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Forecasting integer autoregressive processes of order 1: are simple AR competitive?

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Abstract

In this work we want to clarify, via a Monte Carlo experiment, if (and when) for an integer-valued time series it is really recommended to adopt the coherent forecasting methods from INAR models or if equivalently good predictions can be obtained from the simpler AR models. Results show that INAR models should be preferred.

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1. Introduction

Recently, also in economic research there has been a growing interest in studying non-negative integer-valued time series and, in particular, time series of counts. Examples of this kind of series are the weekly number of guests in a hotel, the daily number of visitors of a website, the daily number of traded stocks. In some cases, the discrete values of the time series are large numbers and may be analysed by using continuous-valued models such as the traditional ARIMA ones with Gaussian errors. However, according to Chatfield (2000), a good model for time series should be consistent with the properties of the data and be unable to predict values which violate known constraints. This means that, when series consists of small non-negative values, like in case of counting data, we have to consider a model that is forecast-coherent and a method of forecasting that produces integer values. In the light of this requirement the well-known linear ARMA processes are of limited use for modeling and especially for forecasting purposes. To take this peculiar feature into account, McKenzie (1985) and Al-Osh and Alzaid (1987) introduce the integer-valued autoregressive process (INAR).

The theoretical properties of INAR models have been extensively studied (e.g. Silva and Silva, 2009; Freeland and McCabe, 2004a), but forecasting from these models is quite a controversial issue. In particular, given the integer nature of INAR models, only integer forecasts of the count variable should be produced. As a coherent method, i.e. capable to preserve the integer nature of the data in obtaining the forecasts, Freeland and McCabe (2004b) suggest to use the median of the $k-$step-ahead conditional distribution, instead of the conditional mean. These authors specifically work with INAR(1) process with Poisson innovations, but after their contribution a number of generalizations appear in the literature, *e.g.* Jung and Tremayne (2006) and Bu and McCabe (2008) who focus on higher order INAR models.[1]

The methods proposed to obtain coherent forecasts have also some disadvantages. On the one hand, they are problem-specific as they depend on the distributional assumption of Poisson error terms, on the other hand, they are computationally not simple. With this particular concern in mind, in the present note we conduct a Monte Carlo experiment to clarify if (and when) it is really recommended to adopt the coherent forecasting methods from INAR models or if equivalently good predictions can be obtained from the simpler AR models. Results show that INAR(1) models clearly exhibit a better forecasting performance compared to AR(1) thus leading to the conclusion that, in case of count time series, the use of AR(1) models to forecast, although appealing for their simplicity, should be avoided and INAR(1) models should be adopted instead.

The remainder is divided as follows. Section 2 briefly reviews the INAR(1) model

[1] There are also contributions that tackle the issue of coherent forecasts from a Bayesian perspective, *e.g.* McCabe and Martin (2005) and Silva *et al.* (2009). In those works, uncertainty due to both model specification and parameters is incorporated into the predictive probability mass function, but this is beyond the scope of the current paper.

and presents the proposal of Freeland and McCabe (2004b) to obtain coherent forecasts. Section 3 describes our Monte Carlo study. In Section 4 an empirical illustration is presented.

2. Forecasting from INAR(1) models

To introduce the class of INAR models we first recall the thinning operator, '$\circ$'. Let Y be a non negative integer-valued random variable, then for any $\alpha \in [0, 1]$

$$\alpha \circ Y = \sum_{i=1}^{Y} X_i$$

where X_i is a sequence of *iid* count random variables, usually Bernoulli, independent of Y, with common mean α.

The INAR(1) process $\{Y_t; t \in \mathbf{Z}\}$ is defined by the recursion

$$Y_t = \alpha \circ Y_{t-1} + \epsilon_t \tag{1}$$

where $\alpha \in [0, 1)$, and ϵ_t is a sequence of *iid* discrete random variables with finite first and second moment. The components of the process $\{Y_t\}$ are the surviving elements of the process Y_{t-1} during the period $(t-1, t]$, and the number of new elements which entered the system in the same interval, ϵ_t. Each element of Y_{t-1} survives with probability α and its survival has no effect on the survival of the other elements, nor on ϵ_t which is not observed and cannot be derived from the Y process in the INAR(1) model. If the error terms are distributed as a $Po(\lambda)$, the marginal distribution of the observed counts is also a Poisson distribution with (unconditional) mean and variance equal to $\lambda/(1-\alpha)$; see Jung *et al.* (2005).

Usually, to obtain predictions from time series models, conditional expectations are adopted, since the derived forecast has minimum mean square error. However, this method does not provide coherent predictions, in the sense that it ignores the restriction that the support should be the set of integer. To circumvent this, Freeland and McCabe (2004b) move by considering the $p-$step ahead predictive probability mass function (pmf) itself which, for the INAR(1) model with Poisson innovations, PoINAR(1), takes this form:

$$\begin{aligned} P(Y_{T+k} = y \mid Y_T = y_T) = \; & \textstyle\sum_{s=0}^{\min(y,y_T)} \binom{y_t}{s} (\alpha^k)^s (1-\alpha^k)^{y_T - s} \times \\ & \tfrac{1}{(y-s)!} \exp\left\{-\lambda \tfrac{1-\alpha^k}{1-\alpha}\right\} \times \left(\lambda \tfrac{1-\alpha^k}{1-\alpha}\right)^{y-s} \end{aligned} \tag{2}$$

where $y_{T+k} \in \{0, 1, 2, \ldots\}$ and $k = 1, 2, 3, \ldots$. Then, in order to obtain coherent predictions for Y_{T+k}, Freeland and McCabe (2004b) suggest using the median of the $k-$step-ahead pmf. Operatively, it is computed as $P_k(Y_{T+k} = y \mid Y_T, \hat{\alpha}, \hat{\lambda})$, where (α, λ) are typically estimated via maximum likelihood (ML).

3. Monte Carlo experiments

In this Section we provide the details of the Monte Carlo experiment we conduct to compare the forecast accuracy of INAR(1) and AR(1) models. To do this, we generate data from INAR(1) DGP's with Poisson errors, whose parameters values are $\alpha = (0.1, 0.25, 0.5, 0.75, 0.9)$ and $\lambda = (1, 3)$. The sample size we consider is $N = (110, 260, 510)$ retaining the last 10 observations for assessing out-of-sample forecasting performance. For each model we generate $s = 2000$ independent realizations.[2]

In practice, to make comparisons, we firstly estimate an AR(1) model for the time series at hand, obtain the forecast with the conditional mean and round it. Then, for the same time series, we calculate the forecasts following the approach depicted in the previous section for PoINAR(1). The forecasting performance of the estimated INAR and AR models is expressed through the Forecast Mean Square Error (FMSE) and Forecast Mean Absolute Error (FMAE) statistics of $k-$step-ahead forecasts, where $k = 1, 2, \ldots, 10$.

Before presenting the core of the results, we report in Table I some summary statistics of the ML estimates of α, λ for PoINAR(1) models and ϕ for AR(1) models relatively to selected cases of our Monte Carlo experiment. We observe that the obtained Monte Carlo mean and standard deviation of the ML estimates perform well, especially for $\alpha > 0.1$ and, as expected, the performance improves when the sample size increases. Interestingly, the value of the estimated parameter ϕ, when we fit an AR model to the data, is very close to the value taken by the thinning parameter.

Moving now to the forecasting performance, in Tables II, III we present, respectively, the FMSE and the FMAE.

Results show that INAR(1) models exhibit a better forecasting performance compared to AR(1) models. In particular, by forecasting with the median of the $p-$step-ahead pmf, a very large gain is obtained in terms both of FMSE and FMAE as the thinning parameter value increases and this gain is even more for $\lambda = 3$. On the contrary, the performance of the AR model greatly worsens with the increase of the thinning parameter and sample size. It seems that only for $\alpha = 0.1$, independently of the value of λ, the two models have similar behaviour, in all other cases the INAR(1) model outperforms AR(1) models, thus leading to the conclusion that, although simple, the latter models should be avoided for this kind of data.

4. An empirical illustration

To illustrate also via an empirical analysis the better forecasting performance of INAR(1) compared to AR(1) models, we applied both approaches to the real time series of the daily count of visits to the web site of the "statistical calendar" (Durante *et al.*, 2012), a

[2]The parameters settings we consider in this experiment are in line with possible real scenarios, see for example Freeland and McCabe (2004a).

Table I: Mean and standard deviations (in parenthesis) of ML estimates across Monte Carlo simulations (columns 1-3), summary statistics of the simulated data (columns 4-6).

λ	N	α	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\phi}$	min	*mean*	max
1	100	0.1	0.0862	1.0105	0.0570	0	1.1083	8
			(0.1116)	(0.1538)	(0.1036)			
		0.25	0.2387	1.0093	0.2080	0	1.3315	10
			(0.0987)	(0.1547)	(0.1313)			
		0.5	0.4922	1.0072	0.4667	0	1.9944	11
			(0.0747)	(0.1665)	(0.0995)			
		0.75	0.7452	1.0060	0.7032	0	3.9833	16
			(0.0409)	(0.1670)	(0.0786)			
		0.9	0.8976	1.0193	0.8416	0	10.0157	26
			(0.0170)	(0.1749)	(0.0617)			
	250	0.1	0.0928	1.0075	0.0739	0	1.0585	7
			(0.0678)	(0.0969)	(0.0793)			
		0.25	0.2459	1.0075	0.2412	0	1.3383	10
			(0.0622)	(0.0998)	(0.0702)			
		0.5	0.4970	1.0064	0.4879	0	1.9842	11
			(0.0452)	(0.1020)	(0.0599)			
		0.75	0.7481	1.0087	0.7329	0	4.0157	17
			(0.0251)	(0.1071)	(0.0462)			
		0.9	0.8990	1.0079	0.8802	0	10.0153	27
			(0.010)	(0.1063)	(0.0313)			
	500	0.1	0.0942	1.0047	0.0846	0	1.1094	9
			(0.0443)	(0.0654)	(0.0561)			
		0.25	0.2454	1.0067	0.2447	0	1.3342	9
			(0.0416)	(0.0685)	(0.0460)			
		0.5	0.4973	1.0044	0.4934	0	1.9999	12
			(0.0323)	(0.0717)	(0.0424)			
		0.75	0.7496	1.0003	0.7409	0	4.0003	16
			(0.0182)	(0.0769)	(0.0319)			
		0.9	0.8997	1.0020	0.8898	0	10.0096	27
			(0.0074)	(0.0736)	(0.0219)			
3	100	0.1	0.0897	3.0322	0.0568	0	3.3323	16
			(0.1042)	(0.3719)	(0.1008)			
		0.25	0.2365	3.0527	0.2076	0	4.0012	16
			(0.0994)	(0.4321)	(1311)			
		0.5	0.4923	3.0398	0.4656	0	6.0030	20
			(0.0715)	(0.4404)	(0.0941)			
		0.75	0.7463	3.0359	0.7032	0	11.9880	30
			(0.0375)	(0.7021)	(0.0786)			
		0.9	0.8982	3.0461	0.8418	9	29.9829	54
			(0.0155)	0.4623	0.0619			
	250	0.1	0.0949	3.0196	0.0751	0	3.3361	12
			(0.0643)	(0.2385)	(0.0771)			
		0.25	0.2456	3.0205	0.2403	0	4.0062	16
			(0.0587)	(0.2504)	(0.0659)			
		0.5	0.4959	3.0193	0.4841	0	5.9929	20
			(0.0437)	(0.2727)	(0.0571)			
		0.75	0.7495	3.0027	0.7344	0	12.0016	33
			(0.0239)	(0.2879)	(0.0458)			
		0.9	0.8988	3.0200	0.8791	9	29.9123	55
			(0.0103)	(0.3026)	(0.0335)			
	500	0.1	0.0977	3.0044	0.0877	0	3.3292	17
			(0.0449)	(0.1709)	(0.0578)			
		0.25	0.2476	3.0179	0.2445	0	4.0112	15
			(0.0429)	(0.1789)	(0.0444)			
		0.5	0.4998	2.9977	0.4930	0	5.9951	22
			(0.0309)	(0.1944)	(0.0411)			
		0.75	0.7486	3.0164	0.7397	0	12.0037	32
			(0.0162)	(0.1927)	(0.0308)			
		0.9	0.8999	3.0027	0.8911	8	30.0091	58
			(0.0068)	(0.2046)	(0.0214)			

Table II: Forecasting Mean Square Error (FMSE)

N	α	FMSE	$\lambda = 1$				$\lambda = 3$			
			$p = 1$	$p = 2$	$p = 5$	$p = 10$	$p = 1$	$p = 2$	$p = 5$	$p = 10$
100	0.1	*median*	1.183	1.150	1.137	1.134	3.543	3.495	3.319	3.519
		mean	1.152	1.150	1.141	1.124	3.483	3.414	3.299	3.443
		ar	1.165	1.153	1.140	1.135	3.640	3.578	3.349	3.590
250		*median*	1.122	1.082	1.084	1.148	3.435	3.517	3.428	3.296
		mean	1.096	1.082	1.081	1.143	3.376	3.402	3.337	3.214
		ar	1.117	1.082	1.084	1.148	3.535	3.544	3.439	3.299
500		*median*	1.063	1.226	1.206	1.139	3.274	3.322	3.380	3.589
		mean	1.048	1.206	1.194	1.127	3.116	3.197	3.303	3.515
		ar	1.058	1.226	1.206	1.139	3.244	3.339	3.391	3.614
100	0.25	*median*	1.338	1.554	1.429	1.443	3.945	4.048	4.037	4.013
		mean	1.242	1.424	1.311	1.357	3.853	3.945	3.985	3.993
		ar	1.518	1.606	1.459	1.504	4.449	3.978	3.992	3.981
250		*median*	1.307	1.450	1.489	1.491	3.875	4.054	4.079	3.824
		mean	1.195	1.331	1.380	1.397	3.735	4.014	4.056	3.832
		ar	1.489	1.493	1.506	1.513	4.110	4.032	4.056	3.825
500		*median*	1.466	1.451	1.407	1.384	3.757	3.988	3.719	3.875
		mean	1.352	1.319	1.329	1.279	3.649	3.957	3.739	3.879
		ar	1.566	1.435	1.420	1.394	4.227	3.934	3.718	3.874
100	0.5	*median*	1.601	2.058	2.187	2.049	4.929	5.701	6.399	6.308
		mean	1.451	1.928	2.151	2.015	4.766	5.578	6.252	6.154
		ar	2.421	2.232	2.134	1.989	7.662	6.389	6.377	6.232
250		*median*	1.587	2.029	2.216	2.140	4.563	5.922	6.150	6.198
		mean	1.482	1.903	2.207	2.135	4.514	5.844	6.075	6.167
		ar	2.490	2.204	2.197	2.107	7.431	6.597	6.138	6.183
500		*median*	1.536	1.961	1.994	2.029	4.744	5.831	5.610	5.783
		mean	1.417	1.838	2.008	2.055	4.659	5.754	5.598	5.831
		ar	2.441	2.094	1.994	2.029	8.382	6.841	5.561	5.778
100	0.75	*median*	1.834	2.872	4.400	4.508	5.733	8.901	12.815	12.488
		mean	1.737	2.782	4.251	4.383	5.696	8.852	12.700	12.338
		ar	6.375	5.610	4.870	4.541	18.683	16.264	14.204	12.464
250		*median*	1.942	2.930	4.110	4.402	5.513	8.636	11.801	12.304
		mean	1.833	2.859	4.000	4.329	5.444	8.550	11.679	12.209
		ar	6.304	5.336	4.534	4.369	18.519	15.906	13.313	12.252
500		*median*	1.779	2.659	3.533	4.134	5.039	7.607	12.097	12.334
		mean	1.671	2.608	3.401	4.076	5.020	7.531	11.939	12.259
		ar	6.019	5.075	4.000	4.087	17.988	14.604	13.401	12.472
100	0.9	*median*	1.935	3.791	6.860	9.781	5.731	10.816	19.773	28.903
		mean	1.862	3.674	6.758	9.664	5.639	10.706	19.659	28.729
		ar	12.281	12.199	11.434	11.258	35.501	34.099	32.272	34.050
250		*median*	2.001	3.761	6.900	9.4115	5.657	10.083	18.768	25.761
		mean	1.961	3.633	6.787	9.323	5.579	10.102	18.700	25.798
		ar	12.732	12.396	11.236	10.971	34.630	33.231	33.255	30.024
500		*median*	1.862	3.442	6.597	9.066	5.655	10.116	19.423	28.381
		mean	1.798	3.333	6.529	8.895	5.529	10.115	19.306	28.375
		ar	12.354	11.449	10.929	10.623	35.270	34.386	31.964	33.283

Table III: Forecasting Mean Absolute Error (FMAE)

			$\lambda = 1$				$\lambda = 3$			
N	α	FMAE	$p = 1$	$p = 2$	$p = 5$	$p = 10$	$p = 1$	$p = 2$	$p = 5$	$p = 10$
100	0.1	*median*	0.793	0.765	0.765	0.781	1.446	1.448	1.403	1.435
		mean	0.838	0.818	0.818	0.828	1.490	1.486	1.461	1.483
		ar	0.786	0.766	0.766	0.782	1.473	1.471	1.420	1.468
250		*median*	0.769	0.752	0.741	0.777	1.415	1.428	1.418	1.402
		mean	0.811	0.801	0.790	0.821	1.462	1.474	1.469	1.449
		ar	0.769	0.752	0.741	0.777	1.442	1.444	1.433	1.406
500		*median*	0.737	0.806	0.802	0.763	1.415	1.428	1.418	1.402
		mean	0.785	0.845	0.843	0.807	1.462	1.474	1.469	1.449
		ar	0.738	0.806	0.802	0.763	1.442	1.444	1.433	1.406
100	0.25	*median*	0.847	0.896	0.851	0.866	1.518	1.574	1.577	1.555
		mean	0.889	0.953	0.915	0.938	1.553	1.586	1.598	1.585
		ar	0.924	0.930	0.882	0.892	1.639	1.561	1.569	1.554
250		*median*	0.847	0.853	0.880	0.867	1.546	1.551	1.575	1.527
		mean	0.874	0.925	0.957	0.953	1.560	1.572	1.590	1.551
		ar	0.911	0.887	0.893	0.881	1.611	1.549	1.571	1.530
500		*median*	0.862	0.841	0.833	0.850	1.546	1.551	1.575	1.527
		mean	0.910	0.912	0.932	0.924	1.560	1.572	1.590	1.551
		ar	0.910	0.843	0.840	0.854	1.611	1.549	1.571	1.530
100	0.5	*median*	0.927	1.078	1.118	1.088	1.736	1.866	1.983	1.957
		mean	0.945	1.101	1.152	1.122	1.744	1.870	1.994	1.969
		ar	1.187	1.142	1.116	1.079	2.189	1.981	1.979	1.954
250		*median*	0.929	1.077	1.151	1.107	1.661	1.880	1.938	1.952
		mean	0.956	1.098	1.174	1.135	1.686	1.908	1.957	1.978
		ar	1.210	1.153	1.146	1.100	2.147	2.018	1.941	1.963
500		*median*	0.930	1.043	1.086	1.105	1.661	1.880	1.938	1.952
		mean	0.952	1.074	1.113	1.129	1.686	1.908	1.957	1.978
		ar	1.215	1.100	1.086	1.105	2.147	2.018	1.941	1.963
100	0.75	*median*	1.006	1.298	1.628	1.645	1.869	2.373	2.844	2.801
		mean	1.036	1.319	1.640	1.662	1.896	2.395	2.852	2.805
		ar	1.965	1.857	1.731	1.669	3.410	3.215	2.981	2.795
250		*median*	1.033	1.305	1.562	1.631	1.832	2.291	2.702	2.754
		mean	1.059	1.335	1.582	1.651	1.852	2.312	2.717	2.770
		ar	1.956	1.809	1.666	1.625	3.413	3.156	2.880	2.764
500		*median*	0.983	1.263	1.455	1.580	1.725	2.149	2.791	2.790
		mean	1.011	1.292	1.468	1.631	1.758	2.169	2.796	2.806
		ar	1.901	1.767	1.544	1.569	3.360	3.024	2.933	2.798
100	0.9	*median*	1.027	1.506	2.051	2.463	1.872	2.596	3.502	4.293
		mean	1.054	1.518	2.066	2.476	1.882	2.604	3.502	4.282
		ar	2.741	2.734	2.641	2.657	4.717	4.621	4.501	4.591
250		*median*	1.032	1.488	2.072	2.423	1.811	2.505	3.476	4.097
		mean	1.072	1.495	2.077	2.437	1.832	2.530	3.478	4.109
		ar	2.780	2.764	2.646	2.625	4.648	4.573	4.360	4.376
500		*median*	0.996	1.410	2.015	2.386	1.863	2.490	3.459	4.233
		mean	1.031	1.430	2.032	2.392	1.871	2.516	3.469	4.246
		ar	2.756	2.595	2.585	2.573	4.670	4.672	4.492	4.591

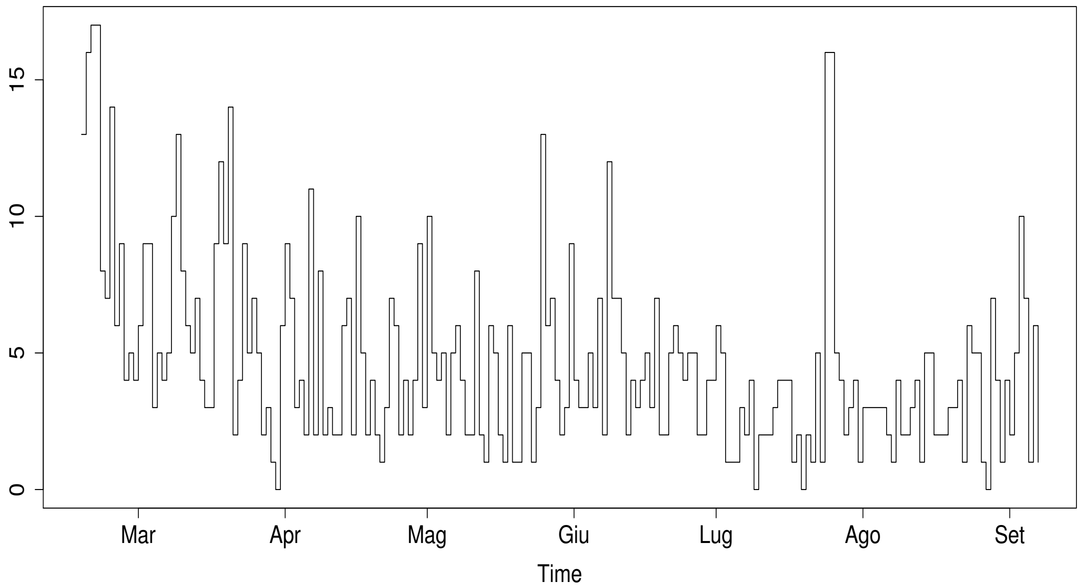

Figure 1: Time series of the daily number of visit to the "statistical calendar" website.

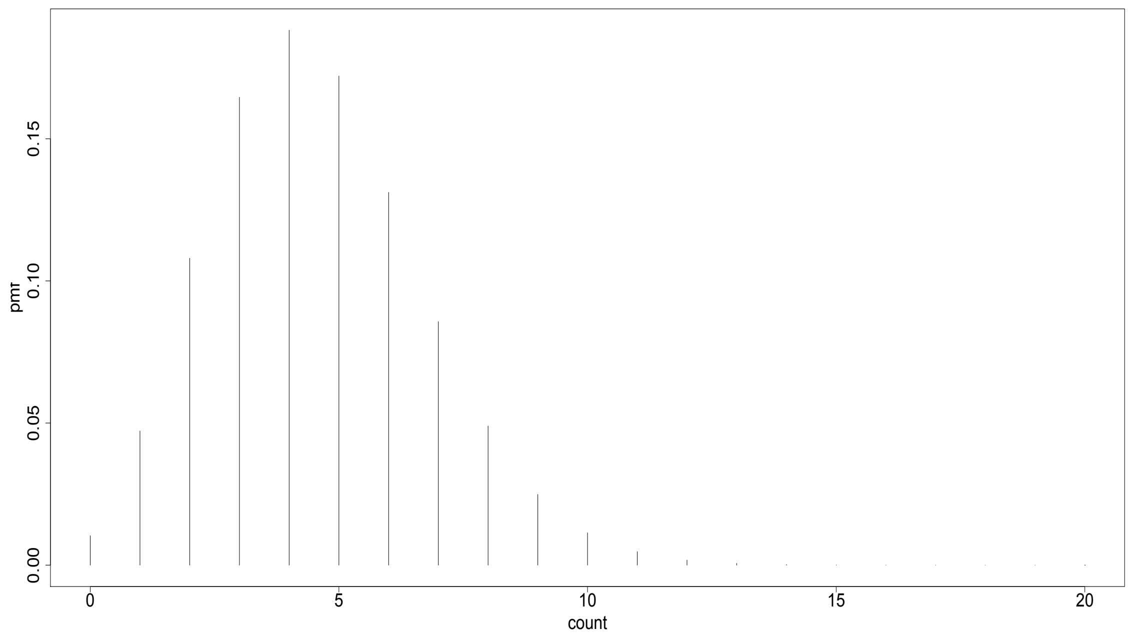

Figure 2: Estimated pmf of the daily number of visit to the "statistical calendar" website.

students' project of the Department of Statistics of the University of Padua, Italy. The data set stretches from February, 27, 2012 to the first week of September, 2012. The site is the output of a contest sponsored by the Italian Statistical Society on the the topic "Statistics and statisticians: ideas to foster and spread the statistical culture". The promotion of the site, after its launch at the end of 2011, motivates a $h-$step-ahead prediction. Indeed, knowing the whole predictive probability mass function allows the webmasters to compute both the median forecast for the following day and the probability of the web site receiving less than k visits, where k is a lower bound, in order to have a fairly large number of visitors. In general, the total number of today visitors is typically given by the loyal visitors from yesterday and new visitors. Thus, the birth-death interpretation of the INAR model fits this particular dataset.

The dataset consists of 203 daily counts, from 0 to 17, but we remove the last 6 observations to obtain out-of-sample forecasts. The series has median 4, mean 4.65 and mode 2. A plot of the series is shown in Figure 1. The empirical autocorrelation functions of the series (not reported here) are coherent with that of an $AR(1)$ and thus of a $INAR(1)$ model. Similarly to what we did in the Monte Carlo experiment, firstly we estimate the $INAR(1)$ and the simple $AR(1)$ models, then we predict the last six observations. The results are shown in Table IV where the better forecasting performance of the INAR model with respect to AR one is evident. Figure 2 displays the estimated probability mass function for $j = 0, 1, 2, \ldots, 20$. At this point, since we know the whole probability mass function, we are able to calculate, for example, the probability to have more than 5 visits which is 0.31.

Table IV: FMSE and FMAE for the series of the daily number of visit to the "statistical calendar" website

Model		FMSE	FMAE
INAR(1)	*Median*	11.33	3
	Mean	10.99	2.95
AR(1)		13.17	3.17

In conclusion, in this paper we show via Monte Carlo simulations that in case of count time series data, forecasting from AR(1) models can be dangerous because of the very poor forecasting performance of those models for this kind of data. Simulations also reveal that, on the contrary, INAR(1) models perform much better and their use is then strongly recommended. Moreover, as illustrated in the application, by forecasting from INAR(1) models, a big amount of information, carried by the knowledge of the entire predictive probability mass function, is available for the researcher. We think these results are very interesting, bearing in mind the variety of realistic empirical economic applications where INAR(1) models can be fruitfully adopted.

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