## Measurement of the $\Lambda_{c}^{+}$lifetime

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 （The Belle II Collaboration）

An absolute measurement of the $\Lambda_{c}^{+}$lifetime is reported using $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}$decays in events reconstructed from data collected by the Belle II experiment at the SuperKEKB asymmetric-energy electron-positron collider. The total integrated luminosity of the data sample, which was collected at center-of-mass energies at or near the $\Upsilon(4 S)$ resonance, is $207.2 \mathrm{fb}^{-1}$. The result, $\tau\left(\Lambda_{c}^{+}\right)=$ $203.20 \pm 0.89$ (stat) $\pm 0.77$ (syst) fs, is the most precise measurement to date and is consistent with previous determinations.

Searches for physics beyond the standard model of particle physics through precise measurements of weakly decaying charm or bottom hadrons often rely on accurate theoretical descriptions of strong interactions at low energy, typically using effective models such as the heavy quark expansion (HQE) [1-7]. The HQE provides a consistent framework for computing the decay widths of heavy hadrons in terms of inverse powers of the heavy quark mass. For bottom hadrons, non-perturbative effects are relatively small and the HQE in $1 / m_{b}$, where $m_{b}$ is the mass of the bottom quark, works well. In contrast, higher-order corrections due to the influence of light valence (spectator) quarks are significant for charm hadron lifetimes, for which the HQE to $1 / m_{c}^{3}$ does not satisfactorily describe lifetimes [7]. The lifetimes of the $\Omega_{c}^{0}$ and $\Xi_{c}^{0}$ were recently measured to be much larger than the previous world average [8 10], inverting the known hierarchy of charm lifetimes. Careful consideration of model-dependent spectator effects is required for theoretical predictions of charm baryon lifetimes to agree with experimental measurements [6, 7]. Improved measurements of charm baryon lifetimes therefore provide refined tests for effective models.

The world average value of the $\Lambda_{c}^{+}$lifetime is $202.4 \pm$ 3.1 fs 11]. Previous measurements include percent-level results from the FOCUS, SELEX, and CLEO collaborations two decades ago [12-14], as well as a more precise measurement, relative to the $D^{+}$lifetime, from the LHCb collaboration [9]. The latter of these has a limiting systematic uncertainty associated with the $D^{+}$lifetime. Relative measurements minimize systematic uncertainties related to event selection that may bias the decay time, particularly at hadron colliders. In contrast, the ability to reconstruct charm hadrons without biasing the decay time allows experiments at electron-positron $\left(e^{+} e^{-}\right)$colliders to precisely determine absolute lifetimes, as demonstrated by the recent measurement of the $D^{0}$ and $D^{+}$lifetimes [15] from the Belle II experiment 16] at the SuperKEKB asymmetric-energy $e^{+} e^{-}$collider 17]. The most recent $\Lambda_{c}^{+}$lifetime measurement at an $e^{+} e^{-}$ collider, from the CLEO collaboration, is in mild tension with other results and increases the uncertainty of the world average 11]. A precise, absolute measurement by Belle II may help to resolve the tension between $\Lambda_{c}^{+}$ lifetime measurements at $e^{+} e^{-}$colliders and other experiments and will substantially improve the world average.

In this Letter, we report a precise measurement of the $\Lambda_{c}^{+}$lifetime using $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}$decays reconstructed in data collected at or near the $\Upsilon(4 S)$ resonance, corre-
sponding to a center of mass energy at or near 10.58 GeV , by the Belle II experiment from 2019 to mid 2021 and corresponding to an integrated luminosity of $207.2 \mathrm{fb}^{-1}$. Unless specified otherwise, charge conjugate decays are implied throughout.

The lifetime of the $\Lambda_{c}^{+}$is determined from a twodimensional fit to the decay time $t$ and its uncertainty $\sigma_{t}$. The decay time is calculated assuming that $\Lambda_{c}^{+}$candidates are promptly produced from continuum $e^{+} e^{-} \rightarrow c \bar{c}$ events and is determined according to $t=m_{\Lambda_{c}} \vec{L} \cdot \vec{p} /|\vec{p}|^{2}$, where $m_{\Lambda_{c}}$ is the world average mass of the $\Lambda_{c}^{+}$11], $\vec{L}$ is the displacement of the $\Lambda_{c}^{+}$decay point from the $e^{+} e^{-}$ interaction point (IP), and $\vec{p}$ is the momentum of the $\Lambda_{c}^{+}$ candidate. The position and size of the IP region is determined using $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$events. Event selection criteria and the fit strategy are optimized and validated using simulated data, but the fit to the collision data does not use any input taken from simulation.

The Belle II detector [16] includes a tracking system comprising a two-layer silicon pixel detector (PXD) surrounded by a four-layer double-sided silicon strip detector (SVD) and a 56 -layer central drift chamber (CDC). The second layer of the PXD had $15 \%$ azimuthal coverage during the collection of the data used in this study. For the $\Lambda_{c}^{+}$decays considered here, the combined PXD and SVD vertexing system provide a decay-length resolution of $40 \mu \mathrm{~m}$, corresponding to an average decay time resolution of 87 fs for an average decay length of $96 \mu \mathrm{~m}$. A time-of-propagation counter in the barrel region of the detector and an aerogel ring-imaging Cherenkov counter in the endcap region provide charged-particle identification (PID) information. An electromagnetic calorimeter consisting of $\mathrm{CsI}(\mathrm{Tl})$ crystals provides energy and timing measurements for photons and electrons. A $K_{\mathrm{L}}^{0}$ and muon detector is installed in the iron flux return yoke of a superconducting solenoid magnet that provides a 1.5 T magnetic field.

We generate $e^{+} e^{-} \rightarrow q \bar{q}$ events with KKMC 18] and hadronize quarks with Pythia 8 [19]. Hadron decays are emulated using EvtGen 20]. The detector response is simulated with Geant4 [21]. Reconstruction of events from simulated and collision data is performed with the Belle II analysis software framework [22]. In addition to the excellent vertexing capability, Belle II benefits from good charged particle tracking performance 23, 24].

Candidate $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}$decays are each reconstructed from one negatively and two positively charged particles, which are required to be well measured with reliable uncertainties to allow for a precise lifetime mea-
surement. Each charged particle must be associated with one or more PXD measurements (hits) in the PXD, at least one hit in the first layer of the SVD, and at least 20 hits in the CDC. Each charged particle must have a distance of closest approach to the IP of less than 0.5 cm in the plane transverse to the beam and 2 cm in the direction parallel to it to remove charged particles not associated with the $e^{+} e^{-}$interaction. A fit constrains the charged particles to come from a common vertex and the $\Lambda_{c}^{+}$candidate to come from the IP [25]. Candidates with a vertex-fit $\chi^{2}$ probability less than 0.01 are rejected. Since the $\Lambda_{c}^{+}$is assumed to originate from the IP, secondary decays in which the $\Lambda_{c}^{+}$originates from a displaced vertex would bias the lifetime measurement. To suppress $\Lambda_{c}^{+}$from $B$ decays, the center-of-mass momentum of each $\Lambda_{c}^{+}$candidate is required to be greater than $2.5 \mathrm{GeV} / c$.

Charged PID information is combined from all subdetector systems except the PXD and SVD. This PID information is used to construct likelihoods $\mathscr{L}(h)$ for a given particle hypothesis $h$. For each candidate, one positively charged particle is required to have $\mathscr{L}(p) /(\mathscr{L}(p)+$ $\mathscr{L}(K)+\mathscr{L}(\pi))>0.9$, the negatively charged particle is required to have $\mathscr{L}(K) /(\mathscr{L}(p)+\mathscr{L}(K)+\mathscr{L}(\pi)+$ $\mathscr{L}(\mu)+\mathscr{L}(e)+\mathscr{L}(d))>0.6$, and the remaining positively charged particle is assumed to be a pion. Here $\mathscr{L}(d)$ is the deuteron hypothesis likelihood. The efficiency of the proton identification is found to be about $88 \%$, with a kaon contamination of less than $2 \%$, and the efficiency of kaon identification is $70 \%$, with a pion contamination of $6 \%$, from studies of $\Lambda^{0} \rightarrow p \pi^{-}$and $D^{*+}{ }_{-}$ tagged $D^{0} \rightarrow K^{-} \pi^{+}$decays. To reduce backgrounds from misidentified charm-meson decays, we reject events with $M\left(\pi^{+} K^{-} \pi^{+}\right)$in $[1.858,1.881] \mathrm{GeV} / c^{2}$ or [2.000, $2.020] \mathrm{GeV} / c^{2}$, with both positively charged particles assumed to be pions. These intervals correspond to three units of resolution, or standard deviations, on the reconstructed mass in both directions around the known $D^{+}$and $D^{*+}$ masses, respectively. Other charm-related backgrounds are suppressed by requiring that the transverse momenta of pions exceed $0.35 \mathrm{GeV} / c$ and those of protons exceed $0.7 \mathrm{GeV} / c$.

Events with multiple candidates, which occur at a rate of $0.2 \%$, are rejected. Analysis of simulated events shows that the event selection criteria do not bias the measurement of the $\Lambda_{c}^{+}$lifetime.

Decays of $\Xi_{c}^{0} \rightarrow \pi^{-} \Lambda_{c}^{+}$and $\Xi_{c}^{+} \rightarrow \pi^{0} \Lambda_{c}^{+}$may bias the measurement of the $\Lambda_{c}^{+}$lifetime, since the $\Xi_{c}^{0}$ and $\Xi_{c}^{+}$have lifetimes of $153 \pm 6 \mathrm{fs}$ and $456 \pm 5 \mathrm{fs}$ [11], respectively, and may shift the production vertex of the $\Lambda_{c}^{+}$away from the IP. The amount of $\Xi_{c}$ contamination is estimated from a fit to the distribution of the $\Lambda_{c}^{+}$vertex displacement from the IP in the plane transverse to the beam line. This distribution depends only on the resolution of the detector for $\Lambda_{c}^{+}$candidates that are produced at the IP. $\Lambda_{c}^{+}$candidates from $\Xi_{c}$ decays
have production vertices that are displaced from the IP and therefore a larger vertex displacement from the IP. The fit to the distribution of the $\Lambda_{c}^{+}$transverse vertex displacement gives a background contamination of 374 $\pm 88$ events, corresponding to $0.003 \%$ of $\Lambda_{c}^{+}$candidates. As this includes both combinatoric backgrounds and $\Xi_{c}$ decays, the central value is taken as an estimate of the maximum number of $\Xi_{c}$ decays. This value is consistent with predictions based on the expected production cross-sections for $\Xi_{c}^{0}$ and $\Xi_{c}^{+}$[26], the measured branching fraction for $\Xi_{c}^{0} \rightarrow \pi^{-} \Lambda_{c}^{+}$[27], and theoretical predictions for $\Xi_{c}^{+} \rightarrow \pi^{0} \Lambda_{c}^{+}$[28]. Backgrounds from $\Xi_{c}$ decays are reduced by restricting the invariant mass of the $\Xi_{c}$ candidate formed by combining the $\Lambda_{c}^{+}$candidate with a $\pi^{-}$or $\pi^{0}$ from the unassigned particle candidates of the event. This restriction is optimized using simulations and the optimal precision on the lifetime measurement is achieved by restricting the mass difference between the $\Xi_{c}$ and $\Lambda_{c}^{+}$candidates to two units of the mass resolution, removing events with $M\left(p K^{-} \pi^{+} \pi^{-}\right)$$M\left(p K^{-} \pi^{+}\right)$in $[0.1834,0.1864] \mathrm{GeV} / c^{2}$ or $M\left(p K^{-} \pi^{+} \pi^{0}\right)$ - $M\left(p K^{-} \pi^{+}\right)$in $[0.1753,0.1873] \mathrm{GeV} / c^{2}$. About $61 \%$ of $\Xi_{c}$ decays to $\Lambda_{c}$ survive this veto, according to studies of simulated events. To account for the effect of the remaining background of this type, the measured $\Lambda_{c}^{+}$lifetime is corrected by subtracting a bias of 0.34 fs , as discussed below.

After all selection criteria, the number of events in the $\Lambda_{c}^{+}$signal range, defined as $M\left(p K^{-} \pi^{+}\right)$in [2.283, 2.290] $\mathrm{GeV} / c^{2}$, within about 1.4 units of the mass resolution around the world average $\Lambda_{c}^{+}$mass, is $1.16 \times 10^{5}$. The relative amount of signal events, determined from a binned least-squares fit to the $M\left(p K^{-} \pi^{+}\right)$distribution (Fig. (1), is $92.5 \%$. In the fit, the $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}$signal is modeled as the sum of Gaussian and Johnson functions [29] with a common mode. The background is modeled as a linear function. Events from the $\Lambda_{c}^{+}$sidebands, defined as $M\left(p K^{-} \pi^{+}\right)$in $[2.249,2.264] \mathrm{GeV} / c^{2}$ and [2.309, 2.324] $\mathrm{GeV} / c^{2}$, are used to constrain the background in the lifetime fit.

The $\Lambda_{c}^{+}$lifetime is measured with an unbinned maximum-likelihood fit to the $\left(t, \sigma_{t}\right)$ distribution for events in the signal region. The signal probability density function (PDF) is the product of an exponential function in $t$ convolved with a Gaussian resolution function, which depends on $\sigma_{t}$, and a PDF for $\sigma_{t}$. The latter is a histogram template formed from signal candidates subtracted by the distribution of sideband candidates after scaling according to the size of the signal and background regions. To account for a possible bias in the decay-time determination, the mean of the resolution function is determined by the fit.

The background PDF, an empirical model of the sideband data, is the sum of two exponential functions convolved with Gaussian resolution functions, which account for backgrounds from long-lived particles, and a zero-


Figure 1: Mass distribution of $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}$candidates with fit projections overlaid. The vertical dashed lines enclose the signal region and the short, vertical dotted lines enclose the sidebands.
lifetime component consisting only of the resolution function, which accounts for combinatorial backgrounds. To account for a possible misestimation of the decay-time uncertainty, the width of the resolution function is given by the per-candidate $\sigma_{t}$ multiplied by a scale factor, $s$, which is a free parameter in the lifetime fit. The mean of the resolution function is common for all terms, but a separate $\sigma_{t}$-scaling parameter is used for the background PDF.

To better constrain the background, a simultaneous fit to the events in the signal region and sidebands is performed, where the $\sigma_{t} \mathrm{PDF}$ for the sidebands is a binned template determined by sideband events. The background fraction in the lifetime fit is Gaussian constrained to $(7.50 \pm 0.02) \%$, as determined from the $M\left(p K^{-} \pi^{+}\right)$ fit.

The lifetime fit is validated both on fully simulated data equivalent to $1 \mathrm{ab}^{-1}$, about five times the integrated luminosity of the collision data, and on simulated distributions generated by randomly sampling the lifetime PDF determined from a fit to the collision data. All validation fits return unbiased results, regardless of the assumed $\Lambda_{c}^{+}$lifetime. Studies of the decay-time distribution in simulation suggest that $\sigma_{t}$ is underestimated by about $10 \%$, which is in good agreement with the results from the lifetime fit to the data, for which the scale parameter is determined to be $\mathrm{s}=1.108 \pm 0.006$. The mean of the resolution function is determined to be 4.77 $\pm 0.63 \mathrm{fs}$.

The $\Lambda_{c}^{+}$lifetime is measured to be $203.20 \pm 0.89 \mathrm{fs}$, where the uncertainty is statistical only. The lifetime fit projection, overlaid on the decay time distribution in the data sample, is shown in Fig. 2, The $\sigma_{t}$ PDF used in the lifetime fit is shown in Fig. 3 The systematic uncertainty is calculated from the sum in quadrature of


Figure 2: Decay-time distribution of $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}$events in the signal region (top) and sidebands (bottom) with fit projections overlaid.


Figure 3: Decay-time uncertainty distribution of $\Lambda_{c}^{+} \rightarrow$ $p K^{-} \pi^{+}$events in the signal region (top) and sidebands (bottom). The $\sigma_{t} \mathrm{PDF}$ used in the fit is shown by the solid blue histogram and the background $\sigma_{t} \mathrm{PDF}$ is shown by the dashed red histogram.
individual contributions from the sources listed in Table $\square$ and described below.

The systematic uncertainty due to backgrounds from $\Xi_{c}$ decays is determined by adding simulated events of

Table I: Systematic uncertainties on the $\Lambda_{c}^{+}$lifetime.

| Source | Uncertainty $[\mathrm{fs}]$ |
| :--- | :---: |
| $\Xi_{c}$ contamination | 0.34 |
| Resolution model | 0.46 |
| Non- $\Xi_{c}$ backgrounds | 0.20 |
| Detector alignment | 0.46 |
| Momentum scale | 0.09 |
| Total | 0.77 |

this type to the $1 \mathrm{ab}^{-1}$ equivalent simulated sample according to the estimated maximum contamination determined from the fit to the distribution of the $\Lambda_{c}^{+}$transverse vertex displacement in data and repeating the measurement. The difference between the simulated $\Lambda_{c}^{+}$lifetime and the measured value is 0.68 fs . Since this is an estimate of the maximum effect of remaining $\Xi_{c}$ backgrounds, half the difference, 0.34 fs , is taken as both a correction to the lifetime and an associated systematic uncertainty.

The resolution model for the lifetime PDF is complicated by correlations between the decay time and the decay-time uncertainty such that it cannot be described by a simple Gaussian function. We neglect these correlations in our model, which consists of a $\sigma_{t}$-dependent Gaussian resolution multiplied by a PDF in $\sigma_{t}$, and include the impact of this approximation as a systematic uncertainty. We fit our model to 1000 sets of signalonly simulated decays, each with a size equivalent to the data. The sets are produced by resampling, with repetition, simulated events in an amount corresponding to an equivalent luminosity of $1 \mathrm{ab}^{-1}$. The difference in the mean lifetime determined from these fits relative to the true value is 0.46 fs , which is taken as a systematic uncertainty due to the resolution model.

To check the resolution model, the lifetime fit is repeated with the Gaussian resolution function replaced with a sum of two Gaussian functions. The difference in the measured lifetime, $0.36 \pm 0.23 \mathrm{fs}$, is covered by the corresponding systematic uncertainty. The bias of the decay-time resolution function for signal events depends on the $\Lambda_{c}^{+}$candidate mass, but cancels if the signal range is centered on the true mass. Differences in the measured lifetime with the signal region varied are consistent with statistical fluctuations and are within the systematic uncertainty due to the resolution model.

Sideband events are included in the lifetime fit to constrain the background PDF. In simulation, sideband events describe the background distribution in the signal region accurately. To account for potential disagreements between the signal region and sidebands in the data, we produce 1000 sets of simulated data by resampling from the $1 \mathrm{ab}^{-1}$-equivalent simulated sample for events in the signal region and from the sidebands of the data sample for events in the sideband region. The mean lifetime
residual is 0.20 fs , which is taken as a systematic uncertainty associated with background contamination.

To check the signal PDF for the $M\left(p K^{-} \pi^{+}\right)$fit, we replace the sum of Gaussian and Johnson functions with a sum of two Gaussian functions. Using the resulting background contribution has a negligible effect on the measured lifetime.

Reconstruction of charged particles at Belle II relies on periodic calibrations to correct for detector misalignment and surface deformations of the internal components of the PXD and SVD, as well as for relative alignments of the tracking system 30]. Detector misalignment can bias measured particle decay lengths and therefore their decay times. To account for imperfections in the detector alignment, sets of signal-only simulated data, each with a size comparable to the collision data, are produced with detectors randomly misaligned according to the alignment precision observed in data. The root mean square dispersion of the lifetime residuals in these misaligned simulated data sets is 0.46 fs , which is taken as a systematic uncertainty due to imperfect detector alignment.

The momenta of charged particles are scaled by a factor, 0.99971 , determined by calibrating the peak positions of abundant charm, strange, and bottom hadron decays. The uncertainty on this scale factor, 0.0009 , results in a systematic uncertainty on the $\Lambda_{c}^{+}$lifetime of 0.09 fs . The uncertainty on the world average of the $\Lambda_{c}^{+}$ mass results in a negligible systematic uncertainty.

As a check of the internal consistency of the lifetime measurement, the full analysis is repeated on subsets of data chosen according to data-collection periods and $\Lambda_{c}^{+}$ momentum ranges, directions, and charge. The result for each subset is consistent with the full result. The lifetime fit is also repeated by selecting the candidate with the best vertex fit probability or randomly selecting a candidate, rather than rejecting events with multiple candidates. The difference in lifetime in each case is negligible. Finally, several events in the data have lifetimes greater than 4 ps , as shown in Fig. 2. Studies of simulated events suggest that these are from long lived charm meson decays and show that they do not bias the lifetime result with the current data-set size.

In conclusion, we measure the $\Lambda_{c}^{+}$lifetime to be $203.20 \pm 0.89$ (stat) $\pm 0.77$ (syst) fs using data with an integrated luminosity of $207.2 \mathrm{fb}^{-1}$ collected by the Belle II experiment at the SuperKEKB asymmetric-energy $e^{+} e^{-}$ collider. This is consistent with the recent, relative measurement by LHCb [9] and other previous results, though the mild tension between the measurement by CLEO [14] and all other measurements remains. The absolute measurement presented here is the most precise $\Lambda_{c}^{+}$lifetime measurement to date and may be useful to test the accuracy of HQE models as theoretical precision improves and discrepancies between theory and experiment become more significant.

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[1] M. Neubert, Adv. Ser. Direct. High Energy Phys. 15, 239 (1998), arXiv:hep-ph/9702375.
[2] N. Uraltsev, in At the frontier of Particle Physics edited by M. Shifman and B. Ioffe (2001) arXiv:hep-ph/0010328.
[3] A. Lenz and T. Rauh, Phys. Rev. D 88, 034004 (2013), arXiv:1305.3588 [hep-ph].
[4] A. Lenz, Int. J. Mod. Phys. A 30, 1543005 (2015). arXiv:1405.3601 [hep-ph].
[5] M. Kirk, A. Lenz, and T. Rauh, JHEP 12, 068 (2017), [Erratum: JHEP 06 (2020) 162], arXiv:1711.02100 [hep-ph].
[6] H.-Y. Cheng, JHEP 11, 014 (2018) arXiv:1807.00916 [hep-ph] .
[7] J. Gratrex, B. Melić, and I. Nišandžić, (2022), arXiv:2204.11935 [physics.hep-ph].
[8] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 121, 092003 (2018), arXiv:1807.02024 [hep-ex].
[9] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 100, 032001 (2019) arXiv:1906.08350 [hep-ex].
[10] R. Aaij et al. (LHCb Collaboration), Science Bulletin 67, 479 (2022).
[11] P. A. Zyla et al. (Particle Data Group), PTEP 2020, 083C01 (2020).
[12] J. M. Link et al. (FOCUS Collaboration), Phys. Rev. Lett. 88, 161801 (2002) arXiv:hep-ex/0202001.
[13] A. Kushnirenko et al. (SELEX Collaboration), Phys. Rev. Lett. 86, 5243 (2001), arXiv:hep-ex/0010014
[14] A. H. Mahmood et al. (CLEO Collaboration), Phys. Rev. Lett. 86, 2232 (2001) arXiv:hep-ex/0011049
[15] F. Abudinén et al. (Belle II Collaboration), Phys. Rev. Lett. 127, 211801 (2021), arXiv:2108.03216 [hep-ex].
[16] T. Abe et al. (Belle II Collaboration), (2010), arXiv:1011.0352 [physics.ins-det].
$[17]$ K. Akai, K. Furukawa, and H. Koiso, Nucl. Instrum. Meth. A 907, 188 (2018), arXiv:1809.01958 [physics.acc-ph].
[18] S. Jadach, B. F. L. Ward, and Z. Was, Comput. Phys. Commun. 130, 260 (2000), arXiv:hep-ph/9912214 [hep-ph].
[19] T. Sjöstrand et al., Comput. Phys. Commun. 191, 159 (2015), arXiv:1410.3012 [hep-ph].
[20] D. J. Lange, Proceedings, 7th International Conference on $B$ physics at hadron machines (BEAUTY 2000): Maagan, Israel, September 13-18, 2000, Nucl. Instrum. Meth. A 462, 152 (2001)
[21] S. Agostinelli et al. (GEANT4 Collaboration), Nucl. Instrum. Meth. A 506, 250 (2003)
[22] T. Kuhr, C. Pulvermacher, M. Ritter, T. Hauth, and N. Braun (Belle II framework software group), Comput. Softw. Big Sci. 3, 1 (2019) arXiv:1809.04299 [physics.comp-ph] .
[23] V. Bertacchi et al. (Belle II tracking group), Comput. Phys. Commun. 259, 107610 (2021), arXiv:2003.12466 [physics.ins-det].
[24] S. Kurz, Proceedings, Connecting the Dots 2020, PROC-CTD2020-40, BELLE2-CONF-PROC-2020-009, NoStop
[25] J.-F. Krohn et al. (Belle II analysis software group), Nucl. Instrum. Meth. A 976, 164269 (2020). arXiv:1901.11198 [hep-ex].
[26] T. Lesiak et al. (Belle Collaboration), Phys. Lett. B 605, 237 (2005)
[27] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 102, 071101 (2020)
[28] P.-Y. Niu, Q. Wang, and Q. Zhao, Phys. Lett. B 826, 136916 (2022).
[29] N. L. Johnson, Biometrika 36, 149 (1949)
[30] T. Bilka et al., EPJ Web Conf. 245, 02023 (2020).

