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Transmission Scheduling for Remote Estimation with Multi-packet Reception under Multi-Sensor Interference

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Abstract: In networked control systems, due to competing demands on bandwidth and energy constraints, sensor scheduling is an important problem for remote estimation and control tasks. Traditionally, a single sensor is scheduled in each resource block to avoid interference or collisions so that the probability of packet loss is reduced. However, receiving multiple packets from different sources under interference is routinely achieved in wireless networks using multi-packet reception techniques. In this work, we explore the problem of sensor scheduling for remote estimation when the estimator is able to simultaneously receive multiple packets. We use the typical signal-to-interference-and-noise-ratio (SINR) based capture model to compute the packet arrival probabilities. Then an optimal scheduling policy is determined by minimizing expected estimation error covariance subject to a constraint on the average number of total transmissions. In the case of two sensors, for a scalar system and for a decoupled two-dimensional system, we show that allowing multiple simultaneous transmissions can improve the quality of the estimation achieving lower energy consumptions and we provide structural results on the optimal policies. Numerical results illustrate the benefits of multi-packet reception in remote estimation.

Keywords: Estimation under communication constraints; Multi-packet reception; Packet loss.

1. INTRODUCTION

The ubiquity of wireless communication networks has paved the way for novel control technologies involving remote processes and networked devices. In particular, remote state estimation of dynamical systems using Wireless Sensor Networks (WSNs) has gained considerable attention in the past years in many different areas, such as home and factory automation, environment monitoring, power distribution, and autonomous vehicles, thanks to its important practical advantages as reduced wiring, increased agility, easy and modular connections. On the other hand, with respect to standard sensing architectures, estimation performances are not necessarily improved through the use of WSNs since, if the network medium access is not suitable designed, resulting information losses and delays cause poor estimates. At the same time, even when losses are avoided, energy consumption becomes an important aspect and frequent communications can drain the batteries of the sensors prematurely. Therefore it is fundamental to devise a scheduling algorithm that manages the access to the network of each device: the solution is not straightforward and many different algorithms have been proposed. When only a single sensor is present, scheduling policies are required to satisfy energy limitations. Shi et al. (2011b) consider a smart sensor with two transmission energy levels: the higher guarantees that packets are always successfully delivered, while the lower has a loss probability greater than 0. The corresponding optimal periodic policy transmits at the higher level as "uniformly" as possible. This structure is shown to be optimal by Ren et al. (2013) also when totally reliable communications are not guaranteed. The case where a single system is observed by multiple sensors has also been studied in many different works. Shi et al. (2011a) consider two sensors with different energy consumptions while, for a general number of sensors, the finite-horizon case is considered by Vitus et al. (2012) and the infinite-horizon case is investigated by Zhao et al. (2014). Without packet losses, the latter shows that the optimal scheduling can be approximated arbitrarily closely by a periodic schedule. The case with multiple sensors with packet losses is studied by Leong et al. (2016). Under the assumptions that the system is observable from each sensor and that local estimates are communicated, optimal scheduling is a time-varying threshold policy (time-invariant in the infinite-horizon case) based on the estimation error covariance matrix at the remote estimator. The general scenario with multiple unstable systems (and multiple sensors) has been studied by Han et al. (2017).

All the above works assume that sensors are not simultaneously scheduled and thus mutual interference is never taken into account. There are two notable exceptions: Gatsis et al. (2018) propose a channel-adaptive optimal random access scheme for remote control of multiple systems, and Li et al. (2019) study the optimal power allocation for remote estimation. In this work, we explore how to exploit multi-packet reception in remote estimation of dynamical

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systems. Indeed, multi-packet reception in the presence of interference is very common in wireless communications by exploiting a suitable receiver equipped to decode multiple simultaneous signals from their superposition in noise. This can be achieved in many ways, such as at the signal modulation level (CDMA), by signal processing based collision resolution methods (see Tong et al. (2001)), or by multiple antennas at transmitter and receiver (MIMO). In particular, massive MIMO is widely adopted by the most recent wireless networks as 5G and last Wi-Fi standards. On one hand, multi-packet reception can yield a great improvement for WSNs because the estimator would be able to simultaneously receive measurements from different sensors but, on the other, simultaneous transmissions would interfere with each other and, even if multi-packet reception is possible, the loss probability of a given packet may be higher. The optimal balance between transmission scheduling and interference mitigation in the context of remote estimation of dynamical systems is an open problem, and largely an unexplored field.

In this work, we aim to partially answer this question. We formulate an optimization problem where the expected trace of the remote estimation error covariance is minimized subject to average transmission energy constraints, but, differently from existing algorithms, we allow multiple simultaneous transmissions. The packet reception model takes into account both the interference due to other incoming communications and external noise. We consider two types of multi-packet reception schemes, one based on the capture property of the wireless receiver (see Zanella and Zorzi (2012)), where any sensor that has a received Signal-to-Interference-and-Noise Ratio (SINR) above a certain threshold at the remote estimator, is successfully decoded. In this scheme, each sensor sees the interference due to other sensor transmissions as noise. We also investigate a more sophisticated receiver based on *Successive Interference Cancellation* (SIC), where the sensor with the strongest received power is decoded first, and its reconstructed signal is subtracted out from the total received signal, so that the sensor that has the second strongest received power can be decoded next where the strongest user's interference is no longer present. In the context of information theory, it is well known that SIC is an optimal scheme that achieves the rate region of a multiple access channel while minimizing the total transmission power (Tse and Viswanath (2005)).

The main contributions of this work can be summarized as follows: (i) in contrast with existing works, we consider multiple simultaneous transmissions, an accurate model of the wireless channel that accounts for mutual interference with the corresponding arrival probabilities, and two different decoding algorithms, namely with and without SIC, (ii) under the considered framework, an optimal scheduling policy is determined by solving an optimization problem that accounts for the estimation quality and penalizes the total number of transmissions, (iii) we provide the general structure of the optimal scheduling policy for a scalar system and a decoupled two-dimensional system for the two-sensor case, showing their threshold-type behaviour and the independence of the decoding algorithm, and (iv) numerical simulations are used to compare the proposed algorithms with a traditional single-transmission scheme.

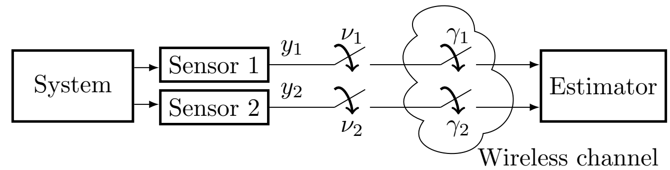

Fig. 1. System model.

2. PROBLEM FORMULATION

In this paper we consider a dynamical system whose state has to be estimated by a remote estimator, as depicted in Fig. 1 for the case of two transmitting sensors. In general, a set of N sensors communicate to the estimator, which plays the role of a fusion centre, through a wireless network. The central node is equipped with a receiver capable of multi-packet reception, thus allowing more than one sensor to transmit simultaneously. To avoid confusion, we denote by *transmission period* the time interval during which all the scheduled sensors transmit their measurements. We assume that there is a transmission period in any sampling period. Moreover, all the simultaneous transmissions are synchronized, starting at the beginning, and finishing at the end of the transmission period.

2.1 System model

Consider the discrete time state-space linear model

$$x(k+1) = Ax(k) + w(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state and $w(k) \in \mathbb{R}^n$ is the process noise modelled as independent and identically distributed (*i.i.d.*) Gaussian random variables $w(k) \sim \mathcal{N}(0, Q)$ with $Q \geq 0$. A set of N sensors is available. At each sampling instant, the i-th sensor measures the output

$$y_i(k) = C_i x(k) + v_i(k) \tag{2}$$

where $y_i(k) \in \mathbb{R}^{m_i}$ and $v_i(k)$ is the measurement noise modelled as i.i.d. Gaussian random variables $v_i(k) \sim \mathcal{N}(0, R_i)$ with $R_i > 0$ and independent of $\{w(k)\}$. During the k-th transmission period, a packet containing the sampled output $y_i(k)$ is communicated to the remote estimator according to the decision variable $\nu_i(k)$: if $\nu_i(k) = 1$, then $y_i(k)$ is transmitted, while it is not transmitted if $\nu_i(k) = 0$. When scheduled, a transmission may not be successfully completed due to interference of other transmissions and channel and receiver noise. We represent this process through the variable $\gamma_i(k)$, which is equal to 1 if the transmission of $y_i(k)$ is successfully completed, 0 otherwise. The information set available at the fusion centre at the time instant k is:

$$\mathcal{I}(k) = \bigcup_{i=1}^{N} \mathcal{I}_i(k), \quad \mathcal{I}_i(k) = \big\{\nu_i(0)\gamma_i(0)y_i(0),\, \nu_i(1)\gamma_i(1)y_i(1),\, \ldots,\, \nu_i(k-1)\gamma_i(k-1)y_i(k-1)\big\}$$

where, with a little misuse of notation, if $\nu_i(t)\gamma_i(t) = 0$ then $\nu_i(t)\gamma_i(t)y_i(t) = \emptyset$, i.e. $y_i(t)$ is missing. Define

$$\widehat{x}(k|k-1) := \mathbb{E}[x(k) \,|\, \mathcal{I}(k)]$$
$$P(k|k-1) := \mathbb{E}[(x(k) - \widehat{x}(k|k-1))(x(k) - \widehat{x}(k|k-1))' \,|\, \mathcal{I}(k)].$$

From Anderson and Moore (2012), $\widehat{x}(k|k-1)$ is the optimal estimator given $\mathcal{I}(k)$, and the matrix $P(k|k-1)$ denotes the corresponding estimation error covariance matrix. In order to easily manage intermittent partial observations we exploit the information form of the optimal estimator given by Hashemipour et al. (1988). Due to space limitations, we report only the update of the error covariance

$$P(k|k)=\left(P^{-1}(k|k-1)+\sum_{i=1}^{N}\gamma_i(k)\nu_i(k)C_i'R_i^{-1}C_i\right)^{-1} \quad (3)$$

$$P(k|k-1)=AP(k-1|k-1)A'+Q. \quad (4)$$

The decision variables $\nu_i(k)$, $i=1,\dots,N$ are chosen at the central node and are communicated back to the sensors within the time interval $(k-1,k)$ without error.

2.2 Channel model

Denote by P_i^{tx} the transmitted power of the i-th sensor, while g_i denotes the slow fading component of the channel power gain (usually dependent on path loss) from the i-th sensor to the remote estimator, and $h_i(k)$ is the fast fading component of the same channel during the k-th transmission period. We assume that P_i^{tx} and g_i are constant, while $h_i(k)$ is modelled as a temporally i.i.d. exponential random variable (this corresponds to Rayleigh fading, a common distribution for a wireless environment with large number of scatterers) with unity mean, i.e. $h_i(k)\sim Exp(1)$, with $h_i(k)\perp h_i(t)$ for $t\neq k$ and $h_i(k)\perp h_j(t)$ for $\forall k,t$ and $i\neq j$. It follows that the received power at the remote estimator from the i-th sensor $P_i^{\text{rc}}(k)$ during the k-th transmission period is

$$P_i^{\text{rc}}(k)=\begin{cases}P_i^{\text{tx}}g_ih_i(k) & \text{if } \nu_i(k)=1\\ 0 & \text{if } \nu_i(k)=0.\end{cases} \quad (5)$$

Given that $\nu_i(k)=1$, the received power is an exponential random variable with mean $\lambda_i^{-1}=g_iP_i^{\text{tx}}$, i.e. $P_i^{\text{rc}}(k)\sim Exp(\lambda_i)$. Due to the nature of the wireless medium, background channel and/or receiver noise is also present. We model it as an Additive White Gaussian Noise (AWGN) whose average power at the estimator is σ^2.

Without SIC, since transmissions are overlapped, the SINR corresponding to the packet containing $y_i(k)$ is

$$\text{SINR}_i(k)=\frac{\nu_i(k)P_i^{\text{rc}}(k)}{\sum_{j\neq i}\nu_j(k)P_j^{\text{rc}}(k)+\sigma^2}. \quad (6)$$

When SIC is employed, we assume without loss of generality that the sensors have been ordered in descending order of received power, i.e. $P_1^{\text{rc}}(k)\geq P_2^{\text{rc}}(k)\geq\ldots\geq P_N^{\text{rc}}(k)$. Since the stronger users are decoded before the weaker users, the SINR for the i-th sensor in this case is given by

$$\text{SINR}_i(k)=\frac{\nu_i(k)P_i^{\text{rc}}(k)}{\sum_{j>i}\nu_j(k)P_j^{\text{rc}}(k)+\sigma^2}. \quad (7)$$

A packet from the i-th sensor at the k-the time slot can be decoded without error if $\text{SINR}_i(k)>\alpha$, where $\alpha>0$ is a threshold depending on the modulation and coding schemes. In order to enable multi-packet reception without SIC we need to have $\alpha\in(0,\,1)$. It can be achieved e.g. by Code Division Multiple Access (CDMA). It follows that the packet arrival process can be expressed as

$$\gamma_i(k)=\begin{cases}1 & \text{if } \text{SINR}_i(k)>\alpha\\ 0 & \text{otherwise.}\end{cases} \quad (8)$$

Since the channel gains are independent across time slots, $\gamma_i(k)$ is also an i.i.d Bernoulli process. However $\gamma_i(k),\gamma_j(k)$ for $j\neq i$ may be dependent on each other due to interference within a given time slot.

3. ARRIVAL PROBABILITIES

In this section we provide the probabilities of the arrival process for the case with 2 sensors. When only 1 sensor is scheduled, we have:

$$q_{10}:=\boldsymbol{P}(\gamma_1(k)=1\,|\,\nu_1(k)=1,\,\nu_2(k)=0)=e^{-\alpha\lambda_1\sigma^2}$$
$$q_{01}:=\boldsymbol{P}(\gamma_2(k)=1\,|\,\nu_1(k)=0,\,\nu_2(k)=1)=e^{-\alpha\lambda_2\sigma^2}$$

When both the sensors are scheduled, denote:

$$p_{11}:=\boldsymbol{P}(\gamma_1(k)=1,\,\gamma_2(k)=1\,|\,\nu_1(k)=1,\,\nu_2(k)=1)$$
$$p_{10}:=\boldsymbol{P}(\gamma_1(k)=1,\,\gamma_2(k)=0\,|\,\nu_1(k)=1,\,\nu_2(k)=1)$$
$$p_{01}:=\boldsymbol{P}(\gamma_1(k)=0,\,\gamma_2(k)=1\,|\,\nu_1(k)=1,\,\nu_2(k)=1)$$
$$p_{00}:=\boldsymbol{P}(\gamma_1(k)=0,\,\gamma_2(k)=0\,|\,\nu_1(k)=1,\,\nu_2(k)=1)$$

It is easy to compute $p_{11}+p_{10}$ and $p_{11}+p_{01}$:

$$p_{11}+p_{10}=\frac{\lambda_2}{\lambda_2+\alpha\lambda_1}\,e^{-\alpha\lambda_1\sigma^2}=\frac{\lambda_2}{\lambda_2+\alpha\lambda_1}\,q_{10}$$
$$p_{11}+p_{01}=\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}\,e^{-\alpha\lambda_2\sigma^2}=\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}\,q_{01}.$$

Under the assumption that $\alpha\in(0,1)$, while different expressions exist, it is convenient to write

$$p_{11}=\left(\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}+\frac{\lambda_2}{\lambda_2+\alpha\lambda_1}-1\right)e^{-(\lambda_1+\lambda_2)\frac{\alpha}{1-\alpha}\sigma^2}$$
$$p_{00}=1-(p_{11}+p_{10})-(p_{11}+p_{01})+p_{11}$$
$$=1-\frac{\lambda_2}{\lambda_2+\alpha\lambda_1}\,e^{-\alpha\lambda_1\sigma^2}-\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}\,e^{-\alpha\lambda_2\sigma^2}+\left(\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}+\frac{\lambda_2}{\lambda_2+\alpha\lambda_1}-1\right)e^{-(\lambda_1+\lambda_2)\frac{\alpha}{1-\alpha}\sigma^2}$$

Finally p_{10} and p_{01} can be found by subtracting p_{11} from $p_{11}+p_{10}$ and $p_{11}+p_{01}$.

When SIC is employed, the corresponding probabilities (denoted by the superscript SIC) are given by

$$p_{11}^{SIC}=\left(\frac{\lambda_2}{\lambda_2+\alpha\lambda_1}e^{-\lambda_1\alpha^2\sigma^2}+\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}e^{-\lambda_2\alpha^2\sigma^2}\right)e^{-(\lambda_1+\lambda_2)\alpha\sigma^2}+\left(1-\frac{\lambda_2}{\lambda_2+\alpha\lambda_1}-\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}\right)e^{-(\lambda_1+\lambda_2)\frac{\alpha}{1-\alpha}\sigma^2}$$
$$p_{10}^{SIC}=\frac{\lambda_2}{\lambda_2+\alpha\lambda_1}e^{-\alpha\sigma^2\lambda_1}\left(1-e^{-\alpha\sigma^2(\lambda_2+\alpha\lambda_1)}\right)$$
$$p_{01}^{SIC}=\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}e^{-\alpha\sigma^2\lambda_2}\left(1-e^{-\alpha\sigma^2(\lambda_1+\alpha\lambda_2)}\right)$$
$$p_{00}^{SIC}=p_{00}.$$

Lemma 1. The following results hold:

(1) $p_{11}+p_{10}<q_{10}$ and $p_{11}+p_{01}<q_{01}$
(2) $p_{00}+p_{10}>1-q_{01}$ and $p_{00}+p_{01}>1-q_{10}$
(3) $p_{00}<\max\{1-q_{10},\,1-q_{01}\}$
(4) $p_{00}<\min\{1-q_{10},\,1-q_{01}\}$ if and only if

$$e^{-(\lambda_1+\lambda_2)\frac{\alpha}{\alpha-1}\sigma^2}\left(\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}-\frac{\alpha\lambda_1}{\lambda_2+\alpha\lambda_1}\right)<\frac{\lambda_1}{\lambda_1+\alpha\lambda_2}\,e^{-\alpha\lambda_2\sigma^2}-\frac{\alpha\lambda_1}{\lambda_2+\alpha\lambda_1}\,e^{-\alpha\lambda_1\sigma^2} \quad (9)$$

As an immediate consequence of previous Lemma, according to point (3), if channels are identical, i.e. $q_{10}=q_{01}=q$, then $p_{00}<1-q$. The same results hold when the arrival probabilities are replaced by their counterparts with SIC.

4. OPTIMAL SENSOR SCHEDULING

In this section, we are interested in finding the optimal sensor scheduling scheme over the finite horizon $[0, K-1]$, where the sum of the trace of expected prediction error covariance matrices $P(k|k-1)$ over the finite horizon is minimized, along with a penalty on the expected number of total sensor transmissions. In particular, the sensor scheduling problem can be formulated as

$$J\left(U_{[0,K-1]}, P_0\right) = \sum_{k=0}^{K-1} \mathbb{E}\left[\mathrm{Tr}\left(P(k|k-1)\right)\right] + \mu \sum_{k=0}^{K-1} \sum_{i=1}^{N} \nu_i(k)$$

where

$$U_{[0,K-1]} = \{\nu_i(k) \;\; i = 1 \cdots, N, \; k = 0, \cdots, K-1\}$$

and μ is a regularization parameter that can be tuned to set the desired trade-off between the estimate performances (i.e the error covariance) and the communication cost (i.e. the number of transmissions). Minimizing the metric for different values of μ corresponds to minimizing the error covariance under different bounds on the mean number of transmissions. The optimal schedule is then

$$U^*(P_0) = \arg\min_U J\left(U, P_0\right).$$

The problem is essentially a stochastic control problem, with the scheduling variables as the control sequence, and can be efficiently solved e.g. through Dynamic Programming. In the following we show some structural results for the two simplest cases: a scalar system and a 2-dimensional system with decoupled process noise, 2 sensors and 1-step horizon ($K = 1$). For these particular cases, we can denote $U = (\nu_1, \nu_2)$. Proofs (reported in Pezzutto et al. (2020)) of the following results are based on simple calculus tools that can not be exploited for the general case, that is let as a future work. Note that the following propositions hold true both with and without SIC once the probabilities p_{11}, p_{10}, and p_{01} are substituted by their counterparts with SIC.

4.1 Scalar system

We say that the sensor are identical if $R_1 = R_2$ and that the channels are identical if $P_1^{\mathrm{tx}} g_1 = P_2^{\mathrm{tx}} g_2$, thus $q_{10} = q_{01}$ and $p_{10} = p_{01}$. We consider $C_1 = C_2 = 1$ w.l.o.g..

Lemma 2. If channels and sensors are identical, then $J((1,0), P) = J((0,1), P)\ \forall P \geq 0$. If channels are identical and $R_1 < R_2$, then $J((1,0), P) < J((0,1), P)\ \forall P \geq 0$. If channels are not identical and $q_{10} > q_{01}$ then

$$J((1,0), P) < J((0,1), P)\ \forall P > 0 \iff R_1 q_{01} < R_2 q_{10}$$

Otherwise if $R_1 q_{01} > R_2 q_{10}$ then

$$J((1,0),\, P) < J((0,1),\, P) \iff P > \hat{P} = \frac{q_{01} R_1 - q_{10} R_2}{q_{10} - q_{01}}$$

Roughly speaking, the previous Lemma states that the sensor with the best channel (highest q) is always preferred for high error covariances, while the sensor whose measurements have the best quality (lowest R) might be preferred for low error covariances. However, there are configurations of the parameters for which the sensor with the worst channel is never preferred to the other. In that case, without multi-packet reception, this sensor is never scheduled. The following Proposition shows that even in this case such a sensor can be exploited when the receiver has multi-packet reception capabilities.

Proposition 3. (Scalar system, 1-step horizon). For a scalar system, if $\mu > 0$, the following hold

1. $\exists \underline{P} > 0$ s.t. if $0 \leq P < \underline{P}$ then $U^*(P) = (0,0)$, while if $P > \underline{P}$ then $U^*(P) \neq (0,0)$
2. $\exists \overline{P} > 0$ s.t. if $P > \overline{P}$ then $U^*(P) = (\nu_1^*, \nu_2^*) = \arg\min_{\nu_1,\nu_2} \boldsymbol{P}(\gamma_1 = 0, \gamma_2 = 0 \,|\, \nu_1, \nu_2)$, namely the scheduling that gives the lowest probability of no new delivered packets. In particular, when condition (9) holds, multiple simultaneous transmissions are optimal, i.e. $U^*(P) = (1,1)$ for $P > \overline{P}$
3. There are no additional thresholds in the interval $(\underline{P}, \overline{P})$ in the following cases:
   - the sensors and channels are identical
   - the sensors are identical and $$p_{00} < \min\{1 - q_{10},\, 1 - q_{01}\}$$
   - if $R_1 > R_2$ but scheduling $(1,0)$ is always better than scheduling $(0,1)$ and $p_{00} < 1 - q_{10}$, i.e. $$R_1 > R_2 \quad q_{10} > q_{01} \quad R_1 q_{01} > R_2 q_{10} \quad p_{00} < 1 - q_{10}$$

The previous Proposition shows that the optimal scheduling has a threshold-type behaviour. In particular, $\underline{P}$ defines the threshold before which no sensor transmit, while $\overline{P}$ is the largest threshold, since for $P > \overline{P}$ the optimal schedule is fixed. In general, it is possible that $\underline{P} = \overline{P}$. Note that, with $R_1 < R_2$ and identical channels, both sensors transmit for $P > \overline{P}$ since (9) holds, despite the fact that the first sensor would be always preferred to second sensor when multiple transmissions are not allowed. It is possible to find also cases where $q_{10} > q_{01}$ and $R_1 q_{01} < R_2 q_{10}$ but condition (9) holds. Two interesting behaviours of the optimal cost and of the optimal scheduling are reported in Fig. 2. In particular, the bottom panel represents the case where sensors and channels are very different and external noise is high. Without SIC, we can see that, in the top panel, simultaneous transmissions are optimal for $P > \overline{P}$, while, in the bottom panel, it is optimal for $P \in (\overline{\overline{P}}, \overline{P})$. In both the cases we can see the improvement given by SIC.

4.2 2-dimensional decoupled system

Consider a 2-dimensional system where matrices A and Q are diagonal. We refer to this system as decoupled. For easy of notation, denote

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \qquad C_1 = [1 \;\; 0] \qquad C_2 = [0 \;\; 1]$$

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \qquad P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

Lemma 4. At the point $P = (P_1, P_2)$,

$$J((1,0),\, P) < J((0,1),\, P) \iff \frac{q_{10} A_1^2 P_1^2}{P_1 + R_1} < \frac{q_{01} A_2^2 P_2^2}{P_2 + R_2}.$$

The curve Γ : $J((1,0),\, P) = J((0,1),\, P)$ divides the positive quadrant of the plane $(P_1,\, P_2)$ in two regions. It passes through $(0,0)$, asymptotically tends to the line r: $q_{10} A_1^2 P_2 = q_{01} A_2^2 P_2$ while always lying underneath it if $q_{10} A_1^2 R_1 > q_{01} A_2^2 R_2$, or always above it otherwise.

The previous Lemma shows that, for a decoupled system, for each sensor there always exists a set of error covariances for which it is preferred to the other, independently of the quality of the channels, the quality of the sensors, and the magnitude of the eigenvalues of the system.

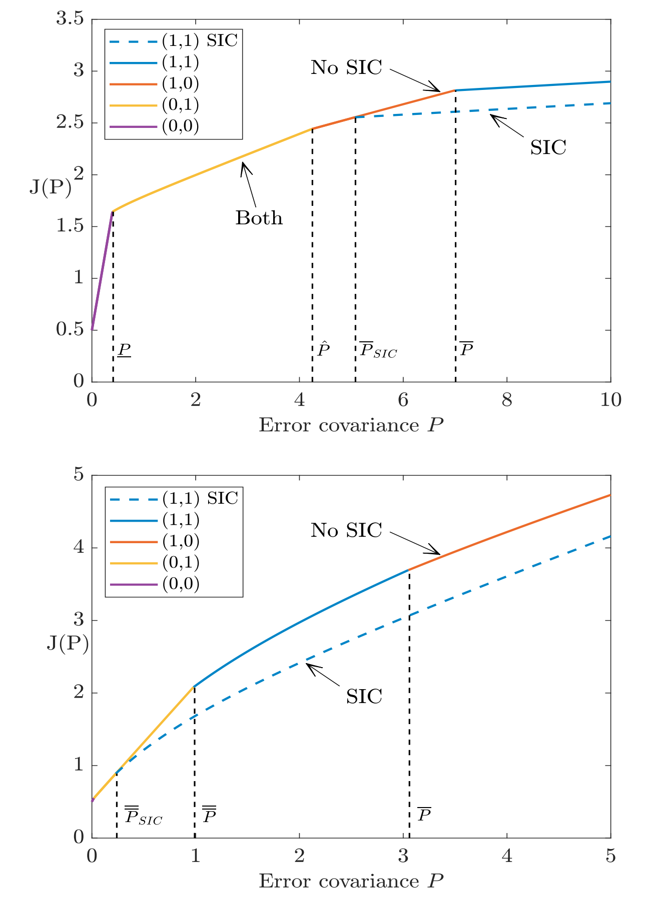

Fig. 2. Examples of optimal scheduling. $A=1.7,\ Q=0.5$. Top: $g_1=1.5,\ g_2=1,\ \sigma=0.1,\ R_1=0.2,\ R_2=0.1,\ \mu=1$ Bottom: $g_1=5,\ g_2=\sigma=1,\ R_1=1,\ R_2=0.01,\ \mu=0.01$

Proposition 5. (2D decoupled system, 1-step horizon). For a 2D decoupled system, if $\mu>0$, the following holds:

(1) $\exists \underline{P}_1>0,\ \underline{P}_2>0$ that define the region
$$R_{00}=\{(P_1,P_2)\,:\,0\le P_1<\underline{P}_1 \text{ and } 0\le P_2<\underline{P}_2\}$$
s.t. if $P\in R_{00}$, then $U^*(P)=(0,0)$, while if $P\notin R_{00}$ then $U^*(P)\neq(0,0)$. The point $(\underline{P}_1,\underline{P}_2)$ belongs to the curve Γ. Moreover, the region R_{00} is strictly included in a set where $U^*(P)\neq(1,1)$.

(2) $\forall P_1\ \exists\overline{P}_2(P_1)>0$ s.t. $U^*(P_1,P_2)=(0,1)$ if $P_2>\overline{P}_2(P_1)$
$\forall P_2\ \exists\overline{P}_1(P_2)>0$ s.t. $U^*(P_1,P_2)=(1,0)$ if $P_1>\overline{P}_1(P_2)$

(3) $U^*(P)=(1,1)$ for $P=(P_1,P_2)$ belonging to the non-empty region R_{11}:
$$\begin{cases}(q_{10}-p_{11}-p_{10})\dfrac{A_1^2P_1^2}{P_1+R_1}+\nu<(p_{11}+p_{01})\dfrac{A_2^2P_2^2}{P_2+R_2}\\(q_{01}-p_{11}-p_{01})\dfrac{A_2^2P_2^2}{P_2+R_2}+\nu<(p_{11}+p_{10})\dfrac{A_1^2P_1^2}{P_1+R_1}\end{cases}$$
The curve Γ intersects the bound of the region R_{11} at a unique point $(\hat{P}_1,\hat{P}_2)$, so that any point (P_1,P_2) s.t. $P_1>\hat{P}_1,\ P_2>\hat{P}_2$ belonging to Γ belongs to R_{11}.

According to point (1) of the previous Proposition, as illustrated in Fig. 3 there exists a rectangular region R_{00} with the origin as bottom-left vertex in which no transmissions is the optimal scheduling. According to (2), we have that, keeping a component of the error covariance fixed and making the other bigger, the optimal scheduling eventually schedules only transmissions to observe the most uncertain state. Point (3) outlines that scheduling $(1,1)$ is optimal in a region R_{11} (partially) containing the curve Γ where scheduling $(1,0)$ and scheduling $(0,1)$ achieve the same cost. As can be seen in Fig. 3, while R_{00} is the same with or without SIC, R_{11} is definitely larger with SIC: multiple simultaneous transmissions are optimal for a wide range of covariance matrices.

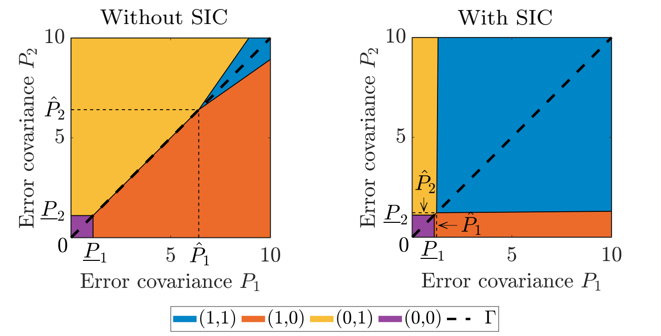

Fig. 3. Example of optimal schedule for a 2D system. Left: without SIC. Right: with SIC. $A_1=A_2=1.7,\ Q_1=Q_2=0.5,\ g_1=g_2=1,\ \sigma=0.1,\ R_1=R_2=0.1,\ \mu=3$.

5. NUMERICAL SIMULATIONS

In this section, we fix the system parameters as follows:
$$A=1.7\quad Q=0.5\quad C_1=C_2=1\quad R_1=R_2=0.1$$
$$P_1^{\text{tx}}=P_2^{\text{tx}}=1\quad g_1=g_2=1\quad \sigma^2=0.1$$
Where not explicitly indicated, $\alpha=0.7$ and $\mu=0.1$. The resulting arrival probabilities are:
$$q_{10}=q_{01}=q=0.932\quad p_{11}=0.110\quad p_{10}=p_{01}=0.438$$
$$p_{11}^{\text{SIC}}=0.862\quad p_{10}^{\text{SIC}}=p_{01}^{\text{SIC}}=0.062\quad p_{00}^{\text{SIC}}=p_{00}=0.014$$
Since channels are identical, according to Proposition 3, $U^*(P)=(0,0)$ if $P<\underline{P}$, $U^*(P)=(1,0)$ if $\underline{P}\le P<\overline{P}$, and $U^*(P)=(1,1)$ if $P\ge\overline{P}$. This section aims to explore the improvement that can be achieved employing a receiver with multi-packet reception capabilities for a scalar system. We compare the optimal (1-step horizon) policy devised in Sec. 4 with the optimal (1-step horizon) policy for the case where at most one single transmission is possible studied by Leong et al. (2016). First we consider the time evolution of the error covariance for the different policies over the same realisations of the processes P_1^{rc} and P_2^{rc}. Results are shown in Fig. 4. We can see that multiple transmissions achieve a lower error covariance when both packets are successfully delivered. Peaks (corresponding to no new packets) are less frequent since $p_{00}<1-q$ with identical channels, and the mean error covariance is further improved using SIC. Fig. 5 reports the mean error covariance for the different policies with varying α, which is a communication parameter that can be properly tuned by changing modulation and coding rate. A low α allows to successfully receive a packet also with a low SINR but it requires a low-order modulation and a high coding rate at the price of a low data-rate. As we can see, simultaneous multiple transmissions always achieve lower mean error covariances. It is worth mentioning that, for small α, the improvement given by SIC is minor, due to the fact that mutual interference affects the packet reception in a negligible way. On the other hand, when

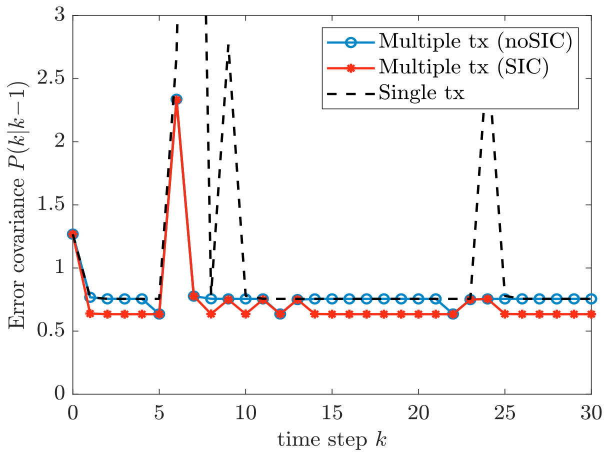

Fig. 4. Evolution of the error covariance

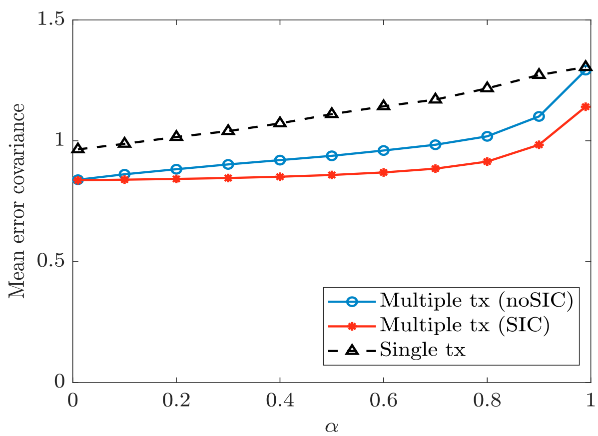

Fig. 5. Error covariance with varying α

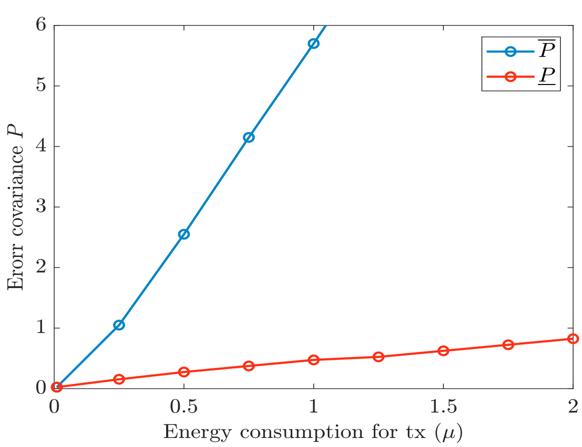

Fig. 6. Thresholds with varying μ

α is close to one, without SIC the improvement given by the multiple transmissions is minor. Interestingly, for the given system, the plot corresponding to SIC is almost flat for $\alpha \in (0, 0.7)$. In Fig. 6 we show for the case without SIC (the case with SIC is analogous) how $\underline{P}$ and $\overline{P}$ behave with varying μ. It is related to the actual energy constraint given by the battery life of the sensors or it can be used to set a trade-off between communication cost and estimation performance. We can see that both $\underline{P}$ and $\overline{P}$ are increasing function of μ: as expected, when the communication cost increases, transmissions are penalized more and only when the error covariance is high, simultaneous transmissions are scheduled.

6. CONCLUSIONS

In this paper we consider a sensor scheduling problem for remote estimation when the receiver is able to decode multiple simultaneous incoming packets. We consider a suitable model for multi-packet reception that takes into account interference and two different decoding algorithms, i.e. with and without SIC. We provide the optimization problem for the general case and we study the structural properties of the optimal schedule for a scalar system and for a 2-dimensional decoupled system. Numerical simulations show that multiple simultaneous transmissions can be beneficial, especially with SIC.

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