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Designing rotation programs: Limits and possibilities ☆

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ABSTRACT

Rotation programs are widely used in our society. For instance, a job rotation program is an HR strategy where employees rotate between two or more jobs in the same business. We study rotation programs within the standard implementation framework under complete information. We introduce the notion of implementation in ordered cycles, where each ordered cycle is a rotation program for an assignment problem. When the designer would like to attain a Pareto efficient goal, we provide sufficient conditions for its implementation in ordered cycles. However, when, for instance, every employee transitions through all different lateral jobs before rotating back to his original one, the conditions fully characterize the class of Pareto efficient goals that are implementable in ordered cycles.

Contents

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1. Introduction

An economics department must distribute the administrative load among its professors. However, most professors want to avoid these tasks due to the workload. Often, departments agree to implement a rotation program to resolve this impasse. Each professor will perform new administrative duties before returning to his original tasks.

Society uses rotation programs widely. The business practice of job rotation is a prominent example, which consists of periodically rotating the jobs assigned to the employees throughout their employment. This practice has been used in many industries for a wide array of employees, from factory line workers to executives (Osterman, 1994, 2000; Gittleman et al., 1998) and for different reasons.[1] Furthermore, rotation programs have been practiced in managing common-pool resources as an alternative to quotas and lotteries. In many areas of the world, rotating groups are formed for farming, grazing, gaining access to water, and allocating fishing spots (Ostrom, 1990; Berkes, 1992; Sneath, 1998). Recently, Ely et al. (2021) show that rotation schemes can be used to prevent the spread of infections. In this view, a rotation scheme is a mechanism to shape social interactions to minimize the risk of contagion. Further, as illustrated by the problem of allocating administrative duties to professors, rotation programs can help in achieving envy-free allocations in assignment problems, which requires that no agent values the object assigned to another agent more than the object assigned to herself.[2] Indeed, human beings tend to solve these conflicts by using lotteries or rotation schemes. It is well known that by using lotteries, it is possible to achieve ex-ante envy-freeness (Hofstee, 1990; Bogomolnaia and Moulin, 2001; Budish et al., 2013). A well-known method is called *Random Serial Dictatorship*: agents are ordered at random according to a uniform distribution, and successively they are let to choose an object in that order. Although no agent envies any other in expectation, as experimental evidence has pointed out (Eliaz and Rubinstein, 2014; Andreoni et al., 2020), this approach does not avoid ex-post envy, since it can happen that an agent receives her most preferred object while others receive their worst.[3]

The RSD algorithm does not always outperform rotations. This can be justified from an ex-ante perspective, as the following example clarifies. The agents are 1, 2, and 3. The objects x, y, and z need to be allocated to them. The agents do not know their preferences. There are six different states of the world, namely UU', UU'', $U'U$, $U'U''$, $U''U$ and $U''U'$. (UU' means that agent 1's utility function is U, 2's utility is U' and 3's utility is U''.) Each state has an equal chance of being the true state. The three types of utility functions are given in the table below. We have that $U(x) = 3$, $U(y) = 4$, $U(z) = 1$, and so forth.

	U	U'	U''
x	3	14	14
y	4	10	10
z	1	9	9

If they use the RSD algorithm, they agree that the priority ordering is determined by a draw from a uniform probability distribution. The six possible orderings are 12, 13, 21, 23, 31, and 32. (12 means that 1 is first and 2 is second.) The probability of each priority ordering is $1/6$. Agent $i = 1, 2, 3$'s expected utility when his utility is U (in states UU' and UU'') is[4]

[1] From one side, employees who rotate accumulate more human capital because they gain a broader range of experiences. On the other side, the employer learns more about its employees if it can observe how they perform at different jobs (Arya and Mittendorf, 2004).

[2] A real life example of achieving ex-post envy-freeness by a rotation program has been due by the municipality of Beijing who rotates teachers among schools to make education more equitable and avoid the concentration of the best staff in only a small number of institutions. See https://www.chinadaily.com.cn/a/202108/26/WS6126cdefa310efa1bd66b254.html.

[3] Recently Aziz et al. (2023) develop a random mechanism to achieve ex-ante and ex-post envy-freeness up to one good. In this case, requiring Pareto efficiency is proven to be impossible.

[4] Note that if the priority ordering is 12 or 13, agent 1 will go first and will choose y. With 21 or 23, agent 2 goes first and will take x. With 21, agent 1 gets the next choice and will choose y. With 23, agent 2 chooses x, 3 choose y, so 1 will take z. With 31 or 32, agent 3 goes first and will take x. With 31, agent 1 gets the next choice and will choose y. With 32, agent 1 will be left with z.

$$\frac{4}{6} \times U(y) + \frac{2}{6} \times U(z) = \frac{4}{6} \times 4 + \frac{2}{6} \times 1 = 3.$$

However, agent i's expected utility when his utility is $U' = U''$ (that is, in states $U'U$, $U'U''$, $U''U$ and $U''U'$) is

$$\frac{3}{6} \times U'(x) + \frac{1}{6} \times U'(y) + \frac{2}{6} \times U'(z) = \frac{3}{6} \times 14 + \frac{1}{6} \times 10 + \frac{2}{6} \times 9 = \frac{35}{3}.$$

Since the probability that agent $i = 1,2,3$'s utility is U is $1/3$ and the probability that it is U' or U'' is $2/3$, agent i's ex-ante utility is

$$\frac{1}{3} \times 3 + \frac{2}{3} \times \frac{35}{3} = \frac{79}{9} = 8.78.$$

If agents opted for the rotation, they would agree that the agent who has utility U would always take y, while the other two agents would rotate between x and z. Thus, the agent who has utility U would always obtain an average utility of 12, while the other two agents would obtain an average utility of

$$\frac{U'(x) + U'(z)}{2} = \frac{U''(x) + U''(z)}{2} = \frac{14+9}{2}.$$

Since the probability that agent $i = 1,2,3$ has utility U is $\frac{1}{3}$ and since, moreover, the probability that i's utility is U' or U'' is $2/3$, agent i's ex-ante utility is

$$\frac{1}{3} \times U(y) + \frac{2}{3}\left(\frac{U'(x) + U'(z)}{2}\right) = \frac{1}{3} \times 4 + \frac{2}{3}\left(\frac{14+9}{2}\right) = 9.$$

Since, for each agent, the ex-ante utility from participating in the RSD algorithm is less than the ex-ante utility he would obtain under rotation, agents will agree that rotating is the best available option from an ex-ante perspective.

In this paper, we propose an implementation approach to studying rotation programs where agents rotate among (Pareto) efficient allocations–not necessarily among *all* efficient allocations. Therefore, our challenge lies in designing a mechanism in which agents' behavior always coincides with the recommendation given by a social choice rule (SCR).

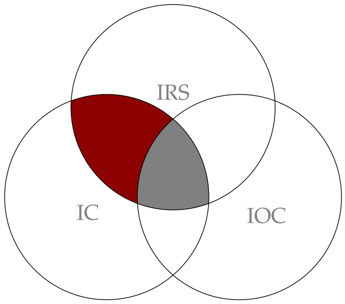
The first difficulty in adopting this approach concerns the choice of the solution concept. Most of the game-theoretical solutions used in literature, such as the core, the (strong) Nash equilibrium, and the stable set (von Neumann and Morgenstern, 1944), satisfy a property called *internal stability*. Roughly speaking, a set of outcomes is internally stable if it is free of inner contradictions, i.e., for every outcome in the set, no agent or group can directly move to another outcome of the set and be better off. However, this property is incompatible with our objective to study how to rotate positions among agents. Thus, a theory of implementation in rotation programs cannot rely on solutions that satisfy internal stability. In contrast, internal stability is relaxed in solution concepts that are modifications, extensions, or generalizations of the stable set. One of the most prominent is the "absorbing set." As Inarra et al. (2005) point out, the notion of absorbing sets appears in the literature under different names and settings. Kalai et al. (1976) study the "admissible set" in various bargaining situations, and Shenoy (1979) defines the "elementary dynamic solution" for coalitional games. More recently, Jackson and Watts (2002) study the "closed cycle" for network formation and Inarra et al. (2013) study the absorbing set for roommate problems. Finally, the myopic stable set (MSS), defined by Demuynck et al. (2019a), includes all previous notions of absorbing sets. The MSS is the smallest set of states such that the following properties are satisfied: 1) There are no profitable deviations from a state inside the set to a state outside the set, and 2) for each state outside the set, there is a sequence of agents' deviations converging to the set. Thus, the MSS is a valid prediction of agents' play, though it violates internal stability because it allows deviations within the set. Furthermore, the prediction offered by the MSS is robust in the following terms: Though agents may reach an agreement on a state outside the set, a sequence of myopic improvements will bring them back to the MSS. Since, for finite environments, the MSS can be equivalently defined as the union of all cycles, the MSS is a good solution concept for studying rotation programs. Indeed, by adopting the MSS as a solution concept, we study implementation in cycles.

From a methodological point of view, we exploit a novel implementation technique, called implementation via *rights structures* (Section 2), a notion introduced by Sertel (2001) and further developed by Koray and Yildiz (2018). A rights structure formalizes power distribution within society. Thus, in contrast to the classical mechanism design exercise, our design exercise consists of allocating rights to agents such that their behavior always coincides with the recommendation given by an SCR. We follow this approach for three reasons. First, a persistent critique in economic design is that canonical mechanisms for implementing socially desirable outcomes have unnatural features (Jackson, 1992). Typically, canonical mechanisms are complex and challenging to explain in natural terms since they rely on tail-chasing constructions. By contrast, agents can easily understand the meaning of a rights structure. Second, though rights structures do not model time, they effectively describe all the paths generated by agents' interactions. Finally, rights structures suit very well the environment of the MSS. Indeed, a rights structure together with a preference profile returns a social environment (Chwe, 1994), which is the natural setting of the MSS (Demuynck et al., 2019a).

However, implementation in cycles does not always guarantee that people rotate among all desired outcomes. Indeed, it does not exclude the possibility that a rotation gets stuck in a cycle, which rules out some desired outcomes from the process. To solve this issue, Section 4 introduces the notion of implementation in ordered cycles, which is an implementation in cycles where the states of every cycle (of the MSS) are arranged in a circle, where there is a one-to-one correspondence between states of the cycle and outcomes selected by SCR, and where agents transition through all different outcomes selected by the SCR.

Synopsis

The paper builds upon three blocks: implementation via rights structures (IRS), implementation in cycles via the myopic stable set solution concept (IC), and implementation in ordered cycles (IOC). The paper's contribution lies in investigating the implications which stem from either $[IRS \cap IC]$ or $[IRS \cap IC \cap IOC]$. The Venn diagram above depicts our contributions.



Section 2 provides the model. Section 3 studies implementation in cycles via rights structures. We show that a condition, which we refer to as *indirect monotonicity*, is sufficient for implementing efficient SCRs in cycles via a finite rights structure.[5] It is worth mentioning that *indirect monotonicity* is weaker than Maskin monotonicity (Maskin, 1977), and that the class of efficient SCRs satisfying *indirect monotonicity* are also implementable in generalized stable sets by a finite rights structure (van Deemen, 1991; Page and Wooders, 2009). Moreover, for marriage problems (Knuth, 1976) and a class of exchange economies with property rights (Balbuzanov and Kotowski, 2019), we show that the set of stable outcomes is implementable in cycles. It is worth stressing that the devised implementing rights structure has well-defined convergence properties (Appendix A presents these properties). In Section 4, we study implementation in ordered cycles via rights structures as a particular case of implementation in cycles. We identify a necessary condition, called *rotation monotonicity*, for implementation in ordered cycles of efficient SCRs. When a multi-valued SCR describes the designer's goal, *rotation monotonicity* fully characterizes the class of implementable SCRs.[6] For assignment problem, since an ordered cycle is a rotation program, we use the notion of implementation in rotation programs.[7] Finally, Section 5 studies job rotation problems, which are a particular kind of assignment problems where the number of jobs to be allocated coincides with the number of agents. For job rotation problems in which agents share the same best/worst outcome, and job rotation problems in which the designer knows that two agents have the same top-outcome, efficient SCRs are implementable in rotation programs.

The main takeaway points can be summarized as follows. First, when the set of allocations is fixed, the implementability of efficient goals through rotation programs is tricky. However, as it happens in the context of auction design (Milgrom, 2004), the design of the set of allocations is crucial for successfully implementing efficient goals. Indeed, by cleverly designing the set of allocations, many significant assignment problems become implementable in rotation programs. Appendix B includes proofs not in the main body.

Clarifying examples

Let us give two assignment examples clarifying our contributions. In both examples, there are three objects and three agents, and each agent cares only about the object she obtains. Example 1 explains *rotation monotonicity* and how the *Random Serial Dictatorship* allocation rule (RSD) can be problematic in our setup. Example 2 explains *indirect monotonicity* and shows how implementation in cycles does not guarantee implementation in rotation programs.

Example 1. Consider an assignment problem where objects $\{a, b, c\}$ must be allocated among agents $\{1, 2, 3\}$. Agents' preferences are specified in Fig. 1. Suppose that designer's goal is to select the allocation (a, c, b) at R and to allow agents to rotate between (b, c, a) and (b, a, c) at R'. That is, he aims to implement a social choice rule F in rotation programs where $F(R) = \{(a, c, b)\}$ and $F(R') = \{(b, c, a), (b, a, c)\}$.

Let us make some observations. First, F always selects Pareto efficient allocations. Second, agent 1 obtains her second-best object while agents 2 and 3 obtain their best object at R, whereas, at R', agent 1 obtains her best object b while agents 2 and 3 rotate between their best object c and their worst object a. Finally, it is worth noting that agents may have incentives to misreport their preferences. Agent 1 has the incentive to claim that the true state is always R', while agents 2 and 3 have the incentive to claim that the true state is always R.[8] That is, the designer cannot trust agents. Nevertheless, F can be implemented in rotation programs

[5] A finite rights structure is a rights structure where the set of states is finite.

[6] See, for instance, Mukherjee et al. (2019).

[7] We thank Hervé Moulin for this terminology.

[8] This can happen, for example, if $u(a) = 0$, $u(b) = 2$, and $u(c) = 3$. Thus the utility from rotating between a and c is 1.5 on average at R', less than the utility from b.

R			R'		
1	2	3	1	2	3
b	c	b	b,c	c	c
a	b	c	a	b	b
c	a	a		a	a

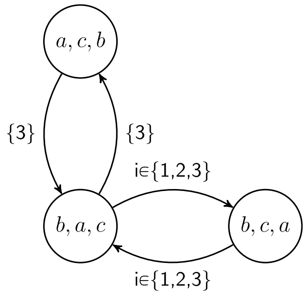


Fig. 1. Rights structure that implements F in rotation programs.

by a finite rights structure displayed in Fig. 1.[9] This rights structure has three states, which are the allocations (a,c,b), (b,a,c) and (b,c,a). When agent i is effective in moving from state x to state y, Fig. 1 denotes it by $x \longrightarrow^{\{i\}} y$. Thus, according to the above rights structure, each agent is effective in moving from (b,a,c) to (b,c,a) and from (b,c,a) to (b,a,c). Only agent 3 is effective in moving from (a,c,b) to (b,a,c) and back to (a,c,b).

In Fig. 1, the rotation program coincides with the MSS at every preference profile.[10] Recall that the MSS is the smallest set of states such that the following properties are satisfied: 1) There are no profitable deviations from a state inside the set to a state outside the set, and 2) for each state outside the set, there is a sequence of agents' myopic deviations converging to the set.

The MSS at R of the above rights structure is the set $\{(a,c,b)\}$, which one can view as a 'degenerate cycle.' Indeed, though $(a,c,b) \longrightarrow^{\{3\}} (b,a,c)$, agent 3 has no incentive to move to (b,a,c) because bP_3c. Moreover, for each other state, there is a sequence of agents' deviations converging to (a,b,c). For instance, we can move from (b,c,a) to (a,c,b) via the following myopic improvement path at R: $(b,c,a) \longrightarrow^{\{3\}} (b,a,c)$ and cP_3a, $(b,a,c) \longrightarrow^{\{3\}} (a,c,b)$ and bP_3c. Using the described myopic improvement path, one can check that a myopic improvement path exists from any state to (a,c,b). Finally, one can also check that the set $\{(a,c,b)\}$ is the unique set having these two properties. By using the above two properties, the reader can check that the MSS at R' consists of a cycle between the states (b,c,a) and (b,a,c).

Rotation monotonicity

To implement F in rotation programs, F must satisfy a Maskin monotonicity-like condition, called *rotation monotonicity*.[11] Maskin monotonicity requires that when an allocation x is desirable at R but not at R' (i.e., $x \in F(R)$ but $x \notin F(R')$), there must exist at least one agent who could be offered a reward z that would give her a strict incentive to move away from x at R', where the reward z would not tempt her if the actual state were R.

First, *rotation monotonicity* requires that desired allocations at R can be arranged in an m-element cycle; i.e., $x_1 \longrightarrow x_2 \longrightarrow \ldots x_{m-1} \longrightarrow x_m \longrightarrow x_1$. Second, when the desired allocations differ at R and R', there exist an x_k at R and an agent who could be offered a reward z that would give her strict incentives to move away from x_k at R', where the reward z would not tempt her if the actual state were R. Finally, for every other desired allocation x_h at R, at least one agent with strict incentives at R' to move from x_h to x_{h+1} exists.

Let us apply these requirements to our example. Let us move from R' to R. Thus, the two-element cycle at R' is $(b,c,a) \longrightarrow (b,a,c) \longrightarrow (b,c,a)$. Since the desired allocations differ at R' and R, we can offer to agent 3 the allocation (a,c,b) as a reward because he has a strict incentive to move from (b,a,c) to (a,c,b) at R but not at R'; that is, bP_3c but cR'_3b. Finally, agent 3 has also a strict incentive to move from (b,c,a) to (b,a,c) at R because cP_3a.

Example 2. Consider an assignment problem where objects $\{a,b,c\}$ must be allocated among agents $\{1,2,3\}$. Agents' preferences are specified in Fig. 2.

Suppose that designer's goal is to select the allocation (a,c,b) at R, to allow agents to rotate between (b,c,a) and (b,a,c) at R', and to allow agents to rotate between (a,c,b), (b,c,a), (b,a,c) at R'', that is $F(R) = \{(a,c,b)\}$, $F(R') = \{(b,c,a),(b,a,c)\}$ and $F(R'') = \{(a,c,b),(b,c,a),(b,a,c)\}$.

F is not implementable in rotation programs since *rotation monotonicity* is violated when preferences move from R' to R''. The reason is that no reward allocation z can be found. However, F is implementable in cycles because it satisfies *indirect monotonicity*. The implementing rights structure is depicted in Fig. 2. Given the discussion presented in Example 1 about the MSS, it can be checked that the MSS at R is the degenerate cycle (a,c,b), at R' consists of a cycle between the states (b,c,a) and (b,a,c), and at R'' consists of two cycles, one between (b,c,a) and (b,a,c) and a degenerate one (a,c,b).

[9] A rights structure is finite when the set of states is finite.

[10] This is not always the case. We show it in Example 2.

[11] In this example, rotation monotonicity is necessary and sufficient for implementation.

R			R'			R''		
1	2	3	1	2	3	1	2	3
b	c	b	b	c	c	b	c	c
a	b	c	c	b	b	a	a	b
c	a	a	a	a	a	c	b	a

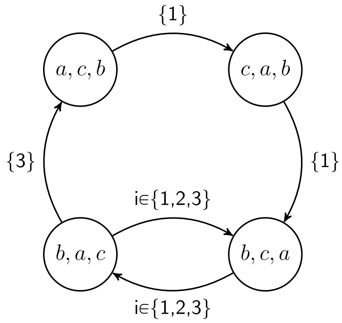


Fig. 2. Preferences and the implementing right structure.

Indirect monotonicity

Indirect monotonicity has a bite when Maskin monotonicity is violated. Thus, suppose $x \in F(R) \setminus F(R')$ and that x has not strictly fallen in preference for anyone when preferences change from R to R'. Then, the condition requires the existence of an agent who could be offered a reward z that would give her a strict incentive to move away from an allocation $\hat{x} \in F(R)$ at R', where z would not tempt her if the actual state were R. Moreover, it requires that x and $\hat{x}$ are connected via a sequence $(x_1, ..., x_K)$ of desired outcomes at R such that $x = x_1$, $x_K = \hat{x}$ and such that for all $k = 1, ..., K - 1$, an agent with strict incentives to move from x_{k+1} to x_k at R' exists.

Let us apply the requirements of *indirect monotonicity* to our example when the preference profile changes from R' to R.[12] Let us consider the allocation $x = (b, c, a) \in F(R') \setminus F(R)$. Note that $x = (b, c, a)$ does not strictly fall in preference for anyone when preferences switch from R' to R. Thus, Maskin monotonicity is violated. We can offer $z = (a, c, b)$ to agent 3 because it would give her a strict incentive to move away from $\hat{x} = (b, a, c) \in F(R')$ at R, where it would not tempt her if the actual state were R'.[13] Finally, $x = (b, c, a)$ and $\hat{x} = (b, a, c)$ are directly connected because agent 3 has a strict incentive to move from $x = (b, c, a)$ to $\hat{x} = (b, a, c)$ at R because $c P_3 a$.

Related literature

To the best of our knowledge, we are the first to study the economic design of rotation programs in an implementation framework that allows agents to rotate among efficient allocations. The previous contributions that come closest to what we are doing are Yu and Zhang (2020a) and Yu and Zhang (2020b). Whereas they study the properties of one particular mechanism for task rotation, we aim to characterize the class of implementable rotation schemes.

Our contribution is also in line with Arya and Mittendorf (2004), who study job rotations within a principal-agent framework. In particular, they identify conditions under which job rotation and specializations are both optimal. However, in contrast to us, their job rotation scheme does not guarantee the rotation of employees among lateral jobs.

Finally, our paper contributes to the literature on implementation via rights structure (Koray and Yildiz, 2018, 2019; Korpela et al., 2019, 2020) and it is broadly related to the literature on assignment problems (Shapley and Shubik, 1971; Hylland and Zeckhauser, 1979; Roth and Sotomayor, 1990; Abdulkadiroğlu and Sönmez, 1998).

2. The setup

We consider a finite (nonempty) set N of *agents*, whose cardinality is denoted by n, and a finite (nonempty) set of *alternatives*, denoted by Z. We endow Z with a metric $\hat{d}$. The power set of N is denoted by $\mathcal{N}$ and $\mathcal{N}_0 \equiv \mathcal{N} - \{\emptyset\}$ is the set of all nonempty subsets of N. Each element $K \in \mathcal{N}_0$ is called a *coalition*. A *preference ordering* R_i is a complete and transitive binary relation over Z. Each agent i $(\in N)$ has a preference ordering R_i over Z. The *asymmetric* part P_i of R_i is defined by $x P_i y$ if and only if $x R_i y$ and not $y R_i x$, whereas the *symmetric* part I_i of R_i is defined by $x I_i y$ if and only if $x R_i y$ and $y R_i x$. A *preference profile* is an n-tuple of preference orderings $R \equiv (R_i)_{i \in N}$. For all R, all $x, y \in Z$ and all $K \in \mathcal{N}_0$, we write $x R_K y$ for $x R_i y$ for all $i \in K$, write $x P_K y$ for $x P_i y$ for all $i \in K$, and write $x I_K y$ for $x I_i y$ for all $i \in K$. As usual, $L_i(x, R) = \{z \in Z | z R_i x\}$ denotes agent i's lower contour set of x at R. The *preference domain*, denoted by $\mathscr{R}$, consists of the set of admissible preference profiles satisfying the following property:

$$R \in \mathscr{R} \iff \text{for all } x, y \in Z: \text{ if } x I_N y, \text{ then } x = y. \tag{1}$$

Remark 1. Our framework is general enough to embed several economic settings. Our main focus is the classical assignment problem where each agent $i \in N$ has strict preferences P_i over a finite set of objects $O = \{o_1, ..., o_m\}$. Our framework encompasses an

[12] The case when we move from R'' to R is based on similar reasoning. However, in all other cases, since Maskin monotonicity is satisfied, *indirect monotonicity* is vacuously satisfied.

[13] Indeed, $b P_3 c$ but $c P'_3 b$.

assignment problem whenever Z equals the set of all feasible allocations $\mathcal{O} = \left\{ o \in O^m | o_k \neq o_l \text{ for all } k, l \in N \right\}$ and for every agent i, preferences P_i over O can be extended to an ordering over $\mathcal{O}$ in the following natural way: $oR_i o' \Leftrightarrow$ either $o_i P_i o'_i$ or $o_i = o'_i$, for all $o, o' \in \mathcal{O}$.

The goal of the designer is to implement an SCR $F : \mathcal{R} \longrightarrow Z$ such that $F(R) \neq \emptyset$ for all $R \in \mathcal{R}$. We refer to $x \in F(R)$ as an F-optimal outcome at R. The *range* of F is the set

$$F(\mathcal{R}) \equiv \{x \in Z | x \in F(R) \text{ for some } R \in \mathcal{R}\}.$$

The *graph* of F is the set

$$Gr(F) \equiv \{(x, R) | x \in F(R), R \in \mathcal{R}\}$$

We impose the following assumption on F:

Definition 1 *(Efficiency).* $F : \mathcal{R} \longrightarrow Z$ is (Pareto) *efficient* if for all $R \in \mathcal{R}$ and all $z \in F(R)$, there does not exist any $x \in Z$ such that $x R_N z$ and $x P_i z$ for some $i \in N$.

In developing our framework, we find it convenient to move away from classical mechanisms or game forms. Indeed, the rights structure is our design variable. Thus, we rely on an implementation framework that models rights distribution within the society. Roughly speaking, we assume that a designer first describes the available alternatives via a set of possible states. Then, he specifies which agent or group has the right to move from a state to another. The rights distribution is such that the prediction of the solution concept returns the socially desirable alternatives for any preference profile. Formally, to implement F, the designer constructs a *rights structure* $\Gamma = ((S, d), h, \gamma)$, where S is the *state space* equipped with a metric d, $h : S \to Z$ the *outcome function*, and γ a *code of rights*, which is a (possibly empty-valued) correspondence $\gamma : S \times S \rightrightarrows \mathcal{N}$. Subsequently, a code of rights specifies, for each pair of distinct states (s, t), the family of coalitions $\gamma(s, t) \subseteq \mathcal{N}$ that is entitled to move from s to t. If $\gamma(s, t) = \emptyset$, then no coalition is entitled to move from s to t. A rights structure Γ is a *finite rights structure* if the cardinality of the space S of Γ is finite. We denote the set of all possible rights structures by $\mathcal{G}$.

The rights structure Γ presented here is an augmented version of the rights structure introduced by Sertel (2001) and Koray and Yildiz (2018) which does not include the metric d. Our formulation would allow us to properly define the solution concept over a possibly infinite state space. From an economic design perspective, the rights structure is the designer's design variable and corresponds to a "mechanism" in the economic theory jargon.

A rights structure together with a preference profile returns a *social environment*, a general framework to model strategic interaction among agents or groups.[14] We assume that in every social environment, the true preference profile is common knowledge among the agents. This is the case of *complete information* among the agents.

Definition 2 *(Social Environment).* For all $(\Gamma, R) \in \mathcal{G} \times \mathcal{R}$, the pair (Γ, R) is a *social environment*, in which there is complete information among the agents.

Next, a model of behavior is needed to predict in what state the agents will end up with. To this end, we need to select a solution concept. Formally, a solution concept is a map Σ, defined over $\mathcal{G} \times \mathcal{R}$, such that for each social environment $(\Gamma, R) \in \mathcal{G} \times \mathcal{R}$, $\Sigma(\Gamma, R)$ is a nonempty subset of S associated with Γ. Elements of $\Sigma(\Gamma, R)$ are the equilibrium states of the social environment (Γ, R). We can now provide a definition of implementation by a rights structure. An SCR is implementable in a solution Σ by a finite rights structure if the set of equilibrium outcomes induced by the social environment coincides with the set of socially optimal outcomes at any preference profile.

Definition 3 *(Implementation).* A finite rights structure Γ implements F in Σ if $F(R) = h \circ \Sigma(\Gamma, R)$ for all $R \in \mathcal{R}$. If such a rights structure exists, F is implementable in Σ by a finite rights structure.

3. Implementation in cycles

3.1. A characterization result: indirect monotonicity

As outlined above, the fundamental idea of our notion of implementation in ordered cycles relies on the Myopic Stable Set (MSS), introduced by Demuynck et al. (2019a). Thus, as a first step, this section presents the MSS and study implementation in cycles.

[14] When S is the set of outcomes and h is the identity function then our social environment coincides with the social environment of Chwe (1994).

To define the MSS, we need the notion of a *myopic improvement path*.[15] There is a myopic improvement path from s to T if a sequence of coalitional deviations from s to a state arbitrarily close to T exists such that every coalition involved in the sequence has the power as well as the incentive to move.

Definition 4 *(Myopic Improvement Path).* Given a social environment (Γ, R), a sequence of states $s_1, \ldots, s_m$ is called a *myopic improvement path* from s_1 to $T \subseteq S$ at R if, for all $\epsilon > 0$, there exist $s \in T$ such that $d(s, s_m) < \epsilon$ and a collection of coalitions $K_1, \ldots, K_{m-1}$ such that, for $j = 1, \ldots, m-1$,

(i) $K_j \in \gamma(s_j, s_{j+1})$

(ii) $h(s_{j+1}) P_{K_j} h(s_j)$.

An MSS can be defined as follows[16]:

Definition 5 *(Myopic Stable Set).* For every social environment (Γ, R), $mss(\Gamma, R) \subseteq S$ is an MSS for (Γ, R) if it is closed and satisfies the following three conditions:

1. *Deterrence of external deviations*: For all $s \in mss(\Gamma, R)$ and all $t \in S \setminus mss(\Gamma, R)$, there is no coalition $K \in \gamma(s, t)$ such that $h(t)\, P_K\, h(s)$.
2. *Asymptotic external stability*: For all $t \in S \setminus mss(\Gamma, R)$, there exists a myopic improvement path from t to $mss(\Gamma, R)$.
3. *Minimality*: There is no closed set $M' \subset mss(\Gamma, R)$ that satisfies the two conditions above.

Deterrence of external deviations requires that, from any state in the set, there are no coalitional deviations to states outside it. *Asymptotic external stability* requires the existence of a myopic improvement path to the set from any state outside it. Finally, *Minimality* requires that the MSS is the smallest closed set satisfying deterrence of external deviations and asymptotic external stability.

Since the MSS is equal to the union of all cycles when the set of states is finite,[17] we now provide a formal definition of a cycle for social environments with a finite state space.

Definition 6 *(Cycle).* Given a social environment (Γ, R) with a finite space state, a sequence of distinct states $s_1, \ldots, s_m$ is called a *cycle* if there exists a collection of coalitions $K_1, \ldots, K_{m-1}$ such that, for $j = 1, \ldots, m$, conditions (i) and (ii) in Definition 4 hold with $s_{m+1} = s_1$, and for all $s \in S$ such that $s \neq s_i$ for all $i = 1, ..., m$, there is no coalition $K \in \gamma(s_i, s)$ such that $s P_K s_i$.

In other terms, a cycle is a particular kind of finite myopic improvement path of length $m > 0$ from s_1 to $T = \{s_m\}$ where $s_{m+1} = s_1$ and from every element of the path no entitled agent has incentive to leave.

Let MSS $(\Gamma,\ R) = \{s \in S \mid s \in mss(\Gamma, R)\}$ be the union of all MSSs at (Γ, R). Thus, according to Definition 3 and the fact that the MSS is equal to the union of all cycles when the set of states is finite, an SCR F is implementable in cycles by a finite rights structure if the outcomes selected by F coincide with the outcomes corresponding to the cycles of the MSS, for each preference profile.

Our characterization result relies on the following definition.

Definition 7 *(Chain).* A chain at $(z, R, R') \in Z \times \mathscr{R} \times \mathscr{R}$ is a sequence of outcomes $z_1, \ldots, z_h$, with $z = z_1$ and $z \neq z_h$ and a sequence of agents $i_1, \ldots, i_{h-1}$ (not necessarily distinct) such that:

(A.0) $z_{k+1} P'_{i_k} z_k$ for all $k \in \{1, \ldots, h-1\}$;

(A.1) $L_i(z_h, R) \nsubseteq L_i(z_h, R')$ for some $i \in N$.

Condition (A.0) states that for each outcome of the sequence, an agent prefers its successor. Condition (A.1) requires that an agent experiences a preference reversal around the last element of the chain when switching from R to R'.

Our first characterization result relies on the following invariance condition.

Definition 8 *(Indirect Monotonicity).*

$F : \mathscr{R} \longrightarrow Z$ satisfies *indirect monotonicity* if for all $(z, R, R') \in Z \times \mathscr{R} \times \mathscr{R}$ the following is true: if $z \in F(R) \setminus F(R')$ and $L_i(z, R) \subseteq L_i(z, R')$ for all $i \in N$, then there exists a chain at (z, R, R') such that $z = z_1$, $z \neq z_h$ and $z_i \in F(R)$ for all $i = 1, ..., h$.

[15] If the state space is finite, then Definition 4 reduces to the following: A sequence of states $s_1, \ldots, s_m$ is called a *myopic improvement path* from s_1 to $T \subseteq S$ at R if $s_m \in T$ and there exists a collection of coalitions $K_1, \ldots, K_{m-1}$ such that, for $j = 1, \ldots, m-1$, (i) $K_j \in \gamma(s_j, s_{j+1})$ and (ii) $h(s_{j+1}) P_{K_j} h(s_j)$.

[16] When the set of states is finite, Condition 2 reduces to the following one: Iterated External stability: For all $t \in S \setminus M$, there exists a direct myopic improvement path from t to M.

[17] See Theorem 3.9 in Demuynck et al. (2019a).

Suppose that z is F-optimal at R. Further, suppose that preferences change from R to R' so that the standing of z improves for every agent. Finally, suppose that z is not F-optimal at R'. Therefore, F violates Maskin monotonicity.[18] *Indirect monotonicity* says that from z, there is a chain at R' among the F-optimal outcomes.

Note that Maskin monotonicity implies *indirect monotonicity* and that they are equivalent when F is single-valued. Also, note that our notion of *indirect monotonicity* resembles Condition α of Abreu and Sen (1990). However, in contrast to Abreu and Sen (1990), we require a sequence of F-optimal outcomes at R. We refer the reader to the Example 1 and Example 2 of the introduction for examples of SCRs satisfying *indirect monotonicity*. Moreover, let us discuss how the rights structure depicted in above implements F in cycles. In this rights structure, states are the outcomes identified by F at any preference profile together with (c, a, b). The rights structure is represented by an oriented graph in which vertices are the states, and the edges illustrate the code of rights. Agent 1 can move from (a, c, b) to (c, a, b), from (c, a, b) to (b, c, a), and from (b, a, c) to (a, c, b). Any agent can move from (b, a, c) to (b, c, a) and vice versa. According to this rights structure, the unique[19] MSS at R, R' and R'' are respectively $mss(\Gamma, R) = \{(a, c, b)\}$ $mss(\Gamma, R') = \{(b, c, a), (b, a, c)\}$, and $mss(\Gamma, R'') = \{(a, c, b), (b, c, a), (b, a, c)\}$. To see this, let us first consider the preference profile R. The set $\{(a, c, b)\}$ satisfies deterrence of external deviations because only agent 1 can deviate–to (c, a, b)– but such a deviation is not profitable for her. It satisfies iterated external stability because from (c, a, b) there is a myopic improvement path to $(c, a, b) \longrightarrow_1 (b, c, a) \longrightarrow_3 (b, a, c) \longrightarrow_1 (a, c, b)$ involving all the states outside the MSS and ending to the MSS. It satisfies trivially satisfies minimality. Next, let us consider the preference profile R'. The set $\{(b, c, a), (b, a, c)\}$ satisfies deterrence of external deviations because only agent 1 can deviate–to (a, c, b)– but such a deviation is not profitable for her. It satisfies iterated external stability because there is a myopic improvement path $(a, c, b) \longleftarrow_1 (c, a, b), \longleftarrow_1 (b, c, a)$ involving all the states outside the MSS and ending to the MSS. It satisfies minimality because any subset of $\{(b, c, a), (b, a, c)\}$ would violate deterrence of external deviations. Finally, let us consider the preference profile R''. The set $\{(a, c, b)(b, c, a), (b, a, c)\}$ satisfies deterrence of external deviations because only agent 1 can leave the set–to (c, a, b)– but such a deviation is not profitable for her. It satisfies iterated external stability because there is a myopic improvement path $(c, a, b) \longleftarrow_1 (b, c, a)$ involving the only state outside the MSS which is dominated by a myopically stable state. It satisfies minimality because taking off (a, c, b) would violate iterated external stability for $mss(\Gamma, R'')$ and taking off any states in $\{(b, c, a), (b, a, c)\}$ would violate deterrence of external deviations for $mss(\Gamma, R'')$.

The following result establishes our characterization result for the implementation in cycles via a finite rights structures.[20]

Theorem 1. *Any efficient $F : \mathscr{R} \longrightarrow Z$ satisfying indirect monotonicity is implementable in cycles by a finite rights structure.*

Indirect monotonicity is a sufficient condition for implementation in cycles via rights structures, though it is not a necessary condition. An example in Appendix A makes the point. The following corollary to Theorem 1 comes from the fact that Maskin monotonicity is a stronger condition than *indirect monotonicity*.

Corollary 1. *Every efficient and Maskin monotonic $F : \mathscr{R} \longrightarrow Z$ is implementable in cycles via a finite rights structure.*

Before moving to implementation in ordered cycles, let us discuss in the following subsections the relevance of Theorem 1. However, the impatient reader can move to Section 4 without loss of understanding.

3.2. Convergence property

As Jackson (1992) and Moore (1992) point out, canonical mechanisms for implementing socially desirable outcomes have unnatural futures: they are highly complex and challenging to explain in natural terms. In particular, when agents are boundedly rational, such mechanisms may lead to the convergence of undesirable outcomes. In our paper, we consider agents who (i) do not make optimal moves but simply aim to improve upon their current situations, and (ii) do not foresee the ultimate consequences that their moves may lead to. Under this assumption, our result shows that agents can reach F-optimal outcomes by using myopic improvement paths. Indeed, Theorem 1 demonstrates that the implementing rights structure guarantees the convergence to a cycle in a finite number of transitions among states. The reason is that our implementation problems are solved by devising a finite rights structure. This property assures that any cycle of the MSS can be reached in a finite sequence of myopic improvements from any state outside the MSS.

Our result can be thought of as the counterpart of recurrent implementation in better-response dynamics studied by Cabrales and Serrano (2011), in which agents myopically adjust their actions in the direction of better-responses. These authors show that a variant of monotonicity is essential for implementing recurrent strategies. Corollary 1 shows that for assignment problems of indivisible goods, monotonicity and Pareto efficiency are sufficient for a similar type of implementability.

We study two models where convergence is desirable in Appendix A. In particular, we consider exchange economies with complex endowment systems recently introduced by Balbuzanov and Kotowski (2019) as well as the class of "pure marriage problems"

[18] Maskin monotonicity says that if an outcome z is F-optimal at the profile R and this z does not strictly fall in preference of anyone when the profile changes to R', then z must remain a F-optimal outcome at R'.

[19] Uniqueness is given by the fact that the implementing right structure is finite, a property of the MSS.

[20] When Z is not a finite set, by using the rights structure designed in the proof of Theorem 1, it is possible to show that it implements F in MSS when F is closed valued and upper hemi-continuous, the set of alternatives Z is compact and the domain $\mathscr{R}$ is also compact.

studied by Knuth (1976).[21] Neither model satisfies any converge property. We show that the direct exclusion core of Balbuzanov and Kotowski (2019) and the solution that selects all stable matchings in the sense of Knuth (1976) can be implemented in MSS.

3.3. Connections to other notions of implementation

We conclude this section by showing that implementation in cycles by a finite rights structure is equivalent to implementation in absorbing sets and implementation in generalized stable sets (van Deemen, 1991; Page and Wooders, 2009). However, before showing this, let us formally introduce these alternative notions of stability.

Definition 9 *(Absorbing Set).* Let us assume that S is finite. The set $A(\Gamma, R) \subseteq S$ is an absorbing set at (Γ, R) if it satisfies the following two conditions:
(a) For all $s, t \in A(\Gamma, R)$, if $s \neq t$, then there exists a finite myopic improvement path from s to $\{t\}$.
(b) For all $s \in A(\Gamma, R)$, there does not exist any finite myopic improvement path from s to $S \backslash A(\Gamma, R)$.

Condition (a) affirms that it is possible for any state in the absorbing set to reach any other state via a myopic improvement path. Finally, by Condition (b), it is impossible to leave the absorbing set via a myopic improvement path from any state in the absorbing set.

van Deemen (1991) and Page and Wooders (2009) propose an extension of the stable set (von Neumann and Morgenstern, 1944) which replace the standard dominance relation with its transitive closure. The following is an equivalent definition of the generalized stable set based on our notion of improvement path.

Definition 10 *(Generalized Stable Set).* Let us assume that S is finite. The set $V(\Gamma, R) \subseteq S$ is a generalized stable set at (Γ, R) if it satisfies the following two conditions:
1. *Iterated Internal Stability:* For all $s \in V(\Gamma, R)$, there is no $t \in V(\Gamma, R)$ with $s \neq t$ such that $s = s_1, ..., s_m = t$ is a myopic improvement path from s to $V(\Gamma, R)$.
2. *Iterated External Stability:* For all $s \in S \backslash V(\Gamma, R)$, there exists a finite myopic improvement path from s to $V(\Gamma, R)$.

Inarra et al. (2005) and Nicolas (2009) studied the relation between the absorbing set and the generalized stable set. Our next result provides further insight into the relationship between the two solution concepts. In particular, we prove that whenever the state space is finite then the union of all stable generalized stable sets equals the union of all absorbing sets, which equals the union of all cycles.

Proposition 1. *Let Γ be a finite rights structure. Then, for all $R \in \mathcal{R}$, $mss(\Gamma, R) = \mathcal{A}(\Gamma, R) = \mathcal{V}(\Gamma, R)$.*

Theorem 1, when combined with Proposition 1, gives us the following significant result.

Corollary 2. *Any efficient $F : \mathscr{R} \longrightarrow Z$ satisfying indirect monotonicity is implementable in absorbing sets by a finite rights structure, and in generalized stable sets by a finite rights structure.*

Recently, Korpela et al. (2023) study implementation in stable set *á la* von Neumann-Moergenstern (von Neumann and Morgenstern, 1944) by rights structures when F is single-valued. They show that Maskin monotonicity is neither sufficient nor necessary for implementation. This contrasts with the result in Corollary 2 on the implementation in generalized stable set. The reason is that when F is single-valued, *indirect monotonicity* is equivalent to Maskin monotonicity, and so the latter is sufficient for the implementation of an efficient single-valued F in generalized stable set.

4. Implementation in ordered cycles

As noted earlier, implementation in cycles is only a preliminary step towards implementation in ordered cycles. Indeed, on the one hand, implementation in cycles gives the designer the ability to design cycles among socially optimal outcomes. However, on the other hand, the question is whether the designer can design cycles that circulate through all socially optimal outcomes. We illustrate this point through the following example.

Example 3. Suppose that $N = \{1, 2, 3\}$, $Z = \{x, y, z\}$, and $\mathscr{R} = \{R, R'\}$. The table below displays agents' preferences.

Let F be such that $F(R) = Z$ and $F(R') = \{x, y\}$. This SCR satisfies *indirect monotonicity* vacuously because $F(R) \setminus F(R') = \{z\}$ and $L_3(z, R) = \{x, z\} \nsubseteq L_3(z, R') = \{z\}$. Fig. 3 also displays a rights structure Γ that implements F in cycles. In this rights structure, the set of states is Z, and the outcome function h is the identity map. The codes of rights γ are such that only agent 3 is effective in moving from x to y, that is, $x \rightarrow^{\{3\}} y$, and only the coalition {1,2} is effective in moving from y to x, that is, $y \rightarrow^{\{1,2\}} x$, and so on.

[21] Pure marriage problems are marriage problems in which no agent can stay single.

R			R'		
1	2	3	1	2	3
x	z	y	x	x	y
y	x	z	y	y	x
z	y	x	z	z	z

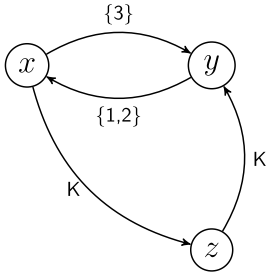


Fig. 3. Implementation in cycles does not imply rotation between desired outcomes. $K \in \mathcal{N}_0$ is such that $\#K \geqslant 2$.

Γ implements F in cycles because $MSS(\Gamma, R) = Z$ and $MSS(\Gamma, R') = \{x, y\}$. Since the social environment (Γ, R) generates a sub-cycle in which the outcome z is ruled out, it follows that Γ does not guarantee that agents rotate through outcomes in Z when agents' preferences are R.

We solve this drawback by focusing on a refinement of the MSS. Let us start by defining our notion of ordered cycles.

Definition 11 *(Ordered Cycle)*. An *ordered cycle* for (Γ, R) is a cycle among the states in $\bar{S} = \left\{s_1, ..., s_m\right\} \subseteq S$ such that for all $s_i \in \bar{S}$:

(i) For all $s \in \bar{S} \setminus \left\{s_i\right\}$, $h\left(s_i\right) \neq h(s)$.
(ii) For all $s \in S \setminus \left\{s_i, s_{i+1}\right\}$ and all $K \in \mathcal{N}_0$, if $K \in \gamma\left(s_i, s\right)$, then not $h(s)\, P_K h\left(s_i\right)$.

Condition (i) says that in an ordered cycle, no two states yield the same outcome. Moreover, since an ordered cycle is a cycle then, for all adjacent states s_i, s_{i+1}, there exists a coalition $K \in \mathcal{N}_0$ which gains by moving from s_i to s_{i+1} and is entitled to make this move— $h\left(s_{i+1}\right) P_K h\left(s_i\right)$ and $K \in \gamma\left(s_i, s_{i+1}\right)$. Finally, this, combined with Condition (ii) implies that the only possible transitions occur among adjacent states.

Let us now state our notion of implementation in ordered cycles.

Definition 12 *(Implementation in Ordered Cycles)*. A rights structure Γ implements $F : \mathscr{R} \longrightarrow Z$ in ordered cycles if the following requirements are satisfied:

- Γ implements F in cycles.
- For all $R \in \mathscr{R}$, states in $MSS(\Gamma, R)$ can be used to form m ordered cycles $\left\{S_1, ..., S_m\right\}$ such that $h \circ S_i = F(R)$ for all $i = 1, ..., m$ and $\bigcup_{i=1}^{m} S_i = MSS(\Gamma, R)$.

If such a rights structure exists, we say that F is *implementable in ordered cycles*.

Conceptually, the above notion of implementation refines our notion of implementation in cycles. Indeed, at each preference profile, the only admissible cycles are those that arrange the outcomes selected by F at that profile in clockwise or anticlockwise order, and the only possible transitions occur among adjacent states in a uni-directional way. Note that a rights structure implementing F in ordered cycles may have an empty core. For assignment problems, implementation in ordered cycles is renamed as implementation in rotation programs.

4.1. Characterization results

In what follows, we introduce two conditions, called *Rotation Monotonicity* and *Property M*, which are at the heart of our characterization results. To this end, we need the notion of *ordered chain*.

Definition 13 *(Ordered Chain)*.

For all $(R, R') \in \mathscr{R} \times \mathscr{R}$ and all sets of ordered outcomes $\{z_1, ..., z_m\}$ of Z, a sequence $z_k, \ldots, z_{k+h}$ (modulo m), with $1 \leqslant k \leqslant m$ and $1 \leqslant h \leqslant m-1$, is an *ordered chain* at (R, R') if there are agents $i_k, \ldots, i_{k+h}$ (not necessarily distinct) and an outcome $z \in Z$ such that the following two conditions are satisfied:

(B.0) $z_{k+1+\ell} P'_{i_{k+\ell}} z_{k+\ell}$ for $\ell \in \{0, ..., h-1\}$;
(B.1) $z_{k+h} R_{i_{k+h}} z$ and $z P'_{i_{k+h}} z_{k+h}$.

An ordered chain reminds the notion of a chain provided in Definition 7, though they are different. Whereas the sequence in Definition 7 does not need to follow any ordering, the sequence in Definition 13 must satisfy restrictions imposed by the given ordering. Moreover, both condition (B.0) and condition (A.0) require that for each outcome in the sequence, there is an agent preferring its successor. However, whereas condition (A.0) requires that also the last agent of the sequence, i_{k+h-1}, prefers z_{k+h-1} to z_{k+h}, condition (B.0) does not require it. Moreover, condition (B.1) requires that the last agent of the sequence, i_{k+h}, has a preference reversal around the last element of the sequence, z_{k+h} when preferences move from R to R'. In contrast to (B.1), condition (A.1) is looser because any agent can have a preference reversal around the last element of the sequence z_h when preferences move from R to R'.

Rotation monotonicity, a necessary condition for implementation, can be stated as follows.

Definition 14 *(Rotation Monotonicity).*

$F : \mathscr{R} \longrightarrow Z$ satisfies *rotation monotonicity* if for all $R \in \mathscr{R}$, $F(R)$ can be ordered as $z_{1,R}, \dots, z_{m,R}$ with $m = |F(R)|$, and for all $R, R' \in \mathscr{R}$, the following requirement is satisfied: if $F(R) \neq F\left(R'\right)$ and either $\#F\left(R'\right) > 1$ or $\left[\#F\left(R'\right) = 1 \text{ and } F\left(R'\right) \notin F(R)\right]$, then for all $z_{k,R} \in F(R)$, the sequence $z_{k,R}, \dots, z_{k+h,R}$ (modulo m), with $1 \leqslant h \leqslant m-1$, is an *ordered chain* at (R, R').

Roughly speaking, when preferences move from R to R' and $F(R) \neq F(R')$, rotation monotonicity requires that from every F-optimal outcome at R starts a sequence involving only F-optimal outcomes at R that leads to an outcome outside $F(R)$, and around which there is a preference reversal when preferences move from R to R'.

More formally, note that *rotation monotonicity* requires that the F-optimal outcomes form an ordered set at every profile. Moreover, for any two profiles, R and R', *rotation monotonicity* applies when $F(R) \neq F(R')$ and either more than one outcome is F-optimal at R' or the unique F-optimal outcome at R' is not F-optimal at R. When these requirements are satisfied (not vacuously), *rotation monotonicity* states that if z_k is an F-optimal outcome at R, then an ordered chain with the following two properties exists. Firstly, the sequence starts from this z_k. Secondly, the sequence involves only F-optimal outcomes at R.

Rotation monotonicity implies *indirect monotonicity* when the SCR F is such that it always selects more than an outcome at each admissible profile. In contrast to *indirect monotonicity*, *rotation monotonicity* requires arranging all F-optimal outcomes circularly.

We now show that only SCRs satisfying *rotation monotonicity* are implementable in ordered cycles.

Theorem 2 *(Necessity). If $F : \mathscr{R} \longrightarrow Z$ is implementable in ordered cycles, then it satisfies rotation monotonicity.*

We refer the reader to Example 1 in the introduction for an example SCR which satisfies *rotation monotonicity*.

As noted earlier, *rotation monotonicity* does not bite when the unique F-optimal outcome at R' is also F-optimal at R and $F(R) \neq F(R')$. Therefore, *rotation monotonicity* cannot be a sufficient condition for implementing F in ordered cycles if no other restrictions are imposed on F. Indeed, *rotation monotonicity*, when combined with an auxiliary condition, termed *Property M*, is sufficient for implementation. *Property M* can be defined as follows.

Definition 15 *(Property M).* $F : \mathscr{R} \longrightarrow Z$ satisfies *property M* if for all $R \in \mathscr{R}$, the set $F(R)$ can be ordered as $z_{1,R}, \dots, z_{m,R}$ for $m = \#F(R)$, and for all $R, R' \in \mathscr{R}$, the following requirement is satisfied: if $F(R) \neq F\left(R'\right)$, $\#F\left(R'\right) = 1$ and $F\left(R'\right) = z_{j,R}$ for some $1 \leqslant j \leqslant m$, then for each $z_{k,R} \in F(R) \backslash F(R')$ such that $k \neq j$ and $1 \leqslant k \leqslant m$,

- either the sequence $z_{k,R}, \dots, z_{k+h,R}$ (modulo m) is an ordered chain;
- or there is a sequence of agents $i_1, \dots, i_\ell$ such that:
  1. $F(R') P'_{i_\ell} z_{j-1,R} P'_{i_{\ell-1}} \dots P'_{i_2} z_{k+1,R} P'_{i_1} z_{k,R}$ and
  2. $L_i\left(z_{j,R}, R\right) \cup \left\{z_{j+1,R}\right\} \subseteq L_i\left(z_{j,R}, R'\right) \quad \forall i \in N.$

Take any profiles R and R' such that $F(R) \neq F(R')$ and such that rotation monotonicity does not bite. For instance, let $F(R') = \{z_{j,R}\} \subseteq F(R)$. *Property M* requires that for each F-optimal outcome at R that is not F-optimal at R', either the conclusion of *rotation monotonicity* holds, or there exists a sequence of agents who myopically prefer to move from $z_{k,R}$ to $F(R')$ at R' via a sequence of F-optimal outcomes at R and for every agent i, there is a monotonic change of agent i's preferences around $z_{j,R}$ when the profile changes from R to R', and the change is such that $z_{j,R} R'_i z_{j+1,R}$.

Theorem 3 *(Sufficiency). If $F : \mathscr{R} \longrightarrow Z$ is efficient, and it satisfies rotation monotonicity and Property M with respect to the same set of ordered outcomes of $F(R)$, for all $R \in \mathscr{R}$, then it is implementable in ordered cycles by a finite rights structure.*

Before discussing an important implication of the above characterization result, it is worth noting that the proof of Theorem 3 does not work without efficiency. The reason is that the implementing rights structure is a variant of the rights structure devised for the proof of Theorem 1.

We conclude this section by focusing on cases where ordered cycles are not trivial, that is, on cases where the designer's goal is multi-valued at each admissible profile. In these cases, *Property M* does not have any bite. It follows that *rotation monotonicity* fully characterizes the class of efficient SCRs that are implementable in ordered cycles.

Corollary 3. *Suppose that $F : \mathscr{R} \longrightarrow Z$ is efficient. Suppose $\#F(R) > 1$ for all $R \in \mathcal{R}$. F is implementable in ordered cycles if and only if F satisfies rotation monotonicity.*

5. Job rotation problems

A job rotation is a well-known organizational technique where employees cycle through different roles in their company according to a fixed schedule. Job rotation benefits include the improvement of the employee's knowledge of the company, the building of new skills, the reduction of boredom among employees, and the increase in productivity (Morris, 1956). From an economic design point of view, a job rotation problem is an assignment problem as defined in Remark 1 such that the number of objects, interpreted as jobs, equals the number of agents. A job rotation problem is thus a tuple (N, J, P) where $N = \{1, ..., n\}$ is a finite set of agents with $n \geqslant 2$, $J = \{j_1, ..., j_n\}$ is a finite set of jobs, $P = (P_i)_{i\in N}$ is a profile of linear orderings such that every $P_i \subseteq J \times J$. For every job rotation problem (N, J, P), every agent i's preferences over J at P_i can be extended to an ordering over the set of all feasible allocations $\bar{J} = \{j \in J^n | j_k \neq j_l \text{ for all } k, l \in N\}$ in the following natural way $j R_i j' \Leftrightarrow$ either $j_i P_i j'_i$ or $j_i = j'_i$, for all $j, j' \in \bar{J}$. Let $\mathscr{R}$ denote the set of all (extended) preference profiles. As for assignment problems, an ordered cycle is a rotation program for job rotation problem. Therefore, in such an environment, implementation in ordered cycles consists of implementation in rotation programs.

Example 4 in Appendix A shows that not every efficient F on $\mathscr{R}$ is implementable in rotation programs. Given this impossibility, we focus on two classes of job rotation problems that are implementable in rotation programs.

5.1. A job rotation problem with restricted domain

There are situations where there is a common best/worst job among the available ones. For instance, suppose that the head of an Economics Department has to allocate a microeconomics course to each of its microeconomics teachers. Courses can be ranked, for example, according to their sizes. The best possible assignment for everyone is to be assigned to the Ph.D. course with the lowest number of students. In contrast, the common worst possible outcome for every teacher is to be assigned to the largest possible class at the undergraduate level.

We consider assignment problems where a common best job exists. Let us denote it by j_1^*.[22] The set of jobs J is given by $\{j_1^*, j_2, ..., j_n\}$. Let $\bar{\mathscr{R}}$ be preference domain such that
$\bar{\mathscr{R}} = \{R \in \mathscr{R} | \text{for all } i \in N,\ \arg\max_J R_i = \{j_1^*\}\}$. With abuse of notation, we also use $\bar{\mathscr{R}}$ to denote the set of all (extended) preference profiles.

The Pareto efficient SCR defined over $\bar{\mathscr{R}}$ is implementable in rotation programs.

Theorem 4. *The efficient $F : \bar{\mathscr{R}} \to \bar{J}$ is implementable in rotation programs.*

Note that, by construction, the efficient F over $\bar{\mathscr{R}}$ is multi-valued. Thus, to check Theorem 4, it suffices to show that the efficient F over $\bar{\mathscr{R}}$ is rotation monotonic.

The intuition behind this theorem is that for each R, elements of $F(R)$ can be arranged circularly as $x(1, R), ..., x(m, R), x(1, R)$ such that no two consecutive allocations of the arrangement allocate j_1^* to the same agent. Thus, an admissible ordered set required by *rotation monotonicity* is $x(1, R), ..., x(m, R)$. Take any R' such that $F(R) \neq F(R')$. Since F is Maskin monotonic, it follows that there exists an $x(i, R) \in F(R)$ such that $x(i, R) R_\ell z$ and $z P'_\ell x(i, R)$ for some agent $\ell \in N$ and an allocation $z \in \bar{J}$. Since, by the way we arranged the elements of $F(R)$, it holds that for all $k \neq i$, $x(k+1, R) P'_j x(k, R)$ for some agent j, it is clear that F satisfies *rotation monotonicity*.

In the context of auction design, Milgrom (2004) states that, in contrast to much of the theoretical literature, the set of outcomes is rarely fixed in practice and is itself subject to design. This observation also extends to our assignment problems.

Let us go back to our problem of allocating courses to teachers to see it. In this context, the head of the department can design syllabuses in a way that there is a common best course, in the sense that it is, for example, the less time-consuming one. Since, in many cases, the designer can design jobs to meet the requirements of Theorem 4, the set of its applications is broad.

5.2. A job rotation problem with partially informed designer

As another application, we consider assignment problems where the designer knows that two agents have the same top-choice. Specifically, we assume that the designer knows that agent 1 and agent 2 have a common top-ranked job, although he does not know it. The domain of admissible profiles of linear orderings is given by $\hat{\mathscr{R}} = \{R \in \mathscr{R} | \tau(R_1) = \tau(R_2)\}$, where $\tau(R_i)$ denote the top-ranked job of agent i at R_i. With abuse of notation, we also use $\hat{\mathscr{R}}$ to denote the set of all (extended) preference profiles over $\bar{J}$.

We are interested in implementing the solution $\phi : \hat{\mathscr{R}} \longrightarrow \bar{J}$. To define the desired allocations at R, that is, $\phi(R)$, let us define the following algorithm.

- **Step 1:** Assign $\tau(R_1)$ either to agent 1 or to agent 2.
- **Step 2:** Assign the remaining jobs $J \setminus \{\tau(R_1)\}$ to $N \setminus \{1, 2\}$ in an efficient way.
- **Step 3:** Assign the remaining job to agent 2 if agent 1 has received his top-ranked job, otherwise, assign it to agent 1.

[22] Since assignment problems where a common worst job exists can be treated symmetrically, we omit their analysis here.

Given a profile R, we can construct a set of allocations at R by using the above algorithm. Let $\phi(R)$ coincide with this set of allocations. The set $\phi(R)$ can be thought of as the set of outcomes generated by an underlying random *serial dictatorship mechanism* (Abdulkadiroğlu and Sönmez, 1998), in which the allowed random priority-orderings are those where either agent 1 has the highest priority and agent 2 the lowest priority or agent 2 has the highest priority and agent 1 the lowest priority.

The next result shows that the solution ϕ is a sub-solution of the Pareto efficient solution.

Lemma 1. *The solution $\phi : \hat{\mathscr{R}} \longrightarrow \bar{J}$ is a sub-solution of the Pareto efficient solution.*

Proof. Assume, to the contrary, that ϕ is not a sub-solution of the Pareto efficient solution. Then, there exist a feasible profile R and an allocation $x \in \phi(R)$ such that x is not a Pareto efficient allocation at R. I.e., there exists an allocation $y \in \bar{J} \setminus \{x\}$ such that $y_i R_i x_i$ for all $i \in N$ and $y_i P_i x_i$ for some $i \in N$. Without loss of generality, let us assume that x is such that it assigns the top-outcome $\tau\left(R_1\right)$ to agent 1, i.e., Step 1 is such that $x_1 = \tau\left(R_1\right)$. Since $y_1 R_1 x_1$, it follows that $x_1 = y_1 = \tau\left(R_1\right)$. Since in Step 2 the remaining jobs in $J \setminus \left\{\tau\left(R_1\right)\right\}$ are assigned to agents in $N \setminus \{1,2\}$ in a Pareto efficient way, it follows that $x_i R_i y_i$ for all $i \in N \setminus \{1,2\}$, and so $x_i = y_i$ for all $i \in N \setminus \{1,2\}$. Since we have established that $x_i = y_i$ for all $i \in N \setminus \{2\}$, it follows that $x = y$, which is a contradiction. □

The solution ϕ is implementable in rotation programs. The reason is that the allocations at R selected by ϕ can be ordered in a way in which agent 1 and agent 2 alternate in enjoying their common top job.

Theorem 5. *$\phi : \hat{\mathscr{R}} \to \bar{J}$ is implementable in rotation programs.*

5.3. A dynamic consideration

In this paper, we have not studied agents' dynamic tradeoffs explicitly and future research should pursue it. However, we can offer a consideration on the efficiency loss that the designer faces when he decides to achieve job rotations via a decentralized decision-making process. To this end, let $\left(N, J, (u_i(\cdot,\theta))_{i\in N}\right)$ be a job rotation problem at θ, where each agent evaluates each sequence of jobs by the discounted sum of the associated sequence of payoffs. More precisely, each agent i has a state-dependent utility function $u_i(\cdot,\theta)$ over $\bar{J}$ and a (common) discount factor $\delta \in (0,1)$ such that he evaluates the infinite sequence of outcomes $\left\{j^t\right\}_{t=1}^{\infty}$ at state θ by its discounted average, which is given by

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}u_i\left(j_i^t,\theta\right),$$

where j_i^t is the job assigned to agent i at time t. Let Θ be the set of possible state-dependent utility functions. It is well-known, that for any given stream $\left\{j^t\right\}_{t=1}^{\infty}$ and any agent i, there is a real number c_i such that agent i is indifferent between the stream $\left\{j^t\right\}_{t=1}^{\infty}$ and the constant stream $\left(c_i, c_i, c_i, \ldots\right)$.[23] The discounted average of the constant stream $\left(c_i, c_i, c_i, \ldots\right)$ is c_i.

Suppose that F over Θ is implementable in rotation programs. Thus, the set of desired outcomes can be arranged in a circle and the sequence of finite length T is repeated over time. Then, the rotation $j^1 \to j^2 \to \ldots \to j^{T-1} \to j^T \to j^1$ generates for each agent i an average payoff given by

$$\frac{1}{T}\sum_{t=1}^{T}u_i\left(j_i^t,\theta\right).$$

If the discount factor δ is close to 1, then agent i's discounted average payoff for the repeated stream $\left\{j^t\right\}_{t=1}^{T}$ is close to his average payoff.[24]

Therefore, if the discount factor δ is close to 1, we can compute the equity and efficiency tradeoffs that the designer faces as follows. For any $\theta \in \Theta$, let a Pareto efficient assignment be denoted by $j^*(\theta)$. Clearly, it is such that

$$j^*(\theta) \in \arg\max_{j\in\bar{J}}\left(\sum_{t=1}^{N}u_i\left(j_i,\theta\right)\right).$$

Since the designer wants to implement a job rotation, he will lose in efficiency. Let us suppose that the length of the rotation is T. The efficiency loss at θ can be expressed as the ratio between the sum of agents' average payoffs for the cycle and the sum of agents' payoffs at the efficient assignment, that is,

[23] To see it, let $c_i = (1-\delta)V$ where $V = \sum_{t=1}^{\infty}\delta^{t-1}u_i\left(j_i^t,\theta\right)$.

[24] In an undiscounted model, the same conclusion can be achieved if we assume that each agent evaluates the stream $\left\{j^t\right\}_{t=1}^{\infty}$ using the limit of means criterion:

$$U_i\left(\left\{j^t\right\}_{t=1}^{\infty}\right) = \lim_{T\to\infty}\left(\frac{1}{T}\sum_{t=1}^{T}u_i\left(j_i^t,\theta\right)\right).$$

The advantage of this criterion is also to ensure that any finite stream of outcomes that is used to reach the rotation among the desired outcomes has no effect on agents' average payoffs.

$$EL(\theta) = \frac{\sum_{i\in N}\left(\frac{1}{T}\sum_{t=1}^{T} u_i\left(j_i^t,\theta\right)\right)}{\sum_{i\in N} u_i\left(j_i^*(\theta),\theta\right)}.$$

6. Concluding remarks

This paper studies rotation programs in an implementation framework. A rotation program is a circular arrangement of the states of an MSS (Demuynck et al., 2019a).

Implementation in cycles is robust in the following sense: at any preference profile, every non-stable allocation converges to a stable allocation via a sequence of myopic deviations. Moreover, implementation in cycles of efficient SCRs by a finite rights structure is equivalent to implementation of those rules in absorbing sets and in generalized stable sets. We identify a sufficient condition for implementing efficient SCRs in cycles, called *indirect monotonicity*. This condition is weaker than (Maskin) monotonicity.

Regarding the implementation in rotation programs, we show that *rotation monotonicity*, when combined with an auxiliary condition, is sufficient for implementing efficient SCRs. Moreover, rotation monotonicity fully characterizes the class of efficient SCRs that can be implemented in rotation programs when the designer's goal is multi-valued at each admissible profile.

Finally, we study some welfare implications of our characterization results. We learn that implementation in rotation programs is somewhat restrictive when the set of outcomes is fixed. However, as in the context of auction design (Milgrom, 2004), the outcome space plays an essential role in assignment problems. Indeed, by cleverly designing the set of outcomes, many significant assignment problems become implementable in rotation programs.

Declaration of competing interest

Authors declare that they have no relevant or material financial interests that relate to the research described in the paper "Designing Rotation Programs: Limits and Possibilities".

Data availability

No data was used for the research described in the article.

Appendix A

Indirect monotonicity is not a necessary condition

Let $N = \{1,2,3\}$, $Z = \{x,y,z\}$, and $\mathcal{R} = \{R, R'\}$. Preferences are defined in the table below.

R			R'		
1	2	3	1	2	3
z	x	y	z	zy	y
y	z	x	y	x	x
x	y	z	x		z

Suppose that the utility of all agents at each state is such that the top ranked outcome gets a value 4, the outcome that is ranked second gets a value of 3 and so on until the bottom ranked outcome gets a value of 0. Let assume that, in this environment a SCR F is socially optimal iff

$$F(R) = \{x,y,z\},\ \ F(R') = \{z,y\}.$$

It is straightforward to see that F can be implemented in MSS via a rights structure representing the mojority rule—$S = \{x,y,z\}$, and for all $s,s' \in S$, $K \in \gamma(s,s')$ if and only if $|K| \geqslant 2$. However, indirect monotonicity is not satisfied for F. Indeed, $z \in F(R)$ experiments a monotonic transformation from R to R'. Moreover, there exists a one step deviation from $z \in F(R)$ to $x \in F(R)$ with x that does not experiment a monotonic transformation from R to R'. Indeed for agent 2 the alternative x becomes the worst option under R'. Then, to satisfy indirect monotonicity we should have $z \notin F(R')$ which is not the case.

Convergence in exchange economies

Let us consider the class of exchange economies studied by Balbuzanov and Kotowski (2019) and consider the notion of *direct exclusion core*. We show, through an example, that the free exchange of goods does not necessarily converge to the direct exclusion core. However, the direct exclusion core is implementable in MSS via a finite rights structure. This result implies that irrespective of the initial allocation of objects, it is possible to converge to a direct exclusion core allocation in a finite sequence of coalitional moves.

An *economy* is a quadruplet (N, H, P, ω) where $N = \{1, ..., n\}$ is a finite non-empty set of agents, $H = \{h_1, ..., h_m\}$ is a finite set of houses that can be allocated among the agents, $P = \left(P_i\right)_{i \in N}$ is a profile of linear orderings, where each linear ordering is defined over $H \cup \left\{h_0\right\}$, and the endowment system $\omega : 2^N \longrightarrow 2^H$ is a function that specifies the houses owned by each coalition. For each coalition $K \in \mathcal{N}_0$, we write $\omega(K) = \bigcup_{T \in \mathcal{K}_0} \omega(T)$. Let us assume that the endowment system ω satisfies the following four properties: (A1) *Agency*: $\omega\left(\emptyset\right) = \emptyset$, (A2) *Monotonicity*: $K \subseteq K' \implies \omega(K) \subseteq \omega\left(K'\right)$, (A3) *Exhaustivity*: $\omega(N) = H$, and (A4) *Non-contestability*: For each $h \in H$, there exists $K^h \in \mathcal{N}_0$ such that $h \in \omega(K) \iff K^h \subseteq K$.

Property A1 restricts ownership to agents or groups. Property A2 requires that a coalition has in its endowment anything that belongs to any sub-coalition. Property A3 states that the grand coalition N jointly owns everything. In property A4, coalition K^h is called the minimal controlling coalition of house h. It guarantees that each house has a set of one or more "co-owners" without opposing and mutually exclusive claims. As Balbuzanov and Kotowski (2019, Lemma 1) show, these properties are needed to assure that the direct exclusion core is nonempty.

We assume that each agent may live in at most one house, and each house $h \in H$ may accommodate at most one agent. A house may be vacant, and an agent can be homeless. We can model this latter outcome by the agent's assignment to an outside option $h_0 \notin H$, which has unlimited capacity.

An allocation $\mu : N \longrightarrow H \cup \left\{h_0\right\}$ is an assignment of agents to houses such that $\#\mu^{-1}(h) \leqslant 1$ for all $h \in H$. We write $\mu(K)$ to denote $\bigcup_{i \in K} \mu(i)$ for any $K \in \mathcal{N}_0$. Let (N, H, P, ω) be an economy. Every linear ordering P_i can be extended to an ordering R_i over the collection $\mathcal{M}$ of allocations in the following way: $\mu R_i \mu' \iff$ either $\mu(i) P_i \mu'(i)$ or $\mu(i) = \mu'(i)$, for all $\mu, \mu' \in \mathcal{M}$. With little abuse of notation, we denote both by R_i. Let $\mathscr{R}$ denote the class of admissible preference profiles of extended preferences.

Definition 16. Given an economy (N, H, R, ω), a coalition $K \in \mathcal{N}_0$ can *directly exclusion block* the allocation μ at R with allocation σ if
(a) $\sigma(i) P_i \mu(i)$ for all $i \in K$ and
(b) $\mu(j) P_j \sigma(j) \implies \mu(j) \in \omega(K)$ for all $j \in N \backslash K$.

In words, a coalition can directly exclusion block an assignment whenever each member strictly gains from an alternative, and anyone harmed by the reallocation is excluded from a house belonging to the coalition. The *direct exclusion core* is the set of allocations that cannot be directly exclusion blocked by any nonempty coalition.

Definition 17 *(Direct Exclusion core).* Given an economy (N, H, R, ω), its *direct exclusion core*, denoted by $CO(R, \omega)$, is defined by $CO(R, \omega) = \{\mu \in \mathcal{M} | \text{no coalition can directly exclusion block } \mu \text{ at } R\}$.

Thus, no coalition can gainfully destabilize a direct exclusion core allocation by invoking their collective exclusion rights. Balbuzanov and Kotowski (2019, Lemma 1) show that the direct exclusion core is never empty, and all its allocations are efficient.

Let us show that the direct exclusion core does not satisfy an external stability requirement. To this end, let us represent an allocation μ by a permutation matrix with columns indexed by elements of N and rows indexed by elements of $H \cup \left\{h_0\right\}$, where h_0 is the last row. If for some $h \in H \cup \left\{h_0\right\}$ and some $i \in N$, entry $\mu_{hi} = 1$, then good h has been assigned to agent i.

Let us consider an economy with three agents and three houses.[25] Each house $i \in H$ is owned by agent i. The table below displays agents' preferences.

R		
1	2	3
2	3	1
3	1	2
1	2	3
h_0	h_0	h_0

$$\mu = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The direct exclusion core at R consists of the allocation μ. Let us consider the following allocations:

$$\sigma^1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \sigma^3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Although the direct exclusion core is not empty, the process of 'free' exchange of houses may not lead to μ because such a process may cycle. Indeed, agents may myopically cycle around σ^1, σ^2 and σ^3.

[25] We borrow this example from Demuynck et al. (2019b, pp. 12-13).

To see it, note that for each agent i, his endowment $\omega(i) = i$ corresponds to his third choice–his last choice is to become homeless. Therefore, given this initial situation, coalition $\{1,2\}$ can trade so that they can achieve the allocation σ^1. At σ^1, agent 1 obtains his first best choice. Thus, coalition $\{2,3\}$ is the only coalition that can achieve a strict improvement. The only allocation that $\{2,3\}$ can move to is allocation σ^2, where agent 2 obtains his first best choice. At σ^2, only coalition $\{1,3\}$ can achieve a strict improvement by moving to the only attainable allocation σ^3, where agent 3 obtains his first best choice. At σ^3, only coalition $\{1,2\}$ can achieve a strict improvement by moving to the only attainable allocation σ^1. Therefore, the free exchange may lock agents in a cycle of exchanges.

A natural question from the preceding example is whether achieving the direct exclusion core through a different exchange process is possible. Corollary 4 answers this question by showing that the direct exclusion core is implementable in MSS via a finite rights structure. To formalize our answer, fix any endowment system ω satisfying the above four properties. Let us define F_{ω}^{CO} by $F_{\omega}^{CO}(R) = CO(R,\omega)$ for all $R \in \mathscr{R}$.

Corollary 4. *Fix any endowment system ω satisfying properties A1-A4. F_{ω}^{CO} is implementable in MSS via a finite rights structure.*

Convergence in matching

As a second application, we consider a two-sided, one-to-one matching model, namely the "marriage problem". A marriage problem is a market without transfers where the sides of the market are, for example, workers and firms (job matching), medical students and hospitals (matching of students to internships), students and advisors (matching of students to thesis advisors). The two-sided markets are referred to as "men" and "women," hence the name "marriage problem." An output of the model is termed a matching, which pairs each woman with at most one man and each man with at most one woman. Roughly speaking, a matching is stable when there is no blocking pair; no pair of agents is better off with each other than with their assigned partners. There are two prominent models describing the marriage problem: the Gale-Shapley model (Gale and Shapley, 1962) and the Knuth model (Knuth, 1976). The former studies stability for marriage problems in which agents can be singles. The latter is a pure matching model in which no agent can be single (thus, the number of men and women is assumed to be the same). Roth and Vande Vate (1990) show that the set of stable matchings in the Gale-Shapley model exhibits a convergence property; for any unstable matching, a myopic improvement path to a stable matching exists. On the contrary, no general convergence result exists for the Knuth model. The reason is that in the convergence process designed by Roth and Vande Vate (1990) agents are allowed to be single, but this is not feasible in the Knuth model. For instance, Tamura (1993) shows that, under the usual matching rules, when there are at least four women, preferences exist such that agents cycle among unstable matchings. Our following result fills the gap. Indeed, since a stable matching in the marriage problem is monotonic and efficient, we establish, as a corollary of Theorem 1, that the set of stable matchings in the Knuth model is implementable in MSS, and so there exists a mechanism such that a converge property in the Knuth model is established.

Corollary 5. *The set of stable matchings in the Knuth model is implementable in MSS via a finite rights structure.*

Note that, under usual matching rules, Demuynck et al. (2019a) show that the MSS is a superset of the set of stable matchings. From this point of view, Corollary 5 further enlightens the relation between the MSS and the set of stable matchings. Moreover, it suggests that implementation by rights structures could represent a tool for refining the MSS whenever its prediction under canonical rules is too loose. Since this conjecture overcomes the aim of the present manuscript, we leave it for future research.

A non-implementable efficient SCR

Example 4. Let F be a SCR defined over $\mathscr{R}$ that selects all efficient outcomes at any preference profile. Suppose that there are three agents. Let the profiles P, P', P'' be defined as follows:

P

1	2	3
j_1	j_1	j_2
j_3	j_2	j_3
j_2	j_3	j_1

,

P'

1	2	3
j_1	j_1	j_3
j_3	j_2	j_2
j_2	j_3	j_1

and

P''

1	2	3
j_1	j_1	j_2
j_3	j_3	j_3
j_2	j_2	j_1

.

It can easily be checked that $F(R) = \{(j_3,j_1,j_2),(j_1,j_2,j_3),(j_1,j_3,j_2)\}$, $F(R') = \{(j_3,j_1,j_2),(j_1,j_2,j_3)\}$ and $F(R'') = \{(j_3,j_1,j_2),(j_1,j_3,j_2)\}$. F is not implementable in rotation programs because it violates *rotation monotonicity*. To see it, assume, to the contrary, that F satisfies *rotation monotonicity*. Then, the elements of $F(R)$ can be ordered as $x(1,R), x(2,R), x(3,R)$ to satisfy *rotation monotonicity*.

Let us consider R''. Select $i \in N$ such that $x(i,R) = (j_3, j_1, j_2)$. We show that $x(i+1,R) = (j_1, j_3, j_2)$. Since $x(i,R)$ has not fallen strictly in anyone's preference ordering because R'' is a monotonic transformation of R at $(j_3, j_1, j_2) = x(i,R)$,[26] it follows that we can only move to the next element of the ordered set, that is, to $x(i+1,R)$. Since the top-ranked job for agent 2 at P'' is j_1 and since, moreover, the top-ranked job for agent 3 at P'' is j_2, it follows that only agent 1 can move to $x(i+1,R)$ at R'', which implies that $x(i+1,R)$ must coincide with (j_1, j_2, j_3), that is, we have that $x(i+1,R)\, P_1'' x(i,R)$ and $x(i+1,R) = (j_1, j_2, j_3)$.[27]

Let us now consider R'. Let us consider the allocation $x(i+1,R) = (j_1, j_2, j_3)$. Since R' is a monotonic transformation of R at $x(i+1,R)$, it follows that we can only move to the next element of the ordered set, that is, to $x(i+2,R)$. Note that the top-ranked job for agent 1 at R' is j_1. Also, note that the top-ranked job for agent 3 at R' is j_3. The preceding discussion implies that only agent 2 can move to $x(i+2,R)$, and so $x(i+2,R)$ must coincide with $(j_3, j_1, j_2) = x(i,R)$, which contradicts the assumption that the elements of $F(R)$ can be ordered as $x(1,R), x(2,R), x(3,R)$. Thus, F does not satisfy *rotation monotonicity*.

Appendix B

Proofs

Proof of Theorem 1. The state space S consists of $S = Gr(F) \cup Z$. Since Z is finite, S is also finite. The outcome function h is defined such that $h(z,R) = z$ for all $(z,R) \in S$ and $h(z) = z$ for all $z \in Z$. The following five rules define the code of rights γ:

RULE 1: $\{i\} \in \gamma((z,R),(x,R))$ for all $R \in \mathcal{R}$, all $z, x \in F(R)$, and all $i \in N$,

RULE 2: $\{i\} \in \gamma((z,R),x)$ if $x \in L_i(z,R)$,

RULE 3: $\{i\} \in \gamma(x,(z,R))$ for all $x,(z,R) \in S$, and all $i \in N$,

RULE 4: $\{i\} \in \gamma(x,y)$ for all $x, y \in S$, and all $i \in N$, and

RULE 5: $\gamma(s,s') = \emptyset$ for any other $s, s' \in S$.

Let us show that the rights structure $\Gamma = (S,h,\gamma)$ defined above implements F in MSS if F is efficient and satisfies *indirect monotonic*. To this end, suppose that F is efficient and indirect monotonic. The following lemmata will help to prove our result. To proceed with our lemmata, we need the following additional definitions. For each $R, R' \in \mathcal{R}$:

$$M(R) \equiv \{(z,R) \mid z \in F(R)\} \subseteq S \qquad U(R) \equiv \{z \in Z \mid Z \subseteq L_i(z,R) \text{ for all } i \in N\};$$

$$Q(R,R') \equiv \left\{ (z',R') \in M(R') \;\middle|\; \begin{array}{l} \text{there does not exist any myopic improvement} \\ \text{path from } (z',R') \text{ to } M(R) \cup U(R) \text{ at } R \end{array} \right\};$$

$$Q(R) \equiv \bigcup_{R' \in \mathcal{R}} Q(R,R').$$

Since S is finite, the property of the asymptotic external stability of Definition 5 is equivalent to the property of iterated external stability, defined in footnote 4. Fix any profile R. The objective of the following lemmata is to show that

$$MSS(\Gamma,R) = M(R) \cup U(R) \cup Q(R) \quad \text{and} \quad F(R) = h \circ (M(R) \cup U(R) \cup Q(R)).$$

Lemma 2. *There is a finite myopic improvement path to $M(R) \cup U(R)$ at R from every state $s \in Z \setminus U(R)$.*

Proof of Lemma 2. Take any $s \in Z \setminus U(R)$. If $U(R) \neq \emptyset$, there is a one-step myopic improvement path from s to $U(R)$, by Rule 4. Otherwise, suppose that $U(R) = \emptyset$. We divide the rest of the proof into two parts according to whether $s \in F(R)$ or not.

Case 1: $s \notin F(R)$. Suppose that $s R_i h(s')$ for all $i \in N$ and all $s' \in M(R)$. Since $s' \in M(R)$ and F satisfies efficiency, it holds that $s I_i h(s')$ for all $i \in N$. Since $R \in \mathcal{R}$ by our domain assumption, then it follows that $s = h(s')$, and so $s \in F(R)$, which is a contradiction. Therefore, it must be the case that there exists an $s' \in M(R)$ such that $h(s') P_i s$ for some $i \in N$. Hence, by Rule 3, there exists a one-step improvement path from s to $M(R)$ at R.

Case 2: $s \in F(R)$. Suppose that there exists an agent $i \in N$ such that $h(s') P_i s$ for some $s' \in M(R)$. By Rule 3, there exists a one step myopic improvement path from s to $M(R)$ at R. Otherwise, suppose that $s R_i h(s')$ for all $s' \in M(R)$ and for all $i \in N$. Efficiency of F implies that $h(s') I_N s$ for all $s' \in M(R)$, and so $h(s') = s$ because $R \in \mathcal{R}$. However, since $U(R) = \emptyset$, there exists $s'' \in Z$ and an agent $i \in N$ such that $s'' P_i s$. Note that agent i has the power to move from s to s'' by Rule 4 and the incentive to do so since $s'' P_i s$. Since F satisfies efficiency and $s \in F(R)$, there must exist another agent $j \in N \setminus \{i\}$ such that $s P_j s''$. Since $s \in F(R)$, by

[26] $L_i((j_3, j_1, j_2), R) \subseteq L_i((j_3, j_1, j_2), R')$ for each agent i.

[27] It cannot be that $x(i+1,R) = (j_1, j_3, j_2)$ because this would lead to the contradiction that $x(i+2,R) = (j_3, j_1, j_2)$. The reason is that there are no preference reversal around (j_1, j_3, j_2) because R'' is a monotonic transformation of R at (j_1, j_3, j_2). Thus, we can only move to the next element of the ordered set at R'' to satisfy *rotation monotonicity*. Since the top-ranked job for agent 1 at P'' is j_1 and since, moreover, the top-ranked job for agent 3 at P'' is j_2, the allocation $x(i+2,R)$ must coincide with (j_3, j_1, j_2) because $(j_3, j_1, j_2)\, P_2''\, (j_1, j_3, j_2)$.

assumption, it follows that $(s,R) \in M(R)$. By Rule 3, agent j can move from s'' to (s,R). Hence, we have established a two-step myopic improvement path at R from s to $(s,R) \in M(R)$—that is, $i \in \gamma(s,s'')$ and $s'' P_i s$ and $j \in \gamma(s'',(s,R))$ and $h(s,R) P_j s''$. $\square$

Lemma 3. *For any $R' \in \mathscr{R}$, the set $Q(R,R')$ satisfies deterrence of external deviations and $h\left(Q(R,R')\right) = \{h(s) \in Z | s \in Q(R,R')\} \subseteq F(R)$.*

Proof of Lemma 3. Suppose that $Q(R,R') \neq \emptyset$ for some $R' \in \mathscr{R}$. Otherwise, there is nothing to be proved. Let us first prove that $h\left(Q(R,R')\right) \subseteq F(R)$. By definition, $Q(R,R') \subseteq M(R')$. Take any $(z',R') \in Q(R,R')$. Assume, to the contrary, that $h(z',R') = z' \notin F(R)$. Suppose that there exists an agent $i \in N$ such that $y P_i z'$ for some $y \in L_i(z',R')$. Then, by Rule 2, agent $i \in \gamma((z',R'),y)$ since $y \in L_i(z',R')$. An immediate contradiction is obtained if $y \in U(R)$ because there is a one-step myopic improvement from $Q(R,R')$ to $U(R)$. Suppose $y \in Z \setminus U(R)$. By Lemma 2, there is a finite myopic improvement path from y to $M(R) \cup U(R)$. Therefore, there exists a finite myopic improvement path from (z',R') to $M(R) \cup U(R)$, which contradicts the definition of $Q(R,R')$. Thus, it has to be that $L_i(z',R') \subseteq L_i(z',R)$ for all $i \in N$.

Let us proceed according to whether $\{z'\} = F(R')$ or not. Suppose that $\{z'\} = F(R')$. Since F satisfies *indirect monotonicity* and $L_i(z',R') \subseteq L_i(z',R)$ for all $i \in N$, it must be the case that $z' \in F(R)$, which is a contradiction. Suppose that $\{z'\} \neq F(R')$. Since $z' \in F(R') \setminus F(R)$ and since $L_i(z',R') \subseteq L_i(z',R)$ for all $i \in N$, *indirect monotonicity* implies that there exist a sequence of outcomes $\{z_1 \ldots, z_h\} \subseteq F(R')$ with $z' = z_1$ and $z' \neq z_h$ a sequence of agents $i_1, \ldots, i_{h-1}$ such that (i) $z_{k+1} P_{i_k} z_k$ for all $k \in \{1, \ldots, h-1\}$ and (ii) $L_i(z_h,R') \nsubseteq L_i(z_h,R)$ for some $i \in N$.

By Rule 1, part (i) of *indirect monotonicity* implies that there exists a finite myopic improvement path from (z',R') to $(z_h,R') \in M(R')$ at R. Part (ii) of *indirect monotonicity* implies that there exists a state $y \in L_i(z_h,R')$ such that $y P_i z_h$. By Rule 2, $\{i\} \in \gamma((z_h,R'),y)$. An immediate contradiction is obtained whenever $y \in U(R)$ because there is a finite myopic improvement path from (z',R') to $U(R)$ at R. Suppose that $y \in Z \setminus U(R)$. Then, by Lemma 2, there exists a finite myopic improvement path from y to $M(R) \cup U(R)$ at R. Therefore, there exists a finite myopic improvement path from (z',R') to $M(R) \cup U(R)$ at R, which contradicts our initial supposition that $(z',R') \in Q(R,R')$. We conclude that $h(Q(R,R')) \subseteq F(R)$.

To complete the proof of Lemma 3, let us show that $Q(R,R') \subseteq M(R')$ satisfies deterrence of external deviations at R. The only way to get out of this set is to use either Rule 1 or Rule 2. Therefore, from any state of $Q(R,R')$, agents can only deviate to $M(R') \setminus Q(R,R')$ or Z. Note that if $M(R') \setminus Q(R,R') \neq \emptyset$, then there exists a myopic improvement path to $M(R) \cup U(R)$ at R, by the definition of $Q(R,R')$. Also, note that from any state in $Z \setminus U(R)$, there exists a finite myopic improvement path to $M(R) \cup U(R)$ at R, by Lemma 2. Hence, if an agent could benefit by deviating from a state $s \in Q(R,R')$ to a state outside of $Q(R,R')$ at R, there would exist a myopic improvement path from s to $M(R) \cup U(R)$ at R, which would contradict the definition of $Q(R,R')$. $\square$

Lemma 4. *If V is a nonempty subset of S satisfying both deterrence of external deviations and iterated external stability at (Γ,R), then $M(R) \subseteq V$.*

Proof of Lemma 4. Let V be a nonempty subset of S satisfying both deterrence of external deviations and iterated external stability at (Γ,R). We show that $M(R) \subseteq V$. We proceed in two steps.

Step 1: $M(R) \cap V \neq \emptyset$. For the sake of contradiction, let $M(R) \cap V = \emptyset$. Then, by iterated external stability of V, there exists a sequence of states $s_1, \ldots, s_m$ with $s_1 \in M(R)$ and a collection of coalitions $K_1, \ldots, K_{m-1}$ such that, for $j = 1, \ldots, m-1$, $K_j \in \gamma(s_j, s_{j+1})$ and $h(s_{j+1}) P_{K_j} h(s_j)$. Moreover, $s_m \in V$. By definition of γ, by the fact that $s_1 \in M(R)$ and that $h(s_{j+1}) P_{K_j} h(s_j)$, we have that only Rule 1 applies, and so it has to be that $\{s_1, ..., s_m\} \subseteq M(R)$. Therefore, $s_m \in M(R) \cap V$ is a contradiction.

Step 2: $M(R) \subseteq V$. Take any $s \in M(R)$. Assume, to the contrary, that $s \notin V$. Since, by Step 1, $M(R) \cap V \neq \emptyset$, take any $s' \in M(R) \cap V$. Since $s, s' \in M(R)$, it must be the case that $h(s) \neq h(s')$. Suppose that for some $i \in N$, $h(s) P_i h(s')$. By Rule 1, agent i can move from s' to s, which contradicts the property of deterrence of external deviations of V. Therefore, it has to be that $h(s') R_N h(s)$. Since $R \in \mathscr{R}$, by our domain assumption, and $h(s) \neq h(s')$, it follows that $h(s') P_i h(s)$ for some $i \in N$. Since F is efficient, it follows that $h(s) \notin F(R)$, and so $s \notin M(R)$, which is a contradiction. Since the choice of s' is arbitrary and since, moreover, $s \in M(R)$, it follows that $M(R) \cap V = \emptyset$, which is a contradiction. Thus, it has to be that $M(R) \subseteq V$. $\square$

Lemma 5. *The set $M(R) \cup U(R) \cup Q(R)$ satisfies both deterrence of external deviations and iterated external stability at (Γ,R). Moreover, $F(R) = h \circ (M(R) \cup U(R) \cup Q(R))$.*

Proof of Lemma 5. By definition of Γ, the set $M(R)$ satisfies deterrence of external deviations. By Lemma 3, the set $Q(R)$ satisfies deterrence of external deviations. By definition, the set $U(R)$ satisfies deterrence of external deviations. Deterrence of external deviations is therefore satisfied by $M(R) \cup U(R) \cup Q(R)$. By Lemma 2, there is a finite myopic improvement path from $Z \setminus U(R)$ to $M(R) \cup U(R)$ at R. For any $R' \in \mathscr{R}$, by the definition of $Q(R,R')$, there is a myopic improvement path from $M(R') \setminus Q(R,R')$ to $M(R) \cup U(R)$ at R. This implies that for any state outside of $M(R) \cup U(R) \cup Q(R)$ there is a myopic improvement path to $M(R) \cup U(R)$ at R, and so iterated external stability is satisfied by $M(R) \cup U(R) \cup Q(R)$. $\square$

Lemma 6. *If V is a nonempty subset of S satisfying both deterrence of external deviations and iterated external stability at (Γ,R), then $M(R) \cup U(R) \cup Q(R) \subseteq V$.*

Proof of Lemma 6. By Lemma 4, we already know that $M(R) \subseteq V$. By iterated external stability of V, it has to be that $U(R) \subseteq V$—the reason is that no myopic improvement path can begin from a unanimously best outcome. We are left to show $Q(R) \subseteq V$. To this end, take any $R' \in \mathscr{R}$. Since $Q(R, R')$ satisfies deterrence of external deviations at (Γ, R) by Lemma 3, it follows that $Q(R, R') \subseteq V$, otherwise, iterated external stability of V is violated by the fact that $Q(R, R')$ satisfies deterrence of external deviations. Since R' is arbitrary, we conclude that $Q(R) \subseteq V$. Thus, $M(R) \cup U(R) \cup Q(R) \subseteq V$. □

Lemma 7. $M(R) \cup U(R) \cup Q(R) = MSS(\Gamma, R)$

Proof of Lemma 7. Lemma 5 implies that the set $M(R) \cup U(R) \cup Q(R)$ satisfies both deterrence of external deviations and iterated external stability at (Γ, R). Lemma 6 implies that the set $M(R) \cup U(R) \cup Q(R)$ is the smallest nonempty set satisfying these two properties. Therefore, the unique MSS of (Γ, R) consists of $M(R) \cup U(R) \cup Q(R)$. □

Lemma 8. $F(R) = h \circ (M(R) \cup U(R) \cup Q(R))$.

Proof of Lemma 8. Let us show that $F(R) = h \circ M(R) \cup U(R) \cup Q(R)$. Clearly, $F(R) \subseteq h \circ M(R)$, and so $F(R) \subseteq h \circ M(R) \cup U(R) \cup Q(R)$. For the converse, Lemma 3 implies that $h \circ Q(R, R') \subseteq F(R)$ for all $R' \in \mathscr{R}$. Since F is efficient, it follows that $U(R) \subseteq F(R)$. Moreover, by definition of $M(R)$, it follows that $h \circ M(R) \subseteq F(R)$. Therefore, $F(R) = h \circ M(R) \cup U(R) \cup Q(R)$. □

Proof of Proposition 1. Fix any Γ and any profile R. First, we show that $mss(\Gamma, R) = \mathcal{A}(\Gamma, R)$. To do this, we prove that $\mathcal{A}(\Gamma, R)$ satisfies deterrence of external deviations, iterated external stability and minimality. Deterrence of external deviations is implied by property (b) of the definition of absorbing sets. To prove iterated external stability, we exploit the topology of an induced graph: Take any $s \notin \mathcal{A}(\Gamma, R)$. Given such an s, let us define the set $H(s, R)$ by

$$H(s, R) = \{t \in S | \text{there is a finite myopic improving path from } s \text{ to } \{t\} \text{ at } R\} \cup \{s\}$$

Note that $H(s, R)$ is nonempty since $s \in H(s, R)$. Let us represent the set $H(s, R)$ by a finite directed graph D, that is, (i) $H(s, R)$ is the set of vertices of D, and (ii) D has a directed arc from t to v if and only if there exists a coalition $K \in \gamma(t, v)$ such that $vP_K t$. A subgraph D' of D is called strongly connected component if each vertex in D' is reachable from any other vertex in D'. By contracting each strongly connected component of D to a single vertex, we obtain a directed acyclic graph $\overline{D}$, which is called the condensation of D. A vertex in $\overline{D}$ is called super vertex. It is well known that a condensation is finite and acyclic.[28] As usual, the number of outgoing directed arcs of a vertex is called the *out-degree* of the vertex. If a vertex does not have any outgoing directed arcs, we say that the vertex has out-degree zero. By Theorem 3.8 in Hararay et al. (1966) we have that $\overline{D}$ has at least one super vertex of out-degree zero, which we name as V^0.

Recall that each super vertex of $\overline{D}$ represents a strongly connected component. Since V^0 is a strongly connected component of $\overline{D}$, it has the property that there are no outgoing arcs from any vertex in V^0 to any other vertex outside V^0. It is straightforward to see that such a V^0 is an absorbing set and, by construction of D, there is a finite myopic improvement path from s to a vertex in V^0 at R. Since the choice of $s \notin \mathcal{A}(\Gamma, R)$ is arbitrary, it follows that iterated external stability for $\mathcal{A}(\Gamma, R)$ is satisfied.

To prove minimality first we show that $A(\Gamma, R) \subseteq mss(\Gamma, R)$. Suppose, toward a contradiction, that there exits $s \in A(\Gamma, R)$ with $s \notin mss(\Gamma, R)$. Then, by iterated external stability of MSS, there exists a finite myopic improvement path from $\{s = s_1, ..., s_m = s'\}$ with $s' \in mss(\Gamma, R)$. By property (a) of the absorbing set, it has to be that $s' \in A(\Gamma, R)$. Moreover, since $s, s' \in A(\Gamma, R)$, property (a) of the absorbing set also implies that there exists a finite myopic improvement path from s' to s, that is, $\{s' = s'_1, ..., s'_\ell = s\}$. Let $s'_k \in \{s' = s'_1, ..., s'_\ell = s\}$ be a state with the property that $s'_k \in mss(\Gamma, R)$ and $s'_{k+1} \notin mss(\Gamma, R)$. By definition of finite improvement path, there is an agent i_k such that $\{i_k\} \in \gamma(s'_{k+1}, s'_k)$ and $s'_{k+1} \succ_{i_k} s'_k$. Thus, deterrence of external deviations is violated for $mss(\Gamma, R)$. Therefore, it has to be that $A(\Gamma, R) \subseteq mss(\Gamma)$. Since the choice of $A(\Gamma, R)$ is arbitrary, it follows that $\mathcal{A}(\Gamma, R) \subseteq mss(\Gamma)$.

Finally, minimality of $\mathcal{A}(\Gamma, R)$ follows by minimality of $mss(\Gamma)$ and by the proved fact that $\mathcal{A}(\Gamma, R) \subseteq mss(\Gamma)$. In the reaming part of the proof we show that the equality $\mathcal{A}(\Gamma, R) = \mathcal{V}(\Gamma, R)$ holds. First, we show that $\mathcal{A}(\Gamma, R) \subseteq \mathcal{V}(\Gamma, R)$. This part of the proof relies on the following statement that has been proved by Nicolas (2009)[29]:

Let $V(\Gamma, R) \subseteq S$ and $\mathcal{A}(\Gamma, R) = \bigcup_{i=1}^{m} A_i(\Gamma, R)$ for some $m \in \mathbb{N}$. $V(\Gamma, R)$ is a generalized stable set if and only if, for all adsorbing set $A_i(\Gamma, R) \subseteq \mathcal{A}(\Gamma, R)$, it holds that

(a) $|V(\Gamma, R) \cap A_i(\Gamma, R)| = 1 \ \forall A_i(\Gamma, R) \subseteq \mathcal{A}(\Gamma, R)$;
(b) $V(\Gamma, R) \subseteq \mathcal{A}(\Gamma, R)$.

This result implies that each $V(\Gamma, R)$ consists of an element of each absorbing set $A_i(\Gamma, R)$. This observation suggests a further characterization of the set $V(\Gamma, R)$. Take an element $v = \{s_1, ..., s_m\}$ of the Cartesian product of each absorbing set at R, that is, $v \in A_1(\Gamma, R) \times ... \times A_m(\Gamma, R)$. Then, define $V \subseteq S$ as the union of the elements of v, that is, $V = \{s \in S | s \in v\}$. Note that, by

[28] See Theorem 3.6 in Hararay et al. (1966).
[29] This result of Nicolas (2009) is a *corrigendum* of Inarra et al. (2005).

construction, property (a) and (b) are satisfied for V, then V is a generalized stable set at R. Since the choice of v is arbitrary, each set constructed in this way is a generalized stable set.

Therefore, we can write $\mathcal{V}(\Gamma, R)$ as the union of the element of the Cartesian product of each absorbing set at R. Formally,

$$\mathcal{V}(\Gamma, R) = \left\{ s \in S | s \in \{s_1, ..., s_m\} \in \bigtimes_{i=1}^{m} A_i(\Gamma, R) \right\}$$

Now, since the finite union of any sets must be a subset of the union of the elements of their Cartesian product, we can write:

$$\{s \in S | s \in A_i(\Gamma, R), \ \ i \in \{1, ..., m\}\} \subseteq \left\{ s \in S | s \in \{s_1, ..., s_m\} \in \bigtimes_{i=1}^{m} A_i(\Gamma, R) \right\}$$

The left hand side is the union of all absorbing sets at R, namely $\mathcal{A}(\Gamma, R)$. The right hand side is the union of the elements of the Cartesian product of the absorbing sets at R, namely $\mathcal{V}(\Gamma, R)$. It follows that $\mathcal{A}(\Gamma, R) \subseteq \mathcal{V}(\Gamma, R)$.

It remains to show that $\mathcal{V}(\Gamma, R) \subseteq \mathcal{A}(\Gamma, R)$. Since we already proved the equality $mss(\Gamma, R) = \mathcal{A}(\Gamma, R)$, it suffices to prove that $\mathcal{V}(\Gamma, R) \subseteq mss(\Gamma, R)$. Suppose, toward a contradiction, that there is an $s \in V(\Gamma, R)$ such that $s \notin mss(\Gamma, R)$. Then, by iterated external stability of MSS, there is finite myopic improvement path $\{s = s_1, ..., s_\ell\}$ with $s_\ell \in mss(\Gamma, R)$. Note that, it has to be that $s_\ell \notin V(\Gamma, R)$ otherwise iterated internal stability is violated for $V(\Gamma, R)$. Then, by iterated external stability of $V(\Gamma, R)$, there is a finite myopic improvement path $\{s_\ell, ..., s_m = t\}$ with $t \in V(\Gamma, R)$. But this means that there is a finite myopic improvement path $\{s = s_1, ..., s_\ell, ..., s_m = t\}$. If $s \neq t$, then the fact that $s, t \in V(\Gamma, R)$ contradicts iterated internal stability of $V(\Gamma, R)$. If $s = t$, then note that $t \notin mss(\Gamma, R)$ and $s_\ell \in mss(\Gamma, R)$. Let $s_k \in \{s_\ell, ..., s_m = t\}$ be a state with the property that $s_k \in mss(\Gamma, R)$ and $s_{k+1} \notin mss(\Gamma, R)$. By definition of finite improvement path, there is an agent i_k such that $\{i_k\} \in \gamma(s_{k+1}, s_k)$ and $s_{k+1} \succ_{i_k} s_k$. Thus, deterrence of external deviations is violated for $mss(\Gamma, R)$, which is a contradiction. □

Proof of Corollary 4. Fix any endowment system ω satisfying properties A1-A4. F_ω^{CO} is Pareto efficient because the direct exclusion core is efficient. In light of Corollary 1, we need only to show that F_ω^{CO} is monotonic. To this end, take any $\mu \in F_\omega^{CO}(R)$ for some $R \in \mathscr{R}$. Take any $R' \in \mathscr{R}$ such that $L_i(\mu, R) \subseteq L_i(\mu, R')$ for all i. Let us show that $\mu \in F_\omega^{CO}(R') = CO(R', \omega)$. Since $\mu \in CO(R, \omega)$, it follows that no coalition can directly exclusion block μ at R. That is, for all $K \in \mathcal{N}_0$ and all $\sigma \in \mathcal{M}$, $\mu(i) R_i \sigma(i)$ for some $i \in K$ or $[\mu(j) P_j \sigma(j)$ for some $j \in N \backslash K$ and $\mu(j) \notin \omega(K)]$. If $\mu(i) R_i \sigma(i)$ for some $i \in K$, it follows from the fact that R' is a monotonic transformation of R at μ that $\mu(i) R_i' \sigma(i)$ for some $i \in K$. If $\mu(j) P_j \sigma(j)$ for some $j \in N \backslash K$ and $\mu(j) \notin \omega(K)$, it follows from the fact that R' is a monotonic transformation of R at μ and the fact that R_j is a linear ordering that $\mu(j) P_j' \sigma(j)$ for some $j \in N \backslash K$ and $\mu(j) \notin \omega(K)$. We have that no coalition can directly exclusion block μ at R'. Thus, F_ω^{CO} is monotonic. □

Proof of Theorem 2. Suppose that Γ implements F in ordered cycle. Fix any R. Then, the set $MSS(\Gamma, R)$ is partitioned in ordered cycles $\{S_1, ..., S_m\}$ such that $h \circ S_i = F(R)$ for all $i = 1, ..., J$. Fix any ordered cycle $S_j = \{s_1, ..., s_m\}$ for some $m \in \mathbb{N}$. Let $x(i, R) = s_i = h(s_i)$ for all $s_i \in S_j$. Thus, $F(R)$ is an ordered set of $\#S_j = m \geqslant 1$ outcomes. Fix any R' such that $F(R') \neq F(R)$. Suppose that either $\#F(R') > 1$ or $[\#F(R') = 1$ and $F(R') \notin F(R)]$. Fix any $s_i \in S_j$. We proceed according to whether $s_i \in MSS(\Gamma, R')$ or not.

Case 1: $s_i \in MSS(\Gamma, R')$ By the implementability of F, $h(s_i) \in F(R) \cap F(R')$. Since by the assumption that $F(R') \notin F(R)$ whenever $\#F(R') = 1$, it must be that $\#F(R') > 1$. Since Γ implements F in ordered cycle, the set $MSS(\Gamma, R')$ is partitioned in ordered cycles $\{\bar{S}_1, ..., \bar{S}_m\}$ such that $h \circ \bar{S}_i = F(R')$ for all $i = 1, ..., m$. Then, there exists a unique j such that $s_i \in \bar{S}_j$. Without loss of generality, let $s_i = s_1 \in \bar{S}_j$.

Since $\bar{S}_j$ is an ordered cycle and since $\#F(R') > 1$, it follows that there exist $s_2 \in \bar{S}_j \backslash \{s_1\}$ and a coalition K_1 such that $K_1 \in \gamma(s_1, s_2)$ and $h(s_2) P_{K_1}' h(s_1)$. Suppose that there exists $i_1 \in K_1$ such that $h(s_1) R_{i_1} h(s_2)$. Then, there exists $h(s_2) \in Z$ such that $h(s_2) P_{i_1}' h(s_1)$ and $h(s_1) R_{i_1} h(s_2)$, where $h(s_1) = h(s_i) = x(i, R)$. Otherwise, suppose that $h(s_2) P_{K_1} h(s_1)$. Since S_j is an ordered cycle, it follows that $s_2 = s_{i+1} \in S_j$ and $h(s_{i+1}) = x(i+1, R)$.

The above argument can be applied to $s_2 = s_{i+1} \in \bar{S}_j$ to derive a state $s_3 \in \bar{S}_j \backslash \{s_2\}$ and a coalition K_2 such that $K_2 \in \gamma(s_2, s_3)$ and $h(s_3) P_{K_2}' h(s_2)$ where $h(s_2) = x(i+1, R)$. Suppose that $s_3 = s_1$. Since $\bar{S}_j$ is an ordered cycle, it follows that $\bar{S}_j = \{s_1, s_2\}$. Since $F(R') \neq F(R)$, it follows that $s_3 = s_1 \neq s_{i+2} \in S_j$. It follows that there exists $i_2 \in K_2$ such that $h(s_1) P_{i_2}' h(s_2)$ and $h(s_2) R_{i_2} h(s_1)$. Thus, $z P_{i_2}' x(i+1, R) P_{i_1}' x(i, R)$ and $x(i+1, R) R_{i_2} z$ where $z = h(s_1) = x(i, R) \in Z$. Suppose that $s_3 \neq s_1$. Then, $s_3 \in \bar{S}_j \backslash \{s_1, s_2\}$. Suppose that there exists $i_2 \in K_2$ such that $h(s_2) R_{i_2} h(s_3)$. Thus, there exists $h(s_3) = z \in Z$ such that $h(s_3) P_{i_2}' h(s_2) P_{i_1}' h(s_1)$ and $h(s_2) R_{i_2} h(s_3)$, where $h(s_1) = h(s_i) = x(i, R)$ and $h(s_2) = h(s_{i+1}) = x(i+1, R)$. Otherwise, suppose that $h(s_3) P_{K_2} h(s_2)$. Since S_j is an ordered cycle, it follows that $s_3 = s_{i+2} \in S_j$ and $h(s_{i+2}) = x(i+2, R)$. And, so on.

Since $\bar{S}_j \neq S_j$, after a finite number $1 \leqslant h \leqslant m$ of iterations, $s_1, s_2, ..., s_{h+1}$ states and $i_1, i_2, .., i_h$ agents can be derived such that $s_1, ..., s_h \in \bar{S}_j \cap S_j$, with $h(s_\ell) = h(s_{i+\ell-1}) = x(i+\ell-1, R)$ for all $\ell = 1, ..., h$, $s_{h+1} \in \bar{S}_j$, $h(s_{h+1}) = z \in Z$ and for all $\ell \in \{1, ..., h\}$, $h(s_{\ell+1}) P_{i_\ell}' h(s_\ell)$ and $h(s_h) R_{i_h} h(s_{h+1})$.

Case 2: $s_i \notin MSS(\Gamma, R')$. By iterated external stability of $MSS(\Gamma, R')$, there exists a finite myopic improvement path from s_i to $t \in MSS(\Gamma, R')$; that is, there are coalitions $\{K_1, ..., K_{q-1}\}$ and states $\{s_i = t_1, t_2, ..., t_q = t\}$ such that for all $p = 1, ..., q-1$,

$K_p \in \gamma(t_p, t_{p+1})$ and $h(t_{p+1})\, P'_{K_p} h(t_p)$. Since Γ implements F in ordered cycle, the set $MSS(\Gamma, R')$ is partitioned in ordered cycles $\{\bar{S}_1, ..., \bar{S}_m\}$ such that $h \circ \bar{S}_i = F(R')$ for all $i = 1, ..., m$. Then, there exists a unique j such that $t_q \in \bar{S}_j$.

Suppose that $t_2 \neq s_{i+1}$. Since S_j is an ordered cycle and $s_i = t_1 \in S_j$, it follows that there exists $i_1 \in K_1$ such that $h(t_1)\, R_{i_1} h(t_2)$ where $h(t_1) = h(s_i) = x(i, R)$. Therefore, $h(t_2)\, P'_{i_1} h(t_1)$ and $h(t_1)\, R_{i_1} h(t_2)$, as we sought. Otherwise, suppose that $t_2 = s_{i+1} \in S_j$. If there exists $i_1 \in K_1$ such that $h(t_1)\, R_{i_1} h(t_2)$, then again $h(t_2)\, P'_{i_1} h(t_1)$ and $h(t_1)\, R_{i_1} h(t_2)$. Otherwise, suppose that $t_2 = s_{i+1} \in S_j$, $h(t_2) = x(i+1, R)$ and $h(t_2)\, P_{K_1} h(t_1)$.

The reasoning used above can be applied to t_3 to conclude that either there exists $i_2 \in K_2$ such that $h(t_2)\, R_{i_2} h(t_3)$ for some $i_2 \in K_2$ or $h(t_3)\, P_{K_2} h(t_2)$ and $t_3 = s_{i+2} \in S_j$.

In the former case, we have that $h(t_3)\, P'_{i_2} h(t_2)\, P'_{i_1} h(t_1)$ and $h(t_2)\, R_{i_2} h(t_3)$, where $h(t_1) = x(i, R)$ and $h(t_2) = x(i+1, R)$. In the latter case, we have that $h(t_3) = x(i+2, R)$ and $h(t_3)\, P_{K_2} h(t_2)$.

Since the myopic improvement path from s_i to $t \in MSS(\Gamma, R')$ is finite, after a finite number $1 \leqslant r \leqslant q-1$ of iterations, we have that $h(t_{p+1})\, P'_{i_p} h(t_p)$ for all $p = 1, ..., r$, and either $[h(t_r)\, R_{i_r} h(t_{r+1})$ for some $i_r \in K_r]$ or $[r = q-1$, $h(t_{p+1})\, P_{K_p} h(t_p)$ and $t_p = s_{i+p-1} \in S_j$ for all $p = 1, ..., r$, and $t_q \in S_j \cap \bar{S}_j]$. In the former case, we have that for all $p = 1, ..., r$, $h(t_{p+1})\, P'_{i_p} h(t_p)$ and $h(t_r)\, R_{i_r} h(t_{r+1})$, where $h(t_p) = h(s_{i+p-1}) = x(i+p-1)$ for all $p = 1, ..., r$. In the latter case, since $t_q \in \bar{S}_j$, it follows that $t_q \in MSS(\Gamma, R')$. Case 1 above can be applied to the outcome $h(t_q) = h(s_{i+q-1}) = x(i+q-1) \in F(R)$ to complete the proof. $\square$

Proof of Theorem 3. The implementing rights structure is a variant of the rights structure constructed in the proof of Theorem 1. The only change concerns the definition of Rule 1. The state space is $S = Gr(F) \cup Z$. The outcome function is $h(x, R) = x$ for all $(x, R) \in Gr(F)$ and $h(x) = x$ for all $x \in Z$. The code of rights γ is defined as follows. For all $i \in N$, all $R \in \mathscr{R}$ and all $s, t \in S$:

RULE 1: If $s = (x(k, R), R)$ and $t = (x(k+1, R), R)$ for some $1 \leqslant k \leqslant m$, then $\{i\} \in \gamma((x(k, R), R), (x(k+1, R), R))$, where the outcomes $x(k, R)$ are those specified by properties 1 and 2.

RULE 2: If $s = (z, R)$, $t = x$ and $x \in L_i(z, R)$, then $\{i\} \in \gamma((z, R), x)$.

RULE 3: If $s = x$ and $t = (z, R)$, then $\{i\} \in \gamma(x, (z, R))$.

RULE 4: If $s = z$ and $t = x$, then $\{i\} \in \gamma(s, t)$.

RULE 5: Otherwise, $\gamma(s, t) = \emptyset$.

Rule 1 allows agent i to be effective only between two consecutive socially optimal outcomes at R, that is, between $(x(k, R), R)$ and $(x(k+1, R), R)$ for all $1 \leqslant k \leqslant m$. Fix any R. Let us show that Γ implements F in ordered cycles. We first show that $F(R) = h \circ MSS(\Gamma, R)$ and then we show that Γ partitions $MSS(\Gamma, R)$ in ordered cycles such that for each ordered cycle S, it holds that $F(R) = h \circ S$. To show that $F(R) = h \circ MSS(\Gamma, R)$ and that $MSS(\Gamma, R) = M(R) \cup U(R) \cup Q(R)$, we need to show that Lemmata 1-7 still hold under the new rights structure Γ. The proofs of Lemma 3 and Lemma 4 need to be amended. As far as the proof of Lemma 4 is concerned, the arguments provided to prove Step 2 of Lemma 4 no longer hold. However, the statement of this step is still true under the new Γ. To show this, take any $s = (x(i, R), R) \in M(R) \cap V$, which exists by Step 1 of the proof of Lemma 4. We show that $M(R) \subseteq V$. Assume, to the contrary, there exists $s' = (x(i', R), R) \in M(R)$ such that $s' \notin V$. To complete the proof of Lemma 4, let us first show that $M(R)$ is a ordered cycle. Since F is efficient and since $\mathscr{R}$ satisfies the restriction in (1), it follows that for all $1 \leqslant k \leqslant m$ and all $(x(k, R), R), (x(k+1, R), R) \in M(R)$, there exists $j \in N$ such that $x(k+1, R)\, P_j x(k, R)$. By definition of Rule 1, it follows that for each $1 \leqslant k \leqslant m$, there exists $j \in N$ such that $\{j\} \in \gamma((x(k, R), R), (x(k+1, R), R))$ and $x(k+1, R)\, P_j x(k, R)$. Moreover, by definition of γ, it follows that $M(R)$ is a ordered cycle because for each $(x(k, R), R)$, there do not exist any $K \in \mathcal{N}_0$ and any $s \in S$, with $s \neq (x(k, R), R)$ and $s \neq (x(k+1, R), R)$, such that $K \in \gamma((x(k, R), R), s)$ and $h(s)\, P_K x(k, R)$. Let us now complete the proof of Lemma 4. Since for each $1 \leqslant k \leqslant m$ there exists $j \in N$ such that $\{j\} \in \gamma((x(k, R), R), (x(k+1, R), R))$ and $x(k+1, R)\, P_j x(k, R)$, it follows that there exist $s_0, s_1, ..., s_{p-1}, s_p$, with $s_0 = s$ and $s_p = s'$, and $i_0, ..., i_{p-1}$ such that $i_h \in \gamma(s_h, s_{h+1})$ and $h(s_{h+1})\, P_{i_h} h(s_h)$ for all $h = 0, ..., p-1$, where $s_h \in M(R)$ for all $h = 0, 1, ..., p$. Since $s_0 \in M(R) \cap V$ and $s_p \in M(R) \backslash V$, there exists the smallest index $h^* \in \{0, ..., p-1\}$ such that $s_{h^*} \in M(R) \cap V$ and $s_{h^*+1} \in M(R) \backslash V$. Since $i_{h^*} \in \gamma(s_{h^*}, s_{h^*+1})$ and $h(s_{h^*+1})\, P_{i_{h^*}} h(s_{h^*})$, this contradicts our initial supposition that V satisfies the property of deterrence of external deviations. Thus, we have that $M(R) \subseteq V$, and so Lemma 4 holds as well.

As far as the proof of Lemma 3 is concerned, it needs to be amended as follows. Fix any $R' \in \mathscr{R}$. The proof of Lemma 3 holds if $\#F(R) \neq 1$ or if $\#F(R) = 1$ and $F(R) \notin F(R')$. The reason is that in these cases *rotation monotonicity* implies *indirect monotonicity*. To complete the proof of Lemma 3, let us suppose that $\#F(R) = 1$ and $F(R) \in F(R')$. Suppose that $F(R) = \{a\} \neq F(R') = \{z(1, R'), ..., z(m, R')\}$. Without loss of generality, let $a = z(1, R')$. Suppose that *Property M* implies that for each $z(i, R') \in F(R') \setminus \{z(1, R')\}$, there exist $x \in Z$ and $i_1, ..., i_h$, with $1 \leqslant h \leqslant m$, such that:

$$z(i+\ell+1, R')\, P_{\ell+1} z(i+\ell, R') \text{ for all } \ell \in \{0, ..., h-1\} \quad \text{and}$$
$$z(i+h, R')\, P_h x \text{ and } x R'_h z(i+h, R').$$

By definition of γ, we have that for each $z(i,R') \in F(R') \setminus \{z(1,R')\}$, there exists a finite myopic improvement path from $(z(i,R'),R')$ to x. Suppose that $U(R) \neq \emptyset$. Since F is efficient and since, moreover, $\mathscr{R}$ satisfies the restriction in (1), it follows that $U(R) = \{z(1,R')\}$. Since by Rule 2 there exists a finite myopic improvement path from x to $z(1,R')$, it follows that there exists a finite myopic improvement path from $z(i,R') \in F(R') \setminus \{z(1,R')\}$ to $M(R) \cup U(R)$. Suppose that $U(R) = \emptyset$. Since Lemma 2 implies that there exists a finite myopic improvement path from x to $M(R) \cup U(R)$, we conclude that there exists a finite myopic improvement path from $z(i,R') \in F(R') \setminus \{z(1,R')\}$ to $M(R) \cup U(R)$. It follows from the definition of $Q(R,R') \subseteq M(R')$ that $Q(R,R') = \emptyset$ if there exists a finite myopic improvement path from $(z(1,R'),R')$ to $M(R) \cup U(R)$, otherwise, $Q(R,R') = \{(z(1,R'),R')\}$. In either case, we have that $h \circ Q(R,R') \subseteq F(R)$ and that $Q(R,R')$ satisfies the property of deterrence of external deviations. Note that $Q(R,R') = \{(z(1,R'),R')\}$ satisfies this property for the following two reasons: 1) Since every agent i is effective in moving the state from $(z(1,R'),R')$ to $(z(2,R'),R')$, it cannot be that $z(2,R')\, P_i z(1,R')$ for some i, otherwise, since we have already shown that there exists a finite myopic improvement path from $(z(1,R'),R')$ to $M(R) \cup U(R)$, it follows that $Q(R,R') = \emptyset$, which is a contradiction; and 2) it cannot be that $x P_i z(1,R')$ for some i and some $x \in L_i(z(1,R'),R')$, otherwise, since Rule 2 implies that $\{i\} \in \gamma((z(1,R'),R'),x)$ and $x P_i z(1,R')$ and since, moreover, Lemma 2 implies that there exists a finite myopic improvement path from x to $M(R) \cup U(R)$, since we have already shown that there exists a finite myopic improvement path from $(z(1,R'),R')$ to $M(R) \cup U(R)$, it follows that $Q(R,R') = \emptyset$, which is a contradiction. Suppose that the above arguments do not hold for some $z(i,R') \in F(R') \setminus \{z(1,R')\}$. Clearly, for each $z(i,R') \in F(R') \setminus \{z(1,R')\}$ such that the above arguments hold, we have that there exists a finite myopic improvement path from $z(i,R') \in F(R') \setminus \{z(1,R')\}$ to $M(R) \cup U(R)$. *Property M* implies that $L_i(z(1,R'),R') \cup \{z(2,R')\} \subseteq L_i(z(1,R'),R)$ for all $i \in N$. For each $z(i,R') \in F(R') \setminus \{z(1,R')\}$ for which the above arguments do not hold, *Property M* implies that there exists a sequence of agents $i_1, ..., i_\ell$ such that

$$z(1,R')\, P_{i_\ell} z(m,R')\, P_{i_{\ell-1}} \cdots P_{i_2} z(i+1,R')\, P_{i_1} z(i,R') \tag{2}$$

Since every agent i can be effective in moving the state from $(z(1,R'),R')$ to $(z(2,R'),R')$, it follows that no agent has an incentive to do so because $z(2,R') \in L_i(z(1,R'),R)$ for all $i \in N$. Since, by Rule 1, each agent $i \in \{i_1, ..., i_\ell\}$ is effective in moving between two consecutive states in $M(R')$, it follows from (2) that there exists a finite myopic improvement path from $(z(i,R'),R')$ to $(z(1,R'),R')$. We conclude that for each $z(i,R') \in F(R) \setminus \{z(1,R')\}$, there exists a finite myopic improvement path from $(z(i,R'),R')$ to either $M(R) \cup U(R)$ or to $\{(z(1,R'),R')\}$. It follows that $Q(R,R') \subseteq \{(z(1,R'),R')\}$. Again, $Q(R,R') = \emptyset$ if there exists a finite myopic improvement path from $(z(1,R'),R')$ to $M(R) \cup U(R)$, otherwise, $Q(R,R') = \{(z(1,R'),R')\}$. In either case, we have that $h \circ Q(R,R') \subseteq F(R)$ and that $Q(R,R')$ satisfies the property of deterrence of external deviations. Since the choice of $R' \in \mathscr{R}$ is arbitrary, it follows that Lemma 3 holds. Since Properties 1-2 imply that Lemmata 1-7 hold, it follows that $F(R) = h \circ MSS(\Gamma,R)$ and that $MSS(\Gamma,R) = M(R) \cup U(R) \cup Q(R)$.

To show that Γ partitions $MSS(\Gamma,R)$ in rotation programs, we proceed according to whether $\#F(R) = 1$ or not. We have shown above that $M(R)$ is a ordered cycle.

Case 1: $\#F(R) \neq 1$. The set $U(R) = \emptyset$. To see it, suppose that there exists $x \in U(R)$. Since F is efficient and since, moreover, $\mathscr{R}$ satisfies the restriction in (1), it follows that $F(R) = \{x\}$, which is a contradiction. Thus, $MSS(\Gamma,R) = M(R) \cup Q(R)$. We have shown above that $M(R)$ is a ordered cycle. Moreover, by its definition, it follows that $F(R) = h \circ M(R)$.

Fix any $R' \in \mathscr{R}$ such that $F(R') \neq F(R)$. We show that $Q(R,R') = \emptyset$. Fix any $z(i,R') \in F(R')$. *Rotation monotonicity* implies that there exist $x \in Z$ and a sequence of agents $i_1, ..., i_h$, with $1 \leqslant h \leqslant m$, such that:

$$z(i+\ell+1,R')\, P_{i_{\ell+1}} z(i+\ell,R') \text{ for all } \ell \in \{0,...,h-1\} \text{ and}$$
$$z(i+h,R')\, R'_{i_h} x \text{ and } x P_{i_h} z(i+h,R').$$

Since, by Rule 1, for each $\ell \in \{0,...,h-1\}$, $\{i_{\ell+1}\} \in \gamma(z(i+\ell,R'), z(i+\ell+1, R'))$ and since, moreover, by Rule 2, $\{i_h\} \in \gamma(z(i+h,R'),x)$, it follows that there exists a finite myopic improvement path from $(z(i,R'),R')$ to x. Since $U(R) = \emptyset$, Lemma 2 implies that there exists a finite myopic improvement path from x to $M(R)$. Therefore, we have established that there exists a finite myopic improvement path from $(z(i,R'),R')$ to $M(R)$, and so $(z(i,R'),R') \notin Q(R,R')$. Since the choice of $z(i,R') \in F(R')$ is arbitrary, we have that $Q(R,R') = \emptyset$.

Fix any $R' \in \mathscr{R}$ such that $F(R') = F(R)$. Nothing has to be proved if $Q(R,R') = \emptyset$. Suppose that $Q(R,R') \neq \emptyset$. We show that $Q(R,R') = M(R')$ and that $Q(R,R')$ is a ordered cycle. Since F is efficient and since $\mathscr{R}$ satisfies the restriction in (1), it follows that for all $(x(k,R'),R'), (x(k+1,R'), R') \in M(R')$, there exists $j \in N$ such that $x(k+1,R')\, P_j x(k,R')$. By definition of Rule 1, it follows that for each $1 \leqslant k \leqslant m$, there exists $j \in N$ such that $\{j\} \in \gamma((x(k,R'),R'),(x(k+1,R'),R'))$ and $x(k+1,R')\, P_j x(k,R')$. If there exists a finite myopic improvement path from some $(x(i,R'),R') \in M(R') \setminus Q(R,R')$ to $M(R) \cup U(R)$, it follows that for each state in $M(R')$ there exists a finite myopic improvement path to $M(R) \cup U(R)$. This implies that $Q(R,R') = \emptyset$, which is a contradiction. Thus, $Q(R,R') = M(R')$. Since Lemma 3 implies that $Q(R,R')$ satisfies the property of deterrence of external deviations, it follows that $Q(R,R')$ is an ordered cycle. Since the choice of $R' \in \mathscr{R}$, with $F(R') = F(R)$, is arbitrary, it follows that $MSS(\Gamma,R)$ is the union of partitioned ordered cycles because, for all $R', R'' \in \mathscr{R}$ such that $F(R') = F(R'') = F(R)$, it holds that $h \circ M(R') = h \circ M(R'')$ and $M(R') \cap M(R'') = \emptyset$. Thus, F is implementable in ordered cycles.

Case 2: $\#F(R) = 1$. Recall that $MSS(\Gamma,R) = M(R) \cup U(R) \cup Q(R)$. Let $F(R) = \{z(1,R)\}$. Note that $M(R) = (z(1,R),R)$. Also, note that if $U(R) \neq \emptyset$, it follows from the efficiency of F and the restriction of $\mathscr{R}$ in (1) that $U(R) = \{z(1,R)\}$. Note that $M(R)$

and $U(R)$ are ordered cycles such that $M(R) \cap U(R) = \emptyset$. To proof is complete if we show that for all $R' \in \mathscr{R}$, either $Q(R, R') = \emptyset$ or $Q(R, R') = \{(z(1, R), R')\}$. To this end, fix any $R' \in \mathscr{R}$. Suppose that $F(R) = \{z(1, R)\} \neq F(R')$. Let us proceed according whether $F(R) \in F(R')$ or not. Suppose that $F(R) \notin F(R')$. Fix any $z(i, R') \in F(R')$. By the same arguments provided in Case 1 above, it follows that there exists a finite myopic improvement path from $(z(i, R'), R')$ to x. If $U(R) \neq \emptyset$, then there exists a finite myopic improvement path from $(z(i, R'), R')$ to $z(1, R) \in U(R)$. Otherwise, if $U(R) = \emptyset$, Lemma 2 implies that there exists a finite myopic improvement path from x to $M(R)$. Therefore, there exists a finite myopic improvement path from $(z(i, R'), R')$ to $M(R) \cup U(R)$, and so $(z(i, R'), R') \notin Q(R, R')$. Since the choice of $z(i, R') \in F(R')$ is arbitrary, we have that $Q(R, R') = \emptyset$. Suppose that $F(R) \in F(R') = \{z(1, R'), ..., z(m, R')\}$. Without loss of generality, suppose that $z(1, R) = z(1, R')$. By arguing as we have done above in the completion of the proof of Lemma 3, we have that either $Q(R, R') = \emptyset$ or $Q(R, R') = \{(z(1, R'), R')\}$, as we sought. □

Proof of Theorem 4 In light of Theorem 2, it suffices to show that F satisfies properties 1 and 2. Since $\#F(R) > 1$ for all $R \in \bar{\mathscr{R}}$, by Corollary 3 it follows that *Property M* is vacuously satisfied. Therefore, let us show that F satisfies *rotation monotonicity* as well. To this end, we need to introduce additional notation.

For all $R \in \bar{\mathscr{R}}$ and all $i \in N$, let $N_i(R)$ denote the set of efficient allocations at R that assign j_1^* to agent i, with $n_i(R)$ representing the number of elements in $N_i(R)$. Since J is a finite set, it follows that $N_i(R)$ is a finite set. For all $R \in \bar{\mathscr{R}}$ and all $i \in N$, let $\tau_2(i, R)$ denote the job ranked second of agent i at R_i. For all $x \in \bar{J}$ and all $R \in \bar{\mathscr{R}}$, let $\bar{x}(R)$ be a permutation of x such that (i) the agent who obtains j_1^* at x, let us say agent i, had his job ranked second $\tau_2(i, R)$ at $\bar{x}(R)$; (ii) the agent who obtains agent i's job ranked second at x obtains j_1^* at $\bar{x}(R)$; whereas (iii) all other agents obtain the same job both at x and at $\bar{x}(R)$. Formally, $\bar{x}_i(R) = \tau_2(i, R)$ if $x_i = j_1^*$, $\bar{x}_j(R) = j_1^*$ if $x_j = \tau_2(i, R)$, and $x_h = \bar{x}_h(R)$ for all $h \in N \setminus \{i, j\}$.

The proof that F satisfies *rotation monotonicity* relies on the following lemmata.

Lemma 9. *For all $R \in \bar{\mathscr{R}}$ and all $i \in N$, $\sum_{j \in N \setminus \{i\}} n_j(R) \geqslant n_i(R)$.*

Proof of Lemma 9. The statement follows if we show that for all $R \in \bar{\mathscr{R}}$ and all $i \in N$, there exists an injective function g_i^R from $N_i(R)$ to $\bigcup_{j \in N \setminus \{i\}} N_j(R)$, that is, if we show that for all $R \in \bar{\mathscr{R}}$ and all $i \in N$, every two distinct elements of $N_i(R)$ have distinct images in $\bigcup_{j \in N \setminus \{i\}} N_j(R)$ under g_i^R. Let us define $g_i^R : N_i(R) \longrightarrow \bigcup_{j \in N \setminus \{i\}} N_j(R)$ by $g_i^R(x) = \bar{x}(R)$. Take any two distinct $x, y \in N_i(R)$. Then, $g_i^R(x) = \bar{x}(R)$ and $g_i^R(y) = \bar{y}(R)$. Suppose that $x_j = y_j = \tau_2(i, R)$ for some $j \in N \setminus \{i\}$. Since $x \neq y$, it follows that $x_h \neq y_h$ for some $h \in N \setminus \{i, j\}$. It follows that $\bar{x}(R) \neq \bar{y}(R)$. Suppose that $x_j = \tau_2(i, R)$ and $y_h = \tau_2(i, R)$ for some $h, j \in N \setminus \{i\}$ such that $h \neq j$. It follows that $\bar{x}(R) \neq \bar{y}(R)$. Thus, g_i^R is an injective function. □

Lemma 10. *For all $R \in \bar{\mathscr{R}}$, elements of $F(R)$ can be ordered as $x(1, R), ..., x(m, R)$, with $m = \sum_{i \in N} n_i(R) > 1$, such that for all $k = 1, ..., m$, if $x_i(k, R) = j_1^*$ for some $i \in N$, then $x_i(k+1, R) \neq j_1^*$ (mod m).*

Proof of Lemma 10. Fix any $R \in \bar{\mathscr{R}}$. Without loss of generality, let us assume that $n_1(R) \geqslant n_2(R) \geqslant ... \geqslant n_{n-1}(R) \geqslant n_n(R)$. Let us apply the following procedure to arrange allocations of $F(R)$ in a way that the statement holds:

Step 0: If $n_1(R) - n_2(R) = 0$, then go to Step 1. If $n_1(R) - n_2(R) = k_0 > 0$, then take any $A \subseteq N_1(R)$ such that $\#A = k_0$. By Lemma 9, there exists $3 \leqslant h \leqslant n$ such that $\sum_{i=h}^n n_i(R) \geqslant k_0$ and $\sum_{i=h+1}^n n_i(R) < k_0$. Then, select any $B \subseteq N_h(R)$ such that $\sum_{i=h+1}^n n_i(R) + \#B = k_0$. List elements of the set A and elements of the set $B \cup \left(\cup_{i=h+1}^n N_i(R)\right)$ in a way that no element of A stands next to another element of set A. Start the list with an element of $A \subseteq N_1(R)$. By construction, no two consecutive allocations of the list allocate j_1^* to the same agent.

Step 1: Then, $n_1(R) - k_0 - n_2(R) = 0$, with $k_0 \geqslant 0$ and $n_1(R) - k_0 = n_2(R) \geqslant ... \geqslant n_h(R) - \#B$, where $B = \emptyset$ and $h = n$ if $n_1(R) = n_2(R)$. Let $n_h(R) - \#B = k_1$. Construct a sequence $\{x_i\}_{i=1}^h$ of elements in $\bigcup_{i=1}^h N_i(R) \setminus (A \cup B)$ (of length equal to h) such that $x_i \in N_i(R)$ for all $i = 1, ..., h$. Thus, the sequence is constructed in a way that no element of $N_i(R)$ stands next to another element of $N_i(R)$, and the last element of the sequence belongs to $N_h(R)$. Since there are k_1 sequences of this type, list these sequences one after the other. By construction, no two consecutive allocations of this arrangement allocate j_1^* to the same agent. Join this linear arrangement to the right end of the arrangement of Step 0. If $n_h(R) - \#B = n_1(R) - k_0$, then the derived linear arrangement can be transformed into a circular arrangement by joining its ends. Otherwise, move to Step 2. For each $i = 1, ..., h-1$, let A_{1i} denote the set of elements of $N_i(R)$ used to construct the sequences. Thus, for each $i = 1, ..., h-1$, $\#A_{1i} = k_1$ and $N_i(R) \setminus A_{1i}$ is the set of allocations that still needs to be arranged.

Step 2: Then, $n_1(R) - k_0 - k_1 = n_2(R) - k_1 \geqslant ... \geqslant n_{h-1}(R) - k_1$. Let $n_{h-1}(R) - k_1 = k_2$. Construct a sequence $\{x_i\}_{i=1}^{h-1}$ of elements in

$$\bigcup_{i=1}^h N_i(R) \setminus \left(A \cup B \cup \left(\bigcup_{i=1}^{h-1} A_{1i}\right)\right)$$

(of length equal to $h-1$) such that $x_i \in N_i(R)$ for all $i = 1, ..., h-1$. Thus, the sequence is constructed in a way that no element of $N_i(R)$ stands next to another element of $N_i(R)$, and the last element of the sequence belongs to $N_{h-1}(R)$. Since there are k_2

sequences of this type, list these sequences one after the other. By construction, no two consecutive allocations of this arrangement allocate j_1^* to the same agent. Join this linear arrangement to the right end of the arrangement of Step 1. If $n_{h-1}(R) - k_1 - k_2 = n_1(R) - k_0 - k_1 - k_2$, then the derived linear arrangement can be transformed into a circular arrangement by joining its ends. Otherwise, move to Step 4. For each $i = 1, ..., h-2$, Let A_{2i} denote the set of elements of $N_i(R)$ used to construct the sequences. Thus, for each $i = 1, ..., h-2$, $\#A_{2i} = k_2$ and $N_i(R) \setminus \left(A_{1i} \cup A_{2i}\right)$ is the set of allocations that still needs to be arranged.

⋮

Step ℓ: Then, $n_1(R) - \sum_{i=0}^{\ell-1} k_i = n_2(R) - \sum_{i=1}^{\ell-1} k_i \geqslant ... \geqslant n_{h-(\ell-1)}(R) - \sum_{i=1}^{\ell-1} k_i$. Let $n_{h-(\ell-1)}(R) - \sum_{i=1}^{\ell-1} k_i = k_\ell$. Construct a sequence $\left\{x_i\right\}_{i=1}^{h-(\ell-1)}$ of elements in $\bigcup_{i=1}^{h-(\ell-1)} N_i(R) \setminus \left(A \cup B \cup \left(\bigcup_{i=1}^{h-(\ell-1)} \bigcup_{j=1}^{\ell-1} A_{ji}\right)\right)$ (of length equal to $h - (\ell - 1)$) such that $x_i \in N_i(R)$ for all $i = 1, ..., h - (\ell - 1)$. Thus, the sequence is constructed in a way that no element of $N_i(R)$ stands next to another element of $N_i(R)$, and the last element of the sequence belongs to $N_{h-1}(R)$. Since there are k_ℓ sequences of this type, list these sequences one after the other. By construction, no two consecutive allocations of this arrangement allocate j_1^* to the same agent. Join this linear arrangement to the right end of the arrangement of Step $\ell - 1$. If $n_{h-(\ell-1)}(R) - \sum_{i=1}^{\ell-1} k_i = n_1(R) - \sum_{i=0}^{\ell-1} k_i$, then the derived linear arrangement can be transformed into a circular arrangement by joining its ends. Otherwise, move to Step $\ell + 1$. For each $i = 1, ..., h - \ell$, Let $A_{\ell i}$ denote the set of elements of $N_i(R)$ used to construct the sequences. Thus, for each $i = 1, ..., h - \ell$, $\#A_{\ell i} = k_\ell$ and $N_i(R) \setminus \left(\bigcup_{j=1}^{\ell} A_{ji}\right)$ is the set of allocations that still needs to be arranged.

⋮

Since the set of allocations is finite, the above procedure is finite, and it produces a circular arrangement of elements of $F(R)$ such that no two consecutive allocations allocate j_1^* to the same agent. □

For each $R \in \bar{\mathscr{R}}$, Lemma 10 implies that elements of $F(R)$ can be ordered as $x(1, R), ..., x(m, R)$, with $m = \sum_{i \in N} n_i(R) > 1$, such that for all $k = 1, ..., m$, if $x_i(k, R) = j_1^*$ for some $i \in N$, then $x_i(k+1, R) \neq j_1^*$ (mod m). Fix any $R' \in \bar{\mathscr{R}}$ such that $F(R) \neq F\left(R'\right)$. We need to consider only the case that $\#F\left(R'\right) > 1$. Suppose that for all $x(i, R) \in F(R)$, there do not exist any agent ℓ and any allocation $z \in \bar{J}$ such that $zP'_\ell x(i, R)$ and $x(i, R) R_\ell z$. This implies that for all $x(i, R) \in F(R)$, $L_\ell(x(i, R), R) \subseteq L_\ell\left(x(i, R), R'\right)$ for all $\ell \in N$. Since F is (Maskin) monotonic, it follows that $F(R) = F\left(R'\right)$, which is a contradiction. Thus, for some $x(i, R) \in F(R)$, there exist an agent ℓ and an allocation $z \in \bar{J}$ such that $zP'_\ell x(i, R)$ and $x(i, R) R_\ell z$. Fix any of such $x(i, R) \in F(R)$. Since by construction of the set $\{x(1, R), ..., x(m, R)\}$ we have that for all $k = 1, ...m$, with $k \neq i$, it holds that $x(k+1, R) P'_j x(k, R)$ for some j, it follows that $x(i, R)$ can be reached via a myopic improvement path at R' by any outcome in $x(k, R) \in \{x(1, R), ..., x(m, R)\} \setminus \{x(i, R)\}$. Thus, F satisfies *rotation monotonicity*.

Proof of Theorem 5. Observe that $\#\phi(R) = 2m$, where m is the number of such allocations at R where all jobs except $\tau\left(R_1\right)$ are assigned to agents $N \setminus \{1, 2\}$ in an efficient way (agent 2 getting the leftover). It follows that *Property M* is always satisfied by ϕ and Corollary 3 applies. Thus it suffices to prove that *rotation monotonicity* is satisfied.

Fix any $R \in \hat{\mathscr{R}}$ and any $x \in \phi(R)$. Let $\hat{x}$ be the allocation obtained from x in which the job assigned to agent 1 under x is assigned to agent 2 under $\hat{x}$, the job assigned to agent 2 under x is assigned to agent 1 under $\hat{x}$, whereas all other assignments are unchanged. That is, $\hat{x}_1 = x_2$, $\hat{x}_2 = x_1$, and $\hat{x}_i = x_i$ for every agent $i \neq 1, 2$. Observe that $\hat{x} \in \phi(R)$ if and only if $x \in \phi(R)$. The following result shows that the efficient solution ϕ is implementable in rotation programs. This result is obtained by requiring that the ordered set

$$\phi(R) = \{x(1, R), x(2, R), ..., x(2n-1, R), x(2m, R)\}$$

satisfies the following properties for all $i \in \{1, ..., 2m\}$: (1) If i is odd, then $x_1(i, R) = \tau\left(R_1\right)$. (2) If i is even, then $x_2(i, R) = \tau\left(R_2\right)$. (3) If $x(i, R) = x$ and i is odd, then $x(i+1, R) = \hat{x}$. $\phi(R)$ is implementable in rotation programs because we can devise a rights structure that allows agent 1 (agent 2) to be effective in moving from the outcome $x(i, R)$ to $x(i+1, R)$ provided that i is even (odd). The reason is that agent 1 (agent 2) has incentive to move from $x(i, R)$ to his top-ranked outcome $x(i+1, R)$ when i is odd (even). To see that *rotation monotonicity* is satisfied, fix any R' such that $\phi(R) \neq \phi\left(R'\right)$. This implies that at least one allocation $x(i, R) \in \phi(R)$ is Pareto dominated at R', that is, there exists an allocation z such that $zR'_j x(i, R)$ for each agent $j \in N$ and $zP'_j x(i, R)$ for some agent $j \in N$. We can proceed according to whether $\tau\left(R_1\right) \neq \tau\left(R'_1\right)$. Suppose that $\tau\left(R_1\right) \neq \tau\left(R'_1\right)$. This implies that $\tau\left(R_1\right) = \tau\left(R_2\right)$ has fallen strictly in agent $j = 1, 2$'s ranking when the profile moves from R to R'. The preference reversal for both agent 1 and agent 2 guarantees that *rotation monotonicity* is satisfied for every $x(i, R) \in \phi(R)$. Suppose that $\tau\left(R_1\right) = \tau\left(R'_1\right)$. We have already observed that at R, it holds that $x(i+1, R) P_2 x(i, R)$ if i is odd, and that $x(i+1, R) P_1 x(i, R)$ if i is even. In other words, there is the following cycle among outcomes in $\phi(R)$:

$$x(1, R) P_1 x(2m, R) P_2 x(2n-1, R) \cdots x(3, R) P_1 x(2, R) P_2 x(1, R)$$

Since $\tau\left(R_j\right) = \tau\left(R'_j\right)$ for $j = 1, 2$, it follows that the above cycle also exists at R'. Since $\phi(R) \neq \phi\left(R'\right)$, we already know that there is at least one allocation $x(i, R) \in \phi(R)$ that is Pareto dominated at R'. Since $x(i, R)$ is efficient at R, it follows that $x(i, R) \in \phi(R)$ has strictly fallen in the preference ranking of at least one agent $j \neq 1, 2$ when the profile moves from R to R'. It follows that *rotation monotonicity* is satisfied. □

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