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CONTROL OF ACOUSTIC VIBRATIONS OF AN ENCLOSURE BY MEANS OF MULTIPLE RESONATORS

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A multiple acoustic resonator is a lumped element device composed of a set of volumes connected by means of narrow ducts. It has a finite number of modes of vibration and natural frequencies which can be tuned to the natural frequencies of the lower order modes of an enclosure. This paper is aimed at studying the acoustic behaviour of a system composed of an enclosure connected to a multiple resonator. An analytical model is developed in which the pressure inside the enclosure is expanded in normal modes. The natural frequencies, modes of vibration, damping ratios and forced responses of the coupled system are calculated. Several locations of the resonator and a range of resonator volumes are considered.

1. INTRODUCTION

Enclosures, which have acoustic modes of vibration, are present in many machines; when sources of noise are able to excite resonance of one of these modes, acoustic emission strongly increases. The resulting noise radiation can be controlled not only by using panels of sound-absorptive material, but also by means of acoustic resonators.

If a sound source has a small bandwidth which encompasses only one natural frequency of the enclosure it is possible to modify the acoustic behaviour of the enclosure by means of a Helmholtz resonator tuned to that natural frequency of the enclosure [1]. A Helmholtz resonator is a device composed of a volume V which is connected to the enclosure by an orifice of area A and effective length l [2]. In a frequency range over which the resonator satisfies the conditions

$$V^{1/3} \ll \lambda, \qquad A^{1/2} \ll \lambda, \qquad l \ll \lambda, \tag{1}$$

where λ is the sound wavelength, the resonator has only one natural frequency, which is given by the ratio between the elastic constant of the volume and the inertia of the fluid contained into the orifice: i.e.,

$$\omega_r = \sqrt{c^2 A / lV}, \tag{2}$$

where c is the sound speed.

The interaction between a Helmholtz resonator and an enclosure was extensively studied by Fahy and Schofield [3] and Cummings [4]. Generally speaking, this is a multi-mode problem, since the resonator can affect many modes of the enclosure. Nevertheless, if the modes of the enclosure are lightly damped, if modal overlap is small and if the resonator frequency ω_r is chosen to be equal to the ith natural frequency of the enclosure ω_i, the multi-mode problem can be reduced to the problem of the coupling between the enclosure

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mode with frequency ω_i and the resonator mode. On the basis of the above-mentioned assumptions, Fahy and Schofield showed that, when the resonator is coupled to the enclosure, the original mode with $\omega_i = \omega_r$ disappears but two new modes arise, the first of which has a lower frequency

$$\omega_{ia} \approx \omega_i(1 - \epsilon/2), \tag{3}$$

and the second of which has a higher frequency

$$\omega_{ib} \approx \omega_i(1 + \epsilon/2). \tag{4}$$

The parameter ϵ determines the separation between the natural frequencies of the two new modes. It is a linear function of the coupling between the resonator and the enclosure and increases with the square root of the resonator/enclosure volume ratio; moreover, it is a maximum for oblique modes and a minimum for axial modes. Damping ratios of the two new modes (ζ_{ia}, ζ_{ib}) are approximately equal in all cases. If the resonator damping ratio (ζ_r) is much greater than the damping ratio of the original mode of the enclosure (ζ_i), then

$$\zeta_{ia} \approx \zeta_{ib} \approx \zeta_r/2: \tag{5}$$

therefore the two new modes can be damped by increasing the damping of the resonator. On the other hand, if the resonator damping ratio (ζ_r) is nearly equal to the damping ratio of the original mode of the enclosure (ζ_i), then

$$\zeta_{ia} \approx \zeta_{ib} \approx \zeta_r \approx \zeta_i: \tag{6}$$

therefore the damping of the two new modes is comparable with the damping of the modes of the enclosure alone.

Often, the spectrum of noise sources located inside a machine excites resonances of two or more modes of the enclosure. If the frequencies of the two modes (i and j) are very close ($2(\omega_i - \omega_j)/(\omega_i + \omega_j) \approx 1\%$) both the resonance peaks can be reduced by means of a single resonator tuned to one of the two frequencies [4]. When the interval between the frequencies of the modes is larger, the acoustic behaviour of the system can be modified by introducing two or more Helmholtz resonators tuned to the natural frequencies of the enclosure [4, 5]. This solution has the drawback of requiring a lot of room and separate openings for the coupling of each resonator to the enclosure. A multiple resonator can be better suited to solve the problem of noise control in this condition because it requires only one opening for the connection to the enclosure and is more compact than a set of Helmholtz resonators. This device comprises a group of chambers connected by means of ducts. If the dimensions of the ducts and the volumes satisfy conditions (1), the multiple resonator behaves like a lumped element device and its natural frequencies can be tuned to the frequencies of the enclosure.

In this paper a study is described of the coupling of a multiple resonator to an enclosure. First the main features of the coupled system will be described; then the influence of resonator location and volume on its capability of modifying enclosure behaviour will be discussed both in terms of free and forced oscillations. Finally, the effect of the resonator on the higher order modes of the enclosure will be studied.

2. THEORY

2.1. MATHEMATICAL MODEL OF A DOUBLE RESONATOR

In this paper the double resonator represented in Figure 1 is considered. V_1 and V_2 are the volumes of the two chambers; A_1 and A_2, l_1 and l_2 are the sections and the effective

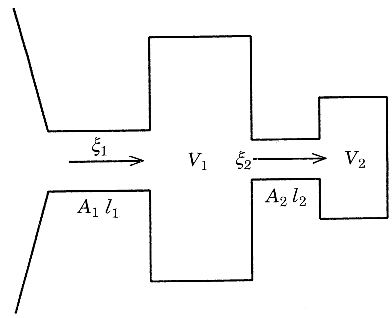

Figure 1. A double resonator.

lengths of the ducts. The displacements ξ_1 and ξ_2 of the gas particles contained into the ducts are the variables which define the dynamic behaviour of the system.

Since the resonator is a lumped element device, its dynamic equations are derived by considering the equilibrium of the gas contained in the two ducts under the action of external pressure p, of the pressures caused by the compression of gas inside the two chambers and of friction forces: thus

$$\rho l_1 A_1 \ddot{\xi}_1 + R_1 A_1^2 \dot{\xi}_1 + \{\rho c^2 (A_1 \xi_1 - A_2 \xi_2)/V_1\} A_1 = p A_1, \tag{7}$$

$$\rho l_2 A_2 \ddot{\xi}_2 + R_2 A_2^2 \dot{\xi}_2 + \rho c^2 (A_2 \xi_2 / V_2) A_2 - \{\rho c^2 (A_1 \xi_1 - A_2 \xi_2)/V_1\} A_2 = 0, \tag{8}$$

where ρ is the fluid density, and R_1 and R_2 are the resistances of the resonator ducts excluding the external radiation resistance. These equations can be arranged in the general matrix form

$$[M]\begin{Bmatrix}\ddot{\xi}_1\\ \ddot{\xi}_2\end{Bmatrix} + [C]\begin{Bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\end{Bmatrix} + [K]\begin{Bmatrix}\xi_1\\ \xi_2\end{Bmatrix} = \begin{Bmatrix}pA_1\\ 0\end{Bmatrix}, \tag{9}$$

where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices respectively. The free oscillations of the resonator are analyzed by setting the external pressure and the damping matrix to zero and introducing the solutions $\xi_1 = \xi_{1_0} e^{i\omega t}$ and $\xi_2 = \xi_{2_0} e^{i\omega t}$. For equation (9) to have a solution it is necessary that

$$\frac{c^4 A_1 A_2}{V_1 V_2} + \left(\frac{-c^2 A_1 l_2}{V_1} + \frac{-c^2 A_2 l_1}{V_2} + \frac{-c^2 A_2 l_1}{V_1}\right)\omega^2 + l_1 l_2 \omega^4 = 0. \tag{10}$$

The two roots of equation (10) are the natural frequencies of the double resonator, and depend on the six geometric parameters of the device. Since the double resonator has to be tuned to two natural frequencies of an enclosure (ω_1 and ω_2) it is necessary to seek the values of volumes and duct dimensions which make the two roots of equation (10) equal to the frequencies ω_1 and ω_2.

To simplify the design of double resonators it is useful to have more insight into equation (10). If the first duct and the first chamber of the double resonator were alone they would make a Helmholtz resonator with natural frequency

$$\omega_1^* = \sqrt{c^2 A_1 / l_1 V_1}. \tag{11}$$

On the other hand, if the second duct and the second chamber were alone they would make a Helmholtz resonator with natural frequency

$$\omega_2^* = \sqrt{c^2 A_2 / l_2 V_2}. \tag{12}$$

Upon dividing both sides of equation (10) by $l_1 l_2$ and introducing the ratio $\alpha = V_2/V_1$, it becomes

$$\omega_1^{*2}\omega_2^{*2} + (-\omega_1^{*2} - \omega_2^{*2} - \omega_2^{*2}\alpha)\omega^2 + \omega^4 = 0. \tag{13}$$

This expression is interesting because it shows that the natural frequencies of the double resonator depend only on the three parameters ω_1^*, ω_2^* and α; if these parameters are kept constant it is possible to find a set of double resonators with the same natural frequencies and different dimensions. After ω_1^*, ω_2^* and α have been fixed, three other parameters have to be chosen in order to define the dimensions of the double resonator. It is useful to specify these parameters in a non-dimensional way with the ratios

$$\beta = l_2/l_1, \qquad \sigma = V_1/V, \qquad \tau = l_1/V^{1/3}, \tag{14}$$

where V is the volume of the enclosure to which the resonator is coupled.

2.2. COUPLING OF A DOUBLE RESONATOR TO AN ENCLOSURE

The equations that describe the dynamic behaviour of the acoustic system comprising an enclosure and a double resonator tuned to the first two natural frequencies of the enclosure are now developed. First, the resonator modes of vibration associated with the natural frequencies ω_1 and ω_2 are determined. The modal matrix $[x]$ relates modal co-ordinates η_1 and η_2 to physical co-ordinates ξ_1 and ξ_2:

$$\begin{Bmatrix}\xi_1\\ \xi_2\end{Bmatrix} = [x]\begin{Bmatrix}\eta_1\\ \eta_2\end{Bmatrix} = \begin{bmatrix}x_{11} & x_{12}\\ x_{21} & x_{22}\end{bmatrix}\begin{Bmatrix}\eta_1\\ \eta_2\end{Bmatrix}. \tag{15}$$

The modal matrix is normalized in order to satisfy

$$[x^{\mathrm{T}}][M][x] = [1], \qquad [x^{\mathrm{T}}][K][x] = [\omega^2], \tag{16}$$

where superscript T designates the transpose of a matrix, [1] is the identity matrix and $[\omega^2]$ is the diagonal matrix of the natural frequencies squared. The resonator equations written in terms of modal co-ordinates are

$$[1]\begin{Bmatrix}\ddot{\eta}_1\\ \ddot{\eta}_2\end{Bmatrix} + [x^{\mathrm{T}}][C][x]\begin{Bmatrix}\dot{\eta}_1\\ \dot{\eta}_2\end{Bmatrix} + [\omega^2]\begin{Bmatrix}\eta_1\\ \eta_2\end{Bmatrix} = [x^{\mathrm{T}}]\begin{Bmatrix}pA_1\\ 0\end{Bmatrix}. \tag{17}$$

In general, the matrix $[x^{\mathrm{T}}][C][x]$ is not diagonal, and therefore the two equations are coupled through velocity terms; however, if the off-diagonal terms are smaller than the diagonal terms, this coupling can be ignored and the equations become:

$$\ddot{\eta}_1 + 2\zeta_{1_{ris}}\omega_1\dot{\eta}_1 + \omega_1^2\eta_1 = x_{11}pA_1, \qquad \ddot{\eta}_2 + 2\zeta_{2_{ris}}\omega_2\dot{\eta}_2 + \omega_2^2\eta_2 = x_{12}pA_1, \tag{18}$$

where $\zeta_{1_{ris}}$ and $\zeta_{2_{ris}}$ are the critical damping ratios of the resonator modes.

The pressure inside the enclosure can be expressed as [6]

$$p(x, y, z, t) = \sum_i P_i(t)\Psi_i(x, y, z), \tag{19}$$

where $\Psi_i(x, y, z)$ are the acoustic modes of the enclosure and $P_i(t)$ are time-varying modal coefficients. If the modes are lightly damped they form an orthogonal set of functions such that

$$\int_V \Psi_i\Psi_j \,\mathrm{d}V = \begin{Bmatrix}0, & i \neq j\\ V_i, & i = j\end{Bmatrix}. \tag{20}$$

The property expressed by equation (20) allows the separation of the enclosure modes; each modal coefficient has to satisfy the wave equation and initial conditions. The following equation thus holds [6]:

$$\ddot{P}_i + 2\delta_i \dot{P}_i + \omega_i^2 P_i = \frac{\rho c^2}{V_i} \int_V \frac{\dot{Q}}{V} \Psi_i(x, y, z)\,\mathrm{d}V - \frac{\rho c^2}{V_i} \int_S \ddot{w} \Psi_i(x, y, z)\,\mathrm{d}S. \tag{21}$$

Here V is the enclosure volume, S is the boundary surface, and ω_i is the ith natural frequency. In general δ_i is a complex quantity $\delta_i = \delta_{iR} + \mathrm{i}\delta_{iI}$; the real part accounts for wall resistivity, while the imaginary part accounts for wall reactance [6]. Since the enclosure is assumed to have rigid walls, only the real part (modal damping) is significant and it accounts for dissipation due to thermal and viscous phenomena. Modal damping δ_{iR} is related to the critical damping ratio ζ_i by the expression

$$\delta_{iR} = \zeta_i \omega_i. \tag{22}$$

The right side of equation (21) represents sources of excitation of the enclosure. $\dot{Q}$ is the time derivative of the volume velocity of the monopole sources assumed to be distributed inside the enclosure volume and $\ddot{w}$ is the vibration acceleration of the boundary surface, which is taken as positive if it is directed outwards from the enclosure.

Since in many real enclosures the main source of excitation is the vibration of a small part A_S of the boundary surface, $\dot{Q}$ is set to zero and the first integral in equation (21) vanishes. The surface acceleration $\ddot{w}$ is assumed to have a constant value $\ddot{w}_s$ over the surface A_S and is equal to $\ddot{\xi}_1$ over the surface A_1 of the external duct of the resonator; elsewhere it is set to zero. Since both A_S and A_1 are small, $\Psi_i(x, y, z)$ can be considered constant over these surfaces and equal to $\Psi_i(S)$ and $\Psi_i(R)$ respectively. Therefore the second integral in equation (21) gives the two terms

$$-\frac{\rho c^2}{V_i} \int_S \ddot{w} \Psi_i(x, y, z)\,\mathrm{d}S = -\frac{\rho c^2 A_1}{V_i} \Psi_i(R)\ddot{\xi}_1 - \frac{\rho c^2 A_S}{V_i} \Psi_i(S)\ddot{w}_S, \tag{23}$$

and equation (21) becomes

$$\ddot{P}_i + 2\zeta_i \omega_i \dot{P}_i + \omega_i^2 P_i = -(\rho c^2 A_1/V_i)\Psi_i(R)\ddot{\xi}_1 - (\rho c^2 A_S/V_i)\Psi_i(S)\ddot{w}_S. \tag{24}$$

To couple the modal equations of the resonator with the modal equations of the enclosure the acceleration $\ddot{\xi}_1$ in equation (24) is expressed as a function of $\ddot{\eta}_1$ and $\ddot{\eta}_2$, whereas the pressure p in equations (18) is expressed as a summation of the pressures caused by the modes of the enclosure. The following set of second order differential equations is obtained:

$$\ddot{P}_i + 2\zeta_i \omega_i \dot{P}_i + \omega_i^2 P_i = -(\rho c^2 A_1/V_i)\Psi_i(R)(x_{11}\ddot{\eta}_1 + x_{12}\ddot{\eta}_2) - (\rho c^2 A_S/V_i)\Psi_i(S)\ddot{w}_S,$$

$$\ddot{\eta}_1 + 2\zeta_{1_{ris}} \omega_1 \dot{\eta}_1 + \omega_1^2 \eta_1 = A_1 x_{11} \sum_i P_i \Psi_i(R),$$

$$\ddot{\eta}_2 + 2\zeta_{2_{ris}} \omega_2 \dot{\eta}_2 + \omega_2^2 \eta_2 = A_1 x_{12} \sum_i P_i \Psi_i(R), \qquad i = 1, n. \tag{25}$$

The coupling between an acoustic mode of the enclosure and the resonator modes depends on the resonator location ($\Psi_i(R)$) on the section A_1 and on V_i, which is related to the type of mode and to the volume of the enclosure. The complexity of this system of equations increases with the number of enclosure modes (n) which are taken into account. To investigate the main features of the coupled system only the first two modes of the enclosure were taken into account; then some calculations were carried out considering the effect of five modes of the enclosure.

TABLE 1

The natural frequencies of the first six modes of the enclosure alone

Frequency (rad/s)	i_x	i_y	i_z
$\omega_1 = 2468{\cdot}3$	1	0	0
$\omega_2 = 3334{\cdot}5$	0	1	0
$\omega_3 = 4048{\cdot}6$	0	0	1
$\omega_4 = 4149{\cdot}2$	1	1	0
$\omega_5 = 4741{\cdot}7$	1	0	1
$\omega_6 = 4936{\cdot}6$	2	0	0

3. EXAMPLES AND RESULTS

3.1. CASE STUDIED

The rectangular enclosure represented in Figure 2 was considered as an example of a machine cavity. It is filled with a refrigerating gas having density $\rho = 3{\cdot}73$ kg/m^3 and sound speed $c = 172{\cdot}3$ m/s. The acoustic mode shapes of a rectangular enclosure are expressed as

$$\Psi_i(x, y, z) = \cos(\pi i_x x/l_x) \cos(\pi i_y y/l_y) \cos(\pi i_z z/l_z). \tag{26}$$

The natural frequencies of the first six modes of vibration of the enclosure are summarized in Table 1. The first mode is an axial mode in the x direction and is described by the function

$$\Psi_1(x, y, z) = \cos(\pi x/l_x). \tag{27}$$

The second mode is an axial mode in the y direction and is described by the function

$$\Psi_2(x, y, z) = \cos(\pi y/l_y). \tag{28}$$

A double resonator tuned to the frequencies of modes 1 and 2 was designed; its parameters are as follows:

$$\omega_1^* = 2843 \text{ rad/s}, \quad \alpha = 0{\cdot}0892, \quad \sigma = 0{\cdot}0214,$$
$$\omega_2^* = 2895 \text{ rad/s}, \quad \beta = 1{\cdot}056, \quad \tau = 0{\cdot}107. \tag{29}$$

3.2. INFLUENCE OF RESONATOR POSITION ON THE FREQUENCIES OF THE COUPLED SYSTEM

Four characteristic positions of Figure 2 were chosen: position 1 ($x = 0{\cdot}5l_x$, $y = 0{\cdot}5l_y$, $z = l_z$) is a node both of mode 1 and of mode 2, and hence $\Psi_1 = 0$ and $\Psi_2 = 0$; position 2 ($x = 0$, $y = 0$, $z = l_z$) is an anti-node both of mode 1 and of mode 2, and hence $\Psi_1 = 1$ and $\Psi_2 = 1$; position 3 ($x = 0$, $y = 0.5l_y$, $z = l_z$) is an anti-node of mode 1 and a node of mode 2, and hence $\Psi_1 = 1$ and $\Psi_2 = 0$; position 4 ($x = 0.5l_x$, $y = 0$, $z = l_z$) is a node of mode 1 and an anti-node of mode 2, and hence $\Psi_1 = 0$ and $\Psi_2 = 1$.

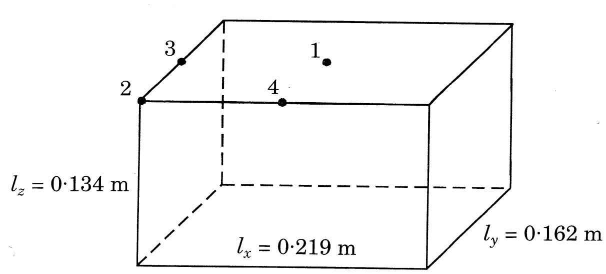

Figure 2. A rectangular enclosure with characteristic points.

TABLE 2

The effect of resonator location on frequency shifts

Ψ_1	Ψ_2	$(\omega_{1a}-\omega_1)/\omega_1$ (%)	$(\omega_{1b}-\omega_1)/\omega_1$ (%)	$(\omega_{2a}-\omega_2)/\omega_2$ (%)	$(\omega_{2b}-\omega_2)/\omega_2$ (%)
0	0	0	0	0	0
1	1	−10·1	+7·7	−4·0	+7·6
1	0	−9·3	+8·7	0	+1·4
0	1	−2·0	0	−4·4	+6·7

In order to calculate the undamped natural frequencies of the coupled system the damping and excitation terms in equations (25) were set to zero, harmonic solutions were introduced and the eigenvalue problem was solved.

If the resonator is located in position 1 ($\Psi_1 = 0$, $\Psi_2 = 0$) it is not coupled with the enclosure and the two systems maintain their former natural frequencies.

If position 2 ($\Psi_1 = 1$, $\Psi_2 = 1$) is chosen the original natural frequencies of the enclosure (ω_1 and ω_2) disappear, but two pairs of new frequencies arise (ω_{1a}, ω_{1b}, and ω_{2a}, ω_{2b}); in each pair the first frequency is lower than the natural frequency of the enclosure alone, and the second frequency is higher than the natural frequency of the enclosure alone.

When the resonator is located in position 3 only the first natural frequency of the enclosure splits into two frequencies, the second natural frequency is not modified and a new natural frequency higher than ω_2 appears. On the other hand, if the resonator is located in position 4, only the second natural frequency of the enclosure splits into two new frequencies. The shifts of the natural frequencies of the coupled system with respect to the original frequencies of the enclosure are summarized in Table 2.

Since both the natural frequencies of the enclosure were split into two when $\Psi_1 = \Psi_2 = 1$, another test was carried out maintaining $\Psi_1 = \Psi_2$ and varying their common value; results are summarized in Figure 3. When $\Psi_1 = \Psi_2$ are reduced both the natural frequencies split in two even if the resonator is connected to the enclosure in a position far from the anti-nodes of the two modes, but the frequency shifts diminishes; their dependence on the common value of $\Psi_1 = \Psi_2$ is approximately linear. This result is in agreement with that found in reference [3] for a Helmholtz resonator.

3.3. INFLUENCE OF RESONATOR VOLUME ON THE FREQUENCIES OF THE COUPLED SYSTEM

A set of double resonators tuned to the rectangular enclosure and having different volumes was designed. Parameters ω_1^*, ω_2^* and α were kept constant in order to maintain

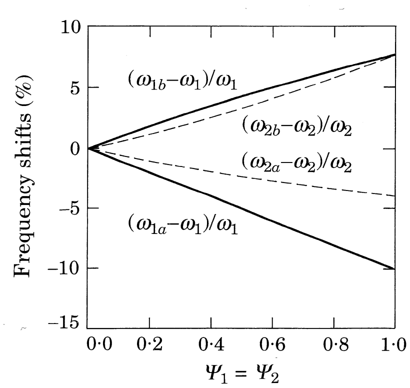

Figure 3. The effect of resonator location on the natural frequency shifts of the coupled system.

TABLE 3

The effect of resonator volume on frequency shifts

σ	$(\omega_{1a}-\omega_1)/\omega_1$ (%)	$(\omega_{1b}-\omega_1)/\omega_1$ (%)	$(\omega_{2a}-\omega_2)/\omega_2$ (%)	$(\omega_{2b}-\omega_2)/\omega_2$ (%)
0·0107	−7·0	+5·8	−3·1	+5·0
0·0161	−8·7	+6·9	−3·6	+6·4
0·0214	−10·1	+7·7	−4·0	+7·6
0·0268	−11·4	+8·4	−4·3	+8·8

the correct frequency relationships. Among the other design parameters β and τ were assumed constant, whereas $\sigma = V_1/V$ was varied, the variation of σ allowing modification of the resonator/enclosure volume ratio, which is given by

$$(V_1 + V_2)/V = \sigma(1 + \alpha). \tag{30}$$

Each resonator was located at an anti-node of both mode 1 and mode 2 (position 2) and the undamped natural frequencies $(\omega_{1a}, \omega_{1b}, \omega_{2a}, \omega_{2b})$ were calculated. Table 3 summarizes the shifts of the natural frequencies of the coupled systems with respect to the natural frequencies of the enclosure alone and shows that the increase of resonator volume widens the frequency shifts. It is interesting to point out that the frequency shifts increase approximately as $\sqrt{\sigma}$; this result is in agreement with the approximate relationship found by Fahy and Schofield for a Helmholtz resonator [3].

3.4. MODES OF THE COUPLED SYSTEM

The double resonator with $\sigma = 0{\cdot}0214$ was considered and the acoustic modes of vibration associated with the four natural frequencies of the coupled system were calculated in the case $\Psi_1 = \Psi_2 = 1$. The elements of the modal vectors X_k have not the same dimension since there are two pressures and two displacements:

$$X_k = \begin{Bmatrix} P_{1_0} \\ P_{2_0} \\ \eta_{1_0} \\ \eta_{2_0} \end{Bmatrix}. \tag{31}$$

A unit value of η_{2_0} was chosen and the other elements were consequently calculated. Pressure inside the enclosure is given by

$$p = P_{1_0} \cos(\pi x/l_x) + P_{2_0} \cos(\pi y/l_y). \tag{32}$$

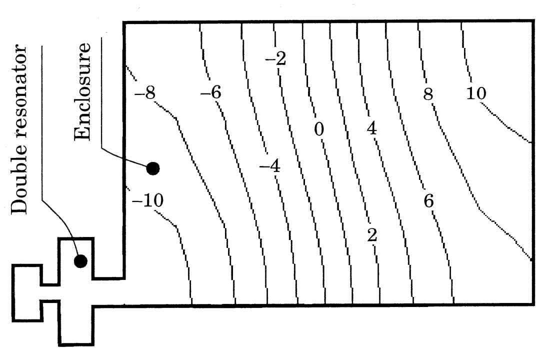

Figure 4. The mode 1a pressure distribution inside the enclosure (the resonator sketch is not to scale).

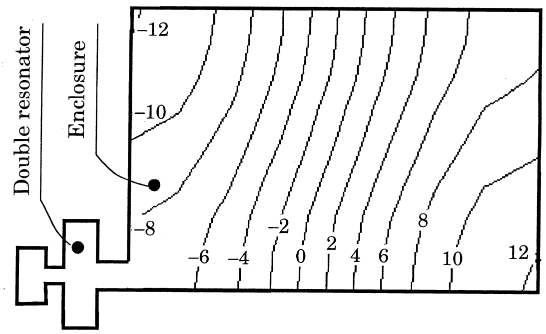

Figure 5. The mode 1b pressure distribution inside the enclosure (the resonator sketch is not to scale).

Modes 1a and 1b have $|P_{2_0}| < |P_{1_0}|$ and, as Figures 4 and 5 show, the pressure inside the enclosure has a distribution similar to that of the original axial mode in the x direction. In mode 1a the resonator is located close to the larger pressure anti-node of the coupled system; in mode 1b it is located close to the smaller pressure anti-node. In modes 2a and 2b $|P_{2_0}| > |P_{l_0}|$ and hence Figures 6 and 7 show that the pressure distributions are similar to that of the original axial mode in the y direction.

3.5. MODAL DAMPING OF THE COUPLED SYSTEM

In order to calculate the natural damped frequencies (ω_{1a_d}, ω_{1b_d}, ω_{2a_d} and ω_{2b_d}) and the modal dampings (δ_{1a}, δ_{1b}, δ_{2a} and δ_{2b}) of the coupled system the forcing terms were set to zero; the solutions

$$P_1 = P_{1_{0d}}\,e^{(\delta + i\omega_d)t}, \qquad P_2 = P_{2_{0d}}\,e^{(\delta + i\omega_d)t}, \qquad \eta_1 = \eta_{1_{0d}}\,e^{(\delta + i\omega_d)t}, \qquad \eta_2 = \eta_{2_{0d}}\,e^{(\delta + i\omega_d)t} \quad (33)$$

were introduced into equations (25) and the complex roots of the frequency equation were calculated. The double resonator with $\sigma = 0{\cdot}0214$ was considered and a maximum critical damping ratio of 5% was assumed for the resonator modes, while a maximum critical damping ratio of 2% was assumed for the modes of the enclosure, these values are in agreement with experimental values reported in references [2–4, 6]. Several cases were considered with different resonator locations and different values of damping; results are presented in Tables 4 and 5.

In the first case the resonator was located in position 2 ($\Psi_1 = \Psi_2 = 1$) and the critical damping ratio of its modes was 5%, whereas the modes of the enclosure were considered to be undamped. Results show that the two original frequencies of the enclosure split in

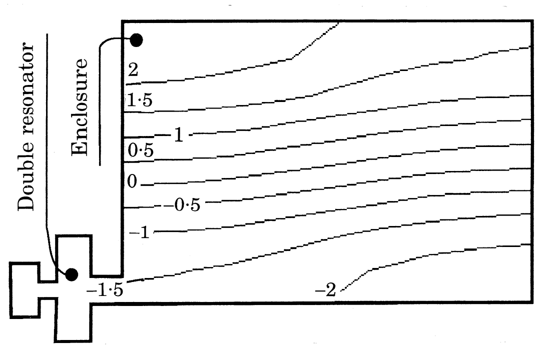

Figure 6. The mode 2a pressure distribution inside the enclosure (the resonator sketch is not to scale).

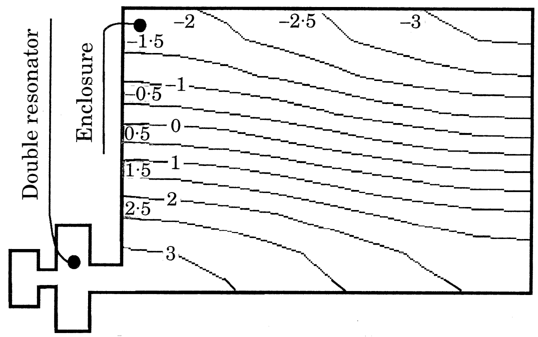

Figure 7. The mode 2b pressure distribution inside the enclosure (the resonator sketch is not to scale).

two as in the undamped case (see Table 4); the critical damping ratios of the new four modes are about one half of the critical damping ratios of the resonator modes (see Table 5). This result is in agreement with that found by Fahy and Schofield for the single resonator [3] and shows that the double resonator can increase the damping of the enclosure.

In the second case the resonator was located in position 2 again, but both the resonator modes and the enclosure modes had a critical damping ratio of 2%. The critical damping ratios of the four modes of the coupled system were found to be equal to the critical damping ratios of the original modes of the resonator and the enclosure; the frequency shifts (with respect to the damped natural frequencies of the enclosure) were equal to those calculated without damping.

In the third case the resonator was located near a node of both enclosure modes ($\Psi_1 = \Psi_2 = 0{\cdot}2$), and a critical damping ratio of 5% was assumed for the resonator, while the enclosure modes were considered undamped. In this case the resonator damping strongly reduces the effect of the resonator on the enclosure; in fact frequency shifts are very small. Modes 1a and 2a have a large critical damping ratio and probably involve large pressure fluctuations inside the resonator, while modes 1b and 2b have a very small critical damping ratio and probably involve large pressure fluctuations inside the enclosure.

In the last case the resonator was located in position 3 ($\Psi_1 = 1$, $\Psi_2 = 0$) and a critical damping ratio of 5% was assumed only for the resonator. As in the corresponding undamped case, the first natural frequency of the enclosure splits in two. The second mode of the enclosure is not coupled to the resonator; it maintains its original frequency and is not affected by resonator damping. The fourth mode, which has frequency higher than the second natural frequency of the enclosure, exhibits a high critical damping ratio.

TABLE 4

Frequency shifts caused by the double resonator in the presence of damping

Ψ_1	Ψ_2	$\zeta_{1_{ris}}$ (%)	$\zeta_{2_{ris}}$ (%)	ζ_1 (%)	ζ_2 (%)	$\frac{(\omega_{1a_d} - \omega_1)}{\omega_1}$ (%)	$\frac{(\omega_{1b_d} - \omega_1)}{\omega_1}$ (%)	$\frac{(\omega_{2a_d} - \omega_2)}{\omega_2}$ (%)	$\frac{(\omega_{2b_d} - \omega_2)}{\omega_2}$ (%)
1	1	5	5	0	0	−9·9	+7·4	−3·6	+7·1
1	1	2	2	2	2	−10·1	+7·7	−4·0	+7·6
0·2	0·2	5	5	0	0	−0·17	−0·05	−0·06	+0·08
1	0	5	5	0	0	−9·0	+8·3	0	+1·3

TABLE 5

The damping ratios of the coupled system

Ψ_1	Ψ_2	$\zeta_{1_{ris}}$ (%)	$\zeta_{2_{ris}}$ (%)	ζ_1 (%)	ζ_2 (%)	ζ_{1a} (%)	ζ_{1b} (%)	ζ_{2a} (%)	ζ_{2b} (%)
1	1	5	5	0	0	2·5	2·3	2·7	2·4
1	1	2	2	2	2	2·0	2·0	2·0	2·0
0·2	0·2	5	5	0	0	4·1	0·8	4·7	0·3
1	0	5	5	0	0	2·4	2·8	0	4·8

3.6. FORCED RESPONSE OF THE COUPLED SYSTEM

The forced response of the coupled system was calculated with the assumption of harmonic excitation. In this case the excitation terms in the dynamic model are

$$\ddot{w}_S = a_0\,\mathrm{e}^{\mathrm{i}\omega_f t}, \tag{34}$$

where ω_f is the frequency of vibration and a_0 is the acceleration amplitude, and the steady state responses of the enclosure modal pressures and of the resonator modal displacements are

$$P_1 = P_{1_{0f}}\,\mathrm{e}^{\mathrm{i}\omega_f t}, \qquad P_2 = P_{2_{0f}}\,\mathrm{e}^{\mathrm{i}\omega_f t}, \qquad \eta_1 = \eta_{1_{0f}}\,\mathrm{e}^{\mathrm{i}\omega_f t}, \qquad \eta_2 = \eta_{2_{0f}}\,\mathrm{e}^{\mathrm{i}\omega_f t}, \tag{35}$$

where $P_{1_{0f}}$, $P_{2_{0f}}$, $\eta_{1_{0f}}$ and $\eta_{2_{0f}}$ are complex quantities which depend on the frequency ω_f. If the excitation terms (34) and the solutions (35) are introduced into equations (25), they become a set of algebraic equations which can be solved to seek $P_{1_{0f}}$, $P_{2_{0f}}$, $\eta_{1_{0f}}$ and $\eta_{2_{0f}}$. The pressure $p(x, y, z)$ at a point (x, y, z) inside the enclosure is

$$p(x, y, z) = (\Psi_1(x, y, z)P_{1_{0f}} + \Psi_2(x, y, z)P_{2_{0f}})\,\mathrm{e}^{\mathrm{i}\omega_f t}; \tag{36}$$

since $P_{1_{0f}}$ and $P_{2_{0f}}$ are complex quantities, the pressure amplitude is the modulus of $(\Psi_1(x, y, z)P_{1_{0f}} + \Psi_2(x, y, z)P_{2_{0f}})$.

The following parameters were chosen: $A_s = 0{\cdot}0005\ \mathrm{m}^2$, $a_0 = 20\ \mathrm{m/s}^2$, $\zeta_1 = 0{\cdot}01$, $\zeta_2 = 0{\cdot}01$, $\zeta_{1_{ris}} = 0{\cdot}05$, $\zeta_{2_{ris}} = 0{\cdot}05$, $\Psi_1(R) = 1$, $\Psi_2(R) = 1$, $\Psi_1(S) = 1$, $\Psi_2(S) = 1$ and pressure was calculated at a corner of the enclosure ($\Psi_1(x, y, z) = \Psi_2(x, y, z) = 1$). In order to have a good insight into the forced behaviour of the coupled system, a wide range of forcing frequencies and four values of resonator/enclosure volume ratio were considered.

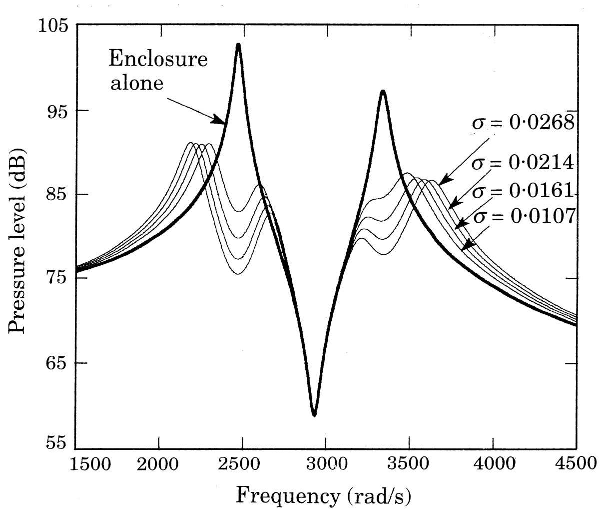

Figure 8. Forced responses.

TABLE 6

The effect of higher order modes on the frequencies of the coupled system

Enclosure alone (rad/s)	Enclosure (two modes) plus resonator (rad/s)	Enclosure (five modes) plus resonator (rad/s)
$\omega_1 = 2468{\cdot}3$	$\omega_{1a} = 2218{\cdot}0$	$\omega_{1a} = 2169{\cdot}1$
—	$\omega_{1b} = 2658{\cdot}1$	$\omega_{1b} = 2621{\cdot}8$
$\omega_2 = 3334{\cdot}5$	$\omega_{2a} = 3201{\cdot}3$	$\omega_{2a} = 3171{\cdot}8$
—	$\omega_{2b} = 3589{\cdot}3$	$\omega_{2b} = 3469{\cdot}5$
$\omega_3 = 4048{\cdot}6$	—	$\omega_3 = 4074{\cdot}9$
$\omega_4 = 4149{\cdot}2$	—	$\omega_4 = 4317{\cdot}9$
$\omega_5 = 4741{\cdot}7$	—	$\omega_5 = 4900{\cdot}4$

The results are summarized in Figure 8, where for comparison the forced response of the enclosure alone is also represented. The double resonators strongly affect the forced response of the enclosure since they cut down the two resonance peaks at frequencies ω_1 and ω_2 and produce four new peaks at frequencies ω_{1a}, ω_{1b}, ω_{2a} and ω_{2b}. When the resonator/enclosure volume ratio is increased (σ increased), the frequency shifts between the frequencies of the new peaks and the frequencies of the original peaks increase; this result is in agreement with the free oscillations analysis presented in section 3.3. The increment of σ deepens the "valleys" at the frequencies ω_1 and ω_2 and significantly reduces the amplitude of the central peaks. Hence an increment of the resonator/enclosure volume ratio reduces the forced response of the coupled system and is advisable.

3.7. INTERACTION BETWEEN THE RESONATOR AND THE OTHER MODES OF THE ENCLOSURE

The undamped natural frequencies of the coupled system were then calculated with account taken of five modes of the enclosure. The resonator with $\sigma = 0{\cdot}0214$ was located at position 2 ($\Psi_1 = \Psi_2 = 1$) and its natural frequencies were tuned to the first and the second natural frequencies of the enclosure. The results are summarized in Table 6, where the seven natural frequencies of the coupled system are compared with the five natural frequencies of the enclosure alone and with the four natural frequencies of the coupled system that were calculated with account taken only of the first two modes of the enclosure (see section 3.2).

The first two modes of the enclosure split in two, as in the previous calculation, but the new frequencies are slightly lower than those calculated in section 3.2, the maximum difference being about 3·5%.

Since every mode of the enclosure has an anti-node at position 2, the resonator influences the other modes of the enclosure and their frequencies increase; this result is in agreement with those found by Cummings in the multi-mode analysis of the coupling of a Helmholtz resonator with an enclosure [4]. In the case treated here, the natural frequency of the third mode increases less than the natural frequencies of the fourth and fifth modes. This result can be explained by considering that the third mode is axial, whereas the fourth and the fifth modes are tangential. Since in a rectangular enclosure of volume V one has

$$\int_V \Psi_i^2 \,\mathrm{d}v = \frac{V}{2}\,(\text{axial mode}), \qquad \int_V \Psi_i^2 \,\mathrm{d}v = \frac{V}{4}\,(\text{tangential mode}), \tag{37}$$

equations (25) show that the fourth and the fifth modes are more tightly coupled to the resonator than the third mode.

4. CONCLUSIONS

A double resonator was tuned to the first two natural frequencies of an enclosure to modify its acoustic behaviour. The effectiveness of the resonator depends strongly on the location. If the resonator is connected to the enclosure at a position near an anti-node of its modes, the original first two modes of the enclosure split and the frequency shifts are large; moreover, resonator damping increases enclosure damping.

Since an anti-node of the first two modes of the enclosure may be an anti-node of higher order modes too, the resonator affects, and is affected by, higher order modes. There is a small effect of higher order modes on the splitting of the first two modes, but the resonator increases the frequencies of the higher order modes; this effect can be useful in some applications.

From the noise control viewpoint the most important result is that the forced response of the enclosure is affected strongly by the double resonator: the original resonance peaks disappear and they are replaced with pairs of peaks having lower amplitude.

The resonator/enclosure volume ratio influences the behaviour of the coupled system. If it increases, the frequency shifts widen and the amplitudes of the forced responses at the frequencies of the original enclosure modes diminish.

Since the construction of a double resonator which is well tuned to two natural frequencies of an enclosure is not easy, further studies will include the influence of tuning errors on the behaviour of the coupled system and the development of methods for optimum design of multiple resonators.

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REFERENCES

1. J. Hersman, L. L. Faulkner, M. Hock and C. Rodman 1983 *Noise-Con* **83**, 407–412. Reduction of combustion-driven acoustic oscillation in a high-pressure steam generation boiler.
2. D. A. Bies and C. H. Hansen 1988 *Engineering Noise Control*. London: Unwin Hyman.
3. F. J. Fahy and C. Schofield 1980 *Journal of Sound and Vibration* **72**, 365–378. A note on the interaction between a Helmholtz resonator and an acoustic mode of an enclosure.
4. A. Cummings 1992 *Journal of Sound and Vibration* **154**, 25–44. The effects of a resonator array on the sound field in a cavity.
5. A. Putnam 1982 *Journal of Engineering for Power* **104**, 867–873. Combustion roar as observed in industrial furnaces.
6. L. L. Beranek and I. L. Ver 1992 *Noise and Vibration Control Engineering*. New York: John Wiley.