity. A first simplified analysis, based on analytical methods, shows that the phase-locking of $m = 1$ tearing modes is the primary responsible for the observed PWI. This phase-locking involves a large number of tearing modes, actually more than those measured by the magnetic sensors of RFX-mod. A more refined analysis is then performed through the calculation of the Connection Length to the wall using the ORBIT code, confirming the role of the $m = 1$ mode phase-locking as the leading loss mechanism toward the first wall, while also revealing an additional contribution of the $m = 0, n = 7$ mode. Furthermore, the calculation of loss times for Maxwellian ions via ORBIT highlights that particles lost through topologically different channels are also characterized by different energy and pitch. This result provides a means to discriminate loss channel topology based on the simulation of thermal content dynamics. Finally, the first implementation in ORBIT of a three-dimensional electrostatic potential calculated by the 3D MHD code SPECYL is shown to be a fundamental key for understanding the thermal content behavior during magnetic reconnection, which had not been fully disclosed by numerical methods. The calculation of highly-resolved, three-dimensional maps of Connection Length and Maxwellian ion loss times presented in this thesis relies on a new parallelized version of ORBIT, implemented for the first time within the ORBIT workflow.

Sommario

Questa tesi è dedicata allo studio dei fenomeni di riconnessione magnetica osservati nell'esperimento RFX-mod durante le operazioni in configurazione *Reversed Field Pinch* (RFP) per il confinamento magnetico di un plasma termonucleare. L'analisi è condotta impiegando un approccio sia numerico che sperimentale. Dal punto di vista numerico, la dinamica del contenuto termico (vale a dire ioni) è simulata tramite il codice Hamiltoniano di centro-guida ORBIT. Dal punto di vista sperimentale invece, viene qui presentata un'attività preliminare di ricondizionamento del sistema di acquisizione della diagnostica *Neutral Particle Analyzer* (NPA), in vista del nuovo RFX-mod2, che entrerà in funzione entro la fine del 2026.

Un esperimento simile a RFX-mod, il *Madison Symmetric Torus* (MST), ha dimostrato sperimentalmente che eventi di riconnessione magnetica sono in grado di accelerare le particelle cariche del plasma. Analogamente, anche in RFX-mod è stato osservato un simile riscaldamento della popolazione ionica, tramite il NPA. L'accelerazione di particelle durante le riconnessioni è un tema rilevante anche in ambito astrofisico, tuttavia una descrizione esaustiva del fenomeno non è stata ancora elaborata. L'obiettivo principale della tesi è pertanto quello di investigare la dinamica delle particelle durante gli eventi di riconnessione magnetica, con particolare attenzione al caso della scarica #29324 al tempo $t = 218$ ms, durante il quale un evento di riconnessione magnetica ha indotto un'interazione plasma parete (PWI) che è stata osservata direttamente mediante una telecamera CCD che osserva la prima parete.

L'analisi segue un approccio graduale di crescente complessità. Una prima analisi semplificata, basata su metodi analitici, mostra che il bloccaggio in fase (o *phase-locking*) dei modi *tearing* caratterizzati da $m = 1$ è il principale meccanismo responsabile della PWI osservata. Tale phase-locking coinvolge un numero elevato di modi, addirittura superiore a quelli che possono essere direttamente misurati tramite i sensori magnetici di RFX-mod. Un'ulteriore analisi, basata sul calcolo della Lunghezza di Connessione tramite il codice ORBIT, conferma il ruolo del phase-locking come principale meccanismo di perdita di particelle verso la prima parete. Tuttavia, rivela al tempo stesso un contributo aggiuntivo di perdita riconducibile al modo $m = 0$, $n = 7$. Inoltre, il calcolo dei tempi di perdita per ioni Maxwelliani (tramite ORBIT) evidenzia che le particelle perse in canali di perdita topologicamente differenti sono caratterizzate da diversa energia e *pitch* (direzione della velocità della particella rispetto al campo magnetico). Questo risultato in realtà fornisce un criterio per poter distinguere la topologia dei canali di perdita basato sulla simulazione della dinamica del contenuto termico. Infine, per la prima volta in ORBIT è stato inserito un potenziale elettrostatico calcolato tramite il codice tridimensionale e MHD SPECYL. La presenza del potenziale elettrostatico nelle simulazioni rappresenta un ingrediente fondamentale per la comprensione del comportamento del contenuto termico durante la riconnessione magnetica, il quale è un aspetto che non era stato ancora riprodotto in maniera

adeguata dai precedenti metodi numerici.

Il calcolo di mappe tridimensionali ad alta risoluzione di Lunghezza di Connessione e di tempi di perdita di ioni Maxwelliani è stato possibile tramite la parallelizzazione di ORBIT, che viene presentata per la prima volta in questa tesi.

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Introduction

Main aspects on Nuclear Fusion

Matter is made of atoms, which in turn are composed by electrons orbiting around a central nucleus made of protons and neutrons, collectively referred as *nucleons*. Nucleons are held together by the nuclear force within an extremely small region of space, of the order of $1 - 10 \text{ fm}^1$. The *binding energy* E_B of a nucleus can be estimated within the framework of the Liquid Drop Model (LDM) [1], which states that a bound nucleus has a mass smaller than the sum of the masses of its constituent nucleons:

$$\Delta m = Zm_p + (A - Z)m_n - M > 0 ,$$

where Z is the atomic number, A the mass number, M the nuclear mass, and m_p and m_n the proton and neutron masses, respectively. The mass difference Δm is directly related to the binding energy through $E = \Delta m \cdot c^2$, being c the speed of light in vacuum. As a consequence, nuclei contain a large amount of stored energy, released through nuclear reactions, hence the term *nuclear energy*.

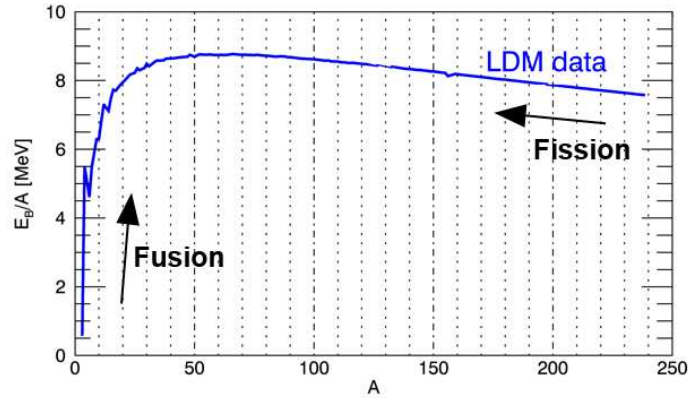


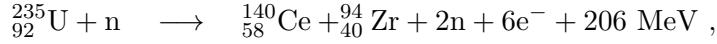
Figure 1: Trend of the binding energy per nucleon as a function of the atomic mass number A . The directions corresponding to fusion and fission reactions are indicated by arrows.

A plot of the binding energy per nucleon is shown in Fig. 1. The most stable nucleus corresponds to $A = 56$ (iron, Fe), which maximizes E_B/A . All other nuclei tend to increase their stability, by moving towards higher binding energies. To do so, they follow two distinct ways:

¹1 *femtometer* (fm) is equivalent to 10^{-15} m. The femtometer is also known as *fermi* in honor of the Italian physicist Enrico Fermi.

- nuclei with $A < 56$ tend to aggregate to form heavier elements: this is the mechanism of *nuclear fusion*;
- nuclei with $A > 56$, tend to split into lighter elements: this is the mechanism of *nuclear fission*.

The energy released in a nuclear reaction appears as kinetic energy of reaction products. In principle, such energy can be stored to be subsequently converted into electric power. While fusion has not achieved yet this goal, fission is routinely exploited in commercial power plants. The reaction [2]:



is currently the most widely used in fission reactors. Because it is triggered by a neutral neutron, it is relatively easy to overcome the Coulomb barrier surrounding the heavy reactive nucleus. Moreover, although only one neutron is required to trigger the reaction, two neutrons are produced, allowing a self-sustaining chain reaction. However, this also implies that loss of control over the chain reaction can lead to severe, long-lasting safety issues. Another major drawback of fission is the production of long-lived radioactive waste, which requires special safety practices for storage.

Instead, the principal reaction commonly considered for nuclear fusion commercial exploitation is the so called *D-T reaction* [3–5] (see Fig. 2) where D denotes deuterium hydrogen isotope (${}^2\text{H}$), and T denotes tritium (${}^3\text{H}$):

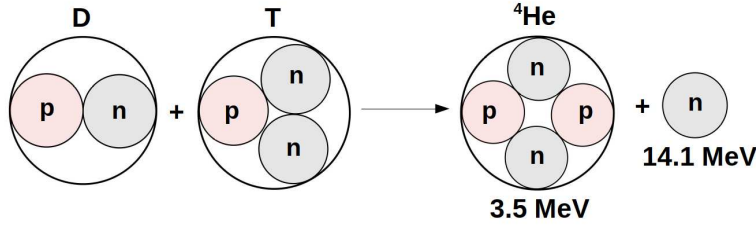
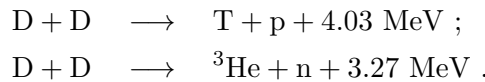
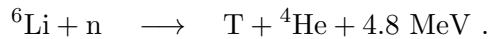


Figure 2: In a D-T fusion reaction, a deuterium nucleus and a tritium nucleus combine to form an α -particle (a ${}^4\text{He}$ nucleus) with a kinetic energy of 3.5 MeV. A neutron with kinetic energy of 14.1 MeV is also produced. The total released energy in a single reaction is thus 17.6 MeV.

The D-T reaction produces an α -particle (a ${}^4\text{He}$ nucleus) with a kinetic energy of 3.5 MeV, and a neutron with a kinetic energy of 14.1 MeV. Alternatively, D-D reactions are triggered in current experiments; with equal probability:



Fusion offers three main potential advantages: fuel reserves, reduced environmental impact, and enhanced safety. Regarding fuel availability, deuterium is naturally abundant, thanks to the presence of one deuterium per 6700 hydrogen atoms, and it can be easily extracted, for example from seawater. Instead, tritium is produced through breeding reactions involving the lithium isotope ${}^6\text{Li}$:



Thus, the long-term exploitation of D-T fusion reactors depends primarily on lithium resources, although alternative materials (such as beryllium, Be, or lead, Pb) are currently under investigation [6].

A further advantage concerns the environmental impact. Fusion reactors do not emit CO₂ or other harmful pollutants during operation. The main environmental issue relates to the periodical disposal of the activated structural materials of the so-called *blanket*, surrounding the vacuum vessel. As mentioned, the D-T reaction produces a high-energy neutron (14 MeV), which activates structural components. However, these materials have relatively short radioactive lifetimes (on the order of 100 years), making their management significantly easier than that of fission waste.

Finally fusion presents important safety benefits. Fusion reaction cannot trigger a chain reaction. This means that the reactor requires to be almost continuously replenished while operating. Consequently, reactor operation can be stopped almost instantaneously if required, eliminating the risk of runaway reactions or meltdowns. For these reasons, nuclear fusion is widely regarded as a promising energy source for future large-scale power production.

Plasma

Despite the efforts devoted to fusion research by the international scientific community over several decades [7], the goal of an operating fusion power plant has not yet been achieved. From a practical standpoint, the main challenge consists of finding effective methods to overcome the Coulomb potential barrier between nuclei in order to trigger the reaction. The condition in which D-T reactions occur at a sufficiently high rate is referred to as *ignition*. Reaching ignition requires that the reacting nuclei be confined for a sufficiently long time, at high densities, and at high temperatures. This requirement is quantitatively expressed by the *triple-product* equation [8]:

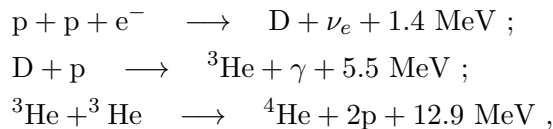
$$nT\tau_E > 3 \cdot 10^{21} \text{ m}^{-3} \cdot \text{keV} \cdot \text{s} ,$$

where τ_E denotes the *energy confinement time*.

The first key requirement for achieving ignition is thus a very high temperature: so high ($T \gtrsim 10\,000^\circ\text{C}$), that matter transits to a new state known as *plasma*. A plasma results from the ionization of a neutral gas into a collection of free charged particles, namely ions and electrons. From a phenomenological point of view, a plasma can be described as a special kind of fluid, that is globally neutral, but composed by electrically charged constituents interacting through long-range Coulomb forces.

Astrophysical plasmas

Plasmas are not uncommon; on the contrary we encounter them frequently in everyday life. They constitute the state of very hot matter, such as lightnings or flames, and last, most importantly, life on Earth is possible thanks to the nuclear fusion reactions occurring in our closest star, the Sun. In the solar core, helium is synthesized from lighter elements through a sequence of fusion reactions known as the *proton-proton* (or *p-p*) *chain* [9]:



where ν_e denotes an electron neutrino and γ a gamma ray photon.

Stars are the natural sites where all chemical elements are synthesized, and nuclear fusion is the fundamental mechanism responsible for their formation. Stellar cores are able to ignite fusion reactions by exploiting their own mass to compress the reactive nuclei into a sufficiently small volume. This mechanism is referred to as *gravitational confinement*.

A wide range of astrophysical phenomena are governed by plasma physics. Of particular interest in contemporary research are *magnetic reconnection events* [10, 11], which provide an efficient framework to explain the dynamics of large-scale processes such as solar flares [12] and the ion heating observed in the solar corona [10, 13, 14] or in the magnetosphere [15, 16].

Laboratory plasmas: magnetic confinement

On Earth, we cannot rely on the Sun gravitational potential. Therefore, alternative methods are employed to ignite the plasmas. In the thesis, we focus on *magnetic confinement*.

Magnetic confinement [17] relies on the principle that a charged particle immersed in an electromagnetic field experiences the Lorentz force:

$$\vec{F} = Ze \left(\vec{E} + \vec{v} \times \vec{B} \right) .$$

This force causes the particle to orbit around the magnetic field line $\vec{B}$, as sketched in Fig. 3. The electric field $\vec{E}$ instead accelerates particles along its direction according to their charge Ze . In the same expression, $\vec{v}$ is the particle velocity vector. The center of the circular orbit is known as the *guiding-center*. The radius of the orbit, perpendicular to the magnetic field, is called the Larmor radius² r_L . The angular frequency of this circular motion, ω_c , is the *cyclotron frequency*:

$$r_L = \frac{mv_{\perp}}{ZeB} \quad ; \quad \omega_c = \frac{ZeB}{m} ,$$

where m is the particle mass, and $v_{\perp}$ is the velocity component perpendicular to the magnetic field.

The simplest magnetic configuration for plasma confinement is a solenoid, where the magnetic field is uniform and parallel to the axis. However, while transverse motion is confined, particles are still free to escape along the magnetic field. To avoid these parallel losses, the magnetic field lines can be closed on themselves, forming a toroidal geometry, essentially a donut-shaped configuration. This can be achieved by arranging coils to form a *toroidal solenoid*. In this geometry, the direction encircling the major axis is referred to as *toroidal* ($\hat{\varphi}$), while the direction encircling the minor axis is the *poloidal* direction ($\hat{\theta}$). The minor radius is denoted by $\hat{r}$, see Fig. 4.

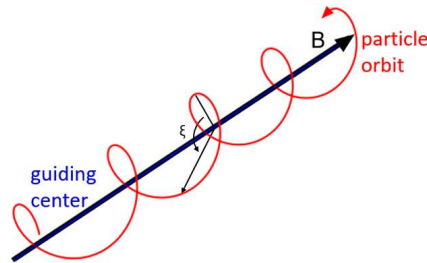


Figure 3: A charged particle immersed in a magnetic field follows a spiral (in red) around the magnetic field line (in black). Particle gyro-phase is indicated with ξ . The guiding-center trajectory in this simplified case coincides with the magnetic field line.

²Also referred to as *cyclotron radius* or *gyro-radius*.

By applying the Ampère law to a torus, it follows that the magnetic field decreases as $\frac{1}{R}$, where R indicates the distance from the torus central axis. It can be shown that, in the presence of a magnetic field gradient, the particle guiding-center drifts [19] with velocity:

$$\vec{v}_d = \frac{mv_{\perp}^2}{2ZeB} \cdot \frac{\vec{B} \times \nabla B}{B^2} + \frac{mv_{\parallel}^2}{ZeB} \cdot \frac{\vec{R}_c \times \vec{B}}{R_c^2 B},$$

where $\vec{R}_c$ is the curvature radius of the magnetic field. By convention, the outward radial direction is taken as positive. In toroidal geometry, the overall effect is that particle drift on the vertical direction. Since $\vec{v}_d$ depends on the sign of the charge, ions and electrons are separated, resulting in the formation of an electric field. This electric field in turn produces an additional drift component:

$$\vec{v}_{E \times B} = \frac{\vec{E} \times \vec{B}}{B^2}.$$

This charge separation would rapidly deteriorate plasma confinement. To counteract this effect, a poloidal field component must be introduced. The superposition of the toroidal (B_{φ}) and the poloidal (B_{θ}) fields yields to a configuration in which the magnetic field lines wind helically around the torus, as illustrated in Fig. 4. This configuration is known as *helical transform*. As a result, the guiding-center trajectory also becomes helical, with each particle executing an arbitrary number of turns in both the toroidal and poloidal directions.

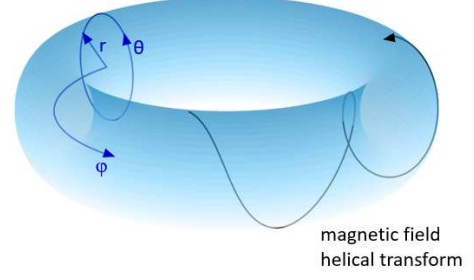


Figure 4: Sketch of the helical transform of magnetic field lines in a toroidal configuration. The toroidal and poloidal directions are indicated by $\hat{\varphi}$ and $\hat{\theta}$, respectively, while $\hat{r}$ denotes the radial direction. Adapted from Ref. 18.

Laboratory plasmas: inertial confinement

An alternative method for laboratory plasma confinement is *inertial confinement* [20]. Here, a small deuterium-tritium solid target is compressed by high-power laser beams. The evaporation of the surface material drives an inward implosion, compressing the fuel to extremely high densities and temperatures, finally leading to thermonuclear burn. The largest inertial confinement based fusion experiment is the National Ignition Facility (NIF), located at Lawrence Livermore National Laboratory (LLNL) in Livermore (California, USA). In 2021 NIF achieved a major milestone by delivering a fusion yield of 1.37 MJ fusion in a single shot [21, 22], surpassing the previous record by approximately a factor eight.

Tokamak, Reversed Field Pinch (RFP), and Stellarators

Tokamak

The best-known magnetic confinement configuration is the Tokamak [23], invented in the USSR in the early 1950s [24]. In a tokamak (schematically represented in Fig. 5), the toroidal component of the magnetic field is generated by a set of poloidal coils, while the poloidal field is produced by the plasma current flowing in the toroidal direction and

driven by the central solenoid. Additional vertical field coils are employed to control plasma shape and position within the vacuum vessel. In this configuration, the magnetic field is predominantly toroidal, i.e. $B_\varphi \gg B_\theta$.

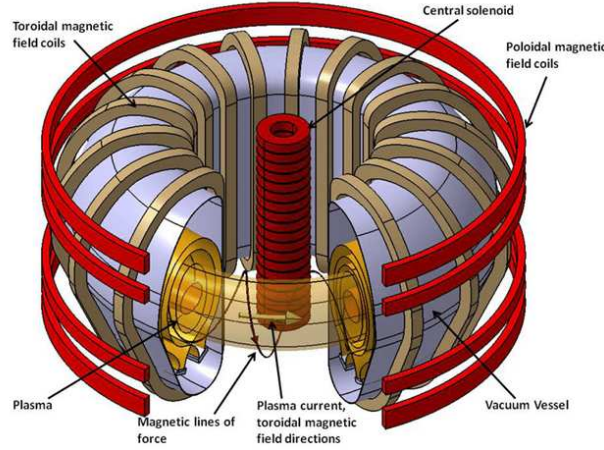


Figure 5: Schematic representation of the tokamak configuration. Taken from Ref. 25.

Tokamaks represent the leading research path of the current European roadmap, lead by the EUROfusion consortium³. The ITER (International Thermonuclear Experimental Reactor) project (currently under construction in Cadarache, France) is conceived as the next milestone in fusion science, and aims at demonstrating the scientific viability of fusion power. ITER is based on the tokamak configuration and it is expected to begin operations by the end of the decade, achieving full performance in the 2040s [26]. The goal of the tokamak community by the end of the century is to build DEMO (DEMONstration Power Plant) [27, 28], aimed at demonstrating the technological feasibility of fusion energy by delivering electricity to the grid, thus paving the way of large-scale exploitation of fusion power.

Reversed Field Pinch (RFP)

An alternative to the tokamak is provided by the Reversed-Field-Pinch (RFP) [29]. Tokamaks and RFPs share several properties, such as the presence of both toroidal and poloidal magnetic fields, each generated by dedicated set of magnetic coils. However, the mechanisms sustaining the discharge differ profoundly, mainly due to different order of magnitudes of magnetic field components. In fact, in the RFP: $B_\varphi \approx B_\theta$, contrary to the Tokamak.

A comparison between the two configurations is shown in Fig. 6. The way RFP sustains high magnetic fields in both directions is the main consequence of having a large plasma current. This feature implies both advantages and drawbacks. On the one hand, the design of RFP is simplified because additional heating systems and superconducting magnetic field coils are not necessary. On the other hand, the RFP is prone to numerous current-driven instabilities, whose nonlinear interactions often lead to magnetic chaos and degraded confinement. The largest RFP device is RFX-mod, to which this thesis

³The EUROfusion Research Roadmap to the Realisation of Fusion Energy is available on: <https://eurofusion.org/eurofusion/roadmap/>

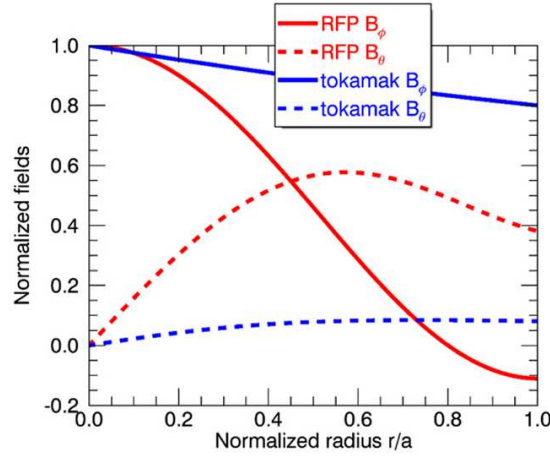


Figure 6: Comparison between tokamak and RFP magnetic fields. Taken from Ref. 29.

is devoted, several key aspects about RFP physics will be discussed in deep in the next chapter.

Stellarators

According to the EUROfusion roadmap, stellarators [30] represent the alternative configuration to be considered for high-performance future reactors. Devised in the 1950s in the USA [31], stellarators are characterized by the absence of a significant plasma current and by the presence of a single elaborately designed magnetic field coil set that directly generates the helical (*rotational* is preferred in stellarator community) transform, see Fig. 7. On one hand, the absence of a strong plasma current makes the stellarator intrinsically capable of steady state operation; however, the experimental design becomes considerably more complex, due to the coil system design and the extensive use of additional heating systems required to compensate for the lack of ohmic heating.

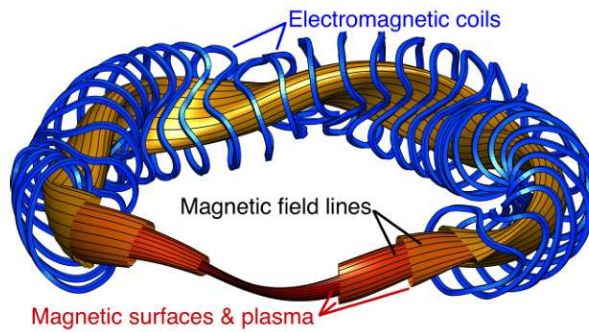


Figure 7: Schematic representation of the stellarator configuration. Taken from Ref. 25.

Wendelstein 7-X (W7-X) [32] is currently the world largest and the most advanced stellarator. It is located in Greifswald, Germany. Its superconducting magnet system consists of two types of coils: non-planar and planar (tilted). The non-planar coils are responsible for generating the rotational transform [33], whereas the planar coils are employed to adjust the toroidal magnetic field, enabling the exploration of a variety of operational

regimes [34, 35]. W7-X is built with a five-field-period geometry: each period contains 10 non-planar coils and 4 planar coils. Within each field period, the coils are arranged in two up-down symmetric half-periods. This highly ordered structure is responsible for the strong *stellarator symmetry* that characterizes W7-X.

This work

RFX-mod (short for modified Reversed Field eXperiment) is a medium-sized device that was operated at Consorzio RFX, in Padova (Italy) between 2004 and 2015 [36, 37]. The device primarily operates in the RFP configuration for plasma confinement [29]. The RFP is characterized by a wide spectrum of tearing modes that nonlinearly interact with each other, exchanging magnetic energy, growing in amplitude and progressively aligning their phase. This dynamics is intrinsically self-sustained and is referred to as *plasma self-organization* [38]. It manifests itself through quasi-periodic transitions from an ordered state, known as the *Quasi-Single Helicity* (QSH) regime to a more chaotic *Multiple Helicity* (MH) regime. During these transitions, the plasma relaxes toward energetically more favorable configurations and undergoes a reorganization of its magnetic topology, a process commonly denoted as *magnetic reconnection* or *crash* [39].

Experiments performed on the Madison Symmetric Torus (MST), a twin device of RFX sited in Madison (Wisconsin, USA) [40], have demonstrated that magnetic reconnection events accelerate charged particles [41, 42]. Similarly to MST, in the RFX-mod plasmas, significant ion heating has been observed to occur during magnetic reconnection events by means of the NPA diagnostics and corroborated by the simultaneous enhancement of the D-D fusion neutron flux [39]. This acceleration was explained as due to strong electric fields, both parallel, $\vec{E}_{\parallel}$ and perpendicular $\vec{E}_{\perp}$ to the magnetic field [43]. In particular, experimental observations in MST suggest that $\vec{E}_{\perp}$ is more important [41, 42], whereas theoretical models based on kinetic Alfvén wave propagation emphasize the importance of $\vec{E}_{\parallel}$ [44]. However, the 3D topology of these electric fields has not been completely validated against measurements.

Particle energization during magnetic reconnection is also fundamental in space plasma physics. For example, ion acceleration has been observed in reconnection exhaust regions using data from the Solar Orbiter mission [45]. Despite this, while space plasma theory provides powerful tools for modeling large-scale global phenomena, it typically lacks the detailed experimental description and precise reconstruction of magnetic configurations that can only be achieved in controlled laboratory experiments [46].

The topic of this thesis is not limited to the RFP configuration. In tokamaks, externally applied *Resonant Magnetic Perturbations*, RMPs [47]) are routinely employed to mitigate the impact of violent instabilities known as *Edge Localised Modes* (ELMs) [48], which arise from steep pressure gradients at the plasma edge [49]. Large-scale devices such as ITER can tolerate only a limited number of such events. Moreover, in medium-sized tokamaks, the nonlinear coupling of internal modes [50, 51] is recognized as a primary cause of stochasticity [52] and weakening of particle confinement prior to disruptions [53, 54]. Therefore, understanding the mechanism of magnetic perturbation interactions driving particle transport is also in perspective of ITER [55]. Finally, the connection between the present work and stellarator physics will be discussed later in the thesis, in particular in Section 4.3.

Within this broader context, the objective of this thesis is to characterize ion acceleration during magnetic reconnection events, both numerically: by developing a modeling

framework using the Hamiltonian, guiding center code ORBIT [56]; and experimentally: through direct measurements of the ion energy distribution function obtained with the Neutral Particle Analyzer (NPA) diagnostic [57, 58]. Special attention is devoted to a specific reconnection event observed during discharge #29324 at $t = 218$ ms, which resulted in a Plasma-Wall Interaction experimentally detected by a fast CCD camera [59]. The primary aim of this work is therefore to discern the magnetic and electric field topology during this event.

The thesis is organized into six chapters. Chapter 1 introduces the RFP magnetic configuration. Chapter 2 describes the RFX-mod setup and details the mathematical core of this thesis: the ORBIT framework. In Chapter 3, the selected case study is analyzed by reconstructing the topology during the magnetic reconnection event. Chapter 4 investigates the impact of different topological structures through simulations in ORBIT of a realistic Maxwellian ion energy distribution. In Chapter 5, the contribution of the electric field to ion acceleration is implemented in ORBIT.

Chapter 1

The Reversed Field Pinch (RFP) configuration

The present chapter introduces the traditional description of toroidal, magnetically confined plasmas, with particular emphasis on the Reversed Field Pinch (RFP) configuration. We first show that, at equilibrium, magnetic field lines are organized into a nested set of toroidal magnetic surfaces as a direct consequence of the force-balance equation. We then extend the discussion to perturbed plasma states, focusing in particular on resonant instabilities, which are localized on magnetic surfaces characterized by rational values of the safety factor. RFP plasmas are resistive and exhibit a monotonically decreasing safety factor profile; as a result, a large number of *tearing modes* can naturally resonate within the plasma, leading to the formation of magnetic islands, where magnetic reconnection events take place. Magnetic reconnection is of particular interest because it drives plasma relaxation [60], which manifests as the transition (also referred to as *crash* [39]), from an ordered Quasi-Single Helicity (QSH) state to a more chaotic Multiple Helicity (MH) state.

1.1 MHD model

Within the framework of *Magneto-Hydro-Dynamics* (MHD), a plasma is described as a fluid composed of charged particles, commonly referred to as a *magnetofluid* [61]. Accordingly, MHD provides a self-consistent dynamical theory of plasmas by combining the conservation equations for neutral fluids with Maxwell equations (in the non-relativistic limit) for the electric field $\vec{E}$ and the magnetic field $\vec{B}$:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0 ; \quad (1.1)$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{1}{nm} \nabla \left(p + \frac{B^2}{2\mu_0} \right) + \frac{1}{nm \mu_0} \nabla \left(\frac{B^2}{2} \right) + \nu \nabla^2 \vec{v} ; \quad (1.2)$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \lambda \nabla^2 \vec{B} . \quad (1.3)$$

The first equation listed above is the *continuity equation*, the second is the *Navier-Stokes equation*, the third is the *induction equation*. Together, they govern the local evolution of the most relevant plasma quantities, namely the particle density n , the velocity field $\vec{v}$, and the magnetic field $\vec{B}$. Here, m is the plasma particle mass, ν is the plasma viscosity. $\lambda = \frac{\eta}{\mu_0}$, where η is the plasma resistivity and μ_0 is the magnetic permeability, is known

as the *magnetic diffusivity*. The simplified case in which resistivity and viscosity are neglected corresponds to *ideal MHD*. This approximation is a good first-order description, since plasma is a very good electrical conductor.

1.2 Plasma static equilibrium

By assuming a static equilibrium, all velocity-dependent terms in the Navier-Stokes equation (1.2) vanish. Under this condition, one obtains the following *force-balance equation*:

$$\nabla p = \vec{j} \times \vec{B} . \quad (1.4)$$

From Eq. (1.4), it follows that both the magnetic field $\vec{B}$ and the current density $\vec{j}$ are perpendicular to the pressure gradient $\vec{\nabla}p$:

- $\vec{B} \cdot \vec{\nabla}p = 0$ implies that the pressure is constant along magnetic field lines and therefore on magnetic flux surfaces;
- $\vec{j} \cdot \vec{\nabla}p = 0$ implies that the pressure is constant along the current density lines, which likewise lie on magnetic surfaces.

If we assume that equilibrium is axisymmetric, we then conclude that the magnetic field will arrange on nested toroidal magnetic surfaces, as schematically illustrated in Fig. 1.1.



Figure 1.1: Magnetic field of a static plasma equilibrium is arranged on a set of nested toroidal magnetic surfaces.

To gain further insight into the physical meaning of the individual terms in Eq. (1.4), consider a cylindrical approximation, i.e. assume to *straighten* the torus into a cylindrical shape¹. By combining this approximation with the Maxwell equations, one obtains:

$$\frac{\partial}{\partial r} \left(p + \frac{B_\theta^2 + B_z^2}{2\mu_0} \right) + \frac{B_\theta^2}{\mu_0 r} = 0 . \quad (1.5)$$

This equation describes the balance between kinetic pressure, magnetic pressure, and field line curvature for a cylindrical magnetofluid in static equilibrium: the kinetic pressure and the magnetic pressure tend to push the magnetic field lines outward, whereas the tension associated with the magnetic curvature counteract this tendency by opposing field-line bending and stretching.

Since magnetic confinement depends on the geometry of the magnetic field lines, a convenient quantity to characterize confinement properties of the magnetic configuration is the ratio between the average kinetic pressure and the magnetic pressure. This parameter defines the so-called "*beta*" *parameter*:

$$\beta = \frac{2\mu_0 \langle p \rangle}{B^2} , \quad (1.6)$$

where the average is taken over the poloidal surface of the plasma.

¹Passing from a toroidal to a cylindrical system of coordinates, the toroidal direction $\hat{\phi}$ turns into the axial one $\hat{z}$.

1.2.1 Grad-Shafranov equation in cylindrical coordinates

Here we adopt a cylindrical coordinate system: (R, φ, Z) , as schematically illustrated in Fig. 1.2, where $R = 0$ corresponds to the major axis of the torus. We further assume axisymmetry, i.e. $\frac{\partial}{\partial \varphi} = 0$. We introduce a scalar function ψ_p such that the radial and vertical components of the magnetic field satisfy the relations:

$$B_R = -\frac{1}{R} \frac{\partial \psi_p}{\partial Z} \quad ; \quad B_Z = \frac{1}{R} \frac{\partial \psi_p}{\partial R} . \quad (1.7)$$

The function ψ_p ensures that $\vec{\nabla} \cdot \vec{B} = 0$:

$$\begin{aligned} \vec{\nabla} \cdot \vec{B} &= \frac{1}{R} \frac{\partial}{\partial R} (R B_R) + \frac{1}{R} \frac{\partial B_\varphi}{\partial \varphi} + \frac{\partial B_Z}{\partial Z} \\ &= \frac{1}{R} \frac{\partial}{\partial R} \left[R \left(-\frac{1}{R} \frac{\partial \psi_p}{\partial Z} \right) \right] + \frac{\partial}{\partial Z} \left(\frac{1}{R} \frac{\partial \psi_p}{\partial R} \right) \\ &= -\frac{1}{R} \frac{\partial^2 \psi_p}{\partial R \partial Z} + \frac{1}{R} \frac{\partial^2 \psi_p}{\partial Z \partial R} = 0 , \end{aligned}$$

where the last step is justified by the fact that $\psi_p \in \mathcal{C}_2$. Moreover, ψ_p is constant along the magnetic field lines, or viceversa, magnetic flux surfaces correspond to the contour lines of the function ψ_p :

$$\begin{aligned} \vec{B} \cdot \vec{\nabla} \psi_p &= B_R \frac{\partial \psi_p}{\partial R} + B_\varphi \frac{\partial \psi_p}{\partial \varphi} + B_Z \frac{\partial \psi_p}{\partial Z} \\ &= -\frac{1}{R} \frac{\partial \psi_p}{\partial Z} \frac{\partial \psi_p}{\partial R} + \frac{1}{R} \frac{\partial \psi_p}{\partial R} \frac{\partial \psi_p}{\partial Z} = 0 . \end{aligned}$$

ψ_p measures the *magnetic field poloidal flux*. Indeed, by calculating the magnetic flux through a toroidal washer-shaped surface lying in the plane $Z = 0$ and radially bounded between R_a and R_b (see the gold-highlighted surface in Fig. 1.2), one obtains:

$$\begin{aligned} \int_0^{2\pi} d\varphi \int_{R_a}^{R_b} dR \, R \cdot B_z(R, Z=0) &= \int_0^{2\pi} d\varphi \int_{R_a}^{R_b} dR \, R \left(\frac{1}{R} \frac{\partial \psi_p}{\partial R} \right) \\ &= 2\pi [\psi_p(R_b) - \psi_p(R_a)] = 2\pi \cdot \psi_p(R_b) , \end{aligned}$$

where we conventionally choose $\psi_p(R_a) = 0$.

Similarly to what was already done for the magnetic field, one can introduce a scalar function f such that:

$$j_R = -\frac{1}{R} \frac{\partial f}{\partial Z} \quad ; \quad j_Z = \frac{1}{R} \frac{\partial f}{\partial R} . \quad (1.8)$$

By substituting the expressions for B_R , B_Z , j_R , and j_Z into the Ampère law, written in the toroidal and radial directions, the following relations are obtained:

$$\begin{aligned} \mu_0 j_\varphi &= \frac{\partial B_R}{\partial Z} - \frac{\partial B_Z}{\partial R} = -\frac{1}{R} \frac{\partial^2 \psi_p}{\partial Z^2} - \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \psi_p}{\partial R} \right) \\ &= -\frac{1}{R} \left[R \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \psi_p}{\partial R} \right) + \frac{\partial^2 \psi_p}{\partial Z^2} \right] = -\frac{1}{R} \Delta^* \psi_p , \end{aligned}$$

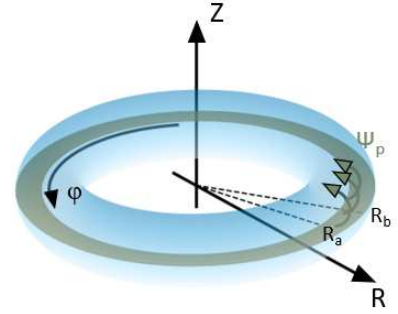


Figure 1.2: Cylindrical coordinate system (R, φ, Z) and reference surface (highlighted in gold) used for the calculation of the poloidal magnetic flux of the poloidal flux.

where, for brevity, the differential operator Δ^* has been introduced,

$$\mu_0 j_R = -\frac{\partial B_\varphi}{\partial Z} = \frac{\mu_0}{R} \frac{\partial f}{\partial Z} .$$

It then follows that:

$$\mu_0 f = F = R B_\varphi , \quad (1.9)$$

where the function f has been rescaled into a new function: F .

The physical meaning of F becomes evident by evaluating the poloidal current as the flux of the current density across the same washer-shaped surface previously considered in the $Z = 0$ plane:

$$I_p = \int_0^{2\pi} d\varphi \int_{R_a}^{R_b} dR \, R \cdot j_Z(R, Z=0) = 2\pi \int_{R_a}^{R_b} dR \, R \left(\frac{1}{R} \cdot \frac{1}{\mu_0} \frac{\partial F}{\partial R} \right) = \frac{2\pi}{\mu_0} F(R) .$$

As already discussed in the previous section, the quantities F , ψ_p and the pressure p are all constant on magnetic surfaces. Therefore, one can write $F \equiv F(\psi_p)$ and $p \equiv p(\psi_p)$. The force-balance equation (1.4) can be thus recast as:

$$\begin{aligned} \vec{\nabla} p &= \frac{dp}{d\psi_p} \vec{\nabla} \psi_p = \vec{j} \times \vec{B} \\ &= \left[j_\varphi \cdot \hat{e}_\varphi + \frac{1}{R\mu_0} \left(\vec{\nabla} F \times \hat{e}_\varphi \right) \right] \times \left[B_\varphi \cdot \hat{e}_\varphi + \frac{1}{R} \left(\vec{\nabla} \psi_p \times \hat{e}_\varphi \right) \right] \\ &= j_\varphi \cdot \hat{e}_\varphi \times \frac{1}{R} \left(\vec{\nabla} \psi_p \times \hat{e}_\varphi \right) + \frac{1}{R\mu_0} \left(\vec{\nabla} F \times \hat{e}_\varphi \right) \times B_\varphi \cdot \hat{e}_\varphi \\ &= \frac{j_\varphi}{R} \vec{\nabla} \psi_p - \frac{B_\varphi}{R\mu_0} \frac{dF}{d\psi_p} \vec{\nabla} \psi_p , \end{aligned}$$

where the following expressions for the current density $\vec{j}$ and the magnetic field $\vec{B}$ have been used:

$$\begin{aligned} \vec{j} &= j_\varphi \cdot \hat{e}_\varphi + \vec{j}_\theta = j_\varphi \cdot \hat{e}_\varphi + \frac{1}{R} \left(\vec{\nabla} F \times \hat{e}_\varphi \right) ; \\ \vec{B} &= B_\varphi \cdot \hat{e}_\varphi + \vec{B}_\theta = B_\varphi \cdot \hat{e}_\varphi + \frac{1}{R} \left(\vec{\nabla} \psi_p \times \hat{e}_\varphi \right) , \end{aligned}$$

where the pedice θ denotes the poloidal component. Finally, recalling that $j_\varphi = -\frac{1}{R\mu_0} \Delta^* \psi_p$ and $B_\varphi = \frac{F}{R}$, one obtains the *Grad-Shafranov equation* [62]:

$$\Delta^* \psi_p + \mu_0 R^2 p' + \frac{1}{2} (F^2)' = 0 , \quad (1.10)$$

where the prime symbol $'$ indicates the derivative with respect to ψ_p . The Grad-Shafranov equation is a nonlinear differential equation whose solutions provide the geometry of magnetic flux surfaces. Together with Maxwell equations for the magnetic field, it forms a closed set describing plasma static equilibrium.

1.2.2 The safety factor

As previously discussed, in toroidal devices magnetic field lines undergo a helical transform due to the superposition of toroidal and poloidal magnetic field components. The helical geometry of field lines is commonly described in terms of the so-called *safety factor*,

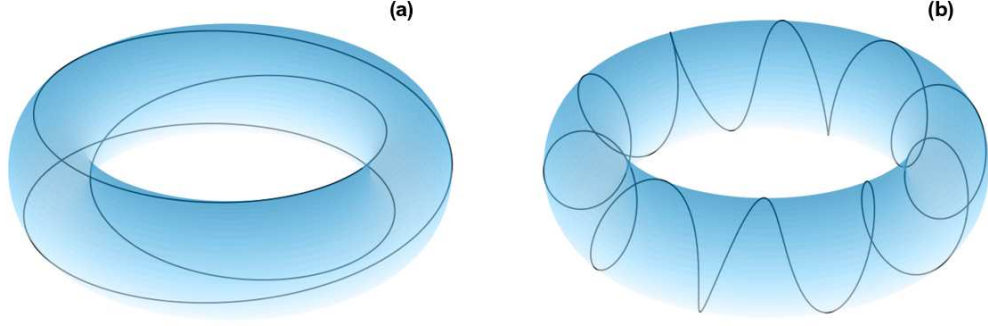


Figure 1.3: Examples of magnetic field lines characterized by different safety factors: **(a)** typical tokamak configuration with $q = 3/1$; **(b)** typical RFP configuration with $q = 1/10$.

q , which is defined as the ratio between the number of toroidal turns and the number of poloidal turns completed by a field line:

$$q = \frac{\Delta\varphi}{2\pi} , \quad (1.11)$$

where $\Delta\varphi$ represents the portion of toroidal angle run by the magnetic field line during a complete 2π poloidal turn. Examples of magnetic field lines associated with different q are shown in Fig. 1.3, including cases with $q > 1$ or $q < 1$. Field lines sharing the same value of q follow identical helical trajectories and therefore belong to the same magnetic surface. The radial profile of the safety factor, $q = q(r)$, can be calculated in cylindrical approximation as:

$$q(r) = \frac{1}{2\pi} \oint d\varphi = \frac{1}{2\pi} \frac{r}{R_0} \frac{B_\varphi}{B_\theta} \oint d\theta = \frac{r}{R_0} \frac{B_\varphi(r)}{B_\theta(r)} , \quad (1.12)$$

where R_0 is the major radius of the torus. In deriving this expression, we used the *field-line equation* [63]:

$$\frac{R_0}{r} \frac{d\varphi}{d\theta} = \frac{B_\varphi}{B_\theta} \quad \longrightarrow \quad d\varphi = \frac{r}{R_0} \frac{d\theta}{B_\theta} B_\varphi . \quad (1.13)$$

From Eq. (1.12), it follows that $q \propto \frac{B_\varphi}{B_\theta}$. Consequently, tokamaks are characterized by an increasing safety factor profile with $q > 1$, and magnetic field lines have the geometry shown in Fig. 1.3(a). In contrast, the RFP configuration exhibits a decreasing safety factor $q < 1$, resulting in the magnetic field line geometry shown in Fig. 1.3(b).

1.3 MHD resonant instabilities

For a plasma in the equilibrium, the two terms appearing in Eq. (1.4) are perfectly balanced. Whenever this balance is violated, the plasma is driven out of the equilibrium and therefore is *perturbed*.

Perturbations may be either stable or unstable. Within the MHD framework, instabilities are commonly classified according to which term in Eq. (1.4) dominates. One distinguishes *pressure-driven* instabilities, when the pressure force term ∇p dominates, and *current-driven* instabilities, when current gradients are the dominant destabilizing mechanism.

In a uniform plasma, perturbations have a wave-like spatial structure. Therefore, they can be decomposed in the Fourier space. In particular, a local magnetic field perturbation $\delta B_x(\vec{r})$ along a generic direction $x = r, \theta$, or φ , can be expressed as a Fourier series as follows:

$$\delta B_x(\vec{r}) = \sum_{\vec{k}} b_{\vec{k}}^x(\vec{r}) e^{i(\vec{k} \cdot \vec{r} + \omega t)} = \sum_{m,n} b_{m,n}^x(\vec{r}) e^{i(m\theta - n\varphi + \phi_{m,n}(t))}, \quad (1.14)$$

where $\vec{k} = (k_r, k_\theta, k_\varphi) = (k_r, m/r, n/R)$ is the wave vector written in toroidal coordinates, and $\vec{r}$ is the position vector in the real space. The integers m and n are respectively referred to as *poloidal mode number* and *toroidal mode number*. As a consequence, the couple (m, n) identifies uniquely a single *mode*.

In the case of a *normal mode*, it can be described by a single plane wave characterized by a given wave vector $\vec{k}$ and a complex frequency $\omega = \omega_R + i\omega_I$. The real part ω_R determines the propagation velocity, while the imaginary part ω_I describes the growth ($\omega_I < 0$) or the attenuation ($\omega_I > 0$) of its amplitude during the time. Time dependence can be resumed in the Fourier phase $\phi_{m,n}$.

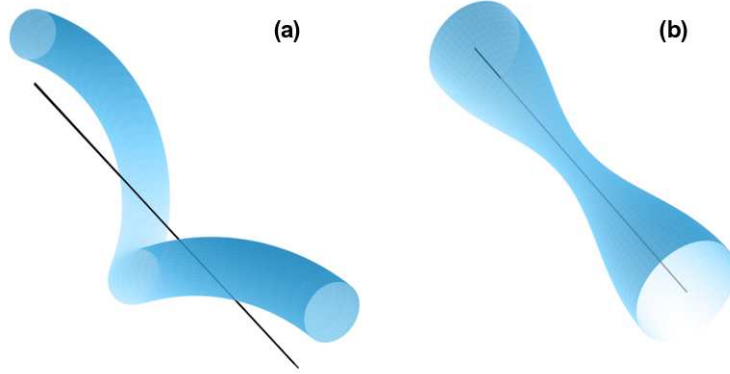


Figure 1.4: Schematic representation of ideal MHD modes acting on plasma column: (a) kink mode and (b) sausage mode. Adapted from Ref. 64.

Considering stationary conditions (namely neglecting time dependence), the perturbation described in Eq. (1.14) reaches its maximum when:

$$m\theta - n\varphi = 0, \quad (1.15)$$

which implies that a perturbed magnetic field line completes a poloidal turn while rotating toroidally by the quantity:

$$\Delta\varphi = \frac{m}{n} \cdot 2\pi \quad \Rightarrow \quad d_\varphi = R_0 \cdot \Delta\varphi = \frac{m}{n} \cdot 2\pi R_0. \quad (1.16)$$

By comparing Eq. (1.16) with the definition of the safety factor (1.11), one readily obtains:

$$q = \frac{m}{n}. \quad (1.17)$$

This result shows that the most dangerous helical perturbations are those occurring at *rational* values of the safety factor, and are therefore referred to as *resonant modes*.

Plasma perturbations can also be classified as *ideal* or *resistive*, depending on whether the plasma resistivity plays a role for their growth time. In the ideal MHD limit, where

$\eta = 0$, it can be demonstrated that magnetic topology is preserved [61] (see next Section). Typical ideal MHD perturbations are the *kink* modes, which correspond to a global bending of the plasma column (see Fig. 1.4(a)), and the *sausage* modes, associated with local restriction of the plasma column (see Fig. 1.4(b)). These modes are non-resonant, and typically stable in RFP plasmas, so they are not addressed in the present thesis.

On the contrary, a resistive plasma with $\eta \neq 0$ is prone to strong current-driven perturbations in the form of *tearing modes*. In this case, resistivity allows changes in the magnetic topology: magnetic field lines reconnect and form coherent structures known as *magnetic islands*. Tearing modes are resonant, and the number of magnetic islands generated by a given mode along the poloidal and toroidal directions, corresponds to its poloidal and toroidal numbers, m and n , respectively. Magnetic islands associated with the (m, n) mode are aligned along the magnetic surface satisfying $q = m/n$, as schematically shown in Fig. 1.5.

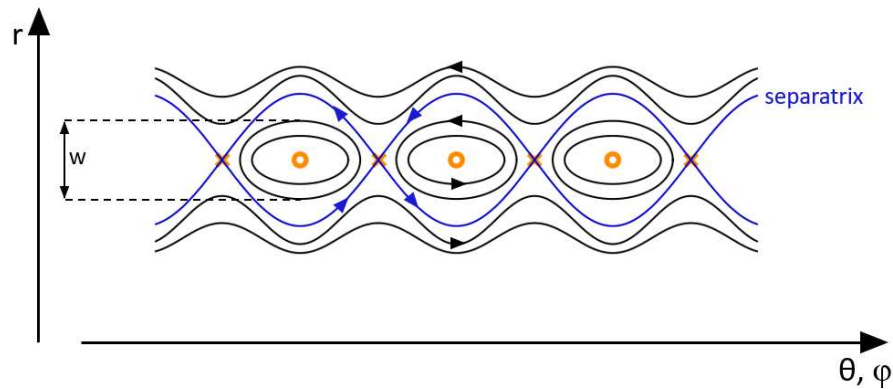


Figure 1.5: Schematic representation of magnetic reconnection leading to the formation of magnetic islands. Arrows indicate the direction of the magnetic field. X-points and O-points are highlighted in orange.

Each magnetic island is characterized by an *O-point*, around which magnetic field lines rotate, and an *X-point* where magnetic reconnection and topological rearrangement take place. The *separatrix* connects X-points and define island volumes against the surrounding plasma. The width of a magnetic island associated with an (m, n) tearing mode, denoted by $w_{m,n}$, is defined as the radial distance between the extremities of its own separatrix [65, 66]. It scales as:

$$w \propto \sqrt{b_{m,n}^r(r_{m,n})} , \quad (1.18)$$

where $b_{m,n}^r$ is the radial amplitude of the mode, $r_{m,n}$ is the radial position of the mode resonance.

Since we focus on tearing modes, unless otherwise specified, the terms *tearing mode*, *mode*, and *perturbation* will be used as synonyms in the remainder of the thesis.

1.4 Modeling of RFP configuration

1.4.1 Magnetic helicity and topology

A very explicative physical description of magnetic topology was introduced by Moffatt in Ref. 67. Let us consider a volume V such that all magnetic field lines are entirely contained within it, with no field line crossing its boundary. Formally, this condition means that the normal component of the magnetic field $B_n = 0$ at the boundary surface. The *magnetic helicity* H in that volume is then defined as:

$$H = \int_V \vec{A} \cdot \vec{B} \, dV, \quad (1.19)$$

where $\vec{A}$ is the magnetic vector potential.

Now, consider two interconnected magnetic flux tubes, labeled C_1 and C_2 , arranged as shown in Fig. 1.6. The quantities Φ_1 and Φ_2 denote the magnetic fluxes through the cross sections of the two tubes, whose volumes are V_1 and V_2 , respectively. The total volume is therefore $V = V_1 + V_2$ which still satisfies the boundary condition: $B_n = 0$.

The total magnetic helicity of the system can be then calculated as the sum of the helicities associated with the two flux tubes:

$$H = H_1 + H_2. \quad (1.20)$$

Focusing on the contribution from the first tube, one obtains:

$$H_1 = \int_{V_1} \vec{A} \cdot \vec{B}_1 \, dV = \Phi_1 \oint_{C_1} \vec{A} \cdot d\vec{x}_1 = \Phi_1 \int_{\Sigma_1} (\nabla \times \vec{A}) \cdot d\vec{S} = \Phi_1 \int_{\Sigma_1} \vec{B}_2 \cdot d\vec{S} = \Phi_1 \Phi_2, \quad (1.21)$$

where, in the second step, we used the Stokes theorem to transform the line integral along the circuit C_1 into a surface integral over the surface Σ_1 . An entirely analogous calculation holds for the contribution H_2 associated with the second tube. As a result, the total helicity reads:

$$H = 2\Phi_1 \Phi_2. \quad (1.22)$$

It is worth noting that the magnetic helicity calculated in this way depends exclusively on the mutual interconnection between the two fluxes Φ_1 and Φ_2 . If one of the tubes were cut and removed, thereby breaking the linkage between C_1 and C_2 , the magnetic helicity of the resulting configuration would vanish. As a consequence, *as long as the magnetic topology remains unchanged, magnetic helicity is conserved and constitutes an invariant of the magnetic configuration.*

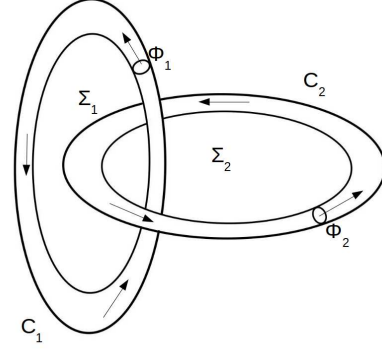


Figure 1.6: Schematic representation of two interconnected magnetic flux tubes.

1.4.2 Woltjer theorems

Other two relevant results have been demonstrated by Woltjer in Ref. 68.

First Woltjer theorem. *The magnetic topology is preserved for a magnetofluid with zero resistivity.*

Proof. Since the magnetic helicity is directly related to the topology, one expects that the magnetic helicity also would be invariant for an ideal magnetofluid. Therefore, to prove the theorem one must verify that:

$$\frac{dH}{dt} = 0 . \quad (1.23)$$

Using the definition of magnetic helicity in Eq. (1.19):

$$\frac{dH}{dt} = \int_V \frac{\partial \vec{A}}{\partial t} \cdot \vec{B} \, dV + \int_V \vec{A} \cdot \frac{\partial \vec{B}}{\partial t} \, dV . \quad (1.24)$$

Let us recall the induction equation (1.3) for an ideal magnetofluid:

$$\nabla \times (\vec{v} \times \vec{B}) = \frac{\partial \vec{B}}{\partial t} = \frac{\partial}{\partial t} (\nabla \times \vec{A}) = \nabla \times \frac{\partial \vec{A}}{\partial t} \longrightarrow \frac{\partial \vec{A}}{\partial t} = \vec{v} \times \vec{B} .$$

By substituting inside the Eq. (1.24), it becomes:

$$\frac{dH}{dt} = \int_V \cancel{(\vec{v} \times \vec{B}) \cdot \vec{B}} \, dV + \int_V \vec{A} \cdot [\nabla \times (\vec{v} \times \vec{B})] \, dV .$$

The first term is canceled because two vectors that are perpendicular each other are scalar multiplied. Instead, the second term can be written in this way:

$$\frac{dH}{dt} = \int_V \cancel{(\vec{v} \times \vec{B}) \cdot \vec{B}} \, dV - \int_V \nabla \cdot [\vec{A} \times (\vec{v} \times \vec{B})] \, dV .$$

The first term can be neglected for the same reason as before. Whereas, the second term can be managed using the Gauss divergence theorem:

$$\frac{dH}{dt} = - \int_S \vec{A} \times (\vec{v} \times \vec{B}) \, dS .$$

By taking into account that $\vec{v}$ and $\vec{B}$ are perpendicular to each other, one finally has:

$$\frac{dH}{dt} = \int_S (\vec{A} \cdot \vec{v}) \cdot \vec{B} \, dS = 0 , \quad (1.25)$$

which is null since perfect boundary conditions ($B_n = 0$) have been assumed at the beginning. □

The second Woltjer result involves the *magnetic energy* (of a charged particles system), which is defined as follows:

$$W = \int_V \frac{B^2}{2\mu_0} \, dV . \quad (1.26)$$

Second Woltjer theorem. *An ideal magnetofluid reaches the minimum of its magnetic energy in the configuration that satisfies:*

$$\nabla \times \vec{B} = \mu \vec{B} . \quad (1.27)$$

Proof. Since, for an ideal magnetofluid, the magnetic helicity is an invariant (as the first Woltjer theorem states), one way to calculate the minimum of the magnetic energy is to employ the method of Lagrange multipliers, namely to impose:

$$\delta W - \frac{\mu}{2\mu_0} \delta \mathcal{H} = 0 , \quad (1.28)$$

where μ is formally a Lagrange multiplier. The ' δ ' symbol represents the differential operator. By substituting in the equation above the expressions for the magnetic energy (1.26), and for the magnetic helicity (1.19), after having manipulated the right-hand side of the equation, one gets:

$$\begin{aligned} 0 &= \delta \left[\int_V \frac{B^2}{2\mu_0} dV - \frac{\mu}{2\mu_0} \int_V \vec{A} \cdot \vec{B} dV \right] \\ &= \frac{1}{\mu_0} \int_V \vec{B} \cdot \delta \vec{B} dV - \frac{\mu}{2\mu_0} \int_V \delta \vec{A} \cdot \vec{B} dV - \frac{\mu}{2\mu_0} \int_V \vec{A} \cdot \delta \vec{B} dV \\ &= \frac{1}{\mu_0} \int_V \vec{B} \cdot \delta \vec{B} dV - \frac{\mu}{\mu_0} \int_V \delta \vec{A} \cdot \vec{B} dV + \frac{\mu}{2\mu_0} \int_V \nabla \cdot (\vec{A} \times \delta \vec{A}) dV \\ &= \frac{1}{\mu_0} \left[\int_V \vec{B} \cdot (\delta \vec{B} - \mu \delta \vec{A}) dV \right] + \frac{\mu}{2\mu_0} \int_V \nabla \cdot (\vec{A} \times \delta \vec{A}) dV , \end{aligned} \quad (1.29)$$

where it has been used the relation: $\delta \vec{B} = \nabla \times \delta \vec{A}$. Namely:

$$\int_V \nabla \cdot (\vec{A} \times \delta \vec{A}) dV = \int_S \vec{A} \times \delta \vec{A} dS = 0 ,$$

and the last term has been neglected because $B_n = 0$. Hence, one has:

$$\delta \vec{B} - \mu \delta \vec{A} = 0 \quad \Rightarrow \quad \vec{B} = \mu \vec{A} .$$

Finally, by applying the curl operator to both sides:

$$\nabla \times \vec{B} = \mu \nabla \times \vec{A} \quad \Rightarrow \quad \nabla \times \vec{B} = \mu \vec{B} . \quad (1.30)$$

□

The process driving plasma to the minimum of the magnetic energy is called *relaxation* [60], and the plasma energy state at its minimum is said *relaxed state*.

The first, immediate consequence of the second Woltjer theorem comes by combining Eq. (1.27) with the Ampère law, thus obtaining:

$$\mu \vec{B} = \mu_0 \vec{j} \quad \longrightarrow \quad \mu = \mu_0 \frac{\vec{j} \cdot \vec{B}}{B^2} . \quad (1.31)$$

Therefore, for an ideal magnetofluid, the magnetic field and the current density are parallel. This means that, according to the force-balance equation (1.4), the pressure gradient must be null:

$$\nabla p = \vec{j} \times \vec{B} = 0 . \quad (1.32)$$

A magnetic field that satisfies Eq. (1.32) is called *force-free field*.

Woltjer theorems applies also for plasma with finite, but sufficiently small, resistivity. This has significant consequences: according to the end of Section 1.3, because of resistivity, tearing modes arise in the plasma, and magnetic field lines can break and reconnect inside the associated islands. Then, the first Woltjer theorem implies that reconnection should sharply change plasma magnetic helicity/topology: this reduces the number of available states during plasma relaxation to those in closer proximity to the starting configuration. Meanwhile, the second Woltjer theorem implies that, through reconnection, the plasma must tend to the most relaxed state among the (topological) available ones. We can finally conclude that, in a slightly resistive plasma, magnetic reconnection is the phenomenon that effectively drives plasma relaxation. RFP experiments, like RFX and RFX-mod later, historically have demonstrated the relevance of the link between reconnection and relaxation. This work aims at exploring with a modern nonlinear approach the relationship between reconnection, magnetic topology and thermal particle dynamics in a magnetically confined plasma.

1.4.3 Toy model of RFP self-organization

As previously mentioned, the RFP magnetic configuration is characterized by a broad spectrum of resonating tearing modes which interact, exchanging magnetic energy with each other. This complex collective dynamics (commonly referred to as *dynamo*) constitutes the main mechanism through which the RFP plasma *relaxes* toward more energetically favorable states². The dynamo occurs intrinsically, as the plasma is a complex system that evolves to states of increased stability. For this reason, the overall process is often described as *plasma self-organization*. The RFX community has historically devoted significant effort to the investigation of this behavior [38, 70–72]. We will focus on some aspects on plasma relaxation in the next chapter. For the moment, let us just introduce a simplified toy model [73] that shows how a perturbed plasma column can naturally evolve toward a state characterized by axial

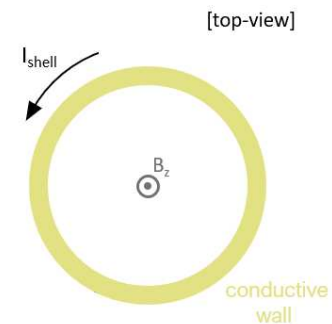


Figure 1.7: Top-view of the cylindrical magnetic configuration considered in the toy model.

Consider the situation depicted in Figs. 1.7-1.8(a). A current I flows along the axis of a straight wire, generating a poloidal magnetic field B_θ . Moreover, an axial magnetic field B_z is imposed by a poloidal current I_{shell} flowing in a conducting shell surrounding the wire.

Now assume that the wire is suddenly perturbed and undergoes a kink deformation, as shown in Fig. 1.8(b). As a consequence of this deformation, the axial magnetic field becomes stronger in the inner region of the column and weaker in the outer region. This unbalance further amplifies the kink deformation, and if the initial axial magnetic field B_z is sufficiently weak, the axial magnetic field in the outer region B'_z may eventually reverse sign.

²A caveat is anyway necessary: the system is not isolated, it acquires energy and then periodically expels the excess energy through relaxation. The behavior is similar to the magnetosphere, where reconnection is stimulated by the energy input of the solar wind [69].

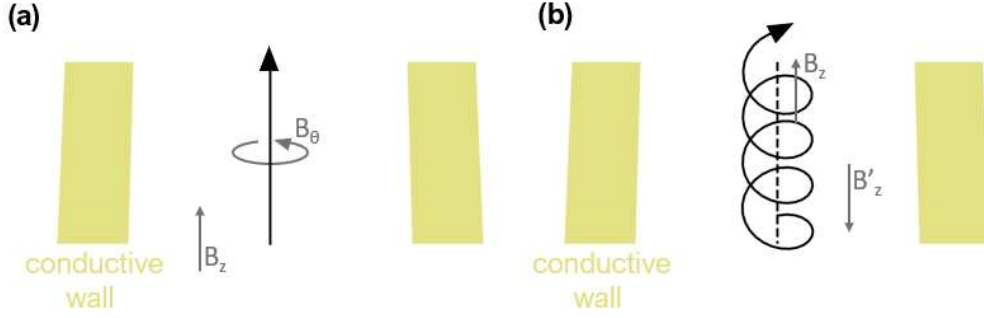


Figure 1.8: Evolution of the wire dynamics described by the wire model: (a) initial unperturbed configuration, (b) perturbed configuration leading to the reversal of the axial magnetic field, B'_z , in the outer region.

Once the reversal reaches a sufficiently high amplitude, the wire can settle into a new equilibrium state, thanks to the stabilizing effect of the conducting shell, which counteracts the variation of magnetic field according to Faraday law. This simple argument shows that the reversal of the axial magnetic field at the plasma edge can be interpreted as a natural outcome of the plasma relaxation toward a lower-energy configuration [38].

1.4.4 The Bessel Function Model

The *Bessel Function Model* (BFM) provides a simplified analytical description of the magnetic configuration of a RFP plasma.

For the sake of simplicity, let us adopt a cylindrical geometry. Assuming the parameter μ to be uniform across the plasma section, the second Woltjer theorem (1.27) applied to the axial and poloidal directions yields:

$$\frac{\partial B_z}{\partial r} = \mu B_\theta \quad ; \quad \frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) = \mu B_z . \quad (1.33)$$

By combining the equations above and introducing the rescaled radial coordinate $r' = \mu r$, one obtains:

$$r' \frac{\partial}{\partial r'} \left(r' \frac{\partial B_z}{\partial r'} \right) + r'^2 B_z = 0 . \quad (1.34)$$

This equation corresponds to the Bessel equation for the axial field component $B_z(r')$, with solution [74]:

$$B_z(r') = B_z(r) = B_0 J_0(\mu r) . \quad (1.35)$$

Here, J_0 denotes the zeroth-order Bessel function, while B_0 is the magnetic field amplitude on axis. Substituting this expression for B_z back into Eqs. (1.33), one also obtains the solution for the poloidal magnetic field component:

$$B_\theta(r') = B_\theta(r) = B_0 J_1(\mu r) , \quad (1.36)$$

where J_1 is the first-order Bessel function.

Fig. 1.9 compares the radial profiles of the Bessel functions J_0 and J_1 with the magnetic field measured during a typical RFX-mod discharge (shot #37537). The comparison shows that the BFM provides a good analytical explanation for the magnetic field in the RFP configuration. In particular, the model predicts the reversal of the toroidal component.

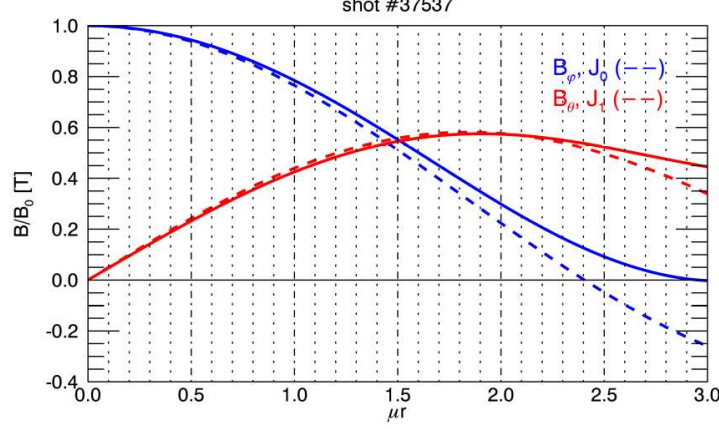


Figure 1.9: According to BFM, the radial profiles of the normalized magnetic field components are described by the first two Bessel functions: J_0 and J_1 . The figure compares J_0 and J_1 profiles with the magnetic configuration realized during RFX-mod discharge #37537.

This means that the same argument we used in this section to describe B_z and B_θ can be extended to toroidal geometries with sufficiently high aspect ratio, such as RFX-mod, for which $A = R_0/a \approx 2 \text{ m}/0.5 \text{ m} = 4$. The residual discrepancies between the analytical Bessel profiles and the experimental magnetic field can be therefore attributed to toroidal effects neglected in the cylindrical approximation. Another source of discrepancy is the resistivity, as we will discuss in the next section.

Two useful parameters commonly used to characterize the RFP magnetic configuration are the reversal parameter F and the pinch parameter Θ , defined as:

$$F = \frac{B_\varphi(a)}{\langle B_\varphi \rangle}, \quad \Theta = \frac{B_\theta(a)}{\langle B_\varphi \rangle}. \quad (1.37)$$

The parameter F quantifies the degree of reversal of the toroidal magnetic field. In the expressions above, $\langle B_\varphi \rangle$ is the toroidal field averaged over the poloidal section. This quantity provides an estimate of the toroidal magnetic flux, since:

$$\langle B_\varphi \rangle = \frac{1}{\pi a^2} \Phi(B_\varphi). \quad (1.38)$$

Considering the cylindrical approximation ($B_\varphi \approx B_z$) for the equation above, one gets:

$$\begin{aligned} \langle B_\varphi \rangle &= \frac{2}{\pi a} \int_0^a B_0 J_0(\mu r) \cdot 2\pi r \, dr \\ &= \frac{2B_0}{a} \int_0^a J_0(\mu r) \cdot r \, dr \\ &= \frac{2}{\mu a} B_0 J_1(\mu a), \end{aligned} \quad (1.39)$$

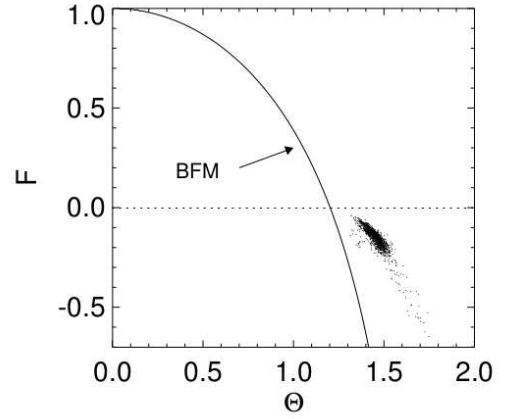


Figure 1.10: Comparison between the $F - \Theta$ curve derived from BFM and experimental measurements obtained in RFX. Adapted from Ref. 75.

where the expression for B_z is taken from Eq. (1.35). By inserting this result into the definition of the pinch parameter Θ (1.37), one finds:

$$2\Theta = \mu a , \quad (1.40)$$

where the expression for B_θ is taken from Eq. (1.36). A similar procedure applied to the reversal parameter yields:

$$F = \Theta \frac{J_0(\mu a)}{J_1(\mu a)} . \quad (1.41)$$

Hence, F and Θ are not independent quantities, but are intimately correlated. The resulting F - Θ curve is shown in Fig. 1.10.

Moreover, Fig. 1.9 shows that the axial magnetic field component associated with J_0 reverses sign when $\mu a > 2.405$, which correspond to $\Theta > 1.2$ according to Eq. (1.40). This result is also consistent with the F - Θ curve.

The safety factor at the plasma boundary can be expressed in terms of the new parameters F and Θ as:

$$q_a = q(a) = \frac{a B_\varphi(a)}{R_0 B_\theta(a)} = \frac{a}{R_0} \frac{F}{\Theta} . \quad (1.42)$$

Furthermore, considering the asymptotic expansion at the first-order of the Bessel functions:

$$\lim_{x \rightarrow 0} J_0(x) = 1 \quad , \quad \lim_{x \rightarrow 0} J_1(x) = \frac{x}{2} , \quad (1.43)$$

and substituting it into the expression for the safety factor on the magnetic axis, one obtains:

$$q_0 = q(0) = \lim_{r \rightarrow 0} \frac{r B_\varphi(r)}{R_0 B_\theta(r)} = \lim_{r \rightarrow 0} \frac{r J_0(\mu r)}{R_0 J_1(\mu r)} = \frac{a}{R_0} \frac{1}{\Theta} = \frac{\varepsilon}{\Theta} , \quad (1.44)$$

where ε denotes the *inverse aspect-ratio*, $\varepsilon = A^{-1}$.

1.4.5 The μ - p model

An additional source of discrepancy between the Bessel-function profiles and the experimentally measured magnetic field in Fig. 1.9 lies in the assumption of force-free field configuration adopted by the BFM. This assumption follows directly from the Woltjer theorems and is strictly valid only for ideal plasmas. In resistive plasmas, however, a finite pressure gradient is still required to guarantee the equilibrium. Historically, a modified BFM was developed to account for a finite edge resistivity: this model is called μ - p model [76].

In this framework, the parameter μ is no longer assumed to be uniform, but is instead treated as a radial function, $\mu \equiv \mu(r)$. In particular, the model assumes a fixed form for the $\mu(r)$ profile:

$$\mu(r) = \mu(0) \left(1 - \frac{r}{a}\right)^\alpha , \quad (1.45)$$

Adopting cylindrical approximation, and combining the force-balance equation (1.4) with the force-free field condition (1.31), one obtains the coupled equations:

$$-\frac{\partial B_z}{\partial r} = \mu_0 j_\theta = \mu B_\theta + \mu_0 \frac{\partial p}{\partial r} \cdot \frac{B_z}{B^2} ; \quad (1.46)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) = \mu_0 j_z = \mu B_z - \mu_0 \frac{\partial p}{\partial r} \cdot \frac{B_\theta}{B^2} . \quad (1.47)$$

These equations can be solved once the radial profiles $\mu \equiv \mu(r)$ and $p \equiv p(r)$ are specified, which motivates the name μ - p model. This model is routinely employed to calculate equilibrium configurations in RFX-mod discharges and can be solved numerically in terms of normalized quantities:

$$\begin{aligned} \tilde{r} &= \frac{r}{a} \quad ; \quad B_z(r) = B_0 \cdot \tilde{B}_z(r) \quad ; \quad B_\theta(r) = B_0 \cdot \tilde{B}_\theta(r) \quad ; \\ p(r) &= p(0) \cdot \tilde{p}(r) = 2n(0)T(0) \cdot \tilde{n}(r)\tilde{T}(r) \quad ; \quad \mu(r) = \mu(0) \cdot \tilde{\mu}(r) = \frac{2\Theta_0}{a} \cdot \tilde{\mu}(r) . \end{aligned}$$

The resulting expressions read:

$$\begin{aligned} \frac{\partial \tilde{B}_z}{\partial \tilde{r}} &= -2\Theta_0 \tilde{\mu} \tilde{B}_\theta - \frac{\beta_0}{2} \frac{\partial}{\partial \tilde{r}} (\tilde{n} \tilde{T}) \cdot \frac{\tilde{B}_z}{\tilde{B}^2} ; \\ \frac{\partial \tilde{B}_\theta}{\partial \tilde{r}} &= -\frac{\tilde{B}_\theta}{\tilde{r}} + 2\Theta_0 \tilde{\mu} \tilde{B}_z - \frac{\beta_0}{2} \frac{\partial}{\partial \tilde{r}} (\tilde{n} \tilde{T}) \cdot \frac{\tilde{B}_\theta}{\tilde{B}^2} . \end{aligned}$$

Here, Θ_0 is referred to as the *on-axis pinch parameter*, while β_0 is the on-axis β parameter. In the limiting case of a force-free field, $p = 0$, and a uniform $\tilde{\mu} = 1$, the model reduces to the BFM with a constant pinch parameter $\Theta \equiv \Theta_0$.

1.5 Phenomenology of magnetic reconnection in RFX-mod

As discussed in Section 1.2.2, in the RFP configuration the safety factor profile $q(r)$ is monotonically decreasing, vanishes at the plasma edge, and becomes slightly negative at the wall. A typical $q(r)$ profile for discharge #29324 at time $t = 218$ ms is shown in Fig. 1.11.

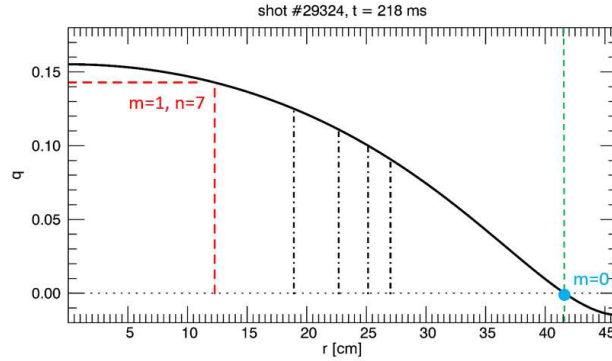


Figure 1.11: Safety factor profile during RFX-mod discharge #29324, at $t = 218$ ms. Dominant (1, 7) mode resonance is indicated by a vertical red dashed line, while resonances of some innermost secondary modes are marked as black dashed lines. The $m = 0$ modes resonate at the reversal surface (green dashed line), where q vanishes (cyan dot).

As a direct consequence of this behavior, a large number of tearing modes can resonate with poloidal mode number $m = 1$ and toroidal mode number $n > n_0$, where the threshold value n_0 is determined by the on-axis safety factor q_0 . In RFX-mod, the safety factor on axis is $q_0 = \varepsilon/\Theta \approx 1/(4 \cdot 1.5) = 1/6 \approx 0.166$, so that the innermost $m = 1$ mode corresponds to $n_0 = 7$, since $q_{1,7} \approx 0.14$. Moreover, the magnetic surface where $q = 0$ is called *reversal* and hosts the resonances of all $m = 0$ tearing modes [77]. In RFX-mod,

the amplitudes of these modes, as defined by Eq. (1.14), are of the order of a few mT, implying the formation of large magnetic islands, since the perturbations amount to a few percent of the on-axis magnetic field.

The spectrum of $m = 1$ modes is characterized by rich dynamics [78] showing a strong interplay between modes. At relatively low plasma current [71, 79] ($I_p \leq 1$ MA) or at high Greenwald fractions [80], the plasma typically is found in the traditional low-confinement regime known as *Multiple Helicity* (MH) [77]. In this state, the mode spectrum is broad [81, 82] and no single mode dominates.

By contrast, at higher plasma currents, a different regime is observed, in which the innermost resonant mode dominates the spectrum. This regime is referred to as the *Quasi-Single Helicity* (QSH) state³. In this case, the $n = 7$ mode is said *dominant*, while modes with $n > 7$ are referred to as *secondary*. The QSH state is associated with improved confinement properties and a more ordered magnetic structure. Initially discovered in RFX [71, 83], QSH states were subsequently observed in several other RFPs, such as MST [84], TPE-RX [85], EXTRAP-T2R [86], and RELAX [87, 88], thereby confuting the traditional view, based on the Taylor relaxation theory [60], that RFP plasmas are inherently dominated by magnetic chaos [89]. A comparison between MH and QSH regimes is presented in Fig. 1.12.

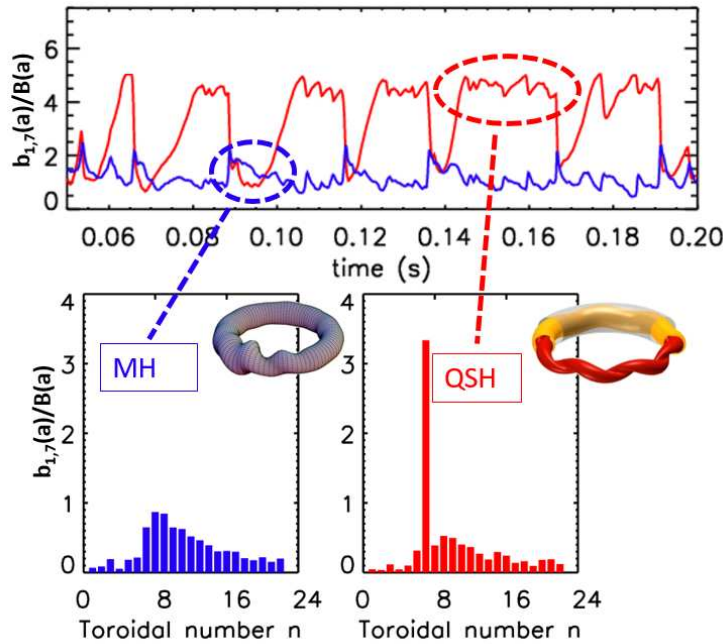


Figure 1.12: The $m = 1$ tearing mode spectrum highlights two distinct confinement regimes. In the *Multiple Helicity* (MH) state, the plasma structure is disordered and characterized by chaos [77]. In the *Quasi-Single Helicity* (QSH) state (red), a single mode (with $n = 7$) dominates the spectrum, resulting in a more ordered structure and improved confinement properties [39]. Figures are adapted from Refs. 90 and 91.

Since the MH state is associated with lower-energy configuration, the plasma tends to relax toward it. As a result, during a single discharge, QSH phases are often intermittently

³In RFX-mod, QSH states are more frequently observed at high plasma current, owing to their favorable scaling with the Hartmann number [72].

interrupted by short relaxation events where MH is temporarily observed [39, 92]. These events are commonly referred to as *crashes*. Interest in crashes is both theoretical, since they are examples of magnetic reconnection events, and practical, since they are associated with a degradation of plasma confinement.

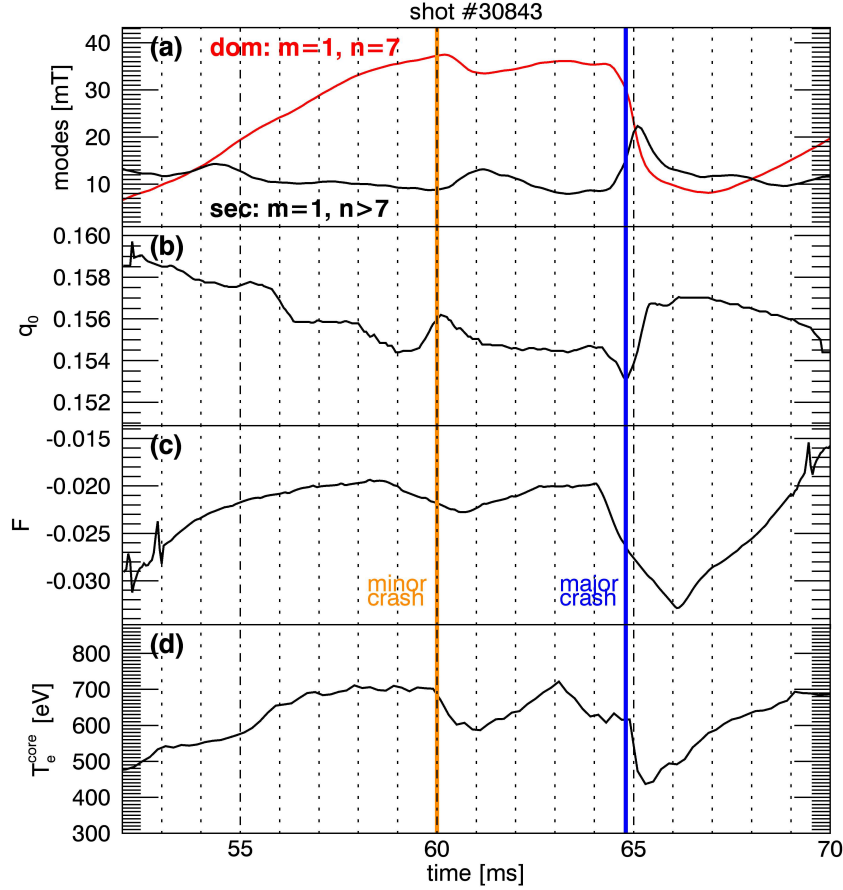


Figure 1.13: QSH-MH cycle observed during RFX-mod discharge #30843. (a) Amplitudes of $m = 1$ modes expressed in terms of the toroidal magnetic perturbation $b_{1,n}^\varphi$: the red curve corresponds to the dominant (1,7) mode, while the black curve represents the root mean square of the secondary mode amplitudes, multiplied by a factor of 2.5 for clarity; (b) on-axis safety factor q_0 , (c) reversal parameter F ; (d) core electron temperature T_e^{core} . The orange vertical line at $t = 60$ ms indicates a minor crash, while the blue line at $t \approx 65$ ms indicates a major crash. Figure adapted from Ref. 39.

The phenomenology of a crash is reported in Fig. 1.13, which shows the evolution of main plasma parameters during discharge #30843: mode amplitudes in panel (a), the on-axis safety factor q_0 in panel (b), the reversal parameter F in panel (c), and the core electron temperature T_e^{core} in panel (d). A crash is characterized by the rapid decrease of the amplitude of the dominant mode, accompanied by the growth of secondary modes. The net effect is the enhanced transport of particles from the plasma core toward the edge. Depending on whether the decrease of the dominant mode amplitude is total or partial, crashes are classified as *major* (highlighted in blue at $t = 65$ ms) or *minor* (highlighted in orange at $t = 60$ ms). One immediate consequence of this process is an overall reduction of

the plasma current. As a consequence, the edge safety factor q_a decreases and the reversal parameter F becomes more negative, in agreement with Eq. (1.42). Another important feature of the crash is the drop in the core electron temperature, indicating a cooling of the plasma core. This behavior is linked to the degradation of the *internal transport barrier* (ITB) [71, 93, 94], a hot and localized thermal structure typically appearing during QSH phases. During crashes, the ITB is partially (minor crash) or completely (major crash) destroyed, leading to enhanced transport. While the qualitative phenomenology is similar, major and minor crashes differ in the amount of energy involved [39]. This is clearly visible in Fig. 1.13.

Chapter 2

Methods

In this Chapter, the experimental setup and the numerical methods employed in this thesis are presented. First, an overview of the RFX experiment [95–97] is provided. Particular attention is devoted to the magnetic probe system, which is used to detect tearing modes driving the crash; to the RFX imaging system, which allows the visualization of eventual Plasma-Wall Interactions (hereafter PWIs) originated by magnetic reconnection events; and to the Neutral Particle Analyzer, which is employed to measure particle acceleration during crash. Subsequently, the numerical methods are introduced, in particular the guiding-center code ORBIT [56], which is used to simulate the magnetic topology and the thermal content dynamics.

2.1 The Reversed-Field eXperiment (RFX)

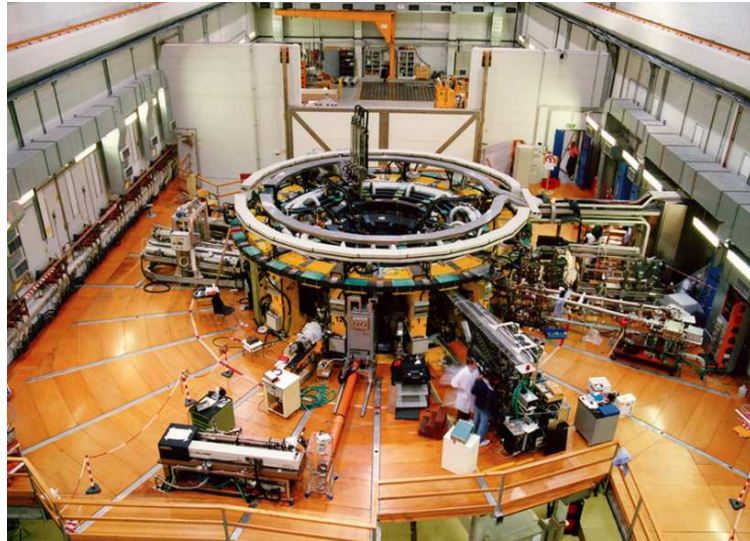


Figure 2.1: View of the RFX-mod device looking toward the assembly hall. Image courtesy of Maria Teresa Orlando.

The Reversed Field eXperiment (RFX) is a medium-sized device that mainly works in the Reversed Field Pinch (RFP) magnetic configuration, and is operated in the CNR research area in Padova, Italy. The main parameters of the machine are shown in Tab. 2.1.

Geometry and plasma features		Power supply	
Major radius	$R_0 = 2.00$ m	Plasma current	$50 \text{ kA} < I_P \leq 2 \text{ MA}$
Minor radius	$a = 0.46$ m	Loop-voltage	$(10 < V_{loop} < 60) \text{ V}$
Electron density	$(1 < n_e < 10) \cdot 10^{19} \text{ m}^{-3}$	Toroidal field	$B_\varphi \leq 0.7 \text{ T}$
Electron temperature	$(0.2 \leq T_e^{\text{core}} \leq 1.2) \text{ keV}$		

Table 2.1: Summary of the main parameters of the RFX-mod device [96].

Historically, the first RFP experiments operating in Padova during the 1970s and 1980s were **ETA-BETA I** and **ETA-BETA II**, described in Refs. [99, 100], and in a recent overview [101]. In the same decades, the origin of the toroidal magnetic field reversal at the plasma edge was studied, leading to the formulation of the relaxation theory by Taylor [60], summarized in Section 1.4.2. Although this theory qualitatively predicted field reversal, it soon became clear that more refined approaches were required to achieve quantitative agreement with the experimental observations. In this context, the **RFX** [95] device began its experimental activity in 1992. Initially, it operated as a RFP with a thick aluminum shell of 65 mm (penetration time of the vertical field $\tau_V \approx 400$ ms) [102], which was devised to passively stabilize ideal MHD modes.

In 1999, an accident involving the Toroidal Field (TF) power supply interrupted experimental operations. Subsequently, the RFX device was upgraded, the thick shell was replaced by a thin copper shell of 3 mm (penetration time of the vertical field $\tau_V = 50$ ms). The new machine was named RFX modified (**RFX-mod**) [36, 103]. This upgrade enabled operation not only in the RFP configuration, but also in tokamak [104, 105] and ultralow-q [106] regimes. Experimental operations of RFX-mod lasted from April 2004 until October 2015.

In RFX-mod, mode stabilization was no longer achieved solely by passive means, but by a complex system of 192 feedback-controlled saddle coils [37, 96, 107]: 4 poloidally arranged $\times$ 48 toroidally arranged, as shown in Fig. 2.2. Each saddle coil acts as an *actuator*, generating a magnetic flux that counterbalances any deviation from the reference level having null radial field. The corresponding *controllers* are magnetic sensors of identical size, arranged in a one-to-one correspondence with the actuators, see Fig. 2.2. The feedback control system therefore mimics the stabilizing effect of eddy currents in an ideal conducting shell. Its algorithm is Fourier based; as a consequence, due to the finite number of coils, whenever the system generates an (m, n) harmonic to cancel the corresponding magnetic perturbation, it also creates aliased sidebands [108]. With a 4×48 coil arrays, the system can effectively control a limited set of modes, namely with $m = 0, 1, 2$, and $0 \leq n \leq 24$.

A further significant improvement in RFX-mod performances was achieved through the real-time implementation of the *clean mode control* (CMC) de-aliasing algorithm, which

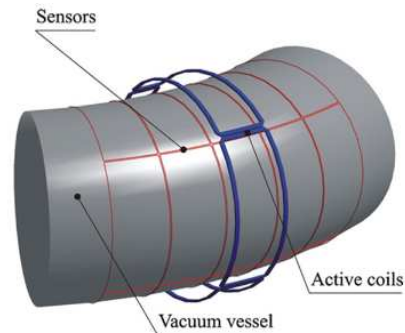


Figure 2.2: Arrangement of the saddle coils forming the active feedback control system of RFX-mod. Adapted from Ref. [98].

enabled the device to reach its design current of 2 MA [36]. With CMC, the edge amplitude of each mode can be reduced to a minimum value by appropriately increasing the corresponding proportional gain [109, 110].

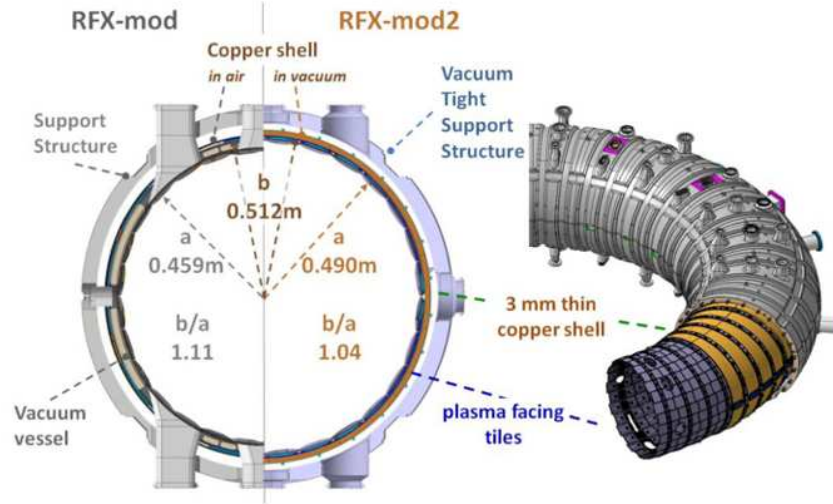


Figure 2.3: Comparison of the device layout in RFX-mod (2004-2015) and RFX-mod2 (2026). Adapted from Ref. 97.

Building on this understanding of the interplay between passive conductive structures and tearing modes in RFP plasmas, an upgrade of the RFX-mod device, named **RFX-mod2**, has been designed and is currently under implementation [97, 111, 112]. RFX-mod2 is expected to begin the commissioning by the end of 2026, and is therefore currently under assembly¹. The primary goal of the upgrade is to modify the resistivity of the magnetic front-end [113, 114] by removing the Inconel 625 vacuum vessel (see Fig. 2.3), which was identified as the main source of tearing mode braking. Therefore, the copper shell will simultaneously act as the first conductive surface for the plasma and the vacuum vessel itself, similarly to the MST experiment [40]. This configuration is expected to reduce the saturated amplitude of tearing modes and the associated magnetic chaos [52, 72], owing to the closer proximity of the shell to the plasma: from $b/a = 1.11$ to $b/a = 1.04$ (see Fig. 2.3). This design choice entails significant challenges for plasma-facing components [115], while preserving the existing magnetic coil system. Innovative solutions have been developed to meet vacuum and electrical requirements for in-vessel components, ensuring their reliability even during abnormal operation events. According to the modeling studies, this new magnetic configuration is expected to reduce the deformation of the Last Closed Flux Surface (LCFS) by a factor about 2-3 compared to RFX-mod [103]. Furthermore, the 3D nonlinear MHD SPECYL code [78], including realistic boundary conditions, predicts a reduction of the energy of tearing modes of about 30% [97].

RFX-mod2 will also be equipped with an entirely revised magnetic probe system [116, 117] aimed at improving mode control and enabling the exploration of new high-current scenarios [118]. Originally, RFX-mod was equipped with two sets of bi-axial pick-up probes (essentially pairs of Rogowsky coils) to measure the toroidal and poloidal components of the magnetic field. The radial field was measured both by the actuator-related

¹Follow the upgrades on the RFX-mod2 assembly on the Consorzio RFX official website: <https://www.igi.cnr.it/benvenuti-su-rfx-mod2/>.

loop probes and by an additional toroidal array of 48 smaller saddle loops located near the shell equatorial gap, where a very localized radial field was expected [119]. These probes were mounted on the copper shell and therefore operated *out-vessel*, i.e. at atmospheric pressure. By contrast, in RFX-mod2, the Inconel vacuum vessel will be removed, and the probes will be installed directly between the graphite tiles and the copper shell, thus operating under vacuum conditions (*in-vessel*). This aspect will improve temporal response but significantly has increased design and testing complexity. Moreover, bi-axial probes will be replaced by tri-axial probes, enabling simultaneous measurements of B_r , B_θ , and B_φ , while the number of saddle loops will be increased to 8×72 . As a result, the number of effectively controllable modes will increase to $m = 0, 1, 2$ and $0 \leq n \leq 36$ [117].

Finally, additional 14×2 local bi-axial pick up probes for B_θ and B_φ and one ring of 8 saddle coils aligned polidally will be anyway mounted ex-vessel in RFX-mod2. The main differences between the magnetic probe systems of RFX-mod and RFX-mod2 are summarized in Tab. 2.2.

RFX-mod		
Measurement	Type of probe	Layout (poloidal $\times$ toroidal directions)
Local B_θ and B_φ	Bi-axial pick-up	$4 \times 48 + 8 \times 6$
Local B_r	Saddle loop	4×48
Averaged B_r	Saddle loop	48 on the equatorial gap
RFX-mod2		
Measurement	Type of probe	Layout
Local B_r, B_θ and B_φ	Three-axial pick-up	14×12
Local B_φ	One-axis pick-up	8×72
Averaged B_r	Saddle loop	8×72

Table 2.2: Comparison between the magnetic probe systems installed on RFX-mod and RFX-mod2. Data on mounted probes are taken from Ref. 119 for RFX-mod and from Ref. 117 plus internal documentation for RFX-mod2.

The RFX-mod device was equipped with a set of CCD cameras covering the first wall both in the toroidal and the poloidal directions [120]. This system, known as the *Optical Camera System* (OCS) eventually observed PWI events occurring during plasma discharges [121]. Since the device first wall is composed of graphite-coated tiles [122, 123], PWI events lead to the release of a significant amount of carbon into the vacuum chamber through *sputtering*. As a consequence, a large fraction of neutral carbon atoms penetrates hotter regions of the plasma, where they become ionized. For this reason, a reliable experimental signature of PWI events is provided by the neutral carbon C-I emission line, at 970 nm, which is also the more intense among the carbon spectral lines.

In this thesis, the analysis focuses on a single PWI case study, namely the event observed during shot #29324 at $t = 218$ ms. This event was recorded by a single fast CCD camera, temporarily installed during the experimental campaign to observe PWI activity around $\varphi \approx 0$. The camera operated with an integration time of $\tau_{int} = 2$ ms and a toroidal field of view (FOV) of approximately 40° .

For the upgraded RFX-mod2 device, the installation of seven new *Basler ace acA720-520um* cameras is planned [124]. Their locations, shown in Fig. 2.4, are designed to cover approximately 70% of the total first-wall surface, corresponding to a total angular coverage of about 260° . Each camera features a pixel dimensions of $6.9 \mu\text{m} \times 6.9 \mu\text{m}$, a nominal focal length of 12 mm, and a maximum acquisition rate of 525 frame/s.

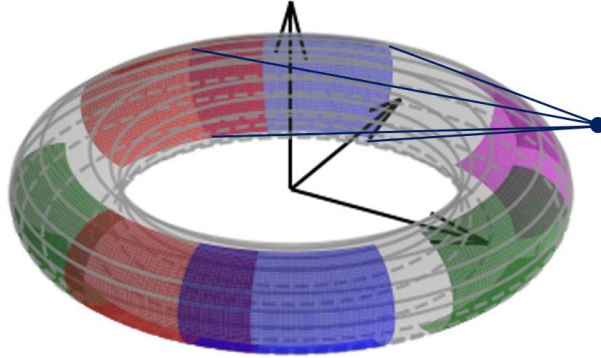


Figure 2.4: Location of the seven new CCD cameras in the RFX-mod2 device.

Moreover, RFX-mod2 will benefit from significant upgrades to several key diagnostics [124]. In particular, the Thomson Scattering (TS) system will be enhanced to achieve higher temporal resolution, thus providing improved measurements of the electron temperature profile, and enabling more detailed investigation of internal transport barrier formation and dynamics [71, 93, 94].

Finally, the experimental counterpart of this thesis is the measurement of the ion energy distribution function during reconnection events in the RFP configuration. The diagnostic dedicated to this purpose is the Neutral Particle Analyzer (NPA), which will be upgraded from RFX-mod and re-installed in the new RFX-mod2. Further details on the work carried out to refurbish the NPA is presented in Appendix A.

2.2 The ORBIT code

The code ORBIT was developed during the 1980s in the **Fortran** programming language at the Princeton Plasma Physics Laboratories (PPPL, Princeton, NJ, USA) by Roscoe White and Morrell Chance [56] starting from the guiding-center Lagrangian formulated a few years earlier by Littlejohn [125]. In practice, ORBIT is an Hamiltonian code that simulates particle motion in an arbitrary electromagnetic field provided as input. As such, it is a versatile tool designed to trace guiding-center trajectories in a variety of fusion-relevant magnetic field configurations: not only in RFPs [126, 127], but also in tokamaks [128–133], stellarators [134, 135], and magnetic dipoles [136, 137]. During the integration, the time step is adaptively controlled to ensure particle energy conservation.

2.2.1 Boozer coordinates

ORBIT employs Boozer coordinates for RFP configuration: (ψ_p, θ, ζ) [138], which constitute a toroidal-like coordinate system: ψ_p acts as a radial coordinate, θ corresponds to

the poloidal angle, and ζ to the toroidal angle². Boozer coordinates *conform to the shape of the magnetic surfaces and trivialize the equations for the field lines* [30], so they represent one of the most suitable choices for describing ion drifts in a 3D magnetic configuration.

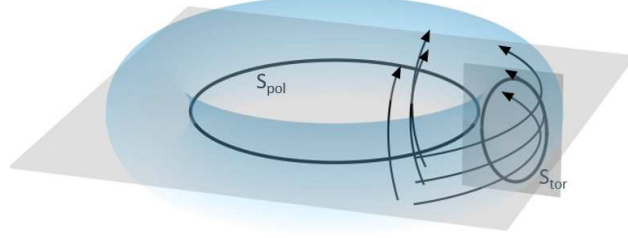


Figure 2.5: Representation of magnetic flux functions. Adapted from Ref. 139.

Referring to the sketch in Fig. 2.5, the toroidal and poloidal flux functions are related to the vector potential $\vec{A}$ through the following relations:

$$\begin{aligned}\psi &= \int_{S_{tor}} \vec{B} \cdot d\vec{S}_{tor} = \int_{S_{tor}} (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}_{tor} = \oint_{\partial S_{tor}} \vec{A} \cdot d\vec{\ell}_\theta = A_\theta \cdot 2\pi a, \\ \psi_p &= \int_{S_{pol}} \vec{B} \cdot d\vec{S}_{pol} = \int_{S_{pol}} (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}_{pol} = \oint_{\partial S_{pol}} \vec{A} \cdot d\vec{\ell}_\zeta = -A_\zeta \cdot 2\pi R_0;\end{aligned}\quad (2.1)$$

where the flux surfaces S_{tor} e S_{pol} are defined as shown in Fig. 2.5. In both derivations, the procedure is the same: first, we substitute the vector potential, $\vec{A}$; second, Stokes theorem is applied to convert the surface integrals into line integrals; finally, it is assumed that $d\ell_\theta$ is positive in the counter-clockwise direction, whereas $d\ell_\zeta$ is positive in the clockwise direction. We then redefine the flux functions as $\psi = \psi/(2\pi a)$ and $\psi_p = \psi_p/(2\pi R_0)$, such that $A_\theta \equiv \psi$ and $A_\zeta = -\psi_p$. With these definitions, the final expression of the vector potential in Boozer coordinates reads:

$$\vec{A} = A_{\psi_p} \cdot \nabla \psi_p + \psi \cdot \nabla \theta - \psi_p \cdot \nabla \zeta = \psi \cdot \nabla \theta - \psi_p \cdot \nabla \zeta. \quad (2.2)$$

In particular, the gauge choice $A_{\psi_p} \equiv 0$ is adopted. Substituting this expression into the definition of the magnetic field yields the following *contravariant representation*:

$$\begin{aligned}\vec{B} &= \nabla \times \vec{A} \\ &= \nabla \times (\psi \cdot \nabla \theta - \psi_p \cdot \nabla \zeta) \\ &= \nabla \psi \times \nabla \theta - \nabla \psi_p \times \nabla \zeta.\end{aligned}\quad (2.3)$$

Analogously, a *covariant representation* is given by:

$$\vec{B} = \delta \cdot \nabla \Psi_p + I \nabla \theta + g \nabla \zeta, \quad (2.4)$$

where $\delta \equiv \delta(\psi_p)$, $I \equiv I(\psi_p)$, and $g \equiv g(\psi_p)$. It can be shown, however, that the function δ produces only a non-secular contribution to the particle motion, which is periodic in the particle velocity and does not affect the shape of the guiding-center trajectory. For this reason, in ORBIT the function δ is neglected.

²RFX historically adopts a left-handed coordinates system for machine coordinates: $r_{\text{RFX}} = r$, $\theta_{\text{RFX}} = \theta$, $\varphi_{\text{RFX}} = -\varphi$. To avoid ambiguity, throughout this thesis, ζ will denote the Boozer (geometric) toroidal coordinate, while φ will indicate the RFX machine toroidal coordinate, i.e. $\varphi_{\text{RFX}} \rightarrow \varphi = -\zeta$.

2.2.2 Hamiltonian of a charged particle in electromagnetic field

The Lagrangian, $\mathcal{L}$, of a charged particle moving in an electromagnetic field can be written in the following general form:

$$\mathcal{L} = \mathcal{L}(\vec{x}, \vec{v}) = \left[\vec{A}(\vec{x}, t) + \vec{v} \right] \cdot \vec{v} - \mathcal{H}(\vec{x}, \vec{v}) , \quad (2.5)$$

where $\vec{x}$ and $\vec{v}$ are the particle position and velocity respectively, whereas $\mathcal{H} \equiv \mathcal{H}(\vec{x}, \vec{v})$ is the Hamiltonian, defined as:

$$\mathcal{H} = \frac{v^2}{2} + \Phi(\vec{x}, t) , \quad (2.6)$$

with Φ indicating the electric potential associated with the electric field. Note that ORBIT employs normalized quantities: particle mass $m = 1$, charge $e = 1$, on-axis magnetic field $B_0 = 1$, and major radius $R_0 = 1$. Time is measured in ω_c^{-1} units, with ω_c the ion cyclotron frequency $\omega_c = (eB)/m$. In the remainder of this thesis, a different expression for the Hamiltonian will be adopted [140], in which the contribution associated with the velocity components parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field are explicitly separated:

$$\begin{aligned} \mathcal{H} &= \frac{1}{2} (v_{\parallel} + v_{\perp})^2 + \Phi \\ &= \frac{1}{2} \frac{v_{\parallel}^2}{B^2} B^2 + \frac{1}{2} \frac{v_{\perp}^2}{B} B + \Phi \\ &= \frac{1}{2} \rho_{\parallel}^2 B^2 + \mu B + \Phi . \end{aligned} \quad (2.7)$$

Here, $\rho_{\parallel} = v_{\parallel}/B$ indicates the fraction of the particle velocity parallel to the magnetic field, while $\mu = v_{\perp}^2/2B$ is the magnetic moment. Following the same decomposition, the Lagrangian changes as follows:

$$\mathcal{L} = \left(\vec{A} + \rho_{\parallel} \vec{B} \right) \cdot \vec{v} + \mu \dot{\xi} - \mathcal{H} . \quad (2.8)$$

In this expression, the notation $\dot{x}$ represents the time derivative of the coordinate x : $\dot{x} = dx/dt$. The variable ξ is the gyro-phase (see Fig. 3), i.e. the angular position of the charged particle as it rotates around its guiding-center, in the plane perpendicular to the magnetic field line.

2.3 Equilibrium

As stated in Section 1.2, the Grad-Shafranov equation implies that the magnetic equilibrium is organized in nested flux surfaces. Under this assumption, VMEC (standing for *Variational Moments Equilibrium Code*) [141] is widely used to reconstruct the magnetic surface geometry by minimizing the plasma energy functional:

$$W = \int_{\Omega_p} \left(\frac{1}{2\mu_0} B^2 + p \right) dV , \quad (2.9)$$

where Ω_p is the toroidal domain representing the plasma volume. VMEC employs a coordinate system analogous to Boozer coordinates and can therefore provide a mathematically consistent input to ORBIT for the description of the magnetic field. Indeed, VMEC has

proven to be an effective tool for computing also helical equilibria in fixed boundary mode with experimental measurements as constraint [142]. Equilibria obtained in such a way can be then analyzed in terms of stability and transport using a suite of well-established numerical codes [143]. For example, the reconstruction of the equilibrium configuration of the stellarator W7-X is based on VMEC and is necessary to provide a magnetic field representation compatible with ORBIT³.

2.3.1 Equilibrium implementation in ORBIT

In the RFX-mod implementation of ORBIT, the equilibrium field is analytical, and is splined from the output of the toroidal μ - p model [90], described in Section 1.4.5. The field is then transformed in Boozer coordinates considering Eq. (2.3) together with the field-line equation. The physical meaning of the magnetic field structure, namely the nested-surface pattern is resumed by the so-called *canonical equations*:

$$\begin{aligned} q &= \frac{d\zeta}{d\theta} = \frac{\partial\psi}{\partial\psi_p} ; \\ \frac{d\psi_p}{d\theta} &= -\frac{\partial\psi}{\partial\zeta} . \end{aligned} \quad (2.10)$$

Their derivation is reported in Appendix B.1. Information on the spline transformation from toroidal coordinates (r, θ, φ) to Boozer coordinates (ψ_p, θ, ζ) is stored in a file named `spdata`, which can subsequently be used to transform quantities back into toroidal coordinates. As an example, Fig. 2.6, shows the poloidal flux function $\psi_p(r)$ for the equilibrium corresponding to shot #29324 at $t = 218$ ms. It is evident that $\psi_p \approx r^2/(2q)$. The value of ψ_p at the wall is $\psi_w = 0.096$, corresponding to the total poloidal flux normalized to $B_0^2 R_0$. This value naturally corresponds to the device minor radius, i.e. $a = r(\psi_w)$.

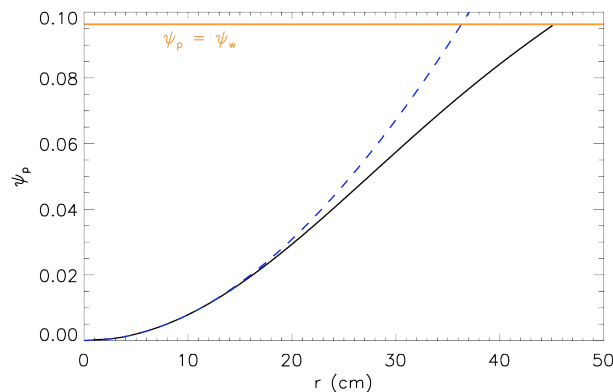


Figure 2.6: Relationship between the Boozer radial coordinate (poloidal magnetic flux) ψ_p and the geometric radial coordinate r for the equilibrium of shot #29324 at $t = 218$ ms. The dashed line represents the limiting expression $\psi_p = r^2/(2q)$. The edge value is $\psi_w = 0.096$, corresponding to the device minor radius $a = r(\psi_w)$.

³This task motivated a research activity not originally foreseen in the Ph.D. project, aimed at developing a workflow for running ORBIT in typical W7-X magnetic configuration including resonant divertor magnetic islands. This work was initiated during a short research period at the Max Planck Institute for Plasma Physics in Greifswald, Germany. However, this topic is actually out of scope for the present thesis, therefore it is deferred to future work.

2.3.2 Guiding center equations of motion

The main goal of ORBIT, however, is to simulate the dynamics of charged particle in an electromagnetic field. It is therefore necessary to derive the corresponding equations of motion. By substituting the magnetic field $\vec{B} = (\delta \approx 0, I, g)$ and the vector potential $\vec{A} = (0, \psi, -\psi_p)$ inside the Lagrangian (2.8), one obtains:

$$\mathcal{L} = (\psi + \rho_{\parallel} I) \dot{\theta} + (\rho_{\parallel} g - \psi_p) \dot{\zeta} + \mu \dot{\xi} - \mathcal{H} . \quad (2.11)$$

This expression is the Lagrangian from which the equations of motion are derived via:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = \frac{\partial \mathcal{L}}{\partial x_i} , \quad (2.12)$$

where x_i is the generic dynamical variable describing the charged particle motion, namely $x_i = \xi, \theta, \zeta, \psi_p$, or $\rho_{\parallel}$. This leads to the following set of equations:

$$-\frac{\partial \mathcal{H}}{\partial \theta} = \left(q + \rho_{\parallel} \frac{dI}{d\psi_p} \right) + I \dot{\rho}_{\parallel} ; \quad (2.13)$$

$$-\frac{\partial \mathcal{H}}{\partial \zeta} = \dot{\rho}_{\parallel} g + \left(\rho_{\parallel} \frac{dg}{d\psi_p} - 1 \right) \dot{\psi}_p ; \quad (2.14)$$

$$\frac{\partial \mathcal{H}}{\partial \psi_p} = \left(q + \rho_{\parallel} \frac{dI}{d\psi_p} \right) \dot{\theta} + \left(\rho_{\parallel} \frac{dg}{d\psi_p} - 1 \right) \dot{\zeta} ; \quad (2.15)$$

$$\frac{\partial \mathcal{H}}{\partial \rho_{\parallel}} = I \dot{\theta} + g \dot{\zeta} . \quad (2.16)$$

The full derivation is reported in Appendix B.2. The equations above can be put in matrix form $\vec{Y} = \mathbb{A} \vec{X}$:

$$\begin{pmatrix} -\partial_{\psi_p} \mathcal{H} \\ -\partial_{\theta} \mathcal{H} \\ -\partial_{\zeta} \mathcal{H} \\ -\partial_{\rho_{\parallel}} \mathcal{H} \end{pmatrix} = \begin{pmatrix} 0 & -A & -C & 0 \\ A & 0 & 0 & I \\ C & 0 & 0 & g \\ 0 & -I & -g & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{\psi}_p \\ \dot{\theta} \\ \dot{\zeta} \\ \dot{\rho}_{\parallel} \end{pmatrix} , \quad (2.17)$$

where the notation ∂_x indicates the partial derivative with respect to the variable x . The coefficients A, C are functions of the equilibrium fields g and I , as well as of the safety factor q , and are defined by:

$$\begin{aligned} A &= q + \rho_{\parallel} \frac{dI}{d\psi_p} = q + \rho_{\parallel} I' ; \\ C &= \rho_{\parallel} \frac{dg}{d\psi_p} - 1 = \rho_{\parallel} g' - 1 , \end{aligned} \quad (2.18)$$

where the prime symbol, $'$, means the derivative with respect to ψ_p .

The final step in obtaining the ORBIT equations of motion consists in inverting the matrix $\mathbb{A}^{-1}$. Owing to the skew-symmetric (or anti-symmetric) characteristic of $\mathbb{A}$ ($a_{i,j} = -\delta_{i,j} a_{j,i}$, with vanishing diagonal elements), it is sufficient to calculate only $n(n-1)/2$ cofactors instead of the full-matrix n^2 entries. In fact, the inverse of a skew-symmetric matrix is skew-symmetric as well.

The determinant of $\mathbb{A}$ is the square of a polynomial in A, g, C , and I , known as the *Pfaffian*:

$$D = Ag - IC . \quad (2.19)$$

The Pfaffian can be explicitly expanded in terms of the equilibrium field functions as:

$$\begin{aligned} D &= g(q + \rho_{\parallel} I') - I(\rho_{\parallel} g' - 1) \\ &= gq + I + \rho_{\parallel}(gI' - Ig') , \end{aligned} \quad (2.20)$$

The inverse matrix therefore reads:

$$\mathbb{A}^{-1} = \frac{1}{D} \begin{pmatrix} 0 & g & -I & 0 \\ -g & 0 & 0 & C \\ I & 0 & 0 & -A \\ 0 & -C & A & 0 \end{pmatrix} . \quad (2.21)$$

Hence, the ORBIT guiding center equations of motion for equilibrium are:

$$\begin{pmatrix} \dot{\psi}_p \\ \dot{\theta} \\ \dot{\zeta} \\ \dot{\rho}_{\parallel} \end{pmatrix} = \frac{1}{D} \begin{pmatrix} 0 & g & -I & 0 \\ -g & 0 & 0 & C \\ I & 0 & 0 & -A \\ 0 & -C & A & 0 \end{pmatrix} \cdot \begin{pmatrix} -\partial_{\psi_p} \mathcal{H} \\ -\partial_{\theta} \mathcal{H} \\ -\partial_{\zeta} \mathcal{H} \\ -\partial_{\rho_{\parallel}} \mathcal{H} \end{pmatrix} , \quad (2.22)$$

whose explicit form is given by:

$$\dot{\psi}_p = -\frac{g}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \theta} + \frac{\partial \Phi}{\partial \theta} \right] + \frac{I}{D} \frac{\partial \Phi}{\partial \zeta} ; \quad (2.23)$$

$$\dot{\theta} = -\frac{C\rho_{\parallel} B^2}{D} + \frac{g}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \psi_p} + \frac{\partial \Phi}{\partial \psi_p} \right] ; \quad (2.24)$$

$$\dot{\zeta} = \frac{A\rho_{\parallel} B^2}{D} - \frac{I}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \psi_p} + \frac{\partial \Phi}{\partial \psi_p} \right] ; \quad (2.25)$$

$$\dot{\rho}_{\parallel} = \frac{C}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \theta} + \frac{\partial \Phi}{\partial \theta} \right] - \frac{A}{D} \frac{\partial \Phi}{\partial \zeta} . \quad (2.26)$$

2.3.3 Canonical momenta

It is straightforward to recast the Lagrangian $\mathcal{L}$ of Eq. (2.8) in the form:

$$\mathcal{L} = P_{\theta} \dot{\theta} + P_{\zeta} \dot{\zeta} - \mathcal{H} . \quad (2.27)$$

The two canonical momenta are P_{θ} and P_{ζ} , defined as:

$$\begin{aligned} P_{\theta} &= \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \psi + \rho_{\parallel} I , \\ P_{\zeta} &= \frac{\partial \mathcal{L}}{\partial \dot{\zeta}} = \rho_{\parallel} g - \psi_p . \end{aligned} \quad (2.28)$$

While P_{θ} can vary along a particle orbit, P_{ζ} is conserved due to the toroidal symmetry of the system, namely $\partial B / \partial \zeta = 0$:

$$\begin{aligned} \dot{P}_{\zeta} &= \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\zeta}} = \dot{\rho}_{\parallel} g + \left(\rho_{\parallel} \frac{dg}{d\psi_p} - 1 \right) \dot{\psi}_p \\ &= \frac{1}{Ag - IC} \left[g \left(C \frac{\partial \mathcal{H}}{\partial \theta} - A \frac{\partial \mathcal{H}}{\partial \zeta} \right) + C \left(-g \frac{\partial \mathcal{H}}{\partial \theta} + I \frac{\partial \mathcal{H}}{\partial \zeta} \right) \right] \\ &= \frac{1}{Ag - IC} \left[\cancel{(Cg - gC)} \frac{\partial \mathcal{H}}{\partial \theta} + (-Ag + IC) \frac{\partial \mathcal{H}}{\partial \zeta} \right] = -\frac{\partial \mathcal{H}}{\partial \zeta} , \end{aligned}$$

where, in the first step, the expressions for $\dot{\rho}_{\parallel}$ and $\dot{\psi}_p$ have been substituted from Eq. (2.22). The calculation is then continued by assuming there is not electric field⁴, i.e. $\Phi = 0$:

$$\dot{P}_{\zeta} = -\frac{\partial \mathcal{H}}{\partial \zeta} = -(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \zeta} = 0 . \quad (2.29)$$

Therefore, for unperturbed, toroidally symmetric systems (such as tokamak or RFP equilibria) the canonical momentum P_{ζ} is a constant of motion. This result follows directly from Noether theorem and can be demonstrated explicitly [144].

The equations for the canonical momenta (2.28) show that the pair of variables P_{θ}, P_{ζ} depends uniquely on the pair of variables $\rho_{\parallel}, \psi_p$, whereas all remaining quantities depend solely on the radial coordinate ψ_p , namely $g = g(\psi_p)$, $I = I(\psi_p)$ and $\partial\psi/\partial\psi_p = q(\psi_p)$. This implies that, when formulating the equations of motion, one may use equivalently (P_{θ}, P_{ζ}) or $(\rho_{\parallel}, \psi_p)$. The choice adopted by White and Chance is to employ $(\rho_{\parallel}, \psi_p)$, in order to avoid an implicit integration scheme involving the canonical momenta. Moreover, $(\rho_{\parallel}, \psi_p)$ have a clear physical interpretation, representing the parallel kinetic energy and the Boozer radial coordinate.

2.4 Perturbations

The equations of motion outlined in the previous Section include only the equilibrium magnetic field, Eqs. (2.3)-(2.4). In this form, they are well suited for neoclassical transport calculations in arbitrary magnetic equilibria. Indeed, ORBIT has been extensively used, for example, to study ripple-induced losses of energetic particles in tokamaks [129, 130], as well as to characterize the poor confinement of alpha particles in stellarator configurations [134]. However, some of the most physically relevant results obtained with ORBIT are those in which low-frequency magnetic perturbations, such as *tearing modes* (typical of RFP plasmas, see Fig. 1.11), are superimposed to the equilibrium field.

All tearing modes can be described as perturbations of the magnetic field $\vec{B}$ orthogonal to the equilibrium field itself. They can be represented through a vector potential perturbation with the gauge choice $\delta\vec{A} = \alpha\vec{B}$, namely:

$$\delta\vec{B} = \nabla \times \alpha\vec{B} , \quad (2.30)$$

where $\alpha \equiv \alpha(\psi_p, \theta, \zeta, t)$ is a scalar function of the Boozer coordinates. From the physical perspective, α corresponds to the perturbation of the poloidal flux associated with tearing mode magnetic islands.

Extensive verification studies have been carried out to assess the validity of the representation given by Eq. (2.30) for tearing modes [127]. By benchmarking ORBIT against the volume-preserving field-line tracing code NEMATO [145] using the same SPECYL input, it was shown that the α -based representation accurately reproduces tearing mode magnetic structures. The radial profile of α closely resembles that of the original δB_r perturbation, as illustrated in Fig. 2.7 for a (0, 4) mode in RFP configuration. Only a minor discrepancy was observed in the longitudinal component δB_{θ} , which does not significantly affect the description of both conserved and chaotic magnetic fields in the RFP [127]. The representation in Eq. (2.30) instead can not be used to describe ideal modes [146]; these modes are however not considered in the present thesis.

⁴This assumption will be relaxed in Chapter 5, where the role of electric fields in ORBIT simulations of particle dynamics is discussed in greater detail.

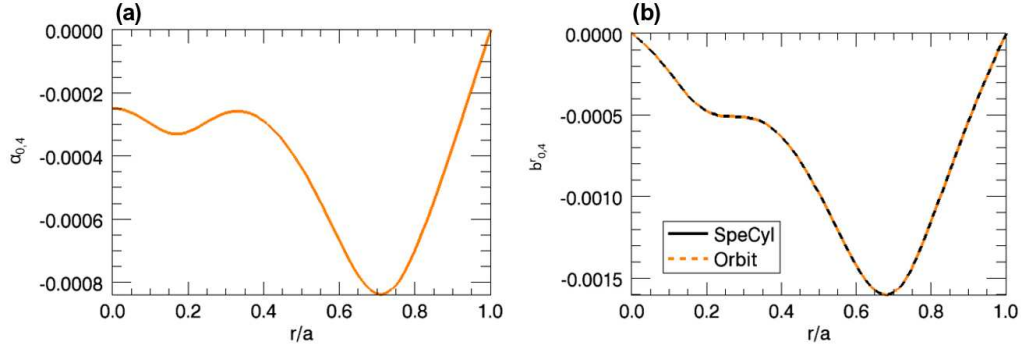


Figure 2.7: Comparison between (a) the normalized α profile and (b) the normalized perturbation components computed with SPECYL and ORBIT for an $m = 0$ $n = 4$ mode. In panel (b), the radial component obtained with ORBIT matches by construction the radial magnetic perturbation from the SPECYL simulation. As expected, the α profile closely resembles the perturbation. The quantity α has dimension of m^{-1} , and is here normalized to the major radius R_0 . Magnetic perturbations are normalized to the on-axis magnetic field B_0 . Adapted from Ref. 127.

Typical inputs which have been employed in studies on the RFX device include:

- the radial magnetic perturbation profile δB_r , obtained as output from the three-dimensional visco-resistive MHD code SPECYL [78];
- the perturbations (i.e. eigenfunctions) computed by solving Newcomb's equations in toroidal geometry with the code NCT [90].

In this thesis, we use the second approach. The perturbations calculated in this way are subsequently supplied to ORBIT using the representation given in Eq. (2.30). ORBIT then computes particle trajectories in the magnetic field: equilibrium and perturbations.

In order to convert the δB_r profiles into the scalar function α , the latter is expanded in a Fourier series, in analogy with the procedure already introduced in Eq. (1.14):

$$\alpha = \sum_{m,n} \alpha_{m,n}(\psi_p) \sin[m\theta - n\zeta + \phi_{m,n}(t)] . \quad (2.31)$$

By matching Eqs. (2.31) and (1.14), one obtains the expression for a single α harmonic in cylindrical approximation:

$$\alpha_{m,n}(r) = \frac{r b_{m,n}^r(r)}{(mg + nI) B_{0,\theta}} . \quad (2.32)$$

The generalized formulation in Boozer coordinates is more complicated and is discussed in detail in Ref. 147. It is worth noting that, by comparing this relation with the expression for the magnetic island width introduced in Section 1.3, the island width can be expressed in terms of the Fourier coefficient $\alpha_{m,n}$ as $w \propto \sqrt{\alpha_{m,n}}$. More precisely, according to Eq. (3.88) in White book [148], the relation is:

$$w_{m,n} = 4 \sqrt{\frac{\alpha_{m,n}}{s}} . \quad (2.33)$$

Here $s = q'/q$ denotes the *local magnetic shear* [149], and the prime symbol $'$ indicates the derivative with respect to the poloidal flux function ψ_p . The derivation of this relation in Boozer coordinates is reported in Appendix B.3 [66].

2.4.1 Perturbation implementation in ORBIT

For the simulations discussed in this thesis, magnetic field perturbations must be provided as input to ORBIT. Information on the magnetic field are stored in a dedicated `spdata` file. The perturbations are computed starting from the pick-up coil measurements [119] using the NCT [90] code for modes with poloidal mode numbers $m = 0, 1$ and toroidal mode numbers $0 \leq n \leq 24$. The radial profiles of the eigenfunctions $b_r^{m,n}(r)$ are then converted into the corresponding scalar functions $\alpha_{m,n}(r)$ using Eq. (2.32). Examples of α and δB_r profiles for the QSH dominant $m = 0, 1$ modes are compared in Figure 2.8. We note that the dominant $m = 1$ mode has a relatively large amplitude: $b_r^{1,7} \approx 50$ mT, corresponding to $\alpha \sim 10^{-3}$. This value is one order of magnitude larger than the typical value of a tearing mode in a tokamak [131]. Finally, the toroidally coupled $m = 0$ mode is smaller by a factor of the order of $\varepsilon = a/R_0$, with a typical amplitude $\alpha \approx 10^{-4}$.

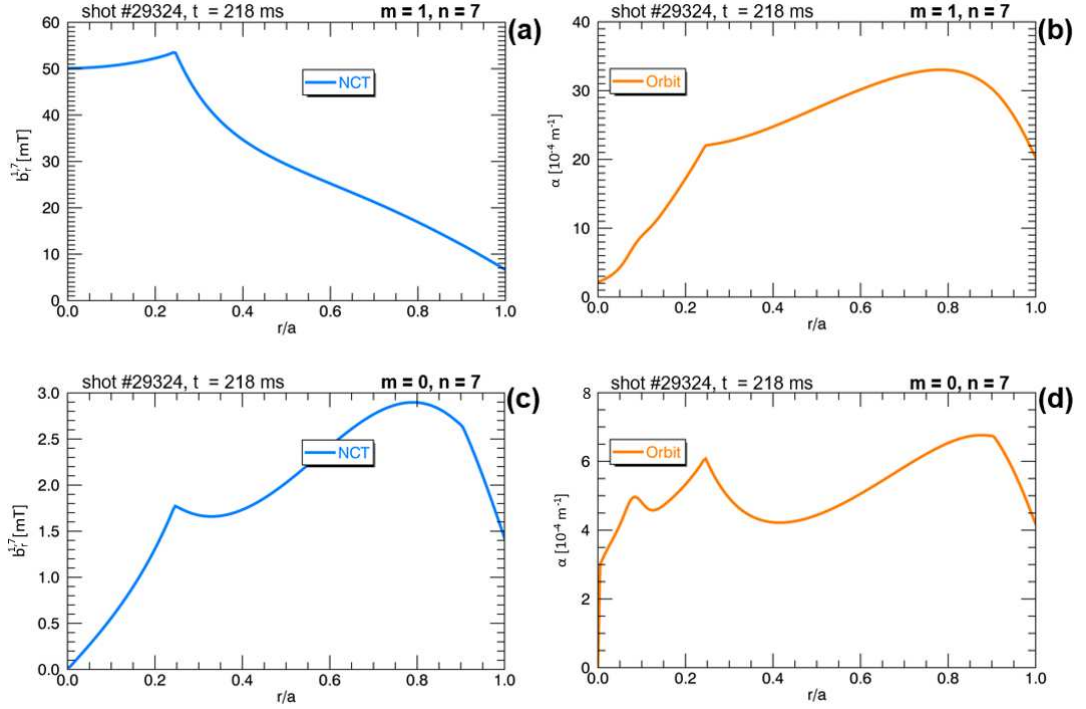


Figure 2.8: Comparison between (a)-(c) the magnetic perturbations (eigenfunctions) computed with the code NCT, and (b)-(d) the corresponding perturbations transformed in ORBIT as α profiles. (a)-(b) refer to the (1,7) mode, while (c)-(d) refer to its (0,7) counterpart.

The canonical equations for the perturbed magnetic field are modified according to Eq. (2.30), with α given by Eq. (2.31):

$$\begin{aligned}
 q &= \frac{d\zeta}{d\theta} = \frac{\partial\psi}{\partial\psi_p} ; \\
 \frac{d\psi_p}{d\theta} &= -\frac{\partial\psi}{\partial\zeta} = \sum_{m,n} (mg + nI) \alpha_{m,n}(\psi_p) \cos(m\theta - n\zeta + \phi_{m,n}) .
 \end{aligned}
 \tag{2.34}$$

The two equations can be integrated as:

$$\begin{aligned}
\psi &= \int d\psi = \int \left(\frac{\partial\psi}{\partial\psi_p} d\psi_p + \frac{\partial\psi}{\partial\theta} d\theta + \frac{\partial\psi}{\partial\zeta} d\zeta \right) \\
&= \int q d\psi_p - \int d\zeta \sum_{m,n} (mg + nI) \alpha_{m,n}(\psi_p) \cdot \cos(m\theta - n\zeta + \phi_{m,n}) \\
&= \int q d\psi_p - \sum_{m,n} \frac{mg + nI}{n} \cdot \alpha_{m,n}(\psi_p) \cdot \sin(m\theta - n\zeta + \phi_{m,n}) . \quad (2.35)
\end{aligned}$$

2.4.2 Guiding center equations of motion with perturbations

The main advantage of introducing a single scalar function α is that it can be added to the guiding center equations of motion (2.22). Formally α appears as an additive contribution to the vector potential, and it can be shown (see in this respect Ref. 148) that the perturbed equations of motion can be obtained from the unperturbed ones (2.22) by substituting:

$$\rho_{\parallel} \rightarrow \rho_{\parallel} + \alpha . \quad (2.36)$$

As a consequence, the Langrangian is modified as:

$$\mathcal{L} = [\psi + (\rho_{\parallel} + \alpha)] \dot{\theta} + [(\rho_{\parallel} + \alpha) g - \psi_p] \dot{\zeta} + \mu \dot{\xi} - \mathcal{H} , \quad (2.37)$$

with the Hamiltonian $\mathcal{H}$ still given by Eq. (2.7).

The Lagrangian equations change correspondingly:

$$\begin{pmatrix} -\partial_{\psi_p} \mathcal{H} \\ -\partial_{\theta} \mathcal{H} - \dot{\alpha} I \\ -\partial_{\zeta} \mathcal{H} - \dot{\alpha} g \\ -\partial_{\rho_{\parallel}} \mathcal{H} \end{pmatrix} = \begin{pmatrix} 0 & -A & -C & 0 \\ A & 0 & F & I \\ C & -F & 0 & g \\ 0 & -I & -g & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{\psi}_p \\ \dot{\theta} \\ \dot{\zeta} \\ \dot{\rho}_{\parallel} \end{pmatrix} , \quad (2.38)$$

where the coefficients A , C , and F are now defined as:

$$\begin{aligned}
A &= q + \frac{\partial\alpha}{\partial\psi_p} + (\rho_{\parallel} + \alpha) I' ; \\
C &= \frac{\partial\alpha}{\partial\psi_p} g + (\rho_{\parallel} + \alpha) g' - 1 ; \\
F &= \frac{\partial\alpha}{\partial\zeta} I - \frac{\partial\alpha}{\partial\theta} g . \quad (2.39)
\end{aligned}$$

Also in this case, the matrix appearing in Eq. (2.38) is skew-symmetric and can be inverted to derive the equations of motion:

$$\begin{pmatrix} \dot{\psi}_p \\ \dot{\theta} \\ \dot{\zeta} \\ \dot{\rho}_{\parallel} \end{pmatrix} = \frac{1}{D} \begin{pmatrix} 0 & g & -I & F \\ -g & 0 & 0 & C \\ I & 0 & 0 & -A \\ -F & -C & A & 0 \end{pmatrix} \cdot \begin{pmatrix} -\partial_{\psi_p} \mathcal{H} \\ -\partial_{\theta} \mathcal{H} - \dot{\alpha} I \\ -\partial_{\zeta} \mathcal{H} - \dot{\alpha} g \\ -\partial_{\rho_{\parallel}} \mathcal{H} \end{pmatrix} , \quad (2.40)$$

where the Pfaffian retains the same formal expression: $D = Ag - IC$ as in Eq. (2.19), even though the coefficient A , C , and F now have different forms.

Finally:

$$\dot{\psi}_p = -\frac{F\rho_{\parallel}B^2}{D} - \frac{g}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \theta} + \frac{\partial \Phi}{\partial \theta} \right] + \frac{I}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \zeta} + \frac{\partial \Phi}{\partial \zeta} \right] ; \quad (2.41)$$

$$\dot{\theta} = -\frac{C\rho_{\parallel}B^2}{D} + \frac{g}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \psi_p} + \frac{\partial \Phi}{\partial \psi_p} \right] ; \quad (2.42)$$

$$\dot{\zeta} = \frac{A\rho_{\parallel}B^2}{D} - \frac{I}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \psi_p} + \frac{\partial \Phi}{\partial \psi_p} \right] ; \quad (2.43)$$

$$\begin{aligned} \dot{\rho}_{\parallel} = & \frac{C}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \theta} + \frac{\partial \Phi}{\partial \theta} \right] + \frac{F}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \psi_p} + \frac{\partial \Phi}{\partial \psi_p} \right] + \\ & - \frac{A}{D} \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \zeta} + \frac{\partial \Phi}{\partial \zeta} \right] - \frac{\partial \alpha}{\partial t} . \end{aligned} \quad (2.44)$$

The equations derived above correspond to Eqs. (3.93)-(3.96) of White book [148]. The terms highlighted in blue vanish in case of an axisymmetric equilibrium: $\partial B/\partial \zeta = 0$. This corresponds to the original formulation, namely the one presented by White and Chance in Eqs. (31) for ψ_p , (27) for θ , (26) for ζ , and (30) for $\rho_{\parallel}$ of Ref. 56.

2.4.3 Canonical momenta with perturbations

The canonical momenta are also modified accordingly. In particular, the toroidal component is given by:

$$P_{\zeta} = (\rho_{\parallel} + \alpha)g - \psi_p , \quad (2.45)$$

which introduces a new term in the equation of its evolution (2.29):

$$\dot{P}_{\zeta} = -(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \zeta} + \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \zeta} . \quad (2.46)$$

It is evident from Eq. (2.46) that the presence on the term $\partial \alpha/\partial \zeta$ breaks the toroidal symmetry of the system, and P_{ζ} is no more conserved, causing particle loss through resonance with the perturbation α .

2.5 Calculation of the Connection Length to the wall

The *Connection Length to the wall* is a metric routinely used to characterize particle dynamics in fusion plasmas. The original term *connection* referred to the length of a magnetic field line physically *connecting* two coils located at different toroidal angles in the ergodic divertor of the TORE Supra Tokamak [150]. Then, the term has been retained in the literature. Currently, this quantity is a standard metric used to characterize the ergodic edge of tokamaks when resonant magnetic perturbations are applied [151], or to describe particle losses and power load on stellarator divertors [152]. In Chapter 3, the Connection Length is used to investigate the magnetic topology during a crash event in a typical high-current RFX-mod discharge. In this section, the calculation of this quantity is introduced.

Referring to the sketch shown in Fig. 2.9, the Connection Length to the wall, $L_{c,w}$, measures the parallel distance traveled by a charged particle along the magnetic field line from a given point $A(r_A, \theta_A, \varphi_A)$ to the wall. Formally, it is defined as:

$$\begin{aligned} L_{c,w}(r_A, \theta_A, \varphi_A) &= \int_{\varphi_A}^{\varphi(r_w)} \frac{B \, d\varphi}{\vec{B} \cdot \nabla \varphi} \quad \text{or} \\ L_{c,w}(\psi_{p,A}, \theta_A, \zeta_A) &= \int_{\zeta_A}^{\zeta(\psi_{p,w})} \frac{B \, d\zeta}{\vec{B} \cdot \nabla \zeta}. \end{aligned} \quad (2.47)$$

Note that the definition of the Connection Length does not depend on the choice of the coordinate system: it can be expressed either in terms of Boozer toroidal angle ζ or RFX toroidal angle φ . This follows directly from the fact that Eq. (2.47) is derived simply from the field-line equation:

$$\frac{d\ell}{B} = \frac{d\zeta}{B_\zeta} \quad \longrightarrow \quad L_{c,w} = \int d\ell = \int \frac{B \, d\ell}{\vec{B} \cdot \nabla \zeta}. \quad (2.48)$$

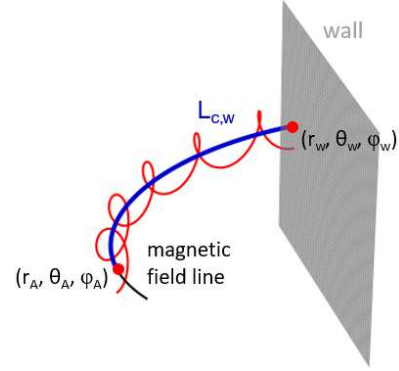


Figure 2.9: Sketch of a particle moving from point $A(r_A, \theta_A, \varphi_A)$ toward the wall. The connection length associated with the particle motion (in red), is the length of the magnetic field line segment highlighted in blue.

In ORBIT, particles with very low energy ($E = 10^{-2}$ eV) are used to represent a field line, since in the limit $r_L \approx 0$ a charged particle closely follows the magnetic field line [127]. A three-dimensional grid of initial positions $(\psi_{p,i}, \theta_j, \zeta_k)$ is therefore defined. Due to the presence of multiple modes whose magnetic islands overlap each other, the edge field of a RFP is chaotic, so for a given initial position, the Connection Length to the wall is estimated by averaging over a bundle of 1000 particles (field lines), starting at $\psi_{p,A} = \psi_{p,i}$, $\theta_A = \theta_j$, and ζ_A taken from 1000 equally spaced positions centered at ζ_k . In this way, a three-dimensional matrix $\mathbb{L}_{i,j,k} = L_{c,w}(\psi_{p,i}, \theta_j, \zeta_k)$ is obtained, where each element represents the mean distance traveled from the initial point $(\psi_{p,i}, \theta_j, \zeta_k)$ to the wall.

In the RFX-mod edge region, the Connection Length $L_{c,w}$ spans from a few centimeters up to tens of kilometers [153]. This wide range is consistent with analogous results obtained in the stochastic edge layer of tokamaks and in the island divertor of stellarators [154].

Associated with the Connection Length, one can also define the *loss time*, namely the time required for a particle to reach the wall:

$$\tau_{loss}(r_A, \theta_A, \varphi_A) = \frac{L_{c,w}(r_A, \theta_A, \varphi_A)}{v_{th}}, \quad (2.49)$$

where v_{th} denotes the thermal velocity of the simulated particle. The loss time is computed in ORBIT following the same procedure adopted for the Connection Length. Unlike $L_{c,w}$, however, the loss time explicitly depends on particle velocity and therefore provides insight into particle dynamical properties. For this reason, it will be extensively employed in Chapter 4 to investigate the energy and pitch distributions of lost ions during the selected case study event.

2.5.1 Parallel computing

For each point of the matrix $\mathbb{L}_{i,j,k} = L_{c,w}(\psi_{p,i}, \theta_j, \zeta_k)$ the workflow consists of reading the corresponding `spdata` input file and producing a `final` output file at the end of the run. In Chapter 3, a three-dimensional map of connection lengths will be presented, computed using $1 \leq i \leq 8$, $1 \leq j \leq 64$, and $1 \leq k \leq 128$. The resulting matrix therefore contains $8 \times 64 \times 128 = 65536$ points. In practice, the calculation of $L_{c,w}$ requires a significant amount of computational time and resources.

During this work, a new workflow has been developed to reduce the computational cost of these calculations by exploiting a *parallel high-performance computing approach* based on *multi-threading* with the `OpenMP` method [155]. This parallelized approach reduces the total computational time required to compute the $\mathbb{L}$ matrix by more than 30% compared to the traditional serial approach. This performance gain should be meant as preliminary, since the workflow still requires to be more efficiently integrated into the shared infrastructure currently used at RFX. At present, the parallelization is achieved through a `C++` script that just manages several code runs at the same time, and therefore corresponds more closely to a multi-process rather than a true multi-threading approach. Nevertheless, further developments are feasible by exploiting the portability of the ORBIT programming language (`Fortran`) to direct parallelization. This would ultimately enable fully parallelized calculation of connection length three-dimensional maps.

Finally, the parallelized approach discussed here represents a robust test case, and constitutes one of the first applications of parallel computing strategies to ORBIT simulations in modern plasma physics. Examples of the exploitation of this workflow can already be found in literature, both for RFX-mod [156] and for the DTT tokamak [133].

2.6 Coupling between ORBIT and SPECYL

Chapter 5 presents the results of the first implementation in ORBIT of a three-dimensional map of electrostatic potential associated with a magnetic reconnection event. The potential is calculated from a 3D nonlinear visco-resistive MHD simulation of a RFP plasma performed with the SPECYL code [78]. SPECYL solves the equations of the visco-resistive model, which are derived in turn from the MHD equations of Section 1.1. In dimensionless form they are expressed by:

$$\begin{aligned} \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} &= \vec{j} \times \vec{B} + \nu \nabla^2 \vec{v} ; \\ \frac{\partial \vec{B}}{\partial t} &= \nabla \times (\vec{v} \times \vec{B}) - \nabla \times (\eta \vec{j}) ; \\ \nabla \cdot \vec{B} &= 0 \quad ; \quad \nabla \times \vec{B} = \vec{j} . \end{aligned} \tag{2.50}$$

where $\vec{v}$ represents the velocity field, while $\vec{j}$ is the current density. Moreover, $\eta = S^{-1}$ and $\nu = M^{-1}$ represent the dimensionless resistivity and viscosity, with S and M being the *Lundquist number* (defined as the ratio between the resistive time τ_R and the Alfvén time τ_A) and the *viscous Lundquist number* (defined as the ratio between the viscous time τ_V and the Alfvén time τ_A) respectively. The model relies on the assumptions of negligible pressure gradients, constant mass density, and effective transport coefficients for both resistivity and viscosity.

The SPECYL simulations discussed in this thesis consider a RFP plasma with $S = 10^6$ and $M = 10^4$. Both resistivity and viscosity are assumed to be stationary in time.

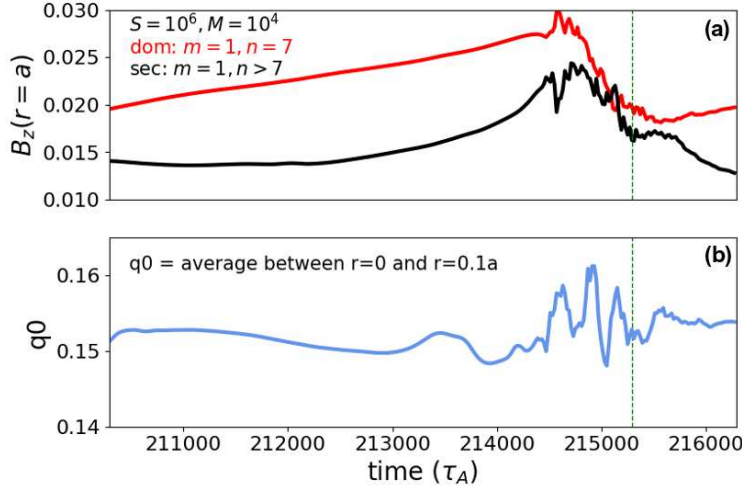


Figure 2.10: Example of reconnection events simulated with SPECYL. The major reconnection event occurs at $t \approx 214500\tau_A$, where τ_A is the Alfvén time and corresponds to $\approx 10^{-7}$ s. Panel (a) shows the time evolution of dominant (in red) and secondary (in black) $m = 1$ modes, while panel (b) shows safety factor on axis q_0 , averaged on a radial slice $0 < r/a < 0.1$. The vertical, dashed green line indicates the time instant chosen for the snapshot of the electrostatic potential Φ used as input to ORBIT. Image taken from Ref. 156, made basing on data from Ref. 157.

Resistivity profile increases towards the edge to mimic the decrease of temperature towards the edge following a Spitzer law, while viscosity is isotropic. Ideal wall boundary conditions are imposed for the magnetic field. Additionally, a seed edge magnetic perturbation with $m = 1$, $n = 7$ topology is introduced to sustain the RFP self-organization, following a well-known approach in literature [158].

Fig. 2.10 shows an example of reconnection events simulated with SPECYL [157] that well reproduces the experimental observations in RFX-mod [39]. Note that intense electric field is observed locally at the reconnection site. In fact, an electric field arises from magnetic flux changes due to reconnection. The presence of a 3D electric field implies the existence of a small but not-negligible electric charge according to the Poisson equation [70]:

$$\nabla \cdot \vec{E} = \rho_c \quad \longrightarrow \quad \nabla^2 \Phi = -\rho_c, \quad (2.51)$$

where Φ denotes the electrostatic potential, while $\rho_c \ll n_e, n_i$ is the charge density. We remind that the Poisson equation is valid only under the Coulomb gauge for the vector potential: $\nabla \cdot \vec{A} = 0$.

The electrostatic potential has been computed inverting the Poisson equation and decomposed in Fourier harmonics in the form:

$$\Phi(\psi_p, \theta, \zeta) = \Phi_{m_p, n_p}(\psi_p) \sin[m_p \theta - n_p \zeta + \phi_{m_p, n_p}(t)], \quad (2.52)$$

where m_p and n_p are the poloidal and toroidal mode numbers characterizing the potential, and ϕ_{m_p, n_p} represents the helical phase related to the (m_p, n_p) mode. The Fourier harmonics Φ_{m_p, n_p} are then provided as input to ORBIT.

Chapter 3

Magnetic Topology

This chapter focuses on the characterization of MHD tearing modes responsible for plasma-wall interaction (PWI). The PWI is monitored using a fast CCD camera [120] viewing the graphite-covered inner wall of the device, whose images reveal two distinct footprints of neutral carbon emission. A preliminary analysis indicates that the enhanced PWI is primarily associated with the phase-locking of the $m = 1$ tearing modes. Several modes contribute to this phase-locking process, indeed more than those directly detected by magnetic probes. A more detailed investigation, based on Poincaré maps, Connection Length to the wall, and recurrence time analysis, performed with the ORBIT code, and considering monoenergetic, low-energy ions, further supports this interpretation and confirms that the phase-locking of the $m = 1$ modes represents the dominant loss channel, which in turn shapes the characteristic PWI pattern. Nevertheless, the contribution of the $(0, 7)$ mode is found to be significant [159].

3.1 The case study: shot #29324, $t = 218$ ms

Discharge #29324 is a high-current RFP plasma, with an average flat-top current of about $\langle I_p \rangle \approx 1.6$ MA. As a consequence, the discharge is predominantly in the Quasi Single Helicity (QSH) state; however, several crash events occur, leading to transient phases of Multiple Helicity (MH) state. During this discharge, a CCD camera was mounted to observe the first wall [59], allowing the direct observation of a well-diagnosed PWI event at time $t = 218$ ms. The corresponding camera frame, shown in Fig. 3.1(c), shows two localized footprints of enhanced radiation, located at toroidal angles $\varphi_0 = -8^\circ$ and $\varphi_1 = 7^\circ$. Since the inner wall tiles of RFX-mod are covered by graphite [123], the observed emission corresponds to the C-I (neutral carbon) line at 970 nm.

By observing Fig. 3.1(a), it can be clearly noticed that the dominant $(1, 7)$ mode amplitude (red curve) undergoes a sudden decrease, while the secondary mode amplitude (black curves) increases. This indicates that the observed PWI is actually induced by a crash. Moreover, Fig. 3.1(a) shows that the amplitude of the $(1, 7)$ mode remains dominant despite the crash, meaning that the QSH survives the event. This is a recurrent observation in RFX-mod [160]: many reconnection events do not completely disrupt the helical configuration and are therefore classified as *minor crashes* [39].

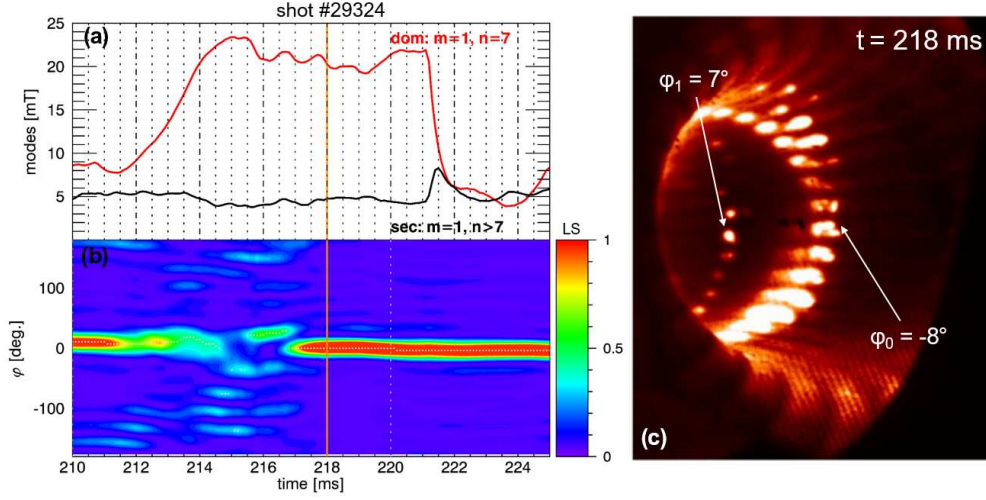


Figure 3.1: (a) Time evolution of the $m = 1$ mode amplitudes during discharge #29324. The red curve corresponds to the dominant $(1, 7)$ mode amplitude, while the black curve represents the root mean square of the secondary mode amplitudes, multiplied by a factor 2.5 to improve visibility. The vertical orange line marks the reference time $t = 218$ ms. (b) Locked-Mode structure, reconstructed through the Locking-Strength (LS) parameter, including harmonics in the range $8 \leq n \leq 23$. (c) CCD camera image of the PWI at the reference time instant $t = 218$ ms, adapted from Ref. 59.

Shot # 29324 at $t = 218$ ms has been selected as the case study of this thesis because it represents a clear example of a relaxation event, making the results presented here broadly applicable to magnetic reconnection phenomena in general. In addition, the availability of direct imaging of the associated PWI pattern, provided by the CCD camera, offers a key instrument to characterize in detail both the magnetic topology and the dynamics of the thermal content, which are the main focus of this work.

3.1.1 The Locking-Strength parameter

The Fourier decomposition of a generic magnetic perturbation reported in Eq. (1.14) employs the following expression for the helical phase¹:

$$\Omega_{m,n}(\theta, \varphi, t) = m\theta - n\varphi + \phi_{m,n}(t), \quad (3.1)$$

where m and n label the individual harmonic, and $\phi_{m,n}$ denotes the Fourier phase associated with the corresponding (m, n) mode. For simplicity, we restrict the analysis to the equatorial plane by fixing $\theta = 0$, thereby reducing the problem to a single degree of freedom. Under this assumption, the helical phase of the (m, n) mode takes the simplified form:

$$\Omega_{m,n}(\varphi, t) = n\varphi + \phi_{m,n}(t).$$

During the crash, secondary modes not only grow in amplitude, but also tend to align their phases [157, 161]. The main consequence of this process is a strong, toroidally localized

¹Here, and in the following of this Chapter, the device toroidal angle φ and the Boozer toroidal angle ζ can be equivalently used.

plasma deformation known as a *Locked Mode* (LM) [162, 163]. Experimentally, the LM causes particle losses associated with strong PWI [164]. The role of the LM in determining PWI will be discussed in Section 3.2. Here, we introduce a historical and widely used metric in RFX experiments to quantify the intensity of the Locked Mode, namely the *Locking Strength* (LS) parameter [82, 165]. The LS is defined as the normalized sum of the cosines of the relative phase difference among the main interacting modes:

$$\text{LS}(\varphi, t) = \frac{1}{1 + 2 + \dots + (n_{\max} - n_{\min})} \sum_{j=n_{\min}}^{n_{\max}-1} \sum_{k=j+1}^{n_{\max}} \cos[\Omega_{1,j}(\varphi, t) - \Omega_{1,k}(\varphi, t)] . \quad (3.2)$$

When the modes are perfectly aligned in phase, i.e. $\Omega_{1,j}(\varphi, t) = \text{const. } \forall j$, the system corresponds to a fully locked configuration, for which $\text{LS} = 1$. Conversely, in the absence of alignment, the phases are randomly distributed and $\text{LS} = 0$, indicating the lack of a Locked Mode. The contour plot reported in Fig. 3.1(b) shows the temporal evolution of the toroidal position of the LM, represented here by the LS maximum, in red. During the crash event, the LM is localized approximately at $\varphi \approx 0$. Notably, the regions of strongest PWI coincide with the locking position, which is a well-established property of RFP discharges [59, 121, 166].

3.1.2 The ideal displacement parameter

Since plasma-wall interactions are found in proximity of the Locked Mode, another approach consists in calculating the deformation of the plasma *Last Closed Flux Surface* (LCFS) induced by the tearing modes during the crash event. This methodology is routinely used in RFX-mod to describe the PWI pattern [121, 166].

For clarity, we consider a simplified framework where resistive effects are neglected. Within this approximation, the local *ideal displacement* $\Delta_{m,n}$ (hereafter also referred to as *shift*) associated with a generic (m, n) mode can be expressed as:

$$\begin{aligned} \Delta_{m,n}(r, \theta, \varphi, t) &= \text{Re} \left[\frac{-i r b_{m,n}^r(r) e^{i(m\theta - n\varphi + \phi_{m,n}(t))}}{[m - nq(r)] B_{0,\theta}(r)} \right] \\ &= \frac{r b_{m,n}^r(r)}{[m - nq(r)] B_{0,\theta}(r)} \cos[m\theta - n\varphi + \phi_{m,n}(t)] \\ &= \Delta_{m,n}(r) \cos[m\theta - n\varphi + \phi_{m,n}(t)] , \end{aligned} \quad (3.3)$$

where $b_{m,n}^r = b_{m,n}^r(r)$ is the radial amplitude of the mode, $q = q(r)$ the safety factor, and $B_{0,\theta} = B_{0,\theta}(r)$ the equilibrium poloidal magnetic field [167, 168]. In RFX-mod, $\Delta_{m,n}$ is reconstructed with the NewComb Toroidal (NCT) code [90]. In ORBIT, the ideal displacement is simply:

$$\Delta_{m,n} = \frac{mg + nI}{m - nq} \alpha_{m,n} , \quad (3.4)$$

where g and I are the toroidal and poloidal covariant magnetic field components (see Eq. (2.4)), and $\alpha_{m,n}$ is the scalar function describing the generic (m, n) harmonic, defined in Eqs. (2.30) and (2.31). Note that one can pass directly from ORBIT shift representation (3.4) to its canonical definition (3.3) and vice versa, simply by using the relation in Eq. (2.32) in cylindrical approximation.

3.2 Preliminary analytical description

3.2.1 The Locking Parameter

The working hypothesis guiding the following analysis is that the locations of the two footprints observed in the PWI pattern of Fig. 3.1(c) coincide with the maxima of the overall LCFS deformation at the plasma edge ($r = a$), associated with the secondary, locked modes. It is calculated by superposing the contributions of the individual modes:

$$\Delta_{sec}(\theta, \varphi) = \sum_{n=8}^{23} \Delta_{1,n}(\theta, \varphi) . \quad (3.5)$$

Here, the analysis is restricted to $m = 1$ modes only, since these resonate in the plasma core and produce a well-defined local displacement at the edge. In contrast, $m = 0$ modes resonate at the reversal surface, which is located close to the plasma edge (see Fig. 1.11), and would cause Eq. (3.3) to diverge.

In order to reproduce the position of the deformation maxima, we parametrize the Locking Strength (LS) to reflect the toroidal profile of the shift Δ_{sec} . By assuming that the Fourier phases of the modes are constant, $\phi_{m,n} = \text{const.}$, we effectively move into the reference frame where the Locked Mode is at rest at $\varphi_{\text{lock}} = 0$ [168, 169]. Under this assumption, the LS defined in Eq. (3.2) can be rewritten as:

$$\text{LS}(\varphi) = \frac{1}{N+1} \sum_{\ell=0}^N \cos(\ell\varphi) , \quad (3.6)$$

where $N = n_{\text{max}} - n_{\text{min}}$ ensures that the normalization condition $\text{LS} = 1$ is satisfied in the case of perfect mode alignment.

To refine this description, we relax the previous assumption that all $m = 1$ modes contribute with equal amplitude. This leads to the definition of the so-called *Locking Parameter* (LP):

$$\text{LP}(\varphi) = \frac{1}{D} \sum_{\ell=n_{\text{min}}}^{n_{\text{max}}} A_{\ell} \cos(\ell\varphi) . \quad (3.7)$$

The LP is therefore a generalized version of the LS that incorporates the spectral distribution of the $m = 1$ modes. In this expression, A_{ℓ} represents the weight associated with the ℓ -th mode, while $D = \sum_{\ell=n_{\text{min}}}^{n_{\text{max}}} A_{\ell}$ is a normalization constant ensuring that $\text{LP} = 1$ at its maximum value, analogously to LS.

A suitable choice for the weights, namely $A_{\ell} = n_{\text{max}} + 1 - \ell$, reproduces the characteristic decay of the secondary $m = 1$ spectrum, as will be discussed in Section 3.7. By setting $n_{\text{min}} \equiv 8$, one obtains $D = (n_{\text{max}} - 6)(n_{\text{max}} - 7)/2$. As a consequence, the LP is an even function of the toroidal angle φ and depends explicitly only on n_{max} , i.e. on the highest toroidal mode number contributing to the LM.

Fig. 3.2 compares the LCFS deformation (i.e. Δ_{sec}) with the LP(φ) profile calculated for $n_{\text{max}} = 23$. The two first minima of the LP profile are well aligned with the maxima of $\Delta_{sec}(\varphi)$. In fact, the angular distance between the two maxima of the shift is $\Delta\varphi^{\Delta} = 25^{\circ}$, while the distance between the corresponding LP minima is $\Delta\varphi_{n_{\text{max}}=23}^{\text{LP}} = 26^{\circ}$. However, this is still larger than the separation of the two footprints observed experimentally in Fig. 3.1(c), where $\Delta\varphi = \varphi_1 - \varphi_0 = 15^{\circ}$. An agreement between the LP profile and the experimental PWI pattern is recovered only by extending the summation to higher n

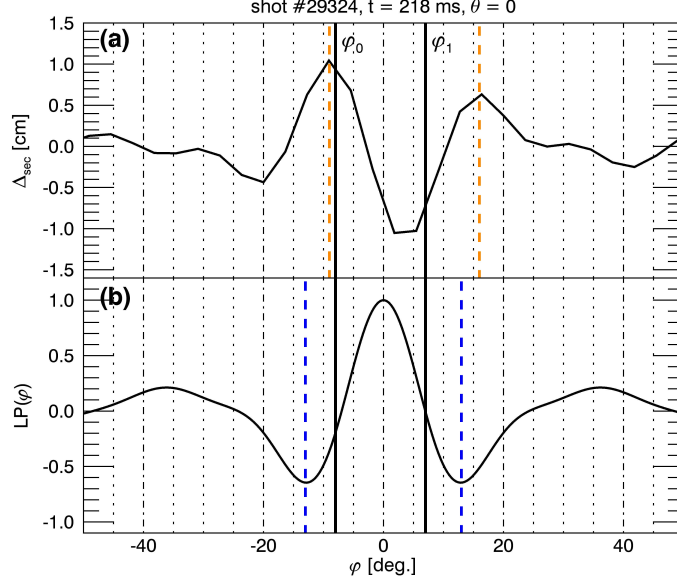


Figure 3.2: Comparison during shot #29324 at $t = 218$ ms between: **(a)** the LCFS deformation due to secondary modes, $\Delta_{\text{sec}}(\varphi)$, and **(b)** the Locking Parameter profile, $LP(\varphi)$. Dashed orange lines indicate the two maxima of $\Delta_{\text{sec}}(\varphi)$, while dashed blue lines indicate the first two minima of $LP(\varphi)$. The experimentally observed PWI footprints are located at toroidal angles φ_0 and φ_1 , highlighted by solid vertical lines.

values, beyond $n_{\text{max}} = 24$. This suggests that a larger set of tearing modes contributes to the locking inducing PWI than those directly resolved by the magnetic probes of RFX-mod [119]. Such evidence is consistent with earlier observations of RFX-mod, which already reported signatures of modes with $n > 24$ [59, 116].

By imposing that the distance between the LP minima coincides with the angular separation between the experimentally observed PWI footprints, namely $\Delta\varphi^{LP} = \Delta\varphi = 15^\circ$, one obtains that:

$$n_{\text{max}} = 45 . \quad (3.8)$$

This result is derived analytically by exploiting the theory of Dirichlet kernels; the full derivation is reported in Appendix C. It confirms that the observed PWI pattern requires contribution of modes with $n > 24$. This finding has already been considered in the upgraded design of the magnetic sensor array of RFX-mod2 [97, 116, 117], which will extend the spectrum of measured harmonics up to $n_{\text{max}} = 36$. Such an improvement is expected to enhance active mode control and reduce likelihood of severe PWI events in future operations.

3.2.2 Three-dimensional reconstruction of the LCFS shape during crash

In Fig. 3.2, we reported the toroidal profile of the LCFS deformation caused by $m = 1$ secondary modes, evaluated at the plasma edge ($r = a$) and at the equatorial plane ($\theta = 0$). However, the LCFS deformation induced by the LM during a crash is inherently three-dimensional, according to the fact that the shift Δ depends on r , θ and φ (see Eq. (3.4)). The deformed LCFS can be parametrized in cylindrical coordinates (R, φ, Z)

in the following way [166]:

$$\begin{aligned} R &= R_0 + \Delta_H + (a_p + \delta_0 + \delta_1) \cdot \cos \theta , \\ Z &= \Delta_V + (a_p + \delta_0 + \delta_1) \cdot \sin \theta . \end{aligned} \quad (3.9)$$

The geometrical meaning of the terms appearing in the equations above is summarized in Fig. 3.3, where a generic poloidal section is sketched. Here, R_0 represents the plasma major radius, which is assumed to be the same of the RFX device ($R_0 = 2$ m); a_p is the plasma column radius, and is indicated in light pink; Δ_H represents the *horizontal shift* corresponding to the Shafranov shift associated with toroidicity, and is highlighted in green in the zoomed panel of Fig. 3.3. Similarly, Δ_V indicates the *vertical shift* associated with vertical instability, and is highlighted in blue in the same zoomed panel. The term $\delta_0 = \delta_0(\theta)$ represents the LCFS deformation associated with the $m = 0$ modes. This contribution modifies the plasma column by locally expanding or contracting its radius and therefore does not depend on the toroidal angle. The resulting deformed plasma column radius, $a_p + \delta_0(\theta)$, is indicated in dark pink in the sketch. Instead, $\delta_1 = \delta_1(\theta, \varphi)$ represents the LCFS deformation due to the $m = 1$ modes, and corresponds to a local shift of the plasma column. The displaced plasma column is colored in red in the sketch. Both δ_0 and δ_1 are reconstructed from magnetic probes measurements and therefore are time-dependent quantities. In the present analysis, they are evaluated at the case study time instant (shot #29324 at $t = 218$ ms).

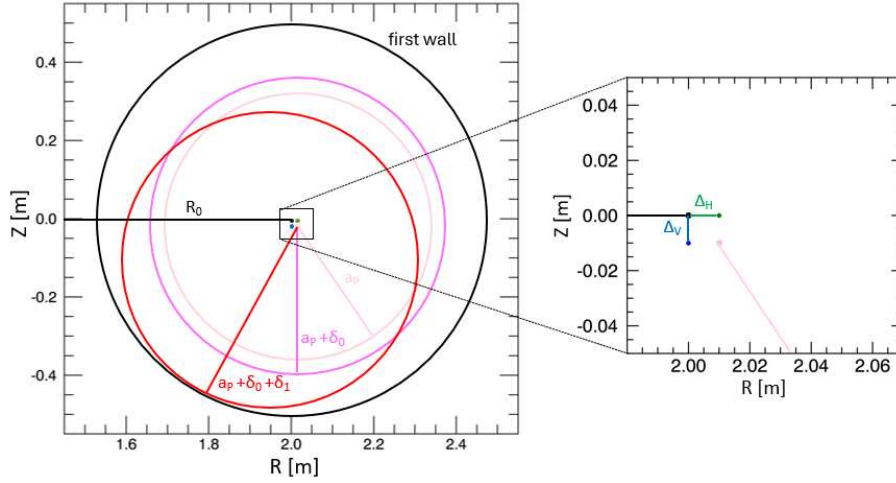


Figure 3.3: Sketch of the poloidal section of the reconstruction of the total Last Closed Flux Surface deformation during shot #29324 at $t = 218$ ms.

The quantity δ_1 coincides with $\sum_{n=7}^{24} \Delta_{1,n}$, where $\Delta_{1,n}$ is defined according to Eq. (3.3), and has a typical amplitude of a few centimeters. In contrast, δ_0 cannot be calculated using Eq. (3.3), since the denominator term $m - nq$ would become very small due to the proximity of the reversal surface (where all $m = 0$ modes resonate) to the wall, leading to a divergence. However, an estimate of the relative profile of the total $m = 0$ deformation can still be obtained by performing a Fourier analysis under the simplifying assumption of cylindrical geometry.

In order to assign a physical plausible amplitude to the $m = 0$ deformation, we impose that the deformed LCFS touches the wall at one only point over the entire wall toroidal surface. Then, we normalize the deformed plasma radius $a_p + \delta_0$ accordingly. The resulting reconstruction of the three-dimensional LCFS shape during the crash is shown in Fig. 3.4. It shows a qualitative highly complex geometry, which can be interpreted as a direct consequence of the contribution of a broad spectrum of tearing modes, as discussed in the previous section.

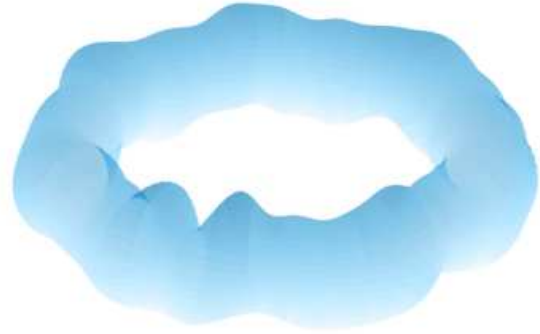


Figure 3.4: Three-dimensional reconstruction of the LCFS shape during shot #29324 at $t = 218$ ms. To simplify visibility, δ_0 and δ_1 have been multiplied by a factor 10.

3.3 Simulations in ORBIT

In the original RFX device, PWI was almost entirely governed by the LM, which was also locked to the wall [170]. In this configuration, commonly referred to as *wall-locked*, experimental measurements scaled directly with the LM toroidal position, and no advanced analysis was required [121, 164, 171] to interpret the PWI pattern induced by LM. With the advent of RFX-mod the situation changed: the LM was arbitrarily set into rotation and its amplitude significantly reduced in size. This revealed the important role of $m = 0$ modes in shaping the PWI, a result obtained through more refined methods, such as the reconstruction of the LCFS deformation [166]. Subsequently, the introduction of the active feedback control system [110] and the exploration of high-current operational scenarios [79], further modified the overall picture: the Quasi-Single Helicity (QSH) state emerged as the dominant regime governing PWI [172–174], while the LM contributed only during crash events [39, 59].

The analysis presented in Section 3.2 relies on the linear superposition of local displacements produced by individual modes. While this approach simplifies calculations, it neglects the nonlinear interaction among modes from the full $m = 0$ and $m = 1$ spectra. The present scenario therefore requires to develop nonlinear, numerical methods capturing the magnetic topology with much more level of detail than what was done in the past. In this work, such an approach is provided through the Hamiltonian, guiding-center code ORBIT [56].

ORBIT is here used to simulate ion dynamics in a perturbed magnetic field, which is reconstructed via the NCT code, as mentioned in Section 2.4. To accurately characterize the magnetic field topology, only ions having very low energy (10^{-2} eV) are considered in the simulations discussed in this chapter, following a past well-established approach [127].

The analysis presented here is also of potential interest to the tokamak community. Indeed, phase-locking of $(2, 1)$ and $(3, 1)$ modes is frequently observed in tokamak plasmas [52], and the resulting magnetic field stochasticity [47, 175, 176] has been identified as a key mechanism preceding disruptions [51, 55] and fast-particle losses [50].

3.3.1 Poincaré plot

Considering a dynamical system, a convenient way to represent particle motion is to record the intersections of its trajectory with a reference surface, which must be perpendicular to the dynamical flux. The set of such intersections defines a Poincaré plot [177]. Formally, a Poincaré section is a surface $F(x, y, z) = 0$ such that it is nowhere tangent to any trajectory of the system:

$$\vec{v} \cdot \nabla F(x, y, z) \neq 0 \quad \forall (x, y, z), \quad (3.10)$$

i.e. the particle velocity is never tangent to the surface [178]. This distinction is represented in the sketch of Fig. 3.5: the blue trajectory ($\vec{v}_1$) satisfies the definition of a Poincaré section, whereas the red one ($\vec{v}_2$) does not. In this chapter, since we are dealing with low-energy particles, trajectories are replaced by magnetic field lines [127], thus composing a so-called *magnetic* Poincaré plot.

The RFP is a toroidal magnetic configuration, for which two types of Poincaré sections are commonly employed. The first is the poloidal Poincaré section, obtained by slicing the torus with a poloidal plane (see Fig 3.6(a)). However, the reversal surface cannot be included, since field lines are tangent to the section there. Alternatively, one can consider a toroidal Poincaré section, obtained by slicing the torus with a toroidal plane (see Fig 3.6(b)), resulting in a plot on the $\varphi - r$ plane.

In this case, the magnetic axis at $r = 0$ cannot be included for the same reason.

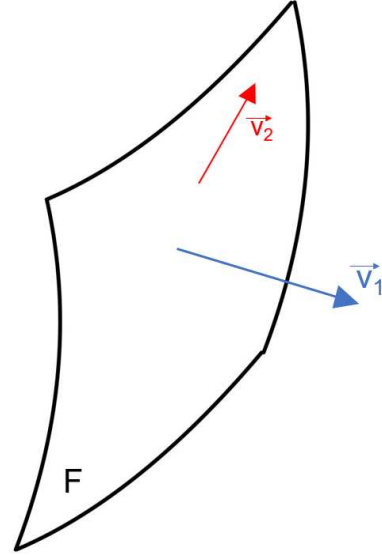


Figure 3.5: The vector $\vec{v}_1$, shown in blue, represents a valid trajectory for the Poincaré section F , whereas the vector $\vec{v}_2$, shown in red, is not compatible with the definition of F as a Poincaré section.

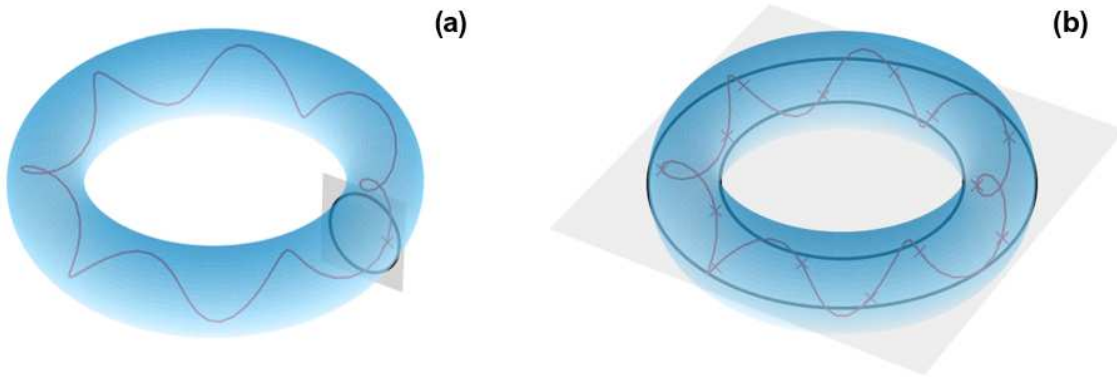


Figure 3.6: Representation of the two possible types of Poincaré plots for a toroidal configuration such as RFX-mod: **(a)** poloidal Poincaré plot and **(b)** toroidal Poincaré plot. The red trajectory represents a generic magnetic field line. Its intersections with the Poincaré section (outlined in black in both panels) are marked by red $\times$ symbols.

3.3.2 Poincaré plots during crash

To describe the PWI case occurring during the crash at #29324 at $t = 218$ ms, Fig. 3.7 reports the toroidal equatorial Poincaré plot calculated with ORBIT. This figure collects the magnetic field lines crossing the equatorial plane at the crash time, thereby providing a direct representation of the magnetic topology during the event. In this plot, magenta points correspond to trajectories of low-energy particles (or magnetic field lines) deposited in the plasma core, while cyan/green points mark trajectories of particles deposited on the plasma edge. The geometrical toroidal axis is shown as a black dotted line. The toroidal positions of the experimentally observed PWI footprints, φ_0 and φ_1 , are also highlighted.

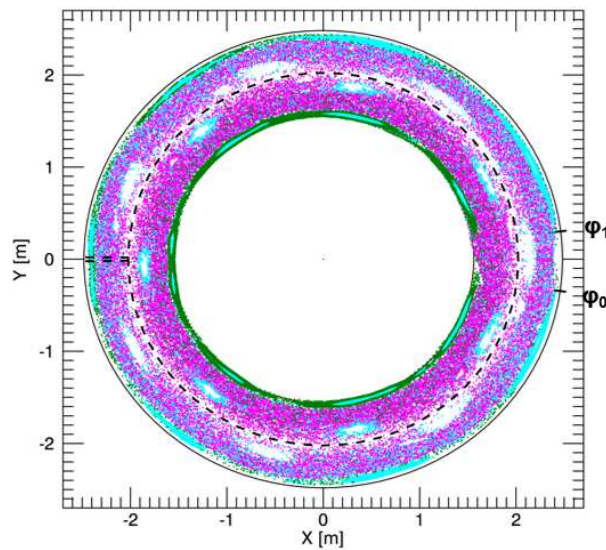


Figure 3.7: Toroidal equatorial magnetic Poincaré plot during discharge #29324 at $t = 218$ ms. Magenta points indicate particles deposited in the plasma core, while cyan and green points correspond to particles deposited near the plasma edge. The geometrical toroidal axis is shown as a black dotted line. The toroidal positions of the experimentally observed PWI footprints, φ_0 and φ_1 , are also highlighted.

For clarity, the Poincaré plot of Fig. 3.7 has been cut along the dotted lines, focusing on the outward equatorial plane, and subsequently warped. The resulting plot, expressed in toroidal coordinates, is shown in Fig. 3.8. The points colored in magenta were deposited on the resonance surface of the $(1, 7)$ mode, black points refer to $m = 1$ secondary modes. The points in cyan and blue were deposited on the reversal surface. The Poincaré plot can be interpreted by distinguishing three radial regions:

- The inner region ($r \leq 25$ cm) is occupied by the chain of magnetic islands associated with the dominant $(1, 7)$ mode. Their presence demonstrates that the QSH state persists despite the crash, consistently with the classification of this event as a *minor* crash. The islands are centered around $r_{1,7} \approx 15$ cm, in agreement with the radial location of the corresponding $q_{1,7}$ surface (see Fig. 1.11). Their toroidal extension is of the order of 1 m, while the radial width is about 10 cm.
- The mid-radius region ($25 \text{ cm} \lesssim r \lesssim 35 \text{ cm}$) is the domain where secondary modes resonate. As a consequence of their phase-locking, the related islands grow in width and overlap with each other generating a chaotic region [179].

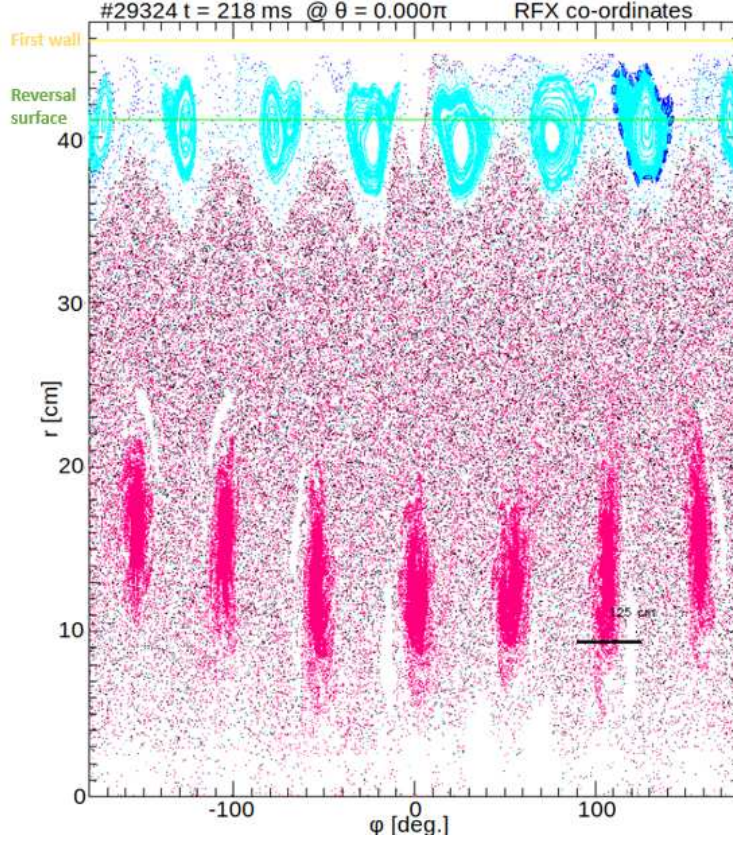


Figure 3.8: Toroidal equatorial magnetic Poincaré plot during discharge #29324 at $t = 218$ ms. The black bar corresponds to a length of 125 cm. Magenta points refer to the dominant $(1, 7)$ mode, black points to the secondary $m = 1$ modes, and cyan points to the $(0, 7)$ mode. Blue points trace magnetic field lines associated with $(0, \text{high-}n)$ modes, forming a Fibonacci-like sequence of secondary $m = 0$ islands. The horizontal green line marks the reversal surface, while the yellow line indicates the position of the device first wall.

- The outer edge region ($r > 35$ cm) is characterized by a chain of $(0, 7)$ mode islands aligned with the reversal surface, indicated by the horizontal green line. They exist because of toroidal coupling between the $(1, 7)$ mode and the Shafranov shift [90]. These islands are smaller than the core $m = 1$ structures by a factor $\varepsilon = A^{-1}$ being $A = R_0/a = 4$ the aspect-ratio of RFX-mod. Each $m = 0$ island is surrounded by a hierarchy of smaller companions. An example is clearly visible in Fig. 3.8 as a set of blue points near $\varphi = 130^\circ$. The distribution follows a Fibonacci-like sequence of secondary structures, forming chains of progressively smaller islands [180]. In this case, a sub-island chain of 19 islands is observed. Since $19 = 7 + 3 \times 4$, we claim that $a_0 = 7$ e $a_1 = 19$ can be interpreted as the first two terms of a generalized recurrence relation: $a_{n+1} = a_n + 3 \times a_{n-1}$. The sequence a_n/a_{n+1} converges to $(\sqrt{13} - 1)/6$, corresponding to the last Kolmogorov-Arnold-Moser (KAM) [181] conserved surface [182, 183] for the $m = 0$ island chain.

To connect this topological picture with the observed PWI, Fig. 3.9(a) shows a Poincaré plot over a zoomed toroidal sector ($-50^\circ \leq \varphi \leq 50^\circ$ and $30 \text{ cm} < r < 46 \text{ cm}$), encompassing the PWI region. Fig. 3.9(b) shows the same plot, but with $m = 0$ modes removed from

the perturbation spectrum provided as input to ORBIT. In this case, the border of the chaotic sea has the same shape as the deformation profile $\Delta_{sec}(\varphi)$ shown in Fig. 3.2(a), thus justifying the use of the local displacement as a first approximation of the magnetic topology. Most notably, Fig. 3.9(b) clearly displays a bundle of magenta and black points escaping from the chaotic sea and reaching the wall at $\varphi \approx 10^\circ$, i.e. near the LM position. This correspondence indicates that the PWI stripe at φ_1 can be attributed to losses caused by the $m = 1$ LM. Similarly, in Fig. 3.9(a) a smaller set of blue points reaches the wall at $\varphi \approx -10^\circ$, suggesting that the second footprint at φ_0 originates from ions leaking along the "tail" of the nearby $m = 0$ island toward the wall.

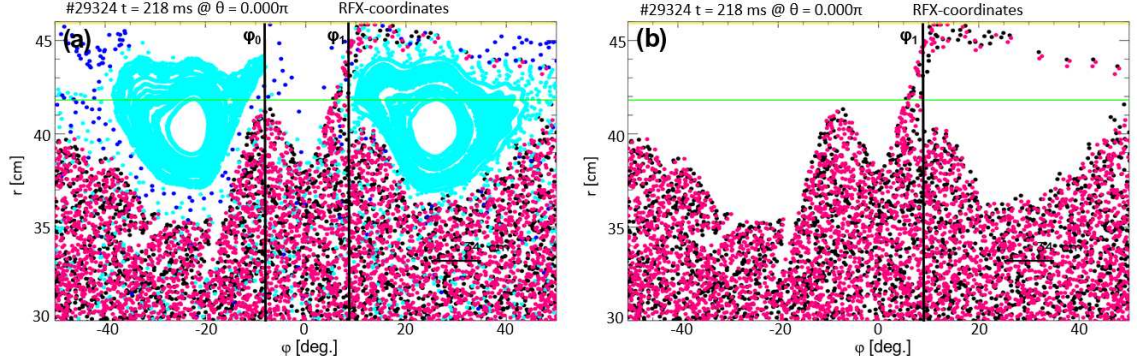


Figure 3.9: Toroidal Poincaré plots on the equatorial plane for discharge #29324 at $t = 218$ ms: (a) including the full spectrum of $m = 0, 1$ modes with $n \leq 24$; (b) including only $m = 1$ modes. The solid vertical lines mark the toroidal positions φ_0 and φ_1 of the experimentally observed PWI footprints.

3.4 Connection Length to the wall

The previous section concluded that the two PWI footprints may be produced from two distinct mechanisms: the stripe at $\varphi_1 = 7^\circ$ is associated with the phase-locking of $m = 1$ secondary modes, whereas the stripe at $\varphi_0 = -8^\circ$ may be related to the tail of the nearest $m = 0$ magnetic island. This interpretation is further supported by the observation that the footprint at φ_0 in Fig. 3.1(c) appears broader than the one at φ_1 . Nevertheless, to validate this hypothesis, a more detailed analysis is required. We use the Connection Length to the wall defined in Section 2.5. As shown in the previous section, the RFP edge is dominated by $m = 0$ magnetic islands embedded in a chaotic sea [174, 184], making it a suitable environment for the calculation of this metric.

Fig. 3.10 shows the contour map of $L_{c,w}(\varphi, r)$ together with the corresponding Poincaré map in the outermost ≈ 10 cm of plasma, focusing on the PWI region $-40^\circ \leq \varphi \leq 30^\circ$. The calculation is performed by considering a grid of initial positions $[r_i, \theta_j, \varphi_k]$ with $1 \leq i \leq 8$, $1 \leq j \leq 32$, $1 \leq k \leq 64$. Then, to reduce numerical error, the Connection Length is estimated by averaging over a bundle of 1000 particles, started at $r = r_i$, $\theta = \theta_j$ and φ sampled from an array of 1000 equally-spaced values centered in φ_k . The simulated particles are monoenergetic and collisionless, with an energy of 10^{-2} eV, as done for Poincaré plots.

The Connection Length spans values from a few centimeters up to ≈ 10 km, which are typical both of the RFP [153] and of tokamak ergodic divertors [154]. Regions of short

$L_{c,w}$ (highlighted in red in the contour plot) correspond to particle fast losses [185]. The LM position at φ_1 emerges as a preferential loss channel, extending deep into the plasma core, in agreement with previous ORBIT simulations in the past [59, 77].

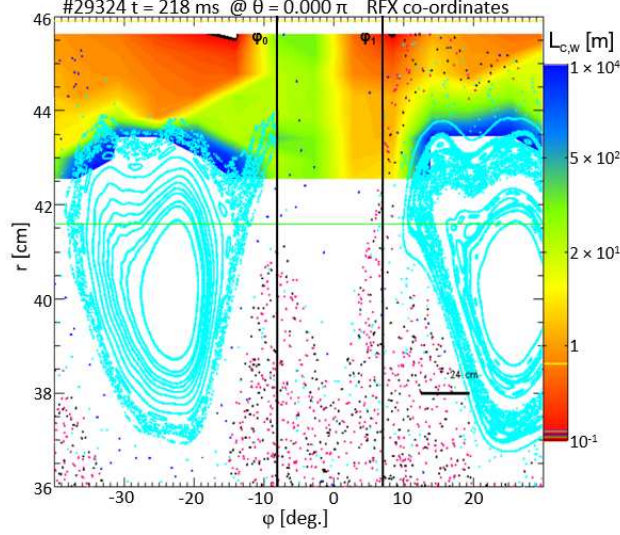


Figure 3.10: Map of the Connection Length to the wall, $L_{c,w}(\varphi, r)$, together with the corresponding equatorial Poincaré plot near the PWI region. Short and long connection lengths are shown in red and blue, respectively. The toroidal positions of the experimentally observed PWI footprints, φ_0 and φ_1 , are indicated by solid vertical lines.

Moreover, this result is independent of particle energy, since ORBIT normalizes time steps to the toroidal transit time; the explicit dependence on energy will be discussed in Section 4.1. In addition to the LM channel, Fig. 3.10 reveals a second loss region at $\varphi \lesssim \varphi_0$, located between the (0, 7) island and the wall [174]. This contributes to the identification of two distinct particle loss mechanisms: parallel losses at the LM toroidal location, at φ_1 , and losses mediated by rotation [184] around the (0, 7) island at φ_0 . Further insight into this rotation-driven loss mechanism will be provided in Section 3.5.2.

3.4.1 Comparison with the experimental image

In Fig. 3.11, the Connection Length is calculated for particles started at $r = 45.9$ cm, by varying the angular coordinates (φ, θ) over a 128×64 grid spanning $-40^\circ \leq \varphi \leq 30^\circ$ and $-50^\circ \leq \theta \leq 50^\circ$. This representation enables a direct mapping of the device first wall in terms of connection lengths. In particular, the $L_{c,w}$ map is displayed as it would appear from the experimental camera viewpoint ($\varphi_{CCD} \approx 277^\circ$), allowing a direct comparison with the experimental CCD image, reported on the right.

The agreement between the two maps is evident: both exhibit two red regions, corresponding to short connection lengths in the simulation and to enhanced PWI footprints in the experiment, sharing almost the same shapes and toroidal locations.

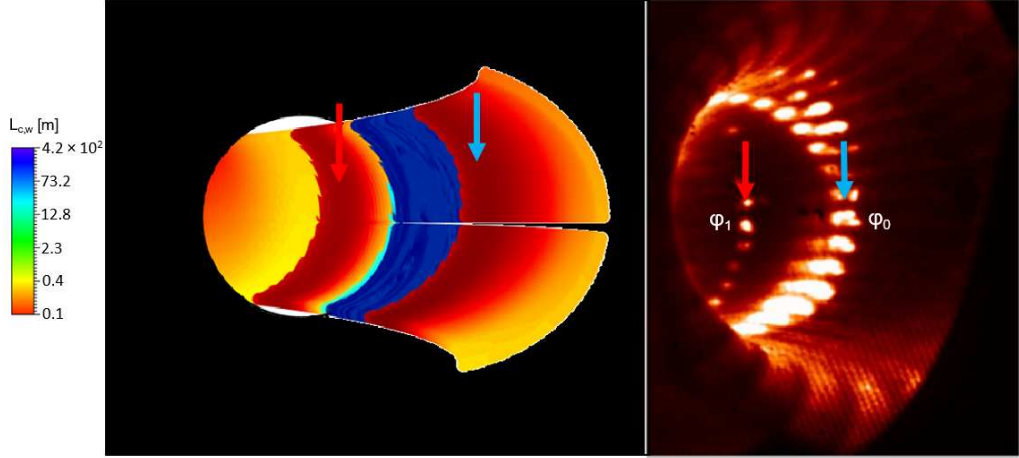


Figure 3.11: Direct qualitative comparison between the $L_{c,w}(\varphi, \theta)$ map at $r = 45.9$ cm (left), represented in toroidal geometry, and the experimental CCD camera image (right). Short connection lengths are colored in red, while long connection lengths are colored in blue. Arrows highlight corresponding PWI stripes in the two images: cyan for the $m = 0$ -related PWI stripe on the right and red for the $m = 1$ -LM-related PWI stripe on the left.

A more quantitative comparison is obtained by considering the two-dimensional projection onto the $\varphi - \theta$ plane. Fig. 3.12 presents the connection length map (panel a) alongside the warped experimental image (panel b). The correspondence is again clear: both maps display a narrower stripe on the right and a broader, slanted stripe on the left. Nevertheless, some discrepancies remain: the simulated stripes appear slightly shifted with respect to φ_0 and φ_1 , likely due to uncertainties in camera alignment with the vessel [173]. Moreover, the spacing between the short- $L_{c,w}$, red regions is larger than that between experimental PWI stripes, reflecting the limited number of modes included in the spectrum sent as input to ORBIT, constrained by the finite resolution of magnetic probes, as already discussed in detail in Sections 3.2 and 3.3.1.

Despite these differences, the overall result is robust: tearing modes determine the spatial pattern of PWI, which can be clearly divided into three regions. The short- $L_{c,w}$ region at φ_1 corresponds to the stripe generated by secondary $m = 1$ mode locking; the second short- $L_{c,w}$ region at φ_0 corresponds to the broader stripe associated with the $(0, 7)$ island; finally, the central region, characterized by longer connection lengths, corresponds to the LM counterpart, which does not produce significant PWI due to the field-line pitch [154], in analogy with the unstable manifold observed for tokamak homoclinic tangles [47, 175].

The observation that the $(0, 7)$ island is responsible for the broader carbon emission stripe detected by the CCD camera confirms the importance of $m = 0$ modes in shaping PWI closely to the LM region. This concluding remark is consistent with earlier results in both in RFX [121] and the Japanese TPE-RX experiment [186].

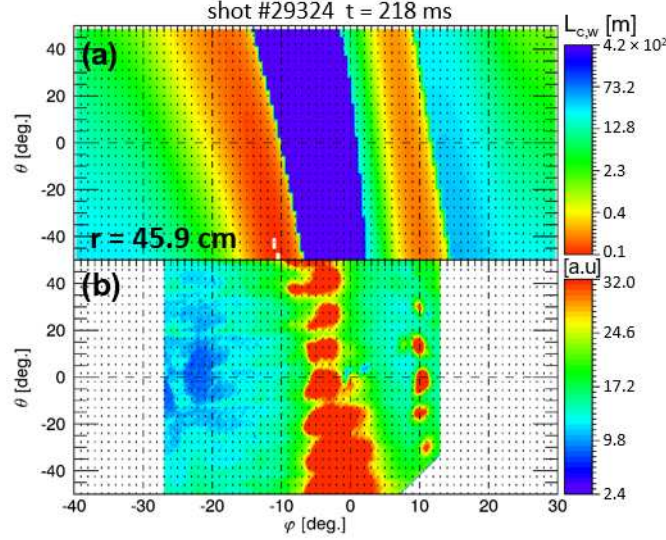


Figure 3.12: (a) High-resolution map of connection lengths $L_{c,w}(\varphi, \theta)$, at fixed radius $r = 45.9$ cm. (b) Warped version of the experimental PWI image of Fig. 3.1(c), where red regions correspond to PWI hot spots.

3.5 Alternative metrics

3.5.1 Linear analysis: impact angles

The field-line pitch provides a useful metric to characterize particle transport in both tokamak and stellarator configurations [154], particularly in the divertor region [152]. Referring to the sketch in Fig. 3.13, the field-line pitch can be defined as: $\Theta_{fld} = dx/d\ell$, where $d\ell$ is the arc length of the particle trajectory (e.g. an ion) towards the wall, and dx is its projection along the direction perpendicular to the wall. Expressed in terms of magnetic field components, this definition becomes: $\Theta_{fld} = B_r/B$, where B_r is the radial component of the magnetic field and B its magnitude. From simple geometrical considerations, the corresponding *impact angle* is given by:

$$\alpha_{imp} = \arcsin\left(\frac{dx}{d\ell}\right) = \arcsin\left(\frac{B_r}{B}\right) = \arcsin \Theta_{fld} . \quad (3.11)$$

In RFX-mod, both B_r and B can be experimentally determined by means of magnetic probes. Accordingly, Eq. (3.11) allows the impact angle α_{imp} of ions to be calculated during the crash of shot #29324 at $t = 218$ ms. The resulting contour plot, namely a map of α_{imp} over the PWI region, is shown in Fig. 3.14.

Positive values of α_{imp} are colored in red, while negative values correspond to bluish regions. The map reproduces with good consistency the pattern already observed

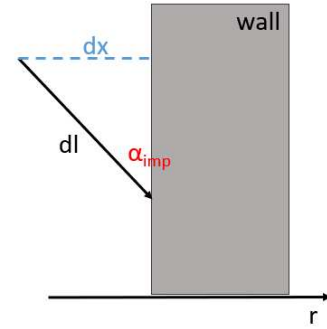


Figure 3.13: Definition of the ion impact angle: $d\ell$ represents the length of the trajectory followed by an ion towards the wall. Its projection along the direction perpendicular to the wall is indicated by dx (in blue). The angle α_{imp} represents the ion impact angle with respect to the wall surface.

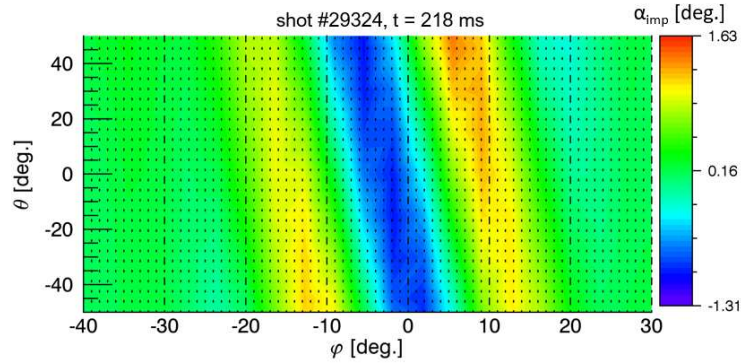


Figure 3.14: Map of ion impact angles during discharge #29324 at time $t = 218$ ms. Positive and negative impact angles are shown in red and blue, respectively.

in the connection lengths map of Fig. 3.12(a): two red regions separated by a central blue zone, reflecting the same underlying physical interpretation. Moreover, the color-scale in Fig. 3.14 indicates that α_{imp} lies within a very narrow range: $-2^\circ \lesssim \alpha_{imp} \lesssim 2^\circ$. This means that the ions lost during the crash event strike the first wall at extremely *grazing* incidence. This result is of primary relevance, since the impact angle of plasma particles strongly influences key plasma-wall interaction processes, such as chemical [187] and physical [164] sputtering, and hydrogen retention [188] with consequently impurity accumulation at the plasma edge, particularly in the presence of coated first wall components.

3.5.2 ORBIT analysis: Recurrence Time

In Section 3.4, we have shown that the Connection Length $L_{c,w}$ provides a suitable metric for characterizing magnetic topology and PWI. Nevertheless, its calculation becomes increasingly demanding in terms of the computational resources when regions far from the wall are considered, especially when high-resolution grids are required. For these reasons, an alternative approach based on the so-called *Poincaré Recurrence Time* (hereafter simply *recurrence time*) has been proposed [189]. Referring to the sketch in Fig. 3.15, let us consider a limited spatial domain Γ . The *escape* time is defined as the time interval elapsed between the first entrance of a particle into Γ at t_{IN1} , and its subsequent exit at t_{OUT} :

$$\tau_{esc} = t_{OUT} - t_{IN1} .$$

The escape time is represented by the green line in Fig. 3.15. Analogously, the *external* time is defined as the interval between the exit time t_{OUT} and the second entrance of the particle into Γ at t_{IN2} :

$$\tau_{ext} = t_{IN2} - t_{OUT} .$$

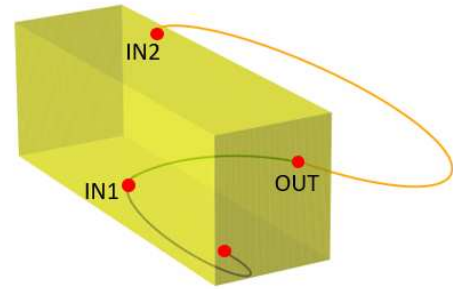


Figure 3.15: Definition of recurrence time. The domain Γ is represented by the yellow parallelepiped. The *escape* time corresponds to the time spent by particle following the green trajectory, while the *external* time corresponds to the time spent by particle following the yellow trajectory. Adapted from Ref. 174.

The external time is represented as a yellow line in Fig. 3.15. The recurrence time, τ_{rec} , is then simply the sum of these two contributions:

$$\tau_{rec} = \tau_{esc} + \tau_{ext} = t_{IN2} - t_{IN1} . \quad (3.12)$$

Hence, the recurrence time corresponds to the interval between two consecutive entrances of the same particle into the chosen domain Γ . A further advantage of the recurrence time, with respect to the Connection Length, is that it depends solely on the geometry of the selected domain and not on the final position reached by the particle. Conversely, it results ill-defined for those particles that are immediately lost to the wall without a second entrance into the selected domain; in such cases, the second entrance time t_{IN2} can not be defined. This makes calculation of recurrence time not reliable for regions at the immediate vicinity of the first wall.

Following the methodology successfully applied in previous RFP studies [174], a map of $\tau_{rec}(\varphi, r)$ was computed for discharge #29324 at $t = 218$ ms. A set of ellipsoidal domains was adopted, centered on points of a grid analogous to that used for the $L_{c,w}(\varphi, r)$ map. Since all test particles are assumed to have the same energy (and thus the same velocity v_{th}), the recurrence time can be converted into a characteristic recurrence length: $L_{rec} = \tau_{rec} \cdot v_{th}$. The resulting map is reported in Fig. 3.16, which can be directly compared with the corresponding Connection Length map shown in Fig. 3.10 [184].

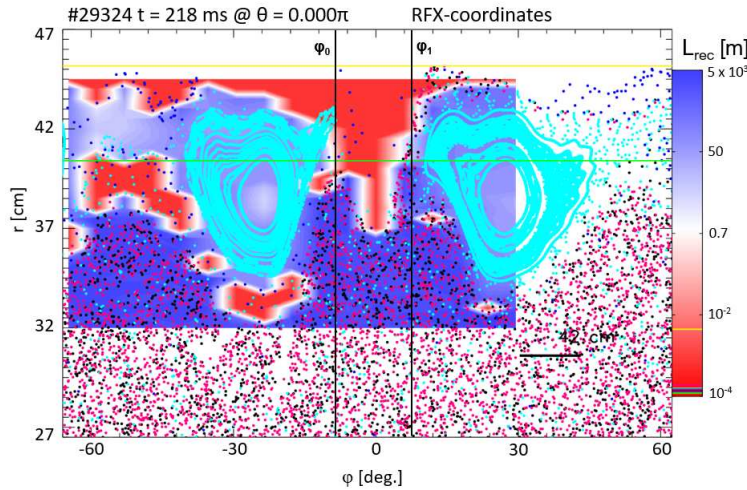


Figure 3.16: Map of the recurrence length, $L_{rec}(\varphi, r)$, together with the corresponding equatorial Poincaré plot on a portion close to the PWI region. Short and long recurrence lengths are shown in red and blue, respectively. The toroidal positions of the experimentally observed PWI footprints, φ_0 and φ_1 , are indicated by solid vertical lines.

The analysis highlights two main features. First, the recurrence method reproduces the topology of the inner plasma, encompassing the entire $m = 0$ island located on the left-hand side of the map. Second, the map reveals a remarkable separation between blue regions, characterized by $L_{rec} \geq 50$ m, and red regions, where $L_{rec} < 1$ m, corresponding to channels of enhanced particle transport. The loss channel associated with the LM is clearly visible at $\varphi \approx 0$. In addition, Fig. 3.16 shows a halo of short recurrence lengths extending from the left-hand side of the (0, 7) island and surrounding it. This halo can be interpreted as the consequence of the clockwise rotation of magnetic field lines around the island and represents an additional particle loss channel. These two mechanisms

correspond precisely to the regions of short connection length identified in Fig. 3.12(a) and to the two PWI footprints observed in the CCD camera image shown in Figs. 3.1(c) and 3.12(b). We therefore conclude that the recurrences can describe the PWI losses in the RFP edge as well as connection lengths.

3.6 The 3D nature of Connection Length

In tokamak studies, it is customary to represent the Connection Length $L_{c,w}$ as a two-dimensional map in either the $\varphi - r$ or $\varphi - \theta$ planes, as done in the previous section. In the RFP configuration, however, the magnetic field is characterized by an intrinsically three-dimensional topology, containing hidden structures, such as *streamers* or *cantori*, that are not easy to be revealed [72, 185]. In this section, we therefore aim to explore the fully three-dimensional nature of the Connection Length by extending the analysis to inner radial positions with respect to that considered in Fig. 3.12(a).

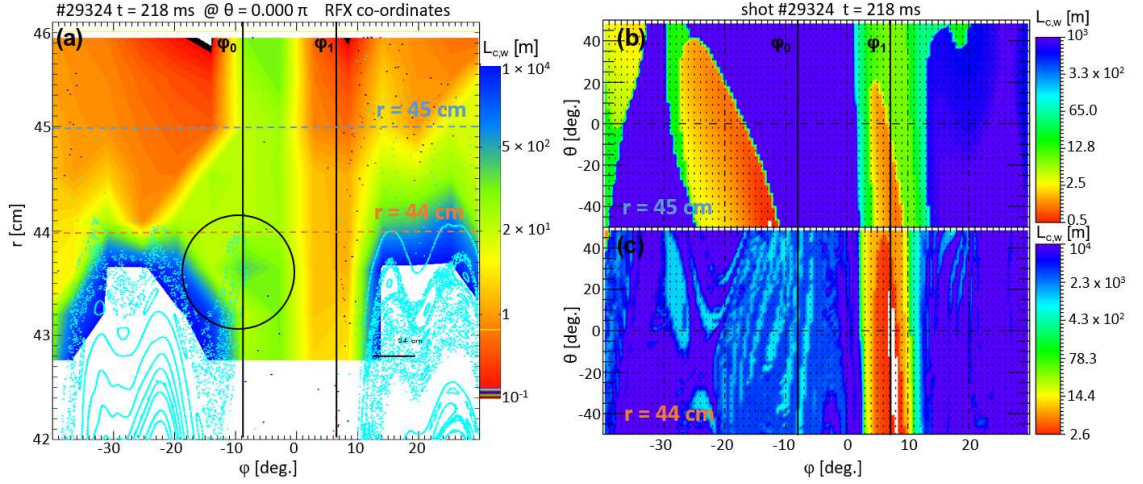


Figure 3.17: (a) Zoomed view of Fig. 3.10 near the PWI region. The two horizontal dashed lines indicate the radial positions to $r = 45$ cm (in blue) and $r = 44$ cm (orange). The tail of the $(0, 7)$ island is highlighted by a circle. (b) $L_{c,w}(\varphi, \theta)$ map at $r = 45$ cm. (c) $L_{c,w}(\varphi, \theta)$ map at $r = 44$ cm.

Fig. 3.17 shows $L_{c,w}(\varphi, \theta)$ maps calculated with ORBIT for particles started at $r = 45$ cm (panel (b)) and $r = 44$ cm (panel (c)) using the same grid of Fig. 3.12(a), together with the (φ, r) map with the overplotted Poincaré plot (panel (a)). As expected, the absolute values of $L_{c,w}(\varphi, \theta)$ increase when moving radially inward. At $r = 45$ cm, both short-connection-length stripes remain clearly visible, although they are shifted toward negative θ values. This displacement is consistent with the helicity of the dominant $(1, 7)$ mode². At $r = 44$ cm, the topology changes significantly. The left stripe, previously associated with the $(0, 7)$ island, disappears from the map, whereas the right stripe at φ_1 , linked to the LM, remains clearly visible in the contour plot. This observation confirms that the LM constitutes a loss channel that extends deeply into the plasma core, in agreement with earlier results obtained in RFX [59, 77]. In place of the vanished left stripe, a set of filamentary, light-blue structures characterized by intermediate values of $L_{c,w}$ appears. These filaments are plausibly associated with the tail of the $m = 0$ island, as highlighted

²This aspect will be further investigated in Section 3.8.

in Fig. 3.17(a). Although such filamentary structures have not yet been directly measured in RFX-mod, the forthcoming RFX-mod2 device will be equipped with an upgraded electrostatic diagnostic system capable of resolving filamentary structures of various origins, including high- n tearing modes and ELMs, in both RFP and tokamak configurations [190].

3.7 Results with an extended MHD spectrum

The analyses presented indicate that the experimentally observed PWI footprints are consistent with tearing-mode spectra including both $m = 0$ and $m = 1$ modes, with toroidal mode numbers extending up to $n_{max} = 45$. This result is remarkable, since in RFX-mod the spectrum can be experimentally resolved with magnetic probes only up to $n = 24$. To test this hypothesis, ORBIT simulations were repeated by constructing a synthetic mode spectrum. This spectrum was obtained by matching the experimentally measured amplitudes for $n \leq 24$, and extrapolating the eigenfunctions $\alpha_{m,n}(r)$ for modes with $m = 0, 1$ and $24 < n \leq 45$ according to a linear decay:

$$\max\{\alpha_{m,n}(r)\} = \frac{n_{max} + 1 - n}{n_{max} + 1 - n^*} \max\{\alpha_{m,n^*}(r)\}, \quad (3.13)$$

with $n_{max} \equiv 45$ and n^* denoting a suitable reference mode close to $n = 24$ used for extrapolation. This approach is consistent with methodologies previously adopted in RFP studies [77]. Fig. 3.18 shows the spectrum of the $m = 1$ secondary modes from which Eq. (3.13) is derived. The linear decay trend is represented by a red line and motivates the choice adopted in Eq. (3.7) for the weights used in the definition of the Locking Parameter.

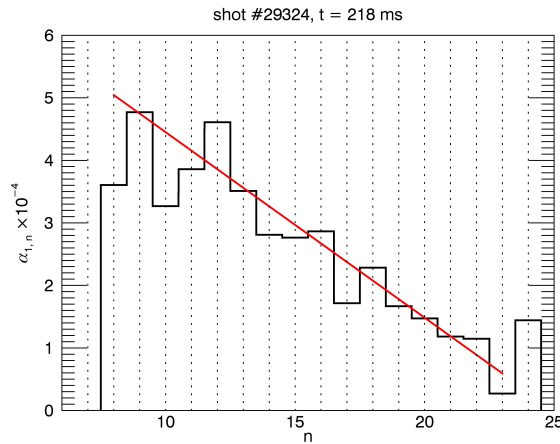


Figure 3.18: $m = 1$ secondary mode spectrum during #29324 at $t = 218$ ms. The linear decreasing trend of the $m = 1$ secondary mode amplitudes is highlighted by a red straight line.

The extrapolated eigenfunctions are reported in Fig. 3.19. For $m = 1$ modes, the reference harmonic is chosen as $n^* = 21$ (solid red line in Fig. 3.19(a)), and higher- n modes are obtained via cubic-spline continuation up to the corresponding resonance surfaces (dashed lines). Similarly, for the $m = 0$ modes the extrapolation is based on $n^* = 22$ (solid red line in Fig. 3.19(b)), and is extended by cubic spline up to the reversal

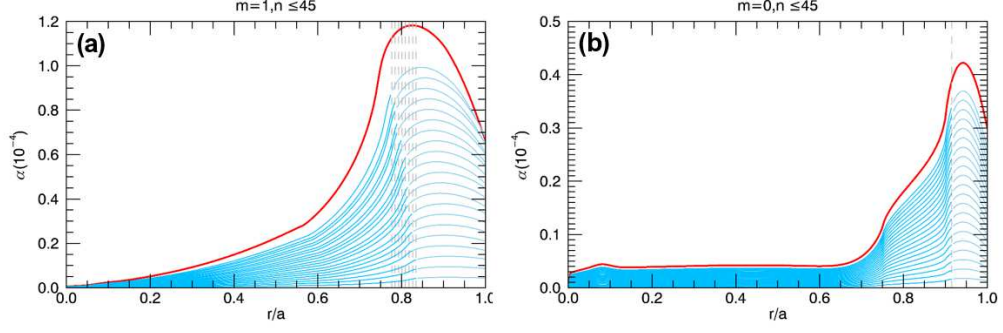


Figure 3.19: (a) Extension of the $m = 1$ mode spectrum in RFX-mod to higher toroidal mode numbers ($24 < n \leq 45$). Vertical dashed lines mark the corresponding resonance surfaces. (b) Extension of $m = 0$ mode spectrum. The vertical dashed line indicates the reversal surface, common resonance to all $m = 0$ modes.

surface, which represents the common resonance for all $m = 0$ modes. The choice $n^* \equiv 24$ was avoided, since this mode corresponds to the highest experimentally resolved harmonic and its measured amplitude is affected by spectral pollution, according to the Nyquist theorem.

Special attention has been devoted to the choice of mode phases. The working ansatz is that modes with $n > 24$ share the same phase as the LM. Accordingly, phases are assigned as:

$$\varphi_{1,n} = n\varphi_{lock} + \text{const.} ; \quad (3.14)$$

$$\varphi_{0,n} = n\varphi_{lock} - \varphi^* , \quad (3.15)$$

with $\varphi^* = \pi/2$, accounting for the asymmetry of the $m = 0$ locking with respect to φ_{lock} , as discussed in Ref. 168.

The resulting equatorial Poincaré plot obtained with the extended spectrum ($m = 0, 1$; $0 \leq n \leq 45$) is shown in Fig. 3.20. As the former Poincaré plot of Fig. 3.8, the magnetic topology is organized into three main regions: a core region characterized by islands associated with the dominant (1, 7) mode, an intermediate chaotic sea due to the overlap of $m = 1$ secondary islands, and an edge region populated by (0, 7) islands aligned with the reversal surface (horizontal green line). Compared with the case of limited spectrum with $n \leq 24$ modes, the inclusion of higher- n harmonics enhances magnetic stochasticity, as visible in the proliferation of secondary structures, including additional Fibonacci-like chains of $m = 0$ islands. A key outcome of the extended spectrum is the reduction of the angular separation between the two PWI-related stripes, in agreement with the analysis based on the Locking Parameter of Section 3.2. In fact, Fig. 3.20(b) provides a zoom into the PWI region, where intersections of field lines with the first wall (yellow line) align closely with the experimentally observed footprint positions φ_0 and φ_1 (solid black lines).

A more quantitative validation is however obtained from the calculation of the connection lengths. The updated map shown in Fig. 3.21(a) reproduces both the correct separation and the correct alignment of the stripes when compared with the warped CCD image of Fig. 3.1(c), represented in panel (b). In particular, the simulated separation $\Delta\varphi_{n_{max}=45}^{L_{c,w}} = 18^\circ$ is fully consistent, with the measured value $\Delta\varphi = 15^\circ$ inferred from CCD camera image. Moreover, the extended-spectrum map reproduces subtle details that are absent in the $n \leq 24$ case, such as a small bulge in the $m = 0$ stripe (circled in

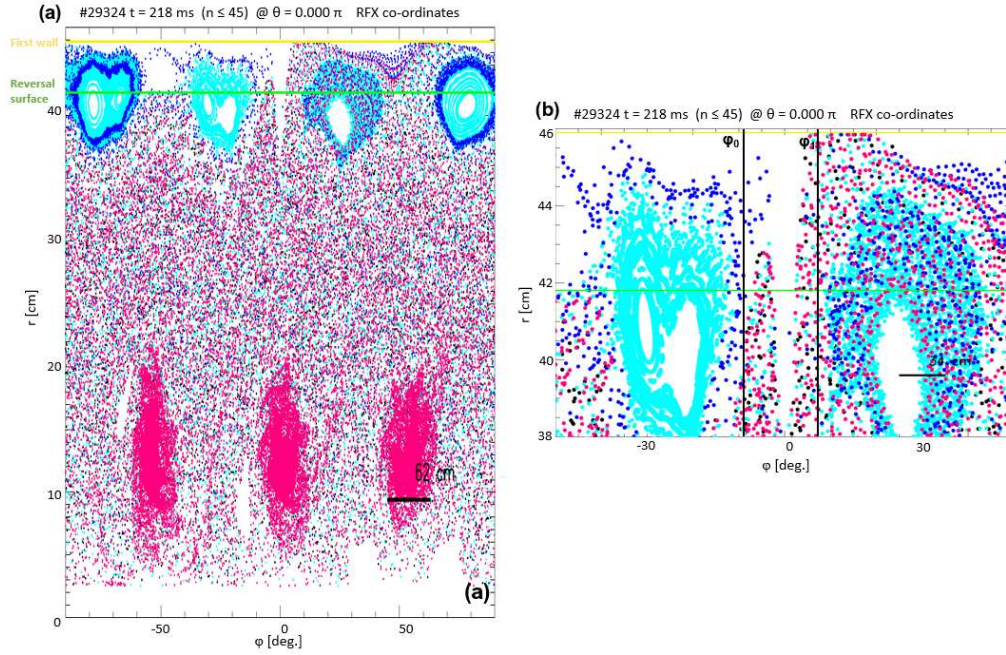


Figure 3.20: (a) Toroidal equatorial Poincaré plot for discharge #29324 at $t = 218$ ms, computed including an extended spectrum of modes up to $n = 45$. (b) Zoom of the same Poincaré plot in the PWI region. Solid vertical lines indicate the toroidal positions ϕ_0 and ϕ_1 of the experimentally observed PWI footprints.

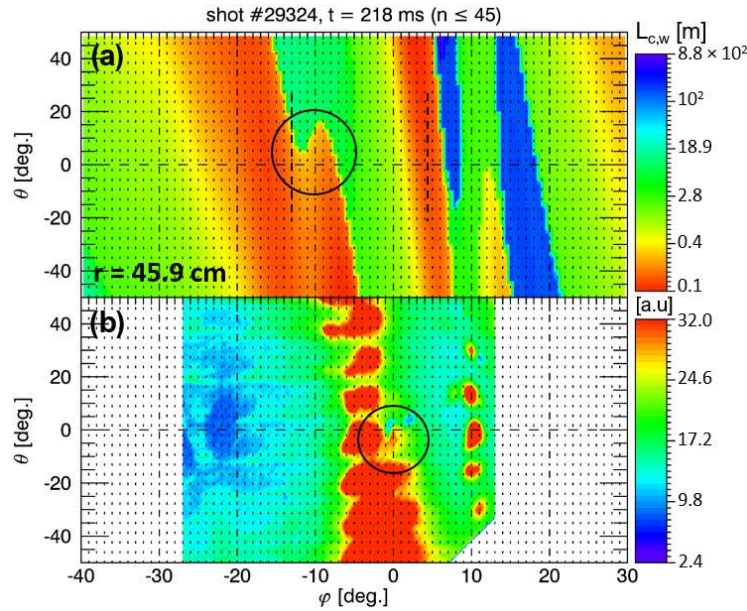


Figure 3.21: (a) High-resolution $L_{c,w}(\phi, \theta)$ map at $r = 45.9$ cm, calculated with extended spectrum with $0 \leq n \leq 45$ on the same grid of Fig. 3.12(a). (b) Warped version of the experimental PWI image of Fig. 3.1(c). Dashed lines indicate the positions of the short- $L_{c,w}$, red stripes in the simulated map. Circles mark correspondences between finer structures observed in the simulated and experimental plots.

both panels of Fig. 3.21), which corresponds to a similar hot-spot observed experimentally. In summary, extending the tearing-mode spectrum beyond the diagnostic limit up to $n_{max} = 45$ yields a synthetic reconstruction that reproduces the PWI footprints both qualitatively and quantitatively, reinforcing the evidence of unresolved high- n modes in RFX-mod.

3.8 Comparison between linear and nonlinear methods

The calculation of connection lengths presented in Section 3.4 can be extended to the entire toroidal surface, namely $-180^\circ < \varphi, \theta \leq 180^\circ$. The result is reported in Fig. 3.22. Panel (a) shows a map calculated on a 256×128 grid of initial positions on the $\varphi - \theta$ plane and considering particles started at radial coordinate at $r = 45.9$ cm, while the map in panel (b) is calculated on a 128×64 grid at $r = 44$ cm.

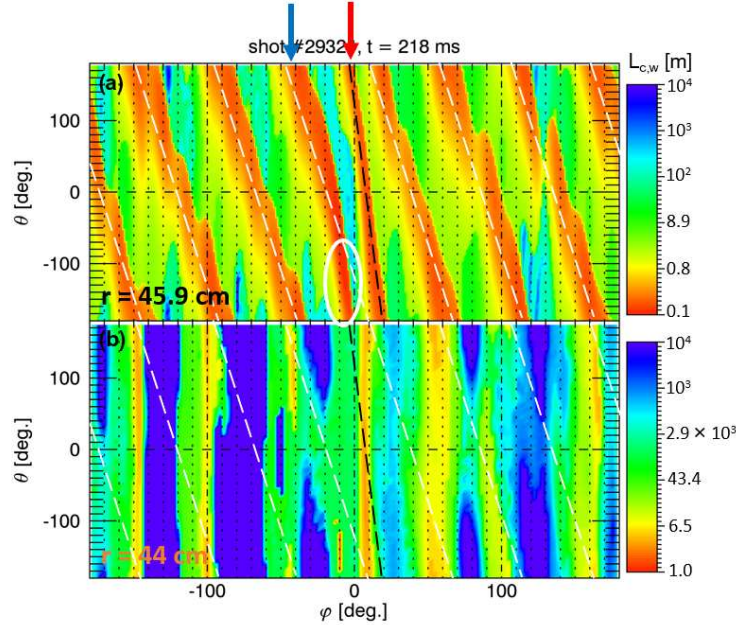


Figure 3.22: $L_{c,w}(\varphi, \theta)$ mapped over the entire first-wall surface considering as initial radial position **(a)** $r = 45.9$ cm and **(b)** $r = 44$ cm. Red and blue regions indicate short and long connection lengths respectively. The red stripe associated with the $m = 0$ island observed PWI footprint is marked by a cyan arrow, while the one related to secondary $m = 1$ mode locking is marked by a red arrow. White dotted lines highlight red regions sharing the same slope as the $m = 0$ island PWI footprint. The black dotted line indicates the mixed slope characterizing the LM PWI stripe.

The map at the outer radius (panel (a)) reveals eight red stripes corresponding to regions of short connection lengths. Among these, the two stripes associated with the experimentally observed PWI footprints are highlighted by arrows: the cyan arrow mark the PWI stripe linked to the $m = 0$ island, and the red arrow indicates the PWI stripe associated with the LM. The remaining red stripes are also related to $m = 0$ island-driven PWI. This interpretation is supported by the $L_{c,w}(\varphi, \theta)$ map calculated at more internal radius (panel (b)): analogously to what discussed in Section 3.6, when moving inward the PWI regions of short connection lengths (in red) disappear and are replaced by region

with intermediate connection lengths (in green). On the contrary, only the red stripe associated with the LM survives in the inner radius map. The resulting global picture is that the magnetic reconnection event under investigation is characterized by seven potential $m = 0$ -island-driven loss channels, and an additional LM-driven loss channel. Only two of these channels were captured by the CCD camera in the experiment. In RFX-mod2, where seven cameras will monitor almost the entire first wall, the full PWI pattern is expected to be observed.

3.8.1 Magnetic topology and geometry of loss channels

Within the framework of linear methods, namely the calculation of the ideal displacement Δ (see Section 3.1.2), the $m = 0$ deformation is independent of the poloidal angle, i.e. $\Delta_{0,n} = \Delta_{0,n}(r, \varphi)$. In this sense, one would expect that the red regions in Fig. 3.22(a), which are associated with $m = 0$ topology, to align along lines of constant φ . However, the $m = 0$ -related loss channels in the connection length map share a common slope, indicated by seven white dotted lines, and this slope corresponds to the periodicity of the dominant (1,7) mode. Although this may seem counter-intuitive, it is consistent with the fact that (0,7) and (1,7) modes are nonlinearly coupled through the Shafranov shift. Since such coupling is inherently toroidal and nonlinear, it cannot be totally understood by linear metrics such as Δ , but it is properly captured by nonlinear analysis such as connection lengths computed with ORBIT.

Furthermore, the LM-related PWI slope (marked by a black dotted line in Fig. 3.22) is characterized by a less well-defined periodicity, which can be interpreted as a consequence of the simultaneous contribution of multiple modes, with different helicities, to the LM structure.

3.8.2 Strong QSH case: shot #37537, $t = 130$ ms

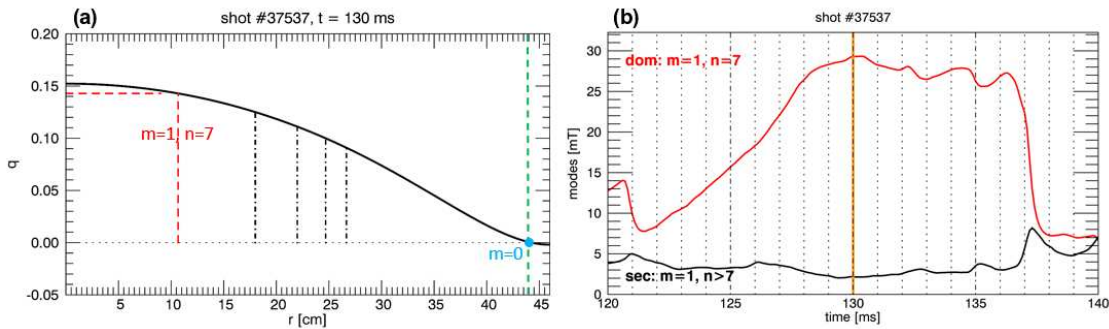


Figure 3.23: (a) Safety factor profile during #37537 $t = 130$ ms: dominant (1,7) mode resonance is highlighted with red dotted lines, main secondary mode resonances are highlighted with dotted black lines, the reversal radius is highlighted with a green dotted line. (b) Time evolution of the $m = 1$ mode amplitudes during discharge #37537. The red curve corresponds to the dominant (1,7) mode amplitude, while the black curve represents the root mean square of the secondary mode amplitudes multiplied by a factor of 2.5 to improve visibility.

By observing Fig. 3.22 we deduce that the QSH regime is responsible for the presence of seven potential $m = 0$ -island-driven PWI regions with (1,7) geometry, while the occurrence

of the LM and the associated crash at $t = 218$ ms opens an additional PWI channel. This new channel appears to distort the geometry of adjacent stripes. For instance, the white circle in Fig. 3.22(a) highlights a mismatch between a red region of short $L_{c,w}$ and the slope expected from its $(1,7)$ geometry. To validate this interpretation, we consider a simpler case in which the secondary $m = 1$ mode phase-locking is absent and the plasma is in a more robust QSH state. This is the case of discharge # 37537 at $t = 130$ ms.

Discharge #37537 represents a typical high-current ($\langle I_p \rangle \approx 1.5$ MA) QSH discharge in RFX-mod [174], where a *slightly-reversed-q* profile (see Fig. 3.23(a)) is exploited to strengthen the QSH state [82, 85, 191]. In this configuration, the reversal surface lies very close to the wall, allowing core magnetic islands to grow without overlapping. As a result, the magnetic topology is less chaotic and more ordered. In fact, at $t = 130$ ms, the plasma experiences a strong QSH state, as shown by the mode-amplitude evolution reported in Fig. 3.23(b).

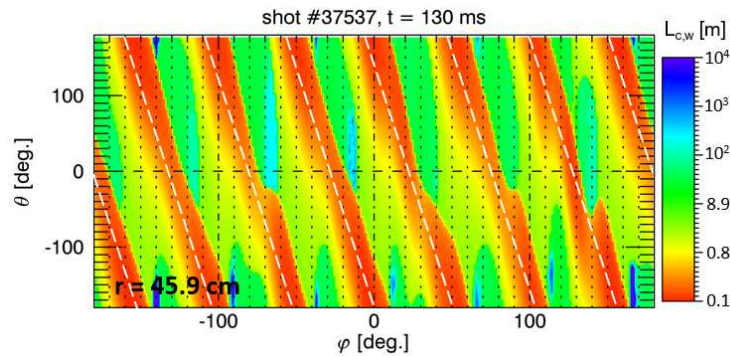


Figure 3.24: $L_{c,w}(\varphi, \theta)$ mapped on the entire first-wall surface for shot # 37537 at $t = 130$ ms. White dotted lines highlight red regions sharing same periodicity, which is the same of $(1, 7)$ mode.

Fig. 3.24 shows a map of connection lengths on the φ - θ plane, calculated on a 256×128 grid of initial positions, with initial radial coordinate $r = 45.9$ cm. Due to stronger QSH, the map related to shot #37537 appears more ordered than the corresponding map in Fig. 3.22(a) for shot #29324. The contour shows seven red stripes of short $L_{c,w}$, without the appearance of any additional stripe attributable to mode locking. Consistently with the discussion in the previous Section, the stripe geometry reflects $(1, 7)$ periodicity, as emphasized by white dotted lines.

As expected in a QSH state, regions of short and long connection lengths are more clearly separated, and several extended regions of long $L_{c,w}$ emerge. To quantify this observation, we isolate the shortest and longest connection length regions and compare their arrangement with the geometry of the $(1, 7)$ and $(0, 7)$ displacements, respectively. Fig. 3.25(a) shows red points corresponding to short connection lengths: defined by $L_{c,w} < L_{c,w}^{*(\text{short})}$. Whereas, the curves marked by asterisks '*' are obtained by applying the $(1, 7)$ periodicity to the main peak of $\Delta_{1,7}(r = a, \theta, \varphi)$ displacement. The agreement in the slope between red points and the asterisked curves confirms that short connection lengths closely follow the $(1, 7)$ geometry. In the absence of a localized LM, the $(1, 7)$ mode thus emerges as the driver of particle losses, pushing neighboring $m = 0$ island towards the wall and causing PWI, so that the overall helicity of short $L_{c,w}$ regions reflects $(1, 7)$ periodical footprint.

Conversely, Fig. 3.25(b) shows blue points corresponding to long connection lengths,

defined by $L_{c,w} > L_{c,w}^{*(\text{long})}$, together with asterisked curves obtained by applying the $m = 0$ periodicity to the highest peaks of $\Delta_{1,7}(r = a, \theta, \varphi)$. In this case, long $L_{c,w}$ regions are clearly organized according to $m = 0$ geometry. These regions correspond to preserved dynamics, where particles remain stably confined rotating around the $m = 0$ island, in continuity with the recurrence length analysis discussed in Section 3.5.2.

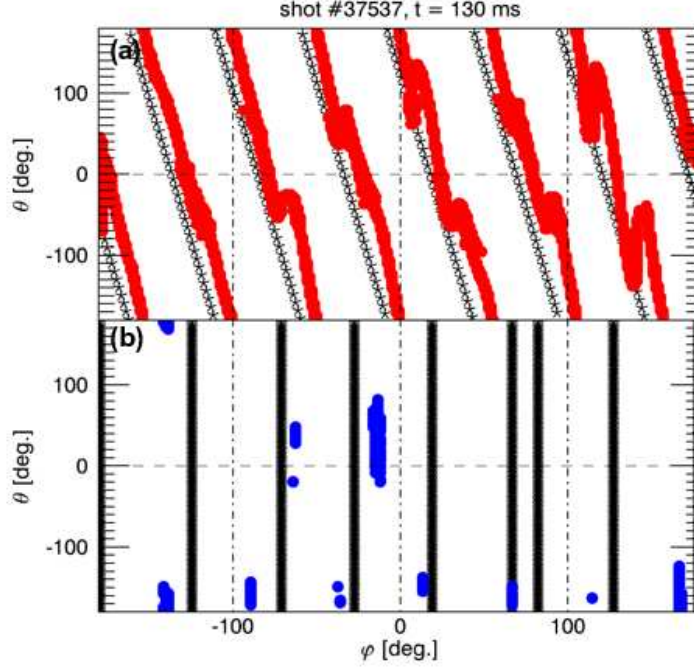


Figure 3.25: (a) Points of short connection lengths extracted from Fig. 3.24 (red) compared with the (1,7) mode periodicity (indicated with '*' symbols). (b) Points of long connection lengths extracted from the same map (blue) compared with the $m = 0$ mode periodicity (indicated with '*' symbols).

In summary, connection length analysis at the RFP edge reveals a mixed helicity structure: regions of short $L_{c,w}$ tend to follow the periodicity of the (1,7) mode, whereas regions of long $L_{c,w}$ follow the $m = 0$ periodicity. Besides this, the presence of a localized LM concur at distorting this helicity pattern.

More generally, this difference in periodicity implies that region of short and long connection lengths are populated by ions undergoing distinct dynamics: predominantly rotational motion around islands for conserved, long- $L_{c,w}$, with $m = 0$ periodicity, and straighter trajectories for lost particles, in short $L_{c,w}$, with $m = 1$ periodicity. It has been already observed that experimental observables reflect this duality [172–174]: for example, floating potential measurements tend to follow the $m = 0$ geometry, whereas core electron temperature tend to follow the $m = 1$ geometry. In this sense, differences among experimental observables may depend on which ion population they predominantly probe.

3.8.3 Relation between shift and Connection Length

In Section 3.1.2, the ideal displacement was introduced as a useful parameter to quantify the deformation of the plasma surface during crash and to infer the toroidal position

of PWI footprints. Similarly, in this chapter, the Connection Length to the wall has been extensively employed to characterize particle losses during the same event.

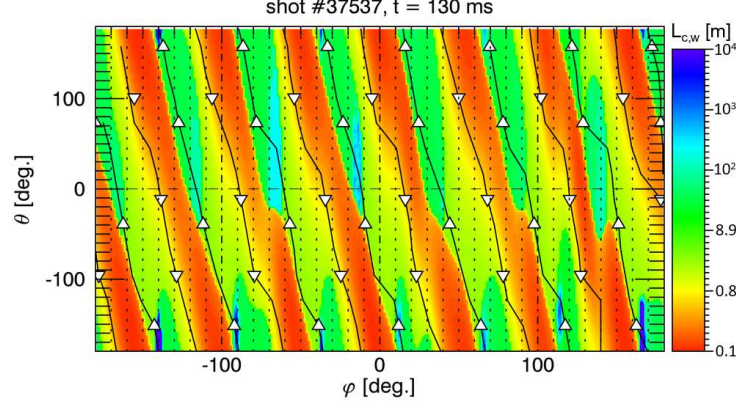


Figure 3.26: Comparison between $L_{c,w}(\varphi, \theta)$ map of shot #37537 at $t = 130$ ms and Δ maxima and minima, indicated with ' Δ ' and ' ∇ ' respectively.

A first quantitative correlation between the two metrics has recently been proposed in the literature (see Ref. 118), showing that the Connection Length and the shift are related through the empirical relation:

$$L_{c,w} = L_0 \cdot \exp\left(\frac{\Delta}{\Delta_0}\right), \quad (3.16)$$

with $L_0 \approx 8.6$ m and $\Delta_0 = 2.5 \cdot 10^{-3}$ m. This scaling has been obtained for a typical RFX-mod2 scenario, where the displacement induced by secondary modes is expected to be reduced and their phase-locking largely suppressed. For simplicity, only data on the equatorial plane ($\theta = 0$) are considered in the fit. The result is that, at least to first order, connection lengths and shift are strictly correlated.

However, it could be useful to consider the entire two-dimensional geometry of the shift and the Connection Length on the φ – θ plane, in order to further understand the relation between the two metrics and directly relate them with the PWI pattern on the wall. In Fig. 3.26, the two-dimensional patterns of $\Delta = \Delta_{1,7} + \Delta_{sec.} \approx \Delta_{1,7}$ is compared to the connection-length map of Fig. 3.24. For each poloidal angle, Δ local maxima and minima along the toroidal direction are determined. Then, Δ maxima are highlighted in Figure by lines tagged with the ' Δ ' symbol, while its minima are indicated by lines tagged with ' ∇ '. Although Δ and $L_{c,w}$ follow a similar two-dimensional-trend, their profiles are shifted between each other. Specifically, moving from left to right across the figure, Δ minima are located between red (short- $L_{c,w}$) regions and green (medium- $L_{c,w}$) regions. Vice versa, Δ

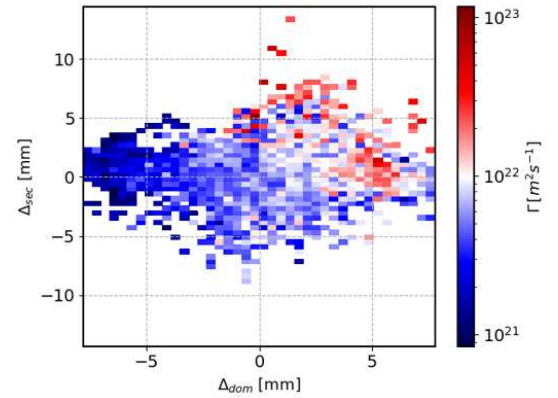


Figure 3.27: Inward flux Γ_{in} as a function of shift Δ . Graph adapted from Ref. 118.

maxima separate green regions from red regions. This behaviour is still coherent with the one-dimensional formula in Eq. (3.16).

In practice, both shift (Δ) and Connection Length to the wall ($L_{c,w}$) are valuable tools for studying crash events that drive PWI in RFP plasmas. In fact, for example, they both enters into the estimate of the outward particle flux:

$$\Gamma_{out} = n_i v_{th} \cdot \frac{\Delta}{L_{c,w}} , \quad (3.17)$$

where n_i is the density of ions, and v_{th} is their thermal velocity. In this expression, the ratio $\Delta/L_{c,w}$ provides an estimate of the average ratio between the perpendicular and parallel distances traveled by an ion before reaching the wall. Since the inward particle flux, Γ_{in} , is proportional to the outward flux, Γ_{out} , it also depends on these quantities (see Fig. 3.27). Consequently, investigating the relationship between the shift and the Connection Length is a critical task to achieve effective reduction of impurities accumulation.

In conclusion, although both metrics offer relevant insights into PWI characterization, their physical meaning is fundamentally different. Owing to its intrinsically three-dimensional nature, the Connection Length $L_{c,w}$ provides a powerful tool to investigate magnetic topology and to interpret the topological origin of PWI footprints. By contrast, the shift does not bring any further information on particle dynamics at the RFP edge comparable to that obtained from connection lengths. The evaluation of Δ is based on a linear-approach, depends on the limited number of experimentally measurable harmonics, and accounts only for the $m = 1$ subset of the tearing mode spectrum. However, it provides effective insights on LM position responsible for PWI.

In addition, this chapter has introduced several complementary metrics that can be employed alongside, or as alternatives to, the Connection Length for characterizing particle losses during magnetic reconnection events. The impact angle α_{imp} , for example, measures the incidence angle at which lost ions strike the wall and is therefore of primary relevance for describing edge processes involving the plasma-facing components. The recurrence length, on the other hand, provides insight into cold ion dynamics during reconnection. Compared with Connection Length, it more accurately captures periodic motion, such as ion rotation around magnetic islands, while remaining ill-defined for particles moving at the very edge, which are directly lost through the wall, as in the LM channel.

Chapter 4

Ion dynamics

In this chapter, the PWI event of interest is further investigated by considering ORBIT simulations of energetic ions distributed according to a Maxwellian distribution and characterized by random particle pitch. The analysis employs the energy-dependent *loss time* metric, τ_{loss} , to map the first wall. The resulting loss time distribution closely resembles the experimentally observed PWI pattern: two distinct stripes of short τ_{loss} , associated with enhanced Plasma-Wall Interaction, clearly emerge. This analysis demonstrates that the reconnection event is responsible for the preferential loss of suprathermal ions. Ions lost through the Locked Mode (LM)-related channel are predominantly co-passing, and are expelled on very short timescales. On the contrary, ions lost via the $m = 0$ -island channel are predominantly counter-passing, and with characteristic loss times approximately one order of magnitude longer. This result further confirms the physical picture already described in the previous chapter: the $m = 1$ -related PWI stripe represents the most efficient mechanism for plasma thermal content loss, whereas the $m = 0$ -related PWI stripe is comparatively milder. This behavior brings the $m = 0$ island-driven PWI in RFP plasmas closer to the concept of stellarator resonant divertor, where particle losses are redistributed over longer timescales and over larger wetted areas.

4.1 Simulations in ORBIT of energetic ions

In the preceeding chapter, ORBIT simulations were performed using low-energy ions, effectively tracing magnetic field lines. However, in order to assess more realistically the impact of magnetic topological structures on experimentally observed PWI, it is necessary to consider higher-energy particles. To this end, we employed the same grid of initial positions $[r_i, \theta_j, \varphi_k]$ as in Fig. 3.17(c) with $r_i = 44$ cm for all the particles, and a 128×64 mesh of (φ_k, θ_j) spanning the intervals $-40^\circ \leq \varphi \leq 30^\circ$ and $-50^\circ \leq \theta \leq 50^\circ$. Instead of monoenergetic cold ions, for each point of the mesh we now consider a Maxwellian population of 1000 ions with an initial temperature $T_0 = 402$ eV and high-energy tail up to $E = 2$ keV. The initial ion temperature is set to the electron temperature T_e measured at $t = 218$ ms, i.e. $T_0 = T_e$. This assumption is motivated by experimental observations in the MST reversed-field pinch, and represents a reasonable approximation during crash events [192]. Ions are initialized with random particle pitch λ , defined as:

$$\lambda = \frac{\vec{v} \cdot \vec{B}}{|v| |B|} . \quad (4.1)$$

Namely, it measures the alignment between the direction of the particle motion and that of the magnetic field. The pitch-angle indicates instead the angle between these two directions. Moreover, simulated ions are allowed to collide with background particles (electrons and ions). Collisions are modeled in ORBIT using a Monte Carlo, energy-conserving, pitch-angle scattering operator [193]. Since particles now are characterized by a broad energy distribution, the Connection Length $L_{c,w}$ is no longer an appropriate metric to characterize losses. Instead, we introduce the *loss time*, τ_{loss} , calculated in ORBIT as the time required for 50% + 1 of the simulated ions to reach the wall [77].

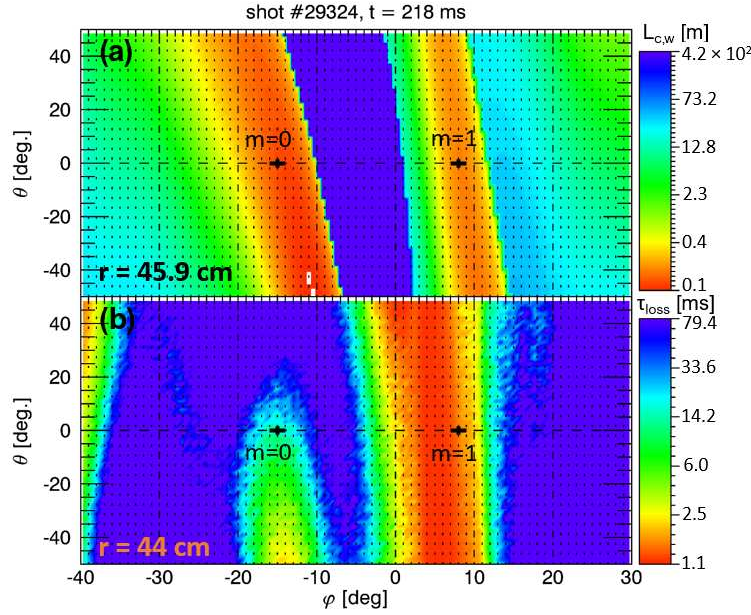


Figure 4.1: Comparison between (a) $L_{c,w}(\varphi, \theta)$ map at fixed radius: $r = 45.9$ cm calculated using monoenergetic cold ions, and (b) $\tau_{loss}(\varphi, \theta)$ at fixed radius: $r = 44$ cm, calculated using Maxwellian ions with $T_0 = 402$ eV. To the left, there is the $m = 0$ -island loss channel. To the right, there is the $m = 1$ LM loss channel. Crosses (+) indicate the runs which are considered for the statistics of energy and pitch of Fig. 4.2.

Fig. 4.1 compares the resulting τ_{loss} map with the previously calculated $L_{c,w}$ map shown in Fig. 3.12(a), which is reproduced in panel (a) for reference. Loss time spans a range from approximately 1 ms to about 80 ms. As expected, energetic ions are lost on very short timescales, $\tau_{loss} \approx 1$ ms, at the toroidal location of the Locked Mode, corresponding to the red stripe on the right-hand side of Fig. 4.1(b). This result further corroborates the fundamental role of the LM in driving PWI, consistently with both experimental evidence [59, 121, 164, 166, 170, 171] and previous numerical investigations on RFX [77]. On the contrary, the $m = 0$ footprint remains visible as a weaker stripe on the left-hand side of Fig. 4.1(b), but it is associated with significantly longer loss times, $\tau_{loss} \approx 10$ ms.

4.2 Thermal content dynamics

Simulations involving energetic ions allow to access to their dynamical properties, such as energy and pitch.

4.2.1 Statistics of lost ions

As concerns lost particles, we refer to Fig. 4.2. Two subsets of simulation runs are sampled (marked by crosses, '+', in Fig. 4.1), associated with the $m = 0$ island and with the Locked Mode (LM) PWI stripe. The energy and pitch histograms for particle lost through the LM channel are shown in Figs. 4.2(a)-(b), while the corresponding histograms for the $m = 0$ -island losses are shown in Figs. 4.2(c)-(d).

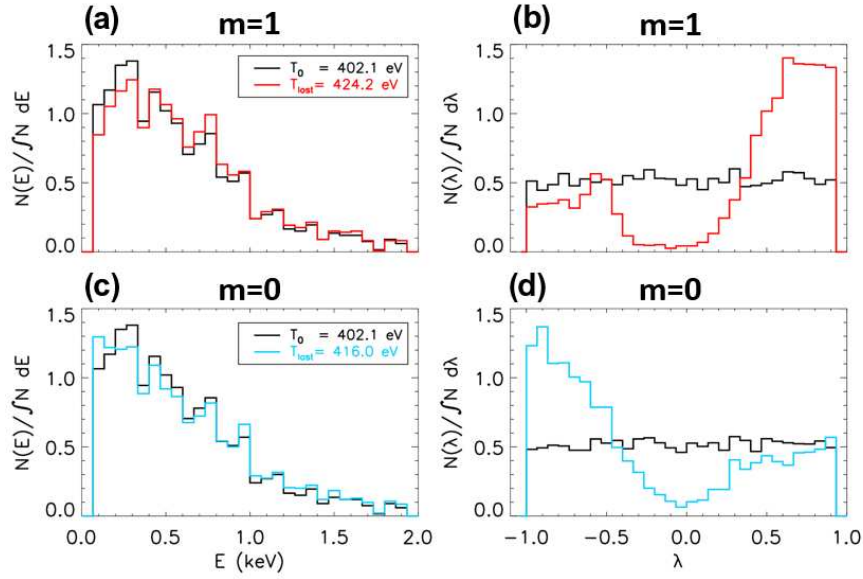


Figure 4.2: (a)-(c) Energy and (b)-(d) pitch histograms. (a)-(b) are related to the LM loss channel, while (c)-(d) are related to the $m = 0$ -island loss channel. The initial Maxwell distribution with $T_0 = 402$ eV is shown in black, while the final distributions are reported in color. The initial pitch distribution is uniform, i.e. with random values in $[-1, 1]$.

From Figs. 4.2(a)-(b) it emerges that co-passing fast ions with $E > T_0$ dominate the PWI losses in the vicinity of the LM and are expelled on very short timescales. The average energy (temperature) of ions lost through the LM channel is indeed $T_{\text{lost}} = 424$ eV $> T_0$, with $T_0 = 402$ eV. Consequently, superthermal ions, being less collisional than thermal ones, are preferentially and rapidly lost. This provides a clear physical explanation for the high efficiency of the LM as a loss channel for hot ions moving from the plasma core. Regarding the pitch distribution, panel (b) shows that trapped particles do not significantly contribute to the loss mechanism, consistently with previous ORBIT simulations performed for RFX plasmas [185, 194, 195].

In the $m = 0$ -island region (see Fig. 4.2(c)-(d)), the loss mechanism is qualitatively similar, but counter-passing fast ions represent the main lost population. The average energy of ions lost in this region is $T_{\text{lost}} = 416$ eV $> T_0$. This behavior is consistent with particle rotation around the magnetic island, deduced through the calculation of

recurrences. The associated loss times are longer than those characterizing the LM-related PWI channel, owing to the increased collisionality with the background plasma, which slows down particle transport toward the wall.

4.2.2 Statistics of conserved ions

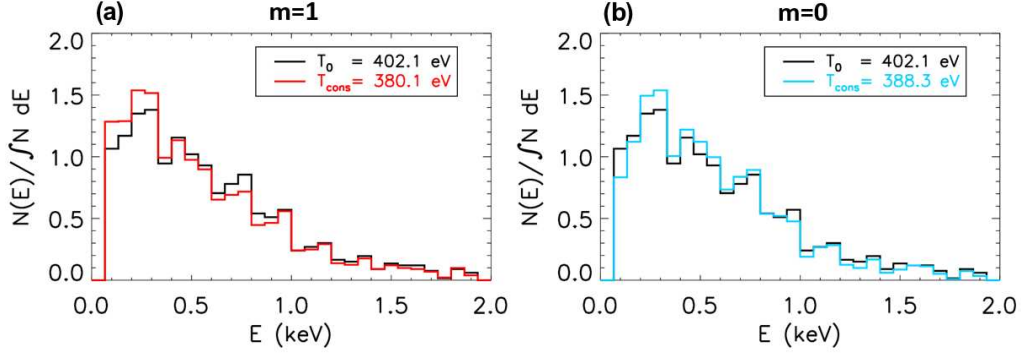


Figure 4.3: Energy histograms of conserved particles related to (a) the LM channel, and (b) the $m = 0$ island. In black the initial Maxwell distribution with $T_0 = 402$ eV, in color the final distributions.

Analogously to the analysis of lost particles, Fig. 4.3 reports the energy statistics of the so-called *conserved* particles, i.e. ions that remain confined within the plasma during the crash. Panel (a) refers to the LM-related PWI region, while panel (b) shows the results for the $m = 0$ -island region. In both cases, a net cooling of the confined plasma population is observed. At the end of the simulation, when the majority of starting ions is lost, the final average temperatures are $T_{cons} = 380.1$ eV $< T_0$ in the LM region and $T_{cons} = 388.3$ eV $< T_0$ in the $m = 0$ -island region. This result is consistent with the temperature decrease typically observed during reconnection events [39, 92] and already evident, for example, in Fig. 1.13(d). We can therefore conclude that *magnetic reconnection events preferentially deplete the suprathermal component of the ion population* [41, 43, 196, 197], leading to a reduction of the mean plasma temperature.

The time evolution of the conserved-particle temperature T_{ave} is shown in Fig. 4.4, which is obtained from a dedicated run of 11000 energetic ions within the magnetic field reconstructed for discharge #29324 at $t = 218$ ms, which is the snapshot chosen. Particles are deposited in proximity of the same grid positions marked with '+' in Fig. 4.1. This provides a direct estimate of the cooling timescale in the magnetic reconnection event of interest. For the $m = 0$ -island case (cyan curve), cooling takes place on ≈ 1 ms. The evolution shows an initial sharp drop, likely corresponding to the fast removal of the suprathermal tail of the energy distribution, followed by stabilization around the final lower average temperature $T_{cons} = 388.3$ eV, whose level is indicated by a dashed line. In contrast, particle losses through the LM channel occur on much shorter timescales, namely they are two orders of magnitude faster. The corresponding temperature evolution, shown in red in Fig. 4.4, indicates a rapid decrease such that ions are expelled within very few timesteps, preventing in this case a proper interpretation.

Despite these differences, the analysis presented here highlights the importance of reconnection-driven cooling. Far from being negligible, this mechanism may have direct physical implications. In particular, it could provide an explanation for the "anomalous" component of

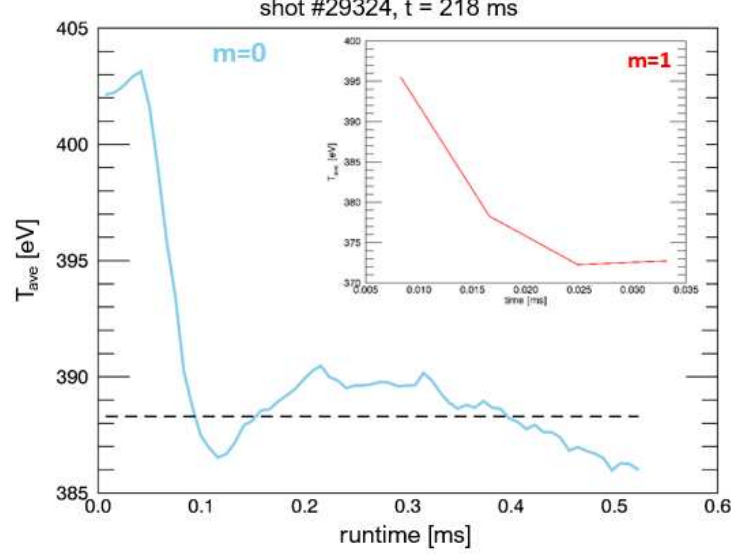


Figure 4.4: Time evolution of conserved ion population average temperature. Cyan curve in the main graph refers to the $m = 0$ -island PWI region, while red curve in the smaller graph refers to the LM-associated PWI region.

the Spitzer resistivity observed in RFP plasmas, which has been argued to be a general signature of collisionless regimes [198].

4.3 Power load on first wall

ORBIT simulations with energetic ions provide valuable insight into plasma dynamics during crash events. In particular, the analysis has focused on two main observables: particle energy and loss time. These quantities can be combined to estimate the thermal load experienced by the first-wall during the magnetic reconnection event [133]:

$$W_{load} = \frac{E_{lost}^{(k,j)}}{A_{wetted}} \cdot \frac{\gamma_{ion}}{N^{(k,j)}} , \quad (4.2)$$

where $E_{lost}^{(k,j)}$ denotes the cumulative energy deposited due to particle lost at the generic grid point (φ_k, θ_j) . This quantity can be also expressed by $E_{lost}^{(k,j)} = \sum_{\ell}^{N^{(k,j)}} E_{\ell}$, where the sum runs over the lost particles started from the point (φ_k, θ_j) . The wetted surface area associated with each grid point is denoted by A_{wetted} and is calculated as the area of the corresponding rectangular cell:

$$A_{wetted} = \Delta\varphi \cdot \Delta\theta \cdot R_0 \cdot a ,$$

where $\Delta\varphi$ and $\Delta\theta$ are the toroidal and poloidal angular distance between two nearest neighbors in the grid, $R_0 = 2$ m and $a \approx 0.5$ m are the major and the minor radii of RFX-mod. The quantity γ_{ion} is the average ion loss rate (in s^{-1}) at the grid point (φ_k, θ_j) , and it can be extrapolated from transport models.

Considering the τ_{loss} map in Fig. 4.1(b), one can calculate the simulated power load

relative to the (φ_k, θ_j) grid point as:

$$W_{load}^{(k,j)} = \frac{E_{lost}^{(i,j)}}{\tau_{loss}^{(k,j)} \cdot A_{wetted}}.$$

Since the number of particles considered in each run was limited ($N^{(k,j)} = 1000$), the calculated thermal load is found though to be too low. Nevertheless, a qualitative comparison between different loss channels can still be drawn. Since particle losses associated with the $m = 0$ -island PWI stripe occur on timescales roughly one order of magnitude longer than those associated with the LM stripe, the corresponding power deposition on the $m = 0$ island is approximately one order of magnitude smaller. As a consequence, PWI mediated by the $m = 0$ -island hotspot is milder and therefore less critical for wall integrity. Interestingly, the $m = 0$ island chain exhibits behavior analogous to stellarator resonant divertors [199–201], where particle losses are redistributed over longer timescales and larger wetted areas [152, 154]. The availability of $m = 0$ islands as naturally occurring divertors in RFP plasmas had already been proposed in earlier studies on RFX-mod [202]. This concept becomes particularly relevant in the context of the upgraded RFX-mod2 device, where a reduction of LM-induced deformations is expected to contribute to effective PWI mitigation [118].

4.4 Particle pitch and magnetic topology of loss channels

We have so far emphasized that the two experimentally observed PWI regions are associated with distinct magnetic topologies, which in turn affect the dynamics of energetic ions. These differences become evident when ORBIT simulation are performed with ions distributed according to a realistic Maxwellian distribution. In particular, Figs 4.2(b)-(c) reveal a sort of “*chirality*” in the lost-particle population: the pitch distribution of lost ions depends on whether particles are expelled by rotating around the $m = 0$ island ($\lambda < 0$) or by crossing the LM loss channel ($\lambda > 0$).

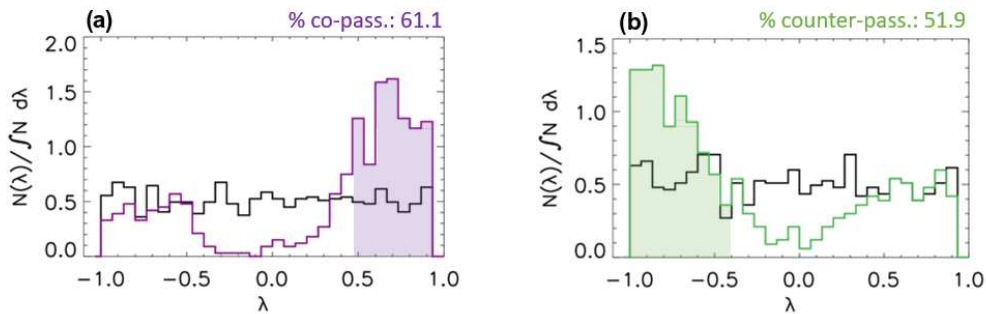


Figure 4.5: Examples of histograms showing prevalent (a) %_{co-pass.} (in purple) and (b) %_{counter-pass.} (in green) lost ion population. In black the initial uniform pitch distribution, in color the final pitch distribution.

In this section, we aim at testing the hypothesis that a one-to-one correspondence exists between the magnetic topology of a loss channel and the pitch of particles lost through it. Specifically, we investigate whether ions lost through $m = 0$ -island-driven

PWI are predominantly counter-passing, while ions lost through the LM-related PWI are predominantly co-pitch.

To this end, we consider a 256×128 grid of initial positions, covering the entire torus surface, i.e. $-180^\circ \leq \varphi, \theta \leq 180^\circ$. As in the runs of Fig. 4.1(b), ions are deposited at $r = 44$ cm, with Maxwellian energy distribution and random initial pitch. For each run, the pitch histogram of lost ions is evaluated. At the end of calculation, two metrics ($\%_{\text{co-pass.}}$ and $\%_{\text{counter-pass.}}$) are obtained. We define ' $\%_{\text{co-pass.}}$ ' as the percentage of lost particles with $\lambda > 0.5$ and ' $\%_{\text{counter-pass.}}$ ' as the percentage of lost particles with $\lambda < -0.5$. Regions are labeled as *co-passing* if $\%_{\text{co-pass.}} > 50$, meaning that the majority of lost particles are strongly co-passing, while regions with $\%_{\text{counter-pass.}} > 50$ are labeled as *counter-passing*. Fig. 4.5 shows two representative examples of this analysis: panel (a) corresponds to a co-passing-dominated lost-ion distribution, at $(\varphi \approx 8^\circ, \theta = 0)$, highlighted in purple, while panel (b) refers to a counter-passing-dominated lost ion distribution $(\varphi \approx -15^\circ, \theta = 0)$ highlighted in green.

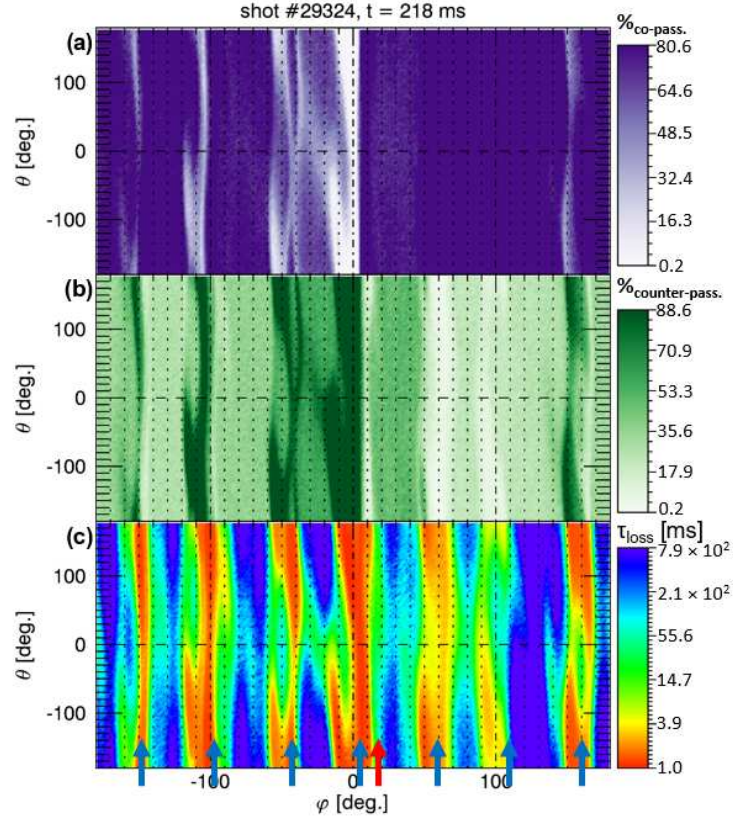


Figure 4.6: (a) $\%_{\text{co-pass.}}$ map in purple color scale and (b) $\%_{\text{counter-pass.}}$ map in green color scale. (c) τ_{loss} -map calculated considering energetic ions deposited over a grid of 256×128 point at fixed $r = 44$ cm. Arrows indicate the PWI regions, with short loss times: in cyan those related to $m = 0$ island, in red that related to LM.

We can now map $\%_{\text{co-pass.}}$ and $\%_{\text{counter-pass.}}$ and compare them directly with the loss time map calculated on the same grid. The resulting contour plots are shown in Fig. 4.6. Panel (a) reports $\%_{\text{co-pass.}}$ in purple, while panel (b) shows $\%_{\text{counter-pass.}}$ in green. Panel (c) shows the corresponding τ_{loss} map. In panel (c), the positions of the $m = 0$ -island loss

channels (cyan arrows) and of the LM-related loss channel (red arrow) are also indicated, following the identification previously deduced from Fig. 3.22. For clarity, the comparison is divided into two parts:

- negative toroidal angles ($\varphi < 0$): panel (c) shows four red stripes of short τ_{loss} associated with $m = 0$ -island-driven loss channels. Accordingly to what we described so far, one would expect the $\%_{counter-pass.}$ map to show four corresponding strongly counter-passing regions. This expectation is confirmed in panel (b), where four well defined dark green regions appear.
- positive toroidal angles ($\varphi > 0$): panel (c) shows one red stripe of short τ_{loss} associated with the $m = 1$ -LM loss channel, along with three additional red regions associated with $m = 0$ -island loss channels. In this case, one would expect panel (a) to display one strong co-passing, dark purple region at the LM position, and panel (b) to show three dark green region corresponding to the $m = 0$ -island related loss channels. However, this expected pattern does not appear in either panel (a) or in panel (b). Instead, panel (a) shows a large, strongly co-passing region that extends over almost the entire $\varphi > 0$ half-plane.

To conclude, the analysis indicates that only a weak correlation exists between short- τ_{loss} regions associated with $m = 0$ magnetic topology and counter-passing particle losses. However, this correspondence gradually fades when approaching the LM loss channel, which instead does not show any evident association with co-passing-dominated regions.

The ability to classify PWI events according to their underlying magnetic topology is of central relevance for RFX-mod. As discussed throughout this work, LM and $m = 0$ -island related loss channels affect the wall differently, producing distinct thermal loads. At present, the most reliable approach for attributing a given magnetic topology to the experimentally observed PWI pattern remains the comparison between connection length maps calculated close to the wall and at more internal radii, as performed in Section 3.6 for shot #29324 at $t = 218$ ms. From this perspective, the Connection Length $L_{c,w}$ represents a powerful metric for interpreting loss mechanisms during reconnection events causing PWI. Nevertheless, this approach is computationally demanding: since the calculation of $L_{c,w}$ relies on simulations of low-energy (and therefore slow) ions, making high-resolution maps particularly expensive in terms of machine time. By contrast, τ_{loss} maps obtained simulating energetic ions are considerably less demanding computationally. In RFX-mod2 upgrade, a natural continuation of this work would be the development of a method capable of reproducing the same information provided by internal $L_{c,w}$ maps while reducing computational cost, possibly by exploiting the outcomes of energetic-ion simulations or by developing more efficient parallelized algorithms for ORBIT calculations. Although this more interpretative aspect lies beyond the main scopes of the present thesis, the results discussed in this last section provide a solid foundation for further investigations.

Chapter 5

Electrostatic potential

The present Chapter is dedicated to the first implementation of ORBIT simulations with a three-dimensional electrostatic potential map. Electrostatic potential arises from magnetic flux changes inside the magnetic islands during reconnection events, and in this case is obtained from a 3D visco-resistive MHD simulation of a RFP plasma performed with the SPECYL code. This approach is employed to investigate in detail the reconnection event considered in this thesis: the inclusion of the electrostatic potential represents a necessary extension of the description presented in the previous chapters focused on the magnetic topology and the thermal content dynamics. By accounting for the electrostatic field contribution, a more comprehensive description of the main physical effects associated with magnetic reconnection can be achieved, including ion acceleration processes that lead to the formation of sizable non-Maxwellian tails [41, 43, 197].

5.1 Calculation of the electrostatic potential map

The electrostatic potential map analyzed in this chapter is calculated from a 3D non-linear MHD simulation of a RFP plasma performed using the SPECYL code [78]. As discussed in Section 2.6, SPECYL solves the equations of the visco-resistive model (2.50). The simulation considered here is characterized by Lundquist number $S = 10^6$ and viscous Lundquist number $M = 10^4$. Both resistivity and viscosity are assumed to be stationary; in particular, resistivity is prescribed to increase toward the plasma edge in order to mimic the temperature drop in that region, while viscosity is taken to be isotropic. An ideal-wall boundary condition is imposed for the magnetic field at the plasma edge, together with a seed magnetic perturbation with (1, 7) helical symmetry introduced to sustain RFP relaxation [158].

The electrostatic potential is computed by inverting the Poisson equation (2.51) and subsequently decomposed into Fourier harmonics. The resulting potential is then inserted in ORBIT according to Eq. (2.52) in the following form, expressed in Boozer coordinates:

$$\Phi(\psi_p, \theta, \zeta) = \Phi_{m_p, n_p}(\psi_p) \sin[m_p \theta - n_p \zeta + \phi_{m_p, n_p}(t)] ,$$

with $m_p = 0$, $0 \leq n_p \leq 25$ and $m_p = 1$, $0 \leq n_p \leq 55$. All Fourier phases are set to $\phi_{m_p, n_p} = 0 \forall m_p, n_p$. This is consistent with the toroidal localization of the Locked Mode at $\varphi = 0$ observed in the RFX-mod discharge #29324 at $t = 218$ ms: the snapshot chosen for SPECYL simulation.

5.2 Ion dynamics during magnetic reconnection

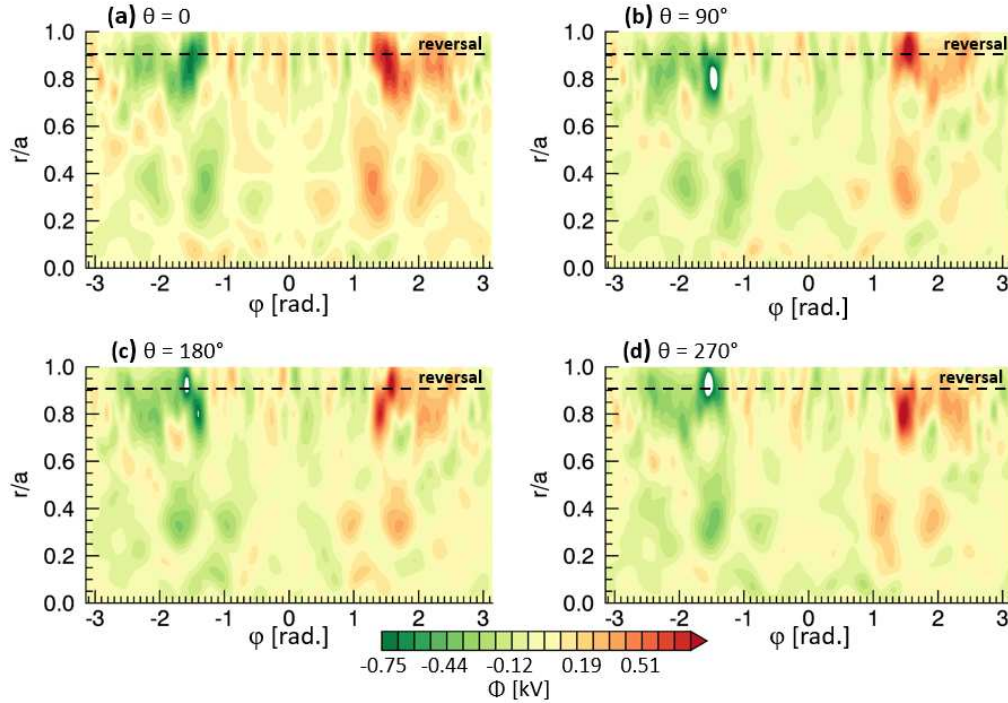


Figure 5.1: Three-dimensional map of electrostatic potential calculated via SPECYL for a RFP plasma, referring to the case study of this thesis: #29324, $t = 218$ ms. The contours show the electrostatic potential Φ at fixed poloidal angles: (a) $\theta = 0$, (b) $\theta = 90^\circ$, (c) $\theta = 180^\circ$, and (d) $\theta = 270^\circ$. Reversal surface is indicated by a dashed line. Regions of positive potential (potential *hills*) are colored in red, while regions of negative potential (potential *wells*) are colored in green.

Fig. 5.1 shows four contour plots of the electrostatic potential map $\Phi(r, \theta, \phi)$ at fixed poloidal angles ($\theta = 0, 90^\circ, 180^\circ$, and 270°), obtained from SPECYL simulations by selecting as time instant the RFX-mod discharge #29324 at $t = 218$ ms [156]. The contours reveal a highly complex structure of the electrostatic potential. However, two main regions can be clearly identified: a potential *well* located at $\phi = -90^\circ$, colored in dark green, and a potential *hill* at $\phi = 90^\circ$, colored in dark red. Both structures are aligned along the reversal surface at $r_{rev.} \approx 41.5$ cm, although slight variations are observed when moving along the poloidal angle. These variations are associated with the presence of a significant $m = 1$ component in the electrostatic potential. Concerning its amplitude, the electrostatic potential spans the range $-1.44 \text{ kV} \leq \Phi \leq 1.44 \text{ kV}$, which is significantly larger than other potential contributions typically measured at the RFX edge, such as the ambipolar potential, which spans the range $-0.14 \text{ kV} \leq \Phi \leq 0.33 \text{ kV}$ (see Fig. 8 in Ref. [153]). It should be noted that this electrostatic potential has not been yet directly measured in RFX-mod; however it is expected to be detected by the upgraded electrostatic probe system foreseen for RFX-mod2. Finally, it is worth emphasizing, that the electrostatic potential associated with the reconnection event is strongly localized both in space and time, as it arises from magnetic flux variations inside the magnetic islands during reconnection. Instead, the ambipolar potential, associated with $(0, 1)$ mode, is

stationary in time [153].

5.2.1 Ion cooling in the electrostatic potential well

First, we simulate a population of thermal ions distributed according to a Maxwellian distribution with initial temperature $T_0 = 402$ eV, random pitches, and initial positions located in the vicinity of the electrostatic potential well (at $r = r_{rev}$, $\theta = 0$, and $\varphi = -90^\circ$). The purpose of this simulation is to investigate how particle dynamics is influenced by the presence of electrostatic field. For simplicity, we consider magnetic field at equilibrium (i.e. magnetic perturbations are taken as null), with only electrostatic perturbations. The boundary conditions for τ_{loss} calculation adopted here are the same of those used in Section 4.2.

The resulting energy distributions are shown in Fig. 5.2. The initial distribution is represented in black, while the final distribution of lost particles is represented in red. By comparing the two distributions, it is evident that the main effect of the presence of the electrostatic potential well on the ion population is a strong cooling. In fact, although the final distribution retains a Maxwellian shape, its average temperature is significantly reduced to $T_{lost}^{well} = 280$ eV. This behavior is consistent with the fact that regions of negative electrostatic potential act as attractors for ion dynamics, thereby tending to confine particles within the potential well. However, the simulations yield a characteristic loss time of $\tau_{loss}^{well} \approx 0.37$ ms, which is considerably shorter than the loss time $1 \text{ ms} \lesssim \tau_{loss} \lesssim 10 \text{ ms}$ obtained in Chapter 4 for energetic ions where only magnetic perturbations were included and the electrostatic potential was neglected. This result indicates that, despite the average cooling of the ion population, the presence of the electrostatic potential further enhances particle losses.

To understand why ions are still lost on short timescales despite the presence of a strongly stabilizing electrostatic potential well, we now examine the trajectory of an individual ion deposited in the potential well with initial energy $E = 404$ eV, neglecting magnetic perturbations and collisions. Fig. 5.3(a) shows the particle trajectory in the (φ, r) plane: the ion follows a periodic orbit along the rim of the potential well on the equatorial plane, but a net radial drift is also clearly visible. This aspect is more evident in Fig. 5.3(b), which reports the temporal evolution of the radial coordinate. The orbit is sinusoidal, consistent with periodic dynamics, but the particle does not return to its initial radial position, revealing the presence of a radial drift. As a consequence, the ion is lost after approximately seven toroidal transits and is not permanently trapped within the potential well, contrary to what would be expected for an electrostatic potential with a purely $m_p = 0$ topology [153].

This behavior can be readily understood by considering the guiding-center equation

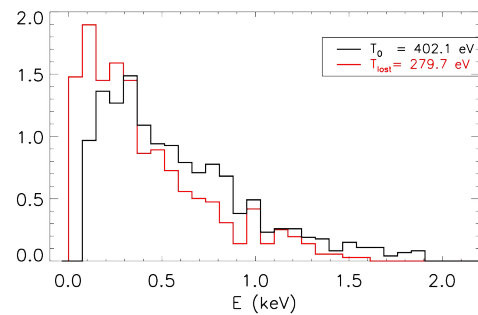


Figure 5.2: Energy histogram from ORBIT simulations of ion dynamics for particles initially deposited inside the electrostatic potential well. The initial Maxwellian distribution with $T_0 = 402$ eV is shown in black, while the final distribution of lost particles is shown in red. Taken from Ref. 156.

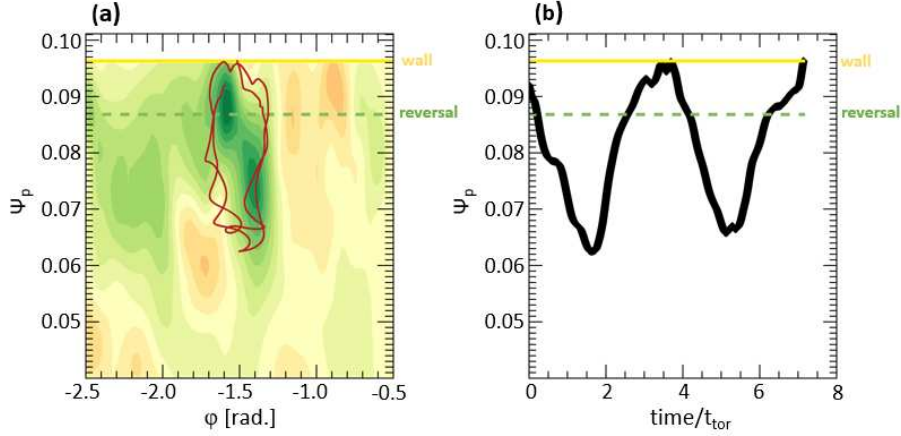


Figure 5.3: (a) Trajectory of a single ion initially deposited near the electrostatic potential well, represented in the (φ, Ψ_p) plane. (b) Evolution of the same trajectory, Ψ_p , with time normalized to toroidal transits, t_{tor} . Taken from Ref. 156.

(2.23) for the radial coordinate in equilibrium conditions (i.e. without magnetic perturbations) implemented in ORBIT [148]:

$$\dot{\psi}_p = -\frac{g}{D} \left[\left(\mu + \rho_{\parallel}^2 B \right) \frac{\partial B}{\partial \theta} + \frac{\partial \Phi}{\partial \theta} \right] + \frac{I}{D} \frac{\partial \Phi}{\partial \zeta}.$$

According to this expression, closed orbits on the toroidal plane slowly drift radially due to the $\partial\Phi/\partial\theta$ term. This implies that an electrostatic potential containing both $m_p = 0$ and $m_p = 1$ components inherently contributes to enhanced particle losses [156].

5.2.2 Ion acceleration in the electrostatic potential hill

In contrast to the behavior observed for ions deposited in the potential well at $\varphi = -90^\circ$, we expect ion acceleration to occur when particles are deposited at the top of the potential hill, located at $r = r_{rev}$, $\theta = 0$, and $\varphi = 90^\circ$. Fig. 5.4 compares the initial ion energy distribution function (which is the same of Fig. 5.2), represented by Maxwellian with average $T_0 = 402$ eV (black curve), with the energy distribution of lost particles (red curve).

It is immediate to see that the main effect of the strong positive electrostatic potential is particle acceleration. In fact, ions are lost on $\tau_{loss}^{hill} = 0.14$ ms, while the final distribution of lost particles exhibits a significantly higher average temperature, $T_{lost}^{hill} = 789$ eV. More importantly, the distribution shows a long, non-Maxwellian tail extending up to ≈ 3 keV. This result

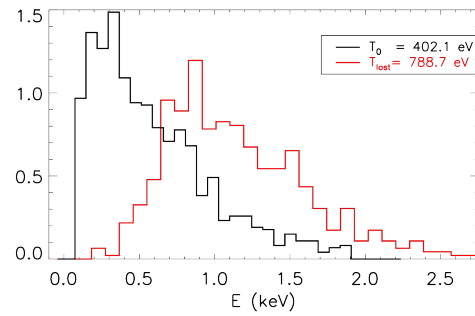


Figure 5.4: Energy histogram from ORBIT simulations of ion dynamics for particles deposited at the top of the potential hill. The initial Maxwellian distribution with $T_0 = 402$ eV is shown in black, while the final distribution of lost particles is shown in red.

is remarkable because, as discussed in the introductory part of this thesis, particle acceleration during magnetic reconnection has been experimentally observed in MST RFP discharges [41, 43, 197], but a consistent numerical reconstruction of these observations has been still far to be properly accomplished. This limitation has hindered the interpretation of experimental measurements, leaving particle acceleration during magnetic reconnection as an open issue in both laboratory and space plasma physics.

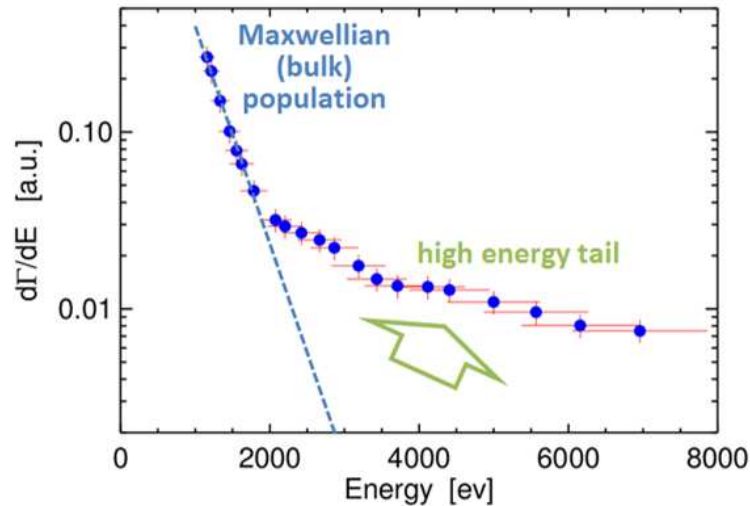


Figure 5.5: Ion energy distribution function measured during a magnetic reconnection event in RFX-mod by means of the Neutral Particle Analyzer (NPA) diagnostics. Taken from Ref. 203.

As a final and significant consideration, we report the experimental ion energy distribution function measured during a reconnection event for a real discharge in RFX-mod. The result is shown in Fig. 5.5. The energy distribution was measured using the Neutral Particle Analyzer (NPA) diagnostic, which is discussed in detail in the following chapter. The measured spectrum clearly exhibits a long high-energy tail up to approximately 7 keV, which hardly deviates from the bulk Maxwellian behavior of the ion population, indicated in the figure by a dashed blue line.

In this section, we have demonstrated that the ORBIT code, when coupled with SPECYL, can properly reproduce ion acceleration during magnetic reconnection events, thereby revealing an additional mechanism of ion heating. The simulation results show good qualitative agreement with experimental observations in the RFX-mod device. However, further investigations are required to unambiguously distinguish between particle acceleration due to magnetic reconnection from other possible contributions, such as those associated with ambipolar potentials. These aspects are anyway left to future work.

5.2.3 Perspectives into future work: the onset of electrostatic chaos

As a final exploratory step of this chapter, we address the question of how the particle loss pattern would appear if both magnetic and electrostatic fields are included in the calculation of the loss times of Maxwellian ions. We therefore use the same grid adopted for the energetic ion map shown in Fig. 4.1: $r = 44$ cm and 128×64 points in the intervals $-40^\circ \leq \varphi \leq 30^\circ$ and $-50^\circ \leq \theta \leq 50^\circ$, considering the combined action of magnetic perturbations (with $m = 0, 1$, $0 \leq n \leq 24$) and electric perturbations (with $m_p = 0$, $0 \leq n_p \leq 25$).

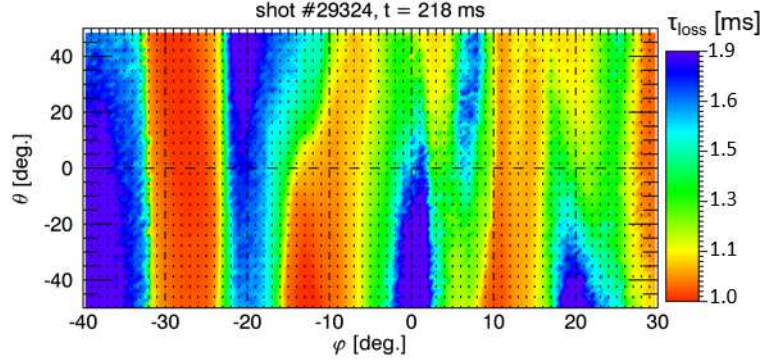


Figure 5.6: Map of loss times $\tau_{loss}(\varphi, \theta)$ at fixed radius: $r = 44$ cm, calculated using Maxwellian ions with initial mean temperature $T_0 = 402$ eV immersed in both magnetically and electrostatically perturbed fields.

and $m_p = 1$, $0 \leq n_p \leq 55$). The resulting loss time map, $\tau_{loss}(\varphi, \theta)$, is shown in Fig. 5.6. The loss times are distributed within a relatively narrow interval: $1 \text{ ms} \leq \tau_{loss} \leq 2 \text{ ms}$. Regions characterized by shorter loss times are colored in red, while longer times are colored in blue. However, the spatial distribution of the loss times appears to be completely uncorrelated with the PWI pattern observed experimentally. This lack of spatial correspondence can be interpreted as a consequence of a sort of *electrostatic chaos*, analogous to that discussed in Section 3.7 for the magnetic field, as caused by the presence of a broad spectrum of electrostatic harmonics. These modes compete with the magnetic perturbations and collectively contribute to enhanced ion transport, with consequences that are still to be understood. Moreover, further investigations will benefit from the renewed design of the electrostatic sensors foreseen for RFX-mod2 [190].

Conclusions and Perspectives

In this thesis, we have investigated the magnetic topology and the thermal content dynamics during Plasma-Wall Interactions (PWIs) triggered by magnetic reconnection events (or crashes) in the RFX-mod Reversed Field Pinch (RFP) device, focusing specifically on discharge #29324 at $t = 218$ ms. This discharge is characterized by two distinct PWI footprints located at $\varphi_0 = -8^\circ$ and $\varphi_1 = 7^\circ$. The primary objectives were to determine which tearing modes (and how many of them) shaped the observed PWI pattern, and to characterize how the ion energy distribution function was modified during the reconnection event.

The topic bears relevance well beyond the RFP configuration. In tokamaks, for instance, nonlinear coupling among internal tearing modes is a precursor to major disruptions, which constitute a critical issue for ITER, given its limited tolerance to such events. Moreover, ion acceleration during reconnection is observed also in astrophysical plasmas. While space-plasma models capture large-scale dynamics, only laboratory devices (such as RFX-mod) enable a controlled, high-resolution reconstruction of the underlying magnetic structure.

The analysis was developed through a progressively more refined sequence of methods. In Chapter 3, we demonstrated that, during the crash, the Locked Mode (LM) formed by secondary $m = 1$ modes (with $7 < n \leq 24$) drives intense, spatially localized particle losses, thus producing the observed PWI. Through the linear reconstruction of the Last Closed Flux Surface (LCFS), we showed that the PWI pattern can only be reproduced by including harmonics up to $n_{max} = 45$ (Eq. (3.8)). A nonlinear analysis performed with the guiding-center code ORBIT confirmed that the experimental PWI image is consistent with the map of connection lengths near the wall (Figs. 3.11 and 3.12). In particular, accurate reproduction of the PWI geometry requires extending the input spectrum to include both $m = 0$ and $m = 1$ modes with $0 < n \leq 45$, revealing the fundamental role of these high- n components, even when their amplitudes are relatively small (Fig. 3.21).

By exploiting the inherently three-dimensional structure of the Connection Length, additional maps were calculated by depositing particles at more internal radii, further from the wall (Fig. 3.17). These maps highlight the significant contribution of the "tail" of the nearby $(0, 7)$ magnetic island to the PWI. As a result, the two experimental footprints can be interpreted as follows: the narrow stripe at φ_1 is caused by $m = 1$ LM-induced losses, whereas the broader stripe at φ_0 is produced by losses mediated by the $(0, 7)$ island. This interpretation is confirmed by recurrence length map (Fig. 3.16). Extending the calculation to the full toroidal surface (Fig. 3.22) further revealed the existence of seven distinct $m = 0$ -island-related loss channels (sharing a common $(1, 7)$ periodicity), in addition to the LM channel observed at φ_1 . Moreover, the internal maps of connection lengths (Fig. 3.17(c)) showed filamentary structures associated with $m = 0$, medium-high n ($n \approx 19$) islands; although not yet measured experimentally, such structures are

expected to become observable with the upgraded electrostatic diagnostic system of RFX-mod2, whose commissioning is planned by the end of 2026.

Moreover, in the future RFX-mod2, an upgrade of magnetic probes will allow for overcoming the limitation in toroidal resolution to $n \leq 24$, bringing improved measurements (up to $n = 36$) and simulations. At the same time, this will allow for better feedback control of tearing modes in order to mitigate/avoid major crashes. In parallel, a set of seven fast cameras covering approximately 70% of the first wall will substantially expand the capability to monitor PWI events.

Owing to its intrinsic three-dimensional nature, the Connection Length provides a powerful tool to investigate magnetic topology, to interpret the topological origin of PWI footprints, and to clarify the nonlinear coupling between tearing modes driving the crash, that otherwise cannot not be fully understood by linear-approaches such as the LCFS deformation analysis. However, Connection Length is very demanding in terms of computational and time resources. The newly parallelized ORBIT algorithm used through this thesis has already demonstrated promising performance and constitutes a solid foundation for future optimization.

To describe magnetic topology, monoenergetic, cold ions ($E = 10^{-2}$ eV) were used to mimic magnetic field lines. Conversely, in Chapter 4, Maxwellian ion population were simulated to properly capture the dynamical properties of lost particles. The analysis revealed that the LM is a highly efficient loss channel for co-passing, fast ions, whereas losses through the $m = 0$ island at φ_0 occur on longer timescales and involve predominantly counter-passing ions (Fig. 4.2). Conserved ions are instead subject to rapid cooling (Figs. 4.3 and 4.4), demonstrating that magnetic reconnection depletes the suprathermal component of the ion population, thus lowering the mean plasma temperature. From the power load perspective, the $m = 0$ -island PWI is relatively milder. Interestingly, the behavior of the $(0, 7)$ island chain is analogous to stellarator resonant divertors, where particle losses are redistributed over extended times and wetted areas. This is particularly relevant for RFX-mod2 device, where reducing LM-induced deformation is expected to mitigate severe PWI.

In Chapter 5, a fully 3D pattern of electrostatic potential, obtained from the MHD code SPECYL for a reconnection event similar to the case study, has been inserted in ORBIT for the first time. The resulting potential pattern clearly exhibits a deep well and a high hill (Fig. 5.1). Ions are initially trapped in the potential well, and then are subsequently lost due to the θ -dependence of the potential (Fig. 5.3), associated with the presence of $m_p = 1$ harmonics. As expected, the potential well slightly cools particle losses (Fig. 5.2), while the potential hill accelerates them (Fig. 5.4). Since loss times in simulations including only electrostatic perturbations are shorter than those obtained with magnetic perturbations, we conclude that the electrostatic potential contributes significantly to enhanced particle losses. This interpretation is corroborated by experimental observations of suprathermal tails in the ion energy distribution, measured via the Neutral Particle Analyzer (NPA) diagnostic, during magnetic reconnection in RFX-mod (Fig. 5.5).

Because electrostatic perturbations enhance particle chaos in competition with magnetic modes, a complete interpretation of the τ_{loss} map (Fig. 5.6), calculated in ORBIT by considering both perturbation contributions, is not yet fully understood and will be matter of future work.

Appendix A

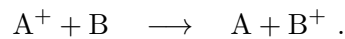
The Neutral Particle Analyzer of RFX-mod2

The global picture emerging from the thesis is that magnetic reconnection events are responsible for localized variation of the magnetic flux and for the consequent induction of electric fields, which can influence the thermal content dynamics and strongly affect the energy distribution function of ion species. For this reason, both in laboratory and in astrophysical plasmas, the characterization of the ion energy distribution function represents a fundamental aspect for understanding plasma dynamics and for clarifying the connection between microscopic particle kinetics and macroscopic plasma stability. Modern magnetic confinement devices are therefore equipped with a variety of dedicated diagnostics aimed at measuring charged particle velocity distribution functions [204].

Within this context, we focus on one of these diagnostics, namely the Neutral Particle Analyzer (NPA) [57, 58]. In this Appendix, we provide an overall description of the NPA functioning and we present the preliminary activity of calibration of the acquisition system of the NPA in perspective of RFX-mod2.

A.1 Charge-exchange reaction

In magnetically confined plasmas, the motion of charged particle is constrained by the Lorentz force. Thermal ions, however, can collide with neutral atoms, which are typically cold and originate from wall recycling or gas fueling. Such interactions may result in a *charge-exchange* reaction:



Since this collision is essentially elastic, the newly formed neutral particle inherits the velocity of the reacting ion. The crucial consequence is that the neutral, being insensitive to the magnetic field, can elude the confinement and escape the plasma. Neutral Particle Analyzer exploits this mechanism by collecting and analyzing these escaping neutrals, thus providing an indirect measurement of the plasma ion velocity distribution.

Several other diagnostics are based on similar charge-exchange principles. Fast Ion D- α (FIDA) spectroscopy, for instance, retrieves information on the fast ion velocity distribution through the Doppler shift and spectral broadening of the Balmer- α emission produced by neutralized deuterium [204]. Fast Ion Loss Detectors (FILD) [205–207] offer complementary measurements by directly collecting escaping fast ions and resolving their energy and pitch-angle via their strike positions on a scintillator plate. NPA measurements

can be complementary also with neutron diagnostics, as both are sensitive to suprathermal ion population. In Reversed Field Pinch plasmas, for example, neutron detectors clearly indicate ion acceleration during reconnection events, as they record enhanced count rates correspondingly [39]. Consistently, NPA observations reveal an energy distribution characterized by a Maxwellian core superimposed with a pronounced energy tail (see Fig. 5.5), attributable to reconnection-driven ion heating [203]. As regards RFX-mod2, NPA and neutron diagnostics will be complemented by direct measurements of the ion distribution function in the velocity space, provided for the first time by the Diagnostic for Ion Velocity Observation (DIVO) system [208]. Moreover, a Gas electron multiplier (GEM) soft-x-ray detection system [209] will be installed to measure bremsstrahlung soft x-rays emitted by electron accelerated up to 20 keV during magnetic reconnection events [42].

A.2 Neutral Particle Analyzer diagnostic in RFX

The Neutral Particle Analyzer employed during RFX-mod operation was developed by the IOFFE Institute of the Russian Academy of Science and installed on a radial port on the low-field side of the machine, in the equatorial plane. Neutral particles escaping the plasma enter the diagnostic line and pass through a nitrogen stripping cell, where they are re-ionized before entering the spectrometer section.

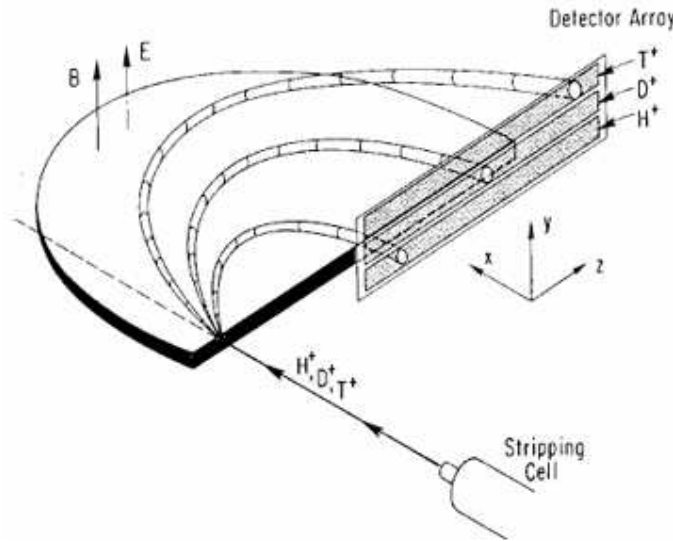


Figure A.1: Scheme of an $E \parallel B$ spectrometer. Neutral particles entering the diagnostic are first re-ionized in a stripping cell. The resulting ions (hydrogen H, deuterium D, and tritium T) then enter the spectrometer chamber, where they are deflected by combined electric and magnetic fields. The ion trajectories depend on both mass and velocity (energy), causing different species and energies to strike distinct detector channels. Taken from Ref. 57.

Fig. A.1 shows the working principle of an $E \parallel B$ spectrometer, originally proposed by Medley and Roquemore [57] and implemented in the RFX NPA. After re-ionization, ions are deflected by a combination of electric and magnetic fields according to their energy and mass, such that particles with different characteristics impinge on distinct detectors. In the configuration shown, ions enter the spectrometer along the x -direction with velocity

$\vec{v} = v_{in}\hat{e}_x$, while the electric field and the magnetic field are parallel and lie along the y -direction: $\vec{E} = E\hat{e}_y$ and $\vec{B} = B\hat{e}_y$.

The electric field accelerates ions according to:

$$y(t) = \frac{1}{2} \frac{eE}{m} t^2 . \quad (\text{A.1})$$

Coordinates $x(t)$, $y(t)$, and $z(t)$ refer to a reference frame centered at the entrance point of particles inside the spectrometer. m and e are the ion mass and charge respectively. Since hydrogen, deuterium, and tritium carry the same charge ($Z = 1$), mass discrimination is achieved through different response to the applied electric field.

Instead, the magnetic field induces ion rotation in the x - z plane with cyclotron frequency:

$$\omega_c = \frac{eB}{m} . \quad (\text{A.2})$$

Detectors are arranged along the z -direction, and ions reach them after half a cyclotron period (at time $t_D = \frac{1}{2\omega_c}$), at a position:

$$z_D = 2r_L = 2 \frac{mv_{in}}{eB} , \quad (\text{A.3})$$

along z -direction. r_L is the Larmor radius of the ion rotation around the spectrometer magnetic field. The corresponding position along y -direction of the hit detector whereas is:

$$y_D = y(t_D) = \frac{1}{2} \frac{eE}{m} \left(\frac{m}{2eB} \right)^2 = \frac{mE}{8eB^2} . \quad (\text{A.4})$$

It is noteworthy mentioning that the position y_D depends only on the ion mass, enabling species separation. In fact, detectors are arranged on arrays, such that each detector array receives ions of a single species, as also indicated by Fig. A.1. Whereas, z_D increases with the entrance velocity, and thus provides energy resolution. In RFX-mod, the NPA is equipped with two detector arrays of 11 detectors each, dedicated to hydrogen and deuterium ions, for a total of 22 channels, resolving an energy range of $0.4 \text{ keV} \lesssim T \lesssim 10 \text{ keV}$. Upgraded electron channeltron multiplier for both species are foreseen in RFX-mod2, allowing to improve electronic response and possibly to increase the number of channels up to 13.

Each detector is connected to an event-counting electronics channel, such that each registered pulse corresponds to one particle hitting the detector. The acquired data therefore consists of an energy histogram with 11/13 bins per ion species. During RFX-mod operation, data acquisition relied on a *VME multi-purpose scaler* with sampling rate of 20 kHz, which provided an effective compromise between temporal resolution and statistical accuracy [203]. In RFX-mod2, the acquisition system will be renewed and based on a *CAEN 2730 digitizer*, capable of operating at sampling rate up to 50 MSamples/second.

A.3 Characterization of CAEN digitizer for the RFX-mod2 Neutral Particle Analyzer

The main outcomes of the activity of characterization of the CAEN 2730 digitizer activity have been summarized in a Technical Report prepared for the RFX internal archives. For completeness, the Technical Report is reproduced in full in the following pages.

 CONSORZIO RFX <i>Ricerca Formazione Innovazione</i> Corso Stati Uniti, 4 - 35127 Padova (Italy) Tel. +39.049.8295000 - Fax +39.049.8700718	Rif:	Date: October 23, 2025
	Document: NT	Authors: P. Porcu, M. Zuin
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Characterization of CAEN digitizer for the RFX-mod2 Neutral Particle Analyzer

Abstract

This technical note presents preliminary tests performed on the acquisition system of the RFX-mod2 Neutral Particle Analyzer (NPA), which is based on the CAEN 2730 digitizer.

The NPA is a diagnostic designed to measure the energy distribution function of charged ions emerging from the plasma edge. The high acquisition frequency offered by the system enables the identification of correlations between particle dynamics and discharge events. In particular, the diagnostic can be used to resolve the time evolution of the energetic ion distribution during relaxation phenomena, which are typical of the Reversed Field Pinch (RFP) configuration [1, 2]. Given these requirements, accurate calibration of both signal amplitude and timing is essential. This report provides initial results aimed at addressing these aspects.

1 Set-up

The CAEN 2730 digitizer is a high-speed acquisition board featuring 32 input channels, each operating at a sampling rate of 50 MSamples/s with 14-bit resolution.

Configuration of acquisition parameters (including discrimination thresholds, signal visualization, and data saving) can be carried out via the COMPASS software provided by CAEN.

Three main acquisition starting modes are supported by the digitizer:

- asynchronous start via software command;
- synchronous start via external NIM or TTL signal connected to the S_IN port;
- remote synchronous control.

For the present tests, the second method was employed, utilizing the digitizer's internal clock for time measurement. Although the digitizer supports synchronization to an external clock, this feature was not activated in the current configuration.

A Stanford Research Systems DG535 pulse generator was used both to let the acquisition start and to generate the input signal, as schematically represented in Fig. 1. The pulse generator allows precise control over signal amplitude, duration, polarity, and frequency.

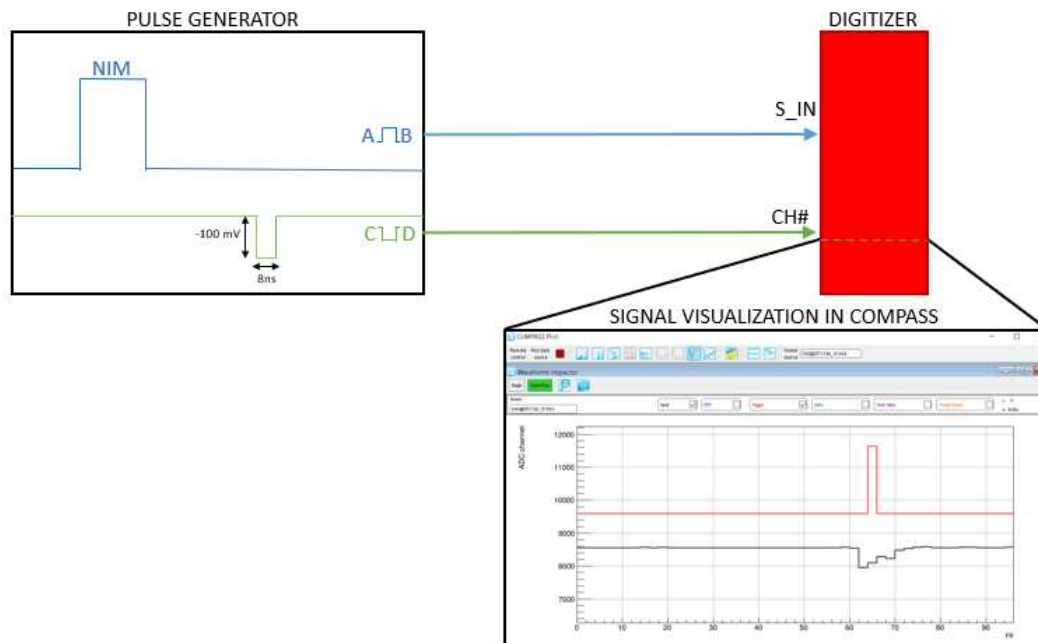


Figure 1: A positive NIM signal (blue) is generated on output channel AB (with positive polarity) of the DG535 pulse generator and sent to the S_IN port of the CAEN digitizer to let data acquisition start. Simultaneously, a negative signal (green), analogous to that expected from the NPA diagnostic, is produced on output channel CD (with negative polarity) and directed to one of the digitizer input channels (CH#, where # = 0 - 31). The input signal has an amplitude of approximately 100 mV, a duration of 8 ns, and negative polarity. Real-time visualization of the acquired waveform is provided via the COMPASS software interface on the host PC.

2 First check channel independence

Initial tests were conducted to verify the independence of the digitizer channels. It was confirmed that signals sent to a specific channel are not observed by adjacent or unrelated channels, which instead display only background noise. This confirms the absence of electrical or software-induced crosstalks under the tested conditions.

Subsequently, channel synchronization was evaluated by assessing whether all digitizer channels reference the same internal clock. A test signal was split using a T-connector and sent to two digitizer channels. The acquired waveforms were compared in terms of shape and temporal alignment.

Fig. 2 illustrates this comparison between channels CH0 and CH1, demonstrating temporal coincidence and waveform consistency. Similar results were obtained for all tested channel pairs, regardless of input signal amplitude, duration, or frequency.

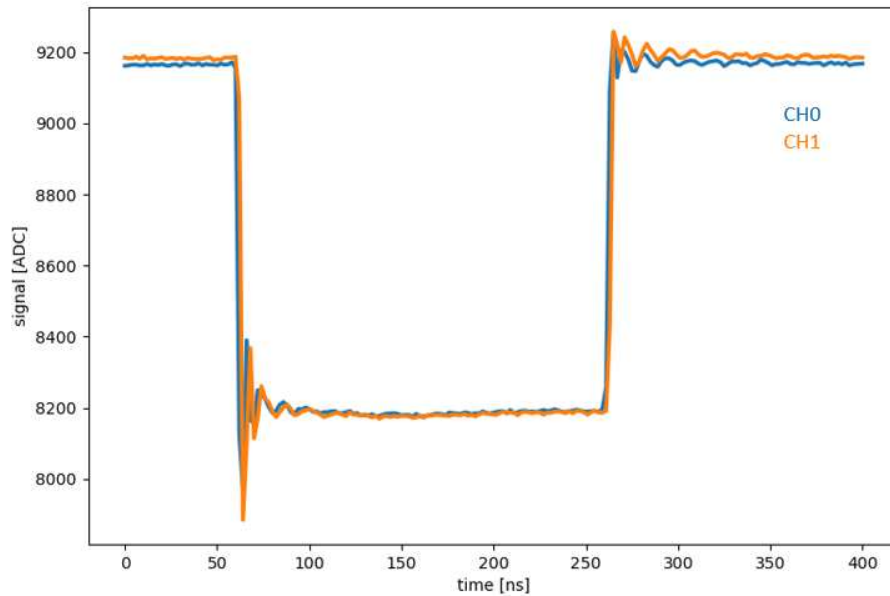


Figure 2: Check on independence between CH0 (blue) and CH1 (orange) channels.

3 Amplitude calibration

3.1 Trigger threshold

The trigger threshold can be configured through the COMPASS software interface, where it is expressed in "lsb" units. A calibration linear fit (see Fig. 3) was established to correlate the physical input voltage $V[V]$ with the corresponding threshold level in lsb. The empirical relation is reported below:

$$V[\text{lsb}] \approx 3881.17 \cdot V[V] + 25.87 \quad . \quad (1)$$

Here, $V[\text{lsb}]$ denotes the maximum trigger threshold at which an input signal of amplitude still activates the digitizer channel, i.e. threshold values exceeding this $V[\text{lsb}]$ level prevent the system from triggering on signals of the corresponding amplitude.

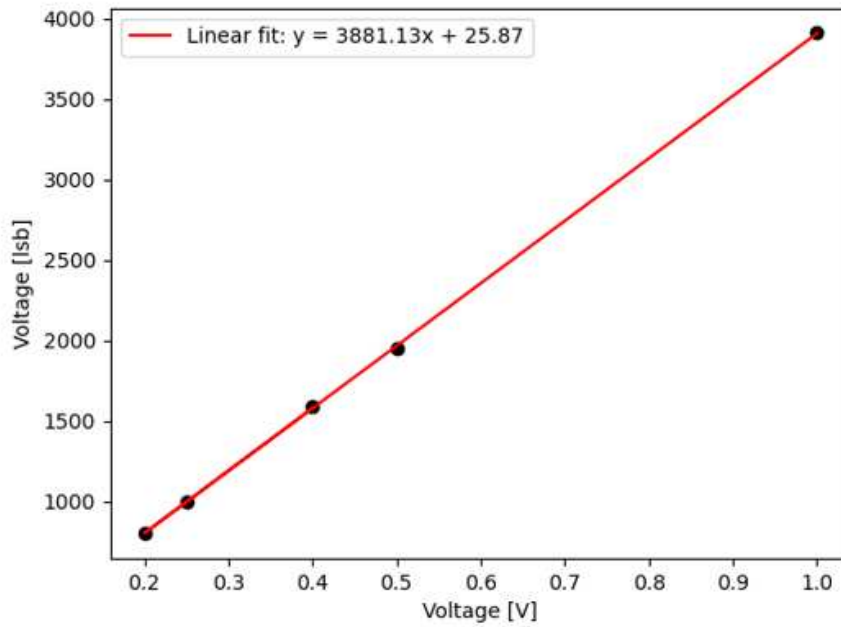


Figure 3: $V[V]$ to $V[\text{lsb}]$ voltage-calibration.

3.2 Signal amplitude calibration

The input signal amplitude, as displayed in COMPASS, is provided in ADC channel units. Therefore, a calibration was performed, and it is shown in Fig. 4. The resulting linear relation is given by:

$$V[\text{ADC}] = (3868 \pm 48) \cdot V[\text{V}] + (-3 \pm 36) \quad , \quad (2)$$

where $V[\text{ADC}]$ refers to the peak-to-peak voltage, after baseline subtraction. It is worth noting that the baseline itself exhibits a dependence on the amplitude of the input signal, a behavior which was consistently observed during testing. However, baseline level dependence on input signal amplitude $V[\text{V}]$ has not been further investigated.

Finally, to convert $V[\text{ADC}]$ values into physical voltages, one must use the inverse formula:

$$V[\text{V}] = (2.59 \pm 0.03) \cdot 10^{-4} \cdot V[\text{ADC}] + (0.00 \pm 0.09) \quad , \quad (3)$$

where errors are obtained by propagation.

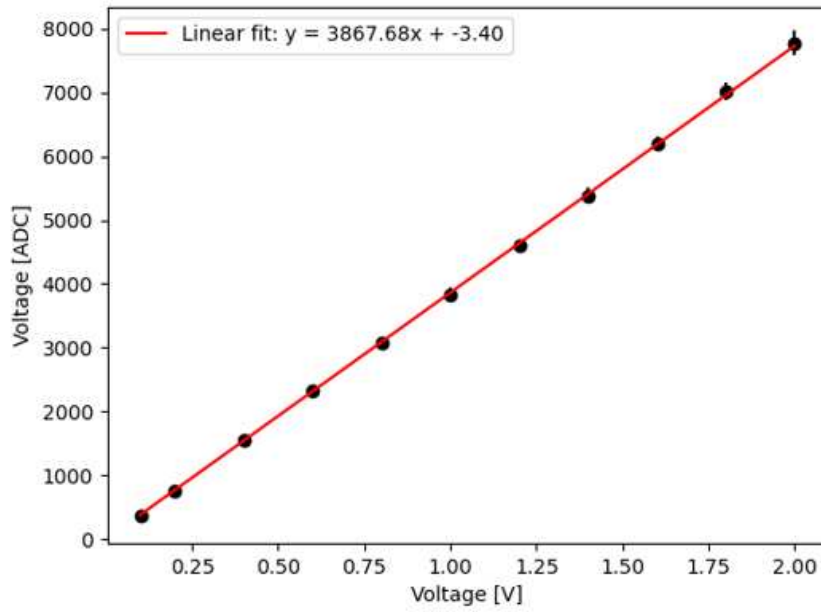


Figure 4: $V[\text{V}]$ to $V[\text{ADC}]$ voltage-calibration.

4 Clock calibration

Last tests focused on the calibration of the digitizer's internal clock. For this purpose, a Meinberg Funkhrehn microSync device (already synchronized via GPS) was employed to generate one NIM pulse per second for a total duration of 1000 seconds. The time tags recorded by the digitizer were then compared against the nominal time provided by the external reference. Fig. 5 shows the result of this comparison. A linear fit of the data yields the following calibration function for the digitizer's internal time base:

$$t_D = 0.9999890 \cdot t - 3.8 \cdot 10^{-7} \text{ s} \quad , \quad (4)$$

where t_D represents the time measured by the digitizer, and t denotes the nominal time provided by the GPS-synchronized microSync clock.

This result confirms a systematic drift in the digitizer's internal clock. Specifically, the slope being less than unity indicates that the digitizer "gains" approximately $11 \mu\text{s}$ per second relative to the reference clock. Additionally, it was verified that this time drift is independent of the shape or frequency of the input signal.

Given these findings, the use of an external clock reference is recommended for future applications in which precise time synchronization is required.

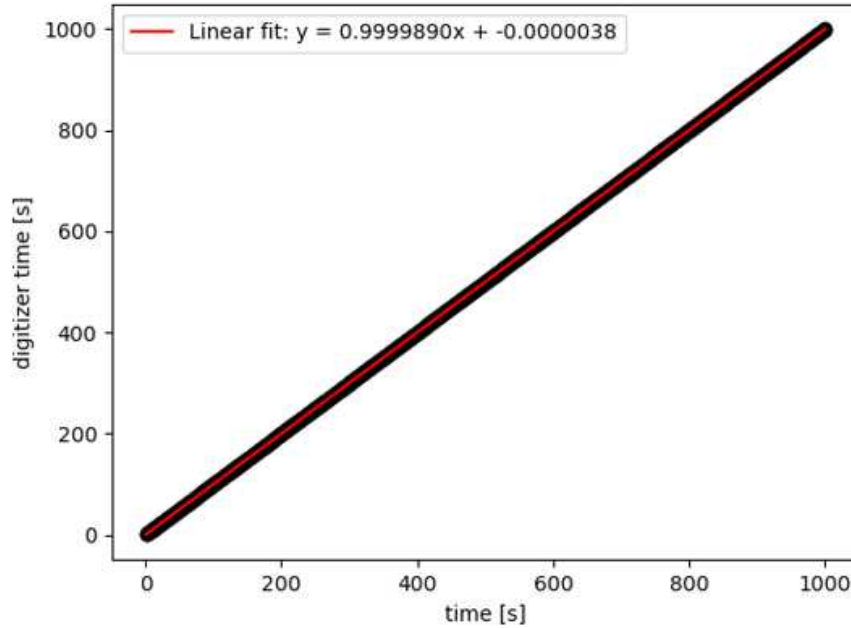


Figure 5: Time-calibration of the digitizer internal clock.

References

- [1] G. Fiksel *et al.*, *Phys. Rev. Lett.* **103**, 145002 (2009)
- [2] J. K. Anderson *et al.*, *Phys. Plasmas* **23**, 055702 (2016)

A.4 Future perspectives

Although the refurbishment, installation, and alignment of the NPA system for RFX-mod2 were originally included among the scopes of the Ph.D. project, these activities have been postponed pending the completion of the machine assembly and the revamping of the dedicated vacuum system. Installation and testing of the new detectors must indeed be performed under controlled vacuum conditions. The full calibration of the detection chain, including the applied electric and magnetic fields for particle detection, will be carried out exploiting a potassium ion source integrated into the diagnostic, and is therefore deferred to future work.

Appendix B

Derivation of Lagrangian equations of motion

In this Appendix, the mathematical framework underlying the description of the equilibrium magnetic field is outlined, and the derivation of the Lagrangian equations of motion reported in Section 2.3.2 of Chapter 2 is presented in detail. These equations are solved by the code ORBIT to describe charged particle dynamics within the guiding center approximation. Moreover, the formula for the island width cited in the thesis is here derived.

B.1 Canonical equations of magnetic field lines

As discussed in Section 2.2.1, in the RFX-mod implementation of ORBIT the equilibrium magnetic field is analytical and is splined from the output of the toroidal μ - p model. Then, it is transformed into Boozer coordinates by exploiting the canonical equations of the magnetic field:

$$\begin{aligned} q &= \frac{d\zeta}{d\theta} = \frac{\partial\psi}{\partial\psi_p} ; \\ \frac{d\psi_p}{d\theta} &= -\frac{\partial\psi}{\partial\zeta} . \end{aligned} \tag{B.1}$$

Both relations are derived from Eq. (2.3) together with the field-line equation. The first canonical equation, which is related to the definition of the safety factor, reads:

$$\begin{aligned} q &= \frac{d\zeta}{d\theta} = \frac{\vec{B} \cdot \nabla\zeta}{\vec{B} \cdot \nabla\theta} \\ &= \frac{(\nabla\psi \times \nabla\theta - \nabla\psi_p \times \nabla\zeta) \cdot \nabla\zeta}{(\nabla\psi \times \nabla\theta - \nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta} \\ &= -\frac{(\nabla\psi \times \nabla\theta) \cdot \nabla\zeta}{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta} \\ &= -\frac{\left[\left(\frac{\partial\psi}{\partial\psi_p} \nabla\psi_p + \frac{\partial\theta}{\partial\theta} \nabla\theta + \frac{\partial\psi}{\partial\zeta} \nabla\zeta \right) \times \nabla\theta \right] \cdot \nabla\zeta}{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta} \end{aligned}$$

$$\begin{aligned}
q &= -\frac{\partial\psi}{\partial\psi_p} \frac{(\nabla\psi_p \times \nabla\theta) \cdot \nabla\zeta}{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta} \\
&= -\frac{\partial\psi}{\partial\psi_p} \frac{(\nabla\zeta \times \nabla\psi_p) \cdot \nabla\theta}{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta} \\
&= \frac{\partial\psi}{\partial\psi_p} \frac{\cancel{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta}}{\cancel{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta}} = \frac{\partial\psi}{\partial\psi_p} .
\end{aligned} \tag{B.2}$$

It is worth noting that, according to the equation above, an alternative covariant representation of the magnetic field can be written as [30]:

$$\begin{aligned}
\vec{B} &= \nabla\psi \times \nabla\theta - \nabla\psi_p \times \nabla\zeta \\
&= \frac{\partial\psi}{\partial\psi_p} \nabla\psi_p \times \nabla\theta - \nabla\psi_p \times \nabla\zeta \\
&= q \nabla\psi_p \times \nabla\theta - \nabla\psi_p \times \nabla\zeta .
\end{aligned} \tag{B.3}$$

By applying the same procedure, the second canonical equation is obtained as:

$$\begin{aligned}
\frac{d\psi_p}{d\theta} &= \frac{\vec{B} \cdot \nabla\psi_p}{\vec{B} \cdot \nabla\theta} \\
&= \frac{(\nabla\psi \times \nabla\theta - \nabla\psi_p \times \nabla\zeta) \cdot \nabla\psi_p}{(\nabla\psi \times \nabla\theta - \nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta} \\
&= -\frac{(\nabla\psi \times \nabla\theta) \cdot \nabla\psi_p}{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta} \\
&= -\frac{\partial\psi}{\partial\zeta} \frac{(\nabla\zeta \times \nabla\theta) \cdot \nabla\psi_p}{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta} \\
&= -\frac{\partial\psi}{\partial\zeta} \frac{\cancel{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta}}{\cancel{(\nabla\psi_p \times \nabla\zeta) \cdot \nabla\theta}} = -\frac{\partial\psi}{\partial\zeta} .
\end{aligned} \tag{B.4}$$

B.2 Lagrangian equations

The Lagrangian equations derived at the end of Section 2.3.2 are calculated through:

$$\frac{d}{dt} \frac{\partial\mathcal{L}}{\partial\dot{x}_i} = \frac{\partial\mathcal{L}}{\partial x_i} , \tag{B.5}$$

where $\mathcal{L}$ is the Lagrangian (2.11), while x_i is the generic dynamical quantity for the charged particle motion, namely $x_i = \xi, \theta, \zeta, \psi_p$, or $\rho_{\parallel}$. Below, the calculation for each x_i is presented:

- $x_i = \xi$:

$$\frac{\partial\mathcal{L}}{\partial\dot{\xi}} = \mu \quad \frac{d}{dt} \frac{\partial\mathcal{L}}{\partial\dot{\xi}} = \frac{d\mu}{dt} \quad ; \quad \frac{\partial\mathcal{L}}{\partial\xi} = 0 .$$

The Lagrangian equation is thus given by:

$$\frac{d\mu}{dt} = 0 , \tag{B.6}$$

which is simply the conservation law for the particle magnetic momentum.

- $x_i = \theta$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} &= \psi + \rho_{\parallel} I & \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} &= \frac{\partial \psi}{\partial \dot{\psi}_p} \dot{\psi}_p + I \dot{\rho}_{\parallel} + \rho_{\parallel} \dot{I} \\ & & &= q \dot{\psi}_p + \rho_{\parallel} \frac{dI}{d\psi_p} \dot{\psi}_p + I \dot{\rho}_{\parallel} \\ & & &= \left(q + \rho_{\parallel} \frac{dI}{d\psi_p} \right) \dot{\psi}_p + I \dot{\rho}_{\parallel} ; & \frac{\partial \mathcal{L}}{\partial \theta} &= -\frac{\partial \mathcal{H}}{\partial \theta} . \end{aligned}$$

Therefore, it follows:

$$\left(q + \rho_{\parallel} \frac{dI}{d\psi_p} \right) \dot{\psi}_p + I \dot{\rho}_{\parallel} = -\frac{\partial \mathcal{H}}{\partial \theta} . \quad (\text{B.7})$$

- $x_i = \zeta$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \dot{\zeta}} &= \rho_{\parallel} g - \psi_p & \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\zeta}} &= \dot{\rho}_{\parallel} g + \rho_{\parallel} \frac{dg}{d\psi_p} \dot{\psi}_p - \dot{\psi}_p \\ & & &= \dot{\rho}_{\parallel} g + \left(\rho_{\parallel} \frac{dg}{d\psi_p} - 1 \right) \dot{\psi}_p ; & \frac{\partial \mathcal{L}}{\partial \zeta} &= -\frac{\partial \mathcal{H}}{\partial \zeta} . \end{aligned}$$

Hence:

$$\dot{\rho}_{\parallel} g + \left(\rho_{\parallel} \frac{dg}{d\psi_p} - 1 \right) \dot{\psi}_p = -\frac{\partial \mathcal{H}}{\partial \zeta} . \quad (\text{B.8})$$

- $x_i = \psi_p$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \dot{\psi}_p} &= 0 \quad , \text{ so } \quad \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\psi}_p} = 0 ; \\ \frac{\partial \mathcal{L}}{\partial \psi_p} &= \left(q + \rho_{\parallel} \frac{dI}{d\psi_p} \right) \dot{\theta} + \left(\rho_{\parallel} \frac{dg}{d\psi_p} - 1 \right) \dot{\zeta} - \frac{\partial \mathcal{H}}{\partial \psi_p} . \end{aligned}$$

Therefore, it results:

$$\left(q + \rho_{\parallel} \frac{dI}{d\psi_p} \right) \dot{\theta} + \left(\rho_{\parallel} \frac{dg}{d\psi_p} - 1 \right) \dot{\zeta} = \frac{\partial \mathcal{H}}{\partial \psi_p} . \quad (\text{B.9})$$

- $x_i = \rho_{\parallel}$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \dot{\rho}_{\parallel}} &= 0 \quad , \text{ so } \quad \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\rho}_{\parallel}} = 0 . \\ \frac{\partial \mathcal{L}}{\partial \rho_{\parallel}} &= I \dot{\theta} + g \dot{\zeta} - \frac{\partial \mathcal{H}}{\partial \rho_{\parallel}} . \end{aligned}$$

Finally:

$$\frac{\partial \mathcal{L}}{\partial \rho_{\parallel}} = I \dot{\theta} + g \dot{\zeta} = \frac{\partial \mathcal{H}}{\partial \rho_{\parallel}} . \quad (\text{B.10})$$

Eqs. (B.7), (B.8), (B.9), and (B.10) respectively correspond to Eqs. (2.13), (2.14), (2.15), and (2.16).

B.3 Island width

This section provide a justification of the expression for the magnetic island width reported in Eq. (1.18). The derivation is adapted from Ref. 66.

Under stationary conditions, namely neglecting time dependence, the radial perturbation to the equilibrium, associated with a generic tearing (m, n) mode can be written as:

$$\delta B_r = b_{m,n}^r \cos(m\theta - n\zeta) = b_{m,n}^r \cos(\Omega) = \frac{dr}{R_0 d\zeta}, \quad (\text{B.11})$$

where, in the last step, the field line equation has been used, and $\Omega = \Omega_{m,n}(\theta, \zeta)$ denotes the helical angle of the tearing mode. In Boozer coordinates, it takes the form:

$$\Omega = m\theta - n\zeta. \quad (\text{B.12})$$

Taking the derivative of both sides with respect to the toroidal angle ζ , one obtains:

$$\frac{d\Omega}{d\zeta} = m \frac{d\theta}{d\zeta} - n \quad \longrightarrow \quad \iota = \frac{1}{q} = \frac{n}{m} + \frac{1}{m} \frac{d\Omega}{d\zeta}.$$

In this step, the first of the canonical equations (B.1) has been substituted, and the notation $\iota = 1/q$ has been introduced¹. An alternative expression for ι can be obtained by performing a first-order Taylor expansion around the resonant surface radius $r_{m,n}$ of the (m, n) mode:

$$\iota(r) = \iota(r_{m,n}) + \frac{d\iota}{dr}(r - r_{m,n}) = \frac{n}{m} + \iota' \Delta r. \quad (\text{B.13})$$

By combining the two expressions for ι derived above, one obtains:

$$\frac{1}{m} \frac{d\Omega}{d\zeta} = \iota' \Delta r. \quad (\text{B.14})$$

Combining Eqs. (B.11) and (B.14), one obtains:

$$\frac{d(\Delta r^2)}{d\zeta} = 2\Delta r \frac{d\Delta r}{d\zeta} = 2\Delta r \frac{dr}{d\zeta} = \frac{2}{\iota' m} \frac{d\Omega}{d\zeta} \frac{dr}{d\zeta} = \frac{2}{\iota' m} \frac{d\Omega}{d\zeta} R_0 b_{m,n}^r \cos(\Omega).$$

Hence:

$$d(\Delta r^2) = 2 \frac{R_0 b_{m,n}^r}{\iota' m} \cos(\Omega) d\Omega. \quad (\text{B.15})$$

By integrating both sides we finally get:

$$\Delta r^2 = 2 \frac{R_0 b_{m,n}^r}{\iota' m} [\sin(\Omega) - \sin(\Omega_0)]. \quad (\text{B.16})$$

For the meaning of Ω and Ω_0 , see Fig. 1 in Ref. 66. The expression above is maximized for:

$$\Delta r_{max} = 2 \sqrt{\frac{R_0 b_{m,n}^r}{\iota' m}}. \quad (\text{B.17})$$

Therefore, the island width is given by:

$$w = 2 \cdot \Delta r_{max} = 4 \sqrt{\frac{R_0 b_{m,n}^r}{\iota' m}}. \quad (\text{B.18})$$

This result coincides with Eq. (8.55) of Zohm book [65], which reports the corresponding derivation for the tokamak case.

¹Indeed, it corresponds to the *rotational transform* ι commonly used in stellarator physics.

Appendix C

Analytical derivation of n_{max}

In this Appendix, we show how to determine the value n_{max} that yields a separation between the minima of the LP function consistent with the experimental observations, i.e. $\Delta\varphi_{n_{max}}^{\text{LP}} \approx \Delta\varphi = 15^\circ$.

For brevity, we will hereafter rename n_{max} as N , so the starting expression for LP results:

$$\begin{aligned} \text{LP}(\varphi) &= \frac{1}{D} \sum_{\ell=8}^N (N+1-\ell) \cos(\ell\varphi) \\ &= \frac{1}{D} \sum_{\ell=1}^N (N+1-\ell) \cos(\ell\varphi) - \sum_{\ell=1}^7 (N+1-\ell) \cos(\ell\varphi). \end{aligned} \quad (\text{C.1})$$

Since the distance $\Delta\varphi^{\text{LP}}$ between minima of LP depends explicitly on N , it is convenient to recast Eq. (C.1) into an analytical form that allows for differentiation. To this end, we exploit the *Dirichlet kernel*, a well known identity, given by:

$$\sum_{k=1}^N \cos(kx) = \frac{1}{2} \left[\frac{\sin\left[\left(N + \frac{1}{2}\right)x\right]}{\sin\left(\frac{x}{2}\right)} - 1 \right]. \quad (\text{C.2})$$

Similarly, one can prove also that:

$$\sum_{k=1}^N k \cdot \cos(kx) = \frac{1}{4 \sin^2\left(\frac{x}{2}\right)} \left\{ (N+1) \cos(Nx) + N \cos[(N+1)x] - 1 \right\}. \quad (\text{C.3})$$

Using this kernel, Eq. (C.1) can be rewritten as:

$$\begin{aligned} \text{LP}(\varphi) &= \frac{1}{D} \left\{ \frac{(N+1)}{2} \left[\frac{\sin\left[\left(N + \frac{1}{2}\right)\varphi\right]}{\sin\left(\frac{\varphi}{2}\right)} - 1 \right] + \right. \\ &\quad \left. - \frac{1}{4 \sin^2\left(\frac{\varphi}{2}\right)} \left\{ (N+1) \cos(N\varphi) + N \cos[(N+1)\varphi] - 1 \right\} - \sum_{\ell=1}^7 (N+1-\ell) \cos(\ell\varphi) \right\}. \end{aligned} \quad (\text{C.4})$$

Since $\text{LP}(\varphi)$ is an even function, the distance between its minima can be expressed as: $\Delta\varphi = 2\hat{\varphi}$ where $\hat{\varphi} > 0$ corresponds to the first positive minimum. The first derivative of

LP (φ) in Eq. (C.4) is:

$$\begin{aligned} \frac{dLP}{d\varphi} = \frac{1}{4D \sin^3\left(\frac{\varphi}{2}\right)} & \left\{ (N+1)(2N+1) \sin^2\left(\frac{\varphi}{2}\right) \cos\left[\left(N+\frac{1}{2}\right)\varphi\right] + \right. \\ & - (N+1) \sin\left(\frac{\varphi}{2}\right) \cos\left(\frac{\varphi}{2}\right) \sin\left[\left(N+\frac{1}{2}\right)\varphi\right] + \\ & - N(N+1) \sin\left(\frac{\varphi}{2}\right) \sin[(N+1)\varphi] + N(N+1) \sin\left(\frac{\varphi}{2}\right) \sin(N\varphi) + \\ & + \cos\left(\frac{\varphi}{2}\right) \{ (N+1) \cos(N\varphi) - N \cos[(N+1)\varphi] - 1 \} + \\ & \left. + 4 \sin^3\left(\frac{\varphi}{2}\right) [(N+1)B(\varphi) - C(\varphi)] \right\}, \end{aligned} \quad (C.5)$$

where $B(\varphi)$ and $C(\varphi)$ are respectively defined as:

$$\begin{aligned} B(\varphi) &= \sum_{\ell=1}^7 \ell \sin(\ell\varphi), \\ C(\varphi) &= \sum_{\ell=1}^7 \ell^2 \sin(\ell\varphi). \end{aligned}$$

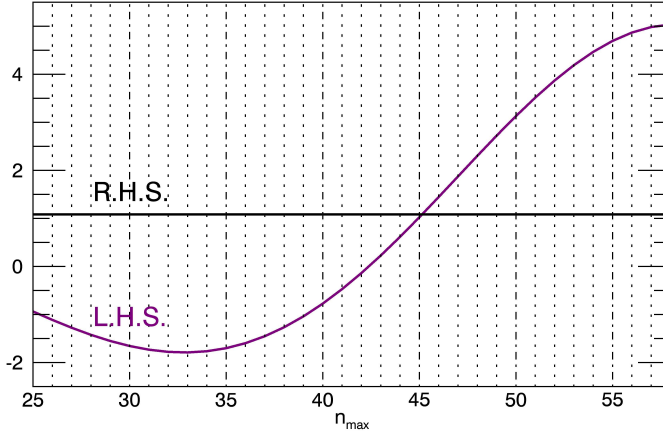


Figure C.1: Graphical solution of Eq. (C.6). Equation left-hand side trend is reported in purple, while equation right-hand side trend is reported in black.

By imposing that the condition of minimum $\left(\frac{dLP}{d\varphi}(\varphi_S) = 0\right)$ is satisfied for the experimentally measured position of the PWI stripe ($\varphi_S = 7.5^\circ$), one gets the following implicit equation to be solved in the variable N :

$$\begin{aligned} \left[2N \sin^2\left(\frac{\varphi_S}{2}\right) + 1\right] \cdot \cos\left[\left(N+\frac{1}{2}\right)\varphi_S\right] + N \sin\left(\frac{\varphi_S}{2}\right) \left[\sin(N\varphi_S) + 4 \sin^2\left(\frac{\varphi_S}{2}\right) B(\varphi_S)\right] \\ = \cos\left(\frac{\varphi_S}{2}\right) - 4 \sin^3\left(\frac{\varphi_S}{2}\right) [B(\varphi_S) - C(\varphi_S)], \end{aligned} \quad (C.6)$$

which we solved graphically, as illustrated in Fig. C.1. The solution provides:

$$N \longrightarrow n_{max} = 45. \quad (C.7)$$

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- P. Porcu, G. Spizzo, M. Veranda, M. Zuin, M. Agostini, M. Gobbin, and D. Terranova. Topology during magnetic reconnection events in RFX-mod. *Physics of Plasmas* **31**, 082506 (2024).
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Posters

- P. Porcu, G. Spizzo, M. Veranda, M. Zuin, M. Agostini, L. Carraro, M. Gobbin, L. Marrelli, P. Scarin, and D. Terranova. *3D Plasma-Wall Interaction during magnetic reconnection events in the RFX-mod device*, presented at 49th EPS Conference, Bordeaux (France) 2023.
- P. Porcu, G. Spizzo, M. Veranda, M. Zuin, M. Agostini, M. Gobbin, and D. Terranova. *Topology during magnetic reconnection events in RFX-mod*, presented at 66th DPP APS Annual Meeting, Atlanta (Georgia, USA) 2024.

- P. Porcu, G. Spizzo, M. Veranda, M. Zuin, M. Agostini, M. Gobbin, D. Terranova. *Magnetic topology of phase-locking driven Plasma-Wall Interaction in RFX-mod RFP discharges*, presented at 51st EPS Conference, Vilnius (Lithuania) 2025.
- P. Porcu, G. Spizzo, M. Veranda, M. Zuin, M. Agostini, M. Gobbin, D. Terranova. *Magnetic topology and ion dynamics during magnetic reconnections in RFX-mod*, presented at Fusenet PhD Event, Aix-en-Provence, ITER (France) 2025.