Int. J. Contemp. Math. Sciences, Vol. 7, 2012, no. 33, 1639 - 1652

A Mathematical View of Matching Food and Wine

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Abstract

The paper presents a mathematical view and some algorithmic tools for analyzing a very important process for sommeliers and wine tasters: matching food and wine. A *Matlab*®package, that essentially analyzes the overlap of two planar polygons, has been developed for this purpose.

Keywords: Matching food and wine, polygon clipping, algorithm for polygon comparison, best matching

1 Introduction

I would like to start with a short quote, taken from [3], which gives a nice idea of the context. *"Food and wine matching is often cast as a rather mysterious science, but in truth it is actually quite simple, and the experimentation involved is great fun. People often thought that the purpose for a wine is for it to be drunk with food in a situation where both complement each other, and it still amazes how often a rather humble wine will synergize with a food match in a profound way."* These words give, at least to me, a perfect idea of what is aimed at by the process of matching food and wine.

The one who is writing this paper is an Italian mathematician that for fun one day decided to learn more about wine tasting and took the course for becoming a sommelier. Italian sommeliers and wine tasters, in matching food and wine, use a diagram (see Figure 1, below) aimed to match as much as possible a given *dish* or a *simple food* with an (the) *appropriate wine*. This diagram is then used to produce two polygons that describe the most important *characteristics* of the food and the wine. Once these characteristics are evaluated, then two polygons will be displayed in order to find the best match between the food and the wine. If these polygons best match each other, after maybe a roto-translation, the sommeliers deduce that the matching is *harmonic* or not.

Mathematically speaking, the problem consists of "comparing" two polygons obtained by connecting the values, indicated in the sommelier diagram, that is somehow reminiscent of a dart board (see the next Section), of the characteristics of the food and those of the wine. How do we compare these polygons? This is the mathematical part described in Section 3.

In Section 5 we describe the matching algorithm, while in Section 6 its implementation in *Matlab*® and some experiments of classical food and wine matching.

At the end of this note, we hope to convince the readers of the necessity of becoming more confident in the ability to choose the proper wine for *(almost) every* dish (or food)!

2 The diagram

First of all we present the diagram that Italian sommeliers use to match a given food with a wine (see Figure 1). There are two different sets of descriptors, that sommeliers call *characteristics*, which are written in the diagram with different characters.

- Words in *normal size*, correspond to the food characteristics that we want to evaluate using a scale of integers from 0 to 10. These are considered in counterclockwise order and are
  - greasiness,
  - juiciness,
  - sweetness tendency,
  - fattiness,
  - sapidity/bitterness tendency/acidity tendency/sweetness,
  - persistence taste-aroma/spiciness/aromatic.
- Words in *capitals*, correspond to the wine characteristics that we want to evaluate, again using a scale of integers from 0 to 10. As above, these are considered in counterclockwise order, opposite to the circles center, as follows
  - ALCOHOL LEVEL,
  - TANNIN CONTENT,
  - SAPIDITY/EFFERVESCENCE, ACIDITY,
  - AROMA INTENSITY/I.A.P. (Intense Aromatic Persistence)
  - SWEETNESS/SOFTNESS

Notice that when we have a group of characteristics, such as those for the food *sapidity/bitterness tendency/acidity tendency/sweetness*, and we give to each of them a value, the one we then consider is the highest, that is the most perceptible of the group.

During the matching process sommeliers, when describing a characteristic of the food or of the wine, quantify the intensity and persistence of a characteristic as follows.

1. The value 0 is used when a characteristic is *absent*.
2. The (integers!) values in the range $1-3$, are used when a characteristic is *barely perceptible*.
3. The values in the range $4-6$ are used when a characteristic is *better perceptible than before, but not clearly*.
4. The values $7, 8$ are considered when a characteristic is *clearly perceptible*.
5. Finally, the values $9, 10$ are given when a characteristic is *perfectly perceptible*.

These values are represented by a set of 11 concentric circles, of radii 0 up to 10, intersected by lines emanating from the inner circle and connecting the characteristics. With respect to the axis $y = 0$ (i.e. the horizontal axis) these lines have angles that can be chosen, in principle arbitrarily, but a reasonable choice is to consider them disposed approximately parallel to the lines $y = 0$, $y = x$ and $y = -x$ (i.e. the horizontal axis and the two lines at 45 degrees to the horizontal one). In Figure 2 we present this diagram, which is not the original one (as found in [1]), but it gives quite well the idea of what Italian sommeliers should handle when performing food and wine matching. Some characteristics of the food match the wine *by opposition* while others *by accordance*. To be more precise, if we associate a set of Cartesian coordinates (x, y), the diagram has essentially *three main symmetries*: one bottom-up that corresponds to the horizontal line $y = 0$ describing a matching by accordance, the other two, describing a matching by opposition, correspond to the bisection lines $y = x$ and $y = -x$.

(a) One is bottom-up through the line $y = 0$. In the bottom part there are the characteristics of the food that should agree with the corresponding ones of the wine, along the same side (left: sapidity, bitterness tendency, acidity tendency, sweetness; right: persistence taste-aroma, spiciness, aromatic), on the top of the diagram for the wine. There is one exception: the sapidity, bitterness and acidity tendency of the food have to be considered as opposed to the SWEETNESS and the SOFTNESS of

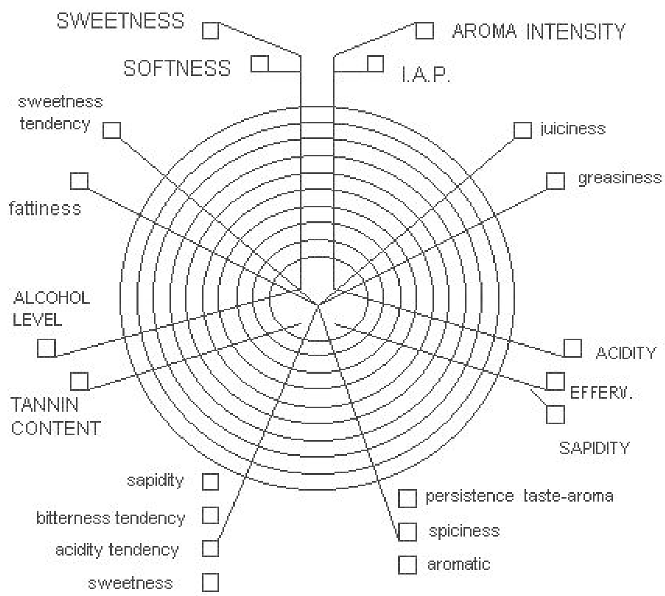

Figure 1: The diagram used by sommeliers to match food and wine

the wine. As an example, if the food has aromatic 6 and spiciness 5, the wine should have AROMA INTENSITY and/or I.A.P. with almost the same values.

(b) The second symmetry is along the bisection line $y = x$. Here the matching is made by opposition. For example a food which is juicy with value 8, must match a wine with similar values of ALCOHOL LEVEL and/or TANNIN CONTENT.

(c) The third symmetry is along the bisection line $y = -x$. The matching is here again by opposition. For example a food which is fat should be matched with a wine with good percentage of ACIDITY and EFFERVESCENCE or SAPIDITY. The reason is simple: EFFERVESCENCE and ACIDITY have the effect of cleaning the fat of the food from the mouth.

In Figure 2 we show a diagram, similar to that that can be found in Italian books for sommeliers (cf. i.e. [1]), where we have identified the characteristics *by opposition* with the colors red and black, while the ones *by accordance* with the colors green and magenta.

In Figure 3, we show a diagram of the characteristics found in a particular food and the corresponding wine, where we have highlighted with different colors the characteristics that match either by opposition or by accordance.

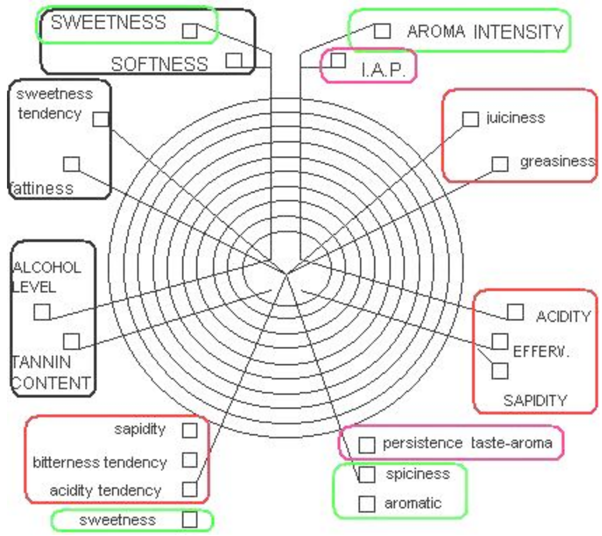

Figure 2: The diagram used by sommeliers to match food and wine by accordance or opposition

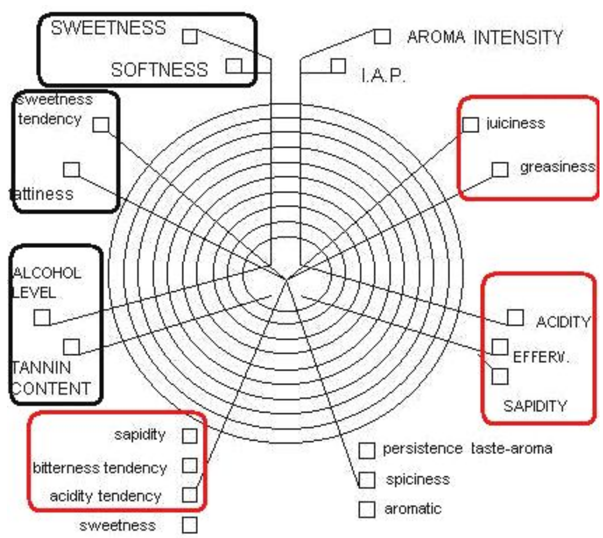

Figure 3: Another example of the diagram used by sommeliers to match food and wine with accordance or opposition

Example 2.1 In Figure 4 we show the diagram for the matching of a slice of S. Daniele ham, Italian prosciutto crudo (in blue), and a red wine from Sicily, DOC Nero d'Avola, year 2002, 14% (in red). Notice: the diagram gives rise to two *polygons*.

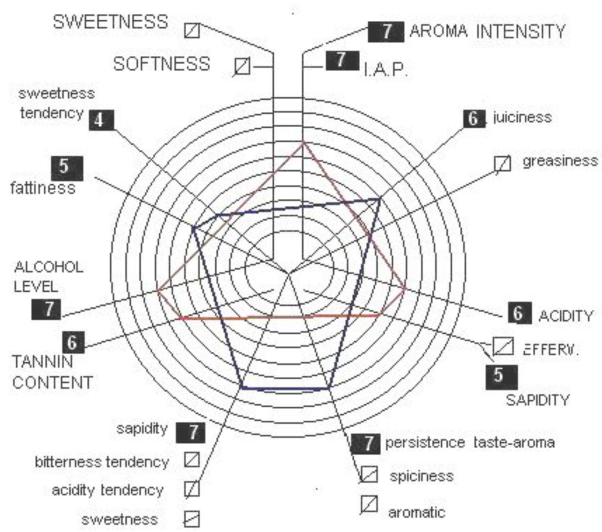

Figure 4: This diagram corresponds to the match of a slice of ham and a glass of Nero d'Avola (Sicilian wine), aged 2002.

3 Why do we match food and wine?

The answer gives rise to the following two other questions.

- For a given food and wine, is the wine *matching* or *not matching* the food?
- Given a food, which characteristics should a wine have for the *optimal* match (or the best possible)?

As we have seen in Example 2.1, the problem can be modeled as a geometrical problem consisting of the "comparison" of two polygons inscribed in 11 circumscribed circles. The comparison is made by analyzing the intersections of their areas and in particular the way they should overlap. Summarizing, our *best matching problem* can be performed, modulo a roto-translation, by looking at how much the polygons overlap! That is, *the shapes of the polygons should not be too different* and they *overlap as much as possible.* The more they overlap the better will be the matching.

4 Mathematical ingredients

In this section we introduce the *mathematical ingredients* that we shall make use of for the best match of our two polygons. We start by constructing our model.

- Put the common center of the circles at the origin.
- Hence, every polygon has vertices at points of the form

  $$(x_s, y_s) = (k\cos\theta_s, k\sin\theta_s)\,,$$

  where $k \in \{0, \dots, 10\}$ and the θ_s are the angles of the lines.

  On looking at the diagram used by Italian sommeliers, for example, for the wine characteristics we may take: $\theta_1 = \pi/2 - \delta/2$, $\theta_2 = \pi/2 + \delta/2$, $\theta_3 = \pi + \delta$, $\theta_4 = \pi + 2\delta$, $\theta_5 = -2\delta$, $\theta_6 = -\delta$ with $\delta = \pi/12$.
- Similarly for the food. Since the food characteristics are symmetric with respect to the center of the circles, a natural choice for the food lines is to take the angles $\alpha_s = \pi + \theta_s$, $s = 1, \dots, 6$, where the θ_s are the angles of the wine.

Letting

$$\begin{aligned} \mathcal{W} &= \{(x_s, y_s),\ s = 1, \dots, S_W\} \\ \mathcal{F} &= \{(u_s, v_s),\ s = 1, \dots, S_F\} \end{aligned}$$

the polygons, obtained by joining the points on the circles giving the values of the analysed characteristics, of the the wine and the food, respectively.

Applying *Green's theorem* (cf. e.g. [6]) to the region enclosed by the polygons, we can easily compute $A_W := |\mathcal{W}|$ and $A_F := |\mathcal{F}|$, i.e. the signed area of the polygons. The vertices must be ordered clockwise or counterclockwise: if they are ordered clockwise, the area will be negative but correct in absolute value.

For example, for the wine polygon we have:

$$A_W = \frac{1}{2}\sum_{i=1}^{S_W} x_i y_{i+1} - x_{i+1} y_i\,, \tag{1}$$

where $x_{S_W+1} = x_1$ and $y_{S_W+1} = y_1$ (cf. i.e. [2]). We observe that equation (1), can be interpreted as the product (crossed) of the two columns array formed by the x and y coordinates of the vertices. For example, again in the case of the wine, formula (1) comes from the product of this array:

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ & \\ x_{S_W} & y_{S_W} \\ x_{S_W+1} & y_{S_W+1} \end{bmatrix}$$

We need also the location of the *centroid* of a polygon, i.e. the geometrical analogue of the center of mass. The general formula is well-known, but for the sake of completeness, we report here the coordinates of the centroid, for example, of the food polygon:

$$u_F = \frac{1}{6A_F}\sum_{i=1}^{S_F}(u_i + u_{i+1})(u_i v_{i+1} - u_{i+1} v_i) \quad (2)$$

$$v_F = \frac{1}{6A_F}\sum_{i=1}^{S_F}(v_i + v_{i+1})(u_i v_{i+1} - u_{i+1} v_i) \quad (3)$$

5 Matching Algorithm (MA)

First, we construct the diagram of Figure 1 and the corresponding polygons. This will be done simply from the values corresponding to the characteristics of the wine and the food. Moreover, in each polygon we indicate the two *moment lines* of the centroidal principal moments about axes parallel to the Cartesian ones. Then we find the centroid, as described above.

Then, the MA does the following steps.

1. Find the equation of the moment lines.
2. Apply a roto-translation to the food polygon. This is aimed to make the polygons having the same centroid.
3. Analyze the difference

   $$E(W, F) = (W \cup F) \setminus (W \cap F) \ , \quad (4)$$

   to check if the wine matches the food.

Remarks.

(a) In step 2, we choose to roto-translate the polygon of the food but we could choose instead the polygon of the wine.

(b) By the formula (4) we have denoted the difference of the union and the intersection of the polygons by $E(W, F)$, as it represents an (absolute) *error*. Indeed, as we shall see, the more polygons overlap the better the matching is and smaller their difference will be.

We want to better specify the step 2 of the MA. With a better look, we observe that the polygon of the food is always upside down with respect to the that of the wine, i.e. "mirror images" of each other through the horizontal line $y = 0$. Thus, step 2 can essentially be performed in two *exclusive* ways.

(i) Reflect the polygon of the food through x-axis and then translate it to the centroid of the wine polygon, as done at step 2 above

(ii) Apply a roto-translation the food polygon so that its centroid matches that of the wine polygon and the characteristics that are to mch are aligned which each other. We describe below exactly how this will be done.

Which one of the two possibilities is preferable? The obvious answer is: "the one that makes (4) smaller". Mathematically speaking, the question can be reformulated as follows: "Does the function $E(W, F)$ have a *minimum*?"

As in linear programming, each (convex) polygon represents a planar region that we can represent in the general form $A\mathbf{x} \leq \mathbf{b}$. Hence, $W := A_1\mathbf{x} \leq \mathbf{b}_1$ and $F := A_2\mathbf{x} \leq \mathbf{b}_2$ be the polygons of the wine and the food, respectively.

Letting $r(\varphi) = \mathrm{e}^{i\varphi}$ the complex exponential function, and let $\tilde{F} := \tilde{A}_2\mathbf{x} \leq \tilde{\mathbf{b}}_2$ be the food polygon obtained either by a reflection or by a rotation of angle φ (obtained multiplying each polygon vertex by $r(\varphi)$) followed, possibly, by a translation of a vector $\mathbf{t}$.

After these transformations, the union of the polygons can be rewritten as

$$W \cup \tilde{F} = (B\mathbf{x} \leq \mathbf{c}) \;\cup\; \bigcup_{j=1}^{m} (C_j\mathbf{x} \leq \mathbf{d}_j) \;. \tag{5}$$

where the first term is $W \cap \tilde{F}$ which gives the (percentage of) overlapping and the second represents the union of the small polygons not belonging to the intersection, that is the difference described by the function (4).

Thus, the minimum of (4) is obtained when $m = 0$ (if possible), which is equivalent to say that $W \cup \tilde{F} = W \cap \tilde{F}$. This means that both the wine polygon and the transformed food polygon overlap perfectly.

As regards to the *optimal* rotation, recalling the fact that the angles of the characteristics lines of the food are the same of the ones of the wine modulo π, in order to maximize the overlap a *heuristic strategy* is to perform six rotations, which correspond to the angles of the six lines of the wine characteristics with the constant δ halved. The angle which maximizes the overlap can then be considered the 'optimal' one for the rotation.

6 Implementation details and examples

The implementation was done in *Matlab*®by the use of the *Matlab*®function `polygeom` by H. J. Sommer, and the function `PolygonClip` from the toolbox `Polygon clipper` by S. Hölz, both downloadable at `Matlab Central File Exchange`. All functions are available from the web page of the author. For details on *Matlab*®, please refer to the monograph [5].

The vertices of the polygons, which lie on concentric circles of radii $0 - -10$, are provided by entering in popup menus the numbers corresponding to the wine and food characteristics. The geometry suggests the use of polar coordinates to locate the vertices and then the lines of the characteristics.

By the function `polygeom` we can compute the areas, the centroids, the perimeters and the moment lines of the polygons. The user can then decide which transformation apply to the food polygon (or reflection plus translation or roto-translation).

The function `PolygonClip` allows us to calculate the overlap of two polygons. Then, by the $Matlab^{®}$function `polyarea` we are able to compute the area of their intersection and compare it with those of the original polygons.

In Table 1 we show the comparison of both strategies in 3 examples. The examples considered, are matching tests that can be realized at the web page of the Italian Association of Sommeliers (see `http://www.sommelier.it`). In Figures 5–7 we have adopted a shorthand notation for food and wine characteristics, by using the initials in lower case or in upper case. The values corresponding to the characteristics are easily deducible just counting the circle to which the vertex belong to. When a characteristic is absent, the corresponding vertex is not drawn.

Example	Area W	Area F	Area R	Opt. angle	Area R-T	Diff.
1	65.5	34.8	34.8	3.1743	32.5	2.3
2	57.7	38.6	37.8	π	38.6	1.2
3	54.1	52.6	43.0	π	44.0	1.0

Table 1: Three examples of wine and food matching. The second and third column give the area of the original polygons. The fourth column gives the area of E(W,F) after reflection. The sixth column the area E(W,F) after roto-translation by the optimal angle (in radians) of the fifth column. The last column is the difference between the values in columns 6 and 4.

Example 6.1 The first example corresponds to the match of an oyster with a sparkling wine, similar to French Champagne, called Franciacorta Brut DOCG (Italy). The corresponding diagram and the polygons after reflection and roto-translation are shown in Figure 5. Notice that in this case the wine polygon has bigger area than that of the food polygon and some characteristics of the food are not balanced by those of the wine. For example, the effervescence of the wine has value 8 which is twice the value of the sweet tendency of the oyster. This means that the matching is not perfect.

Example 6.2 In the second example the matching is between a dish consisting of potato and onion pudding with a Chardonnay Colli Orientali del

Friuli DOC vintage 1995 (a wine from the region Friuli, in the northeastern part of Italy). As in the previous example, the matching is not perfect, due to fact that the polygon of the wine has bigger area than that of the food and, as before, some characteristics of the food are not balanced by those of the wine. The corresponding diagram and the polygons after reflection and roto-translation are in Figure 6.

Example 6.3 The third example, is the matching of a smoked hering fillet with onions, and a red wine (from the region Marche, east-center part of Italy) called Lacrima di Morro d'Alba DOC, vintage 1996. In this case, we have a good match. Sommeliers simply say "the matching is harmonic", even if the polygons are not completely overlapping. The reason is that it is really difficult to find the *complete overlap*. When the overlap is the best possible, as in this case, sommelier concludes with the word "harmonic" matching.

The corresponding diagram and the polygons after reflection and roto-translation are in Figure 7.

A final comment concerns the so-called optimal rotation angle when roto-translation has been applied. In all examples, the optimal angle is π or nearly π radians. This should not be too surprising, since the polygons are constructed on (characteristics) lines which differ of π radians.

We hope to have given an idea of the process of matching food and wine. We invite all readers to try and enjoy ... without getting drunk!!!.

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Received: January, 2012

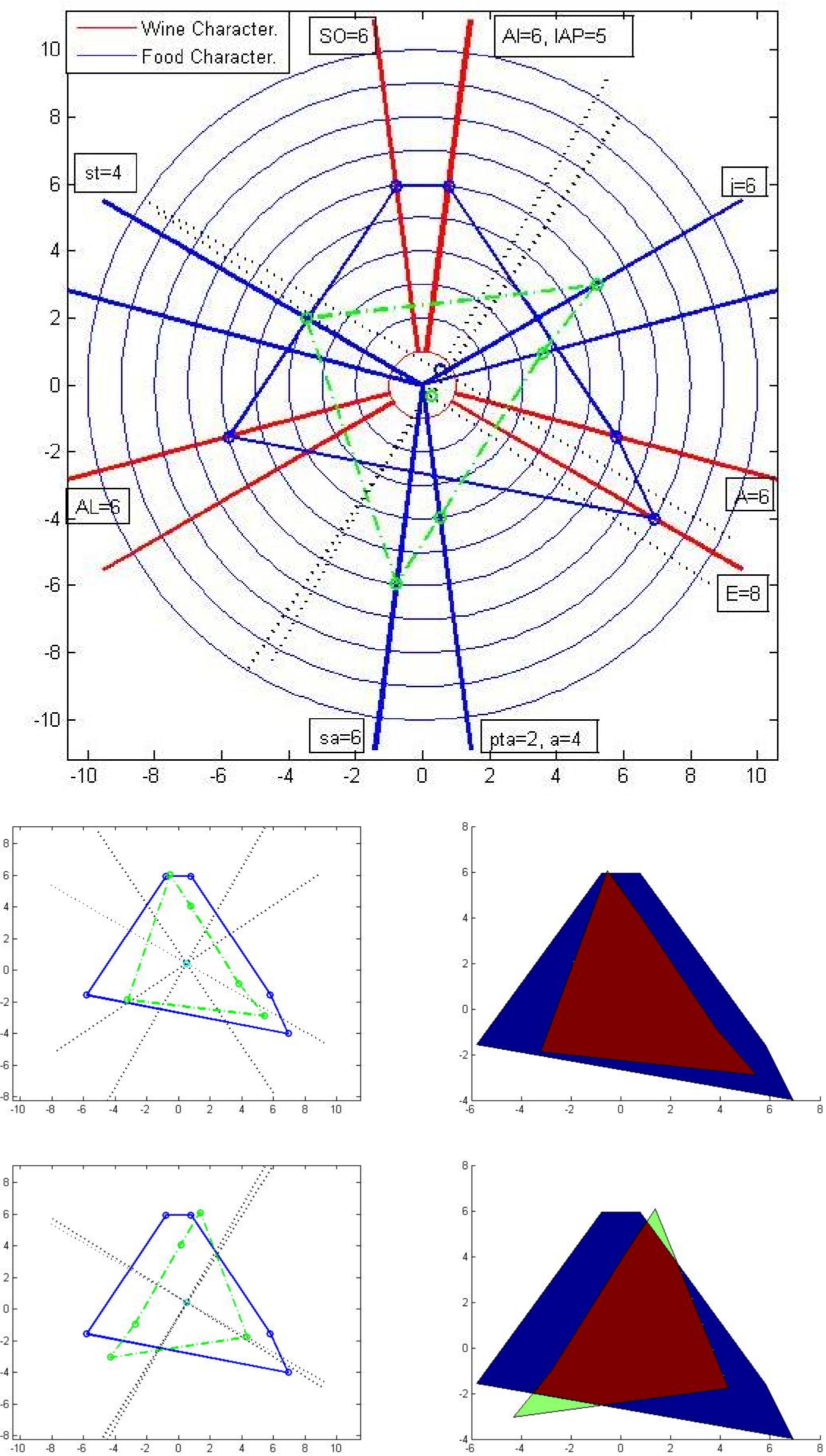

Figure 5: A matching of a oyster with a glass of sparkling wine, Franciacorta Brut DOCG (Italy). First row: the diagram. Second row: the polygons after reflection. Third row: the polygons after 'optimal' roto-translation.

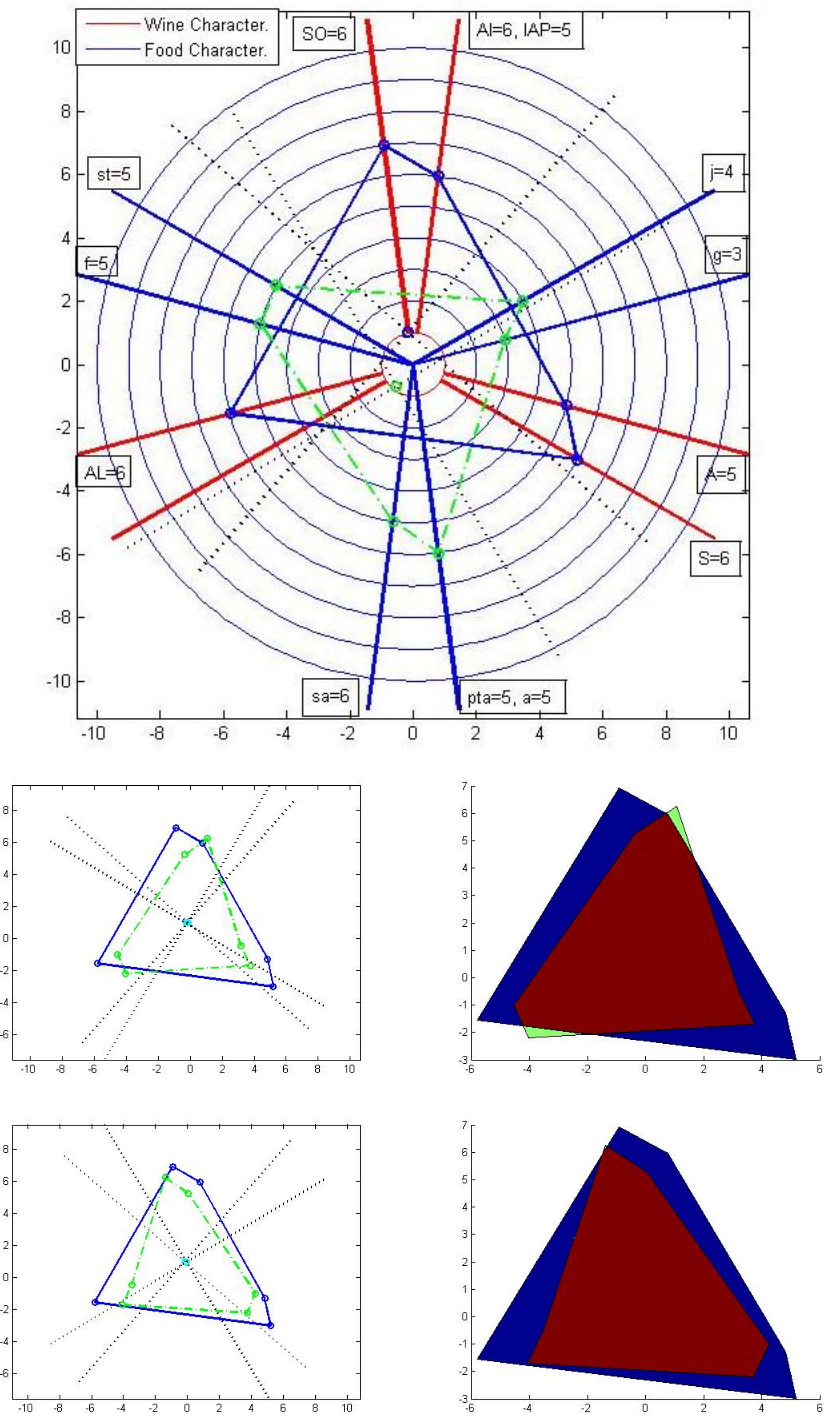

Figure 6: A matching of potato and onion pudding with a Chardonnay 1995, Colli Orientali del Friuli DOC. First row: the diagram. Second row: the polygons after reflection. Third row: the polygons after 'optimal' roto-translation.

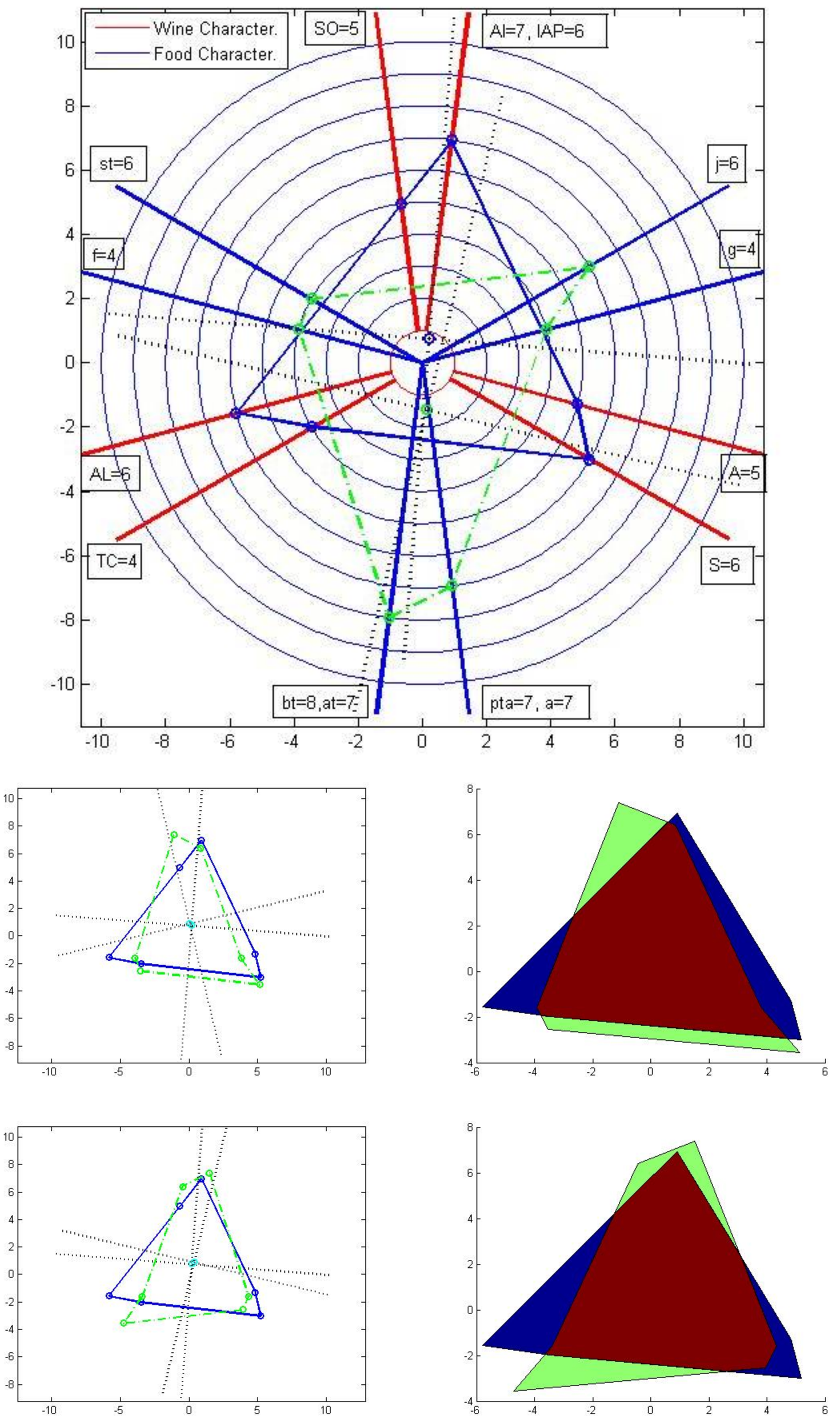

Figure 7: A matching of a hering smoked fillet with onions, and the red wine Lacrima di Morro d'Alba DOC. First row: the diagram. Second row: the polygons after reflection. Third row: the polygons after 'optimal' roto-translation.