Synthesis of the mutual inductor of a Wireless Power Transfer Systems: a field-circuit approach

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Wireless power transfer systems are a viable solution to solve the problems that still delay a widespread diffusion of electric vehicles. Performances of these systems are affected by the magnetic characteristic of the coupling coils, so that they should be carefully designed. This paper presents a two-stage process for the optimal design of the coils. In the first stage, the equivalent circuit of the coupled coils is synthesized using an analytical approach for the computation of the objective functions and a genetic algorithm for their minimization. In turn, in the second stage, the optimization acts on objective functions computed by FEM analysis and identifies the geometrical parameters of the coils with the aim of achieving the circuit parameters recognized as optimal at the end of the first stage. The effectiveness of the proposed method is checked comparing the obtained coil design with a laboratory prototype.

Index Terms— **Coil design, Wireless power transfer, Finite Element Analysis, multi-objective optimization.**

I. Introduction

Wireless Power Transfer Systems (WPTSs) could be used for the charge of the onboard batteries of the electric vehicles instead of the classical battery chargers connected to the vehicle by cables [1], [2].

The theoretical principle at the basis of WPTSs lies on two coupled coils that transfer power from the transmitting to the receiving coil by means of electromagnetic induction ~~from the transmitting to the receiving coil~~; the ~~Fists~~ First Harmonic Equivalent (FHE) circuit of a WPTS is sketched in Fig.1. The transmitting coil is placed under the road soil and is supplied by a current at high frequency by means of a power generator. The receiving coil is installed on the bottom of the vehicle chassis and is coupled to one of the transmitting coils when they ~~coils~~ are aligned [3], [4].

The performance of the WPTSs are related to the power static converters and to the inductive parameters of the coils. The paper is aimed at solving the twofold problem of identifying the inductive parameters that optimize the WPTS performance, first, and to design the coils that implement those parameters, next; both these activities are performed by means of an algorithm of automated optimal design. A pair of coils, designed according to the conventional method [5], and sized to charge the battery of a mini-car, is used as a reference to check the effectiveness of the algorithms themselves.

The circuit synthesis, in order to find improved electrical parameters, like *e.g.* the mutual inductance between the receiving and ~~transferring~~ transmitting coils, is performed by means of NSGA-II (Non-dominated Sorting Genetic Algorithm) starting from a population of N individuals. The geometry optimization in order to synthesize the device with the mutual inductance found in the previous step is performed using both NSGA-II and BiMO (Biogeography-inspired Multi-Objective) [6]–[10] .

The results of the optimization process are a set of non-dominated optimal solutions; each of them offering a feasible coils design for the enhancement of the prototype performance. They can be subsequently analyzed in order to select the most feasible one for the practical realization of a prototype.

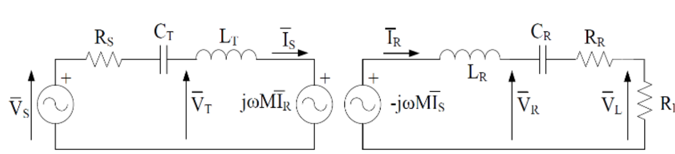

Fig. 1 Electrical FHE circuit of the wireless charger system [1].

II. The forward problem

The design of the windings of the two coils that form the wireless charge device~~,~~ in such a way to increase the efficiency of the circuit, reduce the supply voltage and use as less copper as possible, is here considered. In view of solving the forward problem, a field-circuit approach is developed. In particular, the power circuit supplying the battery is shown in Fig. 1, where V_s is the converter voltage, V_L is the load voltage and R_L represents the battery load.

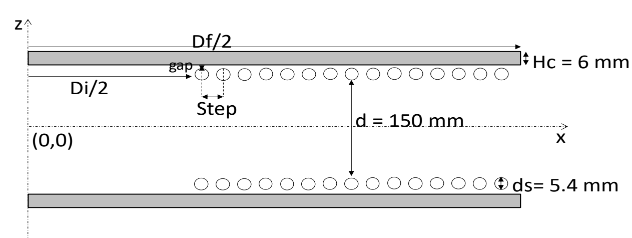

Fig. 2 Geometry of the device (with design variables).

In turn, the geometry of the ~~inductor~~coils, shown in Fig. 2, is simulated by means of an axisymmetric finite-element model, solving the magnetic time-harmonic problem. In particular, the ~~mutual inductor~~coil pair is composed of two aligned pancake

~~inductors~~coils, each of them characterized by N_t =15 turns, internal radius $D_i/2$, and 'Step' turn-step; two ferrite plates are placed at a 'Gap' distance from the coil winding. The ferrite plates have $D_f/2$ radius. The ~~primary~~ transmitting coil~~, generally positioned on the soil,~~ is supplied by a current controlled voltage generator operating at 85 kHz according to the Society of Automotive Engineers (SAE) specification, whereas the ~~secondary~~ reciving coil, ~~generally~~ positioned on the bottom of the car, is placed at 14 cm from the ~~primary~~ transmitting to comply with the ground clearance of the mini-car.

The magnetic problem of the geometry in Fig. 2 was solved in time harmonic condition in the magnetic vector potential, $\dot{\boldsymbol{A}}$, and scalar electric potential $\dot{V}$, using a FE model imposing the gauge of Coulomb, $\nabla \cdot \dot{\boldsymbol{A}} = 0$ [11]–[13]:

$$\nabla \times (\mu_0 \mu_r)^{-1} \nabla \times \dot{A} + j\omega\sigma\dot{A} = -\sigma\nabla\dot{V} \tag{1}$$

$$\nabla \cdot \sigma\left(j\omega\dot{A} + \nabla\dot{V}\right) = 0 \tag{2}$$

where μ_0 and μ_r are the vacuum and relative magnetic permeability respectively ($\mu_r = 1$ in air); σ is the conductivity of the medium (σ= 0 in air) and ω=2πf with f frequency of the supplied current.

III. The inverse problem

The inverse problem is structured in two stages and then solved by means of optimization. The first stage relies on a circuit model of the device and aims at identifying the mutual inductance of the ~~inductor~~ coil windings that minimizes the supply voltage and, simultaneously, maximizes the efficiency. In turn, the second stage relies on a field model of the device and aims at identifying the geometry of the ~~inductor~~ coil windings that implement the mutual inductance selected from the previous stage and at the same time to minimize the copper quantity. The circuit model was originally developed in [3]; the field model is based on an axisymmetric finite-element analysis in time-harmonic conditions (Flux 2D, manufactured by Altair [14] or MagNet manufactured by Mentor Graphics [15]).

The two optimization problems were solved using genetic class algorithm like NSGA-II that simulates the evolution of a population [16]–[18]. The starting population increases generating new individuals and the selection operate in order to decide the individuals that are improved and ~~they~~ that will ~~be~~ form the new population [19]. Both the proposed problems, circuit-based and field-based, are bi-objective problems and they search for minimize a couple of objective functions.

For solving this problem, the μ-BiMO, an optimization algorithm found to be cost-effective, is applied. The μ-BiMO is a modification of the BiMO algorithm, which, in turn, is an extension of the BBO. The BBO algorithm is based on the process of natural immigration and emigration of species between small islands in the search for more friendly habitats, which is observed in nature. Each solution considered is treated as a habitat or island (design vector or individual in genetic algorithms) composed of suitability index variables (SIV, design variables), and each habitat exhibits a quality given by the habitat suitability index (HSI, objective function). The ecosystem, which is the whole set of islands or habitats, is progressively modified by means of two stochastic operators, i.e. migration and mutation: migration improves the HSI of poor habitats by sharing features from good habitats (exploitation step); in turn, mutation modifies some randomly selected SIV of a few habitats in view of a better search in the design space (exploration step).

BBO algorithm has been widely used in the last decade as single-objective algorithm for different applications; in turn, in the last two years, it was extended to multi-objective ~~optimisation~~optimization problems (BiMO algorithm) [20]–[25], thanks to the concept of generalized fitness.

In this new version of BiMO, the role of small rocks in the migration of individuals is considered. As in reality the small rocks help immigrants to colonize islands that otherwise would not be reached, with the concomitant loss of the individuals who would never reach the ground, in the proposed method the rocks have the function not to waste habitats that otherwise would never characterize an ecosystem.

In particular, during the migration procedure it could happen that good habitats are replaced. To recover this, the discarded habitats are stored in a vector (rock vector) that tracks the habitats.

In BiMO, when the number of islands is very small, during the processes of immigration and emigration, the generation of duplicates is a frequent event. Instead of generating new habitats randomly, they are taken from the best habitats belonging to the rock vector.

As far as the degrees of freedom are concerned, in the first stage two design variables, *i.e.* the mutual inductance M in the range [10-62] μH, and the load voltage V_L in the range [76.4, 127.3] V, are searched for [5]. Correspondingly, the following two objective functions, *i.e.* the value of the supply voltage $V_s = V_s(M,V_L)$

$$V_S = 2\omega M \frac{P_B}{V_L} \tag{3}$$

to be minimized, and the efficiency $\eta(M,V_L)$ of the system

$$\eta = \frac{\omega^2 M^2 R_L}{(R_R+R_L)^2 R_S + \omega^2 M^2 (R_R+R_L)} \tag{4}$$

to be maximized, are defined. In (3) P_B is the power transferred to the battery; in the case of the reference coils it is fixed at 560 W because this is the maximum power that can be injected in the mini-car battery during the charge. In (4) R_R and R_S are the parasitic resistances of the coil windings. As a first approximation, considering the results obtained from previous experimental activities [26], [27], both of them have been considered equal to 0.5 Ω.

In the second stage, the geometry of the coil windings is unknown. The following four design variables are selected: diameter of the ferrite plates, D_f [150, 550] mm, internal diameter of the coils, D_i [100, 200] mm, turn step, Step [4.6,10] mm, axial distance between coil and plate, Gap [1, 5] mm.

The optimization problem reads: *identify the coil geometry such that the length of copper-made conductor is minimized, fulfilling the prescribed value of mutual inductance*. The most general solution is represented by the Pareto front trading off mutual inductance and conductor length. The objective functions to be minimized are:

$$f_1 = M - 62\ [\mu H] \tag{5}$$

$$f_2 = \frac{1}{2} D_i N_t + Step\left(\sum_{k=1}^{N_t} k\right) \tag{6}$$

where f_1 represents the discrepancy with the target inductance, whereas f_1 is function of the internal diameter of the winding and turn step and it is proportional to the length of the winding.

IV. Results

For solving the circuit-based problem, both NSGA-II and µBiMO algorithms are applied. The NSGA-II algorithm uses 20 individuals, 100 generations for circuit-based optimization, and 20 individuals, 50 generations for field-based optimization, while µBiMO is based on 5 islands and run for 50 iterations.

Table I Design variables and objective function values for solutions at Pareto front ends (circuit-based problem).

Point	V_L[V]	M [uH]	f_1	f_2
A	100.00	62.00	305.43	0.09
B	100.00	10.00	61.46	0.27
Exp.	100.00	31.61	158.67	0.11

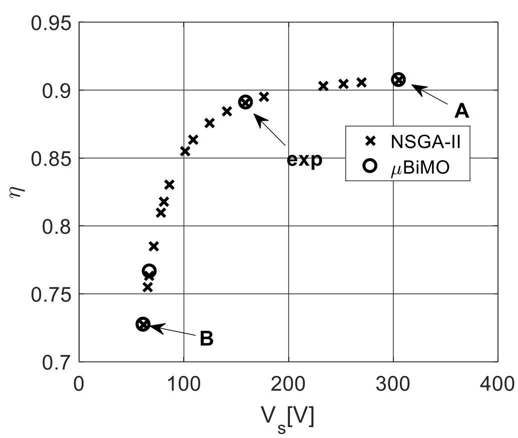

Fig. 3. Pareto front of the circuit-based problem obtained by means of the two optimization algorithms NSGA-II (cross) and µBiMO (circle). The experimental prototype is also highlighted.

The optimization results obtained for the circuit-based problem are reported in Table I and Fig. 3. In particular, Table I lists the values of M and V_L relevant to the extremities A and B of the Pareto front. For the sake of a comparison, in Fig. 3 the values corresponding to the newly sized coils, denoted with circle and cross, are shown together with the values relevant to the experimental laboratory prototype, denoted with the caption "exp"; a picture of the prototypal coil is given in Fig.4. NSGA-II approximated well the Pareto front, while µBiMO, because of the reduced number of islands, approximated it with less points. Both the methods found the two end-points of the front (point A and B in Table I and Fig. 3).

Fig. 4. Experimental coil without upper enclosure plate.

For solving the field-based optimization, both NSGA-II and µBiMO algorithms are used. NSGA-II is based on 20 individuals and run for 50 generations, while µBiMO is run for 100 iterations with 5 islands.

The results of the field-based problem and those relevant to the experimental coil are reported in Table II and Fig. 5, respectively, adopting the same conventions used in Fig. 3 for their representation.

Table II Design variables [mm] and objective function values for at Pareto front ends (field-based problem and laboratory device).

Point	D_i [mm]	Step [mm]	D_f [mm]	Gap [mm]	62-M [µH]	Copper [mm]	M [µH]
A	200.0	10.0	549.56	1.47	2.30	2550.0	59.70
B	100.0	4.6	478.50	1.10	50.09	1233.0	11.91
Exp	140.0	6.6	400.0	1.0	30	1818.00	32

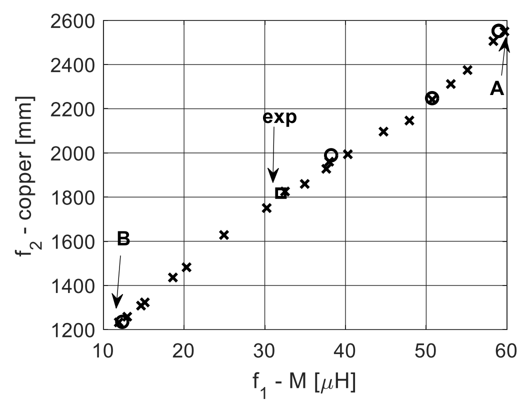

Fig. 5. Pareto front of the fem-based problem obtained by means of the two optimization algorithms NSGA-II (cross) and µBiMO (circle). The experimental prototype is also highlighted with a square.

Both algorithms found a good approximation of the two end-points, A and B in Fig. 5. Nevertheless, NSGA-II well approximated the whole Pareto front instead of µBiMO that is able to identify only 5 points belonging to the Pareto front.

For both the problems, the experimental prototype belongs to the Pareto front i.e. is an optimal solution.

In Fig. 6 the field map relevant to the solutions A and B of Fig. 5 are shown.

In particular, the comparison of Figs. 3 and 5 shows that all the solutions laying between point A and point exp have nearly the same efficiency but the copper requirement decreases of about 30% moving from the first one to the second. It is worth to notice that in computing the efficiency by (2), the parasitic resistances have been considered constant while in a real coil they depend on the actual wire length and section. From this consideration it derives that probably a pair of coils realized according to the design solution Exp behaves better than what is reported in Fig. 3 from the point of view of efficiency while a pair of coils derived from solution A behaves worse.

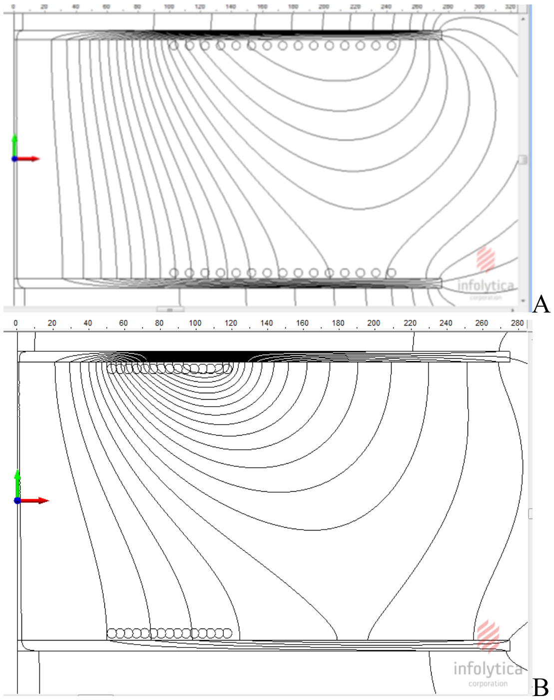

Fig. 6. Field map of the optimal solutions A and B of the fem-based problem (see Fig.5).

V. Conclusion

A two-stage process has been presented for the optimal design of the coupled coils of a WPTS. A set of optimal solutions on a Pareto front has been obtained at each stage. In both cases, comparison of the results optimization process with the parameters of the prototype shows that the latter ones lay on the Pareto fronts, thus confirming the soundness of proposed approach. Designing the prototype required a long, iterative effort made of several circuital simulations and FEM analysis while the presented process took only few hours to reach the same results with, moreover, an additional whole set of possible alternative design solutions among which the best suited to the particular application can be selected.

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