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# Comparative analysis of random utility models and fuzzy logic models for representing gap-acceptance behavior using data from driving simulator experiments

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## Abstract

The paper proposes a comparative analysis of random utility models and fuzzy logic models for representing gap-acceptance behavior at priority intersections, based on data collected from driving simulator tests. Explanatory variables not detectable from on site observations were observed in the experiments. The proposed models include driving styles variables in addition to variables commonly used in gap-acceptance studies. The driving tests have been conducted using STSoftware® fixed-base driving simulator. The comparison between the two kinds of models, performed using the Receiver Operating Characteristic (ROC) curve analysis, indicates that fuzzy models can be considered an alternative to the use of random utility models. Furthermore the ability of driving simulators to provide data not detectable from direct observations is highlighted.

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*Keywords:* driving simulator, gap-acceptance, logit model, fuzzy theory

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## 1. Introduction

In studies of vehicular gap-acceptance behavior, the choice to accept or reject a gap of a certain size is generally considered the result of a driver decision process which includes, as inputs, subjective estimates of a set of explanatory variables, given specific objective factors. These subjective evaluations are usually affected by a high degree of uncertainty, which can be properly treated both by classical probabilistic models, e.g. Logit [1-3] and by fuzzy system theory [4,5].

Calibration and validation of these models are usually based on gap-acceptance data collected at real intersections, generally using observations based on video survey. Objective and subjective variables were found

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to have relevant effects on gap-acceptance behavior [6,7]; however, some of these variables are not detectable from direct observations (e.g. driver's education level, employment status, income, driving styles, etc.). In recent years it has been found that driving simulators can provide reliable observations of drivers' behavior [8, 9]. Based on the findings of Rossi et al. [10], the driving simulator capability to represent real situations has been demonstrated specifically for the case of gap-acceptance behavior.

In this study driving simulator experiments have been designed with the aim to measure the effects of some of these variables. Based on these data, models which include driving styles variables and other variables commonly used in gap-acceptance studies (time interval size, driver's gender) have been estimated and validated. The comparative analysis between these two kinds of models has been carried out using the so-called ROC (Receiver Operating Characteristic) curve analysis.

This work is an extension of previous studies concerning gap-acceptance behavior at priority intersections conducted by the Transportation Laboratory of Padova University [4,5;10-13].

The paper is organized as follows. Section 2 is dedicated to a brief description of the laboratory experimental design. Section 3 describes the estimation of the proposed logit model, section 4 describes the identification of the proposed fuzzy model. Section 5 deals with the comparative ROC curve analysis of the two types of gap-acceptance models. Concluding remarks are presented in Section 6.

## 2. Methodology

In this and in the following sections, we describe in detail the case-study in order to illustrate the collected data and the methods by which we derived information from the data. The analysis has been developed following three steps:

- direct observation, collection and coding of data at a real intersection (field observations);
- development of the virtual scenario using the STSoftware® driving simulator and execution of driving tests;
- estimation of logit and fuzzy gap-acceptance models using simulator data.

### 2.1. Characteristics of the analyzed intersection

The data used in the analysis are gap-acceptance observations (driver decisions) collected from driving simulator experiments, in which the virtual environment has been built with the aim to reproduce a real three-leg priority intersection located in a sub-urban area near Venice (Fig. 1). A high level of detail in the three-dimensional representation of the real context has allowed to create a realistic virtual environment.

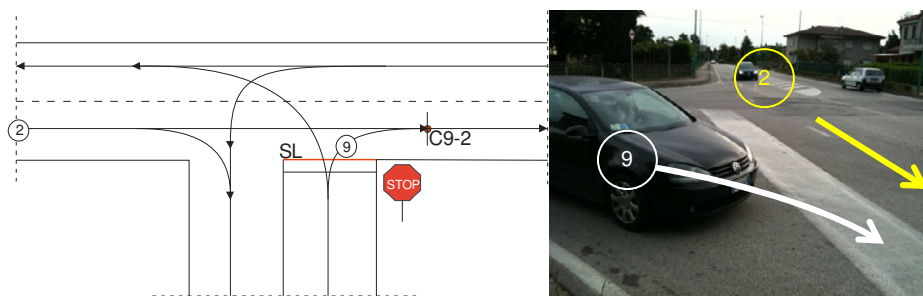


Fig. 1 (a) layout and (b) picture of the real three-leg priority intersection

#### 2.1.1. Driving simulator experiment design

Our experiment explored how subjects (making a right turn maneuver from the minor street) select gaps presented in the same order as observed in the real situation. The driving experiments have been conducted at the

Transportation Laboratory of the Department of Civil, Environmental and Architectural Engineering (University of Padova). The simulation system used is a fixed-base driving simulator produced by STSoftware®. It includes a realistic cockpit, three networked computers, five high definition screens; the system is also equipped with a Dolby Surround® sound system. This simulator configuration allows to produce realistic virtual views of the road network and of the surrounding context (Fig. 2).



Fig. 2. University of Padova Driving Simulator (STSoftware® driving simulator)

In the following sections a description of the experiments tasks is presented.

## 2.2. Characteristics of the sample of participants

The sample of participants was composed by 39 drivers, approximately 80% males and 20% females. Drivers were students, staff of the University and other people having the following characteristics:

- absence of previous experiences with driving simulators;
- at least 3 years of real driving experience;
- average annual driven distance of at least 5,000 km.

A summary of test drivers' characteristics is presented in Table 1.

Table 1. Test drivers' characteristics: age and driving experience

	Mean	Standard Deviation	Range
Age	26.60	3.06	21-36
Years of driving	8.20	3.01	3-17
Km driven per year	11,000	7,700	8,000-40,000

### 2.2.1. Virtual scenarios

To correctly reproduce real traffic flow conditions, field observations were collected during peak-hour periods (7.00-9.00 a.m.) through video camera recorder. The videos were processed using an application software that allows the user to record the primary vehicle arrival time at the conflict point (C9-2 in Fig. 1) together with the vehicle category (e.g. car, van, truck). The data were organized in a database and then processed using a software procedure that extracts the following gap-acceptance information: time interval size and category of major street vehicle closing the interval.

To enhance the similarity of the virtual scenario with the real one, the sequence of vehicle arrivals observed in the field has been reproduced at a point placed 500 meters upstream of the intersection (generation point). The three-dimension rendering software created the vehicle stream at the generation point out of the view of the test

driver, and removed them from the simulation at the destination point, which was placed downstream of the intersection sufficiently far away, so that drivers were not able to see vehicles at the time when they were removed from the simulation.

Primary road speed (for both directions) was constant at 50 km/hr; daytime and good weather conditions, which allow good visibility, were adopted in this scenario.

### 2.2.2. Experiment description

The experiments performed dealt with a relatively simple situation: vehicles on the main road traveled at constant speed, so the gaps between vehicles remained constant once vehicles were created and drivers had to drive adopting their usual behavior, making a right turn maneuver from the minor street if considered safe.

During each test the same driver approached the intersection about 10 times (10 runs) and each driver made at least 4 tests during the experiment. Before starting the test, drivers were subjected to a learning task in the simulator to make them familiar with it.

Each driver involved in the experiment responded to a questionnaire which collects socioeconomic information, such as age, gender, marital status, education, income and driving experience (years of driving, kilometers driven per year). In addition to personal information, the questionnaire included the multidimensional driving style inventory (MDSI) developed by Taubman-Ben-Ari et al. [14].

The MDSI is able to characterize four domains of driving style:

- reckless and careless driving, which refers to deliberate violations of safe driving norms, and the seeking of sensations and thrill while driving;
- anxious driving, which reflects feelings of alertness and tension as well as ineffective engagement in relaxing activities during driving;
- angry and hostile driving, which refers to expressions of irritation, rage, and hostile attitudes and acts while driving, and reflects a tendency to act aggressively on the road, curse, blow horn, or “flash” to other drivers;
- patient and careful driving, which reflects a well-adjusted driving style, and refers to planning ahead, attention, patience, politeness, keeping calm while driving as well as obeying traffic rules.

Essentially, each experiment was divided in four phases:

- driving task on a rural road (10 minutes training)
- rest (5 minutes)
- driving task consisting of right turn maneuver at a priority three-leg intersection
- interview of the participant to collect information about driver and driving task, including physiological reaction during and after the experiment (simulator sickness), other difficulties during the experiments, social and economic characteristics of driver, judgment about the simulation subject’s driving style recognition.

### 2.2.3. Data collected during the experiment

All observations relate to the right turn movement from a minor street controlled by a “yield” sign.

Many parameters characterizing driver behavior provided by the simulator were recorded, such as:

- global position, velocity, acceleration of all vehicles involved (frequency 10 Hz);
- cabin parameters (frequency 10 Hz);
- major-stream vehicles arrival time at the conflict point;
- minor-stream vehicle (test driver) arrival time at the stop line;
- minor-stream vehicle departure time from the stop line (right turn maneuver).

For the purpose of this study, only major stream vehicles arrival time at the conflict point and test driver’s arrival and departure time at the stop line were considered. The data were organized in a database and then processed using a software procedure that allows to extract gap-acceptance information for each driver.

For more detail about the experiment see Rossi et al. [12].

The dataset obtained from the collected data contained a total of 4,384 decisions (gap/lag acceptances and rejections), where 1,914 gaps/lags correspond to acceptances (right turn maneuver completed). The average number of decisions per drivers' approach to the minor street stop line was 2.29 (during a test the same driver approached the intersection about 10 times and each driver made at least four tests during the experiment). A summary of the data collected during the experiment is shown in Table 2.

Table 2. Driving simulator gap-acceptance data. Sample of test drivers' observed decisions

Maneuver	Type of interval	Acceptances	Rejections	Total	Average number of decisions per driver's approach
Right turn from minor street	gap	1,088	1,382	2,470	2.29
	lag	826	1,088	1,914	
	total	1,914	2,470	4,384	

With reference to MDSI, a summary of measures (based on a six level scale of assessment) of test drivers' driving style is presented in Table 3.

Table 3. Sample of drivers' driving style characteristics (MDSI classification)

Gender	Nr.	Reckless/Careless		Anxious		Angry/Hostile		Patient/Careful	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
Male	31	2.32	0.55	2.03	0.41	1.99	0.46	4.46	0.46
Female	8	2.13	0.52	2.34	0.70	1.85	0.45	4.24	0.49
Total	39	2.28	0.54	2.10	0.49	1.96	0.45	4.41	0.47

The identification/estimation of both Fuzzy and Logit models has been carried out using the stratified holdout approach [15]. The full dataset has been divided in a calibration dataset (70% of data) and a validation dataset (30% of data), to identify/calibrate the models and evaluate their performances, respectively. This procedure allows to measure correctly the predictive capabilities of the models, because it is well known that using the same data for estimation and validation could lead to an optimistic evaluation of model performance. Data were randomly sampled from the full dataset, maintaining approximately the same proportion of output classes (acceptance and rejection), type of intervals (gap and lag), and gender of drivers (male and female).

### 3. Logit model calibration

Several Logit models of gap-acceptance behavior were specified and estimated in this study using the HieLoW® program; in which, as expected, the acceptance probability increases with the interval size IS. Gender has an influence in the sense that male drivers seem to accept smaller intervals than female drivers. Also driving styles affect the probability of acceptance: if the driver anxiety increases, the probability of acceptance decreases and if the angry component increases, the probability of acceptance increases. Some examples of these gap-acceptance behaviors are reported in Fig. 3, separately for male and female drivers.

Table 4 shows the results corresponding to the best one, indicated as  $GA_L$ .

The  $GA_L$  model includes, as explanatory variables, the size IS of the time interval (gap or lag), the gender of the driver (represented by a dummy G, which takes the value of one in the case of a male and zero in the case of a female driver), the driver's values of angry/hostile (H) and anxious (A) driving styles, as derived from the

questionnaire. The other driving styles (reckless/careless and patient/careful) are not included in the model because they are not significant.

The estimated  $GA_L$  model has the following expression:

$$P_{acceptance} = 1 / (1 + \exp[-(-7.55 + 1.81 IS + 0.59 G + 0.78 H - 1.16A)]) \tag{1}$$

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Table 4.  $GA_L$  model calibration results: goodness-of-fit indicators, parameter estimates and corresponding Student’s t-statistics (within brackets)

Model	Rho-square	Corrected Rho-square	Percent Right
$GA_L$	0.796	0.794	93.84%

Alternative specific constant (acceptance)	-7.55 (-11.83)
Time Interval Size	+1.81 (21.76)
Gender	+0.59 (2.28)
Angry/Hostile driving	+0.78 (3.11)
Anxious driving	-1.16 (-4.74)

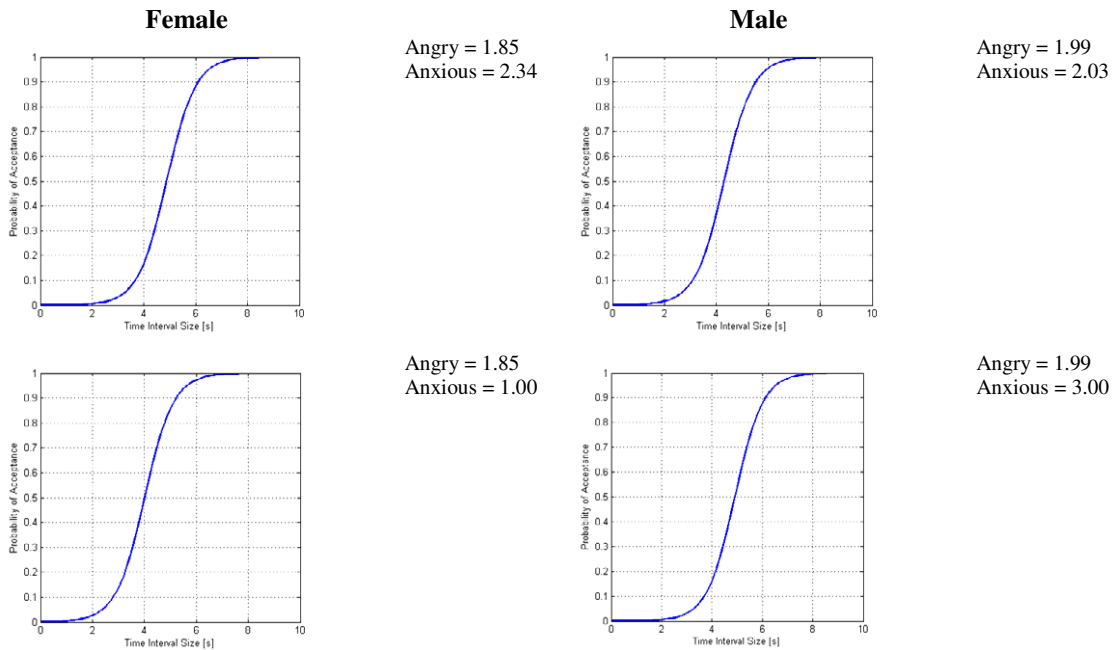


Fig. 3.  $GA_L$  model. Acceptance curves as a function of “Time Interval Size” for different values of driving styles for male and female drivers

#### 4. Fuzzy model identification

Starting from the consideration that the time interval size between vehicles on the primary street is the most important factor affecting gap-acceptance behavior (as widely reported in literature), and considering that drivers evaluate this variable in subjective terms, in this work we consider time interval as a fuzzy variable; other fuzzy variables are driver’s angry/hostile and anxious driving styles. The gender of the driver is an objective factor and therefore it has been treated as a crisp variable in the model.

The fuzzy gap-acceptance model ( $GA_F$ ) has been determined from experimental data using FisPro, an open-source software available for free on the Internet [16]. The membership functions of the premise and consequence fuzzy sets are identified based on K-means algorithm results, and the rules of inference with the so-called FPA (Fast Prototype Algorithm [17]).

The fuzzy system knowledge base so obtained was characterized by five triangular fuzzy sets in the domain of the time interval size, by six triangular fuzzy sets in the domain of the variables “angry/hostile” and “anxious” driving styles and by two “singletons” in the domain of the crisp variable “gender” (Fig. 4).

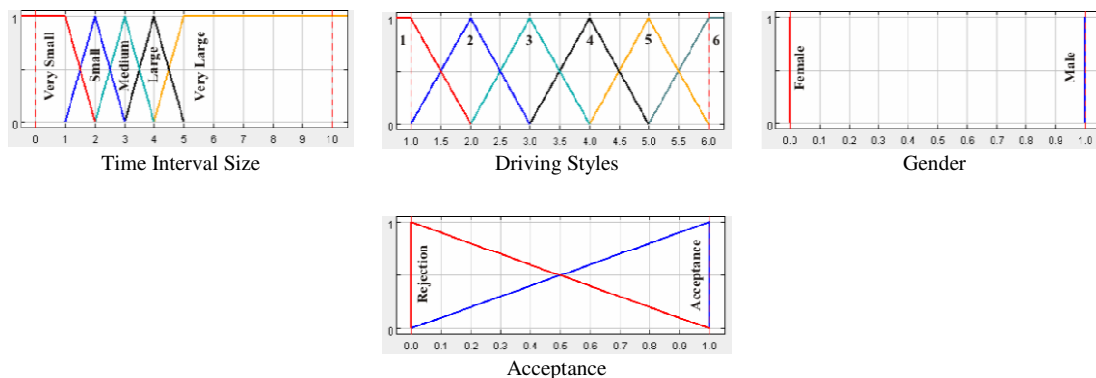


Fig. 4.  $GA_F$  model. Premises and consequence fuzzy sets

Fuzzy sets used for “angry/hostile” and “anxious” variables have been chosen to represent the verbal scale used by the drivers to judge their own driving style, maintaining the same level of aggregation. Unfortunately, the characteristics of driving styles observed for the sample drivers are not representative of all the available combinations (Table 3), but they are concentrated on a limited portion of the domain. As a result the rules could not be developed from data for some combinations of the input variables, in particular for high values of driving style variables.

Sixty rules have been identified using Mamdani’s product-sum inference. A satisfactory value of goodness-of-fit has been obtained ( $R^2=0.80$ ). As an example two rules are:

*If driver is Female And Anxious is 1 And Angry is 2 And Interval Size is Large Then Acceptance*  
*If driver is Male And Anxious is 1 And Angry is 1 And Interval Size is Very Small Then Refusal*

The fuzzy output variable “acceptance” is defuzzified using the centroid method [18] obtaining an “acceptance index” of a certain gap/lag. Using the “acceptance index”, it is possible to build “acceptance curves” that allow to use the model as predictive tool (and to validate it over the validation sample). When a gap/lag of a certain size

has an acceptance index greater than or equal to the 0.5 threshold, it is considered “acceptable”, otherwise it is considered “unacceptable”.

From the acceptance curves provided in Fig. 5, some trends regarding the relationships among the changes in driving styles and the effects on gap-acceptance behavior are shown. For female drivers a reduction in the anxiety value (compared to the mean value) produces a reduction of the size of the minimum accepted intervals. For male drivers, an increase of the anxiety produces an increase in the size of minimum accepted intervals. These trends are similar to the results obtained with the  $GA_L$  model.

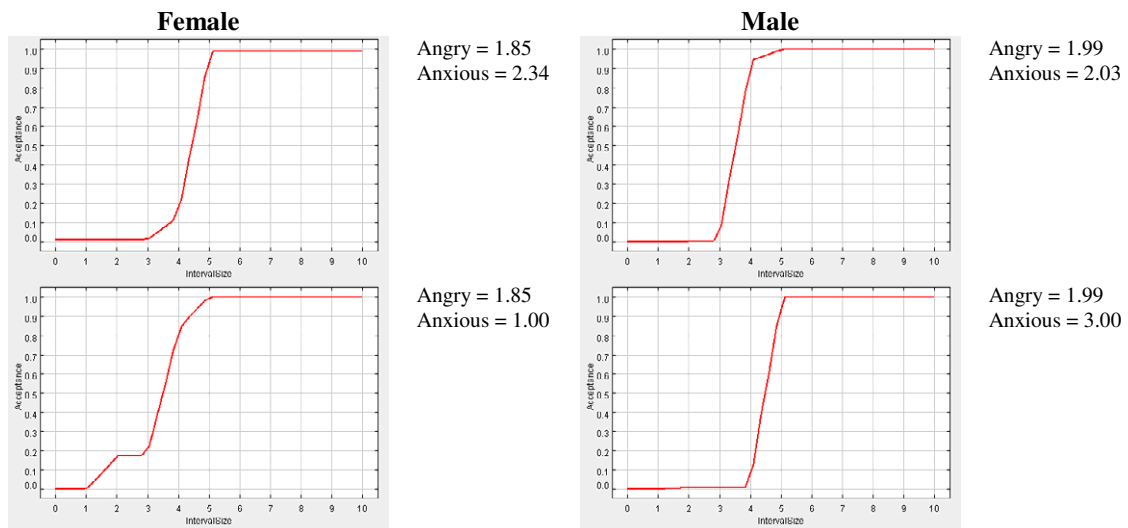


Fig. 5.  $GA_F$  model. Acceptance curves as a function of “Time Interval Size” for different values of driving styles for male and female drivers

## 5. Comparison of the models

The predictive ability of the two models has been tested by means of the ROC curve analysis [19], a method used in various research fields for evaluating and comparing the discriminatory power of models having binary outputs [20, 21], including Logit and Fuzzy models [22]. Few examples are found in the transportation case [23, 11].

The basic idea of ROC curve analysis may be explained by considering an experiment with only two possible outcomes, 1 and 0, that are denoted as positive and negative outcomes. In the  $GA_L$  and  $GA_F$  models the two outcomes are the acceptance (positive) and the rejection (negative) of a certain gap/lag, therefore four cases are possible:

- True Positive (TP): the model predicts an acceptance and the driver accepted a gap/lag of a certain size;
- False Positive (FP): the model predicts an acceptance and the driver rejected a gap/lag of a certain size;
- True Negative (TN): the model predicts a rejection and the driver rejected a gap/lag of a certain size;
- False Negative (FN): the model predicts a rejection and the driver accepted a gap/lag of a certain size.

The probability of correctly identifying positive outcomes is the True Positive Rate (TPR), and the probability of correctly identifying negative outcomes is the True Negative Rate (TNR). They are calculated by:

- TPR (True Positive Rate) = number of TP/(number of TP + number of FN)
- TNR (True Negative Rate) = number of TN/(number of TN + number of FP)

Another metric commonly used is the False Positive Rate (FPR), which is calculated by:



FPR (False Positive Rate) =  $1 - \text{TNR} = \text{number of FP} / (\text{number of TN} + \text{number of FP})$

The discriminatory power of the models increases as both TPR and TNR increase. The ROC curve describes the relationship between TPR, also called “sensitivity”, and  $(1 - \text{TNR})$ , also called “1-specificity”, for all possible classification thresholds. Since the “1-specificity” is the FPR, the ROC curve describes the relationship between the “percentage of hits” and the “percentage of false alarms” obtained with the model.

It is known that the area under the ROC curve (AUC) is related to the accuracy of the model predictions, and increases with it; in particular, when this area is equal to one the model produces perfect forecasts, and when it is equal to 0.5 the model produces random forecasts (no discriminatory power). The AUC is equivalent to the Gini coefficient =  $2 * \text{AUC} - 1$ , and also to the Mann–Whitney–Wilcoxon two-independent sample non-parametric test statistic [24]. Additional performance metrics adopted are precision metric, that represents the percentage of correct acceptance predictions, the F-measure, that is the harmonic average of Precision and PCA; and the percent right (or accuracy), that is the percentage of correct predictions globally made.

The analysis of ROC curve is useful in those cases in which the best cut-off value of the variable of interest is not known a priori by the researcher. In the gap-acceptance case the best cut-off value is expected to be in correspondence of an acceptance probability equal to 0.5 for the  $\text{GA}_L$  model and a value of the acceptance index equal to 0.5 for the  $\text{GA}_F$  model. The results confirm these assumptions, as shown in Fig. 6. The solid line represents the  $\text{GA}_L$  model, while the dashed line the  $\text{GA}_F$  model.

Considering the entire range of threshold values,  $\text{GA}_L$  slightly outperforms  $\text{GA}_F$ , as revealed by the comparison of the AUC metric (Table 5). Nevertheless the cut-off value which maximizes the performances of both models is 0.5, in correspondence of which the two lines are indistinguishable, as can be seen in the squared box of Fig. 6.

Also the other performance metrics obtained by both models in correspondence of the cut-off value of 0.5 (Table 5) are similar, and the two models are equivalent in terms of accuracy.

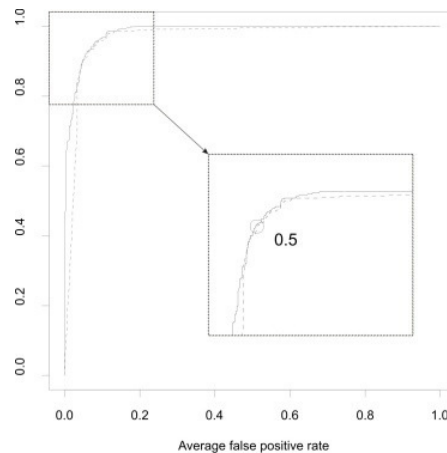


Fig. 6. ROC Curves for  $\text{GA}_L$  and  $\text{GA}_F$  models. Detail for the 0.5 threshold

Table 5.  $\text{GA}_L$  and  $\text{GA}_F$  models. Comparison of performances

Model	AUC	TPR	TNR	Precision	Percent Right	F-Measure	Youden Index
$\text{GA}_L$	0.983	0.920	0.942	0.925	93.2%	0.922	0.862
$\text{GA}_F$	0.967	0.956	0.900	0.881	92.5%	0.917	0.857

## 6. Concluding remarks

In this work data collected from laboratory experiments of driving behavior (questionnaire and driving simulator sessions) have been used to develop a fuzzy model and a logit model of gap-acceptance behavior at priority intersections. Laboratory experiments allowed to observe and record information about explanatory variables not detectable from direct observations (on site), and to include them in models with the aim to better describe, understand and simulate driver's choices. On the other hand the use of a fuzzy model allowed to overtake problems concerning non-homogeneous explanatory variables and uncertain and imprecise information on the system.

The proposed models have included driving styles variables (in particular "angry/hostile" and "anxious"), in addition to some variables commonly used in gap-acceptance studies (time interval size and driver's gender).

The results obtained indicate that:

- both logit and fuzzy models show good capability of representing real driver's gap acceptance behavior, but neither model definitely dominates the other;
  - the fuzzy model appears very simple and easy to generalize to other gap-acceptance situations (changing inference rules or shape and domain of the membership functions);
  - with reference to commonly used variables, the descriptive capability of the models appears substantially coherent with previous results reported in the literature;
- Nevertheless, there are some directions in which this work could be extended:
- identification of fuzzy models and calibration of Logit models with reference to other gap-acceptance situations, such as left turn maneuver from minor street in priority intersections or right turn from minor in roundabouts;
  - intersection capacity analysis using gap-acceptance models in micro-simulation, also testing computation efficiency of fuzzy and logit models;
  - analysis of other factors that could affect gap-acceptance behavior (speed and type of approaching vehicles on the main road, driver's education level, employment status, income, past involvement in car accidents, fatigue, etc.);
  - extension of the sample size (number and stratification) in order to better represent the population of drivers and their driving styles;
  - sensitivity analysis of model results (acceptance probability for logit models and acceptance index for fuzzy models) with respect to model parameters;
  - dynamic calibration of model parameters, to allow model results to reflect "in real time" spatial and temporal variations of driver behavior (for example, the tendency of drivers to accept smaller gaps with increasing waiting times). This aspect, which was not considered in this study, appears to be particularly important for a realistic representation of gap-acceptance behavior within traffic micro-simulation models.

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