Grand Unification of Quark and Lepton Flavor Changing Neutral Currents

M. Ciuchini,[1] A. Masiero,[2] L. Silvestrini,[3] S. K. Vempati,[2] and O. Vives[4]

[1]*INFN, Sezione di Roma III and Dipartimento di Fisica, Università di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy*
[2]*Dipartimento di Fisica "G. Galilei," Università di Padova and INFN, Sezione di Padova, Via Marzolo 8, I-35131, Padua, Italy*
[3]*INFN, Sezione di Roma and Dipartimento di Fisica, Università di Roma "La Sapienza," Piazzale Aldo Moro 2, I-00185 Rome, Italy*
[4]*Department of Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, United Kingdom*
(Received 22 July 2003; published 17 February 2004)

In the context of supersymmetric grand unified theories with soft breaking terms arising at the Planck scale, it is generally possible to link flavor changing neutral current and *CP* violating processes occurring in the leptonic and hadronic sectors. We study the correlation between flavor changing squark and slepton mass insertions in models à la SU(5). We show that the constraints coming from lepton flavor violation exhibit a strong impact on *CP*-violating *B* decays.

DOI: 10.1103/PhysRevLett.92.071801 PACS numbers: 12.10.Dm, 11.30.Er, 11.30.Hv, 12.60.Jv

The phenomenology of low-energy supersymmetric (SUSY) extensions of the standard model (SM) crucially depends on the mechanism responsible for breaking SUSY itself, namely, on the structure of the SUSY soft breaking terms. It is possible to explore these terms by studying the production and detection of SUSY particles. In addition, one can shed light on these terms by analyzing the SUSY effects on rare flavor changing neutral current (FCNC) and *CP* violating processes. Obviously for this latter approach the flavor structure of the soft breaking terms is of utmost importance. Theoretically, we can distinguish two opposite situations. First, we can have SUSY breaking mechanisms which are completely insensitive to flavor (for instance, pure dilaton breaking in supergravities, gauge or anomaly mediated SUSY breaking). Alternatively, SUSY breaking can feel the flavor structure leading to soft breaking terms which are not flavor universal (this is what happens in supergravities which have also moduli breaking). In spite of the original flavor blindness of certain SUSY breaking mechanisms, we can still have large flavor nonuniversalities in the low-energy SUSY breaking sector. This occurs if the universal soft terms appear at some energy scale much larger than the electroweak scale in the presence of new flavor structures (for instance, in grand unified models or in the presence of new Yukawa couplings as in the seesaw model). In the running between these two scales, the soft breaking terms can "pick up" nonuniversal contributions: soft scalar masses and scalar trilinear terms bear a "memory" of the high energy flavor structures through their renormalization group (RG) evolution. In conclusion, either that we start from SUSY breaking mechanisms which are not flavor blind or that we produce flavor nonuniversality through the RG scaling, we can envisage the general situation that soft SUSY breaking terms are flavor nonuniversal at the electroweak scale.

To analyze flavor-violating constraints at the electroweak scale, the model independent mass-insertion (MI) approximation is advantageous [1]. In this method, the experimental limits lead to upper bounds on the parameters (or combinations of) $\delta^f_{ij} \equiv \Delta^f_{ij}/m^2_{\tilde{f}}$, where Δ^f_{ij} is the flavor-violating off-diagonal entry appearing in the $f = (u, d, l)$ sfermion mass matrices and $m^2_{\tilde{f}}$ is the average sfermion mass. In addition, the mass insertions are further subdivided into $LL/LR/RL/RR$ types, labeled by the chirality of the corresponding SM fermions. Detailed bounds on the individual δs have been derived by considering limits from various FCNC processes [2]. As long as one remains within the simple picture of the minimal supersymmetric standard model (MSSM), where quarks and leptons are unrelated, the hadronic and leptonic FCNC processes yield bounds on corresponding δ^q's and δ^l's which are unrelated. The situation changes when one embeds the MSSM within a grand unified theory (GUT).

In this Letter, we study the correlation between the δ^q's and δ^l's in SUSY GUTs and show that lepton flavor-violating processes (as $\mu \to e\gamma$ or $\tau \to \mu\gamma$) can severely constrain the observability of SUSY contributions to hadronic FCNC processes and vice versa. In particular, we stress that we do not exploit the GUT model as a mechanism to generate the amount of flavor violation as has been done several times in the literature, but rather exploit the GUT-symmetric relations among the various flavor-violating terms to achieve such correlations.

In a SUSY GUT, quarks and leptons sit in the same multiplet. As long as the scale associated with the transmission of SUSY breaking to the visible sector is larger than the GUT breaking scale, the quark-lepton unification seeps also into the SUSY breaking soft sector leading to squark-slepton mass squared unification. The exact relations between the mass matrices depend on the choice of the GUT gauge group. As a consequence, the flavor off-diagonal entries Δ^f_{ij} in these mass matrices are also unified at the GUT scale. For instance, in SU(5) $(\Delta^d_{ij})_{RR}$ and $(\Delta^l_{ij})_{LL}$ are equal; in SO(10) all Δ_{ij} are equal at $M_{\rm GUT}$ implying strong correlations within FCNCs at that scale. What is going to happen of such strong correlations when we reach the electroweak scale? In a simple situation of

 0031-9007/04/92(7)/071801(4)$22.50

"big desert," i.e., no new particles or interactions between $M_{\rm GUT}$ and M_W, these relations are left essentially unaffected by the renormalization group equation (RGE) effects and thus the relations between Δ_{ij} can be cast in terms of δ_{ij} at the weak scale. The situation changes if some new particles with intermediate masses interact differently with quarks and leptons, for instance right-handed neutrinos in a seesaw mechanism. In this particular case, the leptonic Δ_{ij} receive new RGE contributions [3], breaking the exact hadron-lepton correlation at the electroweak scale. However, as we show below, it is still possible to extract useful information from these GUT correlations.

The connection between quark and lepton δ parameters can have significant implications on flavor phenomenology. Indeed, using these relations, a quark δ parameter can be probed in a leptonic process or vice versa. In this way, it is possible that constraints in one sector are converted to the other sector where previously only weaker or perhaps even no bounds existed. For example, we find that using the leptonic radiative decay process $l_j \to l_i \gamma$, it is possible to probe some squark δ parameters to a higher precision, unhindered by the hadronic uncertainties. On the other hand, some sleptonic δ parameters, which are unconstrained, can receive bounds from the quark sector.

To be specific, we concentrate on the SUSY SU(5) framework and derive all the relations between squark and sleptonic mass insertions. We then study the impact of the limit from $\tau \to \mu \gamma$ on the $b \to s$ transition observables, such as $A_{CP}(B \to \phi K_s)$.

The soft terms are assumed to be generated at some scale above $M_{\rm GUT}$. Note that even assuming complete universality of the soft breaking terms at $M_{\rm Planck}$, as in mSUGRA, the RG effects to $M_{\rm GUT}$ will induce flavor off-diagonal entries at the GUT scale [4]. Hence we assume generic flavor-violating entries to be present in the sfermion matrices at the GUT scale. The part of the superpotential relevant for quarks and charged lepton masses can be written as

$$W_{\rm SU(5)} = h^u_{ij} T_i T_j H + h^d_{ij} T_i \bar{F}_j \bar{H} + \mu H \bar{H}, \qquad (1)$$

where we have used the standard notation with T transforming as 10 and $\bar{F}$ as $\bar{5}$ under SU(5). The corresponding SU(5) invariant soft potential now has the form

$$-\mathcal{L}_{\rm soft} = m^2_{T_{ij}} \tilde{T}^\dagger_i \tilde{T}_j + m^2_{\bar{F}_{ij}} \tilde{\bar{F}}^\dagger_i \tilde{\bar{F}}_j + m^2_H H^\dagger H + m^2_{\bar{H}} \bar{H}^\dagger \bar{H} + A^u_{ij} T_i T_j H + A^d_{ij} T_i \bar{F}_j \bar{H} + B\mu H \bar{H}, \qquad (2)$$

where the same symbols are used for the scalar components of the superfields. Rewriting the above in terms of the standard model representations we have

$$-\mathcal{L}_{\rm soft} = m^2_{Q_{ij}} \tilde{Q}^\dagger_i \tilde{Q}_j + m^2_{u^c_{ij}} \tilde{u}^{c\star}{}_i \tilde{u}^c{}_j + m^2_{e^c_{ij}} \tilde{e}^{c\star}{}_i \tilde{e}^c{}_j + m^2_{d^c_{ij}} \tilde{d}^{c\star}{}_i \tilde{d}^c{}_j + m^2_{L_{ij}} \tilde{L}^\dagger_i \tilde{L}_j + m^2_{H_1} H^\dagger_1 H_1 + m^2_{H_2} H^\dagger_2 H_2 + A^u_{ij} \tilde{Q}_i \tilde{u}^c{}_j H_2 + A^d_{ij} \tilde{Q}_i \tilde{d}^c{}_j H_1 + A^e_{ij} \tilde{L}_i \tilde{e}^c{}_j H_1 + \ldots \qquad (3)$$

where

$$m^2_Q = m^2_{\tilde{e}^c} = m^2_{\tilde{u}^c} = m^2_T, \qquad m^2_{\tilde{d}^c} = m^2_L = m^2_{\bar{F}}, \qquad A^e_{ij} = A^d_{ji}. \qquad (4)$$

Equations (4) are matrices in flavor space. These equations lead to relations within the slepton and squark flavor-violating off-diagonal entries Δ_{ij}. These are

$$(\Delta^u_{ij})_{LL} = (\Delta^u_{ij})_{RR} = (\Delta^d_{ij})_{LL} = (\Delta^l_{ij})_{RR}, \qquad (5)$$

$$(\Delta^d_{ij})_{RR} = (\Delta^l_{ij})_{LL}, \qquad (\Delta^d_{ij})_{LR} = (\Delta^l_{ji})_{LR} = (\Delta^l_{ij})^\star_{RL}. \qquad (6)$$

The relations between Δ_{LL}'s and Δ_{RR}'s descend from the fermionic representations under SU(5), while the Δ_{LR}'s and Δ_{RL}'s relations also depend on the Higgs responsible for fermion masses. In principle, more than one Higgs can give mass to the fermions. Then the corresponding Clebsch-Gordan coefficients can appear before the relevant entries.

The relations are exact at $M_{\rm GUT}$, however, after SU(5) breaking, quarks and leptons suffer different renormalization effects, and thus they are altered at M_W. It is easy to see from the RG equations that off-diagonal elements in the squark mass matrices in the first two of Eqs. (5) are approximately not renormalized due to the smallness of Cabibbo-Kobayashi-Maskawa (CKM) mixing angles and that the sleptonic entries in them are left unchanged (in the absence of right-handed neutrinos). On the other hand, the last equation receives corrections due to the different nature of the RG scaling of the LR term (A parameter). This correction can be roughly approximated as proportional to the corresponding fermion masses. Taking this into consideration, we can now rewrite Eqs. (5) at the weak scale as

$$(\delta^d_{ij})_{RR} \approx \frac{m^2_L}{m^2_{d^c}} (\delta^l_{ij})_{LL}, \qquad (7)$$

$$(\delta^{u,d}_{ij})_{LL} \approx \frac{m^2_{e^c}}{m^2_Q} (\delta^l_{ij})_{RR}, \qquad (8)$$

$$(\delta^u_{ij})_{RR} \approx \frac{m^2_{e^c}}{m^2_{u^c}} (\delta^l_{ij})_{RR}, \qquad (9)$$

$$(\delta^d_{ij})_{LR} \approx \frac{m^2_{L_{\rm avg}}}{m^2_{Q_{\rm avg}}} \frac{m_b}{m_\tau} (\delta^l_{ij})^\star_{RL}, \qquad (10)$$

where $m^2_{L_{\rm avg}}$ ($m^2_{Q_{\rm avg}}$) are given by the geometric average of left- and right-handed slepton (down-squark) masses $\sqrt{m_L^2 m_{e^c}^2}$ ($\sqrt{m_Q^2 m_{d^c}^2}$), all of them defined at the weak scale.

To account for nonzero neutrino masses, the seesaw mechanism can be implemented within this model by adding singlet right-handed neutrinos. In their presence, additional couplings occur in Eqs. (1)–(5) at the high scale, corresponding to the Dirac and Majorana masses which affect the RG evolution of slepton matrices. To understand the effect of these new couplings, one can envisage two scenarios [5]: (a) small couplings and/or small mixing in the neutrino Dirac Yukawa matrix; (b) large couplings and large mixing in the neutrino sector. In case (a), the effect on slepton mass matrices due neutrino Dirac Yukawa couplings is very small and the above relations Eqs. (7)–(10) still hold. In case (b), however, large RG effects can significantly modify the slepton doublet flavor structure [3] while keeping the squark sector and right-handed charged slepton matrices essentially unmodified, thus breaking the GUT-symmetric relations. Even in this case, barring accidental cancellations among the mass insertions already present at $M_{\rm GUT}$ and the radiatively generated mass insertions between $M_{\rm GUT}$ and M_{ν_R}, there exists an upper bound on the down quark δ parameters of the form

$$|(\delta_{ij}^d)_{RR}| \leq \frac{m_L^2}{m_{d^c}^2} |(\delta_{ij}^l)_{LL}|, \qquad (11)$$

while Eqs. (8)–(10) remain still valid in this case.

The relations (8)–(11) predict links between lepton and quark flavor changing transitions at the weak scale. We summarize these links in Table I. For example, we see that $\mu \to e\gamma$ can be related to $K^0 - \bar{K}^0$ mixing and to $D^0 - \bar{D}^0$ mixing. Similarly, one can expect correlations between $\tau \to e\gamma$ and $B_d - \bar{B}_d$ mixing as well as between $\tau \to \mu\gamma$ and $b \to s$ transitions such as $B \to \phi K_s$.

To demonstrate the impact of these relations, let us assume that all the flavor diagonal sfermion masses are approximately universal at the GUT scale with $m_{\tilde{T}}^2 = m_{\tilde{F}}^2 = m_H^2 = m_{\bar{H}}^2 = m_0^2$ with flavor off-diagonal entries $m_{\tilde{f}}^2 = m_0^2 \mathbb{1} + \Delta_{ij}^f$, with $|\Delta_{ij}^f| \leq m_0^2$. Δ_{ij}^f can be present

TABLE I. Links among various transitions between up-type, down-type quarks and charged leptons. The corresponding δ's are shown.

Transitions (L)		Transitions (D)		Transitions (U)
$(\delta^l)_{RR}$		$(\delta^d)_{LL}$		$(\delta^u)_{LL,RR}$
$\mu \to e$	$\leftrightarrow$	$s \to d$	$\leftrightarrow$	$c \to u$
$\tau \to e$	$\leftrightarrow$	$b \to d$	$\leftrightarrow$	$t \to u$
$\tau \to \mu$	$\leftrightarrow$	$b \to s$	$\leftrightarrow$	$t \to c$
$(\delta^l)_{LL/LR/RL}$		$(\delta^d)_{RR/RL/LR}$		$\cdots$
$\mu \to e$	$\leftrightarrow$	$s \to d$	$\cdots$	$\cdots$
$\tau \to e$	$\leftrightarrow$	$b \to d$	$\cdots$	$\cdots$
$\tau \to \mu$	$\leftrightarrow$	$b \to s$	$\cdots$	$\cdots$

either through the running from the Planck scale to the GUT scale [4] or through some flavor nonuniversality originally present [6]. All gaugino masses are unified to $M_{1/2}$ at $M_{\rm GUT}$. As mentioned above, the Δ parameters do not receive any significant corrections, whereas the diagonal entries are significantly modified (see, for instance, Tables I and IV in [7]). For a given set of initial conditions ($M_{1/2}$, m_0^2, A_0, Δ_{ij}, tanβ) we obtain the full spectrum at M_W with the requirement of radiative symmetry breaking. We then apply limits from direct searches on SUSY particles. Finally, we calculate the contributions of different δ_{23} parameters to both leptonic process and hadronic processes, considering the region in the (m_0, $M_{1/2}$) plane corresponding to a relatively light sparticle spectrum, with squark masses of roughly 350–550 GeV and slepton masses of about 150–400 GeV.

The $b \to s$ transitions have received recently much interest as it has been shown that the discrepancy with SM expectations in the measurements of $A_{CP}(B \to \phi K_s)$ can be attributed to the presence of large neutrino mixing within SO(10) models [8]. Subsequently, a detailed analysis has been presented [9,10] within the context of the MSSM. It has been shown that, for squark and gluino masses around 350 GeV, the presence of a $\mathcal{O}(1)$ $(\delta_{23}^d)_{LL,RR}$ could lead to significant discrepancies from the SM expectations. Similar statements hold for a $\mathcal{O}(10^{-2})$ LR or RL MI. In the present Letter, we study the impact of lepton flavor violation bounds on these δ parameters and subsequently the effect on B-physics observables. In Table II, we present upper bounds on δ_{23}^d within the above mass ranges for three values of the limits on $\mathcal{B}(\tau \to \mu\,\gamma)$. There are no bounds on $(\delta_{23}^d)_{LL}$ because, as is well known [11], large values of $(\delta_{ij}^l)_{RR}$ are still allowed due to the cancellations of bino and higgsino contributions in the decay amplitude.

At present, the constraints coming from B physics are stronger than those obtained for the lepton sector in the cases of $(\delta_{23}^d)_{LL,LR,RL}$. Therefore no impact on B phenomenology is expected even if the present bound on $\mathcal{B}(\tau \to \mu\,\gamma)$ were pushed down to 1×10^{-7}. On the contrary, the bound on $(\delta_{23}^d)_{RR}$ induced by $\mathcal{B}(\tau \to \mu\,\gamma)$ is already at present much stronger than the bounds from hadronic processes, reducing considerably the room left for SUSY effects in B decays. To illustrate this point in detail, we repeat the analysis of Ref. [9] including the bounds coming from lepton processes. We therefore compute at the NLO branching ratios and CP asymmetries for

TABLE II. Bounds on (δ_{23}^d) from $\mathcal{B}(\tau \to \mu\,\gamma)$ for three different values of the branching ratios for tan $\beta = 10$.

Type	$<6 \times 10^{-7}$	$<1 \cdot 10^{-7}$	$<1 \times 10^{-8}$
LL	$\cdots$	$\cdots$	$\cdots$
RR	0.070	0.030	0.010
RL	0.080	0.035	0.010
LR	0.080	0.035	0.010

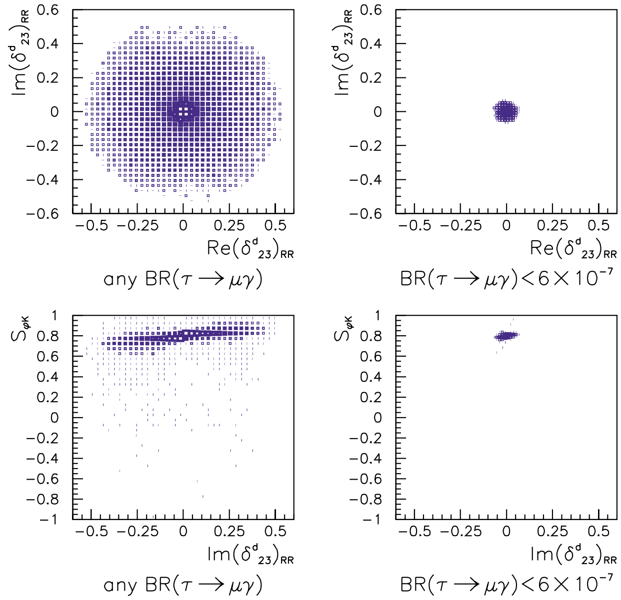

FIG. 1 (color online). Allowed regions in the $\mathrm{Re}(\delta^d_{23})_{RR}$–$\mathrm{Im}(\delta^d_{23})_{RR}$ plane (top) and in the $S_{K\phi}$–$\mathrm{Im}(\delta^d_{23})_{RR}$ plane (bottom). Constraints from $B \to X_s \gamma$, $\mathcal{B}(B \to X_s \ell^+ \ell^-)$, and the lower bound on ΔM_s have been used.

$B \to X_s \gamma$ and $B \to \phi K_s$, $\mathcal{B}(B \to X_s \ell^+ \ell^-)$ and ΔM_s (see Ref. [9] for details). In the first row of Fig. 1, we plot the probability density in the $\mathrm{Re}(\delta^d_{23})_{RR}$–$\mathrm{Im}(\delta^d_{23})_{RR}$ plane for different upper bounds on $\mathcal{B}(\tau \to \mu \gamma)$. Note that making use of Eq. (7) with $|(\delta^l_{23})_{LL}| < 1$ implies $|(\delta^d_{23})_{RR}| \lesssim 0.5$ as the ratio $(m_L^2/m_{d^c}^2)$ varies roughly between (0.2–0.5) at the weak scale, for the chosen high scale boundary conditions. The effect on $(\delta^d_{23})_{RR}$ of the upper bound on $\mathcal{B}(\tau \to \mu \gamma)$ is dramatic already with the present experimental value. Correspondingly, as can be seen from the second row of Fig. 1, the possibility of large deviations from the SM in the coefficient $S_{\phi K}$ of the sine term in the time-dependent $A_{CP}(B \to \phi K_s)$ is excluded in the RR case. Hence, we conclude that in SUSY GUTs the most likely possibility to strongly depart from the SM expectations for $S_{\phi K}$ relies on a sizable contribution from $(\delta^d_{23})_{LL}$ or $(\delta^d_{23})_{LR,RL}$ as long as they are small enough to be within the severe limits imposed by $\mathcal{B}(B \to X_s \gamma)$ [9]. These results would not change significantly if one started with a SO(10) theory instead of a SU(5) theory. The relation in Eq. (7) would be still valid, however, with the additional constraint: $(\delta^d_{ij})_{RR} = m_Q^2/m_d^2(\delta^d_{ij})_{LL} = m_L^2/m_d^2(\delta^l_{ij})_{LL}$. The results of our analysis are therefore valid also for SO(10), although stronger correlations are generally expected.

Exploiting the grand unified structure of the theory, we can obtain similar bounds on other δ^d_{ij} parameters. For example, considering the first two generations, the bound on δ^d_{12} from $\mathcal{B}(\mu \to e \gamma)$ can in many cases compete with the bound from Δm_K [2]. Similar comparisons can be made for the δ^d_{13} from limits on $\mathcal{B}(\tau \to e \gamma)$ and $B^0_d - \bar{B}^0_d$ mixing. A detailed analysis of δ^d_{12} and δ^d_{13} will be presented elsewhere.

In summary, supersymmetric grand unification predicts links between various leptonic and hadronic FCNC observables. Alhough such links have recently been observed in the literature within the context of the seesaw mechanism [12], we would like to stress that the focus of this Letter is quite different. In particular, we do not rely on any GUT-dependent origin of flavor violation but rather study the consequences of any flavor-violating soft terms already present at the GUT scale. Second, seesaw induced effects are strongly dependent on the neutrino Yukawa couplings (see Hisano-Shimizu in Ref. [12]) while our analysis is always applicable independently of this. Thus, to our knowledge, it is the first time that relations between hadronic and leptonic mass insertions have been presented in a general manner. Though such relations can be constructed for any GUT group, we have concentrated on SU(5) and quantitatively studied the implications for transitions between the second and third generations. We have shown that the present limit on $\mathcal{B}(\tau \to \mu \gamma)$ significantly constrains the observability of SUSY in CP violating B decays.

We acknowledge support from RTN EU Project No. HPRN-CT-2000-0148. O.V. acknowledges partial support from the Spanish MCYT FPA2002-00612 and thanks J. March-Russell and G. G. Ross for helpful discussions.

[1] L. J. Hall *et al.*, Nucl. Phys. B **267**, 415 (1986); F. Gabbiani and A. Masiero, *ibid.* **B322**, 235 (1989); J. S. Hagelin *et al.*, *ibid.* **B415**, 293 (1994).
[2] F. Gabbiani *et al.*, Nucl. Phys. **B477**, 321 (1996).
[3] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986); G. K. Leontaris *et al.*, Phys. Lett. B **171**, 412 (1986).
[4] R. Barbieri *et al.*, Nucl. Phys. **B445**, 219 (1995).
[5] A. Masiero *et al.*, Nucl. Phys. **B649**, 189 (2003).
[6] S. K. Soni and H. A. Weldon, Phys. Lett. B **126**, 215 (1983); V. S. Kaplunovsky and J. Louis, *ibid.* **306**, 269 (1993); G. G. Ross and O. Vives, Phys. Rev. D **67**, 095013 (2003).
[7] A. Bartl, *et al.* Phys. Rev. D **64**, 076009 (2001).
[8] T. Moroi, Phys. Lett. B **493**, 366 (2000); N. Akama *et al.*, Phys. Rev. D **64**, 095012 (2001); D. Chang *et al.*, Phys. Rev. D **67**, 075013 (2003).
[9] M. Ciuchini *et al.*, Phys. Rev. D **67**, 075016 (2003).
[10] R. Harnik *et al.*, hep-ph/0212180; S. Khalil and E. Kou, Phys. Rev. D **67**, 055009 (2003); G. L. Kane *et al.*, Phys. Rev. Lett. **90**, 141803 (2003); D. Chakraverty *et al.*, Phys. Rev. D **68**, 095004 (2003).
[11] J. Hisano *et al.*, Phys. Rev. D **53**, 2442 (1996); J. Hisano and D. Nomura, *ibid.* **59**, 116005 (1999); I. Masina and C. A. Savoy, Nucl. Phys. **B661**, 365 (2003).
[12] S. Baek *et al.*, Phys. Rev. D **63**, 051701 (2001); **64**, 095001 (2001); J. Hisano and Y. Shimizu, Phys. Lett. B **565**, 183 (2003); C. S. Huang *et al.*, Nucl. Phys. **B673**, 331 (2003).