

# Mathematical Analysis of the Packet Delay Statistics in Bluetooth Piconets under Round Robin Polling Regime

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**Abstract**—Personal Area Network technologies like Bluetooth and its subsequent derivations and evolutions (Bluetooth v1.2, v2.0+EDR) are valid candidates to realize the mobile and pervasive communication paradigm that is considered in several recent research projects. Although the delay performance of the basic Bluetooth network configuration (piconet) has been widely evaluated through numerical simulations, no satisfactory analytical framework has been yet proposed in the literature. In this paper we present an analysis of the packet delay statistic in Bluetooth piconets, for a limited–1 (round robin) polling strategy. The mathematical model proposed in this paper extends the other models presented in the literature by providing more accurate results for a wider range of traffic patterns, under the assumption of a marked Poisson arrival process. Our analysis provides a complete statistical characterization of the packet delay, by means of Laplace-Stieltjes transform, for generic traffic patterns. Furthermore, expressions for the estimation of the average packet delay for unbalanced and asymmetric traffic are derived, thus improving existing results based on the theory of  $M|G|1$  queues with vacations. Such expressions are, however, rather complex. Therefore, we propose an approximation, based on a renewal argument, which leads to a closed–form expression for the access delay statistic. The proposed analysis permits an accurate estimation of the packet delay under a wide range of network load conditions.

**Index Terms**—Bluetooth, polling, delay analysis, multislot packets, asymmetric traffic, unbalanced traffic

## I. INTRODUCTION

The vision of ubiquitous and pervasive communications have been recently embraced by several research projects [1], [2], [3]. Such a vision opens the way to an entire world of potential new applications and business models. An example is the Virtual Community, which encompasses all the situations where people may wish to establish a spontaneous and self-organized network for sharing information. For instance, people waiting in an airport lounge or in a waiting-room of medical clinic may desire to spend some time by playing a distributed game, exchanging ringing tones or background pictures for the cellular phone or, more generally, sharing

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information. Home networking and Smart Environments provide other examples of applications enabled by the pervasive communications paradigm.

The realization of the *anywhere anytime* communication paradigm requires that the user is surrounded by communication– and computation–enabled devices, embedded in the environment. In this context, technologies for personal area networks, such as Bluetooth [4] and its evolutions, play a primary role, since they offer moderate-to-high transmission rates, reduced energy consumption, low cost and support of self–management functionalities in small radio networks.

Unfortunately, the penetration of the Bluetooth technology in the market has been initially slowed down by some implementation and compatibility problems. Most of such problems are now solved and Bluetooth is undertaking the expected success, being integrated in hundreds of portable electronic devices [5]. Furthermore, the Bluetooth Special Interest Group [4] is promoting the enhanced data rate version of the standard, Bluetooth v2.0, that will permit higher bit rates (up to 3 Mbps) and faster node connections. Though not yet commercialized, these latest versions of the standard will enlarge the potentialities of the system and promote its diffusion in the context of pervasive communications.

The realization of the aforementioned visionary scenarios will require a deep understanding of the fundamentals that determine the performance of a Wireless Personal Area Network (WPAN). In particular, the analysis of the performance achieved by different packet scheduling policies under various traffic loads assumes a role of primary importance, as stated in many studies on this topics [6], [7]. The research community working on Bluetooth-related topics is focusing more and more on the problems involved in the construction, maintenance and management of Bluetooth-based multihop networks, the so-called *scatternets* [8], [9]. On the other hand, the literature still lacks a complete understanding of the performance attainable by the basic building block of Bluetooth technology, i.e., a single-hop network referred to as *piconet* in the Bluetooth lexicon.

The Bluetooth standard encompasses a medium access control (MAC) mechanism inspired to a classic and simple polling scheme, which greatly impacts on system performance.

Therefore, several polling strategies proposed for Bluetooth have been analyzed in literature, mostly via simulation [10], [6], [7], [11], [12], [13], [14]. However, a mathematical performance characterization of the basic polling mechanisms for Bluetooth would permit a much deeper understanding of the actual effect of different design choices. Unfortunately, the specific characteristics of this technology, such as the tight correlation among the service time of different queues, prevent a straightforward application of well-known results obtained for classical polling systems [7].

Misic and Misic [15] presented a model, based on the theory of  $M|G|1$  queues with vacations, which provides an approximate expression for the mean packet delay. In their model, however, some dependencies among random variables are not taken into account, leading to an underestimation of the delay up to 40% for a heavily loaded piconet operating under the limited-1 (Pure Round Robin) regime. These problems have been pointed out in [16], where the authors propose a clever and elegant model that permits to reduce the problem to the analysis of a classical gated limited-1 polling system with non-zero switch-over time. Furthermore, the authors showed that inaccurate modeling may lead to significant performance estimation errors, in particular when the system approaches saturation. Unfortunately, the method proposed in [16] leads to exact results only under specific traffic patterns, namely when the same amount of traffic is offered by each slave to the master (symmetrical traffic) and by the master to each slave (bi-directional traffic). Some generalizations are considered in [17], which relaxes some of the assumptions of the previous work, but still maintains the constraint on the symmetrical traffic pattern. In [18] the authors presented another model, also based on  $M|G|1$  queuing theory, that slightly improves the one presented in [15]. Nevertheless, it also ignores some dependencies among the service time of the different queues, that affects the precision of the delay estimation as the system approaches saturation.

In this paper we present a novel model for the limited-1 polling policy that provides better performance estimation under a wide range of operating conditions. Although an exact model for systems of multiple interacting queues is still far from being defined, we prove that an accurate modeling of *some* of the interdependencies neglected in the previous works may drastically improve the accuracy of the model in predicting the packet delay. The price paid for such an improvement is a greater model complexity. Therefore, we propose, starting from such enhanced model, an approximation that leads to a closed-form expression for the estimation of the average packet delay. Such an estimation is proved to be very accurate and, hence, it improves over the existing literature.

In summary, the model presented in this manuscript extends the models already proposed in the literature in that it is able to closely track the behavior of the network delay for asymmetric and unbalanced traffic pattern, also when the system approaches saturation. Furthermore, in the scenarios where expressions for the average delay have been derived in the literature ([16], [17]), the model returns exact results.

The paper is organized as follows: Sec. II provides a brief overview of the Bluetooth baseband layer characteristics.

Sec. III presents the system model we use for our analysis. In Sec. IV we evaluate the network performance in terms of packet delay distribution. Sec. V provides numerical results, presenting comparison with the results obtained in [15], [18] and [16], whenever possible. Sec. VI concludes the paper pointing out some open issues for future research.

## II. THE BLUETOOTH TECHNOLOGY

The basic network configuration, in the Bluetooth world, is the so-called piconet, a cluster of no more than eight devices sharing a common frequency-hopping radio channel. The access to the shared medium is regulated by one of the units, called master, which cyclically polls the other devices, named slaves. Full-duplex communication is achieved by means of a time-division duplex (TDD) mechanism. The standard does not specify the polling scheme to be adopted. Even if offering poor performance, limited-1 polling is the current choice, due to its simplicity and low implementation cost. The protocol encompasses two types of links. One, synchronous connection oriented (SCO), is aimed at the transport of real-time services (mainly voice), and is based upon a periodical reservation scheme. The other, asynchronous connectionless (ACL), is designed for the transport of elastic data traffic. In this paper we focus on ACL only. For ACL links, the standard offers six different Basic Rate (BR) packet formats, which differ in length and error protection. Unprotected packet formats, which provide High nominal Data rate, are denoted by  $DHn$ . Similarly,  $DMn$  is used for protected formats which provide Medium Data rate by adopting a (15, 20) shortened Hamming Forward Error Correction (FEC) code over the payload field. Both unprotected and protected formats can extend over  $n = 1, 3$  or  $5$  consecutive slots, where a slot is of 0.625 ms.

The version  $v2.0 + EDR$  of the Bluetooth specifications introduces the Enhanced Data Rate (EDR), which obtains higher data rates by using more powerful modulation schemes at the physical layer.

For backward compatibility, the synchronization and frame-control fields of EDR packets are identical in format and modulation to Basic Rate (BR) packets. However, these fields are followed by a guard time of approximately  $5 \mu s$  and by a synchronization field (SYNC), whose length depends on the specific EDR packet type considered. The SYNC field and the following payload and trailer fields are transmitted by using one of the two higher rate modulation schemes. Nevertheless, the slot occupancy of EDR packet formats is still limited to 1, 3 or 5 consecutive slots.

A resume of the characteristics of the different ACL packet formats provided by Bluetooth v2.0+EDR is reported in Table I.

It is worth stressing that, in this paper we are interested in evaluating the performance of the 1-limited polling strategy only. Therefore, we limit our analysis to the case of error-free channels and consider only unprotected packet formats.

## III. SYSTEM MODEL

We introduce a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ , on which all the stochastic processes of interest are defined.

Type	Rate	Slot occupancy	Max. payload length [bytes]	FEC rate
DM1	BR	1	17	2/3
DM3	BR	3	121	2/3
DM5	BR	5	224	2/3
DH1	BR	1	27	–
DH3	BR	3	183	–
DH5	BR	5	339	–
2-DH1	EDR	1	54	–
2-DH3	EDR	3	366	–
2-DH5	EDR	5	678	–
3-DH1	EDR	1	83	–
3-DH3	EDR	3	552	–
3-DH5	EDR	5	1021	–

TABLE I  
PACKET CHARACTERISTICS FOR ACL LINKS (BR=BASIC RATE; EDR= ENHANCED DATA RATE)

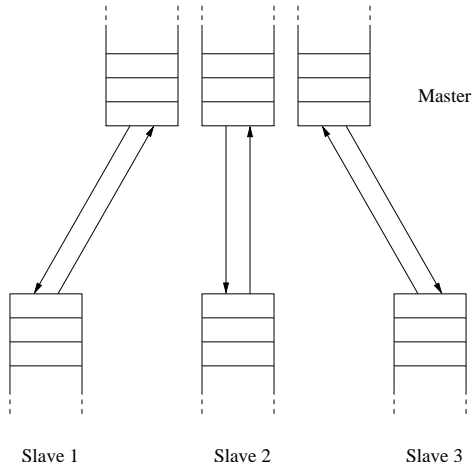


Fig. 1. Equivalent queuing model of a Bluetooth piconet with  $N = 3$  slaves.

The statistical expectation operator (taken with respect to the measure induced by  $\mathbb{P}$ ) is denoted by  $\mathbb{E}[\cdot]$ . For any random variable  $X$ , we denote its mean by  $x = \mathbb{E}[X]$ , its statistical power by  $x^2 = \mathbb{E}[X^2]$ , its zeta-transform by  $\mathcal{X}(z) = \mathbb{E}[z^X]$  and its Laplace-Stieltjes transform (LST) by  $\mathcal{X}^*(s) = \mathbb{E}[e^{-sX}]$ .

A piconet consisting of  $N$  slaves may be modeled as a system of  $2N$  interacting queues, as depicted in Fig.1 for the case of  $N = 3$ . Let us enumerate each queue in a piconet from 0 to  $2N - 1$ , so that the suffix  $i \in \{0, \dots, 2N - 1\}$  will be used to denote the  $i$ -th queue. (Notice that, for each slave node, uplink and downlink queues are indexed separately). Arrivals to the system are modeled by means of a Poisson point process  $\{t_n\}_{n \in \mathbb{N}}$ , defined on  $(\Omega, \mathcal{F}, \mathbb{P})$ , where  $t_n$  represents the arrival epoch of the  $n$ -th packet [19]. We may associate to  $\{t_n\}$  a counting process  $N(\cdot)$ , defined as:

$$N[0, t) = \sum_{n \in \mathbb{N}} \delta_{t_n}([0, t)),$$

where  $\delta_{t_n}(\cdot)$  is the Dirac measure at  $t_n$ . The process  $N[0, t)$  counts the number of packet generated in the interval  $[0, T)$ . We assume that the process has a finite non-zero intensity, such that:

$$0 < \lambda = \mathbb{E}[N[0, T)] < +\infty, \quad (1)$$

where  $T = 0.625$  ms is the slot length.

To each point we then associate a mark  $\sigma_n$ , defined on the same probability space, of the form  $(i, l)$ , which takes value in  $\{0, \dots, 2N - 1\} \times \{1, 3, 5\}$ . The marks represent, respectively, the queue to which the packet arrives and the packet length. Notice that, we are assuming that only one-hop communications take place.<sup>1</sup> Further, we consider the marks to be independent identically distributed (i.i.d.). Then, we may define the arrival intensity at link  $i$  as:

$$\lambda_i = \lambda \cdot \mathbb{P}[\sigma_0 \in \{(i, \{1, 3, 5\})\}], \quad i = 0, \dots, 2N - 1. \quad (2)$$

Note that, the arrival processes at the various queues turn out to be independent Poisson processes with intensities  $\lambda_i$ .

Similarly, we may define the packet length probability for the queue  $i$ :

$$\pi_i(l) = \frac{\mathbb{P}[\sigma_0 = (i, l)]}{\mathbb{P}[\sigma_0 \in \{(i, \{1, 3, 5\})\}]}, \quad l = 1, 3, 5. \quad (3)$$

Clearly, we have  $\sum_{l=1,3,5} \pi_i(l) = 1$  for any  $i$ . The system may thus be completely described by means of the traffic vector  $\Lambda(s)$ , having entries:

$$\Lambda(s)_i = \lambda_i \sum_{l=1,3,5} \pi_i(l) e^{-sl}, \quad i \in \{0, \dots, 2N - 1\}. \quad (4)$$

Note that the LST of the transmission time  $Z_i$  of a packet of queue  $i$  is given by  $Z_i^*(s) = \frac{\Lambda(s)_i}{\lambda_i}$ .

#### IV. PERFORMANCE ANALYSIS

Our aim is to derive a mathematical expression of the average packet delay, i.e., the average time that a packet spends in queue and in transmission before being received by the next-hop node. The average delay is, hence, given by the sum of the access delay, i.e., the time spent in queue waiting for the service to begin, and the service time.

To this end, we first determine the statistic of the cycle time, defined as the time interval between two successive polls of the same node. Then, we consider the statistic of the access delay, i.e., the time between a packet arrival and its service beginning. Finally, we combine the results to get the statistic of the packet delay.

##### A. Cycle time statistics

The cycle time  $T_C$  is defined as the time interval between two successive polls of the same node. Let  $B_i$  be the part of the cycle time dedicated to the queue  $i$ . Furthermore, let  $\rho_i$  denote the equivalent load factor of the  $i$ -th queue, defined as:

$$\rho_i = \lambda_i t_C = \lambda_i \sum_{i=0, \dots, 2N-1} b_i. \quad (5)$$

In other words, the load factor gives the average number of packets generated by a given transmission queue in a polling cycle. It is easy to verify that the load factor is also equal to the stationary probability that the served queue is found empty

<sup>1</sup>The analysis can be extended to take into account slave-to-slave communications as done in [18], where the classical tool of statistical routing is applied in order to get estimates of the mean packet delay.

$$b_i = \frac{(2N-1)\lambda_i [2\pi_i(3) + 4\pi_i(5)] + 1 - \sum_{m \neq i} \lambda_m [2\pi_m(3) + 4\pi_m(5)]}{1 - \sum_m \lambda_m [2\pi_m(3) + 4\pi_m(5)]}. \quad (10)$$

[20].

Considering that a 1-slot long POLL/NULL packet is sent whenever a queue is found empty, the probability mass distribution of the random variable (r.v.)  $B_i$  is given by:

$$P[B_i = k] = \begin{cases} \rho_i \pi_i(1) + 1 - \rho_i & k = 1, \\ \rho_i \pi_i(3) & k = 3, \\ \rho_i \pi_i(5) & k = 5, \\ 0 & \text{otherwise.} \end{cases}$$

Taking expectation, we get:

$$b_i = \rho_i (2\pi_i(3) + 4\pi_i(5)) + 1. \quad (6)$$

Together with (5) this defines a system of  $2N$  equations in  $\rho_i$ :

$$\rho_i = \lambda_i \cdot \sum_{m=0, \dots, 2N-1} [\rho_m (2\pi_m(3) + 4\pi_m(5)) + 1], \quad (7)$$

which (the computation is trivial) solves for

$$\rho_i = \frac{2N\lambda_i}{1 - \sum_m \lambda_m [2\pi_m(3) + 4\pi_m(5)]}. \quad (8)$$

We recall that the piconet is stable if and only if  $\rho_i < 1$  for each  $i$ . Then, (8) defines the achievable rate regions of a Bluetooth piconet, providing a characterization of the limiting performance in terms of throughput (see [18] for some examples and [21] for an extension to fading channels). It is worth noting that the stability condition applies to a more general stationary ergodic framework, where the expectations are taken with respect to the measure induced by the corresponding Palm probability [22].

Replacing (8) in (5), the average cycle time is given by:

$$t_C = \frac{2N}{1 - \sum_{i=0}^{2N-1} \lambda_i [2\pi_i(3) + 4\pi_i(5)]}. \quad (9)$$

The expression for the mean cycle time can be shown to hold in a more general stationary ergodic framework [23], and the same happens for the mean station times  $b_i$ , reported in (10), where, again, the expectations are taken with respect to the Palm probability  $\mathbb{P}^0$  [19].

If the system is stable, the throughput on the link  $i$ , expressed in bit/s, can be easily found. Denoting by  $\gamma(l)$  the payload length (in bits) of the packets of length  $l$ , we get:

$$S_i = \lambda_i \sum_{l=1,3,5} \pi_i(l) \gamma(l). \quad (11)$$

This clearly applies to the case in which just the same Physical Rate and Error Correction Code (if any) is used in each traffic flow. In the more general case, to deal with all the packet types outlined in Table I we just need to extend the marks by adding a field specifying the kind of packet under consideration.

While no characterization of the complete cycle time statistics is possible in general, we may get an approximation by considering the  $B_i$  to be independent random variables. In this way, we get for the cycle time LST:

$$\mathcal{T}_C^*(s) = \prod_{i=0, \dots, 2N-1} \mathcal{B}_i^*(s). \quad (12)$$

For example, such an approximation would lead to the following (optimistic) estimate of the cycle time variance

$$\sigma_{T_C}^2 = \sum_{i=0, \dots, 2N-1} \sigma_{B_i}^2,$$

which could be used to obtain bounds on the cycle time distribution by means of the classical Chebyshev's bound.

### B. Delay analysis

The access delay,  $W$ , is defined as the time between a packet arrival and its service time beginning, and may be thought as the sum of two terms, as depicted in Fig. 2. The first,  $V$ , denotes the time interval between the arrival of the packet to the queue and the time instant the queue gets the token. The second describes the time needed to serve all the packets found waiting in queue, whose number is indicated by  $L$ . Denoting by  $U(k)$  the inter-visit time for the  $k$ -th packet, i.e., the time needed to get rid of the  $k$ -th queued packet, we can express the access delay as:

$$W_i = V_i + \sum_{k=1}^{L_i} U_i(k). \quad (13)$$

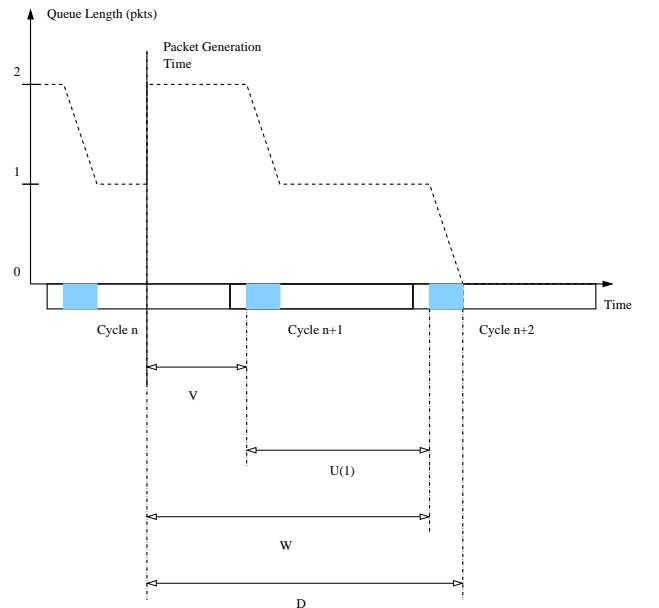


Fig. 2. Decomposition of the packet delay for the case  $L = 1$ .

$$\tilde{w}_i = \frac{t_C^2}{2t_C \cdot \left\{ 1 - \lambda_i \left[ 2N + 2\pi_i(3) + 4\pi_i(5) + \sum_{m \neq i} \tilde{\rho}_{m,i} \cdot (2\pi_m(3) + 4\pi_m(5)) \right] \right\}}. \quad (30)$$

Since packet arrivals follow a Poisson process, we easily obtain the following relationship between the zeta-transform of  $L$  and the LST of  $W$  [24]:

$$\mathcal{L}_i(z) = \mathcal{W}_i^*(\lambda_i - \lambda_i z). \quad (14)$$

Note that, once we solve for  $\mathcal{W}^*(s)$ , by means of (14) we can get the statistics of the packet length distribution at polling instants, which can be used to correctly dimension the system buffers.

As previously done for the cycle time, let us assume  $U_i(k)$  to be i.i.d., so that we can work with transforms. It is worth stressing that this represents just an approximation of the extremely complex network behavior; however, it enables us to get an exact expression for the first order moments, while providing (optimistic) estimates of the higher order moments. Transforming both members of (13) and applying relationship (14), we thus find

$$\mathcal{W}_i^*(s) = \mathcal{V}_i^*(s) \cdot \mathcal{W}_i^*(\lambda_i - \lambda_i \mathcal{U}_i^*(s)). \quad (15)$$

The packet delay is the sum of the access delay and the transmission delay. The two terms are independent and, thus:

$$\mathcal{D}_i^*(s) = \mathcal{W}_i^*(s) \cdot \mathcal{Z}_i^*(s). \quad (16)$$

Note that, in general, (15) cannot be inverted directly, so that we should resort to numerical methods to obtain the access delay probability density function [25]. However, statistical moments of any order may be obtained by deriving both members, easily allowing one to get bounds on the delay distribution. To this end, we need to compute the LSTs of  $V_i$  and  $U_i$ . Although the inter-visit time  $U_i$  is technically a polling cycle, its statistic is conditioned on the fact that there is at least one packet waiting at queue  $i$  and, thus, the other queues experience a greater load. Therefore, we can write

$$U_i = Z_i + \sum_{m \neq i} \tilde{B}_{m,i}, \quad (17)$$

where  $\tilde{B}_{m,i}$  is distributed like  $B_m$  but with an equivalent load factor  $\tilde{\rho}_{m,i}$ . Clearly, in stationary regime,  $\tilde{\rho}_{m,i} = \lambda_m u_i$ . Therefore, we get a system of equations that, after some algebra, can be shown to solve for

$$\tilde{\rho}_{m,i} = \frac{\lambda_m [z_i + 2N - 1]}{1 - \sum_{r \neq i} \lambda_r [2\pi_r(3) + 4\pi_r(5)]}, \quad (18)$$

where  $z_i$  is the average service time at queue ( $i$ ),  $z_i = \pi_i(1) + 3\pi_i(3) + 5\pi_i(5)$ .

As for the cycle time, we consider the terms of (17) to be independent. Thus, we get:

$$\mathcal{U}_i^*(s) = \mathcal{Z}_i^*(s) \cdot \prod_{m \neq i} \tilde{\mathcal{B}}_{m,i}^*(s). \quad (19)$$

For the computation of  $\mathcal{V}_i^*(s)$  we follow the footprints of [26], generalizing their results in terms of LST upon an

independence approximation. First, we condition  $V_i$  on the link  $m$  that was in service when the packet arrived (at queue  $i$ ). We denote such quantity by  $V_{i|m}$ . By the total expectation theorem, we get:

$$V_i = \sum_m V_{i|m} p_m, \quad (20)$$

where, upon the Poisson assumption and the renewal theorem, it can be readily understood that the probability of queue  $m$  being in service when the packet arrived is:

$$p_m = \frac{b_m}{t_c}. \quad (21)$$

The random variable  $V_{i|m}$  can be written as the sum of two quantities,

$$V_{i|m} = \Omega_m + Y_{i,m}. \quad (22)$$

The first quantity, denoted by  $\Omega_m$ , is the residual service time at queue  $m$ . Thanks to the Poisson assumption for the arrival process, its LST can be shown to be equal to [24]:

$$\Omega_m^*(s) = \frac{1 - \mathcal{B}_m^*(s)}{b_m t_c}. \quad (23)$$

The other term,  $Y_{i,m}$  is the time spent serving the links that, in the polling cycle, come *after*  $m$ , but *before*  $i$ . This is conditioned on the fact that there was an arrival (at queue  $i$ , but this is immaterial due to the Poisson assumption) *while* queue  $m$  was in service. In symbols:

$$Y_{i,m} = \sum_{i > r > m} \hat{B}_{r,m}. \quad (24)$$

Again, the distribution of  $\hat{B}_{r,m}$  is the same as  $B_r$ , but with a higher load due to the conditioning. The quantity  $\hat{\rho}_{r,m}$  equals the probability of having an arrival at queue  $r$  given that there was an arrival (at queue  $i$ ) while queue  $m$  was in service. This leads us to defining the following system of  $2N$  equations:

$$\hat{\rho}_{r,m} = \lambda_r \left\{ \frac{b_m^2}{b_m} + 2N - 1 + \sum_{u \neq m} \lambda_u [2\pi_u(3) + 4\pi_u(5)] \right\}, \quad (25)$$

where we used the fact that, again due to the Poisson assumption on the arrival process and the renewal theorem, the average service time at link  $m$  given that it was in service when a packet arrived equals  $\frac{b_m^2}{b_m}$  [24]. The system turns out to solve for:

$$\hat{\rho}_{r,m} = \frac{\lambda_r \left( \frac{b_m^2}{b_m} + 2N - 1 \right)}{1 - \sum_{u \neq m} \lambda_u [2\pi_u(3) + 4\pi_u(5)]}. \quad (26)$$

Assuming the quantities of interest to be independent and passing to LSTs, we find:

$$\mathcal{V}_i^*(s) = \sum_m \frac{1}{t_C} \cdot \frac{1 - \mathcal{B}_m^*(s)}{s} \cdot \mathcal{Y}_{m,i}^*(s), \quad (27)$$

$$\mathcal{Y}_{m,i}^*(s) = \prod_{i>r>m} \hat{\mathcal{B}}_{r,m}^*(s). \quad (28)$$

It is apparent that the derivation of  $\mathcal{V}_i^*(s)$  is lengthy and cumbersome. It seems then natural to look for a further approximation, which, on the one hand, is able to track the network behavior till high loads, while, on the other one, leads to a closed form expression for the mean packet delay, so that it can be easily used for dimensioning purposes.

The idea is to approximate the r.v.  $V$  as the residual life in a renewal process having renewal period equal to  $T_C$ . Thus, we have [24]:

$$\tilde{\mathcal{V}}_i^*(s) = \frac{1 - \mathcal{T}_C^*(s)}{st_C}, \quad (29)$$

which turns out to be independent of the index  $i$ . For example, plugging (29) into (15), after some easy algebra we get a closed-form approximation, reported in (30), for the mean packet delay. We may also use such expression to provide some (rough) buffer dimensioning guidelines. For example, applying Little's formula, we may get the approximate mean buffer length,  $\tilde{l}_i = \lambda_i \tilde{w}_i$ . Then, using Markov bound, we get  $P[\tilde{L}_i \geq \alpha] \leq \frac{\tilde{l}_i}{\alpha}$ . A better estimate can be found by deriving the second order moments of  $\tilde{W}_i$  and applying Chebyshev's bound.

## V. NUMERICAL RESULTS

In this section, we present some numerical results which show the accuracy of our analytical delay estimation both in the balanced (symmetrical and asymmetrical) and unbalanced case. It is worth recalling that, for a general limited-1 polling system with non-zero switchover times, exact expressions for the mean delay are known (for the case of Poisson arrivals) only for the balanced case, where all the queues experience the same load [20], [27].

Therefore, in the case of balanced-traffic scenario, the results provided by our model can be compared with both results obtained by other analytical models presented in the literature [15] [18] and the exact values, which can be computed as in [16] or by means of (15). On the other hand, in the unbalanced-traffic case, the proposed formula is validated through simulation results, since no exact analytical expression is available.

Let us first consider a balanced scenario, where the packet size distribution and the generation rate are equal for all the traffic flows. Both symmetrical and asymmetrical configurations are evaluated. In the first one, both uplink and downlink queues are active, whereas, in the asymmetrical configuration, only downlink traffic is present. The resulting curves are plotted in Fig. 3 and Fig. 4, respectively. It is apparent that the approximation introduced in (30) leads to accurate results (underestimation of the access delay of the order of a few % for loads above 0.9), clearly overcoming those obtained by using the techniques in [15] and [18], which

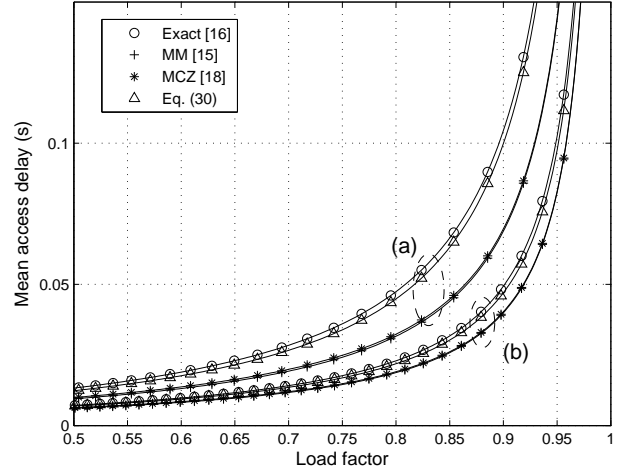


Fig. 3. Mean access delay for a balanced symmetrical scenario  $N = 3$ ; (a):  $\pi(1) = \frac{1}{9}, \pi(3) = \frac{3}{9}, \pi(5) = \frac{5}{9}$ ; (b):  $\pi(1) = \frac{5}{9}, \pi(3) = \frac{3}{9}, \pi(5) = \frac{1}{9}$ .

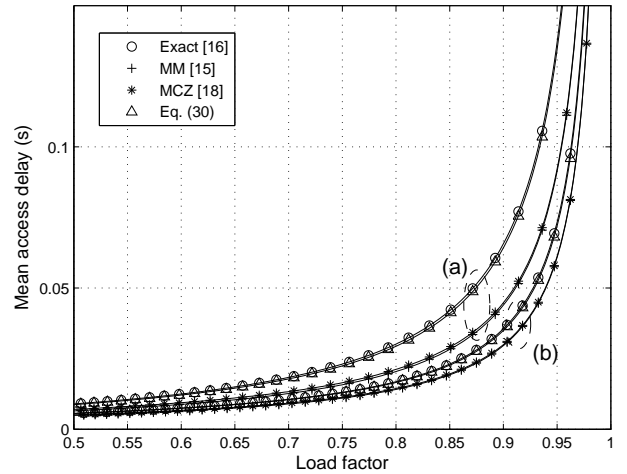


Fig. 4. Mean access delay for a balanced asymmetrical scenario  $N = 3$ ; (a):  $\pi(1) = \frac{1}{9}, \pi(3) = \frac{3}{9}, \pi(5) = \frac{5}{9}$ ; (b):  $\pi(1) = \frac{5}{9}, \pi(3) = \frac{3}{9}, \pi(5) = \frac{1}{9}$ .

show relevant underestimation of the delay. From these two figures, it is possible to appreciate the ability of the proposed approximation to track the network behavior in an accurate way, for a wide range of traffic conditions.

Let us now consider an unbalanced scenario, where the traffic intensity and the packet length distribution of the various flows can differ. Since no exact results are known, the analytical results are compared with the outcomes of extensive numerical simulations. The comparison also includes the results provided by the model proposed in [18], which was the only model for the delay estimation in the unbalanced case previously presented in the literature.

The first set of results refers to a piconet with  $N = 3$  slaves, with an equal packet length distribution for all the flows ( $\pi_i(l) = \frac{1}{3} \forall i, l$ ), but different traffic intensities ( $\lambda_1 = 0.2, \lambda_2 = 0.4, \lambda_3 = 0.6$ ). Fig. 5 shows the delay experienced by the three nodes. Simulation results are plotted with a statistical

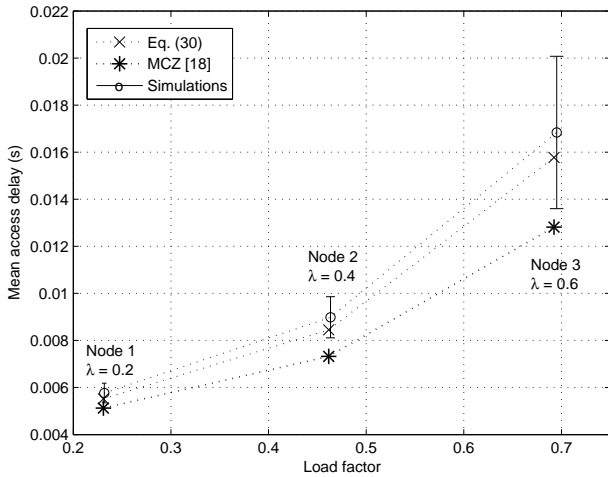


Fig. 5. Mean access delay for an unbalanced symmetrical scenario,  $N = 3$ ,  $\pi_i(l) = \frac{1}{3} \forall i, l$ . Comparison with analytical (Eq. (30), [18]) and simulative results (98% confidence interval) is shown.

confidence interval of 98%, and are compared with those given by (30) and obtained in [18]. As expected, the higher the load of a node, the higher the delay experienced by that node. However, it can be seen that the model proposed in this paper leads to results that fall within the confidence interval, while the approach described in [18] returns less accurate results.

The second set of results shows how our analysis might be used to provide an estimation of the delay-limited capacity regions of a piconet. The delay-limited capacity region for a piconet of  $N$  slaves is defined as the convex  $2N$ -dimensional polyhedron characterizing all the achievable rates able to *guarantee* an upper-bounded packet access delay. As an example, in Fig. 6 it is reported the delay-limited capacity region of a piconet with  $N = 2$  slaves, for two different packet length distributions ( $\pi_i(l) = \frac{l}{9}$  and  $\pi_i(l) = \frac{6-l}{9}$ ,  $l \in \{1, 3, 5\}$ ) and for a delay of 5 ms.

## VI. CONCLUSION

In this paper, we have presented a novel mathematical model for performance evaluation of the 1-limited polling policy in a Bluetooth piconet. The proposed analysis improves the previous models in that (i) it enables to deal with unbalanced asymmetric traffic patterns (ii) it provides better performance estimations for a wide range of operating scenarios. The proposed model returns the exact average delay in two specific scenarios, for which the exact expression of the mean packet delay is known. A simplification, derived upon a renewal approximation, has been introduced, leading to a simple closed-form expression for the mean packet delay. Such formula has been shown, by comparison with the outcomes of extensive simulations, to offer a good match for a wide range of operating conditions.

Future research directions include, on the one hand, the inclusion of channel errors in the framework and, on the other one, the introduction of a more realistic traffic model (of particular interest is the case of batch arrivals, where a

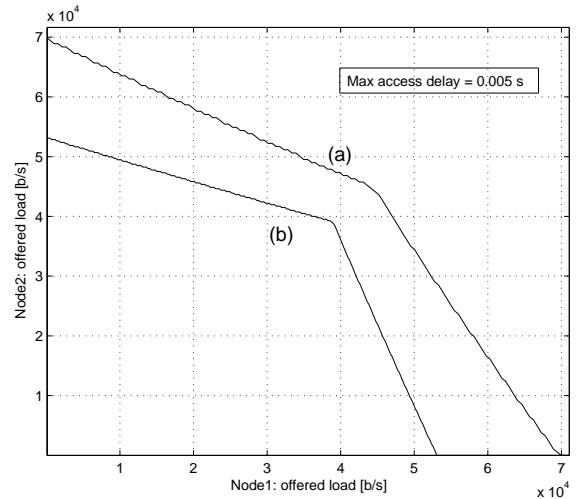


Fig. 6. Delay limited capacity region for a symmetrical scenario, calculated using Eq. (30) with  $N = 2$ , (a):  $\pi(1) = \frac{1}{9}, \pi(3) = \frac{3}{9}, \pi(5) = \frac{5}{9}$ ; (b):  $\pi(1) = \frac{5}{9}, \pi(3) = \frac{3}{9}, \pi(5) = \frac{1}{9}$ .

batch corresponds to the segmentation of a single L2CAP layer packet).

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## REFERENCES

- [1] "Mobile Life," FP6-IST, Project Reference: 511607. [Online]. Available: <https://www.ist-mobilife.org/>
- [2] "My personal Adaptive Global NET," FP6-IST, Project Reference: 507102. [Online]. Available: <http://www.telecom.ece.ntua.gr/magnet/>
- [3] "Reconfigurable Ubiquitous Networked Embedded Systems," FP6-IST, Project Reference: 4536. [Online]. Available: <http://www.ist-runes.org/>
- [4] Specification of Bluetooth system, ver. 1.1, February, 22 2001. [Online]. Available: <http://www.bluetooth.com>
- [5] Bluetooth wireless - consumer products. [Online]. Available: <http://www.bluetooth.com/products/>
- [6] S. Baatz, M. Frank, C. Kuhl, P. Martini, and C. Scholz, "Bluetooth scatternets: An enhanced adaptive scheduling scheme," in *Proc. of IEEE INFOCOM*, New York, 2002.
- [7] A. Capone, M. Gerla, and R. Kapoor, "Efficient polling schemes for Bluetooth picocells," in *Proc. IEEE ICC*, Helsinki, 2001.
- [8] K. E. Persson, D. Manivannan, and M. Singhal, "Bluetooth scatternet: criteria, models and classification," *Ad Hoc Networks*, vol. 3, no. 6, pp. 777–794, Nov. 2005.
- [9] R. M. Whitaker, L. Hodge, and I. Chlamtac, "Bluetooth scatternet formation: a survey," *Ad Hoc Networks*, vol. 3, pp. 403–450, Jul. 2005.
- [10] N. Johansson, U. Korner, and J. P., "Performance evaluation of scheduling algorithms for Bluetooth," *Broadband Communications: Convergence of Network Technologies*, Kluwer, vol. 159, no. ISBN:0-7923-8677-9, pp. 139–150, 2000.
- [11] A. Das, A. Ghose, A. Razdan, H. Saran, and R. Shorey, "Enhancing performance of asynchronous data traffic over the Bluetooth wireless ad hoc network," in *Proc. of IEEE INFOCOM*, Anchorage, Alaska, 2001.
- [12] L. Har-Shai, R. Kofman, A. Segall, and G. Zussman, "Load adaptive inter-piconet scheduling in small-scale Bluetooth scatternets," *IEEE Comm. Mag.*, vol. 42, pp. 136–142, July 2004.
- [13] R. Kapoor, A. Zanella, and M. Gerla, "A fair and traffic dependent polling scheme for Bluetooth," in *Proc. of the joint IEEE conferences ICWLHN 2002 and ICN 2002*, Atlanta, USA, 2002.
- [14] M. Kalia, D. Bansal, and R. Shorey, "Data scheduling and SAR for Bluetooth MAC," in *Proc. IEEE VTC (Spring)*, Tokyo, 2000.
- [15] J. Misić and V. B. Misić, "Modeling Bluetooth piconet performance," *IEEE Comms. Lett.*, vol. 7, no. 1, pp. 18–20, Jan 2003.

- [16] G. Zussman, U. Yechiali, and A. Segall, "Exact probabilistic analysis of the limited scheduling algorithm for symmetrical Bluetooth piconets," in *Proc. of PWC2003*, Venice, Italy, 2003.
- [17] G. Zussman, A. Segall, and U. Yechiali, "Bluetooth time division duplex – analysis as a polling system," in *Proc. of IEEE SECON*, Santa Clara, CA, 2004.
- [18] D. Miorandi, C. Caimi, and A. Zanella, "Performance characterization of a Bluetooth piconet with multi-slot packets," in *Proc. of WiOpt03*, Sophia-Antipolis, France, 2003.
- [19] F. Baccelli and P. Bremaud, *Elements of Queueing Theory*. Berlin: Springer-Verlag, 1994.
- [20] H. Takagi, *Analysis of Polling Systems*. Cambridge, MA: M.I.T. Press, 1986.
- [21] D. Miorandi and A. Zanella, "Achievable rate regions for Bluetooth piconets in fading channels," in *Proc. IEEE VTC (Spring)*, Milan, Italy, 2004.
- [22] S. G. Foss and N. I. Chernova, "On polling systems with infinitely many stations," *Siberian Math. J.*, vol. 37, no. 4, pp. 940–956, 1996.
- [23] E. Altman, P. Kostantopoulos, and Z. Liu, "Stability, monotonicity and invariant quantities in general polling systems," *Queueing Systems*, vol. 11, pp. 35–57, 1992.
- [24] L. Kleinrock, *Queueing Systems*. New York: John Wiley & Sons, 1975.
- [25] J. Abate and W. Whitt, "Solving probability transform functional equations for numerical inversion," *Oper. Res. Letters*, vol. 12, pp. 36–43, 1992.
- [26] O. C. Ibe and X. Cheng, "Approximate analysis of asymmetric single-service token passing systems," *IEEE Trans. Comms.*, vol. 37, no. 6, pp. 572–577, Jun 1989.
- [27] H. Takagi, "Queueing analysis of polling models: an update," in *Stochastic analysis of computer and communication systems*, H. Takagi, Ed. Amsterdam: North Holland, 1990, pp. 267–318.