

Population of mixed-symmetry states via α transfer reactions

C. E. Alonso,¹ J. M. Arias,¹ L. Fortunato,² N. Pietralla,³ and A. Vitturi²

¹*Departamento de Física Atómica, Molecular y Nuclear, Facultad de Física, Universidad de Sevilla, Apartado 1065, E-41080 Sevilla, Spain*

²*Dipartimento di Fisica Galileo Galilei and INFN, Via Marzolo 8, I-35131 Padova, Italy*

³*Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*

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Within the neutron-proton interacting boson model we study the population of mixed-symmetry states via α transfer processes. Closed expressions are deduced in the case of the limiting $U_{\pi+\nu}(5)$ and $SU_{\pi+\nu}(3)$. We find that the population of the lowest mixed-symmetry 2^+ state, vanishing along the $N_\pi = N_\nu$ line, depends on the number of active bosons and is normally smaller than that of the lowest full symmetric 2^+ state. In particular, for deformed nuclei where the number of bosons is normally large, the relative population of the mixed-symmetry 2^+ state is of the order of a few percent. More favorable cases can be found near shell closures, as in the case of α transfer leading to ^{140}Ba .

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One of the most interesting features of the neutron-proton version of the interacting boson model (IBM-2) [1] is the occurrence, in addition to all the states already present in the original IBM-1 version, of states that are not fully symmetric in the proton-neutron degree of freedom [2]. In spherical nuclei, the lowest of such mixed-symmetry states is expected to be a 2^+ state (normally indicated as 2^+_{M1}) [3], while in the well-deformed case one expects a mixed-symmetry $K = 1^+$ band head [4], with a spin quantum number that is outside the simple IBM-1. These mixed-symmetry states display particular properties, such as an electromagnetic decay by a strong $M1$ transition. They have been populated in many nuclear reactions such as inelastic scattering of electrons [4], photons [5], or hadrons [6]; β decay [3]; fusion evaporation reaction [7]; and Coulomb excitation [8]. For a review article, see Ref. [9]. The possibility of populating these states by ($^{12}\text{C}, ^8\text{Be}$) reactions, namely, via α transfer processes, has recently been suggested [10].

The reaction mechanism associated with the α transfer reaction is not fully understood [11]. Whether the four particles are transferred sequentially due to the action of the one-body field or, as the other extreme, are transferred as a unique cluster in a single shot is a matter for discussion. In the former case the particle correlations act via constructive interference of the different paths in the intermediate states due to the correlated features of the initial and final states. In the latter case, one assumes that the correlations have preformed an α particle in the initial and final states, with a probability given by the so-called α spectroscopic factors, and the process is then formally described as the transfer of a single object. In addition, each approach relies on the choice of a number of ingredients, such as the proper optical potentials, coupling form factors, etc. Therefore, in the end, different descriptions of the reaction mechanism yield estimates of the absolute cross sections that, in addition to the dependence on the bombarding energy, can vary even by orders of magnitude from one approach to the other. As far as relative cross sections are concerned, on the other hand, one can profitably assume that the transfer intensities will in all cases scale as the square of the

matrix element of the four-particle creation (or annihilation, in the case of pickup reactions) operator.

In this article we therefore calculate four-particle creation matrix elements, comparing those involving fully symmetric or mixed-symmetry final states.

In the framework of the interacting boson model the operator associated with the α -particle creation does not simply acquire a two-boson creation form. A mapping procedure from the underlying fermion space will in fact map the four fermion creation operator into a sum of boson operators of increasing order. At the lowest order, however, we assume the simplest expressions for the $L = 0$ and $L = 2$ α -particle creation operator,

$$A_\alpha^\dagger(L = 0) = c_1 s_\pi^+ s_\nu^+ + c_3 (d_\pi^+ \times d_\nu^+)^{(0)} \quad (1)$$

and

$$A_\alpha^\dagger(L = 2) = c_2 [s_\pi^+ \times d_\nu^+ + d_\pi^+ \times s_\nu^+]^{(2)} + c_4 (d_\pi^+ \times d_\nu^+)^{(2)}. \quad (2)$$

The parameters c_i can in principle be derived microscopically from the mapping procedure. In practice, and for the purpose of this article, we can treat them as phenomenological parameters to be determined from experimental data. Note also that one-step transfer processes can only populate natural-parity states [11]; so the transfer to 1^+ states is not allowed. On the other hand, the dipole $L = 1^-$ transfer operator has vanishing matrix elements within the IBM-2 space, characterized by states with positive parity only. These simple forms, Eqs. (1) and (2), are close to the original operators introduced by Bennett and Fulbright [12] and applied by the authors to different isotopic and isotonic chains in the f - p shell.

We consider as the initial state the ground state of an even-even nucleus characterized by the boson number $N_i = N_\pi + N_\nu$ and as the final states the lowest fully symmetric states and the first mixed-symmetry 2^+ states in a nucleus with boson number $N_f = N_\pi + N_\nu + 2$. In general, matrix elements have to be determined after the diagonalization of the IBM-2 Hamiltonian, from the explicit wave functions.

Analytic expressions can, however, be obtained in the case of limiting situations. We first consider the case of the $U_{v+\pi}(5)$ limit. The group chain in this case is

$$U_v(6) \times U_\pi(6) \supset U_{v+\pi}(6) \supset U_{v+\pi}(5) \\ \supset O_{v+\pi}(5) \supset O_{v+\pi}(3) \supset O_{v+\pi}(2) \quad (3)$$

and the $U(5)$ wave functions are characterized by the 12 quantum numbers that classify uniquely the basis states, namely,

$$[[N_v] \times [N_\pi]; [N - f, f]\{n_1, n_2\}(v_1, v_2)\alpha LM), \quad (4)$$

where α stands for the two missing labels necessary to completely specify the $O(5) \supset O(3)$ reduction, omitted when the reduction is unique.

With this notation the initial state is

$$|0_1^+\rangle = [[N_v] \times [N_\pi]; [N, 0]\{0, 0\}(0, 0)0] = |s_v^{N_v} s_\pi^{N_\pi}; 0\rangle, \quad (5)$$

where $N = N_v + N_\pi$. The final states that can be populated via the operators (1) and (2) are the lowest fully symmetric states [2] and the lowest mixed-symmetry state. Their labels and full expressions are given in the first three columns of Table I. The associated reduced matrix elements of the α transfer operator are given in the last column. One can see from these expressions that the α transfer intensities are expected to scale approximately as $N_\pi N_v$ to the ground state, as $N_\pi N_v/N$ to the one-phonon state, and as $N_\pi N_v/N^2$ to the two-phonon states. Note that the population of the mixed-symmetry state, which scales as $(N_\pi - N_v)^2/N$, will in general be much lower than that of the corresponding symmetric one. The ratio of α transfer cross sections to the 2_M^+ state and to the 2_1^+ state does not depend on the parameters of the transfer operators and amounts to $(N_\pi - N_v)^2/4(N_\pi + 1)(N_v + 1)$. In particular, for $N_v = N_\pi$ we obtain the selection rule that the population of the mixed-symmetry state is completely forbidden.

We consider now the case of $SU(3)$. In this limit the group chain is

$$U_v(6) \times U_\pi(6) \supset U_{v+\pi}(6) \supset SU_{v+\pi}(3) \\ \supset O_{v+\pi}(3) \supset O_{v+\pi}(2) \quad (6)$$

and the $SU(3)$ wave functions are characterized by the 12 necessary quantum numbers, namely,

$$[[N_v] \times [N_\pi]; [N - f, f]\rho(\lambda, \mu)\kappa LM), \quad (7)$$

where ρ (set of three) and κ are missing labels necessary to completely specify the $U(6) \supset SU(3)$ and $SU(3) \supset O(3)$ reductions, omitted when these are unique. In the following we omit the $O_{v+\pi}(2)$ label, M . With this notation the initial state is

$$|0_1^+\rangle = [[N_v] \times [N_\pi]; [N, 0](2N, 0)0), \quad (8)$$

where $N = N_v + N_\pi$. The final states that can be populated via the operators (1) and (2) are the lowest fully symmetric states and the lowest 2^+ mixed-symmetry state (see Table II, first and second columns).

The evaluation of the α transfer matrix elements is more feasible in the intrinsic frame. The ground, β , γ , and mixed-symmetry bands can be associated with intrinsic states of the form [13]

$$|\Phi(N_\pi + 1, N_v + 1; \text{gs})\rangle \\ = (B_g^\pi)^{N_\pi+1} (B_g^v)^{N_v+1} |\Phi(N_\pi + 1, N_v + 1; \beta)\rangle \\ = n(\hat{N}_\pi (B_g^\pi)^{N_\pi} B_\beta^\pi (B_g^v)^{N_v+1} + \hat{N}_v (B_g^\pi)^{N_\pi+1} (B_g^v)^{N_v} B_\beta^v) \\ \times |\Phi(N_\pi + 1, N_v + 1; \gamma)\rangle \\ = n(\hat{N}_\pi (B_g^\pi)^{N_\pi} B_\gamma^\pi (B_g^v)^{N_v+1} + \hat{N}_v (B_g^\pi)^{N_\pi+1} (B_g^v)^{N_v} B_\gamma^v) \\ \times |\Phi(N_\pi + 1, N_v + 1; M)\rangle \\ = n(\hat{N}_v (B_g^\pi)^{N_\pi} B_M^\pi (B_g^v)^{N_v+1} - \hat{N}_\pi (B_g^\pi)^{N_\pi+1} (B_g^v)^{N_v} B_M^v), \quad (9)$$

where $n = (N + 2)^{-1/2}$ and where we have used the general notations $\hat{N}_\alpha = \sqrt{N_\alpha + 1}$ and

$$(B_i^\alpha)^{N_{\alpha,i}} = \frac{(b_{\alpha,i}^\dagger)^{N_{\alpha,i}}}{\sqrt{N_{\alpha,i}!}} |0\rangle, \quad (10)$$

TABLE I. Notation and explicit expressions of $U(5)$ states (first three columns) and α transfer reduced matrix elements.

$ f\rangle$	$[[N_v + 1] \times [N_\pi + 1]; [N - f, f]\{n_1, n_2\}(v_1, v_2)\alpha LM)$	Explicit expression	$\langle f A_\alpha^\dagger(L) 0_1^+(N_v, N_\pi) \rangle$
$ 0_1^+\rangle$	$ \cdots; [N + 2, 0]\{0, 0\}(0, 0)0\rangle$	$ s_v^{N_v+1} s_\pi^{N_\pi+1}; 0\rangle$	$c_1 \sqrt{(N_v + 1)(N_\pi + 1)}$
$ 0_2^+\rangle$	$ \cdots; [N + 2, 0]\{2, 0\}(0, 0)0\rangle$	$\frac{1}{\sqrt{(N+2)(N+1)}} (\sqrt{N_v(N_v+1)} s_v^{N_v-1} d_v^2 s_\pi^{N_\pi+1}; 0\rangle + \sqrt{2(N_v+1)(N_\pi+1)} s_v^{N_v} d_v s_\pi^{N_\pi} d_\pi; 0\rangle + \sqrt{N_\pi(N_\pi+1)} s_v^{N_v+1} s_\pi^{N_\pi-1} d_\pi^2; 0\rangle)$	$c_3 \sqrt{\frac{2(N_v+1)(N_\pi+1)}{(N+2)(N+1)}}$
$ 2_1^+\rangle$	$ \cdots; [N + 2, 0]\{1, 0\}(1, 0)2\rangle$	$\sqrt{\frac{1}{N+2}} (\sqrt{N_v+1} s_v^{N_v} d_v s_\pi^{N_\pi+1}; 2\rangle + \sqrt{N_\pi+1} s_v^{N_v+1} s_\pi^{N_\pi} d_\pi; 2\rangle)$	$2c_2 \sqrt{\frac{5(N_v+1)(N_\pi+1)}{N+2}}$
$ 2_2^+\rangle$	$ \cdots; [N + 2, 0]\{2, 0\}(2, 0)2\rangle$	$\frac{1}{\sqrt{(N+2)(N+1)}} (\sqrt{N_v(N_v+1)} s_v^{N_v-1} d_v^2 s_\pi^{N_\pi+1}; 2\rangle + \sqrt{2(N_v+1)(N_\pi+1)} s_v^{N_v} d_v s_\pi^{N_\pi} d_\pi; 2\rangle + \sqrt{N_\pi(N_\pi+1)} s_v^{N_v+1} s_\pi^{N_\pi-1} d_\pi^2; 2\rangle)$	$c_4 \sqrt{\frac{10(N_v+1)(N_\pi+1)}{(N+2)(N+1)}}$
$ 2_M^+\rangle$	$ \cdots; [N + 1, 1]\{1, 0\}(1, 0)2\rangle$	$\sqrt{\frac{1}{N+2}} (\sqrt{N_\pi+1} s_v^{N_v} d_v s_\pi^{N_\pi+1}; 2\rangle - \sqrt{N_v+1} s_v^{N_v+1} s_\pi^{N_\pi} d_\pi; 2\rangle)$	$c_2 \frac{\sqrt{5(N_\pi - N_v)}}{\sqrt{N+2}}$

TABLE II. Explicit expressions of $SU(3)$ states (first and second columns) and α transfer reduced matrix elements.

$ f\rangle$	$[[N_\nu] \times [N_\pi]; [N - f, f]\rho(\lambda, \mu) \kappa LM)$	$\langle f A_\alpha^\dagger(L) 0_1^+(N_\nu, N_\pi) \rangle$
$ 0_1^+\rangle$	$[[N_\nu + 1] \times [N_\pi + 1]; [N + 2, 0](2N + 4, 0)0)$	$\frac{1}{3}\sqrt{(N_\pi + 1)(N_\nu + 1)} \left[c_1 + \frac{2c_3}{\sqrt{5}} \right]$
$ 2_1^+\rangle$	$[[N_\nu + 1] \times [N_\pi + 1]; [N + 2, 0](2N + 4, 0)2)$	$\frac{2}{3}\sqrt{2(N_\pi + 1)(N_\nu + 1)} \left[c_2 - \frac{c_4}{\sqrt{7}} \right]$
$ 0_\beta^+\rangle$	$[[N_\nu + 1] \times [N_\pi + 1]; [N + 2, 0](2N, 2)0)$	$\frac{2}{3}\sqrt{\frac{2(N_\pi + 1)(N_\nu + 1)}{(N + 2)}} \left[c_1 - \frac{c_3}{\sqrt{5}} \right]$
$ 2_\beta^+\rangle$	$[[N_\nu + 1] \times [N_\pi + 1]; [N + 2, 0](2N, 2)\kappa = 02)$	$\frac{2}{3}\sqrt{\frac{(N_\pi + 1)(N_\nu + 1)}{(N + 2)}} \left[c_2 + \frac{2c_4}{\sqrt{7}} \right]$
$ 2_\gamma^+\rangle$	$[[N_\nu + 1] \times [N_\pi + 1]; [N + 2, 0](2N, 2)\kappa = 22)$	$2\sqrt{\frac{2(N_\pi + 1)(N_\nu + 1)}{3(N + 2)}} \left[c_2 + \frac{2c_4}{\sqrt{7}} \right]$
$ 2_M^+\rangle$	$[[N_\nu + 1] \times [N_\pi + 1]; [N + 1, 1](2N + 2, 1)2)$	$\sqrt{\frac{2}{3(N + 2)}}(N_\nu - N_\pi) \left[c_2 - \frac{c_4}{\sqrt{7}} \right]$

with $\alpha = \nu, \pi$ and $i = \{g, \beta, \gamma, M\}$. The basis (proton or neutron) bosons b_g, b_β, b_γ , and b_M are given by

$$\begin{aligned} b_g^\dagger &= \frac{1}{\sqrt{3}}(s^\dagger + \sqrt{2}d_0^\dagger), & b_\beta^\dagger &= \frac{1}{\sqrt{3}}(-\sqrt{2}s^\dagger + d_0^\dagger), \\ b_\gamma^\dagger &= \frac{1}{\sqrt{2}}(d_2^\dagger + d_{-2}^\dagger), & b_M^\dagger &= \frac{1}{\sqrt{2}}(d_1^\dagger + d_{-1}^\dagger). \end{aligned} \quad (11)$$

Once the matrix elements are obtained in the intrinsic frame a projection to the laboratory system is performed. After that projection, we obtain, for the reduced matrix elements to the fully symmetric states and to the mixed-symmetry 2_M^+ state, the expressions in the third column of Table II. It is worth noting that more involved expressions than Eq. (10) for the structure of the intrinsic ground, β , γ , and mixed-symmetry bands can be found in the literature [14] but they give the same results for the α transfer intensities in leading order of an expansion in N .

In the $SU(3)$ case, therefore, one expects that the α transfer intensities scale approximately as $N_\pi N_\nu$ to the ground-state band, as $N_\pi N_\nu / N$ to the one-phonon (either β or γ) state, and as $(N_\pi - N_\nu)^2 / N$ to the mixed-symmetry states. This is much lower than the values for the corresponding symmetric states. The ratio of α transfer cross sections to the 2_M^+ state and to the 2_1^+ state does not depend on the parameters of the transfer operators and amounts to $3(N_\pi - N_\nu)^2 / (4(N + 2)(N_\pi + 1)(N_\nu + 1))$. Again, the population of the mixed-symmetry state is forbidden for $N_\nu = N_\pi$. For the sake of clarity, in Fig. 1

the $L = 2$ α transfer transitions studied in this work are schematically presented.

We finally mention the case of the $O(6)$ dynamical symmetry. In this case the straightforward procedure is to express the IBM-2 $O(6)$ states in terms of products of proton and neutron $O(6)$ states (see Ref. [2]) and then expand each IBM-1 $O(6)$ state in the IBM-1 $U(5)$ basis using the transformation brackets given, for example, in Ref. [15]. The evaluation of the matrix elements of the α transfer operator is reduced to a summation of $U(5)$ matrix elements. A trivial simplification occurs, due to the selection rules of the boson creation operators, but the remaining sum does not seem to be easily reducible to a compact expression.

As an example of the spherical limit, we discuss the case of ^{140}Ba , considered in Ref. [10]. In this case the considered reaction is $^{136}\text{Xe}(^{12}\text{C}, ^8\text{Be})^{140}\text{Ba}$. As already discussed in Ref. [3] one can assume a proper description of the low-lying spectrum of ^{140}Ba within the simple $U(5)$ limit. Assuming as cores the $Z = 50$ and $N = 82$ shell closure, one has in the initial system $N_\pi = 2$ and $N_\nu = 0$. This leads to the relative α transfer intensities, with respect to the cross section to the ground state, equal to $\frac{c_2^2}{c_1^2}$ for the first 2_1^+ state, to $\frac{c_2^2}{3c_1^2}$ for the mixed-symmetry 2_M^+ state, to $\frac{c_3^2}{6c_1^2}$ for the 0_2^+ state, and finally to $\frac{c_4^2}{6c_1^2}$ for the 2_2^+ state. In this rather fortunate and promising case, due to the small values of the boson numbers N_π and N_ν , the population of the first 2_1^+ state is expected to be of the same order of magnitude as that of the ground state, but

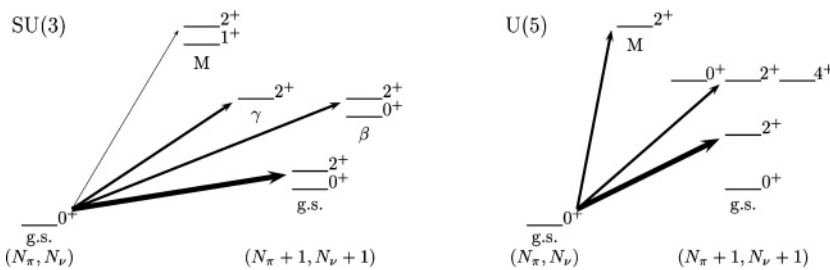


FIG. 1. Schematic representation of the $L = 2$ α transfer intensities studied in this work in the $SU(3)$ (left) and $U(5)$ (right) states.

more importantly the transition to the mixed-symmetry 2_M^+ state is expected to be only a factor of three smaller than the population of the first 2_1^+ state.

As an example of the deformed limit, we consider ^{156}Gd , a typical $SU(3)$ case in the rare earth region where 1^+ states corresponding to the scissors mode with mixed symmetry have been observed. The corresponding α -transfer reaction is $^{152}\text{Sm}(^{12}\text{C}, ^8\text{Be})^{156}\text{Gd}$. In this case the initial nucleus has $N_\pi = 6$ and $N_\nu = 4$. These higher boson numbers, typical of the $SU(3)$ limit, imply that the population of the mixed-symmetry 2_M^+ state relative to the population of the first 2_1^+ state is less than 1%.

In summary, we have evaluated the matrix elements of α transfer as two-boson operators in the $U(5)$ and the $SU(3)$ dynamical symmetries of the F -spin limit of the IBM-2 from the initial ground state with boson numbers $N_i = N_\pi + N_\nu$ to selected final states with boson numbers $N_f = (N_\pi + 1) + (N_\nu + 1)$ and spin quantum numbers 0 or 2. Lowest order expressions for α transfer cross sections to

full-symmetry states and to the lowest mixed-symmetry 2^+ state have been derived this way. Corresponding experiments have been proposed. We stress the importance of these experiments with respect to two points. First, α transfer reactions might turn out to be an efficient way to populate mixed-symmetry states, in particular in spherical nuclei. Second, the unique boson number dependence of relative α transfer cross sections might be a new and useful test either for the assignment of mixed-symmetry character to particular excited nuclear states or, even more importantly, for the predictive power of the interacting boson model with proton-neutron degree of freedom.

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