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Bit Error Rate Evaluation of a Dual-Filter Heterodyne FSK Optical System

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Summary

The frequency separation between the two channels of a dual-filter heterodyne FSK optical system is a very important parameter to evaluate in the presence of laser phase noise. In this work the impairment due to the crosstalk interference between the two transmitted tones of a weakly coherent FSK optical system is investigated. In order to make accurate computations, the method of moments seems to be the most suitable: to this aim, in this work an algorithm for the moments evaluation is proposed. The numerical results are then compared with a Monte Carlo simulation approach.

1 Introduction

The interest in coherent optical communication systems is mainly addressed to their capability of very narrow channel spacing in multichannel systems with frequency division multiplexing. Among the different modulation/demodulation techniques, the heterodyne frequency shift keying (FSK) is one of the most promising [1–3] for its different advantages (possibility of direct modulation, simplicity of the receiver structure, etc.). In a dual-filter FSK system, one of the most important parameters to evaluate is represented by the frequency distance between the two channels, in order to have an acceptable crosstalk between them. The characterization of the contribution of the phase noise at the intermediate frequency is therefore very important.

In this work we present an algorithm for the evaluation of the joint moments of the vector process representing the useful signal and the interfering signal.

The moments are then used in the performance evaluation of a dual-filter heterodyne FSK optical system in order to take into account the effects of both laser phase noise and crosstalk interference.

2 Receiver scheme

The receiver scheme is depicted in Fig. 1. The signal coming from the fiber is combined with the local oscillator power and then sent to a photodetector. The output current at the intermediate frequency (IF) is then filtered with two bandpass filters centered at the frequencies f_0 and f_1. The filters are assumed to be integrate and dump filters since they are the only ones to allow an accurate analysis and they can be taken as a benchmark for different type of filters [4]. The resulting signals are sent to square-law envelope detectors; then, the difference between the two signals is filtered again with a low pass post-detection filter. We follow the model generally accepted in the literature [1, 2] in which the intermediate frequency filter has integration time T′ which is an integer fraction of the bit period T,

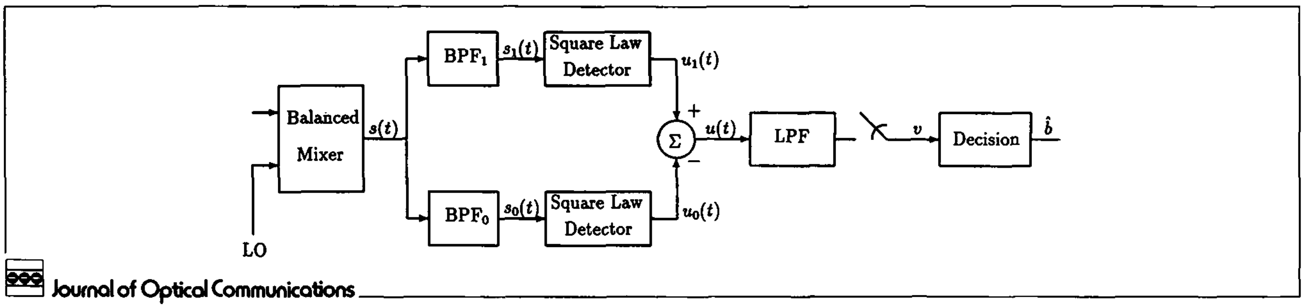

Fig. 1: Reference scheme for the receiver

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i.e. $T' = T/K$, and the post-detection filter is modeled as a discrete time filter taking the average of the K samples. Finally the decision on the transmitted bit is taken by comparison with a zero threshold.

3 Phase noise characterization

The characterization of phase noise at the output of the intermediate filters play a fundamental role in the analysis of the crosstalk interference. From a mathematical point of view, the problem reduces ot the statistical characterization of the following random processes [4]:

$$z_1(t) = x_1(t) + i y_1(t) = \int_0^t e^{i\theta(\tau)} d\tau, \tag{1}$$

$$z_2(t) = x_2(t) + i y_2(t) = \int_0^t e^{i[\theta(\tau) + 2\pi\Delta f \tau]} d\tau, \tag{2}$$

where the process $\theta(\cdot)$ represents the phase noise, modelled as the integral of a Gaussian white noise process with spectral density D, related to the 3 dB bandwidth B of the laser ($D = 2\pi B$), and $\Delta f = |f_1 - f_0|$ is the frequency deviation between the transmitted tones of the FSK system. The complete statistical characterization of this processes is a very hard task even in the simplified case in which the crosstalk is neglected [4].

In this section we give a method for the moments evaluation of the vector process **z** whose components are given by (1) and (2). It stems from an approximation of the random process $z_2(t)$ which represents the interfering signal: this approach is very accurate only for small phase noise, i.e. when the variance of the process $\theta(t)$ is small. The reason to present an approximation method is due to the impractical numerical complexity in the evaluation of the exact joint moments of the vector process **z**.

In fact, following the same approach carried out in [5], it is possible to find a recursive relation of the Laplace transform of the exact joint moments. The numerical calculation of the exact moments is an extremely heavy task. From a computational point of view, the only possibility is represented by the evaluation of the marginal moments of $z_1(t)$ and $z_2(t)$.

3.1 Approximate moments derivation

An approximate solution to the problem can be obtained applying the approximations made by Foschini and Vannucci in [4] to the process (2) only.

The random process $z_2(t)$ in equation (2) has the following Taylor series expansion:

$$z_2(t) = i\left[\frac{1 - e^{i\omega t}}{\omega}\right] + i\int_0^t e^{i\omega\tau}\theta(\tau)\, d\tau - \frac{1}{2}\int_0^t e^{i\omega\tau}\theta^2(\tau)\, d\tau + \ldots \tag{3}$$

Note that the first term (3) is due to the deterministic crosstalk from channel #2 into channel #1. Now, let us consider the integral:

$$\int_0^t e^{i\omega\tau}\theta(\tau)\, d\tau. \tag{4}$$

Following the same approximation method used in [6], the leading asymptotic behaviour of (4) is obtained integrating by parts; on this way we get:

$$\int_0^t e^{i\omega\tau}\theta(\tau)\, d\tau \approx \frac{1}{i\omega}\left[e^{i\omega t}\theta(t) - \theta(0)\right] - \frac{1}{i\omega}\int_0^t e^{i\omega\tau}\frac{d\theta(\tau)}{d\tau}\, d\tau. \tag{5}$$

The last integral vanishes more rapidly than $1/\omega$, thus the second side of (5) can be approximated as:

$$\frac{1}{i\omega} e^{i\omega t}\theta(t), \tag{6}$$

since it can be assumed $\theta(0) = 0$. On this way we get:

$$z_2(t) \approx \frac{1}{\omega}\left[i(1 - e^{i\omega t}) + e^{i\omega t}\theta(t)\right]. \tag{7}$$

In the following, the assumption $\Delta f t = n$, with n integer, is assumed. This corresponds to the condition of orthogonal signals in the absence of phase noise. Then, we obtain:

$$z_2(t) \approx \frac{1}{\omega} e^{i\omega t}\theta(t), \quad \Delta f \gg B, \tag{8}$$

where $B = D/2\pi$.

In conclusion:

$$|z_2(t)|^2 \approx \frac{1}{\omega^2}\theta^2(t). \tag{9}$$

In [5] it has been shown that:

$$\mathfrak{L}\{E[e^{iu\theta(t)} z_1^h(t)\, z_1^{*k}(t)]|s\} = M[h, k, u; s] \tag{10}$$

where the functions $M[h, k, u; s]$ satisfy the following recursive equation

$$M[h, k, u; s] = \frac{1}{s + u^2}(hM[h-1, k, u+1; s] + kM[h, k-1, u-1; s]) \tag{11}$$

with initial condition

$$M[0, 0, u; s] = \frac{1}{s + u^2}, \tag{12}$$

and $D = 2$. From the series expansion of $\exp(iu\theta)$ one can then obtain:

$$\begin{aligned} M[h, h, u; s] &= \mathfrak{L}\{E[e^{iu\theta}|z_1|^{2h}]|s\} \\ &= \mathfrak{L}\left\{E\left[\sum_{r=0}^{\infty}\frac{(iu\theta)^r}{r!}|z_1|^{2h}\right]\middle| s\right\} \\ &= \sum_{r=0}^{\infty}\frac{(iu)^r}{r!}\mathfrak{L}\{E[\theta^r|z_1|2^h]|s\} \\ &= \sum_{r=0}^{\infty}\frac{\partial^r M[h, h, u; s]}{\partial u^r}\bigg|_{u=0}\frac{u^r}{r!}. \end{aligned} \tag{13}$$

Thus, since the last equality holds for every value of the real variable u, we get:

$$i^r \mathfrak{L}\{E[\theta^r|z_1|^{2h}]|s\} = \left.\frac{\partial^r M[h,h,u;s]}{\partial u^r}\right|_{u=0}, \tag{14}$$

and

$$m_{hr}(s) \triangleq \mathfrak{L}\left\{E\left[|z_1|^{2h}\left(\frac{\theta}{\omega}\right)^{2r}\right]\Bigg| s\right\} = \left.\frac{(-1)^r}{\omega^{2r}}\frac{\partial^{2r} M[h,h,u;s]}{\partial u^{2r}}\right|_{u=0}. \tag{15}$$

On the basis of the recursive equation (11) and the results obtained in [5] one can get the following formula for the joint moments:

$$\boxed{m_{hr}(t) = (h!)^2 \frac{(-1)^r}{\omega^{2r}} \mathfrak{L}^{-1}\{W[2h,h,s;2r]|t\}} \tag{16}$$

where the functions $W[n,q,s;j]$ satisfy the equation

$$W[n,q,s;j] = \sum_{l=0}^{j}\binom{j}{l}\left.\frac{\partial^l}{\partial u^l}\frac{1}{s+(2q-n+u)^2}\right|_{u=0} (W[n-1,q,s,j-l] + W[n-1,q-1,j-l]) \tag{17}$$

with initial condition:

$$W[0,0,s;l] = \left.\frac{\partial^l}{\partial u^l}\frac{1}{s+u^2}\right|_{u=0} = \begin{cases} 0, & l \text{ odd} \\ (-1)^{l/2} l! \left(\dfrac{1}{s}\right)^{l/2+1}, & l \text{ even}. \end{cases} \tag{18}$$

The proof is deferred to Appendix A.

4 Systems analysis

The complex envelope of the signal after the photodetector is given by

$$s(t) = A\exp\{i\theta(t) + 2\pi b\Delta f t\} + n(t), \tag{19}$$

where $b \in \{0,1\}$ is the transmitted symbol, $\theta(t)$ is the combined phase noise of the transmitting and local oscillator lasers, Δf is the frequency deviation between the two tones and $n(t)$ is a white Gaussian noise taking into account the effect of both shot and thermal noise.

Owing to the system simmetry we may consider the case in which the transmitted symbol is 1. Then, the equivalent complex envelopes at the output of the IF filters are given by

$$s_1(t) = \frac{A}{T'}\int_{t-T'}^{t} e^{i\theta(\tau)}d\tau + n_1(t), \tag{20}$$

$$s_0(t) = \frac{A}{T'}\int_{t-T'}^{t} e^{i[\theta(\tau)+2\pi\Delta f\tau]}d\tau + n_0(t), \tag{21}$$

where $n_0(t)$ and $n_1(t)$ are complex Gaussian noises. The first term of $s_0(t)$ represents the crosstalk interference. In the absence of the phase noise this term vanishes if $1/T'$ is submultiple of Δf. The samples at the output of the envelope detectors are given by

$$u_{1k} \triangleq u_1(kT') = |Az_k + n_{1k}|^2 \tag{22}$$

$$u_{0k} \triangleq u_0(kT') = |Aw_k + n_{0k}|^2 \tag{23}$$

where

$$z_k = \frac{1}{T'}\int_{(k-1)T'}^{kT'} e^{i\theta(\tau)}d\tau \tag{24}$$

$$w_k = \frac{1}{T'}\int_{(k-1)T'}^{kT'} e^{i[\theta(\tau)+2\pi\Delta f\tau]}d\tau \tag{25}$$

and n_{1k}, n_{0k}, $k=1,\ldots,K$, are independent complex Gaussian variables, whose real and imaginary parts are independent with zero mean and equal variance σ^2. Having defined the averages

$$v_i = \frac{1}{K}\sum_{k=1}^{K} u_{ik}, \quad i=0,1 \tag{26}$$

the final decision gives

$$\hat{b} = \begin{cases} 1, & v_1 > v_0 \\ 0, & v_1 \le v_0 \end{cases}. \tag{27}$$

Then the bit error probability is given by

$$P_e = P[v_1 \le v_0]. \tag{28}$$

The random variables u_{ik} in (26) are mutually independent as shown in [3]. Conditioning on the values of Z and W

$$Z = |z_1|^2 + \ldots + |z_K|^2 = \lambda \tag{29}$$

$$W = |w_1|^2 + \ldots + |w_K|^2 = \mu, \tag{30}$$

the variables v_0 and v_1 are non centrally χ-squared distributed random variables [7]. Then, the error probability can be written as

$$P_e = \int_0^K\int_0^K P[v_1 < v_0|Z=\lambda, W=\mu]\, f_{ZW}(\lambda,\mu)\, d\lambda\, d\mu, \tag{31}$$

where $f_{ZW}(\lambda,\mu)$ is the joint probability density function of Z and W. The inner probability in (31) can be expressed as [9]

$$P[v_1 < v_0|Z=\lambda, W=\mu] = \int_0^\infty\int_a^\infty f_{v_1}(a|\lambda)\, f_{v_0}(b|\mu)\, db\, da$$

$$= Q(\sqrt{\varrho\mu}, \sqrt{\varrho\lambda}) - I_0(\varrho\sqrt{\lambda\mu})\exp\left(-\varrho\frac{\lambda+\mu}{2}\right)\left[1 - \frac{1}{2^{2K-1}}\sum_{j=0}^{K-1}\binom{2K-1}{j}\right]$$

$$+ \frac{1}{2^{2K-1}}\exp\left(-\varrho\frac{\lambda+\mu}{2}\right)\sum_{n=1}^{K-1} I_n\left(\varrho\sqrt{\lambda\mu}\right)\left[\left(\frac{\lambda}{\mu}\right)^{\frac{n}{2}} - \left(\frac{\mu}{\lambda}\right)^{\frac{n}{2}}\right]\sum_{j=0}^{K-n-1}\binom{2K-1}{j} \tag{32}$$

where $I_n(x)$ denotes the n-th order modified Bessel function [8], ϱ is the signal to noise ratio (SNR) defined as

$$\varrho \triangleq \frac{A^2}{2\sigma^2}, \tag{33}$$

and $Q(\alpha, \beta)$ denotes the Marcum Q-function [7].

5 Numerical evaluation of the bit error rate

The error probability can be expressed as the integral

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y)\, f_{ZW}(x, y)\, dx\, dy, \tag{34}$$

where $g(x, y)$ is a known analytical function, while $f_{ZW}(x, y)$ is the joint probability density function of which only a finite set of moments is known. Actually, Z and W are the sum of K independent random variables related to the random process $z_1(t)$ and $z_2(t)$ described in Section 3.

A suitable approximation of the integral is given by a cubature formula

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\lambda, \mu)\, f(\lambda, k)\, dx\, dy \approx \sum_i \sum_j g(x_i, y_j)\, W_{ij} \tag{35}$$

where $\{x_i, y_j\}$ are the abscissas and W_{ij} the weights of the cubature formula. The abscissas and the weights of the cubature can be derived from the moments via a sub-optimum approach due to Dogliotti et al. [10], since no optimum method is known for a set of non-symmetric moments. Shortly, from the first 2M. marginal moments of Z and W one can find the sets of orthogonal polynomials $p_i(x)$ and $q_i(y)$ corresponding to the probability density functions $f_Z(\lambda)$ and $f_W(\mu)$. As known, the roots of the M-th polynomial density give the M abscissas of the corresponding quadrature rule and can be obtained by the algorithm of Golub and Welsch [11]. Then, the abscissas of the cubature are given by the cartesian product of the two sets of abscissas evaluated separately with the marginal moments. This leads to a sub-optimum sets of point for the cubature unless the two random variables are independent. The joint moments are used to obtain the weights expressing the integrand function in terms of the orthogonal polynomials $p_i(x)$ and $q_j(y)$. In conclusion, the abscissas are obtained on the basis of the marginal moments and the weights on the basis of the joint moments.

6 Numerical results

The error probability is mainly related to the following system parameters:

- BT: normalized laser linewidth (B denotes the sum of the 3-dB linewidths of the local oscillator and the transmitting laser).
- ΔfT: normalized frequency deviation between the two tones.
- SNR: the SNR is defined by (33).

The error probability has been evaluated via the cubature method once that the joint moments have been obtained with the method described in Section 3. The results obtained are compared with a Monte Carlo simulation of the joint probability density function of the random variables Z and W. The simulation is just taken as a reference because of the computational complexity needed to achieve accurate results. The case considered for the simulation is $K = 1$. An example of the joint probability density obtained by simulation is given in Fig. 2 for $BT = 0.4$ and $\Delta fT = 2$.

A comparison between the results obtained with the different methods is reported in Figs. 3 and 4: it is evident that the approximation gives accurate results (within 0.5 dB at $P_e = 10^{-9}$) for small values of the normalized linewidth BT, while for larger values the accuracy tends to decrease for small normalized frequency deviation ΔfT. This is in agreement with the analysis carried out in Section 3 (see eq. (8)).

Note that for such values of the normalized linewidth the performance is extremely poor, so that a greater value of K is necessary. For $K > 1$ the random variables Z and W are the sum of K contributions obtained for a

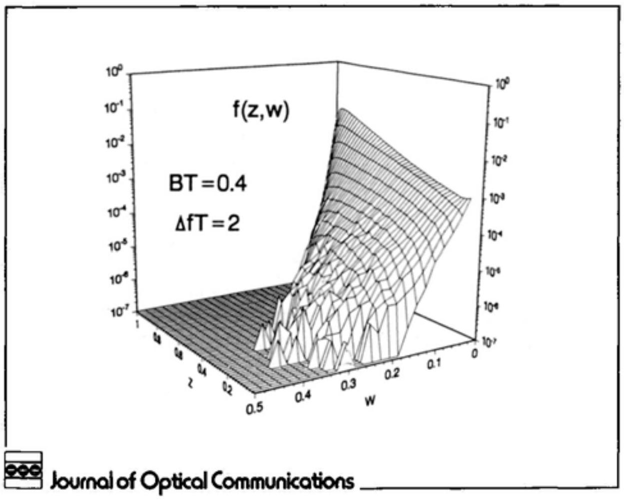

Fig. 2: Joint probability density of Z and W obtained by simulation for $K = 1$, $BT = 0.4$ and $\Delta fT = 2$

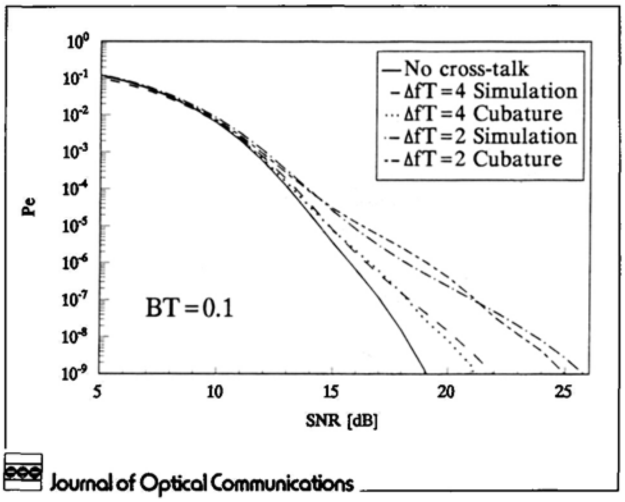

Fig. 3: Error probability versus SNR for $K = 1$, $BT = 0.1$ and different values of ΔfT: comparison between the different approaches

Journal of Optical Communications
15 (1994) 6

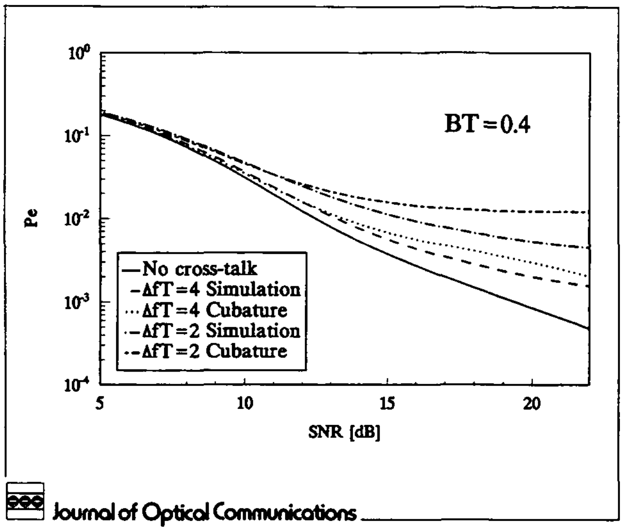

Fig. 4: Error probability versus SNR for K = 1, BT = 0.4 and different values of ΔfT: comparison between the different approaches

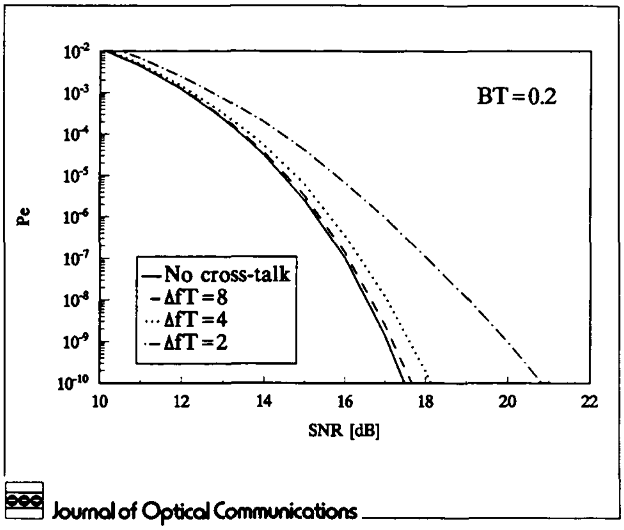

Fig. 5: Error probability versus SNR with optimum value of K, for BT = 0.2 and different values of ΔfT

normalized linewidth BT/K, with a subsequent increase in the accuracy of the approximation.

In Fig. 5 the error probability is presented for a fixed normalized linewidth BT = 0.2 and for different values of the frequency separation. All the computation refer to an optimal value of the parameter K which represents the bandwidth expansion factor of the IF filters.

It is evident the penalty due to the crosstalk interference arising when the two tones are not sufficiently separated.

In Fig. 6 the error probability is shown for different values of BT and for a fixed frequency separation $\Delta fT = 4$.

Finally, Fig. 7 shows the penalty, with respect to the ideal case of no phase noise and no crosstalk, as a function of BT and for different values of the normalized frequency deviation. A dramatic penalty is observed for increasing values of the normalized linewidth BT and for decreasing valus of the frequency deviation.

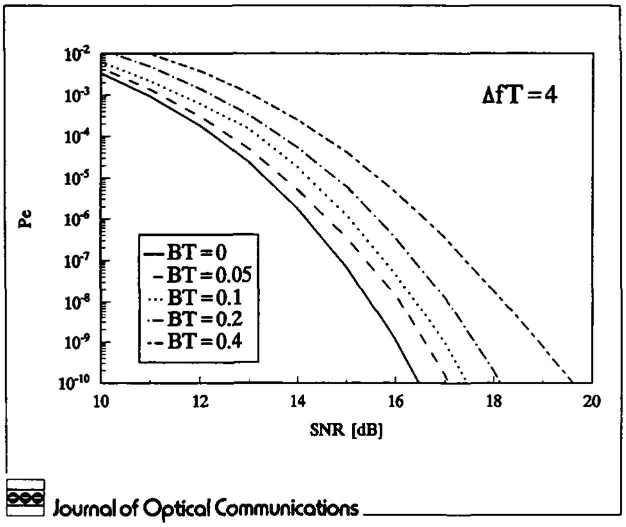

Fig. 6: Error probability versus SNR with optimum value of K, for $\Delta fT = 4$ and different values of BT

7 Conclusions

The analysis of the penalty due to the crosstalk interference between the two branches of an FSK receiver has been presented. The method of moments is very promising to get accurate results. Anyway, the approximation of the crosstalk components is necessary to reduce the numerical complexity.

The good agreement between the moments based technique and the simulation approach show that the method just proposed can be usefully used for the performance evaluation, without the computational complexity of a Monte Carlo simulation.

The results obtained show that the phase noise represents a strong impairment to reduce the spacing between the transmitted tones. Therefore the spectral properties of lasers play a fundamental role among the requirements for multichannel transmission systems to reduce the channel spacing.

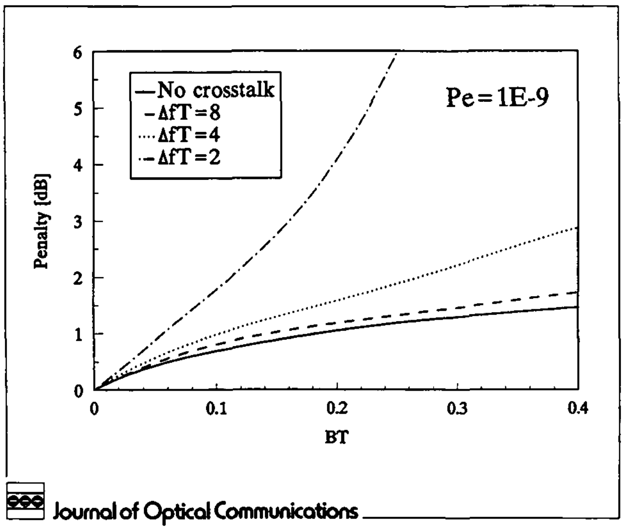

Fig. 7: Penalty at $P_e = 10^{-9}$ versus the normalized linewidth BT for different values of ΔfT

8 Appendix A: On the derivation of (16)

The functions M[h, k, u; s] in (10), related to the $\mathfrak{L}\mathfrak{T}$ of the moments, are given by (see [5]) M[h, k, u; s] $= h!\,k!\,Q_h^{h+k}(u,s)$ where $Q_q^n(u,s)$ are rational function satisfying the relation

$$Q_q^n(u,s) = \frac{1}{s+(2q-n+u)^2}\left[Q_q^{n-1}(u,s) + Q_{q-1}^{n-1}(u,s)\right], \qquad \text{(A1)}$$

with boundary conditions:

$$Q_0^n(u,s) = \frac{1}{s+(u-n)^2}\, Q_0^{n-1}(u,s), \qquad \text{(A2)}$$

$$Q_n^n(u,s) = \frac{1}{s+(u+n)^2}\, Q_{n-1}^{n-1}(u,s). \qquad \text{(A3)}$$

On this way, from (15), we get

$$m_{hr}(s) = (h!)^2 \frac{(-1)^r}{\omega^{2r}} \frac{\partial^{2r} Q_h^{2h}(u,s)}{\partial u^{2r}}\bigg|_{u=0}. \qquad \text{(A4)}$$

Let us introduce the new functions W[n, q, s; j] defined by

$$\frac{\partial^j Q_q^h(u,s)}{\partial u^j}\bigg|_{u=0} \triangleq W[n,q,s;j]. \qquad \text{(A5)}$$

One gets

$$W[n,q,s;j] = \sum_{l=0}^{j} \binom{j}{l} \frac{\partial^l}{\partial u^l} \frac{1}{s+(2q-n+u)^2}\bigg|_{u=0}$$
$$(W[n-1,q,s,j-l] + W[n-1,q-1,-s;j-l]) \qquad \text{(A6)}$$

with initial condition:

$$W[0,0,s;l] = \frac{\partial^l}{\partial u^l}\frac{1}{s+u^2}\bigg|_{u=0} = \begin{cases} 0, & l \text{ odd} \\ (-1)^{l/2}\, l!\left(\dfrac{1}{s}\right)^{l/2+1}, & l \text{ even}. \end{cases} \qquad \text{(A7)}$$

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