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A SIMPLE NUMERICAL APPROACH FOR OPTIMUM SYNTHESIS OF A CLASS OF PLANAR MECHANISMS

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Abstract—In this study a numerical method for optimum synthesis of planar mechanisms, generators of functions, paths and rigid motions, is presented. Design parameters have wide variability ranges, inside which first guesses, demanded by the iterative minimization procedure, can be chosen at random. Kinematic analysis is carried out by decomposition of the mechanism into Assur groups; mechanism assembly is managed by the construction of a proper penalty function. Optimization is carried out by using a non-derivative and a quasi-Newton method in series. Some optimum design examples are presented to illustrate the power of the method.

INTRODUCTION

Many closed-loop mechanisms are used to guide a rigid body, to generate a path, or to realize a functional relation between the motion of a motor-link and the follower-link.

The aim of dimensional synthesis is to determine the geometric characteristics of the links which allow the mechanism to perform the desired task.

Dimensional synthesis based on optimization techniques has developed substantially since the 1960s. The least square method was initially used [1–3] and followed by many others, such as the penalty function approach [4], geometric programming approach [5] and sensitivity coefficient method [6]. In these approaches, mechanism assembly was nearly always managed by considering the compatibility equations of the mechanisms as constraint equations to be satisfied by design variables.

In a paper on the synthesis of a six-bar linkage, Pakes *et al.* [7] proposed an original and simple method for handling mechanism assembly, embedding the assembly criterion in the penalty function.

More recently, methods based on analysis of design sensitivity have been introduced [8–10]: the penalty function is minimized subjected to the state equations of the mechanism and to equality and inequality design constraints; the input parameter (e.g. crank rotation) is discretized into a finite set of points; and a special technique is used to calculate the derivatives of the constraint equations.

Selective precision synthesis (SPS) and stochastic formulation must also be mentioned.

In the SPS approach [11–13] the kinematic chain is considered to be composed of dyads, "accuracy neighbourhoods" are constructed around the precision points, and a mechanism that goes through each accuracy neighbourhood is found with an optimization technique.

In stochastic formulation [14–16] not only structural but also mechanical errors are minimized, because of the importance of manufacturing and assembly errors in the practical construction of mechanisms.

The major difficulties of optimum synthesis, as shown by various authors, essentially deal with the non-linearity of position analysis and management of mechanism assembly during the iterative minimization procedure.

In this study, the assembly criterion is embedded in the penalty function as in Ref. [7], and kinematic analysis is carried out by mechanism decomposition into Assur groups.

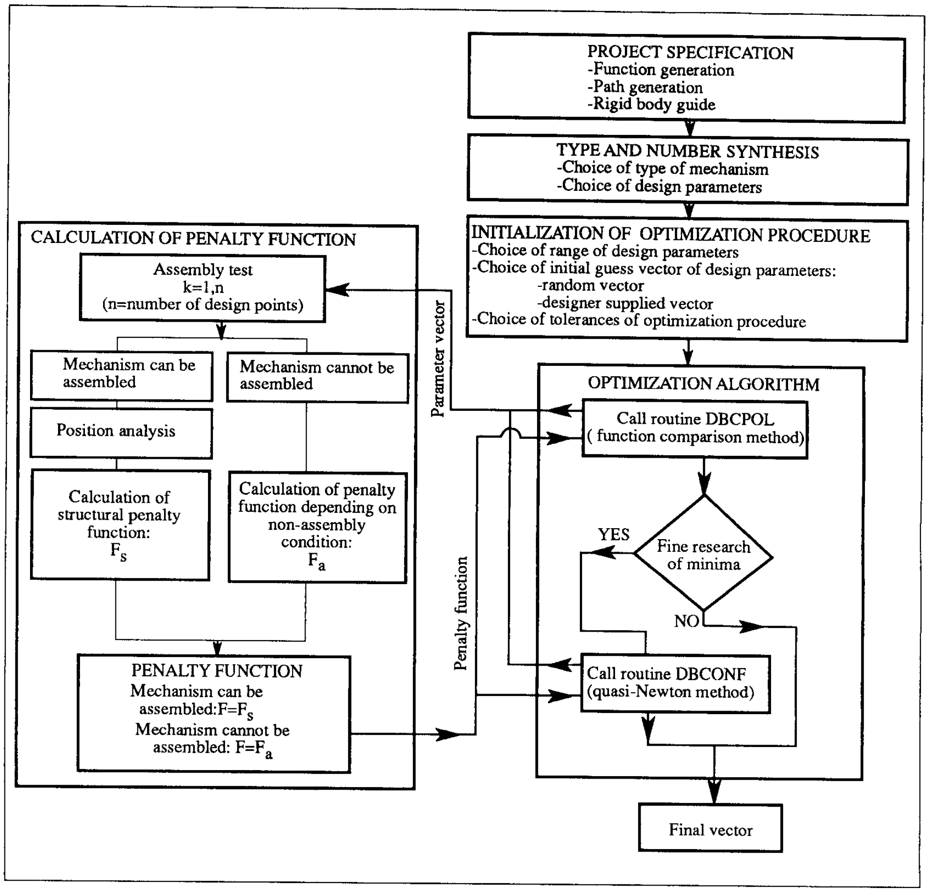


Fig. 1. Flow chart for method of optimum synthesis.

PROPOSED METHOD

Figure 1 shows the flow chart of the proposed method. The project specifications in the various types of synthesis are:

- —number of design points;
- —coordinates of the tracer point in the design points for path generation;
- —coordinates and rotations of the rigid body in the design points to guide the rigid body;
- —rotations of the follower-link in the design points for function generation. In any case, the design points are correlated with motor-link positions (synthesis with prescribed timing).

The designer must then choose the type of mechanism (e.g. a four-bar mechanism) and fix the design variables, which may be link lengths, coordinates of fixed pivots, initial rotations of motor cranks, or the position of a particular point of a link.

The following step, the initialization of the design procedure, is relatively easy, because the proposed method allows wide variability ranges of design parameters. Although the choice of a guess vector for the design parameters ensuring mechanism assembly is advisable, the following examples show that (within the variability ranges) good solutions can be reached by starting from a random guess vector.

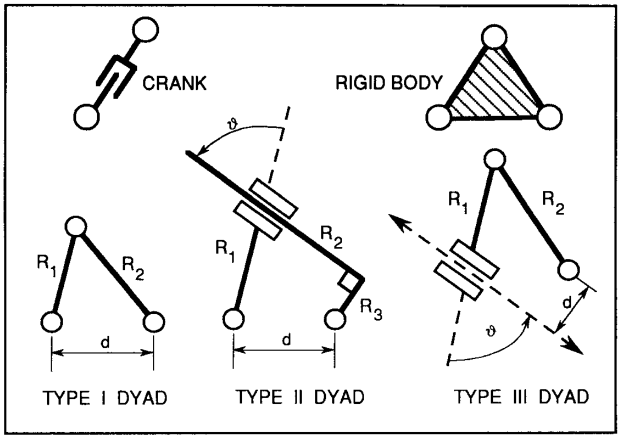


Fig. 2. Assur groups.

The choice of tolerances of the optimization procedure depends on the accuracy required for output motion.

Automatic search for the optimum mechanism is based on the search for a vector of design parameters minimizing the penalty function.

In the proposed method, the penalty function not only considers the error between the desired motion and the motion obtained with the current vector of design parameters (the so-called structural error), but also embeds the assembly criterion. If a vector of design parameters does not allow mechanism assembly, a sharp increase in the global penalty function follows, shifting the search to other directions.

A key feature of the proposed method is decomposition of the mechanism into Assur groups, in order to perform position analysis and the assembly tests required to calculate the penalty function. This approach usually speeds up kinematic analysis considerably.

The minimization procedure is carried out by using two subroutines in series, the first for a coarse search and the second for a fine one. The use of a stable subroutine for coarse search is the key feature that allows such great variability in design parameters.

Kinematic analysis

Problems due to non-linearity of the closure equations are overcome by carrying out kinematic analysis by decomposition of the mechanisms into elementary groups, also known as Assur groups [17–19]. This method consists of considering the articulated system as made up of motor-cranks to which elementary groups, which do not modify the degrees of freedom of the mechanism, are then added.

The solution of the complete mechanism is performed by solving first the equations of the motor-cranks and then those of the Assur groups. The advantage is that the equations of each single group are dealt with separately. Moreover, the equations of the first- and second-class Assur groups do not require iterative procedures for their solution. In this work, therefore, only the class of mechanisms that can be made up with such groups is considered.

Figure 2 shows the elementary groups implemented in the module that executes kinematic analysis during the optimization iterative procedure.

Assembly is checked in every position by verifying the assembly conditions of all the elementary groups making up the mechanism.

If a group cannot be assembled, the residual of the assembly conditions is calculated. This residual is the difference between actual distance d and the value of this distance that allows group assembly.

Type I dyad.

Assembly conditions: $\begin{cases} d \leqslant (R_1 + R_2) \\ d \geqslant |R_1 - R_2| \end{cases}$, residuals: $\begin{cases} r = d - (R_1 + R_2) \\ r = |R_1 - R_2| - d \end{cases}$.

Type II dyad.

Assembly condition: $d \geqslant |R_1 \sin\theta - R_3|$, residual: $r = |R_1 \sin\theta - R_3| - d$.

Type III dyad.

Assembly conditions: $\begin{cases} d \leqslant (R_2 + R_1 \sin\theta) \\ d \geqslant |R_2 - R_1 \sin\theta| \end{cases}$, residuals: $\begin{cases} r = d - (R_2 + R_1 \sin\theta) \\ r = |R_2 - R_1 \sin\theta| - d \end{cases}$.

Penalty functions

If the mechanism can be assembled in all positions, penalty function F is equal to the structural penalty function. This function takes into account the error between the desired motion and the motion obtained with the current vector of design parameters. It is defined as follows:

$$F_s = w_x f_x + w_y f_y + w_\theta f_\theta.$$

Terms f_x, f_y and f_θ are the sums of the squares of the errors in n design points on x, y and on angle θ respectively:

$$f_x = \sum_{i=1}^{n} (x_i - x_{d_i})^2; \quad f_y = \sum_{i=1}^{n} (y_i - y_{d_i})^2; \quad f_\theta = \sum_{i=1}^{n} (\theta_i - \theta_{d_i})^2,$$

where $(x_{d_i}, y_{d_i}, \theta_{d_i})$ indicate design coordinates and (x_i, y_i, θ_i) are the coordinates supplied by kinematic analysis of the mechanism with the current vector of design parameters.

Weights w_x, w_y, w_θ allow the desired type of synthesis to be performed:

—if $w_x = w_y = 0$ and $w_\theta \neq 0$, function generation is obtained, θ being the follower-link rotation correlated to motor-link rotation;
—if $w_x \neq 0$, $w_y \neq 0$ and $w_\theta = 0$, path generation is obtained;
—if $w_x \neq 0$, $w_y \neq 0$ and $w_\theta \neq 0$, rigid body guide is obtained, and in this case θ is rigid body rotation.

Information about the transmission angle may be inserted into the penalty function by adding a term that increases when the transmission angle decreases.

If the mechanism cannot be assembled in all positions, function F_a related to the non-assembly condition is introduced. This function must depend on number m of positions in which the mechanism cannot be assembled and on the extent of the modifications which must be carried out to obtain a closed-loop configuration. It is therefore:

$$F_a = w_a \sum_{i=1}^{m} r_i^2,$$

where $\Sigma_{i=1}^{m} r_i^2$ is the sum of the squares of the residuals of the first group that cannot be assembled and $w_a \geqslant 1$ is an amplifying coefficient.

The penalty function is thus:

$$F = F_a + F^*,$$

where F^* is the penalty function calculated in the previous iteration. The addition of F^* has the purpose of keeping F_a large, even when the sum of the squares of the residuals is small.

Some tests have shown that number m of positions in which the mechanism cannot be assembled is the essential factor. Therefore, the term $\Sigma_{i=1}^{m} r_i^2$ can be replaced with a conventional value of the structural penalty function, calculated by assuming all null values for (x_i, y_i, θ_i) variables in m positions.

With this approach, if starting conditions correspond to a mechanism that cannot be assembled, the penalty function is continually changed until the minimization algorithm determines a vector of parameter values that does allow mechanism assembly. However, when starting from a mechanism that can be assembled, the algorithm selects the directions of parameter variations to avoid non-assembly areas.

The error between the desired motion and the obtained motion of the synthesized mechanism is represented in the following examples by mean errors:

$$\epsilon_{x_{\text{mean}}} = \frac{1}{n}\sum_{i=1}^{n}|x_i - x_{d_i}| \quad \epsilon_{y_{\text{mean}}} = \frac{1}{n}\sum_{i=1}^{n}|y_i - y_{d_i}| \quad \epsilon_{\theta_{\text{mean}}} = \frac{1}{n}\sum_{i=1}^{n}|\theta_i - \theta_{d_i}|$$

$$\epsilon_{xy_{\text{mean}}} = \frac{1}{n}\sum_{i=1}^{n}[(x_i - x_{d_i})^2 + (y_i - y_{d_i})^2]^{1/2}$$

and by maximum errors:

$$\epsilon_{x_{\max}} = |x_i - x_{d_i}|_{\max} \quad \epsilon_{y_{\max}} = |y_i - y_{d_i}|_{\max} \quad \epsilon_{\theta_{\max}} = |\theta_i - \theta_{d_i}|_{\max}$$

$$\epsilon_{xy_{\max}} = [(x_i - x_{d_i})^2 + (y_i - y_{d_i})^2]^{1/2}_{\max}.$$

Minimization algorithm

Minimization is first carried out by using the BCPOL subroutine of the IMSL library [20], based on a direct search Complex algorithm; no derivative information is taken into account.

A Complex is a set of $2N$ points in an N-dimensional space; the algorithm iterates by replacing the point of the Complex with the highest function value with a new point with a lower function value, and stops when the difference among the function values in the points of the Complex is less than given tolerance δ. The algorithm also stops when a second criterion, based on the standard deviation of the function values, is satisfied.

This non-derivative method turns out to be very suitable for this type of problem, since it gives the design parameters a wide range of variability.

If the accuracy of the solution obtained with the BCPOL subroutine is not sufficient it may be improved by using a more accurate subroutine in the neighbourhood of the minimum found by the BCPOL subroutine.

Subroutine BCONF of the IMSL library is used for this search. It is based on a quasi-Newton method, and the gradient of the function is calculated with a finite-difference technique. The minimization procedure is stopped when:

$$\|g(h_i)\| \leqslant \lambda,$$

where $g(h_i)$ is the function gradient in point h_i and λ is a gradient tolerance.

Subroutine BCONF allows good accuracy, but it performs well only if the variability range of the design parameters is about ten times shorter than the variability range allowed by subroutine BCPOL.

NUMERICAL EXAMPLES AND DISCUSSION

Example 1

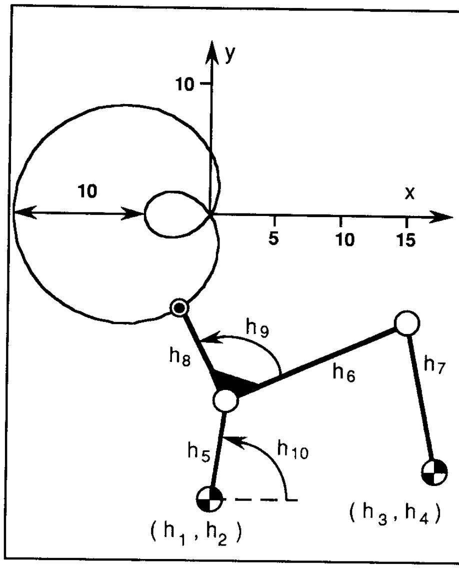
As a first example, a four-bar mechanism was synthesized with a point belonging to coupler plane able to generate a conchoid path represented by the law:

$$r = \rho\cos(\alpha) + \frac{\rho}{2}$$

$$P(x_{d_i}, y_{d_i}) = P[-r\cos(\alpha_i); -r\sin(\alpha_i)] \quad \alpha_i = (i-1)\frac{2\pi}{50}(i = 1, 50),$$

where ρ was assumed equal to 10 units and α was variable from 0.0 to 6.28 radians. The conchoid curve was described with 50 precision points.

Figure 3 shows the desired path, defined with respect to the reference frame, the sketch of the four-bar mechanism to be synthesized, and the 10 design parameters which were the position of the two ground pivots (h_1, h_2, h_3, h_4), link lengths (h_5, h_6, h_7, h_8) angle (h_9) and initial crank rotation



PARAMETERS	RANGE OF DESIGN PARAMETERS		OPTIMUM DESIGN: INITIAL GUESS VECTOR	OPTIMUM DESIGN: FINAL VECTOR
h_1	-25	25	-13.743	-4.205
h_2	-25	25	-2.486	4.784
h_3	-25	25	8.771	-2.911
h_4	-25	25	20.028	4.479
h_5	10	60	16.285	59.997
h_6	0.01	60	21.051	7.780
h_7	0.01	60	34.559	59.439
h_8	0.01	60	48.067	56.935
h_9	-3.14	3.14	-2.9751	-1.4993
h_{10}	-3.14	3.14	-1.5620	-0.8683
ERRORS: $\varepsilon_{xy\,max}$			66.71	0.441
ERRORS: $\varepsilon_{xy\,mean}$			51.98	0.250

Fig. 3. Four-bar conchoid path generator mechanism.

(h_{10}). The variability ranges of the design parameters are also shown in Fig. 3. Note that the ranges are greater than the size of the path to be achieved.

Optimization was carried out 50 times, starting from 50 different initial guess vectors determined by a random procedure. Minimization was performed with subroutine DBCPOL, with a tolerance factor of $\delta = 10^{-3}$ and the parameters of the penalty function $w_x = 10$, $w_y = 10$, $w_\theta = 0$, $w_a = 10$.

The optimization procedure was able to find 50 mechanisms that could be assembled, although only seven of the 50 guess mechanisms could be assembled.

The conchoid path was traced with very good accuracy ($\epsilon_{xy_{mean}} \leqslant 0.4$) by 16 mechanisms, and with sufficient accuracy ($0.4 \leqslant \epsilon_{xy_{mean}} \leqslant 0.6$) by 28 mechanisms; only six mechanisms had poor accuracy ($0.6 \leqslant \epsilon_{xy_{mean}}$).

All mechanisms were double-crank linkages with the two ground pivots and tracer point close together.

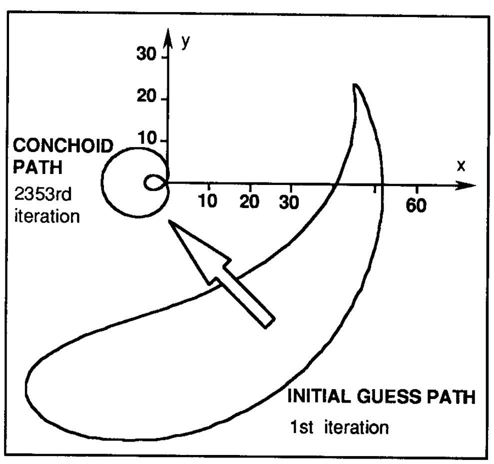
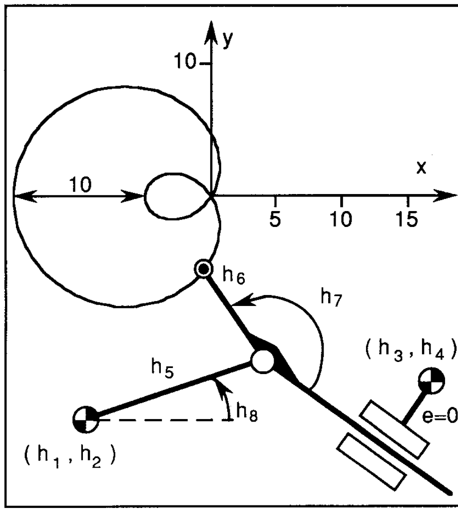


Fig. 4. Initial and final paths of four-bar path generator mechanism of Fig. 3.



PARAMETERS	RANGE OF DESIGN PARAMETERS		OPTIMUM DESIGN: INITIAL GUESS VECTOR	OPTIMUM DESIGN: FINAL VECTOR
h_1	-25	25	-19.299	-9.564
h_2	-25	25	-17.662	1.771
h_3	-25	25	-5.791	-0.288
h_4	-25	25	-15.045	-1.805
h_5	10	700	105.091	326.875
h_6	10	700	138.852	321.835
h_7	-6.28	6.28	-0.7689	-0.0029
h_8	-6.28	6.28	-0.6691	-0.2461
ERRORS: $\varepsilon_{xy\,max}$			567.04	0.382
ERRORS: $\varepsilon_{xy\,mean}$			549.58	0.088

Fig. 5. Inverted slider crank mechanism.

The solutions showed smaller errors when the tolerance factor was reduced. Improved performance was obtained by keeping the two ground pivots closer, by shortening the coupler-link, and by lengthening the cranks, which quickly reached the extreme values of their variability ranges.

Some mechanisms with similar configurations were also found. Among the groups having different configurations linkages similar to cognate mechanisms were noted.

Figure 3 shows the parameters of the best solution mechanism with the corresponding guess vector, and the maximum and mean errors obtained.

Figure 4 shows the paths described by the initial guess mechanism and by the best mechanism. Note how the iterative procedure did converge, although it started from a configuration describing a very different path.

Example 2

As a second example, a mechanism composed of a motor crank and a type II dyad (Fig. 5) was synthesized, assuming the conchoid curve of the previous example as a design path.

The position of the ground pivots (h_1, h_2, h_3, h_4), length and initial rotation of the motor crank (h_5, h_8), and position (h_6, h_7) of the point describing the path, were assumed as design parameters.

The optimization procedure, the best results of which are shown in Fig. 5, led to configurations having link h_6 tending to line up with the sliding axis and the line joining the two ground pivots tending to pass through the origin.

The results were satisfactory (subroutine DBCPOL was used, with a tolerance factor of $\delta = 10^{-3}$ and the parameters of the penalty function $w_x = 1$, $w_y = 1$, $w_\theta = 0$, $w_a = 10$). With lower tolerance factors, better results were obtained, but they tended to lengthen h_5 and h_6 up to the allowed limits.

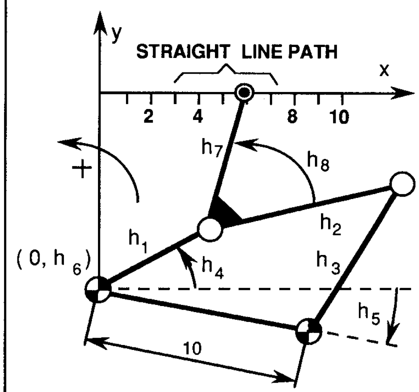
Example 3

To compare the accuracy of the method with the results obtained by other authors [8], the problem of the synthesis of a four-bar mechanism describing a rectilinear path was considered.

Figure 6 shows a sketch of the four-bar mechanism with its eight variables. The distance between the fixed pivots was constant and equal to 10 units.

For a crank rotation of 0.6982 radians, the coupler point had to describe a rectilinear path from $x_1 = 2.93$ to $x_2 = 7.08$, which was divided into 19 equidistant points corresponding to 18 crank rotations.

The values given in Ref. [8] were used as first guess variables. Different weights were adopted for the penalty functions related to the errors along coordinates x and y ($w_x = 0.01$ $w_y = 1$, respectively), to favour path rectilinearity.



PARAMETERS	RANGE OF DESIGN PARAMETERS		OPTIMUM DESIGN	
			INITIAL GUESS VECTOR	FINAL VECTOR
h_1	20	1	3.5	3.5552
h_2	20	1	9.0	5.8597
h_3	20	1	10.0	2.1862
h_4	3.14	-3.14	-1.3491	-1.3987
h_5	3.14	-3.14	-1.0	-1.0407
h_6	10	-10	-2.5	-2.7541
h_7	20	0	6.5	6.6746
h_8	3.14	-3.14	1.0	1.7559
ERRORS:		εy_{max}	1.36032	0.00036
		εy_{mean}	0.94006	0.00018

Fig. 6. Four-bar straight-line generator mechanism.

Optimization was conducted first by subroutine DPCPOL (tolerance factor $\delta = 10^{-8}$) and then, to improve the results with little variation in the design parameters, by subroutine DBCONF (tolerance factor $\lambda = 10^{-9}$).

The results are shown in Fig. 6, with the initial vector and maximum and mean errors obtained along coordinate y.

The maximum error obtained here was nearly three times lower than that obtained in Ref. [8] for the same problem (0.001175). The maximum error in the position along the axis x was 0.00393.

In a second simulation, 30 minima searches were carried out, starting from random-generated guess vectors inside the variability ranges of Fig. 6. It can be observed that the amplitudes of the variability ranges are nearly five times larger than the length of the rectilinear path and may reach 360° for the angles.

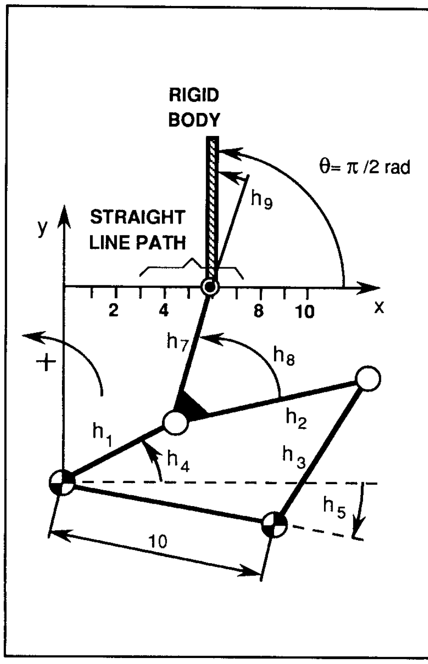
Table 1 shows the four best solutions obtained in this simulation, with a DBCPOL tolerance factor of $\delta = 10^{-5}$ and a DBCONF tolerance factor of $\lambda = 10^{-8}$.

The solutions correspond to mechanism configurations which were very different from one another: in particular, the fourth solution had a very long crank (reaching the maximum allowed value) with a pivot positioned above the rectilinear path to be constructed.

Maximum errors along axis y were about 10 times higher than those of the solution shown in Fig. 6. In the fourth case the reach of a boundary for parameter h_1 caused larger mean and maximum errors.

Table 1. Optimum results for four-bar straight-line path generator

PARAMETERS	RANGE OF DESIGN PARAMETERS		OPTIMUM DESIGN			
			FIRST FINAL VECTOR	SECOND FINAL VECTOR	THIRD FINAL VECTOR	FOURTH FINAL VECTOR
h_1	20	1	3.1505	3.0429	2.4093	20.0000
h_2	20	1	6.3664	7.7982	13.3617	14.7037
h_3	20	1	6.0663	8.9634	7.2850	10.6531
h_4	3.14	-3.14	-1.3740	-1.3658	-1.3406	-0.9316
h_5	3.14	-3.14	-1.0920	-1.1082	-0.5059	0.2811
h_6	10	-10	-4.6563	-5.2170	-9.8005	7.5324
h_7	20	0	8.0853	8.5157	12.3781	12.3056
h_8	3.14	-3.14	1.3134	1.0078	1.1550	0.0900
ERRORS		εy_{max}	0.00352	0.00277	0.00447	0.00881
		εy_{mean}	0.00067	0.00074	0.00089	0.00136



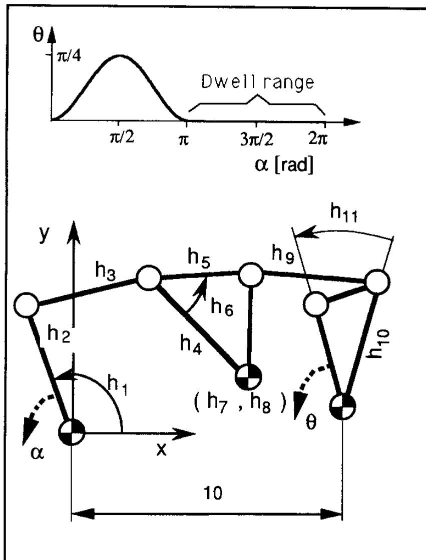
PARAMETERS	RANGE OF DESIGN PARAMETERS		OPTIMUM DESIGN: INITIAL GUESS VECTOR	OPTIMUM DESIGN: FINAL VECTOR
h_1	1	20	18.5845	5.8910
h_2	1	20	7.1545	12.6436
h_3	1	20	17.8077	8.6868
h_4	-3.14	3.14	1.9561	-1.9180
h_5	-3.14	3.14	3.8151	-0.50525
h_6	-10	10	1.3719	5.0937
h_7	0	20	11.3135	5.0315
h_8	-3.14	3.14	-0.2408	-0.7182
h_9	-3.14	3.14	1.1097	1.4400
ERRORS:	εx_{max}		cannot be assembled in 6 positions	0.0777
	εx_{mean}			0.0544
	εy_{max}			0.0415
	εy_{mean}			0.0230
	$\varepsilon \theta_{max}$			0.0409
	$\varepsilon \theta_{mean}$			0.0200

Fig. 7. Four-bar rigid body guidance mechanism.

Example 4

In the fourth example, the problem of rigid body guide was treated. A four-bar linkage had to move a rigid body with a given constant inclination (around axis x) along the rectilinear path of Example 3.

The inclination of the rigid body with respect to link h_7 was added as a design parameter (h_9).



PARAMETERS	RANGE OF DESIGN PARAMETERS		OPTIMUM DESIGN: FINAL BEST VECTOR
h_1	-3.14	3.14	-0.9350
h_2	0.1	10	0.9406
h_3	0.1	10	3.8059
h_4	0.1	10	4.1576
h_5	0.1	10	6.3000
h_6	-3.14	3.14	0.5519
h_7	0.0	10	5.7462
h_8	-10	10	-0.9047
h_9	0.1	10	5.9308
h_{10}	0.1	10	5.3474
h_{11}	-3.14	3.14	-0.6820
ERRORS:	$\varepsilon \theta_{max}$		0.0174
	$\varepsilon \theta_{mean}$		0.0067

Fig. 8. Watt II six-bar linkage function generator.

Optimization was carried out starting from 30 first guess vectors generated by a random procedure. Subroutine DBCPOL was used with $\delta = 10^{-5}$, and the weights in the penalty function were $w_x = 1$, $w_y = 1$, $w_\theta = 10$, $w_a = 10$. The parameters of the four-bar mechanism having the lowest value of the penalty function are shown in Fig. 7.

Example 5

The last example deals with optimum synthesis of a sine function generator mechanism with a dwell range. This problem can be found in looms where combs have to perform a sine beat-up diagram with a dwell during grasping, cutting and weft exchange.

A Watt II mechanism was chosen; its sketch and the design parameters with their variability ranges are shown in Fig. 8. The function to be generated, also shown, had a sinusoidal part and a dwell range of the same duration.

Minima search, starting from 30 random generated guess configurations, provided some good results, one of which is shown in Fig. 8. Subroutine DBCPOL was used with $\delta = 10^{-5}$, and the weights in the penalty function were $w_x = 0$, $w_y = 0$, $w_\theta = 10$, $w_a = 10$.

CONCLUSIONS

The proposed method for the optimum synthesis of planar mechanisms was tested in some problems of path generation, rigid body motion and function generation. It performs well and ensures good accuracy, especially when the two minimization subroutines are used in series.

The method is able to cope with non-assembly conditions, and very extended variability ranges can be selected for design parameters.

First guess vectors were determined at random inside their variability ranges, to demonstrate the robustness of the method.

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REFERENCES

1. N. I. Levitiskii and K. K. Shakvasian, *Int. J. Mech. Sci.* **2,** 76 (1960).
2. N. I. Levitiskii and Y. L. Sarkisyan, *J. Mech.* **3,** 3 (1968).
3. G. H. Sutherland and B. Roth, *Trans. ASME Jl Engng Ind.* **97B,** 303 (1975).
4. R. I. Alizade, A. V. Mohan Rao and G. N. Sandor, *Trans. ASME Jl Engng Ind.* **97B,** 785 (1975).
5. A. C. Rao, *Mech. Mach. Theory* **14,** 141 (1979).
6. M. Huang, ASME Paper No. 74-DET-59.
7. H. W. Pakes, R. M. Parry and G. R. Wray, *Mech. Mach. Theory* **14,** 171 (1979).
8. E. J. Haug and V. N. Sohoni, *Computer Aided Analysis and Optimization of Mechanical System Dynamics* (Ed. E. J. Haug), Vol. F.9, p. 499, NATO ASI Series. Springer, Heidelberg (1984).
9. E. J. Haug, R. Wehage and N. C. Barman, *Trans. ASME Jl Mech. Des.* **103,** 560 (1981).
10. E. J. Haug and J. S. Arora, *Applied Optimal Design.* Wiley, New York (1979).
11. S. N. Kramer and G. N. Sandor, *Trans. ASME Jl Engng Ind.* **97B,** 689 (1975).
12. R. S. Schaefer and S. N. Kramer, *Mech. Mach. Theory* **14,** 161 (1979).
13. A. Bajpai and S. N. Kramer, *Mech. Mach. Theory* **20,** 521 (1985).
14. S. S. Rao and C. P. Reddy, *Mech. Mach. Theory* **14,** 413 (1979).
15. G. H. Sutherland and N. R. Karwa, *Mech. Mach. Theory* **13,** 311 (1978).
16. M. Choubey and A. C. Rao, *Mech. Mach. Theory* **17,** 91 (1982).
17. I. Artobolevskii, *Mechanisms in Modern Engineering Design.* Mir, Moscow (1976).
18. H. Suh and C. W. Radcliffe, *Kinematics and Mechanisms Design.* Wiley, New York (1978).
19. R. Ghigliazza and C. U. Galletti, *Meccanica Applicata alle Macchine.* UTET, Torino (1986).
20. *IMSL Manual*, Math/Library, Vol. 8, Version 1.0, IMSL Inc., Houston, Tex. (1987).

UNE SIMPLE METHODOLOGIE NUMERIQUE POUR LA SYNTHESE OPTIMALE DES MECANISMES DECOMPOSABLE EN GROUPES D'ASSUR

Résumé—Cet étude présente une méthodologie numérique pour la synthèse optimale des mécanismes plans générateurs de fonctions et de trajectoires et pour le guidage des corps rigides. Les paramètres de projet, représentés par les charactéristiques géometriques de chaque membre et par les positions des pivots fixés au chassis, peuvent changer entre les domaines de variabilité dans l'intérieur desquels sont choisis d'une manière casuelle les valeurs de prémier essai requis par le procès itératif de minimisation. L'analyse cinématique est conduite par le moyen de décomposition du mécanisme en groupes d'Assur. L'assemblage des mécanismes est gérée par une construction opportune de la fonction pénalité sans l'imposition d'équations sur les paramètres du projet. L'optimisation est conduite par une méthode non-dérivative et par une quasi-Newton. Quelques exemples de projet optimal sont présentés, pour illustrer les potentialités de la méthode.