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A Mathematical Solution for Food Thermal Process Design

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Abstract

A new mathematical procedure was developed to correlate g (the difference between the retort and the coldest point temperatures in canned food at the end of the heating process), f_h/U (the ratio of the heating rate index to the *sterilizing value*), z (the temperature change required for the thermal destruction curve to traverse one log cycle) and J_{cc} (the cooling lag factor). These are the four heat penetration parameters of 57 Stumbo's tables (18,513 datasets) in canned food. The quantities f_h/U, z and J_{cc} are input variables to determine the g values, which is used in Ball's formula to calculate the heating process time B at constant retort temperature. The new procedure was based on three equations; the first was obtained by the inversion of the function that expresses the *process lethality*, *F*, and hence the f_h/U parameter. However, the inversion was possible for a sub-domain of the function. The inverse function $g = g(f_h/U, z, J_{cc})$ was then extended to the entire domain ($10°C \le z \le 111°C$, $f_h/U \ge 0.3$ and $0.4 \le J_{cc} \le 2$) using two polynomials (second and third equation) obtained with articulated multiple regressions starting from the Stumbo's datasets. A comparison between the calculated value of g and desired Stumbo's values of g provided the following values: a determination coefficient R^2=0.9999, a mean relative error *MRE*=0.85±0.91% and a mean absolute error *MAE*=0.06°±0.09°C (0.11°±0.16°F). The results obtained by applying the mathematical procedure of this work, namely the g values using the three equations and the process time B using Ball's formula, closely followed the process time calculated from tabulated Stumbo's g values (root mean square of absolute errors *RMS*=0.393 min, average absolute error=0.259 min with a standard deviation *SD*=0.296 min). The high accuracy and simplicity of the procedure proposed here, make it useful in the development of mathematical algorithms for calculating and controlling, by computer, of food thermal processes. These algorithms replace the 57 look-up tables and 18,513 data sets needed in the Stumbo formula method. As such, this work offers a computerized formula method as an alternative to existing computerized numerical methods for this purpose.

Keywords: Mathematical modeling, Thermal process design, Canned food, Food engineering

1. Introduction

The canned food has a nutritional value similar to that of fresh or frozen food [1], provided that the thermal process (for example, sterilization) is preliminarily calculated in order to obtain maximum food safety with minimal damage to the organoleptic quality and nutritional value [2].

After Bigelow [3] found the two mathematical laws of the influence of temperature and time on the destruction of a microbial population, it was seen that the same laws could mathematically describe the alteration of the constituents (enzymes, proteins and vitamins) with only the foresight to assume different values of the kinetic parameters (D_T and z) compared to those of microbial thermal death.

Consequently it is preferable to speak generically of thermal processes and not of simple sterilization.

However, for the canned food, the two previous laws Bigelow on microbial destruction or alteration of the constituents, are insufficient for modeling and subsequent calculating the thermal processes. In fact, it is necessary to consider also the mathematical law that describes the penetration of the heat in the mass of canned food.

The first method, known as general method, was proposed by Bigelow [3]. This graphical method combined the two laws on microbial destruction with the experimental heat penetration curve, to calculate the optimum time (B) that allowed the attainment of a given sterilization.

In later times, various authors [4, 5, 6, 7, and 8] proposed contributions for the improvement of the general method. However, the general methods, although very accurate, propose an approach by "trial and error" that is not suited to the fast solution of the design problem.

These difficulties led Ball [9] to the proposal of a method based on the use of some equations representing the curves of heat penetration. The method of Ball, known as the formula method, initially restricted to certain values of the parameters of thermal death, was first improved [10] and then greatly expanded by Stumbo [11] to cover the whole range of values of these parameters.

Even if additional formula methods were proposed [12 and 13], Smith and Tung [14] established, by means of a comparative evaluation, that the method of Ball with the use of 57 Stumbo’s tables gave the most accurate estimations in all the different conditions of food thermal processes.

Therefore it is clear that, among the formula methods, the Ball-Stumbo is the most accurate, namely that calculates the heating time ensuring the desired microbial *lethality*, but preserving the organoleptic and nutritional qualities. However, it involve the consultation of 57 Stumbo's tables with 18,513 datasets, and then it shows serious difficulties on its computerization that is necessary to make quick and automated the canned food thermal processes calculations.

To solve this problem, over the past three decades, several mathematical models based on heat transfer were proposed [15, 16, 17, 18], but in recent years, for a better solution, it was also followed the path of mathematical modeling by computational thermo-fluid dynamics. The consequent use of CFD methods proved to be a valuable tool to ensure food safety and nutritional quality [19, 20, 21, 22 and 23]. The dawn side of most of these numerical approaches is the need of high computing power and in any case very long calculation times [24]. In addition, their use requires some experience in the use of CFD (right choice of mesh, etc.) and the precise knowledge of multiple input data related to the food product and system, such as the heat transfer coefficient of the heating and cooling medium, thermal diffusivity of the food product, can shape and dimensions and processing conditions. One exception among these numerical methods is the work of Teixeira, et al. [25], which requires only the heating rate and heating lag factors from heat penetration tests as input variables.

Also in recent years, in parallel and alternatively, it was proposed the use of artificial neural networks (ANNs) [24, 26 and 27]. They consist in the sequential solution of a high number of algebraic equations connected to an information processing system (black box) that learns from 18,513 Stumbo's datasets.

In a previous work [28], the 57 Stumbo's tables, which forms the basis for the application of the modern formula method, were transformed into a mathematical model, easy to handle and to computerize for the solution of check problem, i.e. the verification of the attainment of desired microbial *lethality F*. Computerization was performed using an analytical approach based on a modified and expanded Ball's model, comprising ten equations, converging to Stumbo's datasets, to predict, in check problem, the *lethality value F*, and hence the f_h/U ratio, for a given g value.

Nevertheless, the system of ten equations was implicit in the g value which is necessary to solve the design problem. Therefore the g value could be only calculated in numerical form, for example using a spreadsheet by putting the f_h/U value into the equations system and searching for the corresponding g value that ensures the validity of the ten equations. Recall that the g value is necessary to compute directly the thermal process time B with Ball's formula, as it will be seen later, if the *process lethality F* is known a priori (design problem).

As a result of these difficulties, due to the lack of an analytical approach that provides direct g values, it is necessary to seek a new mathematical solution. This new solution would enable easier development of the software to automate the calculation of the thermal process. Therefore, the overall objective of this paper was to redefine the mathematical problem of food process thermal design and then to develop a new mathematical procedure, with the solution converging to Stumbo's datasets and predicting the *g* value for a given f_h/U value.

2. The mathematical problem

For various microorganisms, the experimental values of the number of decimal reductions, *n*, necessary to achieve desirable sterilization are known, $n = \log\left(\frac{N_0}{N}\right)$, where *N* is the number of viable microorganisms at time *t* (min), and N_0 corresponds to the initial number at time $t_0 = 0$.
Therefore, the total heating time (min) at a constant reference temperature (equal to 121.1°C), which is known as the *process lethality*, is defined by the symbol F_o and is simply calculated by:

$$F_o = n \cdot D_{121.1} \tag{1}$$

where *n*, the number of decimal reductions, and $D_{121.1}$, the decimal reduction time at reference temperature 121.1°C, are experimentally known as function of the residual microbial population and respectively of the target microorganism. Therefore, the *process lethality F* also becomes known through the equation (1). When *z*=10°C, and the temperature is equal to reference temperature,121.1°C, the *process lethality* is indicated by the F_0 symbol [10].
During sterilization and, more generally during thermal treatments (for example cooking, pasteurisation etc..) the temperature inside the canned food slowly increases over the heating time and then slowly decreases as well over the subsequent cooling time (Fig. 1).
Often it can also occur variability of temperature in the interior space between the various points of the canned food. For the purposes of achieving the desired sterility, the spatial variability of temperature is not a problem because it is enough to consider the situation more restrictive, i.e. the temperature over the time of the coldest point in the canned food.
The change in temperature *T* vs. time *t* in the coldest point of the canned food produces a change in the decimal reduction time, now called D_T instead $D_{121.1}$. Consequently the *process lethality F* is now obtained as follows [28]:

$$F = n \cdot D_{121.1} = \int_0^t 10^{\frac{T-121.1}{z}} dt \tag{2}$$

In this integral equation, *F* is known from equation (1), and the time *t* = *B* of heating of the canned food is unknown. This is the design problem.
The solution of the integral (2), which must be done for both the heating and cooling phase, requires the relationship between the coldest point temperature and the time. This relationship is also called temperature-time history or heat penetration curves.
To obtain the temperature-time relationship, Ball [9] considered that, after a possible

initial lag period, the difference of temperature between the retort and coldest point of the can $(T_R - T)$had an exponential decay with respect to time *t*:

$$(T_R - T) = (T_R - T_0) \cdot J_{ch} \cdot e^{-\frac{2.3 \cdot t}{f_h}} \quad (3)$$

where T_R (°C) is the retort temperature, T_0 (°C) is the initial food temperature, J_{ch} is the heating rate lag factor at the can center (coldest point), and f_h(min) is the heating rate index (the heating time required for the log temperature vs. time plot to traverse one log cycle). From equation (3), as did Ball and Olson [10] and Stumbo [11], it is now possible to obtain the heating process time *t* = *B* (min) (fig. 1) at a constant retort temperature T_R:

$$B = f_h \cdot \log\left[\frac{J_{ch}\left(T_R - T_0\right)}{g}\right] \quad (4)$$

where $g = \left(T_R - T_g\right)$ (°C) is the difference between the retort temperature and coldest point temperature T_g(°C) at the end of the heating process (figure 1). Equation (4), called Ball's formula, requires preliminary experiments to evaluate the factor J_{ch} and the index f_h.

The heating lag factor J_{ch} may assume different values (range of values close 1 to 2), depending on the rheological and thermal properties of the canned food.
When the retort temperature T_R is different from the conventional 121.1°C, often is preferable an higher temperature, also for improving exergetic efficiency [29], the time required to accomplish a heat process of some given *F* value is defined as the *sterilizing value U*:

$$U = F \cdot 10^{\frac{121.1 - T_R}{z}} \quad (5)$$

As indicated by Stoforos [30], in the design problem of a thermal process, it is required the prediction of process time *B* to obtain a required *lethality* (*F* or *U*). This involves the experimental determination of the parameters of the heating and cooling curves (f_h, J_{ch}, f_c, J_{cc}), the calculation of f_h/U and the determination of the *g*-value, using the appropriate tables initially created by Ball [9 and 10] and then expanded by Stumbo [11] as it will be seen below. Then, using equation (4), it is possible to calculate *B*.

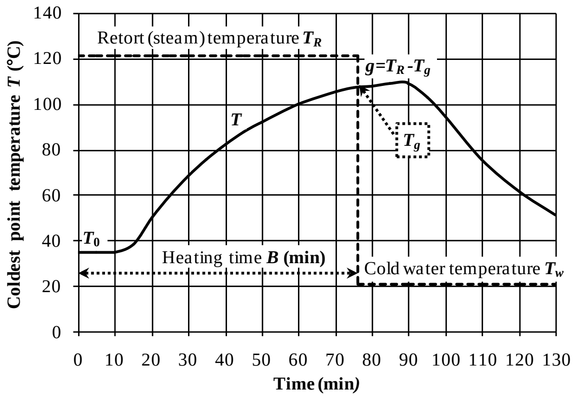

FIGURE 1 - Heat penetration curve or temperature-time history of the coldest point of the canned food during the entire thermal process from a first heating period to a second cooling period.

As was noted previously, the semi-analytical methods of the formula are based on the integration of equation (2). Thus, it needs to define the temperature-time relationship.

In the *heating phase*, beginning from the time when the retort temperature is considered constant, introducing the temperature T, obtained by equation (3), in to equation (2) and then integrating, the result of Ball and Olson [10] is obtained as follows:

$$F_h = -\frac{f_h}{2.3}\cdot 10^{\frac{T_R-121.1}{z}}\left[\mathrm{Ei}\left(\frac{-2.3\cdot g}{z}\right)-\mathrm{Ei}\left(\frac{-2.3\cdot 44.4}{z}\right)\right] \tag{6}$$

where the function Ei is called the Exponential integral and F_h is the *lethality* during the heating phase.

The lower limit of integration, that Ball and Olson [10] imposed equal to 44.4°C, is the initial temperature difference of the heating process. It assures to take into account all the contributions to the *lethality* of the temperature-time values.

When z is lower than 15°C (26°F), then $\mathrm{Ei}\left(-\frac{2.3\cdot 44.4}{z}\right)\rightarrow 0$ and it is possible to simplify the equation (6), with no significant error:

$$F_h = -\frac{f_h}{2.3}\cdot 10^{\frac{T_R-121.1}{z}}\left[\mathrm{Ei}\left(\frac{-2.3\cdot g}{z}\right)\right] \tag{7}$$

However, recalling equation (5), under the condition $z \le 15°C$ and during the *heating phase*, the *sterilizing value* can be written as follows:

$$U_h = -\frac{f_h}{2.3}\cdot \mathrm{Ei}\left(\frac{-2.3\cdot g}{z}\right) \tag{8}$$

In the *cooling phase* (fig. 1) Ball considered the cooling lag factor J_{cc} constant and equal to 1.41, and, therefore, the presence of a cooling lag. It is then possible to mathematically represent the cooling temperature-time history using an equation similar to the function (3) only with a lag behind the introduction of cold water into the retort at the temperature T_w.

During this lag, Ball [9] represented the cooling temperature-time history by a hyperbola. However, the hyperbolic function fits the experimental data only for *z* between 3.3° and 15°C (6°-26°F) and J_{cc}=1.41.

Ultimately, by introducing first the hyperbolic function of cooling and then the exponential function of cooling in equation (2) and then integrating, Ball obtained the equation for *cooling process lethality* F_c. This equation was more complicated than that of the *heating process lethality,* but still contained the Ei function. Here, this equation is shown briefly as follows:

$$F_c = -\frac{f_c}{2.3} \cdot 10^{\frac{T_R - 121.1}{z}} \cdot F_c\left(g, z, T_w\right) \tag{9}$$

where $F_c\left(g, z, T_w\right)$ is the influence of the *g* value, z value and the temperature T_w of cold water used in the retort during cooling.

Ball considered the temperature T_w constant and equal to 21.1°C (70°F). In addition the cooling rate index $f_c = f_h$ is considered a valid assumption as it was later verified by Stumbo [11]. The *sterilizing value* during the *cooling phase* is then:

$$U_c = -\frac{f_h}{2.3} \cdot F_c\left(g, z\right) \tag{10}$$

Adding U_h and U_c, it provides the *sterilizing value U* of the entire thermal process:

$$U = U_h + U_c = -\frac{f_h}{2.3}\left[\mathrm{Ei}\left(\frac{-2.3 \cdot g}{z}\right) + F_c\left(g, z\right)\right] \tag{11}$$

For each value of *z*, equation (11) becomes:

$$\frac{f_h}{U} = f\left(g\right) \tag{12}$$

Ball solved it, and because the Ei function is a tabulated function, he obtained a table that provided, for the different $\frac{f_h}{U}$ values, the corresponding *g* value. The table was made for the various *z* values between 3.3 and 15°C (6-26°F).

Recall that in computing values of $f_h/U{:}g$ using equation (11), Ball considered the cooling lag factor J_{cc} constant and equal to 1.41 and a limited range of *z* values (6-26°F).

It is clear that with the lag factor J_{cc} and *z* value outside these limits, the values of f_h/U are different.

Based on this feedback, Stumbo [11] produced 57 tables of $f_h/U{:}g$, for a wide range of z values (4.4°-111.1°C; 8°-200°F) and cooling lag factor J_{cc} values (0.4-2), keeping valid the hypothesis that the cooling rate index $f_c = f_h$.
Recall that Stumbo developed the integration to obtain the *sterilizing values U*, using the general method and that the U values obtained were the *sterilizing values* of the entire thermal process, accounting for the lethal heat during both heating and cooling. Therefore, the g values from 57 Stumbo's tables, to be used in Ball's formula (4), accounts for lethal heat values of both heating and cooling.
Finally, Stumbo produced a very important work that greatly expanded the applicability of Ball's formula method. The method became able to determine, with equation (4), the process time B for any kind of thermal process (z value) and for any canned food (J_{cc} value). However, the method remained limited due to the lack of ability to automate it. Rather, Stumbo has expanded this problem because the Ball's table has turned into 57 Stumbo's tables representing the function:

$$\frac{f_h}{U} = f\left(g, z, J_{cc}\right) \tag{13}$$

3. Proposal of a solution method

The problem of the impossibility of automation of Ball's formula method, i.e., computer-based application of formula (4) with Stumbo's tables, can be translated into the problem of the inversion of function (13), $g = g\left(\frac{f_h}{U}, z, J_{cc}\right)$.
Looking at the 18,513 datasets from Stumbo's tables, within the range of z values between 10°C and 111°C and with the values of $f_h/U \geq 0.3$, it was observed that g values range from $1.2 \cdot 10^{-3}°C - 30°C$. Moreover, it was always clear from analysis of Stumbo's datasets that the function $\frac{f_h}{U} = f\left(g, z, J_{cc}\right)$ is continuous and monotonically increasing and therefore invertible.
For any given J_{cc} and from equation (11), function (13) can be written as:

$$\frac{2.3}{\frac{f_h}{U}} = -\mathrm{Ei}\left(\frac{-2.3 \cdot g}{z}\right) - F_c\left(g, z\right) \tag{14}$$

Using a previous mathematical model [28], it was possible to note that when $g \rightarrow 1.2 \cdot 10^{-3}$ the values of the function $F_c\left(g, z\right)$ become negligible compared to those of the function $\mathrm{Ei}\left(\frac{-2.3 \cdot g}{z}\right)$.
Moreover, the values of Ei function can be calculated with the following series:

$$-\mathrm{Ei}\left(-x\right) = -\gamma - \ln\left(x\right) + x - \frac{x^2}{2 \cdot 2!} + \frac{x^3}{3 \cdot 3!} - .... + \frac{x^p}{p \cdot p!} \tag{15}$$

where γ is Euler's constant (γ=0.5772…) [31] and $x = \frac{2.3 \cdot g}{z}$. However, when $g \rightarrow 1.2 \cdot 10^{-3}$ and $x \rightarrow 0$, the above series converges on the following function:

$$-\mathrm{Ei}(-x) = -\gamma - \ln(x) \tag{16}$$

from which, by combining with equation (14), the inverse function for any given value of J_{cc} can easily be obtained:

$$g = \frac{z}{2.3} \exp\left(-\gamma - \frac{2.3}{f_h/U} \right) \tag{17}$$

The g values that are obtained from equation (17) come close to those of Stumbo's tables for small values of J_{cc} and z and for reduced f_h/U values. That is, for the cases of small cans (reduced f_h) and/or high *sterilizing values U*. Outside these f_h/U, z and J_{cc} values, equation (17) is insufficient and must be corrected by the introduction of polynomials for the f_h/U, z and J_{cc} variables. As a result of a careful analysis of the data in Stumbo's tables and as a consequence of a trial-and-error approach, the influence of the f_h/U parameter has been solved by means of a third-degree polynomial in the variable $\ln(f_h/U)$.

However, to extend the validity of this polynomial to all Stumbo's tables, for each z value from $10°C \le z \le 111°C$, it was necessary to multiply the third-degree polynomial in $\ln(f_h/U)$ with another third-degree polynomial in the variable $\ln(z)$. The product of two polynomials, understood as correction factor H of equation (17), is the following:

$$H = \left(au^3 + bu^2 + cu + d\right) \cdot \left(Ay^3 + By^2 + Cy + D\right) \tag{18}$$

where $u = \ln(f_h/U)$, $y = \ln(z)$. After the product of two polynomials was carried out, the products of the coefficients *a, b, c, d, A, B, C* and *D* were obtained (Table 1) using multiple regression analysis [32] (R^2=0.999) from Stumbo's datasets.

A similar approach was made to evaluate the influence of J_{cc} identifying a polynomial function K as follows:

$$K = \left[1 + \left(J_{cc} - 0.4\right) \cdot \left(pu^2 + qu + r\right) \cdot \left(Pz^2 + Qz + R\right)\right] \tag{19}$$

where $u = \ln(f_h/U)$. After the product of two polynomials was carried out, the products of the coefficients *p, q, r, P, Q* and *R* were obtained (Table 2) using multiple regression analysis (R^2=0.999) from Stumbo's datasets.

Ultimately, the g values are calculated with the following equation:

$$g = \frac{z}{2.3}\exp\left(-\gamma - \frac{2.3}{f_h/U}\right)\cdot H \cdot K \qquad (20)$$

Equation (20) is valid in the domain: $10°C \le z \le 111°C$, $f_h/U \ge 0.3$ and $0.4 \le J_{cc} \le 2$.

TABLE 1 – Coefficients of polynomial *H*

$a \cdot A$	-0.004402
$a \cdot B$	0.048989
$a \cdot C$	-0.162490
$a \cdot D$	0.160914
$b \cdot A$	0.014952
$b \cdot B$	-0.136467
$b \cdot C$	0.355817
$b \cdot D$	-0.128237
$b \cdot A$	0.038973
$c \cdot B$	-0.411237
$c \cdot C$	1.373832
$c \cdot D$	-1.310923
$d \cdot A$	-0.032731
$d \cdot B$	0.252513
$d \cdot C$	-0.697395
$d \cdot D$	1.614456

TABLE 2 – Coefficients of polynomial *K*

$p \cdot P$	$-2.1736 \cdot 10^{-5}$
$p \cdot Q$	$3.6527 \cdot 10^{-5}$
$p \cdot R$	$-4.6221 \cdot 10^{-3}$
$q \cdot P$	$-1.0870 \cdot 10^{-4}$
$q \cdot Q$	$7.0356 \cdot 10^{-3}$
$q \cdot R$	$-7.0012 \cdot 10^{-2}$
$r \cdot P$	$8.7693 \cdot 10^{-5}$
$r \cdot Q$	$1.6666 \cdot 10^{-2}$
$r \cdot R$	0.2322

4. Results and discussion

4.1 Calculation of g

The method proposed in this work, made of the previous three equations (18), (19) and (20) to be sequentially solved, was implemented in a spreadsheet to calculate *g* values by varying f_h/U values obtained from Stumbo's tables. Figure 2 shows the calculated values of *g* compared with the *g* Stumbo's values. The indices of comparison were: a determination coefficient R^2=0.9999, a mean relative error *MRE*=0.85±0.91% and a mean absolute error *MAE*=0.06°±0.09°C (0.11°±0.16°F). These results are better with respect to the previous mathematical model [28], where the following values were found: a determination coefficient R^2=0.9990, a mean relative error *MRE*=2.67±2.69% and a mean absolute error *MAE*=0.16°±0.20°C (0.29°±0.36°F). Recall that the previous mathematical model [28], made of ten equations, was implicit in the *g* value which is necessary to solve the design problem. Therefore the *g* value was only calculated in numerical form, for example using a spreadsheet by putting the f_h/U value into the equations system and searching for the corresponding *g* value that ensures the validity of the ten equations.

4.2 Mathematical procedure validation

Ball's formula (equation 3) was used to calculate the thermal process time *B* for various heating conditions. For comparison purposes, these conditions correspond to those imposed by Sablani and Shayya [26]: *z* values of 10° and 44.4°C (18° and 80°F), T_0 values of 65.5°C (150°F), f_h values of 30 and 90 min, J_{ch} values of 1 and 2, J_{cc} values of 0.4 and 2, T_R values of 111.1°, 121.1° and 140°C (232°, 250° and 284°F) and *F* values of 5, 15 and 25 min.

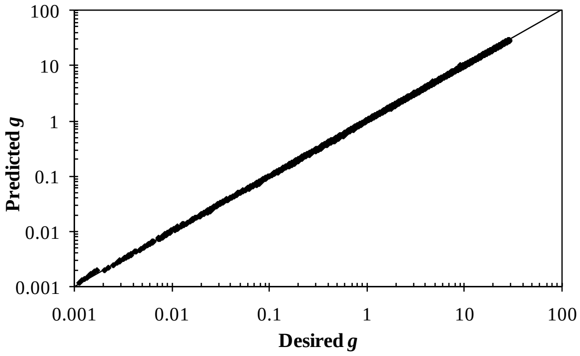

FIGURE 2. Values of *g* predicted using the method proposed in this work vs. the desired Stumbo's values of *g*.

The comparison between the process time *B*, calculated using *g* values of this work, and *g* values from Stumbo's tables, is shown in Fig. 3. Table 3, last row, shows the corresponding comparison indices, such as the root mean square of deviations *RMS* equal to 0.393 min, the average absolute error equal to 0.259 min and the standard deviation *SD* equal to 0.296 min. Table 3 also shows that, the same comparison indices, obtained with the previous mathematical model [28] and, respectively, with the neural networks models [26 and 27], are higher, confirming the best fitting of Stumbo's datasets by the method presented in this work. All of these comparisons were significant at the 95% confidence level.

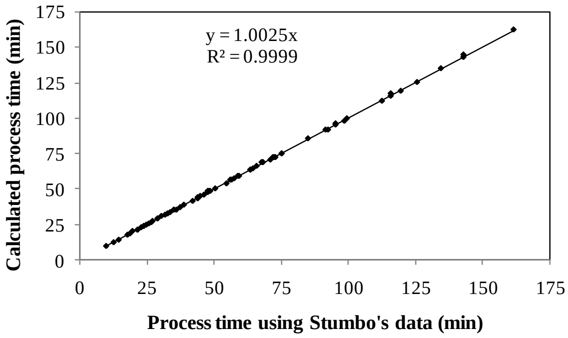

FIGURE 3. Comparison of the process time *B* calculated using *g* values from Stumbo's tables [11] and the semi-analytical method proposed in this work.

TABLE 3. Comparison of process times calculated using *g* values from Stumbo's [11] tables, the ANN model of Sablani & Shayya [26], the ANN model of Mittal & Zhang [27], the mathematical model [28] and the semi-analytical method proposed in this work.

	RMS (min)	Deviations or absolute error (min)			*S.D.*	Slope of line in Fig. 5
		Max	Min	Ave		
Sablani & Shayya [26]	1.164	2.57	0.05	0.936	0.697	1.011
Mittal & Zhang [27]	0.612	1.63	0.01	0.466	0.400	0.999
Friso[28]	0.567	1.61	0.003	0.421	0.380	1.001
This work	0.393	1.58	0.003	0.259	0.296	1.002
RMS = Root Mean Square of deviations, *S.D.* = Standard Deviations						

5. Conclusions

The Ball's formula method, in its most complete version, requires the consultation of 57 Stumbo's tables for the check problem as well as for the design problem of food thermal processes.

In a previous work [28] it was proposed a mathematical model comprising ten equations to be solved sequentially, especially for the fast solution of the check problem on the microbial *lethality* achieved from a given process time as an alternative to the consultation of the 57 Stumbo's tables.

In this work was instead developed a mathematical procedure for the solution of the design problem, that is determining the process time needed to achieve a specified microbial lethality. This was done with the use of a rapid and computerized calculation of the *g* value, for a given f_h/U value, and then used to determinate the thermal process time *B* by the Ball's formula.

This mathematical procedure, which consists of only three equations (18, 19 and 20), eliminates the consultation of 57 Stumbo's tables, both manual as well as through

the computerized storage and interpolation of 18,513 Stumbo's datasets.
Equation (20) is based on the inversion of the function that expresses the *process lethality* obtained from a previous mathematical model [28], as an extension of the work of Ball. However, the inversion was possible for a sub-domain of the function. The inverse function $g = g(f_h/U, z, J_{cc})$ was then extended to the entire domain using polynomials in equations (18) and (19) and obtained with articulated multiple regressions starting from Stumbo's datasets [11].
The results obtained by applying the mathematical procedure of this work, namely the *g* values using the three equations (18, 19 and 20) and the process time *B* using Ball's formula, closely followed the process time calculated from tabulated Stumbo's *g* values.
The high simplicity and accuracy of the procedure proposed here, better than ANNs models [26 and 27] and than previous mathematical model [28], make it useful in the development of algorithms for calculating and controlling, using computer, of food thermal processes.

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