On the alleged innocence of mereology

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Abstract

In *Parts of Classes* [Lewis 1991] David Lewis attempts to draw a sharp contrast between mereology and set theory and to assimilate mereology to logic. He argues that, like logic but unlike set theory, mereology is "ontologically innocent". In mereology, given certain objects, no further ontological commitment is required for the existence of their sum. On the contrary, by accepting set theory, given certain objects, a further commitment is required for the existence of the set of them. The latter – unlike the sum of the given objects – seems to be an abstract entity whose existence is not directly entailed by the existence of the objects themselves. The argument for the innocence of mereology is grounded on the thesis of "Composition as identity". Lewis analyses two different versions of the thesis: the first is the *Strong composition thesis*, according to which certain objects *are* their sum, where the use of "are" would mean that composition is *literally* identity. The second version is the *Weak composition thesis*, according to which composition is *analogous*, under some aspects, to identity. He criticises the first version of the thesis and argues for the second one.

In the paper we argue that (T1) arguments for the ontological innocence of mereology are not conclusive. An obvious objection to the *Strong composition thesis* is that – given certain objects Xs – they cannot be their sum because none of them is the sum. One could reply to this objection by observing that the "are" in the sentence "The Xs are their sum" is to be understood collectively and not distributively. But the crux is that the collective reading fails to generate a new entity, whereas mereology, in particular in Lewis' use for the reconstruction of set theory as "megethology", needs to consider sums as real objects. Besides, we contend that Lewis' argument for the innocence of mereology based on the *Weak composition thesis* is a *petitio principii*. The reason is that the aspects of the analogy between composition and identity, which Lewis emphasises, obtain under the presupposition of the existence of sums. But this is just what a denier of innocence would refuse.

(T2) Some arguments against the ontological innocence of mereology show a certain ambiguity in the innocence thesis itself. Some defences of the innocence seem to implicitly presuppose that the sum of certain objects Xs is not a genuine entity. Speaking of the sum of the Xs would be just another way of speaking plurally of the Xs. However, the relevant use of sums in mereology treats them as well determined objects. The relevant innocence thesis takes for granted that, though sums are genuine objects, nevertheless their existence does not require any further commitment.

(T3) The innocence thesis, apart from Lewis' defence, seems to depend on a general conception of the nature of objects and on how the notion of ontological commitment is understood. We think that the thesis is the manifesto of a realistic conception of parts and sums. This conception consists of the following clauses: (i) given any object x, it is well determined which parts it possesses; these are in turn objects whose existence is a necessary consequence of the existence of x. (ii) However any objects Xs are given, they

automatically constitute a well determined object x which is their sum; (iii) We can refer singularly and plurally to parts and sums of given objects. Obviously, one might wonder if such a conception is really ontologically innocent. One could object that it is not innocent because clauses (i) – (iii) are not. For example, clause (i) could be considered as an ontological commitment to the existence of sums. But the innocence at issue does not concern the above-sketched conception. The innocence is embedded in the conception itself. In other words, someone who argues for clauses (i) – (iii) takes a point of view from which mereology *appears* to be innocent. For, such a point of view forces us to consider as well determined the parts of any object and does not allow us to separate the existence of certain objects form the existence of their sum.

(T4) is the claim that the alleged innocence of mereology is subject to Quine's notorious criticisms of the set-theoretical interpretation of second order logic. To the purpose, we construct a mereological model of a substantive fragment of set theory, i.e. the one that grounds the principal model semantics of second order logic. First, we construct a mereological model under the assumption of the existence of infinitely many atoms. Then, we replace this assumption with that of the existence of any infinite object (with or without atoms).

Finally, let us make a general point about the innocence thesis of mereology. A conclusive argument for that would be a refutation of the thesis that there are only denumerably many entities. For, since the parts of an infinite object constitute a non-denumerable infinity, such an argument would entail that there could be no infinite without a non-denumerable infinity. However, the thesis that any genuine infinity is a denumerable one has had some important advocates. So, a conclusive argument for the innocence of mereology seems to be highly implausible.

0. In *Parts of Classes* [Lewis 1991] David Lewis attempts to draw a sharp contrast between mereology and set theory and to assimilate mereology to logic. He argues that, like logic but unlike set theory, mereology is "ontologically innocent". Consider the following sentences:

(1) There is a cat, Mina, which is sleeping.

(2) There is a mouse, Gino, which is dancing.

Whoever asserts (1) is committed to the existence of a cat whose name is Mina. Whoever, after the assertion (1), asserts (2) is committed to the existence of a mouse whose name is Gino. Whoever, after the assertion (1) and (2), accepts set theory is further committed to the existence of an entity – a set – whose elements are Gino and Mina. On the contrary, if one accepts logic no further commitment is required apart from a commitment to Mina and Gino. Lewis argues that the same is for mereology: given certain objects, no further ontological commitment is required for the existence of their sum (or fusion). On the contrary, by accepting set theory, given certain objects, a further commitment is required for the existence of the set of them. The latter – unlike the sum of the given objects – seems to be an abstract entity whose existence is not directly entailed by the existence of the objects themselves.

The goal of this paper is to analyze arguments pro and cons the ontological innocence of mereology. We argue that:

(T1) arguments for the ontological innocence of mereology are not conclusive.

(T2) Some arguments against the ontological innocence of mereology show a certain ambiguity in the innocence thesis itself.

(T3) The innocence thesis, apart from Lewis' defence, seems to depend on a general conception of the nature of objects and on how the notion of ontological commitment is understood. Specifically, we think that the thesis is the manifesto of a *realistic* conception of parts and sums.

(T4) the alleged innocence of mereology is subject to Quine's notorious criticisms of the set-theoretical interpretation of second order logic. To the purpose, we construct a mereological model of a substantive fragment of set theory, i.e. the one that grounds the principal model semantics of second order logic.

The paper is divided into six sections. In the first one we recapitulate Lewis' version of mereology. In the second section we analyze Lewis' argument for the innocence of mereology: an argument grounded on the thesis of "Composition as identity". Lewis analyses two different versions of the thesis: the first one is the *Strong composition thesis* (StrongCom), according to which certain objects *are* their sum, where the use of "are" would mean that composition is *literally* identity. The second version is the *Weak composition thesis* (WeakCom), according to which composition is *similar* or *analogous*, under some aspects, to identity. In the third section we analyse some arguments pro and cons (StrongCom), specifically Lewis, Yi, and Van Inwagen arguments against (StrongCom), and we argue for (T1) and (T2). In the fourth section we analyse arguments pro and cons (WeakCom). Specifically, we analyse arguments pro (WeakCom) given by Lewis. Again, some arguments pro (T1) and (T2) are given in this section. In section five we construct a mereological model of a substantive fragment of set theory, i.e. the one that grounds the principal model semantics of second order logic, and we argue for (T4). In the last section (6) we give an argument for (T3).

1. "Mereology", literally the "science or theory of parts", stands for theories analyzing the relation "... is a part of ...". There are different formulations of mereology depending on the language adopted. Due to the fact that we would like to consider Lewis' defence of the ontological innocence of mereology we propose his formulation of

mereology, suited to point out some relevant aspects of the problem we are analysing.

Lewis treats mereology in a plural language, a language extending that of first logic, including singular and plural reference, singular and plural quantification [for an introduction to a plural language see Boolos 1984]. In such a language we consider logical terms:

(a) Plural terms, for example the pronoun "them", or plural variables, for example "X" as symbolic counterpart.

(b) Plural quantifiers, for example, "there are some things… such that".

(c) A special two-place predicate "… is one of …". This predicate admits a singular term in its first place and a plural one in its second place.

By adding to this vocabulary the non-logical predicate, "… is a part of …" we obtain a language rich enough to formulate mereology. By means of the predicate "… is a part of …" one could define sums (or fusions) and the overlapping relation.

(Def.1) y and x overlap if and only if there is a z such that it is part of x and part of y.

(Def.2) y is a sum of the X if and only if each of the X is a part of y and each part of y overlaps one of the X.

(Def.3) The X compose y if and only if y is the sum of the X.

(Def.5) x and y are disjointed if and only if they do not overlap.

Mereology consists of the logical consequences of the following axioms:

(Reflexivity) x is part (non proper part) of itself

(Transitivity) If x is part of some part of y, then x is part of y.

(Unrestricted Composition) If there are some X there is a sum of the X.

(Uniqueness of Composition) If y and z are sums of the same X then $y = z$.

For example, the following theorem is a logical consequences of the above axioms:

(Theorem) If there are two objects, neither of which is part of the other, then there is something else that is not identical with either of them.

2. Consider again the sentences:

   (1) There is a cat, Mina, which is sleeping.

   (2) There is a mouse, Gino, which is dancing.

Whoever asserts (1) is committed to the existence of a cat whose name is Mina. Whoever, after the assertion (1), asserts (2) is committed to the existence of a mouse whose name is Gino. Suppose that someone – after the assertion of (1) and (2) – asserts:

   (3) There is a sum of the mouse Gino and the cat Mina, Gina.

Is she committed to the further existence of the sum of the mouse Gino and the cat Mina?
If we follow the Quinian motto (see Quine 1939: 708) that to exist is to be the value of the bound variables, the answer should be positive: since sums are values of bound variables, mereology is committed to the existence of the sum of whatever plurality of objects X, no matter how they are given and at however they are heterogeneous.
Question: is the ontological commitment to the existence of the sum of the X a further commitment? Specifically, is the commitment to the existence of the sum of the cat and the mouse a further commitment besides the existence of the cat and the mouse? Lewis' answer is: no. One could answer that (3) is a logical consequence of (1), (2) and mereology, in the specific case of the (Theorem 1). But, such an answer is trivial. In fact it does not say anything at all about the ontological commitment of mereology. Lewis' point is that with (3) we have not introduced a new entity: "Given a prior commitment to cats, say, a commitment to cat-sums is not a *further* commitment. The sum is nothing over and above the cats that compose it. It just *is* them. They just *are* it. Take them together or take them separately, the cats are the same portion of Reality either way. Commit yourself to their existence all together or one at a time, it is the same commitment either way… I say that composition… is like identity. The 'are' of composition is, so to speak the plural form of the 'is' of identity. Call this the Thesis of *Composition as Identity*. It is in virtue of this thesis that mereology is ontologically innocent: it commits us only to things that are identical, so to speak, to what we were committed to before" [Lewis 1991: 81-82].

Lewis' argument for the innocence of mereology is the following:

(P1) Composition – a many-one relation – is like identity.

(P2) The commitment to sums is already presupposed in the acceptance of the objects that are summed.

(P3) Nothing could be considered more ontologically innocent than the request to accept something identical to things already accepted.

(C) Mereology is ontologically innocent.

For Lewis, the sum of certain objects is the very same objects: sum is that things and nothing more. Speaking of sums of heterogeneous and/or scattered objects might seem to be inappropriate. But mereology is not concerned with that: the generality of the theory does not permit to exclude certain sums for reasons concerning the nature or the location of the taken objects.
Lewis' argument rests on the thesis (P1) of *composition as identity*. What does it mean that composition is like identity? The answer depends on the reading of (P1) one accepts. In fact, there are two of them: a strong reading (StrongCom) and a weak one (WeakCom). For (StrongCom):

   (StrongCom) The predicate "are" used for the composition relation is literally the plural for of the "is" of identity.

Formally:

  $\forall X \ \forall y \ ((y \text{ is the sum of the } X) \rightarrow y = X)$

Those who accept (StrongCom) argue that the sum of some things is literally identical to that things: things are their sum, the sum is that things. If so, it is obvious that there is no further commitment to anything else apart from the commitment to parts. In this perspective the predicate "are" of composition is just a different form of the "is" of identity in the same way in which predicates "am" and "are" in sentences as:

   (4) I am Pino.

   (5) You are Dino

are alternative forms of "is" in a sentence as:

(6) She is Dina.

According to the above thesis the cat Mina and the mouse Gino are literally identical to its sum, Gina, even if none of the two is identical to it.

In the second reading of (P1) – the weak reading of composition (WeakCom) – the composition predicate is only analogous to identity. (WeakCom) is formulated in the following way:

(WeakCom) The predicate "are" used for the composition relation is *analogous* to the plural form of the "is" of identity.

The strength of Lewis' argument for the innocence of mereology strongly depends on the truth of (P1), i.e. on the truth either of (StrongCom) or of (WeakCom). In the next two sections we analyze some arguments pro and cons the two readings of the *composition as identity* thesis. Specifically, in the next section we analyze Lewis, Yi, and Van Inwagen's arguments against (StrongCom).

3. Lewis formulates two arguments against (StrongCom) in [Lewis 1991: 87]. The first one concerns the difficulties for a generalization of the definition of identity. Given the definition of identity between singular individuals in the following way:

(IS) $x = y =_{df.} \forall Z$ (x is one of the Z $\leftrightarrow$ y is one of the Z)

one could try to generalize it to the case of a plurality and a single individual obtaining:

(ISP) $X = y =_{df.} \forall Z$ (each of the X is one of the Z $\leftrightarrow$ y is one of the Z).

But if y is the sum of the X (where X are two or more disjointed objects) there are some things – the X themselves – such that each of the X is one of them but *y* is none of them. *Viceversa*, there are some things – *y* itself – such that *y* is one of them but none of the X. For example: let the X be Mina the cat and Gino the mouse and y their sum, Gina. Taken X for Z then each of Mina and Gino is one of the Z but Gina is not. On the other side, taken Gina for Z then Gina is one of the Z but neither Gino nor Mina is one of Z.

Yi has proposed an argument against (StrongCom) in [Yi 2001] similar to Lewis' argument. Consider, again, the cat Mina, the mouse Gino and their sum Gina. Given (StrongCom) and mereology one could say that:

(13) Gino and Mina are (identical to) Gina

but:

(14) Gina is not identical to Gino

and

(15) Gina is not identical to Mina.

From (14) and (15) one obtains that:

(16) Gina is not identical neither to Gino nor to Mina

Moreover, the predicate "...is one of..." could be extended to a predicate with singular places so defined:

t is one of $u \leftrightarrow \forall X(t$ is one of $(u$ and X)).

So, we can say:

(17) Gina is one of Gina

And, given (17) and (13),

(18) Gina is one of Mina and Gino.

But

(19) Gina cannot be one of Gino and Mina

By (16). Then, (StrongCom) is wrong.
Lewis' second objection concerns the indiscernability of identical (InId) i.e.:

(IdIn) $\forall x \, \forall y \, (x = y \rightarrow \forall F \, (Fx \leftrightarrow Fy))$

where the third universal quantification is of second order and "F" is a predicative variable. "Even though – Lewis argues – the many and the one are the same portion of Reality, and the character of that portion is given once and for all whether we take it as many or take it as one, still we do not really have a generalized principle of indiscernability of identicals… What is true of the many is not exactly

what is true of the one. After all they are many while it is one" [Lewis 1991: 87].

Consider the following example. Suppose that the number of the X is *n,* where *n* >1. Then, the plural predicate "…are exactly *n*" should apply – given (InId) – to *y* too, but the number of *y* is one.

Wallace in (manuscript) replies to both Lewis' arguments. On the first one she observes that:

$$X = y$$

in

(ISP) $X = y =_{df.} \forall Z$ (each of the X is one of the Z $\leftrightarrow$ y is one of the Z)

has a distributive reading, i.e. each of the X is identical to *y*, whereas when *y* is a sum of the X identity has a collective reading. With reference to the above example: Gina is not distributively identical to Mina and Gino, but it is collectively identical to them.

Problem: if – as Wallace suggests – we read identity collectively, it becomes a primitive notion, indefinable in terms of plural quantification, as Lewis has observed. Moreover, the crux is that collective identity is not a genuine many-one relation. For, to hold that the sum of the X is collectively identical to the X amounts to denying that the sum of the X is a genuine entity: speaking of the sum of the X would be nothing but a device for referring to the X collectively. On the contrary mereology, in particular in Lewis' use for the reconstruction of set theory as "megethology", needs to consider sums as genuine objects.

One reply to Yi's argument is – again – the distinction between a *collective* and a *distributive* reading of conjunction. Consider a sentence as:

(20) Dino and Lino have lifted the piano

and suppose that Dino and Lino have lifted the piano all together. For sure, in (20) the "and" is not used as a propositional connective. In fact if it is so, from (19) we could infer that:

(21) Dino has lifted the piano and Lino has lifted the piano

Assume that the piano is too heavy to be lifted only by Dino or by Lino. One can conclude that (20) is true whereas (21) is false. If it is so then the "and" in (20) should function as a connective yielding a plural term "Gino and Pino".

Yi admits that the plural term "Dino and Lino", in the sentence:

(20) Dino and Lino have lifted the piano

does not refer singularly neither to Dino nor to Lino. That does not mean it does not refer at all. Suppose that a plural term as "Dino and Lino" does not refer singularly to none of the two individuals, but that it plurally refers to both of them. Then, the mereologyst could argue that the sum is identical to Dino and Pino.

If Yi thinks that the commitment to plural terms is ontologically innocent he should say, arguing in the same way, that mereology is ontologically innocent.

When Yi argues that there is a sum whose name is Gina such that it is neither Dino nor Mina, the mereologyst could reply that there is a plurality of objects, Gino and Mina, which is neither a cat nor a mouse. In other terms, even if the mereologyst could accept Yi's conclusion that there are some things which are neither a cat nor a mouse, he could reply that they are a cat and a mouse.

An easy reply to the above objection is to say that who has lifted the piano is not the sum of Dino and Lino. It is an action that Dino and Lino take together, simultaneously, an action not involving the presence of a new entity.

Likewise, saying that Mina and Gino are a cat and a mouse is just saying one is a cat and the other is a mouse and it is not saying that a single entity is a cat and a mouse. Saying that the term "Gina and Pino" possesses, after all, a reference, even if it does not refer neither to Mina nor to Gino, does not mean that we are referring to a different entity from Mina and Gino; it simply means that the term – just because it is a plural term – does not singularly refer to one of them. It refers simultaneously to both of them. This plural reference does not commit us to the alleged entity Gina.

Wallace reply to Lewis' second argument, the indiscernability argument, is recovered, with substantial modifications, by Baxter (in [Baxter 1988]).

For Baxter a way to maintain (InId) and (StrongCom) is arguing for two kinds of identity, a *strict* and a *popular* one. Baxter gives the following exemplification of the above distinction: "Suppose a man owned some land which he divides into six parcels. Overcome with enthusiasm for [the denial of composition as identity] he might try to perpetrate the following scam. He sells off the six parcels while retaining ownership of the whole. That way he gets some cash while hanging on to his land. Suppose the six buyers of the parcels argue that they jointly own the whole and the original owner now owns nothing. Their argument seems right. But it suggests that the whole was not a seventh thing" [Baxter 1988: 579].

A justification of (StrongCom) is to argue that to strictly count the many is to loosely count the one.

(BT) The whole is the many parts counted as one thing [Baxter 1988: 579].

Even if Baxter argues that (BT) does not deny the existence of the whole, but just the *additional* existence of the whole, it seems to us that this popular mood does not reify the whole. Baxter's example demonstrates a *weak* use of the sum, not involving the existence of it as an entity. It seems to be a use of sums similar to the one of sets in a sentence as:

(7) The set of the Germans camping in Pinarella has cardinality six hundreds.

A sentence one can reformulate without the introduction of the notion of set, saying that:

(8) The Germans camping in Pinarella are six hundreds.

Likewise, the sentence:

(9) I have seen a flock of six geese

can be rewritten in this way:

(10) I have seen six geese

so that (9) does not involve that "flock" stands for a certain specific entity.
For Baxter speaking of the sum of the X would be just another way of speaking plurally of the X. Unfortunately, mereology does not have just this eliminative use of the sums, since *each* individual in mereology is the sum of its parts. If there are individuals, there are sums too!
Van Inwagen replies to Baxter's in ([Van Inwagen 1994]). Consider Baxter's example of the land and its six parcels and express the fact that there are two parcels with a different size. We will use a quantified sentence with the following form:

(11) $\exists x\ \exists y\ \exists z\ (y < x \land z < x \land \neg G(y, z))$

where "<" stands for the relation "… is part of …" and "G" stands for the relation "… has the same size of …". On how many objects do we quantify? It seems that we must quantify on seven entities,

because the first existential quantifier is exemplified by the whole. End of van Inwagen's argument.

One could reply by arguing that, using the plural language (11) could be rewritten quantifying – singularly and plurally – just on the six parts:

(11*) Among the X two of them have a different size.

Formally:

(11**) $\exists x \, \exists y \, (x$ is one of the X $\land$ y is one of the X $\land \neg G(x, y))$

However, it seems to us that it is possible to revive van Inwagen's criticism simply modifying his example. Suppose I would like to express the fact that the whole land is larger than each of its parcels. The singular variables range on every parcel of the land (included the very same land) so that:

(12) $\exists x \, \forall y \, (y < x)$.

A second kind of objection to (StrongCom) has been formulated by van Inwagen in [van Inwagen 1994]. It is an objection concerning the very intelligibility of (StrongCom). Consider Lewis' sentences:

(22) It (the sum) is just them (the cats composing it),

(23) They (the cats composing it) just are it (the sum).

In a semi-formal way, using the plural language, one could translate (22) and (23):

(22') The sum y of the X is just the X,

(23') The X are just the sum y of the X.

For van Inwagen it is easy to observe that the "is" of identity is used in a correct way (from a syntactical point view) when there are singular terms on the right and left side of the relation. So for example, we say:

(24) Tully is Cicero,

(25) x is y.

Alternatively, in the natural language we use the plural form of identity: the "are" of (plural) identity. Such a term is used in a correct way (from a syntactical point view) when there are plural terms on the right and left side of the relation. So for example, we say:

(26) Fichte, Schelling and Hegel are German idealists,

(27) The X are Y.

Problem: what is the meaning of a sentence where the "is" and "are" of identity are placed by a singular term on one side and a plural term on the other?

Of course, we can define both the singular and plural form of identity in terms of the relation "…is one of…". The singular one should be:

(IS) x is y =$_{df.}$ $\forall Z$ (x is one of the Z $\leftrightarrow$ y is one of the Z)

and the plural one:

(IP) X are Y =$_{df.}$ $\forall z$(z is one of the X $\leftrightarrow$ z is one of the Y)

Problem: how should we define the "hybrid" form "is/are" in terms of "… is one of…" or in some other similar way such as the definition of identity in terms of overlapping? If we follow this train of thought, Lewis' tentative explanation with the sentence:

The "are" of composition is, so to speak, the plural form of the "is" of identity

seems to be false: whatever one could mean by the "are" of composition, it cannot be the plural form of the "is" of identity because the plural form of the "is" of identity is the "are" of identity. Van Inwagen's conclusion is that (StrongCom) is unintelligible because the sentences exemplifying it are ungrammatical.

A first reply to van Inwagen's argument has been to argue that in natural language there are "hybrid" uses of is/are (the examples are in Wallace [manuscript]). Consider, for example, the sentences:

(28) Two cups are a pint

(29) One pint is two cups

(30) One kilometer is thousand meters

(31) Thousand meters are one kilometer

....

Unfortunately, it is easy to reply that in these mixed uses the predicate in question is not really the identity one. One pint – differently from the cups – is a unit of measurement. A kilometer and a thousand meters are different measurements (expressed by different numbers) of the same size. The above examples can be easily rephrased in the following way:

(28') Two cups have the capacity of one pint.

(29') One pint has the capacity of two cups.

(30') A kilometer and 1000 meters measure the same distance.
...

But, the problem does not seem connected with the hybrid form. In a plural language a plural term could denote a singular individual. The formula:

(32) y is the X

naturally means that y is the only one X according to our definition (ISP):

(ISP) $X = y =_{df.} \forall Z$ (each of the X is one of the Z $\leftrightarrow$ y is one of the Z).

The same result is obtained with an example taken from the natural language. Suppose there is a bell with a German name written: Wuerms, and Pino says:

(33) There are some Germans.

The sentence is true even if just a German stays in the apartment. van Inwagen's criticism is better interpreted as a criticism to those who read (32) as:

(34) y is the sum of the X.

To say that y is the only X (when the X are n, with $n > 1$) does not mean that y is the sum of the X.

4. The second reading of (P1) the weak one (WeakCom), says that composition is just *similar* or *analogous*, under some aspects, to identity. The composition relation is so formulated:

(WeakCom) The predicate "are" used for the composition relation is *analogous* to the plural form of the "is" of identity.

In this second reading of the thesis of *composition as identity* one is confined to argue for a certain similarity between composition and identity. Similarity has many aspects. The aspects of the similarity Lewis shows are the following:

- (Unrestricted composition) Just as everything is identical to something, likewise given anyway some X they compose something.

So, for example, there is no special condition Gino must satisfy for being identical to himself. Likewise, there is no special condition Gino and Mina must satisfy for composing something.

- (Uniqueness of Composition). Just as there cannot be anything identical with two distinct objects, likewise there cannot be two distinct sums of the same objects.

For example, there cannot be two distinct things both identical to Gino. Likewise, there cannot be two distinct objects both composed by Gino and Mina.

- (Ease of Describing Sums) Just as if you fully describe the thing *x* you fully describe something identical to x, likewise if you fully describe the X you fully describe their sum.

For example, you can fully describe the object identical to Gino describing Gino. Likewise you can fully describing the object Gina composed by Gino and Mina fully describing Gino and Mina.

- (The spatial coincidence) just as *x* and *y* have to occupy the same spatio-temporal region if the first object is identical to the second one, likewise *y* and X have to occupy the same spatio-temporal region if the first one is the sum of the second ones.

For example, if there is an object Gino in a certain place at a certain time, Gino exists in the same place-time. Likewise, Gina is in the same region occupied by Mina and occupied by Gino.
On the base of the following analogies Lewis proposes a defense of the ontological innocence of mereology. In fact, from:

(1) There is a cat, Mina, which is sleeping.

and

(2) There is a mouse, Gino, which is dancing.

it follows that:

(1') There is something the cat Mina is

(2') There is something the mouse Gino is

For Lewis, from (1') and (2'), just considering "are" as a plural form of "is", it follows that:

(3') There is something the mouse Gino and the cat Mina are

Where the "something" in (3') is – for Lewis – the sum, Gina. End of the argument.
Objections. First of all, the sentence (3') does not seem to be a consequence of (1') and (2'). If you mean "are" as the plural from of "is", from (1') and (2') follows:

(3'') There are the cat Mina and the mouse Gino.

But (3'') simply says that they both exist. In fact, we can paraphrase (3'') in the following way:

(3''') There is something the cat Mina is and there is something the mouse Gino is.

The problem is that (3''') does not entail (3'). Formally, from:

(35) $\exists x \, (x = a) \wedge \exists x \, (x = b)$

does not follow that:

(36) $\exists x \, (x = a \wedge x = b)$.

It seems to us that those who object that (36) is not the correct paraphrase of (3') – because in (3') one says that there is something Mina and Gino collectively, not distributively – are wrong.
In fact, the conjunction of (1) and (2) says that Mina and Gino are

each of them something, and it does not say that they are collectively something.

Moreover, let us observe that there is a strong non-parallelism between composition and identity: while the description of an object identical to *x* describes *x*, the sum of the X does not describe the X at all. Given certain X and Y the sum of the X could be identical to the sum of the Y even if the X are not identical to the Y. Consider, for example, a rectangle A:

A   A

One could see the rectangle A as the mereological sum either of two squares or of two triangles. But if both squares and triangles were identical to their common sum, i.e. the rectangle, then for the transitivity of identity, the squares should be identical to the triangles, that is absurd.

Besides,all aspects of the similarity at issue, apart from that concerning spatio-temporal regions, are applicable to the membership relation too. Nevertheless no mereologyst would like to assimilate sums to sets.

The conclusive objection to Lewis' argument of the similarity is that it is just a *petitio principii*.

Lewis' analogy between composition and identity rests just on the assumption of the existence of sums. For example, if one argues, in order to show an aspect of the similarity in question, that, as everything is identical to something, so too, however some X are given, they compose something, she presupposes just the existence of the sum of arbitrarily given objects. But that is just what is in question and the similarity should demonstrate. Whoever challenges the ontological innocence of mereology denies the innocence of the alleged existence of sums of arbitrarily taken objects.

To conclude: arguments for the innocence of the mereology – both those based on (StrongCom) and those based on (WeakCom) – are not conclusive (our first T1 thesis). Moreover, we have argued that some arguments against the ontological innocence of mereology show a certain ambiguity in the innocence thesis itself. Some defences of the innocence seem to implicitly presuppose that the sum of certain objects X is not a genuine entity. Speaking of the sum

of the X would be just another way of speaking plurally of the X. However, the relevant use of sums in mereology treats them as well determined objects (T2).

5. Let us formulate a mereological model for sets of individuals. The goal of this section is to argue that the alleged innocence of mereology requires the ontological innocence of a substantive fragment of set theory, i.e. the one that grounds the principal model semantics of second order logic. Then, the ontological innocence of mereology is subject to Quine's notorious criticisms of the set-theoretical interpretation of second order logic.

Let T be a theory of sets of individuals. The language L of T is a first order language with identity and with two kinds of variables:

x, y, z, … variables for individuals;
α, β, γ,… variables for sets;
∈ is the membership symbol.

Atomic formulas have the following form:

x = y
α = β
x ∈ α.

Complex formulas are defined in the usual way. Axioms of T are:

Extensionality (ES) $\alpha = \beta \leftrightarrow \forall x \, (x \in \alpha \leftrightarrow x \in \beta)$

Comprehension (Com) $\exists \alpha \, \forall x \, (x \in \alpha \leftrightarrow A(x))$,

where A(x) is any propositional function of L. It is possible to give the following mereological interpretation of T.

Let D be any domain of atoms (finite or infinite). Le D' be the sum of the atoms of D with a further atom j. Let us interpret the variables for individuals in the atoms of D and the variables for sets in the parts of D' containing j. Let us interpret ∈ in the mereological relation < ("… to be a part of …").

The presence of j has the effect of introducing the null set, j itself, and the singletons, the singleton {x} of x, being the sum of x and j. And it is easy to verify the axioms (ES) and (Com).

(ES) Suppose that $\forall x \, (x \in \alpha \leftrightarrow x \in \beta)$. Then, α and β are sums of j and of the same atoms of D. So, they have the same atomic parts. So, they are identical.

(Com) Let A(x) be any propositional function. We must prove that there is a set whose elements are the individuals satisfying A(x). The searched set is just the sum of j and the atoms satisfying A(x).

So,assuming the existence of infinitely many atoms, we get a model of the power set of an infinite domain.

To the purpose of obtaining a mereological infinite model of T, the assumption of the existence of infinitely many atoms is replaced with that of the existence of infinitely many pairwise disjointed objects O (with or without atoms).
Let the objects O be interpreted as individuals. Let F(O) be the sum of the O and a further object j disjointed from each of them. Then the role of sets can be played by the parts of F(O) containing j and not "cutting off" any of the O. That means that we take as set each part α of F(O) such that:

(i) α contains j

and

(ii) Each of the O either is a part of α or it is disjointed from α.

Observe that, using Lewis'definition of an infinite object, the existence of infinitely many disjoint objects follows from the existence of a single infinite object. For, consider the following definition:

Def (infinite). An object x is infinite if and only if x is the sum of some things, each of which is a proper part of another.

Given the above definition Def (inifinite) one can argue that:

(Theorem 3): If there is an infinite object there are infinitely many pairwise disjointed objects.

Proof. Let a be an infinite object that is the sum of some X each of which is a proper part of one of them. From X we can extract an infinite sequence of objects b_0, b_1…, b_n,… such that each of them is a proper part of the subsequent. Then, objects b_1-b_0, b_2-b_1,…, b_{n+1}-b_n…(where b_{n+1}-b_n is the complement of b_n in b_{n+1}) are pairwise disjointed.

Let us observe that T is a substantive fragment of set theory, i.e. the one that grounds the principal model semantics of second order logic. Because of such ground Quine (in Quine 1970) notoriously

argues that second order logic is a wolf in sheep's clothing. That means that second order logic is set theory in logic's clothing.

Specifically, detractors of second order logic criticize the use of thecomprehension principle (Com) as a *logical* principle. They hold that it does not possess the peculiar features of a logical principle. (Com) seems to concern the notion of set in the nowadays sense of set theory, where sets are understood as entities constituted by their elements. But, such a notion of set is highly problematic, and it does not seem to have a logic nature.

Since it is possible to give a mereological interpretation of T, Lewis' assimilation of mereology to logic seems to be subject to the same objections (our T4 thesis).

6. What about the ontological innocence of mereology?

First of all Lewis' argument for the innocence of mereology shows a certain ambiguity in the use of the term "sum".

On one side, Lewis seems to argue that, given certain objects X, referring to their sum is ontologically innocent because there is not a *new* entity as referent of the expression "the sum of the X". So, talking of the sum of the X is simply a different way of talking of the X, *looking at them as a whole*. This seems to be the only way to make intelligible, and plausible, the statement that:

(37) The X are their sum.

However, on the other side, Lewis' innocence is not understood as a mere linguistic use, where sums are not reified. It is not an innocence thesis comparable to that of plural reference where the reference to some objects does not require the existence of a single entity picking up them in a whole. Consider what Lewis says on this last issue: "Plural quantification is innocent: we have many things, we do mention one thing that is the many taken together. Mereology is innocent in a different way: we have many things, we do mention one thing that is the many taken together, but this one thing is nothing different form the many. Set theory is not innocent. Its trouble has nothing to do with gathering many into one. Instead, its trouble is that when we have one thing, then somehow we have another wholly distinct thing, the singleton. And another, and another ,... *ad infinitum*. But that is the price for mathematical power. Pay it" [Lewis 1991.87].

In general, it is difficult to say what else could be the act of taking together the many as one if it is not an act of plural reference, an act that – according to Lewis – does not engage any singular entity apart from the many taken together. For sure, mereology and Lewis' use of it, specifically in his reconstruction of set theory as "meghetology" (on

meghetology see Lewis 1993), requests that sums are taken as real objects.

Lewis seems to suggest that even if the sum of the X is a well determined individual, distinct from the X, the existence of such individual has to be necessarily accepted from whom has already accepted the existence of the X. In other words, committing oneself to the existence of the X would be an implicit commitment to some other entities and – among them – the sum of the X.

The problem is that arguing for this thesis implies a premise (P1) inadequate both to the (StrongCom) and to the (WeakCom): to the strong one (StrongCom) because the sum of the X is not literally identical to the X, to the weak one (WeakCom) because the analogy between composition and identity is – as we have argued – a *petitio principii*.

Moreover, we do not think that there are some conclusive arguments for the thesis that whoever accept the existence of the X is committed to the acceptance of the existence of the sum of the X (T1). For, since the parts of an infinite object constitute a non-denumerable infinity – for example the existence of natural numbers would imply the automatic existence of the continuum – such an argument would entail that there could be no infinity without a non-denumerable infinity. However, the thesis that any genuine infinity is a denumerable one has had some important advocates (see for example Kroeneker or Poincaré). So, a conclusive argument for the innocence of mereology seems to be highly implausible. This seems to be a general point about the innocence thesis of mereology.

Last, we think that the thesis of the ontological innocence of mereology is the manifesto of a *realistic* conception of parts and sums.

This conception consists of the following clauses:

(i) given any object x, it is well determined which parts it possesses; these are in turn objects whose existence is a necessary consequence of the existence of x.
(ii) However any objects X are given, they automatically constitute a well determined object x which is their sum;
(iii) We can refer singularly and plurally to parts and sums of given objects.

Obviously, one might wonder if such a conception is really ontologically innocent. One could object that it is not innocent because clauses (i) – (iii) are not. For example, clause (i) could be considered as an ontological commitment to the existence of sums. But the innocence at issue does not concern the above-sketched conception. The innocence is embedded in the conception itself. In other words, someone who argues for

clauses (i) – (iii) takes a point of view from which mereology *appears* to be innocent. For, such a point of view forces us to consider the parts of any object as well-determined by the object itselfand does not allow to separate the commitment to certain objects from that to their sum (T3).

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