Multi-physical analysis of a rainfall energy harvester

I. Palomba, M. Bottin, G. Rosati and A. Doria

Abstract The kinetic energy of raindrops is a renewable source of energy that can be scavenged by piezoelectric harvesters. Experimental tests have shown that a water layer covering the surface hit by the raindrops strongly enhances the energy transfer from the rain to the harvester. A mathematical model able to explain this phenomenon has been recently developed. This paper focuses on the ability of the model to cope with variations in the characteristics of the raindrop impact. Comparisons with experimental results are made.

1 Introduction

The recent developments in the field of vibration energy harvesting make it possible to exploit the kinetic energy of raindrops when they impact on a piezoelectric membrane [1] or on a cantilever beam [6] and excite their free vibrations.

Sometimes a plate is mounted at the free end of a cantilever beam to increase the area hit by raindrops and the generated power. Some studies showed that when a water layer accumulates on the surface of the piezoelectric element or on the surface of the tip plate there are significant effects in impact mechanics and in the performance of the harvester [6].

To completely exploit these phenomena a novel cantilever beam equipped with a spoon was developed and experimentally tested in [2, 5]. Experimental tests showed a large increase in the generated voltage caused by the presence of the water layer. Recently, a mathematical model able to explain the measured phenomena was proposed in [5]. Numerical results obtained in a specific test condition showed a good agreement with experimental results. The mathematical model requires some parameters of the impact between the raindrop and the water layer that can be derived from experimental tests or from empirical correlations based on experiments. Mea-

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surements are affected by errors and the operating conditions of the harvester change over time due to water accumulation, wind and rain rate variations. The aim of this paper is to evaluate the robustness of the mathematical model considering variations in impact parameters. The paper is organized as follows. Section 2 describes the multi-physical model for the rainfall harvester that takes into account the coupling between the mechanical, electrical and fluid-dynamic domain. In Section 3, experimental tests on two different harvesters are used to validate the model, then an extensive sensitivity analysis is presented. Conclusions are drawn in Section 4.

2 Rainfall harvester multi-physical model

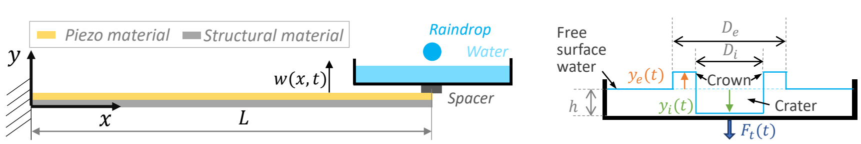
The studied rainfall harvester has a cantilever structure consisting of a thin piezoelectric layer and of a metallic layer fixed at one end to achieve a structure operating in a flexural mode. At the free end of the cantilever, a container (or spoon) is rigidly attached allowing for water accumulation and hence for the impact of the raindrop on a wet surface. The mechanical schematic of the harvester is shown in Fig. 1a.

The fundamental equation that describes the vibrations of the cantilever harvester in the mechanical domain is:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + c_s I\frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + m\frac{\partial^2 w(x,t)}{\partial t^2} + c_a\frac{\partial w(x,t)}{\partial t} + \theta V(t) = F_t(t) \tag{1}$$

where $w(x,t)$ is the transverse displacement of any point along the cantilever; I is the equivalent area moment of inertia of the composite cross section; EI is the bending stiffness of the composite cross section as shown in [3]; m the mass per unit length of the cantilever; c_s and c_a are the strain rate and air damping coefficients, respectively, which are assumed to satisfy the proportional damping criterion; θ is the backward piezoelectric coefficient that couples the mechanical problem with the electrical problem; $V(t)$ is the voltage.

It is worth noting that the equation of vibrations is coupled with the electrical domain by the term $\theta V(t)$. The coupling with the fluid-dynamic domain takes place through the force $F_t(t)$ that represents the force deriving from the impact of the raindrop on the water layer that covers the spoon.



(a) (b)

Fig. 1: Equivalent mechanical schematic of the harvester (a). Schematic of the simplified splashing mechanism (b).

The electrical behaviour of the harvester is described by the equation:

$$C_{pu}\dot{V}(t) + \frac{1}{R}V(t) = -e_{31}h_{pc}b\int_0^L \frac{\partial^3 w(x,t)}{\partial x^2 dt}dx \tag{2}$$

where C_{pu} is the capacitance, R the resistance, e_{31} the piezoelectric constant, b the width of the beam, and h_{pc} the distance between the neutral axis of the composite cross section and the center of the piezoelectric layer.

The coupling between the mechanical domain and the fluid-dynamic domain is only one-way, since water dynamics has a relevant effect on the impact force on the harvester, but vibrations have a very small effect on water dynamics inside the spoon. This phenomenon is due to the different scales of vibration amplitude and water motion that differ each other of about two orders of magnitude.

The generation of the impact force on the harvester due to the impact of the raindrop on the water layer covering the spoon is a very complex fluid-dynamic problem, because several phenomena take place: formation of the crater, generation of a crown around the crater, collapse of the crown and possibly generation of water ripples [4, 7]. A simplified model has been proposed in [5] to represent these phenomena. The dynamics of the impact of a drop on a liquid layer is influenced by the ratio between the layer depth h and the drop diameter D_d. In the range $0.1 < h/D_d < 2$ the drop impact generates in the liquid layer a cylindrical crater with a flat bottom [7]. A one-dimensional model of the water motion after the impact was developed assuming that the quantity of water that moves from the impact area, generating the cylindrical crater, progressively produces the crown surrounding the crater, which is assumed to have an annular shape (see Fig. 1b). The water leaving the crater has velocity $\dot{y}_i$ directed downwards, whereas the water inside the growing crown has velocity $\dot{y}_e$. The following continuity equation holds, similar equations hold for water displacements and accelerations, since the areas are constant:

$$\rho_w A_e \dot{y}_e(t) = \rho_w A_i \dot{y}_i(t), \tag{3}$$

where ρ_w is water density and A_e, and A_i are the areas of the crown and of the crater, respectively. The fluid velocity in the radial direction is not considered, since only the variation in the linear momentum in the vertical direction (perpendicular to the cantilever harvester) is able to excite the harvester. With the above-mentioned assumptions the linear momentum $p(t)$ of the water after the impact of the drop is:

$$p(t) = (m_i - \rho_w A_i y_i(t))\,(-\dot{y}_i(t)) + (m_e + \rho_w A_e y_e(t))\,(\dot{y}_e(t)), \tag{4}$$

where $m_e = \rho_w A_e h$ and $m_i = \rho_w A_i h$ are the masses contained in the external and the internal cylinders, respectively, before the impact. The force $F_t(t)$ exerted by the moving water on the spoon is obtained from the Newton's equation:

$$\begin{aligned} F_t(t) = -\frac{dp(t)}{dt} = &-(m_i - \rho_w A_i y_i(t))\,(-\ddot{y}_i(t)) - \rho_w A_i y_i(t)\dot{y}_i^2(t) + \\ &-(m_e + \rho_w A_e y_e(t))\,\ddot{y}_e(t) - \rho_w A_e y_e(t)\dot{y}_e(t)^2 \end{aligned} \tag{5}$$

If equation (3) is introduced into equation (5), the following result is obtained:

$$F_t(t) = -\rho_w A_e \left(1 + A_e/A_i\right) \left(y_e(t)\ddot{y}_e(t) + \dot{y}_e^2(t) \right) \tag{6}$$

It is worth noticing that the initial masses disappear in this equation. The calculation of the force $F_t(t)$ in (6) requires the knowledge of the areas of the crown and crater (A_e and A_i) and of the motion of the crown $y_e(t)$. In the previous research carried out by the authors [5] the areas were calculated with an energetic approach [7] assuming that a fraction α of the energy of the drop (kinetic and surface energy) is transformed into gravity potential energy and surface energy of the cavity and crown. The motion of the crown was described as a damped harmonic oscillation that fits the data measured during the experimental tests by means of a high frame-rate camera.

3 Experimental validation and sensitivity analysis

The voltage produced by the vibration of cantilever harvesters excited by the raindrops was experimentally investigated. In particular, the two prototypes of rainfall harvesters shown in Fig. 2 have been built and adopted. They are both based on a commercial piezoelctric harvester PPA 1001 built by MIDE in cantilever configuration and are both provided with a spoon containing a thin layer of water at the free end. The main differences between the two prototypes rely on the spoon geometry, mass and position of the center of mass with respect to the harvester. The prototype in Fig. 2a is equipped with a cylindrical container having an inner radius of 21 mm, and a mass of 3.65 g. The center of the container is fixed on the tip of the harvester and a small spacer is used to avoid the contact between the bottom surface of the container and the top surface of harvester, which could limit harvester bending. The spoon is filled with 1 mL of water, corresponding to a water depth, h, of 2.88 mm. A hole is made on the container to allow water spill when the water layer increases above the selected value.The prototype in Fig. 2b is equipped with a spoon having a square section of 27.2 mm inner side, sidewall height of 3 mm, and a mass of 1.28 g. An edge of the container is directly glued at the free end of the cantilever, and

(a) (b)

Fig. 2: Prototypes of the harvester with: cylindrical spoon (a) and square spoon (b).

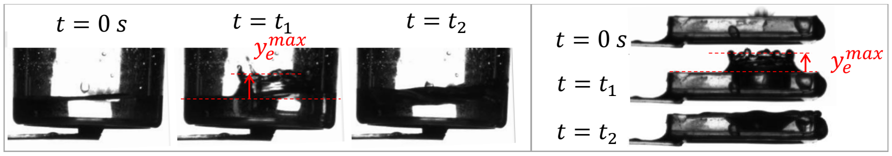


Fig. 3: Footage of the cylindrical (left) and square (right) spoon prototypes.

hence its center of mass is 14.5 mm from the harvester tip. The full volume of the container is filled with water so a hole for water spill is not necessary.

The experimental tests were carried out indoor using simulated rain. Regular drops with a diameter of about 2 mm were generated by means of an intravenous drip set. The drop falling height was set to 1 m, corresponding to an impact velocity of about 4 m/s. The generated voltage was measured by means of an acquisition module for vibrations analysis (NI9234). The motion of the water inside the container was recorded by means of Teledyne Dalsa Genie Nano G3-GM10-M0640 camera, having an acquisition rate of 600 fps and an exposure time of 1000 ms. A telecentric lens Computar TEC-55 was mounted on the camera. A light source of 625 nm and the corresponding red filter installed on the camera lens made it possible to minimize external disturbances. The camera footage were processed by a computer to infer some of the parameters needed for the calculation of the force F_t in equation (6) [5]; they are: the maximum crown height y_e^{max}, the instant t_1 at which the crown reaches its maximum height, and the instant t_2 at which the crown collapses (see Fig. 3).

Fig. 4 shows the estimated forces acting on the two harvesters due to the impact of a raindrop on the liquid layer. The same figure also shows the two nonlinear terms of Equation 6: the first part of the oscillation is mainly driven by the inertial term $\propto y_e\ddot{y}_e$, which rapidly decreases in favor of the velocity contribution $\propto \dot{y}_e^2$. It can be noted that the highest value of $F_t(t)$ neither coincide with the maximum of the velocity term, nor with the inertial term. The harvester voltage estimated using the

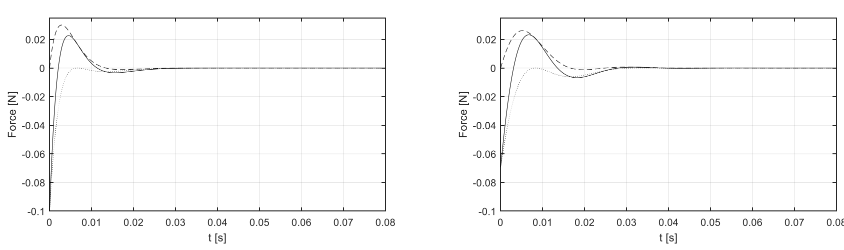


Fig. 4: Force acting on the harvester with cylindrical (a) and square (b) spoon. The continuous line denotes the total force F_t, the dashed line the term depending on $y_e\ddot{y}_e$, and the dotted line the term depending on $\dot{y}_e^2$.

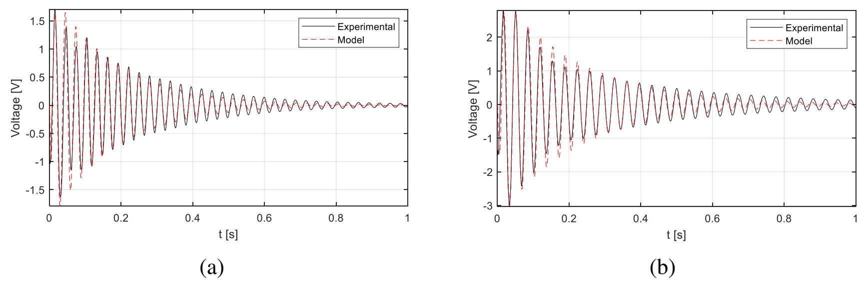


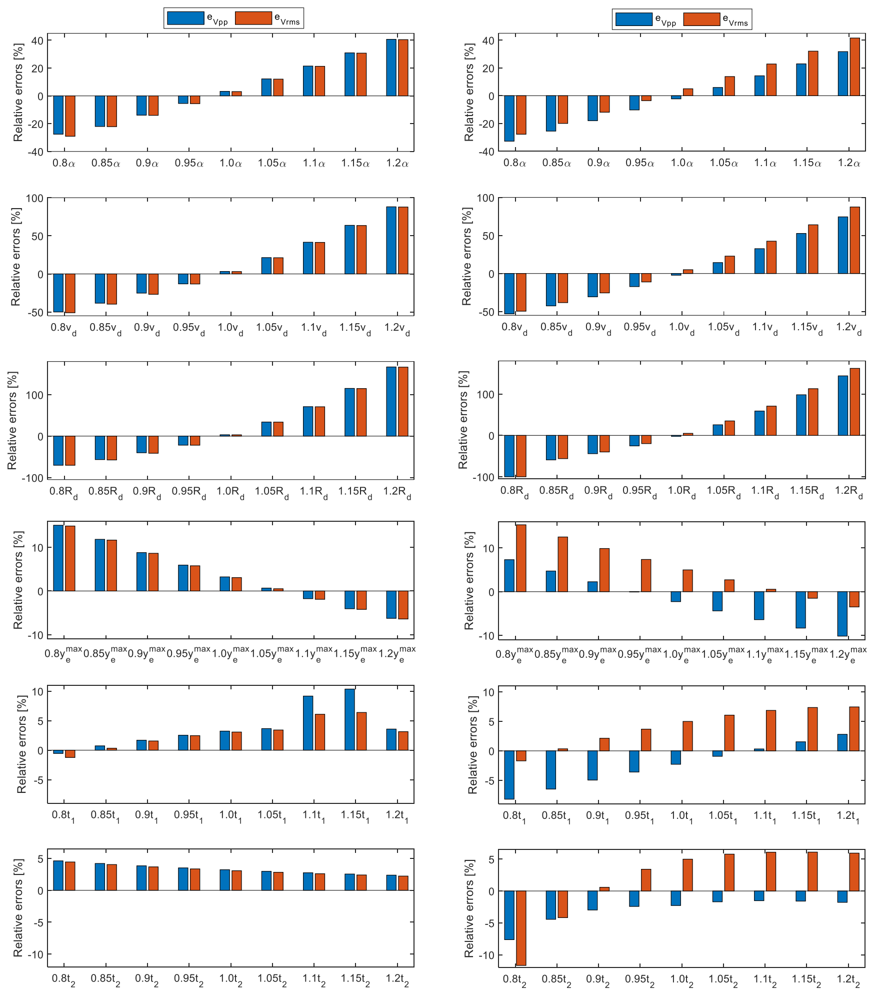
Fig. 5: Comparison between the experimental (black) and the estimated (red) harvester voltage: (a) cylindrical spoon, (b) square spoon.

model and measured data is compared with the measured voltage in Fig. 5, which highlights a good agreement between the signals.

To deepen the accuracy of the model and its robustness to the uncertainty of the parameters inferred from the footage, the voltage estimated considering variations in the impact parameters is compared with the measured one. The considered parameters are: the energy fraction α, the drop impact velocity v_d, the drop radius R_d, the maximum crown height y_e^{max}, the instants of the maximum crown height t_1 and collapse t_2. The effect of these parameters on the energy content of the estimated signals is evaluated in terms of maximum peak-to-peak voltage and root mean square (rms) voltage; while the effect on the voltage waveform is evaluated through the amplitude of the first three peaks. The comparison between all these quantities is made in terms of relative percentage error $e_X = 100 \cdot (\tilde{X} - X)/X$, where X and $\tilde{X}$ represent the measured and estimated quantities, respectively. Fig. 6 shows that only the parameters related to the impact energy (α, v_d and R_d) are able to generate variations in the peak-to-peak and rms voltage larger than the variations in the parameters itself. The effect increases moving form α to v_d and to R_d, because these quantities are related in linear, quadratic and cubic way to impact energy, respectively. Fig. 7 deals with the amplitudes of the voltage peaks and shows that the parameters α and y_e^{max} affect all peaks, whereas the other parameters chiefly affect specific peaks.

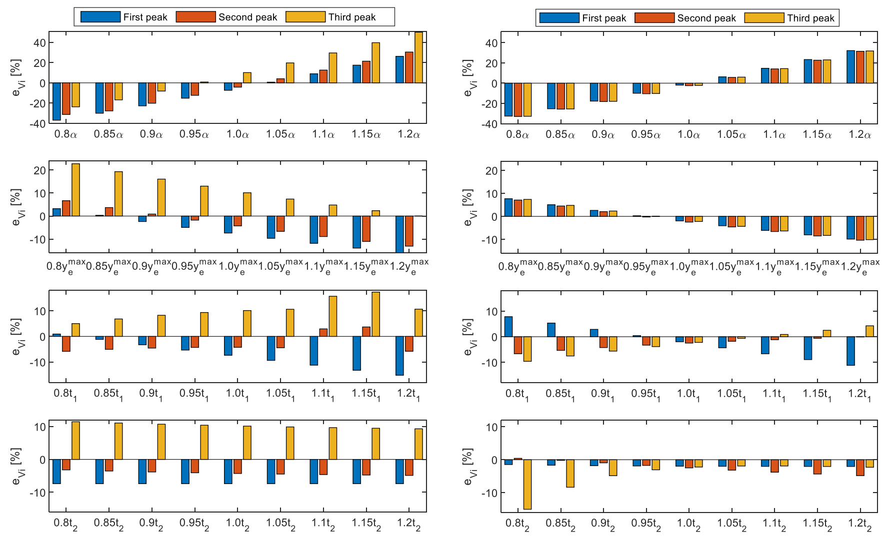
4 Conclusions

The multi-physical model has been validated by means of experimental tests carried out on two different harvesters. The output of the model depends on some impact parameters (e.g. raindrop radius and velocity, maximum crown height) that depend on operating conditions and are affected by measurement errors. A parametric analysis has been carried out to analyze the effect of variations in these parameters both on the quantities that are directly related to the energy transfer from the impacting raindrop to the harvester (maximum peak-to-peak and rms voltage) and on the quantities that determine the waveform of the generated voltage (heights of the successive peaks). Results show that there are large errors in the estimated voltage only when the impact parameters related to the kinetic energy of the drop significantly change.



(a) (b)

Fig. 6: Peak-to-peak and rms voltage errors for a variation of ±20% of the impact parameters: (a) cylindrical spoon ($\alpha = 0.55$; $v_d = -4.03\,m/s$; $R_d = 1.00\,mm$; $y_e^{max} = 3\,mm$; $t_1 = 6.7\,ms$; $t_2 = 41\,ms$); (b) square spoon ($\alpha = 0.58$; $v_d = -4.05\,m/s$; $R_d = 1.05\,mm$; $y_e^{max} = 2.5\,mm$; $t_1 = 8.3\,ms$; $t_2 = 25\,ms$).



(a) (b)

Fig. 7: Amplitude errors on the first three voltage peaks for a variation of ±20% of the impact parameters: (a) cylindrical spoon, (b) square spoon.

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