education sciences

Article

"Romeo and Juliet: A Love out of the Shell": Using Storytelling to Address Students' Misconceptions and Promote Modeling Competencies in Science

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Abstract: In this paper, we present the design of a Teaching—Learning Sequence (TLS) based on storytelling. The TLS has a twofold goal: to address students' misconceptions about atomic models and to promote students' development of modeling skills. The story is titled "Romeo and Juliet: a love out of the shell", and the characters are electrons living inside an atom. The TLS was tested with upper secondary school students. A qualitative analysis of the data shows that the TLS was able to engage students and helped them reconstruct the atomic model, while the story improved students' understanding of specific concepts related to the atomic model. The use of storytelling in the context of our research is discussed, together with the limitations of the story and possible future research developments.

Keywords: STEM education; storytelling and narrative; atomic models; modeling competencies; interdisciplinarity

Citation: Aquilina, G.; Dello Iacono, U.; Gabelli, L.; Picariello, L.; Scettri, G.; Termini, G. "Romeo and Juliet: A Love out of the Shell": Using Storytelling to Address Students' Misconceptions and Promote Modeling Competencies in Science. *Educ. Sci.* **2024**, *14*, 239. https://doi.org/10.3390/educsci14030239

Academic Editor: Myint Swe Khine

Received: 3 January 2024
Revised: 18 February 2024
Accepted: 19 February 2024
Published: 25 February 2024

1. Introduction

The literature shows that students' misconceptions about atomic models arise when they tackle the topic in school courses. After studying the planetary and Bohr atomic models, students cannot easily move beyond them, probably because they rarely feel the need to develop new atomic models [1]. In this paper, we use storytelling both to address students' misconceptions about atomic models and to promote students' development of modeling skills [2].

Storytelling is the art of narration and fulfills the need of humans to communicate, to entertain and self-entertain, to explain the world around us, and to communicate events or the actions of our ancestors [3]. Narration elicits a highly engaging relationship with the text, as the audience is actively involved through their cognitive, affective, and practical aptitudes and subjective abilities. From an educational perspective, storytelling is an effective tool for promoting the development of analytical and problem-solving skills. It serves an important educational role, as it preserves and transmits practical knowledge, thereby influencing human action. In this regard, it constitutes a significant teaching and learning method. For this reason, storytelling is used in many domains, for example, in mathematics education (e.g., the DIST-M model [4,5]), physics education (e.g., [6,7]), chemistry education (e.g., [8–10]), biology education [11,12] and sciences education [13,14], and it is also a powerful tool for interdisciplinary learning [15].

In this research, we explore storytelling precisely as an interdisciplinary tool capable of building bridges and forming pathways between different disciplines. In particular, the story we introduce aims to address questions related to physics, chemistry, and also mathematics. The plot of the story is inspired by Shakespeare's well-known literary work "Romeo and Juliet". However, the characters of our story are not humans but electrons living inside an atom. Through the tragicomic adventures of these electrons, we aim to address students' misconceptions about atomic models and to promote their modeling skills [2]. More specifically, we designed a Teaching–Learning Sequence (hereinafter named TLS) whose primary aim is to dispel the main misconceptions of high school students about atomic models by using storytelling as a tool to address and solve these issues. The TLS was tested with a sample of students attending two different classes at a technical computer science high school.

Our research questions were as follows:

(RQ1) to what extent did the TLS help students reconstruct the atomic model?

(RQ2) to what extent did the TLS engage students?

In the following, we describe the theoretical background on which the design of our TLS is based. Then, we describe the research method, that is, the story problem, the TLS design, the participants, the experimental setting, and the data analysis criteria. Thereafter, we show the preliminary findings of a qualitative analysis of the data collected during the experimentation. We conclude this paper by discussing the results obtained and possible future developments.

2. Theoretical Background

In this section, we describe the theoretical aspects underlying our TLS. In particular, we begin by introducing the students' main misconceptions about the atomic model, which our TLS intends to address, grouping them into categories functional to the development of the TLS (Section 2.1). Then, in Section 2.2, we explain the role of storytelling in the teaching–learning of sciences and the properties that a narrative should have to be effective. Since our TLS aims to promote students' development of modeling skills, in Section 2.3, we introduce the concept of mathematical modeling and the multifaceted aspects of modeling competence. Finally, in Section 2.4, we present the inquiry-based approach to science education, and we start introducing some of the choices we made while structuring our TLS.

2.1. Students' Misconceptions about the Atomic Models

There are many misconceptions that need to be addressed to promote understanding of atomic models. However, we have chosen to structure our story to address some of those most widely discussed in the literature. We grouped the misconceptions addressed in our work into three groups, named "On planetary and Bohr's models", "On energetic levels" and "On orbitals", as shown in Table 1.

Ivanjek et al. [16] report that students struggle to associate spectral lines with transitions between energy levels by linking a spectral line to a single energy level and to describe photon emission processes properly, that is, they fail to identify the electron as the object that goes through the transition [1]. Moreover, Taber [17] found out that students do not clearly understand the concept of an orbital and confuse orbitals with planetary orbits or concentric shells. Thus, they find it difficult to understand atomic quantum-mechanical models, that is, they fail to imagine the electrons as being "located" in orbitals defined in terms of probability and not subject to well-defined boundaries.

Table 1. Overview of the misconceptions addressed through the introduction of the story presented in this work.

On planetary and Bohr's models [1]	• Students, after studying planetary and Bohr's atomic models, cannot move beyond them easily. • Students rarely reflect on and/or understand the need for the development of new atomic models.
On energetic levels [16]	• Students find it difficult to associate spectral lines with transitions between energy levels. • Students do not describe photon emission processes properly.
On orbitals [17]	• Students do not clearly understand the concept of an orbital. • Students find it difficult to understand atomic quantum-mechanical models.

2.2. The Role of Narrative and Storytelling

According to Fuchs, "stories have the power to propose concepts and models and, therefore, elements of theory" [18] (p. 947). Recent research on the application of narrative techniques in the social sciences and its findings have elevated narrative to a methodological instrument within pedagogical practices. This tool is instrumental in cultivating conceptualization, aimed at nurturing the development of modeling and simulation skills [6].

Bruner [19,20] identifies the narrative properties that a story should have in a problem in order for the question to arise naturally from the story. These include the following:

- narrative diachronicity: a story needs to be plunged into a temporal dimension as to be present in a problem;
- normativeness: a story is about five key elements: at least one character, a goal to be achieved, an action, a useful tool to accomplish the action, and a specific situation as a set for the action.

These features are interconnected around a crisis or obstacle that should make it difficult for the character to achieve his/her goal. This concept is closely related to Herman's classification [21], which highlights a tension necessary for the events of the story among the constitutional elements of the story itself.

Mathematical story problems are problems written in text form in which the question lies in a situation familiar with the reader, often called "context", and play an important role in education practices [22]. The properties highlighted by Bruner prove to be very useful in the construction of a well-structured narrative context that avoids the emergence of narrative fractures [23], that is, obstacles that arise at the narrative level and prevent the development of the solving process (e.g., information necessary for solving a problem that is narratively implausible). More precisely, in a narrative problem, narrative fractures mean that the question does not refer to the story that has been narrated: this interrupts the bond between the elements of the narration and makes the comprehension useless for the reader to understand. In most cases, the narrative fracture is right between the context and the question. Therefore, "the greater the connection between the question and the story narrated in the context, the more the understanding of the story will promote the understanding of the question and ultimately the problem" (translation by authors) [23] (p. 438). Consequently, the final question has to narratively arise from the story. In this sense, Rosetta Zan created a model called "The C&Q (Context and Question) Model" [23] outlining the production of a problem text, which takes into account what has been said so far (Figure 1).

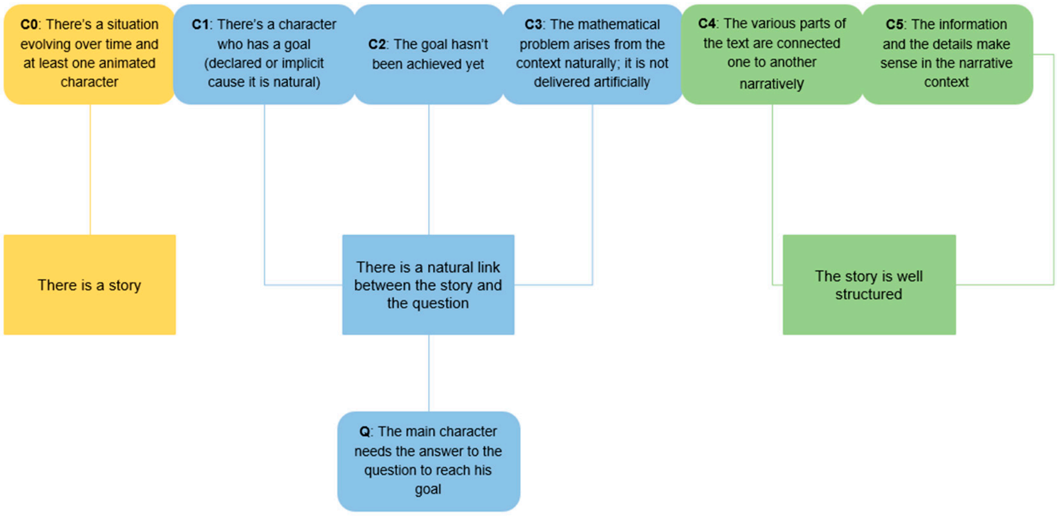


Figure 1. The C&Q model, as in [23].

The C&Q model is aimed at formulating a problem text that takes into account the elements highlighted by Bruner for a good story and the need for the question to arise naturally from the problem.

In order to describe the model, it can be divided into three blocks. The first block (C0) refers to narrative diachronicity and defines the necessary conditions for there to be a story; the second (C1, C2 and C3) is composed of all the properties that a story must have in order for a natural connection between context and question to emerge, namely, the presence of a character with an unachieved goal and the need for the mathematical problem to arise from the context; and the third (C4, C5) refers to the structure of the story by specifying that all its parts are connected narratively and that the information contained (the problem data) makes sense in the narrative context.

What emerges, particularly from the second block, is that, in order for the question to arise naturally from the context, the answer should be necessary for the achievement of the protagonist's purpose.

2.3. Modeling Competency

A model is often used to display only certain features of an object, so it serves as a simplified representation of the object itself [24]. In this sense, a mathematical model represents certain characteristics of a non-mathematical object through mathematical entities [2].

During the 1990s, the Danish KOM project [25] introduced an organizational model of competencies in mathematics. Among these, modeling competency refers to the use of mathematics to deal with extra-mathematical issues, contexts, and situations, and it is considered one of the core competencies. According to Niss and Højgaard, we mean modeling competency as "being able to construct such mathematical models, as well as to critically analyze and evaluate existing or proposed models, whilst taking purposes, data, facts, features and properties of the extra-mathematical domain being modelled into account" [24] (p. 16). Modeling competency actually means the interweaving of two main aspects. The first aspect is the ability to build models in different contexts and situations, a process that goes through the activation of different stages. The second aspect encapsulated by modeling competence is the ability to analyze a mathematical model by recognizing its purposes and validity in relation to the context.

In a pedagogical environment, the modeling process requires making choices, simplifying, and creating mental images of the studied situations. In this context, our aim is also to reflect on the limits of a model and on possible misconceptions that could arise when students build or use models.

2.4. Inquiry-Based Science Education

Inquiry-based pedagogy can be defined as a teaching approach which encourages students to work in a way that replicates the method of scientific research.

In the United States, this inquiry-based approach to science education has played a significant role in education since the early 20th century when the first criticisms of the traditional education model—which identified the sole purpose of schools as knowledge transmission—emerged. During those years, John Dewey (1852–1952) became an advocate for progressive education that supported "learning by doing". In 1909, in a speech at the American Association for the Advancement of Science (AAAS), he stated that the teaching of sciences emphasizes the accumulation of information over the conception of science as a way of thinking, as a mental habit [26]. Dewey believed that science was much more than a body of knowledge to be learned and that the scientific method should not be exclusive to scientists alone [26]. Instead, it could serve as a learning method that fosters free thinking in anyone [27]. This marked the beginning of discussions about teaching not only the corpus of scientific facts and knowledge but also, and especially, the process of scientific inquiry. In this way, teachers, accustomed to transmitting their knowledge, become facilitators of student learning, with students becoming active protagonists in the process of knowledge building [28]. In this context, it is important to distinguish between two different pedagogical approaches to sciences, the deductive and the inductive approaches: the deductive approach is still the most commonly adopted one in European and Italian schools and is referred to as "top-down transmission" because knowledge is transmitted from the teacher, who presents the content and its deductive logic, to the students. The inductive approach, on the other hand, allows for the reconstruction of knowledge by students themselves, with the guidance and support of teachers, and for this reason, it is called "bottom-up" ("from the bottom to the top").

Recent studies [29,30] that analyzed data provided by the PISA 2015 surveys have compared the impact of Inquiry-Based Learning (IBL) and teacher-directed practices on students' attitudes towards science and their academic performance. Both studies advocate for the use of inquiry-based activities that are accompanied and guided by the teacher, while casting doubt on the effectiveness of overly unrestricted inquiry. For this reason and according to the inquiry-based approach to science education, in order to foster learners' engagement in the proposed TLS, we decided to follow the 5E cycle model [28], with the guidance of the teacher-researcher part of our team. This model promotes collaborative and active learning in which students work together to solve problems and investigate new concepts by asking questions, observing, analyzing, and drawing conclusions. To provide more solid guidance to the students, we decided to structure our TLS with the alternation of group work and class discussion orchestrated by the teacher. In Section 3.2 we describe the phases of the TLS, coordinating them with the features and the objectives of the phases of the 5E cycle.

3. Method

In this section, we describe how all the elements of the theoretical framework outlined in the previous section enter the various parts of which our TLS is composed. Our TLS consists of five different phases according to the 5E model. In order to better illustrate the link between the phases of our TLS and those of the model, we begin by presenting the story problem we developed, providing a rationale for our choices (Section 3.1) and aligning them with the students' misconceptions regarding atomic models and Zan's C&Q model. Moreover, using the 5E model, in Section 3.2 we outline the phases of the Teaching and Learning Sequence (TLS) in line with our goal of enhancing modeling competence. Additionally, we provide information about the participants and the experimental setup (Section 3.3). We conclude this section by outlining the research methodology we adopted for the data analysis (Section 3.4).

3.1. The Story Problem

In the following tables, we present the story (left column) together with a rationale of the design principle used while inventing it (right column).

PROLOGUE

Prologue	Rationale of design principles
Romeo is a bold and dynamic electron found in an atom with seven energy levels. He is at the 4s energy level, together with the faithful Mercutio, his companion on raids. Always upside down compared to him, but then there is no place for two equal electrons in their crew. The two are part of the Montague family, known for being particularly lively. *Juliet is an electron in 2s, she is more tied to her nucleus and in fact she is a Capulet, a rival family to that of the Montagues and decidedly more calm. Juliet is always accompanied by her nurse; they too are turned upside down with respect to each other.* *There is a grand ball to which everyone is invited, and, to better organize their arrangement, there is a need to schematize their position.* Discuss with your classmates what should be the design of the atom where the two families «are» and build a model.	As highlighted in Section 2.1, some misconceptions can arise in students' understanding of atomic models. To identify these misconceptions, the initial part of the story involves students in identifying the main characteristics of atoms and modeling them. Consequently, they are encouraged to connect their prior knowledge with the essential elements of the narrative. Notably, the story provides subtle hints, such as Romeo being depicted as a bold and dynamic electron (an idea students are familiar with due to their understanding of electrons moving around nuclei) or his friendship with Mercutio, with whom he exhibits a contrasting relationship (which relates to students' knowledge of the Pauli Exclusion Principle from their background in chemistry). Meanwhile, Juliet is portrayed as more closely tied to the nucleus since she resides in a 2s orbital. In this context, the teacher's objective is to pinpoint any misconceptions related to the atomic model that students may have developed from their previous studies. Following the C&Q model, the Prologue introduces the characters and immerses them in a plot, featuring a possible twist—the grand ball. The specific location of the grand ball remains implicit, as it is challenging to conceive of electrons dancing outside the metaphorical context of "moving swiftly". However, all the other character details are essential for initiating the story and allowing mathematical and physical problems and situations to emerge.

CHAPTER 1

Chapter 1—part 1	Rationale of design principles
At one point during the dance, Romeo notices Juliet in her orbital, and, even if he occasionally gets close to her, he is unable to stay there permanently: quivering with love, he asks who knows her and what her tastes are in terms of radiations (electrons are well known to be romantics). He discovers that Juliet is obsessed with color harmony and that the color she prefers is purple "486 nm". To get noticed he wants to perform his famous photon—spectroscopic serenade and jump to emit a purple trail. Discuss with your teammates to help Romeo understand how far he will have to jump and whether or not he would have gotten closer to Juliet in this way.	In this chapter, we delve into the concept of interatomic orbital distances, exploring the inherent limitations imposed by electron position uncertainties. Much like the iconic star-crossed lovers, Romeo and Juliet, who yearn to be close but find themselves unable to maintain a permanent proximity, electrons face similar challenges in their spatial arrangements. To elucidate the intriguing connection between emission and electron transitions to different energy levels, we introduce a romantic–comedic twist, employing Juliet's passion for color harmony as a plot device. Juliet's preference for the color purple is strategically chosen to align with her energy level, prompting students to contemplate the intriguing relationship between spectroscopy lines and electron energy transitions. As students ponder the impossibility of Romeo and Juliet's enduring closeness, they begin to discern potential disparities between their mental models and theoretical concepts. Through calculations of energy transitions and the resulting orbital distances, students gain insight into the quadratic proportionality that underlies these phenomena, prompting a gradual reshaping of their personal notions regarding orbital distances.

CHAPTER 1	
Chapter 1—part 2	Rationale of design principles
The two are deeply in love and would like to spend the rest of their days together. But Juliet's family hinders them, crying scandal: a Montague cannot be so tied to the nucleus! What to do? The nurse offers Romeo the chance to take her place, but, for her, this would mean losing her place next to Juliet. Romeo and Juliet, very hesitant, then decide to move towards the orbitals occupied by the Montagues. But how to get up there? While the couple is tormented by this problem, an enlightened friar, Lory, arrives to their rescue with two THz 457s, offering to give them a lift. Despite this help, Romeo and Juliet are unable to reach the Montague orbital, so they loudly invoke another friar, Enzo, asking for new help. *Discuss with your teammates to understand how far they will jump thanks to the first photons and which photons Fra Enzo will have to carry for the two lovers to reach the Montague orbital*.	In this section, students confront the challenges posed by the spatial separations between orbitals, compelling them to grasp two essential aspects. Firstly, they must discern the specific energy jump that a photon of a given frequency can facilitate, (the choice of Friar Lory in the plot is due to the role of Frair Lorenzo in "Romeo and Juliet"). Secondly, they need to determine the frequency of the photon required to reach the Montague orbital, so as to reflect on the inverse problem. In the framework of the C&Q model, it is strikingly evident that the characters are profoundly influenced by their individual roles and aspirations. Take, for instance, Romeo's desire to gain Juliet's attention and their joint pursuit of a life away from their feuding families. This narrative intricately parallels the fundamental interplay of orbitals within the model, establishing a direct and compelling link between the characters' human drama and the pivotal role of orbitals in the model. Following the previous chapter, a deliberate choice was made to foster a collaborative learning environment among the students. The primary aim of this approach is to collectively cultivate a shared knowledge base within the entire class, enabling them to collaboratively synthesize the concepts explored. In this pedagogical strategy, students direct their attention towards specific facets of their engagement and exploratory experiences, thereby demonstrating their grasp of the concepts and their proficiency in employing related process skills. In this context, educators introduce a given concept, process, or skill, and learners are encouraged to articulate their understanding of these elements. Teachers play a pivotal role in guiding students towards achieving a deeper comprehension of the topic. Furthermore, students are tasked with constructing arguments that reflect their unique ideas, integrating various forms of representation such as verbal, graphic, and analytical. Simultaneously, they assess whether the atomic model they hold aligns with the theoretical framework under examination or if it necessitates adjustment.

CHAPTER 2	
Chapter 2 and Epilogue	Rationale of design principles
Juliet's escape has thrown the entire atomic balance into crisis, forcing some Montagues to change levels in order to maintain overall stability. Then, when the couple comes to the Montagues, they cry out for revenge, and the couple is then forced to flee again. *The Montagues set out in search of Romeo and Juliet but fail because it is not possible to reconstruct the trajectory followed by the two lovers.* *The story unfortunately ends in tragedy: the two do manage to free themselves from the influence of their families, but they still understand that they cannot be together. Now condemned to separation, the two lovers decide to draw up a schema of the place (the atom) where they met to remember it forever.* Discuss with your teammates why this trajectory cannot be reconstructed. End the story with a tragic ending, explaining the reasons for the separation sentence. EPILOGUE Construct with your teammates a possible model of the scheme realized by Romeo and Juliet.	In this second chapter, students begin to grapple with the consequences of the two electrons escaping and the inherent uncertainties that the uncertainty principle introduces. The tragic conclusion of the classic novel finds a reflection in the fate of these two electrons; once they are no longer bound to the atom, their paths are inevitably set on a collision course due to their opposite charges, leading them to be absorbed in a vast sea of electrons. At the outset, the consequences of Romeo and Juliet's choices become apparent: the voids within the nucleus are replenished with new electrons, ultimately disturbing the equilibrium of the two feuding families. This disruption leads them to share orbits, not fueled by anger but by fate. The Montagues seek revenge, yet they grapple with the inability to reconstruct the electrons' orbitals due to the uncertainty principle. As part of their learning journey, students are encouraged to collaboratively sketch their envisioned final atomic model following the narrative's progression and to contemplate how they might populate it with electron positions. This final chapter is meticulously structured to allow students to solidify their acquired knowledge, particularly regarding the uncertainty principle and its relationship with trajectories. It seamlessly weaves this scientific understanding with the timeless drama of the story. In particular, a poignant concept is introduced towards the end, one that resonates deeply with the tragic essence of the narrative: the impossibility of two electrons remaining together outside the nucleus, mirroring Romeo and Juliet's inability to coexist within their own narrative and drama. Instead, they find fulfillment in their tragic demise, existing beyond the scope of the other characters who continue to live. In this sense, the proposed narrative gains a profound, implicit connection with the dramatic elements, as the drama itself prompts us to question the fate of an electron outside the nucleus, without its nucleus as a reference point. Ultimately, even the identities of Romeo and Juliet lose their significance beyond their respective contexts, much like the indistinguishability of electrons except for the energy levels they occupy. While this concept may seem tangential to the primary objective of the story, it remains coherent with the underlying physics, providing a solid foundation for future developments in the atomic theory of nuclear physics, atoms, and electricity.

3.2. The Teaching—Learning Sequence (TLS)

As we partly described in the previous section (Section 3.1), we structured our TLS into five different phases (Prologue, Chapter 1, in-between collective discussion, Chapter 2, Epilogue and final collective discussion) with the alternation of group work and class discussions. During the group work, students deal with the story and the different tasks related to it, while, during the class discussions, the teacher supports students to share and to discuss the work undertaken in groups in order to create a class common knowledge. The structure we introduced is consistent with the inquiry-based learning goals, as it aims to create a learning environment similar to that of a research community, and, in particular, it follows the 5E learning cycle.

In this section we make explicit the connection between the phases of our TLS and the phases of the 5E cycle (see Figure 2).

Figure 2. The 5E cycle and the phases of our story.

In the first phase of the cycle, that is, the engagement phase, teachers assess the learners' prior knowledge and help them become engaged in a new concept through the use of short activities that foster curiosity and elicit prior knowledge [28]. In such a context, storytelling proves to be a very effective tool to promote students' interest and engagement, especially when they deal with difficult and tricky topics.

The first part of our story, the "Prologue", is built to activate the engagement phase of the cycle, as the students need to connect their knowledge with the critical elements of the story in order to identify the main characteristics of the atom and build a first model of it. The final task of this first section goes exactly in this direction: "Discuss with your classmates what should be the design of the atom where the two families «are» and build a model." This way, the teacher should be able to recognize possible students' misconceptions about the atomic model and to evaluate their beginning competencies.

In the explore phase students carry out activities within which current concepts (i.e., misconceptions), processes, and skills are identified, and conceptual change is facilitated. Students should be involved in activities that help them use prior knowledge to generate new ideas and to design and conduct a preliminary investigation [28].

With the first Chapter of our story, we enter in the explore phase of the 5E cycle, in which students have to use their prior knowledge and the data given by the story to explore the properties of the atoms and start to confront the atomic model they have in mind with the theoretical one, in a process of continuous self-assessment. In particular, "Chapter 1" is built to make them reason about the concept of orbitals, the link between spectral lines and the transition between energy levels, and the role of photons in these transitions (see Section 2.1 about students' misconceptions on atomic models). In particular, we gave the students two different tasks: in "Chapter 1—part 1", they have to determine the quantum jump that an electron in a given orbital has to perform to emit a purple trail and to understand whether it would move closer to or farther from the nucleus; in "Chapter 1—part 2", they have to calculate the quantum jump enabled by a given photon and to determine the energy of a second photon, which should lead two electrons to a given orbital.

At the end of the first Chapter, we chose to include a collective discussion within the different groups, orchestrated by the teacher. In this phase the teacher stimulates the students to construct arguments to support their solutions to the tasks and to compare their choices with those of the other groups. The purpose of the collective discussion is to build know-how, knowledge shared by the whole class, and to make sense, collectively, of the concepts explored. In this sense, this part of our TLS is the explain phase of the 5E cycle, in which students focus on a particular aspect of their engagement and exploration experiences and demonstrate their conceptual understanding and process skills, while the teacher formalizes the concepts brought up by the proposed activity [28]. In particular, we expect that the students will focalize the main properties of the atoms and the atomic models, coordinating different representation registers, and that they will recognize if the atomic model they had in mind is in line with the theoretical one.

In the elaborate phase, teachers challenge and extend students' conceptual understanding and skills. Through new experiences, students develop deeper understanding, more information, and adequate skills [28].

The second Chapter of our story leads the students to elaborate and deepen their understanding of the situation explored, as this time they have to use the atomic model to construct conjectures and to clarify some unclear passages of the story. In particular, we gave the students two consequent tasks: first, they have to understand the reason behind the impossibility of reconstructing the trajectories drawn by Romeo and Juliet; second, they are asked to formulate the possible reasons why the story ends in tragedy, consolidating the difference between the concept of orbitals and orbits and familiarizing them with the quantum-mechanical model of the atoms.

According to the 5E model [28], the evaluate phase is transversal to all the phases of the cycle, as the students do an auto-evaluation during their entire learning process, in the perspective of formative assessment. At the end of the activities proposed, it is advisable to make students draw their conclusions, while the teacher evaluates their scientific comprehension of the topic explored.

In this view, at the end of the second Chapter, the two lovers express their last wish, which opens the road to the "Epilogue" of the TLS. The students are again asked to construct a model of the atom, considering everything they have seen and learned from Romeo and Juliet's adventures. This task will allow students to undertake an auto-evaluation of their progress, confronting the first atomic model they drew with the final one and recollecting all the moments of self-assessment they have performed during the different phases of the story.

At the end of the final group work, the teacher leads a collective final discussion to help the students to meta-reflect on their own learning processes and has the opportunity to evaluate if they have reached the expected educational goals.

3.3. Participants and Setting

To assess the feasibility and efficacy of the proposed activity, an exploratory study was carried out. The study involved a sample of 41 students from two distinct 10th-grade classes, with the participants' average age being between 15 and 16, attending a technical computer science high school. Since one of the authors of this paper was a teacher in these classes, the activity was implemented during regular school hours.

The students were affiliated with so-called "digital classes", where iPads are used as primary tools for studying and taking notes.

The students who took part in the activity had already studied the atomic model in their chemistry classes during the first half of the year. This was an essential feature for conducting our study, as one of the goals of the activity was to address some of the misconceptions resulting from traditional study methods.

The first class was composed of 20 males, while the second class had 17 males and four females. This gender imbalance was because technical industrial institutes specializing in computer science typically have very few female students.

Groups of three or four students were formed to work on the same tasks, without pre-structured roles. The activity was carried out for 5 h. Students were actively engaged in solving tasks, with assessments carried out at each step. Groups were organized by the teacher to ensure heterogeneity in terms of social interaction: students who typically found it more challenging to engage with others were paired with more collaborative students, to ensure effective interaction within all groups. Students' proficiency levels in chemistry or physics were not taken into account during the formation of the groups.

During the activities, students received the reading material and corresponding tasks via Google Classroom. The teacher played the role of an observer and, when requested, a facilitator. While moving between different groups, the teacher observed how students were working but refrained from interrupting their work, intervening only when explicitly requested by the students to clarify a task, explain an unfamiliar term, or provide encouragement for their ongoing work.

All the activities took place in a school laboratory characterized by spacious areas, chairs/desks on wheels that students could easily move around to form small workstations, and multiple whiteboards where some groups could write down calculations. The choice of location was made to optimize teamwork, providing ample space to prevent disruption due to noise, and allowing for easy sharing of work through whiteboards or file sharing via iPads.

As we explained in Section 3.2, the activities proposed in this exploratory study followed the planned framework of the 5E model. The division of the narrative remained consistent with the originally intended structure, including the same tasks, and all the activities adhered to the same framework. The discussions held within the groups and with the entire class were recorded, along with pictures and work produced by the students, for subsequent analysis.

3.4. Data Collection and Analysis Criteria

We collected the following data:

1. Recordings of student discussions, both within small groups and plenary sessions during the entire cycle;
2. Drawings of the atomic models created by students in the "Prologue" phase and in the "Epilogue" phase;
3. Written reports produced by students regarding the calculation of energy levels and radii in "Chapter 1", and the conclusion of the story in "Chapter 2";
4. A diary maintained by the supervising teacher during the entire cycle;
5. Interviews conducted with two chemistry teachers who participated in the final discussion during the "Epilogue" phase;
6. A satisfaction questionnaire given to the students upon completing the activity.

To address the first research question (RQ1), "To what extent does the learning activity help students in reconstructing the atomic model?", we analyzed the drawings representing the atom based on the atomic models owned by the students at the beginning and at the end of the activity, and we examined if there were significant changes in these representations. Additionally, we analyzed audio recordings of the final discussions to gain a better understanding of students' drawings.

The drawings created by the students were categorized based on five indicators: the size and shape of orbitals, the distance between orbitals, the representation of electrons, the shape and size of the nucleus, and how the spin of electrons was depicted.

Furthermore, drawings within the same group were compared to identify any changes regarding the aforementioned indicators.

Regarding the analysis of audio recordings, all audio files were transcribed and associated with each drawing or task to which the student referred while speaking. Within the transcribed texts, phrases such as "Here we did this because. . .", "We wanted to indicate that. . .", "The concept we wanted to convey was. . .", were identified, highlighting the intention to create a specific drawing to express a particular concept.

To address the second research question (RQ2), namely, "To what extent did the narrative engage students?", we analyzed the following:

- answers to a satisfaction questionnaire administered to students at the end of the activity, to gauge how the students themselves experienced the entire activity;
- the observations conducted by the teachers participating in the activity (the instructor who conducted the activity and two chemistry teachers who took part in the final discussion), to understand if the teacher's perception was corroborated or not.

In the analysis of the supervising teacher's diary and in the audio recordings of the observations made by the chemistry teachers, phrases or expressions indicating student engagement, such as "I noticed that the students participated/were involved", were sought. Similarly, in the written responses of the students, phrases such as "I felt involved", "I enjoyed it", or "It was nice" were searched for.

4. Preliminary Findings

A qualitative data analysis shows that students were engaged in the activity and the story helped them to better understand specific concepts related to the atomic model. At the beginning of the activity, nearly all students exhibited a misconception regarding linear proportionality, namely, the belief that orbits/orbitals are equidistant from each other, and that there is a direct proportionality between the principal quantum number (n) and the radius. In the "Prologue" phase, students held the planetary model in mind, while in the "Epilogue", although some students still retained the idea of equally spaced orbits, most of them achieved a correct understanding of the concept of orbit/orbital going beyond the "old" idea of distance.

4.1. Students' Models

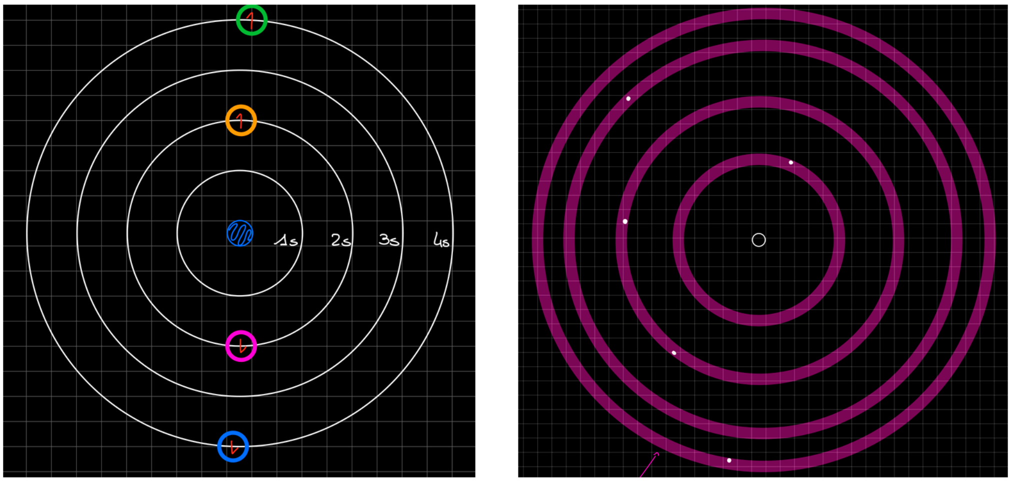
From the analysis of the drawings, it emerges that the students' final drawings can be traced back to three different types of atom representation (R):

- R1: orbits/orbitals represented at varying distances to convey the concept of energy levels more effectively;
- R2: orbits/orbitals represented at correct distances according to the radius;
- R3: attempt to depict the concept of orbitals and the correct distances between them.

The preliminary analysis of the drawings created by the students shows that the Bohr model predominates. Despite their discussion about orbitals, all groups drew orbits, representing them as lines depicting the trajectories of electrons. The drawn orbits are equidistant, thereby confirming the misconception associated with direct proportionality. Sometimes, students introduced elements of the atomic quantum-mechanical models, such as spin. However, even if they represented the spin with arrows "acting" in different directions, from these representations it seems they did not have a clear understanding of this concept.

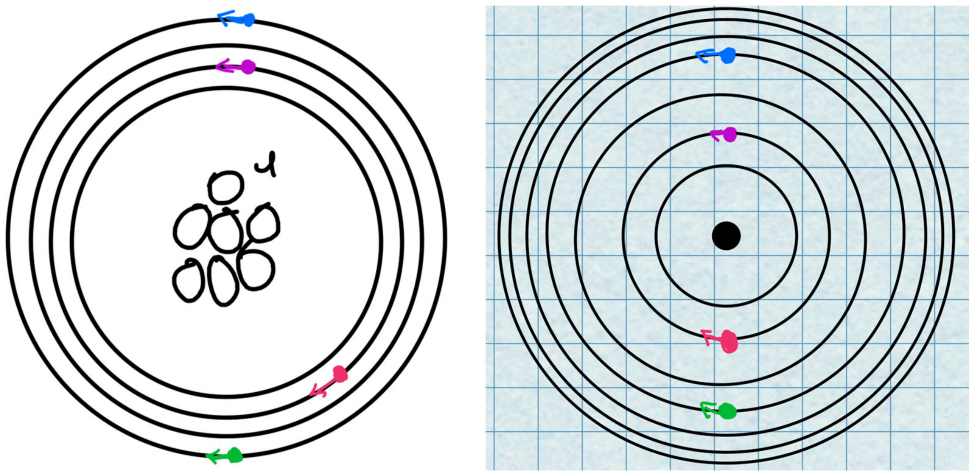
4.1.1. Atom Representation (R1)

Examples of graphic representations of the atom made by students with their iPads are shown in Figures 3 and 4.



(**a**) (**b**)

Figure 3. Drawings of the atomic model created by Group 3 students with their iPads: (**a**) in the "Prologue" phase; (**b**) in the "Epilogue" phase. Colours have no particular meaning.



(**a**) (**b**)

Figure 4. The drawings of the atomic model created by Group 8 students with their iPads: (**a**) in the "Prologue" phase; (**b**) in the "Epilogue" phase.

Students of Group 3 (see Figure 3) initially represented the atom according to a planetary model: well-defined orbits, delineated with lines, and electrons positioned on these lines. In accordance with the literature, it is evident that the Bohr model is deeply rooted in the students, although other atomic models have been introduced during school chemistry courses. In the final representation (Figure 3b), it is noticeable that they used a thicker and more transparent stroke, rather than a simple line, to convey the concept of orbitals. The electrons are now depicted as smaller dots and are not positioned in the middle of the lines. Instead, they are illustrated at different locations within the space representing the orbital. The orbital distances do not correspond to the actual radii. Nevertheless, students justify this choice as follows:

Student 1: *Since we know that electrons don't move in precise, let's say, circles... With the Bohr atom, it was said that electrons followed specific orbits.*

Teacher: *How did you draw them then? (Referring to the initial drawing.)*

Student 1: *In a circle, we drew lines. But we know that electrons don't follow that precise path; they exist in orbitals, which are regions where electrons are more likely to be found. So, we don't know the precise radius because it's a region. Therefore, in my opinion, since the radius can always vary, you can't use the radius to depict the atomic model; it's more accurate to use energy levels.*

The students, therefore, made a deliberate and informed choice in line with their comprehension during the activities. They constructed a new representation that could better convey the concepts they wanted to express, giving new meaning to the representation.

Other groups independently arrived at the same conclusion, as can be observed in Figure 4b and from the discussion that took place to explain the drawing:

Student 2: *We thought of representing the atom with varying sizes.*

Teacher: *Here you have drawn the distances increasingly closer. Why?*

Student 2: *Because it represented differences in energy levels.*

4.1.2. Atom Representation (R2)

Several groups struggled to convey the concept of orbitals (see Figure 5), continuing to represent the orbitals in the form of lines (henceforth, when students draw lines resembling electron trajectories and orbits, we shall refer to them as ”orbitals” for the sake of simplicity). However, the radii distances are now correct and no longer appear at regular intervals as they did initially in the initial drawings. Furthermore, they also emphasized that the energy difference between one orbital and another decreased as one moved farther away. Some drawings are presented here to facilitate a better understanding of the similarities across different representations.

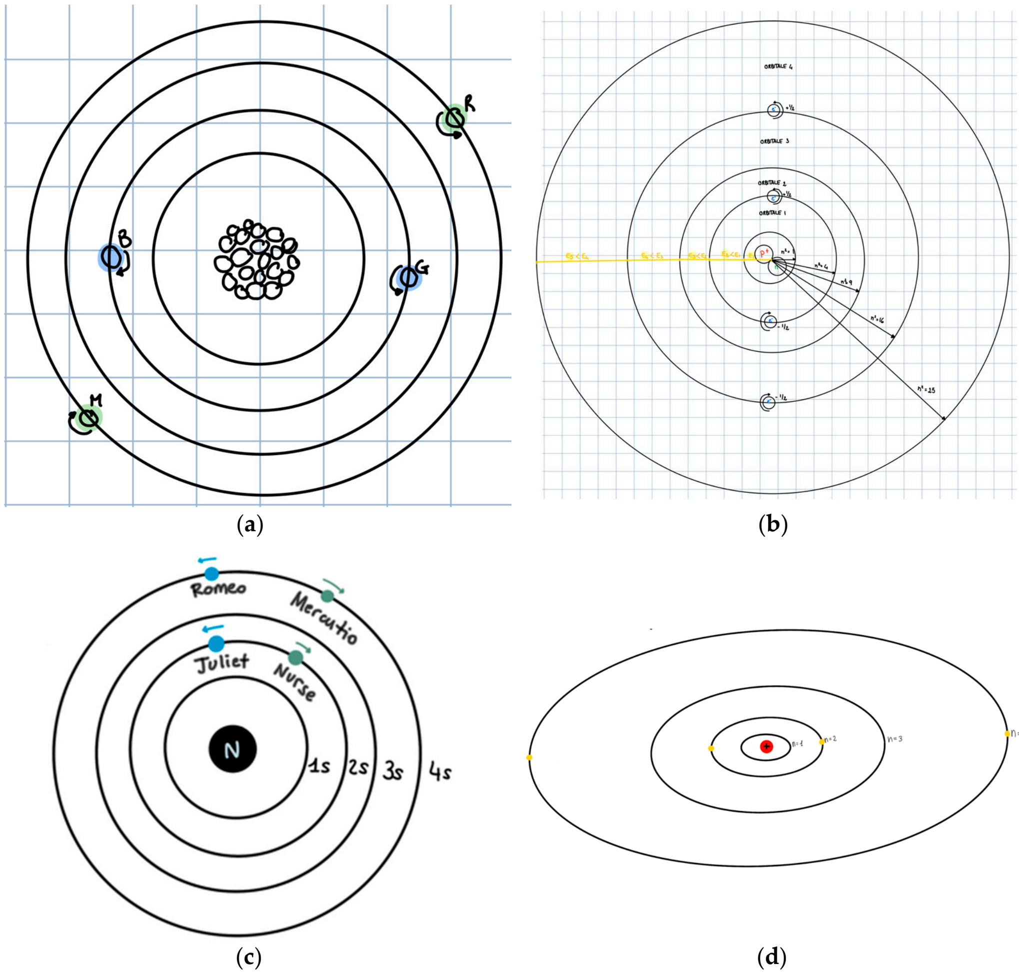
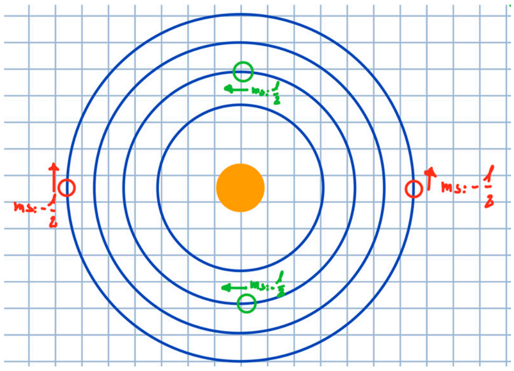
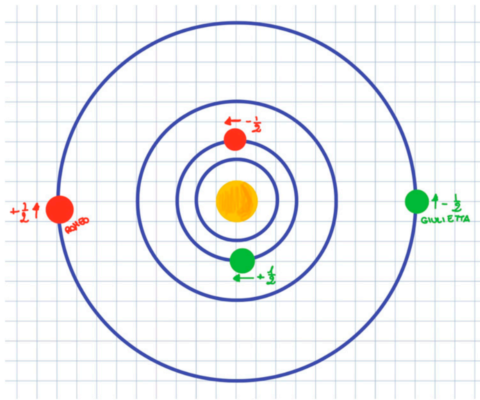
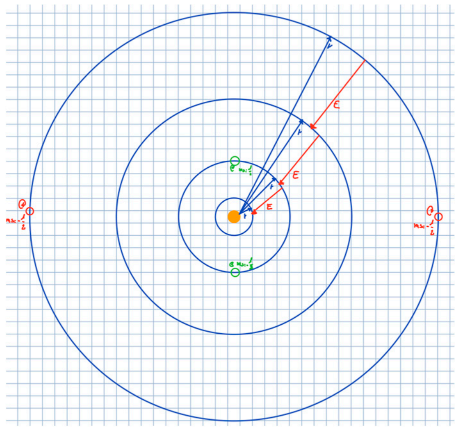


Figure 5. *Cont.*

(**e**) (**f**)



(**g**)

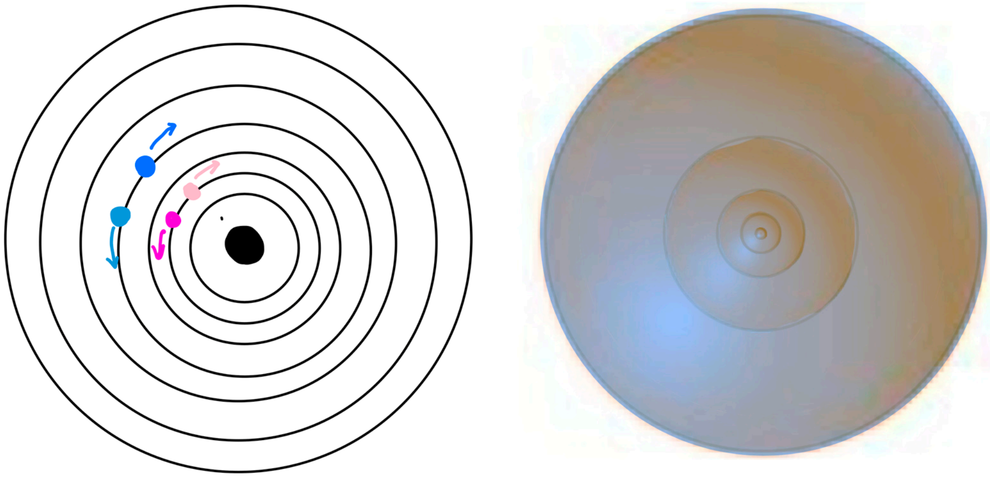
Figure 5. The drawings of the atomic model: created by Group 10 students with their iPads in the "Prologue" phase (**a**) and in the "Epilogue" phase (**b**); created by Group 6 in the "Prologue" phase (**c**) and in the "Epilogue" phase (**d**); created by Group 5 in the "Prologue" phase (**e**), in "Chapter 1"(**f**), and in the "Epilogue" phase (**g**).

Student 3, referring to the representation in Figure 5b, stated: *In our representation, we increased the distance from the nucleus, and this increase follows a pattern of n^2 based on the orbital. However, we also noted that the energy difference (E_4, E_5, etc.) decreases as one moves farther away*.

4.1.3. Atom Representation (R3)

On the other hand, another group presented different final drawings. As can be observed in Figure 6a, the initial representation exhibits the same issues observed previously: circular orbits were delineated by lines, and the spins of the electrons were depicted as if the electrons were moving in different directions. The distances between the orbits follow a sort of regularity, more noticeable in the last three orbits, which are equidistant.

In the final drawing (Figure 6b), students attempted to represent concentric spheres, with electrons no longer prominently visible, as they aimed to convey the idea that the precise position could not be determined. The distances between the spheres reflect the correct distances according to n^2.



(**a**) (**b**)

Figure 6. The drawings of the atomic model created by Group 2 students with their iPads: (**a**) in the "Prologue" phase; (**b**) in the "Epilogue" Phase.

4.2. Students' Engagement

From the final questionnaire, it emerged that students enjoyed working in this mode: 84.5% reported that the proposed approach allowed them to be engaged and active. These responses were corroborated by observations recorded by the instructor who conducted the activity in her diary.

During the initial activity, she reported: "*Everyone worked very well*" or "*The students were happy to do this activity; they took it seriously and even asked if they could stay for another two hours*".

However, students did encounter some difficulties. The "Chapter 1" and "Chapter 2" tasks posed challenges for second-year high school students who had not yet covered electric charge and electric energy in physics. Of the students, 36.9% stated that the activity was too difficult. This observation was also noted by the instructor: "*By the end of the hour, their brains were fried, and they couldn't work anymore*", "*They didn't want to do calculations anymore, maybe giving them so many calculations was too much, and we needed to simplify them a bit*", or "*Only a few of them worked because the task was perhaps too high*".

Nevertheless, at the end of the activity, all groups managed to calculate the energies and distances between energy levels, which gave them some encouragement. In the final discussion, this difficulty no longer emerged. Of the students, 84.2% reported that the proposed approach allowed them to be engaged and active, which was evident in the attentive and participatory final discussion. This sentiment was also echoed by the two chemistry teachers who observed:

Chemistry Teacher 1: "*It was great; I must admit I was skeptical at first. I had read the story and thought «I wonder where this is going». But there was indeed a lively debate*".

Chemistry Teacher 2: "*I didn't think the students would be so interested; everyone was following along. Unlike when they're not interested, and you see them fidgeting*".

In response to the open-ended question in the satisfaction survey, "One thing I liked was:", several students provided the following responses:

- "*The story*";
- A student with a specific learning disorder (SLD), mentioned, "*The connection of a fairly complicated topic with such a simple story*";
- "*Doing a lesson different from the usual, not sitting at desks in the classroom but collaborating in small groups*";
- "*The organization of the work and the narrative we worked on*";
- "*The originality of the story and the opportunity to engage in a suitable environment*";
- "*The story and the location; the classroom we were in is very advanced and allowed us to interact effectively*".

In response to the question, “One thing I didn’t like was:”, many students mentioned:

- “*The groups*”;
- “*The difficulty and challenges of working in groups*”;
- “*The inability to choose the groups*”.

These criticisms were directed not at the activity format itself but rather at the challenges of working together with classmates. Despite being classes that occasionally worked in groups, the complexity of the task may have brought about some tensions among students. However, others mentioned, “*Nothing, I liked everything overall*”.

Finally, students had the opportunity to freely add comments. One student’s comment was, “*I was disappointed to learn that you cannot determine the precise position of the electron and, consequently, the path it follows*”, which highlights the growth in understanding the topic and the internalization of the concept.

Another student wrote, “*It would be great to have more lessons like this*”, confirming that the activity was well-received and found a favorable consensus.

5. Discussion and Conclusions

The analysis of the data collected during the experimentation of our TLS showed that, at the beginning of the TLS (”Prologue” phase), the students had the planetary Bohr’s model in mind as an atomic model. Moreover, most students had a misconception concerning the direct proportionality between the principal quantum number (n) and the atomic radius. We found no references to this misconception in the literature. At the end of the TLS (“Epilogue” phase), some students still maintained the idea of equidistant orbitals, but most of them achieved a correct understanding of the concept of orbital. Specifically, the students used three different types of atom representation in the final drawings: orbitals represented at varying distances to more effectively convey the concept of energy levels (see Figures 3 and 4); orbitals represented at radius-corrected distances (see Figure 5); and attempts to represent the concept of orbitals and the correct distances between them (see Figure 6). Several groups continued to represent the orbitals as lines, but with ray distances no longer appearing at regular intervals as in the initial drawings. In the final drawing, some groups tried to convey the idea that the position of electrons cannot be precisely determined. To do so, they represented the orbitals as concentric spheres, with the electrons not clearly visible (see Figure 6). From the exploratory analysis, it seems that all the groups underwent an evolution in their representation of atomic models. Not all of them were able to solve all the misconceptions addressed, but there was a positive change in each group. This suggests that the TLS helped students in reconstructing their mental model of the atom, guiding them closer to the correct one. These results were further corroborated by the students themselves through the final satisfaction survey, in which 89.4% of them expressed that the proposed approach allowed them to develop a clearer mental image of the atomic model. With reference to the research question (RQ1), therefore, it seems that the TLS helped students to reconstruct the atomic model. The story helped students better understand specific concepts related to the atomic model. However, it can be observed that there was no evolution in the initial and final drawings regarding the size of the nucleus or how they represented spin, since the TLS did not focus on these two aspects.

To address the research question (RQ2), we can draw upon two primary sources: observations reported in their diary by the teachers involved (the instructor who conducted the TLS and two chemistry teachers who observed the final discussion) and a satisfaction questionnaire administered to the students at the end of the TLS. Most students (84.5%) reported that they engaged during the TLS. For some students (36.9%) the tasks proposed in the TLS were too difficult, and this was also noted by the instructor. The biggest challenge for students was working together with classmates. However, in the final discussion, this difficulty no longer emerged for most students (84.2%), and this was also observed by chemistry teachers. Moreover, the story seems to have engaged all students, even a student with SLD who pointed out “*the connection of a rather complicated topic with such a simple story*” (see Section 4.2). This is an interesting aspect, highlighting how storytelling can be an effective tool for inclusive learning of science in general and, in particular, physics and mathematics [31]. Therefore, it seems that the TLS and, in particular, the story we created were also able to engage students from the perspective of inclusion.

On the basis of the observation that some students found the proposed tasks complex and that some of them struggled to internalize complex concepts such as orbitals, it should be useful to think about how to deal with these difficulties. It could be interesting to increase the opportunities for discussion between students from different groups. Peer interaction could help students clarify the most debated issues and could lead them to the negotiation of a shared solution.

Furthermore, it should be useful to reduce the length of the tasks to enable students to have more time to reflect on them deeply. Another way to help students solve the tasks they perceive as complex consists of asking them to present the same situation from multiple points of view. For example, students could be asked to provide three different types of solutions for the same task: graphic, written, and through formulas/diagrams, to allow them to answer using the tools with which they are most familiar.

In our work, we used storytelling differently from known models in the literature. Our narrative certainly is useful for clarifying physical concepts. However, our story is embedded within a TLS and aims at engaging students actively. In this respect, the way we use storytelling is very different from that of Fuchs et al. [6], who use stories to introduce, for example, forces of nature. Our TLS is closer to the DIST-M model [4], despite the fact that we introduced some important differences. Some similarities with the DIST-M model are that students are involved in tasks that follow the evolution of the story and that our TLS aims to develop skills in students. However, the DIST-M requires students to assume roles within the collaborative group and to impersonate characters in the story, and it is an online learning model, which requires the use of digital technologies to be fully implemented [32]. In our TLS, students do not take on specific roles and impersonate any characters in the story, and the use of digital technologies is not necessarily expected but is a choice left to the teacher depending on the educational context. We believe that this different use of storytelling is an added value of our research. However, a possible future development could be to extend TLS in the context of gamification and to investigate its possible benefits mainly in terms of student engagement.

We conclude this paper by highlighting a limitation of the story we have designed from a physical point of view. Our story does not fit the real atomic structure. Indeed, we chose to consider a hydrogen atom with multiple electrons because we thought it was easier for the students to manipulate. We are aware of the fact that this may represent a critical point of our story, but in the classes where we experienced the activity it has not created problems, since the students noticed this inconsistency and talked about it with the teacher. As mentioned in Section 2.3, a model necessarily simplifies the reality it represents, and we think it is important for students to experience this fact to become aware of it. Even if no relevant issues related to the representation of the atom emerged from the experimentation, we believe it could be interesting to investigate this question further.

Author Contributions: Writing—original draft preparation, G.A., U.D.I., L.G., L.P., G.S., G.T.; writing—review and editing, G.A., U.D.I., L.G., L.P., G.S., G.T. All authors have read and agreed to the published version of the manuscript.

Funding: The APC was funded by EduSpaces-MultiLab of the Faculty of Education of the Free University of Bozen-Bolzano, Project leader, Prof, Dr. Federico Corni.

Institutional Review Board Statement: The data used in this study were collected and anonymized by the school teachers according to the school policy, and do not allow any conclusions to be drawn about the participating individuals. The procedure complies with the data protection law (GDPR).

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: The data presented in this study are valuable on request for the corresponding author.

Acknowledgments: The paper resulted from a collaboration starting during the BrEW Math 01 (Brixen Education Workshop on Storytelling in STEM disciplines at the crossroads of science and humanities) held at the MultiLab of the Faculty of Education, Free University of Bozen-Bolzano, 8-10/11/2022.

Conflicts of Interest: The authors declare no conflict of interest.

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