

Probing Isospin mixing with the Giant Dipole Resonance in the ^{60}Zn compound nucleus

G. Gosta,^{1,2} A. Mentana,^{1,2} F. Camera*,^{1,2} A. Bracco†,^{1,2} S. Ceruti,^{1,2} G. Benzoni,² N. Blasi,² S. Brambilla,² S. Capra,^{1,2} F.C.L. Crespi,^{1,2} A. Giaz,² S. Leoni,^{1,2} B. Million,² S. Riboldi,^{1,2} C. Porzio,^{1,2} S. Ziliani,^{1,2} O. Wieland,² A. Nannini,³ M. Rocchini,³ N. Marchini,^{4,3} M. Ciemala,⁵ M. Kmiecik,⁵ A. Maj,⁵ B. Wasilewska,⁵ M. Zieblinski,⁵ D. Filipescu,⁶ J. Kaur,⁷ N. Marginean,⁸ S. Pascu,⁸ T. Glodariu,⁶ D. Ghita,⁷ V. Zamfir,⁷ J.J. Valiente-Dobón,⁹ G. de Angelis,⁹ F. Galtarossa,⁹ A. Goasduff,⁹ T. Bayram,⁹ A. Gadea,¹⁰ A. Montaner,¹¹ I. Zanon,⁹ D. Brugnara,^{9,11} A. Gozzelino,⁹ G. Pasqualato,^{9,11} R. Menegazzo,¹² A. Gottardo,⁹ G. Jaworski,^{9,13} S. Lenzi,^{12,11} D. Napoli,⁹ D. Testov,^{9,11} M. Siciliano,⁹ T. Marchi,⁹ D. Mengoni,^{11,12} D. Bazzacco,¹² A. Boso,^{11,12} P.R. John,^{11,12} F. Recchia,^{11,12} R. Raabe,¹⁴ O. Poleshchuk,¹⁴ and J. Yang¹⁴

¹*Dipartimento di Fisica dell'Università degli Studi di Milano, Milano, Italy*

²*INFN, Sezione di Milano, Milano, Italy*

³*INFN, Sezione di Firenze, Italy*

⁴*Università degli Studi di Camerino, Unicam, Italy*

⁵*Institute of Nuclear Physics, Polish Academy of Sciences, Krakow, Poland*

⁶*Extreme Light Infrastructure (ELI-NP) and Horia Hulubei National Institute of Physics and Nuclear Engineering - IFIN HH, Bucharest, Romania*

⁷*Extreme Light Infrastructure (ELI-NP) and Horia Hulubei National Institute of Physics and Nuclear Engineering - IFIN HH, Bucharest, Romania*

⁸*Horia Hulubei National Institute of Physics and Nuclear Engineering - IFIN HH, Bucharest, Romania*

⁹*INFN, Laboratori Nazionali di Legnaro, Legnaro, Italy*

¹⁰*Instituto de Física Corpuscular, CSIC-Universidad de Valencia, Valencia, Spain*

¹¹*Dipartimento di Fisica dell'Università degli Studi di Padova, Padova, Italy*

¹²*INFN, Sezione di Padova, Padova, Italy*

¹³*Heavy Ion Laboratory, University of Warsaw, Warsaw*

¹⁴*KU Leuven, Department of Physics and Astronomy, Leuven, Belgium*

An experimental study of the isospin mixing in the mass region $A=60$ was made by measuring the γ -decay from the Giant Dipole Resonance in the compound nuclei ^{60}Zn and ^{62}Zn . These compound nuclei were populated at two different excitation energies, $E^* = 47$ MeV and $E^* = 58$ MeV using the fusion evaporation reactions $^{32}\text{S} + ^{28}\text{Si}$ at the bombarding energy of 86 MeV and 110 MeV and $^{30}\text{Si} + ^{30}\text{Si}$ at 75 MeV and 98 MeV. In the experiment, performed at the laboratory LNL-INFN, the γ -rays were measured with GALILEO detection system in which large-volume $\text{LaBr}_3(\text{Ce})$ detectors were added to the HPGe detectors. The Coulomb spreading width was obtained from the comparison of the two reactions and then isospin mixing parameter at zero temperature and the isospin-symmetry-breaking correction for beta decay were deduced. The present results were compared with data of the same type in other mass regions and with data from mass and beta decay measurements and with theory. The present data allow to deduce for the first time a consistent picture for mass dependence of isospin mixing and for the corresponding correction for the beta decay, supporting a reliable extension to the very interesting region of ^{100}Sn .

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One of the basic symmetries of the strong nuclear force is the isospin symmetry, introduced to handle theoretically that the interaction between protons and neutrons was found to be the same. This symmetry, known to be broken by the Coulomb force, manifests itself beyond the nucleon-nucleon interaction in the structure of nuclei and in nuclear reactions which selectively populate the Isobaric Analog States (IAS) [1].

The necessity to investigate experimentally the size of isospin-mixing in different mass regions stems mainly

from two questions. One concerns the detailed knowledge of nuclear structure in mirror nuclei pair (two nuclei for which the number of protons(neutrons) in one is equal to the number of neutrons(protons) of the other) and isobaric triplets (having $(N-Z)/2=-1,0,+1$ and the same A). The other is related to the precise information on the weak interaction in beta decay which involves the up and down quarks. For the study of the weak interaction from beta decay lifetime, several efforts are made to improve the knowledge of the quantity ft . This quantity is related to the first element of Cabibbo-Kobayashi-Maskawa matrix, namely the V_{ud} term. Indeed, to obtain the ft values for β -decay of $0^+ \rightarrow 0^+$ super-allowed Fermi type, the isospin mixing value is an important correction to be

*Corresponding author: Franco.Camera@mi.infn.it

†Corresponding author: Angela.Bracco@unimi.it

made for the nuclear matrix element [2–6].

The breaking of isospin symmetry induces a mixing between states with different isospin values. The theoretical treatment of the isospin symmetry violation involves a subtle balance between the attractive short-range strong force and the repulsive long-range Coulomb interaction, which polarizes the entire nucleus. Consequently, a no-core framework should be used in dealing with nuclear structure problems, such as that of the energy difference in mirror nuclei and in isobaric triplets [7]. Recently, using the nuclear Density Functional Theory DFT [6] containing the no-core feature, new predictions were obtained for the mirror energy difference [7]. The same framework was also used to compute the isospin mixing coefficient as a function of mass. It was found that there is a need of contributions from isospin mixing to reproduce the mirror energy data. Furthermore, it turned out that use of the DFT model is ideal for medium mass nuclei, as for them a reliable shell-model interpretation is not always feasible, due to the large size of the required valence space [8].

Values of isospin mixing should be deduced from experiments to constrain theory and this can be achieved with observables sensitive to this quantity, as the E1 transitions in $N=Z$ nuclei (as described in the text below). This is necessary both to obtain the matrix element of the beta decay and to understand nuclear structure features.

In this Letter a new experimental work providing data on isospin mixing coefficient for $A=60$, obtained through the γ -decay of the Giant Dipole Resonance (GDR), is presented. The main motivations are: i) to test predictions of the mass dependence of the isospin mixing by providing data in a region where these effects become important; ii) to provide a stronger base to the technique that uses the GDR decay in compound nuclei which allows to populate $N=Z$ with stable ions up to $A=80$; iii) to extract an overall picture by comparing the results from this technique with mass measurements, beta decay, and E1 transitions among low-lying levels.

The technique used in the present work consists in detecting the E1 decay in nuclei with $N=Z$ which, for the selection rules of this symmetry, is forbidden unless there is a mixing of states with different isospin values. Since these mixings are rather small, the GDR, concentrating almost 100% of the E1 strength, represents a good probe to search for forbidden decays and to find a signature of the isospin mixing in nuclear states.

For $N = Z$ nuclei with medium mass, as they are unstable, the approach that can be used is to form, via fusion reactions, compound nuclei (CN) with $N = Z$ at finite temperature (T) and then deduce isospin mixing at $T=0$ using the model reported in Ref. [9]. In the long wavelength approximation, E1 transitions, as those from the GDR, are possible only between states with difference in isospin value equal to 1 ($\Delta I = 1$). In heavy ion reactions around the Coulomb barrier using self-conjugate projec-

tile and target nuclei, namely both with an equal number of protons and neutrons, the compound nucleus (CN) has $N=Z$ and thus is populated with isospin $I=0$ and can decay by E1 transitions only to states with isospin $I=1$, characterized by a level density lower than that of $I = 0$ states. However, if the initial state has some degree of isospin mixing (and thus it has a small $I=1$ component) its E1 decay to the more numerous $I=0$ states can occur. This technique, proposed firstly in Ref. [11], was employed so far only in few other works concerning the $A \approx 30$ [10–13] and $A=80$ [14, 15] mass regions. However, only in the case of $A=80$ a complete analysis of the data at two different temperatures was made, allowing to infer the isospin mixing coefficient at zero temperature by using the model of Ref. [9]. One should also point out that the GDR in nuclei at finite T and angular momentum was investigated in many experimental and theoretical works and, thus, a solid base exists for the use of statistical analysis of the measured spectra (see e.g. [16–18]). In addition, at finite temperature, one expects a partial restoration of the isospin symmetry because the degree of mixing in a CN is limited by its finite lifetime, as predicted by Wilkinson [19].

The present experiment was performed at the Legnaro National Laboratory LNL (INFN, Italy) employing heavy ion beams from the TANDEM accelerator to measure the γ decay from the fusion-evaporation reaction $^{32}\text{S} + ^{28}\text{Si}$ at two different bombarding energies, 86 and 110 MeV, leading to the CN ^{60}Zn with isospin $I=0$, as both beam and target have $N=Z$. A second reaction, namely $^{32}\text{S} + ^{30}\text{Si}$ at 75 and 98 MeV, leading to the CN ^{62}Zn with non-zero isospin, was measured and used as a reference. The bombarding energies were chosen so that both compound nuclei were populated at the same excitation energies, namely $E^* = 47$ MeV and $E^* = 58$ MeV. The corresponding temperature of the CN on which the GDR is built, is 2 MeV and 2.4 MeV, as deduced from the expression $T = \sqrt{(E^* - E_{\text{GDR}} - E_{\text{rot}})/a}$ where E_{GDR} is the GDR energy, E_{rot} is the rotational energy, $a = A/8$ MeV $^{-1}$ is the level density parameter and A the mass number.

The experimental setup consisted of the GALILEO detection system, including 25 HPGe detectors [20], coupled to an array of 10 LaBr $_3$ (Ce) scintillator detectors. These LaBr $_3$ (Ce) detectors were used to measure γ decay up to 25 MeV and were calibrated using 15.1 MeV γ -rays from the reaction $^{11}\text{B} + d \rightarrow ^{12}\text{C} + n$ at 19.1 MeV. The HPGe detectors were used to identify the type of residual nuclei produced in the reactions. By inspecting the low-energy spectra measured with the HPGe detectors, an oxygen contamination in the targets was seen. Therefore, some of the experimental runs were repeated with better quality targets. In addition, by comparing statistical model calculations (see below for details) for the $^{60,62}\text{Zn}$ and ^{48}Cr compound nuclei, the latter populated by fusion with oxygen, it was found that the GDR decay

in the region of interest (10-18 MeV) is not significantly affected by the target contamination.

The analysis of the measured high energy γ -ray spectra was made using the statistical model and it was mainly based on three steps: i) the fit of the ^{62}Zn data to obtain the GDR parameters, namely strength, width and centroid; ii) the fit of the ^{60}Zn spectra using the parameters from step 1 and varying the Coulomb spreading width as the only free parameter; iii) the evaluation of the isospin mixing coefficient using the parameter obtained from step 2. For the statistical model calculations, the version of the code CASCADE [22, 23] including the isospin formalism (as in Ref. [14]) was employed. The isospin mixing is included according to the parametrization of Harney, Richter and Weidenmüller [24] in which the mixing between the state $I_{<} = I_0$ and $I_{>} = I_0+1$ is considered, where I_0 is isospin of the initial CN state. At finite excitation energy, the compound nucleus exhibits a decay width Γ_{\geq}^{\uparrow} and a mixing probability, α_{\geq}^2 , that are defined as

$$\alpha_{\geq}^2 = \frac{\Gamma_{\geq}^{\downarrow}/\Gamma_{\geq}^{\uparrow}}{1 + \Gamma_{\geq}^{\downarrow}/\Gamma_{\geq}^{\uparrow} + \Gamma_{\leq}^{\downarrow}/\Gamma_{\leq}^{\uparrow}} \quad (1)$$

where, α_{\geq}^2 represents the mixing of states with I_{\geq} with states with I_{\leq} . In this expression $\Gamma_{\geq}^{\downarrow}$ is the Coulomb spreading width of the states \geq . $\Gamma_{\geq}^{\downarrow}$ is rather constant with excitation energy while Γ_{\geq}^{\uparrow} increases rapidly, thus a partial restoration of isospin symmetry at high excitation energy is expected. All calculations were folded with the detector response function and normalized to the experimental data in the energy region 5-6 MeV.

The GDR parameters were deduced by fitting the ^{62}Zn data with statistical model calculations, in which the Coulomb spreading width was set to zero, and minimizing the Figure Of Merit (FOM), defined as χ^2 over the number of counts [17], between 12 and 17 MeV. The obtained GDR parameters, $E_{GDR} = 18.4 \pm 0.1$ MeV and $\Gamma_{GDR} = 11.6 \pm 0.2$ MeV for $T=2$ MeV and $E_{GDR} = 18.1 \pm 0.1$ MeV and $\Gamma_{GDR} = 12.6 \pm 0.2$ MeV for $T=2.4$ MeV, are in line with the existing systematics. The corresponding computed spectra are shown together with the data in panel (a) of Fig.1. In the inset, the occurrence distribution for which the FOM of the $E_{GDR}-\Gamma_{GDR}$ pair is minimum, is displayed. The error bars are deduced from a Gaussian fit of the projections of this distribution.

As previously described, the Coulomb spreading width is an important quantity which is connected to the density of states with isospin 1 and to the matrix element of the Coulomb interaction from a state with isospin 0 to a state isospin 1. It can be obtained from experiment and in this case it was deduced from the ratio of the experimental data for ^{60}Zn and for ^{62}Zn , where only the first ones depend on the isospin mixing. Therefore, in this fit the Coulomb spreading width was the only free

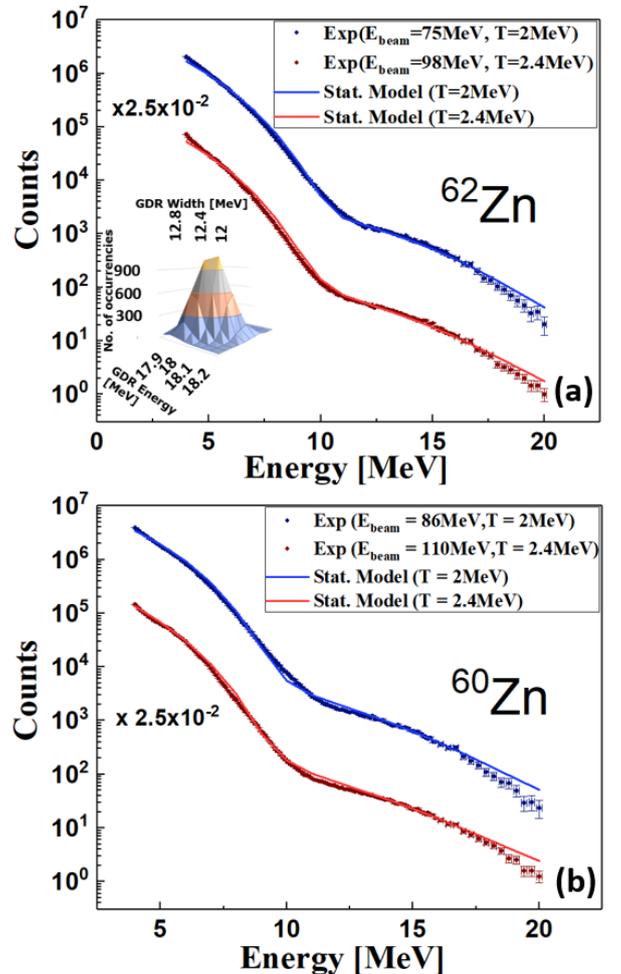


FIG. 1: *Panel (a)*: experimental spectra for ^{62}Zn at $T=2$ MeV (blue points) and $T=2.4$ MeV (red points) and the best fitting statistical model calculations corresponding to the minimum of the FOM for the variation of energy and width of the GDR (E_{GDR}, Γ_{GDR}). The occurrences distribution for which the FOM is minimized is displayed in the inset for the ^{62}Zn nucleus at $T=2$ MeV.

Panel (b): experimental spectra for ^{60}Zn at $T=2$ MeV (blue points) and $T=2.4$ MeV (red points) and statistical model calculations corresponding to the values of the Coulomb spreading width obtained by fitting the ratio spectra of Fig.2.

parameter since the GDR parameters were fixed to be those obtained from the fit of the ^{62}Zn data. The values corresponding to the minimum of the χ^2 are $\Gamma^{\downarrow} = 5(3)$ keV at $T=2$ MeV and $\Gamma^{\downarrow} = 7(3)$ keV at $T=2.4$ MeV. The error bars were obtained as combination of two different errors: one comes from the propagation of the errors of the GDR parameters and the other from the minimization procedure of the Γ^{\downarrow} . The calculations corresponding to the best fitting values of Γ^{\downarrow} are shown in panel (b) of

Fig.1. The ratio spectra from these measurements are presented in Fig.2 together with calculations for different values of Γ^\downarrow . The insets show the occurrence distributions for which the χ^2 is minimized.

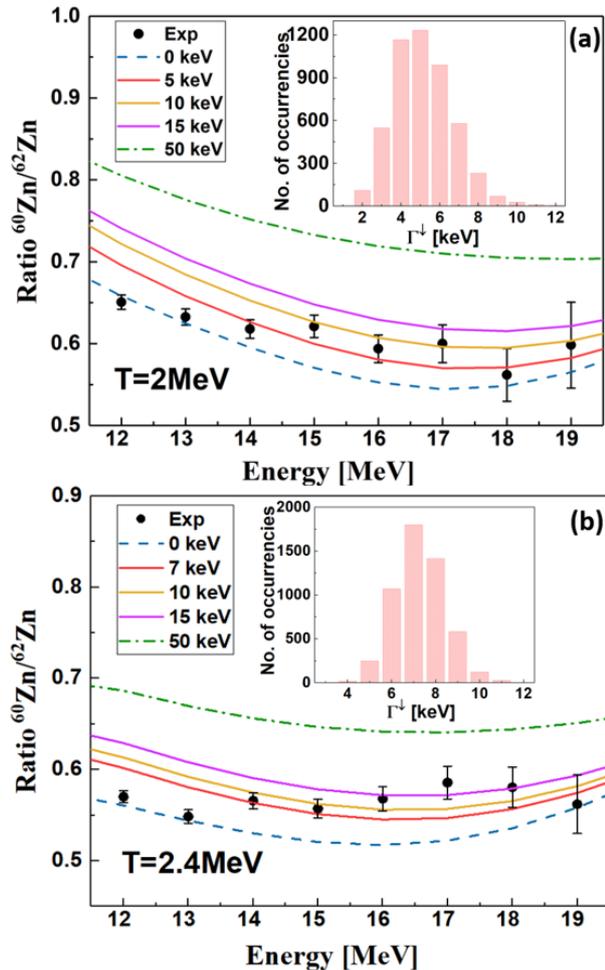


FIG. 2: The experimental and calculated ratio spectra $^{60}\text{Zn}/^{62}\text{Zn}$ are shown in panels (a) and (b), for $T=2$ and 2.4 MeV, respectively. The curves (shown with different colors as indicated in the legend) are statistical model calculations corresponding to different values of the Coulomb spreading width, namely $\Gamma^\downarrow=0, 5$ keV for $T=2$ MeV, 0 and 7 keV for $T=2.4$ MeV, and $10, 15, 50$ keV for both temperatures. In the insets the occurrence distributions for which the χ^2 is minimized are displayed.

It is interesting to compare the value of Γ^\downarrow extracted from the GDR decay at finite temperature with those existing in the literature [24, 25], mostly concerning the IAS width and also from other methods. Fig.3 shows these data for mass values $A=30, 60$ and 80 for which GDR data exist. The present work confirms that Γ^\downarrow is a quantity that does not depend on temperature. This is consistent with the finding at $A=80$ (Ref.[15]), having a similar error bar, and also with that at $A\approx 30$ where

the datum from GDR has a larger error but still within the large spread of values from other observables [13]. In addition, the similarity of the results from different methods indicates that they come from the same physical mechanism [26, 27].

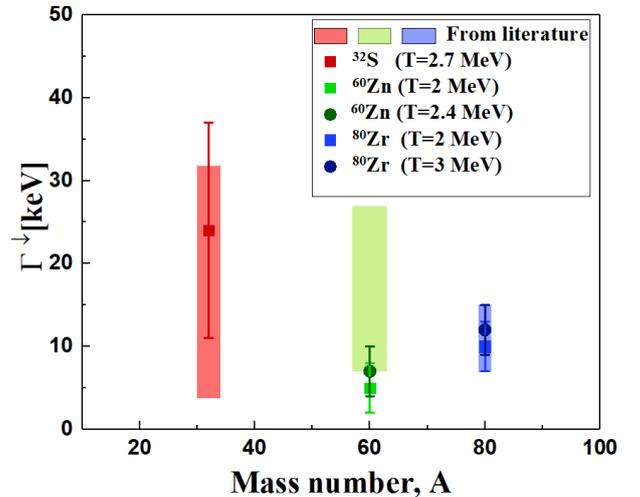


FIG. 3: The Coulomb spreading width obtained in the present work shown together with the values for other mass regions deduced from GDR γ -decay measurements in compound nucleus reactions [13–15]. The colored bars indicate the spread of values reported in literature and obtained with other methods.

Following the prescriptions used in Ref. [14, 15], we deduced the degree of mixing at angular momentum $J=0$ and we found $\alpha^2_{>} = (2.1 \pm 1.2)\%$ at $T=2$ MeV and $\alpha^2_{>} = (1.8 \pm 0.8)\%$ at $T=2.4$ MeV. These results of the isospin mixing are rather constant with temperature, mainly due to its small change of only 0.4 MeV for the two measurements. In general, the isospin mixing is expected to decrease with temperature as a result of a dynamical mechanism in the nucleus governed by the lifetime of the system, which decreases with excitation energy. In the case of $Z=40$, for which data were obtained at two very different temperatures, this effect of the decrease with increasing temperature of the isospin mixing was instead clearly seen.

The isospin mixing for the ground state was deduced from its value at finite temperature by using the model of [9], already tested for the nucleus ^{80}Zr [15]. Within this approach, the variation of the mixing probability with T is given by

$$\alpha^2_{>}(T) = \frac{1}{I_0 + 1} \frac{\Gamma^\downarrow_{\text{IAS}}}{\Gamma_{\text{CN}}(T) + \Gamma_{\text{IVM}}(\text{IAS})} \quad (2)$$

where $\Gamma^\downarrow_{\text{IAS}}$ is the width of the IAS, to be considered equal to $\Gamma^\downarrow_{>}$, $\Gamma_{\text{IVM}}(\text{IAS})$ is the width of the Isovector Monopole Resonance (IVM) at the excitation energy of

the IAS, which is expected to be constant with T and Γ_{CN} is the compound nucleus decay width that increases with T . $\Gamma_{\text{IVM}}(\text{IAS})$ is not expected to vary significantly in the mass interval 60-80 so that the value of 240 keV [1, 9, 14, 15], previously deduced for ^{80}Zr , was here used. For the nucleus ^{60}Zn the isospin mixing probability at $T = 0$ deduced from this experiment is $\alpha_{>}^2 = 2.5\% \pm 0.8\%$.

With this new experimental point for $\alpha_{>}^2$ at $A=60$, obtained with the same technique used for the existing datum at $A=80$, we can provide a good test to theory in this particular mass region where a sharp increase is expected. Moreover, this finding for $\alpha_{>}^2$, consistent with that for ^{64}Ge [28] from a low-lying state analysis, supports the validity of this technique involving a well established statistical model analysis of the GDR spectra and the use of a model to extract the zero temperature values. In figure 4 the data from γ -decay are compared with two predictions [4], based on isospin- and angular momentum-projected DFT calculations, which are indicated with Theory 1 (Theory 2) and they were obtained after (before) performing the rediagonalization on the isospin basis. This comparison indicates that the rediagonalization is important to describe the isospin mixing data. The isospin mixing, within the extended mean-field approach of these not perturbative predictions, takes into account long-range polarization effects due to the Coulomb interaction.

The correction term from isospin-mixing breaking δ_{C} for the first term (V_{ud}) of the CKM matrix was deduced from the $\alpha_{>}^2$ value from both experiments and predictions. According to the prescription of Ref. [30] δ_{C} is connected to $\alpha_{>}^2$ via the expression:

$$\delta_{\text{C}} = 4(I + 1) \frac{V_1}{41\xi A^{2/3}} \alpha^2 \quad (3)$$

where $V_1 = 100$ MeV, $\xi = 3$, and I is the isospin of the nucleus. This quantity is shown in the inset of Fig.4 for the mass region 50-100. The curve is extracted, through Eq. 3, from the predictions of $\alpha_{>}^2$ ($T=0$) of Theory 1 while the data are from the GDR experiments (violet and red stars), beta-decay experiments (black points) and a mass measurement (blue triangle). The isospin breaking correction δ_{C} is sizable in this mass region and increases rather sharply following the trend of the isospin mixing.

In summary this experiment has allowed to deduce for the first time the isospin mixing coefficient for $A=60$ using the GDR γ -decay from compound nuclei. This new finding supports the validity of this experimental technique based on the measurement of the GDR γ -decay in $N=Z$ compound nuclei to give insight into the problem of isospin mixing. A good test of theory in the mass region $A=60-80$ is provided by the GDR data making more reliable the evaluation of the isospin-mixing correction necessary to deduce the features of the weak interaction from beta decays. In addition, a better knowledge of

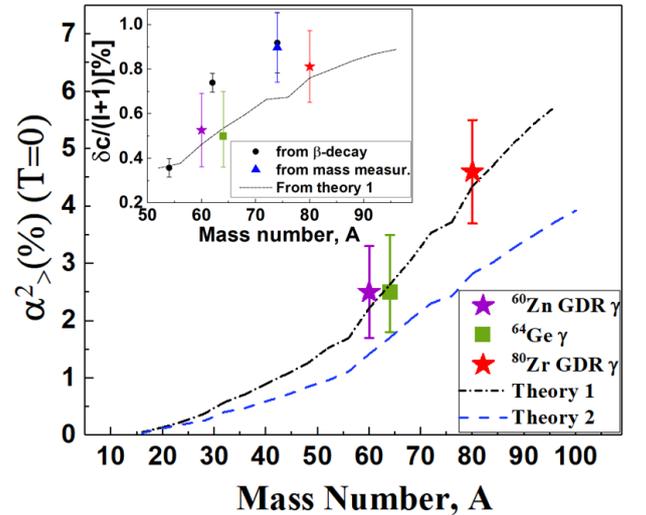


FIG. 4: Data and predictions for the isospin mixing coefficient $\alpha_{>}^2$ at $T=0$. The violet and red stars, from GDR γ -decay, are for ^{60}Zn (this work) and ^{80}Zr [15], respectively. The green square shows the value for ^{64}Ge from Ref. [28] deduced from a low-lying E1 transition.

The curves are calculations from Ref. [4]. The dash-point (dash) line, indicated as Theory 1 (Theory 2), corresponds to calculations obtained after (before) performing a rediagonalization on the isospin basis. The inset displays the isospin mixing correction δ_{C} , related to $\alpha_{>}^2$, which is employed to extract the Ft values from beta decay. Black circles, from Ref. [2], were deduced from β -decay, the blue triangle was obtained from the mass measurement [29], the red star is from Ref. [15] while the violet star is from the present work. The black line in the inset is extracted, through Eq. 3, from the predictions of $\alpha_{>}^2$ ($T=0$) of Ref. [4], here denoted as Theory 1.

the isospin-mixing correction has implications for the predictions of the structural difference of mirror nuclei. Future efforts should go in the direction of studying heavier CN towards ^{100}Sn using radioactive beams by exploiting this technique which gives access to regions not directly accessible at $T = 0$.

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