Nonperturbative Renormalization Group Equation and Beta Function in $N = 2$ Supersymmetric Yang-Mills Theory

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We obtain the exact beta function for $N = 2$ supersymmetric SU(2) Yang-Mills theory and prove the nonperturbative renormalization group equation $\partial_\Lambda \mathcal{F}(a, \Lambda) = (\Lambda/\Lambda_0)\partial_{\Lambda_0}\mathcal{F}(a_0, \Lambda_0) \times \exp[-2\int_{\tau_0}^{\tau} dx\,\beta^{-1}(x)]$. [S0031-9007(96)00260-8]

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Montonen-Olive duality [1] and related versions suggest the existence of deep structures underlying relevant quantum field theories (QFT's). As a remarkable example, the Seiberg-Witten exact results about $N = 2$ supersymmetric (SUSY) Yang-Mills theory [2] (see [3] for reviews and related aspects), extensively studied in [4–31], are strictly related to topics such as uniformization theory, Whitham dynamics, and integrable systems.

In the case of $N = 2$ SUSY Yang-Mills theory with compact gauge group G, the terms in the low-energy Wilsonian effective action with at most two derivatives and four fermions are completely described by the so-called prepotential $\mathcal{F}$ [33]

$$S_{\text{eff}} = \frac{1}{4\pi}\,\text{Im}\left(\int d^4x d^2\theta d^2\bar{\theta}\,\Phi_D^i\bar{\Phi}_i + \frac{1}{2}\int d^4x d^2\theta\,\tau^{ij}W_iW_j\right), \tag{1}$$

where W_i is a vector multiplet, $\Phi_D^i \equiv \partial\mathcal{F}/\partial\Phi_i$ is the dual of the chiral superfield Φ_i, $\tau^{ij} \equiv \partial^2\mathcal{F}/\partial\Phi_i\partial\Phi_j$ are the effective couplings, and $i \in [1, r]$ with r the rank of G.

The prepotential $\mathcal{F}$ plays a central role in the theory. The most important property of $\mathcal{F}$ is holomorphicity [32,33]. Furthermore, it has been shown in [33] that $\mathcal{F}$ gets perturbative contributions only up to one loop. Higher-order terms in the asymptotic expansion comes as instanton contribution implicitly determined in [2].

We stress that the exact results obtained by Seiberg and Witten concern the Wilsonian effective action in the limit considered in (1). In this context it is useful to recall that, when there are no interacting massless particles, the Wilsonian action and the standard generating functional of one-particle irreducible Feynman diagrams are identical. In the case of supersymmetric gauge theories the situation is different. In particular, due to IR ambiguities (Konishi anomaly), the 1PI effective action might suffer from holomorphic anomalies [34].

An interesting question concerning the Seiberg-Witten theory is whether, using their nonperturbative results, it is possible to reconstruct the full quantum field theoretical structure. In this context we note that in [17], where a method to invert functions was proposed. A nonperturbative equation [see Eq. (7)] which relates in a simple way the prepotential and the vacuum expectation values of the scalar fields has been derived. In [20] Sonnenschein, Theisen, and Yankielowicz conjectured that the above relation should be interpreted in terms of renormalization group ideas.

In this Letter we will prove this conjecture. In particular, we will obtain the nonperturbative renormalization group equation (RGE) and the exact expression for the beta function of $N = 2$ SUSY SU(2) Yang-Mills theory.

Let us denote by $a_i \equiv \langle\phi_i\rangle$ and $a_D^i \equiv \langle\phi_D^i\rangle$ the vacuum expectation values of the scalar component of the chiral superfield. For gauge group SU(2) the moduli space of quantum vacua, parametrized by $u \equiv \langle\text{tr}\,\phi^2\rangle$, is $\Sigma_3 = \mathbf{C}\backslash\{-\Lambda^2, \Lambda^2\}$, the Riemann sphere $\hat{\mathbf{C}} = \mathbf{C} \cup \{\infty\}$ with punctures at $\pm\Lambda^2$ and ∞, where Λ is the dynamically generated scale. It turns out that [2]

$$a_D(u, \Lambda) = \partial_a\mathcal{F} = \frac{\sqrt{2}}{\pi}\int_{\Lambda^2}^{u}\frac{dx\sqrt{x-u}}{\sqrt{x^2-\Lambda^4}}, \qquad a(u, \Lambda) = \frac{\sqrt{2}}{\pi}\int_{-\Lambda^2}^{\Lambda^2}\frac{dx\sqrt{x-u}}{\sqrt{x^2-\Lambda^4}}. \tag{2}$$

A crucial step in recognizing the full QFT structures underlying the Seiberg-Witten theory is the fact that [8] (see also [17])

$$\left[\frac{\partial^2}{\partial u^2} + \frac{1}{4(u^2-\Lambda^4)}\right]a_D = 0 = \left[\frac{\partial^2}{\partial u^2} + \frac{1}{4(u^2-\Lambda^4)}\right]a, \tag{3}$$

which is the "reduction" of the uniformizing equation for Σ_3, the Riemann sphere with punctures at $\pm\Lambda^2$, and ∞ [6,8,17]

$$\left[\frac{\partial^2}{\partial u^2} + \frac{u^2+3\Lambda^4}{4(u^2-\Lambda^4)^2}\right]\sqrt{\Lambda^4-u^2}\,\frac{\partial a_D}{\partial u} = 0 = \left[\frac{\partial^2}{\partial u^2} + \frac{u^2+3\Lambda^4}{4(u^2-\Lambda^4)^2}\right]\sqrt{\Lambda^4-u^2}\,\frac{\partial a}{\partial u}. \tag{4}$$

A related aspect concerns the transformation properties of $\mathcal{F}$. It turns out that [17,35]

$$\begin{aligned}\gamma \cdot \mathcal{F}(a) &= \tilde{\mathcal{F}}(\tilde{a})\\ &= \mathcal{F}(a) + \frac{a_{11}a_{21}}{2}a_D^2 + \frac{a_{12}a_{22}}{2}a^2\\ &\quad + a_{12}a_{21}aa_D\\ &= \mathcal{F}(a) + \frac{1}{4}v^t\left[G_\gamma^t\begin{pmatrix}0&1\\1&0\end{pmatrix}G_\gamma - \begin{pmatrix}0&1\\1&0\end{pmatrix}\right]v\,,\end{aligned} \tag{5}$$

where $v = \binom{a_D}{a}$ and $G_\gamma = \binom{a_{11}a_{12}}{a_{21}a_{22}} \in$ SL$(2,\mathbf{C})$. Observe that $\gamma_2 \cdot [\gamma_1 \cdot \mathcal{F}(a)] = (\gamma_1\gamma_2) \cdot \mathcal{F}(a)$ and

$$G_\gamma^t\begin{pmatrix}0&1\\1&0\end{pmatrix}G_\gamma - \begin{pmatrix}0&1\\1&0\end{pmatrix} = 2\begin{pmatrix}a_{11}a_{21} & a_{12}a_{21}\\ a_{12}a_{21} & a_{12}a_{22}\end{pmatrix}. \tag{6}$$

We stress that if $G_\gamma \in \Gamma(2)$ then $\tilde{\mathcal{F}} = \mathcal{F}$, that is, $\gamma \cdot \mathcal{F}(a) = \mathcal{F}(\tilde{a})$. The transformation properties of $\mathcal{F}$ have been obtained for more general cases in [20,35,36]. Equation (5) implies that $2\mathcal{F} - a\partial_a\mathcal{F}$ is invariant under SL$(2,\mathbf{C})$. In particular, it turns out that [17]

$$2\mathcal{F} - a\,\partial\mathcal{F}/\partial a = -8\pi i b_1\langle \mathrm{tr}\,\phi^2\rangle, \tag{7}$$

where, as stressed in [20,21], $b_1 = 1/4\pi^2$ is the one-loop coefficient of the beta function. Relevant generalizations of the nonperturbative relation (7) have been obtained by Sonnenschein, Theisen, and Yankielowicz [20] and by Eguchi and Yang [21].

We note that the relation (7) turns out to be crucial in obtaining the Seiberg-Witten theory from the tree-level type II string theory in the limit $\alpha' \to 0$ [37].

In Ref. [20] it has been suggested that Eq. (7) should be understood in terms of RG ideas. In particular, it was suggested to consider the left hand side of (7) as a measure of the anomalous dimension of $\mathcal{F}$. Actually, we will see that $\langle \mathrm{tr}\,\phi^2\rangle$ involves the nonperturbative beta function in a natural way. This allows us to find the RGE for $\mathcal{F}$.

In order to specify the functional dependence of u we use the notation of [18] by setting $u = \Lambda^2\mathcal{G}_1(a)$ and $u = \Lambda^2\mathcal{G}_3(\tau)$, where $\tau = \partial_a^2\mathcal{F}$. Equation (3) implies

$$(1 - \mathcal{G}_1^2)\partial_a^2\mathcal{G}_1 + (a/4)\,(\partial_a\mathcal{G}_1)^3 = 0\,, \tag{8}$$

and by (7) and (8) [17,18]

$$\begin{aligned}\partial_a^3\mathcal{F} = (a\partial_a^2\mathcal{F} - \partial_a\mathcal{F})^3/4[64\pi^2 b_1^2\Lambda^4\\ + (a\partial_a\mathcal{F} - 2\mathcal{F})^2],\end{aligned} \tag{9}$$

which provides recursion relations for the instanton contribution [17]. By (2) we have $a(u = -\Lambda^2, \Lambda) = -i4\Lambda/\pi$ and $a(u = \Lambda^2, \Lambda) = 4\Lambda/\pi$ so that the initial conditions for the second-order equation (8) are $\mathcal{G}_1(-i4\Lambda/\pi) = -1$ and $\mathcal{G}_1(4\Lambda/\pi) = 1$.

Equations (7) and (9) are quite basic for our purpose. For example, by (7) we have [18]

$$\mathcal{F}(a,\Lambda) = 8\pi i b_1\Lambda^2 a^2\int_{4\Lambda/\pi}^{a} dx\,\mathcal{G}_1(x)x^{-3} - \frac{ib_1\pi^3}{4}a^2, \tag{10}$$

and [18]

$$\partial_{\hat{\tau}}\langle \mathrm{tr}\,\phi^2\rangle = \langle\phi\rangle^2/8\pi i b_1\,, \tag{11}$$

which is the quantum version of the classical relation $u = a^2/2$. In this context we observe that $\hat{\tau} = a_D/a$ has the same monodromy of τ, and their fundamental domains differ only for the values of the opening angle at the cusps [38]. These facts and (11) suggest considering τ and $\hat{\tau}$ as dual couplings. In particular, a "dual theory" should exist with $\hat{\tau}$ playing the role of gauge coupling.

As noticed in [20,21], the fact that $\tau = \partial_a^2\mathcal{F}$ is dimensionless implies

$$a(\partial_a\mathcal{F})_\Lambda + \Lambda(\partial_\Lambda\mathcal{F})_a = 2\mathcal{F}\,. \tag{12}$$

Thus, according to (7), we have

$$\Lambda\partial_\Lambda\mathcal{F} = -8\pi i b_1\langle \mathrm{tr}\,\phi^2\rangle. \tag{13}$$

In [18] it has been shown that $\mathcal{G}_3$ satisfies the equation

$$2(1 - \mathcal{G}_3^2)^2\{\mathcal{G}_3, \tau\} = -(3 + \mathcal{G}_3^2)\,(\partial_\tau\mathcal{G}_3)^2, \tag{14}$$

with initial conditions

$$\mathcal{G}_3(-1) = \mathcal{G}_3(1) = -1, \qquad \mathcal{G}_3(0) = 1\,. \tag{15}$$

The solution of (14) is

$$u = \Lambda^2\mathcal{G}_3(\tau) = \Lambda^2\{1 - 2[\Theta_2(0|\tau)/\Theta_3(0|\tau)]^4\}, \tag{16}$$

that by the "inversion formula"(7) implies [18]

$$2\mathcal{F} - a\frac{\partial\mathcal{F}}{\partial a} = 8\pi i b_1\Lambda^2\left\{2\left[\frac{\Theta_2(0|\partial_a^2\mathcal{F})}{\Theta_3(0|\partial_a^2\mathcal{F})}\right]^4 - 1\right\}, \tag{17}$$

showing that such a combination of theta functions acts on $\partial_a^2\mathcal{F}$ as integral operators.

Before considering the beta function, we observe that the scaling properties of a_D and a suggest introducing the following notation:

$$\begin{aligned}\Lambda^{-1}a_D(u,\Lambda) &= a_D(v,1) \equiv b_D(v)\,,\\ \Lambda^{-1}a(u,\Lambda) &= a(v,1) \equiv b(v)\,,\\ v &\equiv u/\Lambda^2.\end{aligned} \tag{18}$$

We now start to evaluate the nonperturbative beta function. First of all, note that in taking the derivative of τ with respect to Λ we have to distinguish between $\partial_\Lambda\tau$ evaluated at u or a fixed. We introduce the following notation:

$$\beta(\tau) = (\Lambda\partial_\Lambda\tau)_u\,, \qquad \beta^{(a)}(\tau) = (\Lambda\partial_\Lambda\tau)_a\,. \tag{19}$$

Acting with $\Lambda\partial_\Lambda$ on $\mathcal{G}_3(\tau) = u/\Lambda^2$, we have

$$\beta(\tau)\mathcal{G}_3'(\tau) = -2u/\Lambda^2, \tag{20}$$

so that

$$\beta(\tau) = -2\mathcal{G}_3/\mathcal{G}_3'. \tag{21}$$

Integrating this expression and considering the initial condition $\mathcal{G}_3(0) = 1$ in (15), we obtain

$$\langle \mathrm{tr}\,\phi^2\rangle_\tau = \Lambda^2 e^{-2\int_0^\tau dx\beta^{-1}(x)}, \tag{22}$$

or equivalently

$$\langle \mathrm{tr}\phi^2\rangle_\tau = (\Lambda/\Lambda_0)^2\langle \mathrm{tr}\phi^2\rangle_{\tau_0} e^{-2\int_{\tau_0}^\tau dx\beta^{-1}(x)}. \tag{23}$$

Using once again the relation (7), we obtain

$$(a\partial_a - 2)\mathcal{F}(a,\Lambda) = 8\pi i b_1 \Lambda^2 e^{-2\int_0^\tau dx\beta^{-1}(x)}, \tag{24}$$

or equivalently

$$(a\partial_a - 2)\mathcal{F}(a,\Lambda) = (\Lambda/\Lambda_0)^2 (a_0\partial_{a_0} - 2) \times \mathcal{F}(a_0,\Lambda_0) e^{-2\int_{\tau_0}^\tau dx\beta^{-1}(x)}, \tag{25}$$

which provides the anomalous dimension of $\mathcal{F}$. Note that in (25) we used the notation a_0 to denote a at $\tau_0 \equiv \tau(\Lambda_0)$. By Eqs. (12), (24), and (25) we obtain the nonperturbative RGE

$$\partial_\Lambda \mathcal{F} = -8\pi i b_1 \Lambda e^{-2\int_0^\tau dx\beta^{-1}(x)}, \tag{26}$$

that is,

$$\partial_\Lambda \mathcal{F}(a,\Lambda) = \frac{\Lambda}{\Lambda_0}\partial_{\Lambda_0}\mathcal{F}(a_0,\Lambda_0) e^{-2\int_{\tau_0}^\tau dx\beta^{-1}(x)}. \tag{27}$$

We note that, due to the $\tau(\Lambda)$ dependence, this equation is highly nonlinear, reflecting its nonperturbative nature.

We now start in deriving from Eq. (9) an alternative expression for the beta function. Let us consider the differentials

$$\begin{aligned} d\tau &= (\partial_\Lambda \tau)_a d\Lambda + (\partial_a \tau)_\Lambda da \\ &= (\partial_\Lambda \tau)_u d\Lambda + (\partial_u \tau)_\Lambda du, \end{aligned} \tag{28}$$

$$\begin{aligned} da &= (\partial_\Lambda a)_u d\Lambda + (\partial_u a)_\Lambda du \\ &= b d\Lambda + \Lambda b' dv = (b - 2vb')d\Lambda + \Lambda^{-1} b' du. \end{aligned} \tag{29}$$

Equations (28) and (29) yield

$$(\partial_\Lambda \tau)_u = (\partial_\Lambda \tau)_a + (b - 2vb')(\partial_a \tau)_\Lambda. \tag{30}$$

By (12) we have $\Lambda(\partial_\Lambda \tau)_a = -a(\partial_a \tau)_\Lambda$, so that

$$\beta(\tau) = -2vb'\Lambda(\partial_a\tau)_\Lambda = 2v(b'/b)\beta^{(a)}(\tau). \tag{31}$$

Let us introduce $\mathcal{G}$ and σ defined by $b_D = \partial_b \mathcal{G}$ and $\sigma = \partial_b^2 \mathcal{G} = b_D'/b'$. By a suitable rescaling of (9), it follows that $(\partial_b \sigma)_\Lambda = 1/2\pi i b'^3(1 - v^2)$. On the other hand, $\mathcal{G} = \Lambda^{-2}\mathcal{F}$ and $\sigma = \tau$, so that

$$(\partial_b \tau)_\Lambda = 1/2\pi i b'^3 (1 - v^2). \tag{32}$$

Being $\Lambda(\partial_\Lambda \tau)_a = -b(\partial_b \tau)_\Lambda$, we have

$$\beta(\tau) = v/\pi i b'^2 (v^2 - 1), \tag{33}$$

$$\beta^{(a)}(\tau) = b/2\pi i b'^3 (v^2 - 1). \tag{34}$$

By using Eqs. (2), (16), (18), (33), and (34) and Riemann's theta relation $\Theta_3^4 = \Theta_2^4 + \Theta_4^4$, where $\Theta_i \equiv \Theta_i(0|\tau)$, we obtain

$$\beta(\tau) = \frac{2\pi i(\Theta_4^4 - \Theta_2^4)}{(\Theta_2^8 - \Theta_2^4\Theta_3^4)[\int_{-1}^1 dx \sqrt{1/(x^2-1)(x\Theta_3^4 + \Theta_2^4 - \Theta_4^4)}]^2}, \tag{35}$$

$$\beta^{(a)}(\tau) = \frac{2\pi i \int_{-1}^1 dx \sqrt{(x\Theta_3^4 + \Theta_2^4 - \Theta_4^4)/x^2 - 1}}{(\Theta_2^8 - \Theta_2^4\Theta_3^4)[\int_{-1}^1 dx \sqrt{1/(x^2-1)(x\Theta_3^4 + \Theta_2^4 - \Theta_4^4)}]^3}. \tag{36}$$

Let us discuss some properties of $\beta(\tau)$ and $\beta^{(a)}(\tau)$. First of all, by (34), it follows that $\beta^{(a)}(\tau)$ is nowhere vanishing. This is a consequence of the fact that $|b|$ has a lower bound that, as noticed in [39], is given by $b(0) \sim 0.76$. Both $\beta(\tau)$ and $\beta^{(a)}(\tau)$ diverge at $u = \pm\Lambda^2$, where dyons and monopoles are massless. This happens at $\tau \in \mathbf{Z}$, corresponding to a divergent gauge coupling constant.

By (33), the $\beta(\tau)$ function is vanishing at $u = 0$. We can find the corresponding values of τ by (16). On the other hand, by uniformization theory, we know that $u = 0$ corresponds to $\tau_n = (i + 2n + 1)/2$, $n \in \mathbf{Z}$.

As a by-product of our investigation we observe that (21) and (35) yield

$$\Theta_2'\Theta_3 - \Theta_2\Theta_3' = \frac{\Theta_2^5\Theta_3 - \Theta_2\Theta_3^5}{8\pi i} \times \left[\int_{-1}^1 dx \sqrt{\frac{1}{(x^2-1)(x\Theta_3^4 + \Theta_2^4 - \Theta_4^4)}}\right]^2, \tag{37}$$

where $\Theta_i' \equiv \partial_\tau \Theta_i(0|\tau)$.

We note that, in a different context, an expression for the beta function was derived in [40], whereas very recently Minahan and Nemeschansky [41], using different techniques, obtained an expression for the beta function which has the same critical points of $\beta(\tau)$ in (35). If one identifies (up to normalizations) $\beta(\tau)$ with that in [41], one obtains a relation involving the four Θ_i's (including Θ_1).

The beta function also has a geometrical interpretation. To see this, we use the Poincaré metric on Σ_3 expressed in terms of vacuum expectation values in [18]. In terms of β we have

$$ds_P^2 = |\beta/2v\mathrm{Im}\tau|^2|du|^2 = e^{\varphi}|du|^2, \tag{38}$$

so that β/v is the chiral block of the Poincaré metric. We observe that (3) is essentially equivalent to the Liouville equation $2\partial_u\partial_{\bar{u}}\varphi = e^{\varphi}$ (see, e.g., [42]).

An important aspect of the Seiberg-Witten theory concerns the structure of the critical curve C on which $\mathrm{Im}a_D/a = 0$. The structure and the role of this curve have been studied in [2,18,38,39,43,44]. In particular, in [18], using the Koebe 1/4 theorem and Schwarz's lemma, inequalities involving the correlators and $\Lambda\partial_\Lambda\mathcal{F} = -8\pi iu$ have been obtained. Expanding the beta function in the regions of weak and strong coupling, one has to consider Borel summability for which the inequalities in [18] should provide estimations for convergence domains.

Finally, we observe that the way the results in this paper have been obtained suggest an extension to more general cases.

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[1] C. Montonen and D. Olive, Phys. Lett. **72B**, 117 (1977).
[2] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); Nucl. Phys. **B431**, 484 (1994).
[3] N. Seiberg, hep-th/9506077 (unpublished); D.I. Olive, hep-th/9508089 (unpublished); C. Gómez and R. Hernández, hep-th/9510023 (unpublished); P. Fré, hep-th/9512043 (unpublished); A. Bilal, hep-th/9601007 (unpublished).
[4] A. Klemm, W. Lerche, S. Yankielowicz, and S. Theisen, Phys. Lett. B **344**, 169 (1995).
[5] P.C. Argyres and A.E. Faraggi, Phys. Rev. Lett. **74**, 3931 (1995).
[6] A. Ceresole, R. D'Auria, and S. Ferrara, Phys. Lett. B **339**, 71 (1994).
[7] D. Finnell and P. Pouliot, Nucl. Phys. **B453**, 225 (1995).
[8] A. Klemm, W. Lerche, and S. Theisen, hep-th/9505150 (unpublished).
[9] U.H. Danielsson and B. Sundborg, Phys. Lett. B **358**, 273 (1995).
[10] M. Douglas and S. Shenker, Nucl. Phys. **B447**, 271 (1995).
[11] H. Brandhuber and K. Landsteiner, Phys. Lett. B **358**, 73 (1995).
[12] A. Hanany and Y. Oz, Nucl. Phys. **B452**, 283 (1995).
[13] P. Argyres, M. Plesser, and A. Shapere, Phys. Rev. Lett. **75**, 1699 (1995).
[14] A. Gorsky, I. Krichever, A. Marshakov, A. Mironov, and A. Morozov, Phys. Lett. B **355**, 466 (1995).
[15] J. Minahan and D. Nemeschansky, hep-th/9507032 (unpublished).
[16] M. Billó, A. Ceresole, R. D'Auria, S. Ferrara, P. Fré, T. Regge, P. Soriani, and A. Van Proeyen, hep-th/9506075 (unpublished).
[17] M. Matone, Phys. Lett. B **357**, 342 (1995).
[18] M. Matone, hep-th/9506181 [Phys. Rev. D (to be published)].
[19] K. Ito and S.-K. Yang, Phys. Lett. B **366**, 165 (1996).
[20] J. Sonnenschein, S. Theisen, and S. Yankielowicz, Phys. Lett. B **367**, 145 (1996).
[21] T. Eguchi and S.-K. Yang, hep-th/9510183 (unpublished).
[22] M. Henningson, Nucl. Phys. **B458**, 445 (1996); **B461**, 101 (1996).
[23] E. Martinec and N. Warner, Nucl. Phys. **B459**, 97 (1996); hep-th/9511052 (unpublished).
[24] T. Nakatsu and K. Takasaki, Mod. Phys. Lett. **A11**, 157 (1996).
[25] R. Donagi and E. Witten, Nucl. Phys. **B460**, 299 (1996).
[26] E. Martinec, Phys. Lett. B **367**, 91 (1996).
[27] H. Itoyama and A. Morozov, hep-th/9511126, hep-th/9512161, hep-th/9601168 (unpublished).
[28] M. Alishahiha, F. Ardalan, and F. Mansouri, hep-th/9512005 (unpublished).
[29] O. Aharony and S. Yankielowicz, hep-th/9601011 (unpublished).
[30] B. de Wit, M.T. Grisaru, and M. Roček, hep-th/9601115 (unpublished).
[31] K. Ito and N. Sasakura, hep-th/9602073 (unpublished).
[32] S.J. Gates, Jr., Nucl. Phys. **B238**, 349 (1984).
[33] N. Seiberg, Phys. Lett. B **206**, 75 (1988).
[34] M.A. Shifman and A.I. Vainshtein, Nucl. Phys. **B277**, 456 (1986); **B359**, 571 (1991).
[35] B. de Wit, V. Kaplunovsky, J. Louis, and D. Lüst, Nucl. Phys. **B451**, 53 (1995).
[36] B. de Wit, hep-th/9602060 (unpublished).
[37] S. Kachru, A. Klemm, W. Lerche, P. Mayr, and C. Vafa, Nucl. Phys. **B459**, 537 (1996).
[38] P. Argyres, A. Faraggi, and A. Shapere, hep-th/9505190 (unpublished).
[39] F. Ferrari and A. Bilal, hep-th/9602082 (to be published).
[40] V. Novikov, M. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. **B299**, 381 (1983).
[41] J.A. Minahan and D. Nemeschansky, hep-th/9601059 (unpublished).
[42] M. Matone, Int. J. Mod. Phys. **A10**, 289 (1995).
[43] A. Fayyazuddin, Mod. Phys. Lett. **A10**, 2703 (1995).
[44] U. Lindström and M. Roček, Phys. Lett. B **355**, 492 (1995).