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Theoretical Analysis of the Capture Probability in Wireless Systems with Multiple Packet Reception Capabilities

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***Abstract*—In this paper, we address the problem of computing the probability that r out of n interfering wireless signals are "captured," i.e., received with sufficiently large Signal to Interference plus Noise Ratio (SINR) to correctly decode the signals by a receiver with multi-packet reception (MPR) and Successive Interference Cancellation (SIC) capabilities. We start by considering the simpler case of a pure MPR system without SIC, for which we provide an expression for the distribution of the number of captured packets, whose computational complexity scales with n and r. This analysis makes it possible to investigate the system throughput as a function of the MPR capabilities of the receiver. We then generalize the analysis to SIC systems. In addition to the exact expressions for the capture probability and the normalized system throughput, we also derive approximate expressions that are much easier to compute and provide accurate results in some practical scenarios. Finally, we present selected results for some case studies with the purpose of illustrating the potential of the proposed mathematical framework and validating the approximate methods.**

***Index Terms*—Capture, wireless, collision, successive interference cancellation, multi-packet reception**

I. Introduction

One of the main problems in wireless systems is the mutual interference produced by overlapping radio signals emitted by different transmitters, that might prevent the correct decoding of some or all of the signals involved, an event that is often referred to as *collision*. This situation may be observed, for instance, in random access systems, where transmissions from different sources take place without coordination, or in dense wireless sensor networks, where multiple sensor nodes may require to transmit their data to the sink node in the same time slot, or yet in ad hoc networks, in particular in the presence of hidden nodes. When the various signals are received with significantly different powers, the so-called *capture effect* may take place, i.e., the strongest signals may survive the collision and be correctly decoded despite the interference due to the other signals [1].

The capture phenomenon may significantly impact the system performance, in particular for systems with "Multi-Packet Reception" (MPR) capabilities, i.e., capable of decoding multiple overlapping packets out of a collision event, provided that the received signals satisfy the capture condition.[1] MPR has been recently shown to be a promising solution for high-capacity wireless networks [2], [3]. In fact, a better understanding of the ability of the receiver to correctly decode one or more signals, as a function of the statistical distribution of the received signal powers, may enable a more effective design of the transceiver architecture and the optimization of the transmission protocols.

A. Related literature

The relevance of the signal capture phenomenon in mobile radio systems has been recognized since long, as testified by the rather rich literature on the topic.

In [4] the authors assume that a signal is captured whenever the strongest interferer is sufficiently far apart from the designated receiver, according to a statistical geometry approach. In [5], capture is assumed to occur if the arrival instants of the first and second signals are sufficiently apart. A capture model based on the number of simultaneous transmissions is considered in [6], whereas in [7] it is assumed that a packet is captured only if during its overall transmission period no other signal is received with higher power. The stability of the slotted Aloha system with MPR capabilities is studied in [8], where the number r of captured signals is modeled as a random variable whose probability mass distribution depends only on the collision size n, i.e., the overall number of overlapping transmissions. The authors show that the MPR capability can stabilize Aloha and the maximum stable throughput when n goes to infinity is equal to the mean value of r. The paper, however, does not explicitly focus on the derivation of the capture distribution, which is instead obtained for some sample cases by using simple capture models, as those described above.

Successively, the analysis of the capture phenomenon was extended to include basic physical propagation aspects. In this case, we find two different approaches for modeling signal capture in radio systems, one based on the *protocol model* and the other on the *physical model*. The protocol model gives a geometric interpretation of signal propagation, according to which the capture of a signal only depends on the distance between the different transmitters and the common receiver. In [2], [3], in particular, it is assumed that the receiver

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[1] In this paper, we use the term "capture" to indicate the condition under which a signal can be decoded by an MPR system. However, if the number of captured signals exceeds the MPR capability of the receiver, the signals in excess are actually not decoded, even though they experience the capture condition.

can capture multiple signals transmitted within its reception range, provided that all other (interfering) transmitters are at a distance from the receiver larger than a given interference range. This approach makes it possible to carry out elegant performance analysis and to derive closed form bounds for the system capacity in different scenarios, but relies on an idealized and rather unrealistic model.

On the other hand, the physical model, which we adopt in this paper, explicitly includes the physical propagation phenomena and the cumulative character of interference in the capture model, considering the random distribution of the signal powers at the receiver and introducing the Signal-to-Interference-plus-Noise-Ratio (SINR) criterion to determine the capture probability [9], [10]. If P_j denotes the power of the jth signal at the receiver, the SINR for that signal is defined as

$$\gamma_j = \frac{P_j}{\sum_{h \neq j} P_h + N_0} \tag{1}$$

where N_0 represents the background noise power. A signal j is said to be captured and, hence, it is potentially decodable despite the interference produced by the other overlapping signals, if $\gamma_j > b$, with $b > 0$ representing the so-called *capture threshold* of the system.

The capture threshold b is a system parameter, whose value depends on the structure of the receiver and, more generally, on the properties of the communication system. For instance, conventional narrowband systems with a single antenna necessarily have capture threshold $b \geq 1$ and, as a consequence, at most one signal at a time can be captured by the receiver [10], [11]. Conversely, in Code Division Multiple Access (CDMA) systems, the capture threshold can be significantly less than 1, depending on the length of the spreading codes that are used to distinguish the signal of each user. These systems, in fact, trade the spectral efficiency of each user with a SINR gain proportional to the length of the spreading code, which may result in a capture threshold $b < 1$. Hence, CDMA systems are capable of decoding up to $\lfloor 1/b \rfloor$ overlapping signals, thus exhibiting *multi-packet reception capabilities*.

In general, however, the signals at the receiver are affected by random attenuation factors, so that the number of signals that can be actually captured is also random. An analysis of the capture probability for $b < 1$ has been proposed in [11], [12]. In particular, in [11] the authors derive an expression for the probability that there is *at least* one signal above the capture threshold, which is significantly more difficult to compute than in the case $b \geq 1$.

A different approach to enhance the system capacity in the presence of interference is based on Successive Interference Cancellation (SIC), which was first proposed in [13]. Broadly speaking, SIC is an iterative reception scheme where signals are generally decoded one at a time, starting from the strongest, i.e., the one with the largest SINR [14]. After the signal is decoded, it is canceled from the aggregate received signal and, then, the next strongest user is decoded in the subsequent decoding iteration. Therefore, SIC systems are inherently capable of multi-packet reception [15]. A recent analysis of the network capacity of SIC systems is presented in [16], where the authors provide bounds on the transmission capacity of a wireless ad hoc network applying a statistical geometry approach. In [17], it is shown that SIC can increase the stable throughput of Aloha systems up to 0.793 packets per slot by measuring the aggregate signal power at the receiver and using fine-grained power control at the transmitters. However, the effect of non-ideal channel state information or power control on the capture probability with a SIC receiver is not addressed.

B. Novel contributions

In this work, we advance the state of the art in the analysis of the capture phenomena by proposing a novel mathematical framework that, compared to the existing mathematical approaches, provides more general results and scales better with the size of the problem, i.e., the number of overlapping signals. This framework makes it possible to readily evaluate the performance of random access systems with MPR capabilities in various scenarios, and provides a useful tool for system design and dimensioning. More specifically, in this paper we provide the following original contributions.

We first consider the simpler and more classical case of pure MPR systems without SIC. For these types of systems, we derive an analytical expression of the *complete capture probability distribution*, i.e., we give the expression of the probability $C_n(r)$ that exactly r signals out of n are above the capture threshold for any $0 \leq r \leq n$. The numerical evaluation of this expression is scalable with the values of both n and r, unlike the expression in [11] that involves n nested integrations, and whose complexity is therefore exponential in n. From the capture probability, we obtain the exact expression of the *normalized system throughput* $S_n(R)$, defined as the mean number of successfully decoded packets out of a collision of size n, when there is a limit R (called MPR capability) to the number of signals that can be simultaneously decoded.[2]

Second, we extend the analysis to SIC systems, where the decoding process involves successive decoding iterations. At each iteration, the signals with power above the capture threshold are decoded and, then, subtracted from the compound received signal, leaving a fraction z of residual interference power. This process is repeated sequentially, until no further signal is captured or the maximum number K of interference cancellation iterations is reached. We provide the expression of the probability $C_n^{(s)}(r; K)$ that r signals out of n are decoded by a receiver capable of performing up to K interference cancellation iterations. Once again, from the capture probability we can derive the normalized system throughput as a function of the maximum permitted number K of SIC iterations.

Third, we derive simple approximate expressions, based on the central limit theorem, for a lightweight computation of the capture probabilities. In particular, we propose a novel, coarse approximation of the capture probability that can be computed without resorting to numerical integration. Furthermore, we

[2]As a side result, we generalized the capture probability expression to a heterogeneous scenario, where users may have different capture thresholds corresponding, for instance, to different modulation schemes or spreading factors. For space constraints, however, this generalization has not been reported here, and can be found in [19].

propose a simple recursive method to obtain an accurate approximation of the normalized throughput for SIC systems with extremely low computational complexity.

Finally, to assess the potential of the proposed method and illustrate the type of analysis it enables, we provide a selection of results obtained in some reference scenarios.

II. System model

The most natural application scenario for this study is the uplink of a wireless access network, such as a slotted Aloha system or an IEEE 802.11 cell. In this scenario, n radio terminals simultaneously transmit their signals to a common receiver. For the sake of simplicity, we only consider the case of synchronized transmissions of equal duration, which is a classical assumption in the related literature [16]–[18]. An analysis of the capture phenomenon in case of asynchronous transmissions of different length for IEEE 802.11 cells is presented in [20], where however the capture model is based on the number of simultaneous transmissions, without considering the SINR aspect.

The received powers P_j, with $j = 1, 2, \dots, n$, are assumed to be independent and identically distributed (iid) random variables, with Probability Density Function (PDF) $f_P(x)$ and Cumulative Distribution Function (CDF) $F_P(x)$ that depend on the transmit powers, the statistical distribution of the distance between the transmitter and the receiver, and the stochastic phenomena (fading, shadowing) that affect the signal propagation. The compound signal at the receiver is the superposition of the n overlapping radio signals transmitted by the users, with power equal to

$$\Lambda = \sum_{j=1}^{n} P_j + N_0 \, , \tag{2}$$

where N_0 accounts for the noise power. For the sake of simplicity, in the sequel we omit the noise term that is expected to be negligible with respect to the other terms.[3]

A signal with power P_j is captured if its SINR γ_j, given by (1), is larger than the capture threshold b,

$$\gamma_j = \frac{P_j}{\Lambda - P_j} > b \, . \tag{3}$$

For pure (no SIC) MPR systems, we assume that the receiver successfully decodes all the captured signals, up to the MPR capability R: if the number r of captured signals exceeds R, the remaining signals are actually not decoded.

For SIC systems, we suppose that at each iteration the receiver is capable of decoding all the signals that experience the capture condition. The decoded signals are then subtracted from the aggregate received signal before performing the next decoding iteration. The signal cancellation requires the receiver to reconstruct the waveform of the decoded users, an operation that involves the accurate estimation of the channel impulse response and errorless message decoding. When these operations are imperfect, the signal cancellation leaves some residual power that increases the noise level experienced at the successive decoding iterations. To model this idiosyncrasy of the interference cancellation process we assume that the cancellation of a signal received with power P leaves a residual interference power of zP, with $z \leq 1$. This proportional model for the residual interference power, though simple, is rather common in the literature [16], [21], and has been justified for some specific modulation schemes [22], [23]. In any case, the derivation presented in this paper can be adapted to more general models as well, at the cost of a more cumbersome notation.

The iterative SIC reception process stops when all signals are decoded, or no signals satisfy the capture condition in a decoding cycle. Furthermore, we assume that the SIC process can be performed up to K times, with $K = 0$ corresponding to the case of a pure MPR system with no SIC.

[3]The analysis can be extended to include the noise term, though at the cost of a more complex notation with no additional insight.

III. Analysis of pure multi-receiver systems

In this section, we consider pure MPR systems, lacking any SIC capability. The aim is to determine the expression of the probability

$$C_n(r) = \Pr\left[r \text{ signals out of } n \text{ are captured}\right] . \tag{4}$$

Computing (4) in MPR systems is difficult because the SINR of the different users are coupled. One possible way to handle this interdependency is to apply the law of total probability, conditioning on each random variable $\{P_j\}$, as done in [12]. However, this method generates a number of nested integrals that grows linearly with the number of users and, therefore, the resulting expression is not practical for more than a few users. With our approach, instead, we obtain for any n an expression with at most three nested integrals, which can be easily computed with numerical methods.

To begin with, we define the capture condition in terms of the overall aggregate received power Λ by rewriting (3) in the following form

$$P_j > \Lambda b' \tag{5}$$

where $b' = b/(1+b)$ is termed *modified capture threshold*. In this way, we express the capture condition in terms of minimum received signal power, $\Lambda b'$, which is the same for all the users.

Now, the derivation of the expression of $C_n(r)$ develops along four basic steps. First, we apply basic combinatorial analysis to express the unordered joint probability function $C_n(r)$ in terms of the ordered joint probability $c_n(r)$ of capturing the *first* r signals and missing the remaining $n - r$. Second, we condition on $\Lambda = x$ in such a way that the capture threshold $\Lambda b'$ on the right-hand side of (5) becomes deterministically equal to xb' so that, due to the independency of the received signal power from different nodes, we can easily compute the probability that r nodes are received with power above xb' and the others fall below the threshold. Third, we apply Bayes' rule to get the conditional PDF of the total received power Λ being x, given that r received signals have power in the interval (xb', ∞), and the other $n-r$ have power in the interval $[0, xb']$. The conditional received signal powers constrained in these intervals maintain their independency and are represented as auxiliary random variables $\alpha_h(u,v)$, where u and v denote the extremes of the interval. We can then express the conditional PDF of the aggregate power Λ evaluated in x, given that r signals have power in the interval (xb',∞) and $n-r$ in the interval $[0,xb']$, as the n-fold convolution of the PDFs of the auxiliary random variables $\alpha_h(u,v)$. This convolution can be efficiently performed in the frequency domain, which represents the last step. We can now state the final result and, then, detail the derivation.[4]

Theorem 1: For any positive n, and any values $0 \le r \le n$, the probability of capturing r out of n packets can be computed as

$$C_n(r) = \binom{n}{r} \int_0^\infty (1-F_P(xb'))^r \; F_P(xb')^{n-r} \times \left[\int_{-\infty}^{\infty} \left[\Psi_{\alpha(xb',\infty)}(\xi)\right]^r \left[\Psi_{\alpha(0,xb')}(\xi)\right]^{n-r} e^{i2\pi x\xi}\, d\xi\right] dx \tag{6}$$

where

$$\Psi_{\alpha(u,v)}(\xi) = \int_u^v \frac{f_P(a)}{F_P(v)-F_P(u)} e^{-i2\pi\xi a} da \tag{7}$$

for $u \le v$ and zero otherwise, and $i = \sqrt{-1}$.

Proof: The proof of the theorem develops along the four basic steps described above.
First step: Let U_0 be the set of captured signals and U_1 the set of missed (non-captured) signals. Due to the symmetry of the problem, the r captured signals can be arbitrarily chosen. Hence, without loss of generality, we can write

$$C_n(r) = \binom{n}{r} c_n(r) \tag{8}$$

where $c_n(r)$ is the probability that signals $U_0 = \{1,2,\ldots,r\}$, are captured and signals $U_1 = \{r+1,\ldots,n\}$ are missed. In formula:

$$c_n(r) = \Pr\left[\mathbf{P}_0 > \Lambda b', \mathbf{P}_1 \le \Lambda b'\right] \tag{9}$$

where, for brevity, we adopted the compact notation $\mathbf{P}_0 > v$ in place of $\{P_j > v, \forall j \in U_0\}$ and similarly for the opposite inequalities.
Second step: Applying the total law of probability on Λ, we get

$$\begin{aligned} c_n(r) &= \int_0^\infty \Pr\left[\mathbf{P}_0 > xb', \mathbf{P}_1 \le xb' | \Lambda = x\right] f_\Lambda(x) dx \\ &= \int_0^\infty \Pr\left[\boldsymbol{\mathcal{E}} | \Lambda = x\right] f_\Lambda(x) dx \end{aligned} \tag{10}$$

where $f_\Lambda(x)$ is the PDF of the aggregate received power Λ, and we set $\boldsymbol{\mathcal{E}} = \{\mathbf{P}_0 > xb', \mathbf{P}_1 \le xb'\}$, for compactness.
Third step: Applying the rule of Bayes, we obtain

$$\begin{aligned} c_n(r) &= \int_0^\infty f_\Lambda(x|\boldsymbol{\mathcal{E}}) \Pr\left[\boldsymbol{\mathcal{E}}\right] dx \\ &= \int_0^\infty f_\Lambda(x|\boldsymbol{\mathcal{E}}) (1-F_P(xb'))^r F_P(xb')^{n-r} dx \end{aligned} \tag{11}$$

where

$$f_\Lambda(x|\boldsymbol{\mathcal{E}}) = \lim_{h\to 0} \frac{\Pr\left[\sum_{j=1}^n P_j \in (x-h,x] \,\middle|\, \mathbf{P}_0 > xb', \mathbf{P}_1 \le xb'\right]}{h}$$

is the *conditional* PDF of the aggregate received signal power Λ, given $\boldsymbol{\mathcal{E}}$, i.e., given that the first r signals have power above the threshold $\Lambda b' = xb'$, and the remaining $n-r$ have power below such a threshold.
Fourth step: We now introduce the parameterized random variable $\bar{\Lambda}(y)$ defined as

$$\bar{\Lambda}(y) = \sum_{h=1}^{r} \alpha_h(y,\infty) + \sum_{h=r+1}^{n} \alpha_h(0,y) \,, \tag{12}$$

where, for any $0 \le u \le v$, $\alpha_h(u,v)$ are iid random variables with common PDF

$$f_{\alpha(u,v)}(a) = \frac{f_P(a)}{F_P(v)-F_P(u)} \,, \quad \text{for} \quad a \in (u,v] \,; \tag{13}$$

and zero otherwise. In practice, (13) is the conditional PDF of P given that $P \in (u,v]$.[5]

Due to the statistical independence of the terms in (12), the PDF $f_{\bar{\Lambda}(y)}(a)$ of $\bar{\Lambda}(y)$ is equal to the multi-fold convolution of $f_{\alpha(y,\infty)}(a)$ and $f_{\alpha(0,y)}(a)$. In the frequency domain, the Fourier Transform (FT) $\Psi_{\bar{\Lambda}(y)}(\xi)$ of $f_{\bar{\Lambda}(y)}(a)$ becomes

$$\Psi_{\bar{\Lambda}(y)}(\xi) = \left[\Psi_{\alpha(y,\infty)}(\xi)\right]^r \left[\Psi_{\alpha(0,y)}(\xi)\right]^{n-r} \tag{14}$$

where $\Psi_{\alpha(u,v)}(\xi)$ is the FT of $f_{\alpha(u,v)}(a)$, which is given by (7). The function $f_{\bar{\Lambda}(y)}(x)$ can be obtained from (14) through inverse FT, that is

$$f_{\bar{\Lambda}(y)}(x) = \int_{-\infty}^{\infty} \left[\Psi_{\alpha(y,\infty)}(\xi)\right]^r \left[\Psi_{\alpha(0,y)}(\xi)\right]^{n-r} e^{i2\pi x\xi}\, d\xi \,. \tag{15}$$

We now notice that, for any x, the function $f_{\bar{\Lambda}(y)}(x)$ with $y = xb'$ is equal to $f_\Lambda(x|\boldsymbol{\mathcal{E}})$. Hence, (11) can be expressed as

$$c_n(r) = \int_0^\infty f_{\bar{\Lambda}(xb')}(x) \left(1-F_P(xb')\right)^r \; F_P(xb')^{n-r} dx \,. \tag{16}$$

Replacing (15) into (16) and the result into (8) we finally get (6). ■

Note that this result is completely general and holds for any spatial distribution of the transmitters and any propagation model, provided that the received powers are iid. The actual evaluation of (6) might require numerical methods for the computation of the two nested integrals and of the FT (7), when it cannot be expressed in closed form.[6]In any case, the computational complexity of (6) is limited for all the cases of interest and, most importantly, it is essentially independent of r and n, so that our method is very scalable. On the other hand, the expression provided in [11, Eq. (19)] only gives the probability of capturing *at least* one signal (which is equal to $1-C_n(0)$), and requires the explicit computation of n nested

[4]Even though the statement and the proof of Theorem 1 are given here with reference to continuous distributions of the received powers, the same result can be shown to hold true for any probability distributions as well.

[5]If $\Pr[P=0] > 0$, the PDF $f_{\alpha(0,v)}(a)$ must be defined for $a \in [0,v]$ rather than $a \in (0,v]$.

[6]An efficient way to compute the FTs and their inverse is described in [18], [19]

integrals, whose complexity grows exponentially with n, and therefore cannot be used except for very small collision sizes.[7]

We now turn our attention to the *normalized system throughput*, defined as the expected number of packets that can be successfully decoded in a slot in which n users transmit. This performance figure has been deeply analyzed in the literature, mainly for: (i) systems with single reception capability ($R = 1$), i.e., able to decode only one packet even when multiple signals experience $SINR > b$, or (ii) systems with infinite reception capability ($R = \infty$), i.e., capable of correctly receiving all the packets that satisfy the capture condition [11].

In this work, we generalize the analysis to systems that can actually decode at most $R \geq 1$ simultaneous signals (e.g., due to hardware limitations), even when the number of signals above the capture threshold is larger than R. Denoting by $S_n(R)$ the normalized throughput of a system with MPR capability $R \leq n$, we have

$$S_n(R) = \sum_{r=1}^{R-1} r\, C_n(r) + R \sum_{r=R}^{n} C_n(r) = \sum_{r=1}^{R-1} r\, C_n(r) + R Q_n(R) \tag{17}$$

where $Q_n(R) = \sum_{r=R}^{n} C_n(r)$ is called *full-capacity reception probability*, since it denotes the probability that R or more signals are above the capture threshold and, consequently, the multi-reception capability of the receiver is fully exploited. Using (6) into (17), we can compute the normalized system throughput for any value of the reception capability R. In particular, the normalized throughput of single reception systems is equal to $S_n(1) = Q_n(1) = 1 - C_n(0)$, whereas the normalized throughput of infinite-reception systems is $S_n(\infty) = \mathtt{E}[r|n]$, where $\mathtt{E}[r|n]$ denotes the expected value of the number of captured signals (out of n) and can be computed as in [10].

IV. Analysis of Successive-Interference-Cancellation systems

The derivation of the capture probability to systems with SIC basically follows the same rationale as in Sec. III, but is slightly more complex and requires a more cumbersome notation due to the iterative nature of the decoding process.

Let k be the iteration at which the reception process ends, i.e., in which the last capture occurs. Furthermore, let U_h be the set of signals that are decoded at the hth iteration, with $h = 0, 1, \ldots, k$. The signals that remain to be decoded at the end of the reception process, if any, are collected in the set U_{k+1}. The aggregate power of the signals in set U_h will be denoted by[8]

$$\Gamma_h = \sum_{j \in U_h} P_j\,, \quad h = 0, 1, \ldots, k+1\,. \tag{18}$$

Since the decoded signals are cancelled from the overall received signal, leaving a fraction z of their power as residual interference, the overall signal power at decoding iteration h can be expressed as

$$\Lambda_h = z \sum_{j=0}^{h-1} \Gamma_j + \sum_{\ell=h}^{k+1} \Gamma_\ell + N_0\,,$$

where the first sum is zero when $h = 0$. Once again, in the following the noise term will be omitted. A signal with power P_j will be captured at decoding iteration h if

$$P_j > \Upsilon_h = \Lambda_h b' \tag{19}$$

where Υ_h is called *absolute capture threshold* for iteration h.

We wish to determine the expression of the probability

$$C_n^{(s)}(r; K) = \Pr[r \text{ signals out of } n \text{ are captured within at most } K \text{ SIC iterations}]\,. \tag{20}$$

The derivation unfolds along the four phases described in Sec. III that, however, becomes slightly more involved.

First step: To begin with, we observe that $C_n^{(s)}(r; 0) = C_n(r)$ and, for any K, $C_n^{(s)}(0; K) = C_n(0)$, which can be computed using (6). In the following, we hence assume $K \geq 1$ and $r > 0$. The capture probability can thus be expressed as

$$C_n^{(s)}(r; K) = \sum_{k=0}^{\min(r-1, K)} C_n^{\star}(r; k)\,, \tag{21}$$

where $C_n^{\star}(r; k)$ is the probability of capturing r signals with the last capture occurring at iteration k. It shall be noted that, according to the definition of k, it must be $r \geq k+1$, since *at least* one signal has to be decoded at each iteration for the reception process to continue. Hence, let r_h denote the number of signals decoded at the hth iteration, with $h = 0, 1, \ldots, k$, and let $r_{k+1} = n - r$ denote the number of undecoded signals at the end of the reception process. Due to the symmetry of the problem, we can write

$$C_n^{\star}(r; k) = \sum_{r_0=1}^{r-k} \sum_{r_1=1}^{r-r_0-k+1} \cdots \sum_{r_{k-1}=1}^{r-r_0-\ldots-r_{k-2}-1} \mathcal{A} c(\mathbf{r}) \tag{22}$$

where, for compactness, we used $\mathbf{r}$ to denote the vector $\{r_0, r_1, \ldots, r_{k+1}\}$, and we set $\mathcal{A} = \frac{n!}{r_0! r_1! \ldots r_{k+1}!}$. The function $c(\mathbf{r})$ is the *ordered* capture probability for the vector $\mathbf{r}$, i.e., the probability that signals 1 to r_0 are captured at iteration zero, signals $r_0 + 1$ to $r_0 + r_1$ are captured at iteration one, and so on, and that signals from $\sum_{h=0}^{k} r_h + 1$ to n remain undecoded at the end of the reception process.

We now need to distinguish the case $k < K$, when the reception ends because no signal is captured at iteration $k+1$, from the case $k = K$, when the reception process is terminated because the maximum allowed number of SIC iterations has been reached. In the following, we derive the expression of $c(\mathbf{r})$ for $k < K$, and then we explain how to adjust the result for the case $k = K$.

Recalling (19), we can express the ordered capture probability for $k < K$ as

$$c(\mathbf{r}) = \Pr\left[\mathbf{P}_0 > \Upsilon_0 \geq \mathbf{P}_1 > \Upsilon_1 \geq \cdots \geq \mathbf{P}_k > \Upsilon_k, \mathbf{P}_{k+1} \leq \Upsilon_{k+1}\right] \tag{23}$$

[7] The approach proposed in this section can be easily extended to a heterogeneous scenario, where both the capture threshold and the PDF of the received signal powers may differ among users. The extension, which unfolds exactly as in the homogeneous case but requires a more cumbersome notation, can be found in [19].

[8] If the set U_{k+1} is empty, the aggregate power Γ_{k+1} is conventionally set to 0.

where we adopted the same compact notation introduced in Sec. III.[9]

Second step: Applying the total probability theorem with respect to the random variables Γ_h, we obtain

$$c(\mathbf{r}) = \iint f_{\mathbf{\Gamma}}(\mathbf{g}) \Pr\Big[\mathbf{P}_0 > \lambda_0 \geq \mathbf{P}_1 > \lambda_1 \geq \cdots \quad (24)$$
$$\cdots \geq \mathbf{P}_k > \lambda_k, \mathbf{P}_{k+1} \leq \lambda_{k+1} \Big| \mathbf{\Gamma} = \mathbf{g}\Big] dg_0 \cdots dg_{k+1}$$

where we used $\mathbf{\Gamma}$ and $\mathbf{g}$ in place of $\{\Gamma_0, \cdots, \Gamma_{k+1}\}$, and $\{g_0, \cdots, g_{k+1}\}$, respectively, and we set[10]

$$\lambda_h = \left(z \sum_{j=0}^{h-1} g_j + \sum_{\ell=h}^{k+1} g_\ell\right) b', \quad h = 0, \ldots, k+1\,.$$

The function $f_{\mathbf{\Gamma}}(\mathbf{g})$ in (24) denotes the joint PDF of $\mathbf{\Gamma}$ evaluated in $\mathbf{g}$.

Third step: Now, applying the rule of Bayes we get

$$c(\mathbf{r}) = \iint f_{\mathbf{\Gamma}}(\mathbf{g}|\boldsymbol{\mathcal{E}}) \Pr[\boldsymbol{\mathcal{E}}]\, dg_0 \cdots dg_{k+1} \quad (25)$$

where $\boldsymbol{\mathcal{E}} = \bigcap_{h=0}^{k+1} \mathcal{E}_h$ with[11] $\mathcal{E}_h = \{\mathbf{P}_h \in (\lambda_h, \lambda_{h-1}]\}$ for $h = 0, \ldots, k$, and $\mathcal{E}_{k+1} = \{\mathbf{P}_{k+1} \in [0, \lambda_{k+1}]\}$. The function $f_{\mathbf{\Gamma}}(\mathbf{g}|\boldsymbol{\mathcal{E}})$ is the *conditional* joint PDF of $\mathbf{\Gamma}$ given $\boldsymbol{\mathcal{E}}$, evaluated in $\mathbf{g}$, i.e.,

$$f_{\mathbf{\Gamma}}(\mathbf{g}|\boldsymbol{\mathcal{E}}) = \lim_{\mathbf{h}\to 0} \frac{\Pr[\mathbf{\Gamma} \in (\mathbf{g}-\mathbf{h}, \mathbf{g}] | \boldsymbol{\mathcal{E}}]}{h_0 h_1 \cdots h_{k+1}}\,.$$

We observe that the events in $\boldsymbol{\mathcal{E}}$ are mutually independent, so that we can write

$$\Pr[\boldsymbol{\mathcal{E}}] = F_P(\lambda_{k+1})^{r_{k+1}} \prod_{h=0}^{k} [F_P(\lambda_{h-1}) - F_P(\lambda_h)]^{r_h}\,. \quad (26)$$

Furthermore, the conditional PDF of $\mathbf{\Gamma}$ given $\boldsymbol{\mathcal{E}}$ can also be factorized as

$$f_{\mathbf{\Gamma}}(\mathbf{g}|\boldsymbol{\mathcal{E}}) = \prod_{h=0}^{k+1} f_{\Gamma_h}(g_h|\mathcal{E}_h)\,. \quad (27)$$

Fourth step: We now introduce the family of parameterized auxiliary random variables

$$\bar{\Gamma}_h(u,v) = \sum_{\ell=1}^{r_h} \alpha_{h,\ell}(u,v)\,, \quad h = 0, \ldots, K+1\,; \quad (28)$$

where $\alpha_{h,\ell}(u,v)$ are iid random variables with PDF as in (13).

The PDF $f_{\bar{\Gamma}_h(u,v)}(x)$ of $\bar{\Gamma}_h(u,v)$ is hence given by the r_h-fold convolution of $f_{\alpha(u,v)}(x)$ and can be obtained as

$$f_{\bar{\Gamma}_h(u,v)}(x) = \int_{-\infty}^{\infty} \left[\Psi_{\alpha(u,v)}(\xi)\right]^{r_h} e^{i2\pi x\xi}\, d\xi\,. \quad (29)$$

We now notice that, for any x, the function $\bar{\Gamma}_h(u,v)(x)$ is equal to $f_{\Gamma_h}(x|\mathcal{E}_h)$ when u and v correspond to the limits of the interval in $\mathcal{E}_h$. Hence, for $k < K$ we have

$$\begin{aligned} c(\mathbf{r}) = \iint_0^\infty & F_P(\lambda_{k+1})^{r_{k+1}} \left[\prod_{h=0}^{k} (F_P(\lambda_{h-1}) - F_P(\lambda_h))^{r_h}\right] \\ & \left[\prod_{h=0}^{k} \int_{-\infty}^{\infty} \left[\Psi_{\alpha(\lambda_h,\lambda_{h-1})}(\xi)\right]^{r_h} e^{i2\pi g_h\xi}\, d\xi\right] \quad (30) \\ & \left[\int_{-\infty}^{\infty} \left[\Psi_{\alpha(0,\lambda_{k+1})}(\xi)\right]^{r_{k+1}} e^{i2\pi g_{k+1}\xi}\, d\xi\right] dg_0 \cdots dg_{k+1}\,. \end{aligned}$$

For $k = K$, the ordered capture probability becomes

$$c(\mathbf{r}) = \Pr[\mathbf{P}_0 > \Upsilon_0 \geq \mathbf{P}_1 > \Upsilon_1 \geq \cdots \geq \mathbf{P}_K > \Upsilon_K, \mathbf{P}_{K+1} \leq \Upsilon_K] \quad (31)$$

which is very similar to (23), except for the fact that the upper bound of $\mathbf{P}_{K+1}$ is now Υ_K rather than Υ_{K+1}. Hence, (30) can be directly extended to the case $k = K$ by defining

$$\lambda_{K+1} = \lambda_K = \left(z \sum_{j=0}^{K-1} g_j + \sum_{\ell=K}^{K+1} g_\ell\right) b'\,.$$

Putting all the pieces together, we can finally express (21) as[12]

$$\begin{aligned} C_n^{(s)}(r;K) = & \sum_{k=0}^{\min\{r-1,K\}} \sum_{r_0=1}^{r-k} \sum_{r_1=1}^{r-r_0-k+1} \cdots \sum_{r_{k-1}=1}^{r-r_0-\ldots-r_{k-2}-1} \mathcal{A} \\ & \iint_0^\infty F_P(\lambda_{k+1})^{r_{k+1}} \left[\prod_{h=0}^{k} (F_P(\lambda_{h-1}) - F_P(\lambda_h))^{r_h}\right] \\ & \left[\prod_{h=0}^{k} \int_{-\infty}^{\infty} \left[\Psi_{\alpha(\lambda_h,\lambda_{h-1})}(\xi)\right]^{r_h} e^{i2\pi g_h\xi}\, d\xi\right] \quad (32) \\ & \left[\int_{-\infty}^{\infty} \left[\Psi_{\alpha(0,\lambda_{k+1})}(\xi)\right]^{r_{k+1}} e^{i2\pi g_{k+1}\xi}\, d\xi\right] dg_0 \cdots dg_{k+1}\,. \end{aligned}$$

It shall be noted that the number of nested integrals grows proportionally to (and hence the computational complexity grows exponentially with) K, which is however expected to be limited in practical systems for complexity, latency and efficiency reasons. Conversely, the computational complexity of (32) grows much more slowly as a function of r and n, so that our method is very scalable with respect to the number of users in the system, which can take also large values.

For SIC systems it makes sense to consider the mean normalized throughput as a function of the maximum permitted number of SIC iterations K. Hence, in this case the normalized system throughput becomes

$$S_n^{(s)}(K) = \sum_{r=1}^{n} r C_n^{(s)}(r;K)\,. \quad (33)$$

[9]If U_{k+1} is empty, the inequality $\mathbf{P}_{k+1} \leq \Upsilon_{k+1}$ is trivially verified.

[10]When $h = 0$, the first sum is zero.

[11]For preserving the expression symmetry, we introduced the dummy parameter $\lambda_{-1} = \infty$.

[12]Note that, for $K = 0$, (32) returns $C_n(r)$, though with a slightly different expression with respect to (6).

V. Low-complexity approximations of capture probability and throughput

Although in most cases the numerical evaluation of the capture probability distributions (6) and (32), and of the normalized throughput functions (17) and (33) is affordable, sometimes it might be preferable to resort to approximate methods that provide fairly good results at a much lower computational cost. In the following we propose some possible approximations that trade off results accuracy for numerical complexity. We first propose approximations for the capture probability distributions and, thereafter, we turn our attention to the normalized throughput expressions.

A. Capture probability approximations

The main issue in computing (6) and (32) consists in the numerical evaluation of the PDF of $\bar{\Lambda}(xb')$ and $\bar{\Gamma}_h(u,v)$, respectively. However, the computation of these functions can be greatly simplified by resorting to the Central-Limit-Theorem (CLT). In fact, for sufficiently large r_h, the distribution of terms like $\sum_{\ell=1}^{r_h}\alpha_{h,\ell}(u,v)$ that appear in the expressions of $C_n(r)$ and $C_n^{(s)}(r;K)$ can be approximated by a Gaussian distribution, with mean $r_h m_{\alpha(u,v)}$ and variance $r_h\sigma^2_{\alpha(u,v)}$, where $m_{\alpha(u,v)}$ and $\sigma^2_{\alpha(u,v)}$ are the mean and variance of the random variable $\alpha(u,v)$, provided that they exist and are finite.

For instance, the PDF of the parameterized random variable $\bar{\Lambda}(y)$, defined in (12), can be approximated as

$$\begin{aligned} f_{\bar{\Lambda}(y)}(a) &\simeq \frac{1}{\sqrt{2\pi\sigma_r^2(y)}}\exp\left(-\frac{(a-m_r(y))^2}{2\sigma_r^2(y)}\right) \\ &= \frac{1}{\sigma_r(y)}\phi\left(\frac{a-m_r(y)}{\sigma_r(y)}\right) \end{aligned} \tag{34}$$

where $\phi(\cdot)$ is the standard normal PDF and

$$\begin{aligned} m_r(y) &= r m_{\alpha(y,\infty)} + (n-r)\, m_{\alpha(0,y)} \;; \\ \sigma_r^2(y) &= r\sigma^2_{\alpha(y,\infty)} + (n-r)\sigma^2_{\alpha(0,y)} \;. \end{aligned} \tag{35}$$

With a change of variable ($y = xb'$) and a simple rearrangement of the terms, the capture probability $C_n(r)$ for pure MPR systems can thus be approximated as

$$\tilde{C}_n(r) = \binom{n}{r}\int_0^\infty \phi\left(\frac{y-b'm_r(y)}{b'\sigma_r(y)}\right)\frac{[1-F_P(y)]^r F_P(y)^{n-r}}{b'\sigma_r(y)}dy \tag{36}$$

The numerical solution of (36) requires a single integration, which is generally much faster than the numerical solution of (6), and can therefore be used as a simple approximation. In particular, the approximation is excellent for $r=0$, and $\tilde{C}_n(0)$ turns out to be very close to the correct value $C_n(0)$ already for $n>4$. This result is of particular interest because it provides a very simple way to have an accurate estimate of the probability that *at least* one signal is captured, which corresponds to the normalized throughput for single reception systems, $S_n(1) = Q_n(1) = 1 - C_n(0)$, and is the performance metric considered in most of the previous literature on the subject [9]–[12].

The same approach can be used to avoid the computation of the innermost numerical integrals in (32). In particular, for sufficiently large values of r_h, the CLT approximation for the expression (29) yields

$$f_{\tilde{\Gamma}_h(u,v)}(x) \simeq \frac{\exp\left(-\frac{(x-r_h\, m_{\alpha(u,v)})^2}{2\, r_h\, \sigma^2_{\alpha(u,v)}}\right)}{\sqrt{2\pi\, r_h \sigma^2_{\alpha(u,v)}}} \,. \tag{37}$$

The expression (36) can be further simplified, at the cost of some additional loss of accuracy. For sufficiently small values of the coefficient of variation $\sigma_r(y)/m_r(y)$, the Gaussian distribution function is very narrow around the mean $b'm_r(y)$, which depends on y as shown by (35). Accordingly, the terms $\phi\left(\frac{y-b'm_r(y)}{b'\sigma_r(y)}\right)$ in (36) are generally very small when y lies far away from the mean $b'm_r(y)$, whereas for $y \simeq b'm_r(y)$ the Gaussian term can be approximated by an impulse with area $b'\sigma_r(y)$. Using this approximation in (36), we obtain this new approximate expression of the capture probability distribution:

$$\hat{C}_n(r) \simeq \binom{n}{r}\sum_{y^\star\in Y^\star}[1-F_P(y^\star)]^r F_P(y^\star)^{n-r} \,, \tag{38}$$

where $Y^\star$ is the set of points such that

$$y^\star = b'm_r(y^\star)\,. \tag{39}$$

The approximate expression (38) does not involve any numerical integration, though computing $Y^\star$ may still require numerical methods in some cases. Even though this approximation is rather coarse in general, it gives an idea of the shape of the probability distribution and provides a good estimate of some useful metrics. In particular, the zero-capture probability $C_n(0)$ and the full capture probability $C_n(n)$ are quite well approximated by (38), that thus enables the asymptotic analysis of these two important performance measures, as will be discussed later on.

B. Throughput approximations

We here propose a first-order approximation of the capture probabilities that yields a simple, recursive method to estimate the normalized throughput of both pure and SIC-capable MPR systems.

1) Pure multi-receiver case: Here we focus on the normalized throughput $S_n(\infty)$ of pure MPR systems with infinite reception capability. The number of decoded signal can be expressed as $r = \sum_{j=1}^n \chi_j$ where $\chi_j = 1$ if the jth signal is captured and zero otherwise. Hence, we can write[13]

$$S_n(\infty) = \mathtt{E}\left[r|n\right] = n\mathtt{E}\left[\chi_j\right] = n\Pr\left[P_1 > \mathcal{I}_0\right] \tag{40}$$

where $\mathcal{I}_0 = (N_0 + \sum_{j=2}^n P_j)b$. The right-most term of (40) is the probability that a certain signal, say the first one, is captured. This probability can be computed in different ways. A first possibility is to determine the PDF of $\mathcal{I}_0$ as the $(n-1)$-fold convolution of $f_P(\cdot)$ (neglecting, as usual, the noise term). Otherwise, as in the previous section, we can resort to the central limit theorem and approximate $\mathcal{I}_0$ with a Gaussian random variable, with mean $(n-1)m_{\alpha(0,\infty)}b$ and variance $(n-1)\sigma^2_{\alpha(0,\infty)}b^2$. However, we here prefer to follow yet another strategy that yields an even simpler solution. In practice,

[13] Note that here we consider the capture threshold b rather than the modified capture threshold b'.

we approximate $\mathcal{I}_0$ with its mean $I_0 = (n-1)m_{\alpha(0,\infty)}b$. Hence, from (40), we get the following normalized throughput approximation

$$\hat{S}_n(\infty) = n\big[1 - F_P\big((n-1)m_{\alpha(0,\infty)}b\big)\big]\,, \qquad (41)$$

which can be directly computed from the CDF of P in an extremely simple manner. As we will see in Sec. VII, despite its simplicity, (41) provides an accurate approximation of $S_n(\infty)$ in many cases, in particular when the number of nodes is large and the capture threshold b is small. More importantly, this trivial normalized throughput approximation for pure MPR systems paves the way for the derivation of the throughput approximation in SIC systems, which is presented next.

2) Successive Interference Cancellation case: The basic idea consists in computing an estimate of the mean number $\tilde{r}_h$ of signals decoded at each iteration h, taking into account the effect of interference cancellation. For the generic iteration h, we define the following parameters: n_h is the number of signals that are not yet decoded at the beginning of the iteration, I_h is the approximated value of the absolute capture threshold $\mathcal{I}_h$ for these signals and ρ_h is the additional residual interference that will result from the cancellation of the $\tilde{r}_h$ signals at the end of decoding iteration h. For $h = 0$, we have $n_0 = n$ and

$$\tilde{r}_0 = n_0 \Pr[P_j > I_0] = n_0[1 - F_P(I_0)]\,, \qquad (42)$$

as given by (41). The residual interference left by the cancellation of the $\tilde{r}_0$ signals, in turn, will be approximated as

$$\rho_0 = z\tilde{r}_0 \mathtt{E}[P|P > I_0] = z\tilde{r}_0 m_{\alpha(I_0,\infty)}\,.$$

Notice that, when computing the residual interference, the mean power of a captured signal is constrained to be larger than the (approximated) absolute capture threshold I_0. Analogously, the power of the signals considered in the subsequent iteration will be constrained to be lower than I_0, and the same applies to the following iterations. Hence, for the generic iteration $h > 0$, we define

$$n_h = n - \sum_{j=0}^{h-1}\tilde{r}_j\,; \quad I_h = \left[\sum_{j=0}^{h-1}\rho_j + (n_h - 1)m_{\alpha(0,I_{h-1})}\right]b\,;$$
$$F_{\alpha(0,I_{h-1})}(I_h) = \Pr[P \le I_h | P \le I_{h-1}]\,; \qquad (43)$$
$$\tilde{r}_h = n_h\big(1 - F_{\alpha(0,I_{h-1})}(I_h)\big)\,; \quad \rho_h = z\tilde{r}_h m_{\alpha(I_h,I_{h-1})}\,.$$

Finally, the approximate normalized throughput for K SIC iterations is given by

$$\tilde{S}_n^{(s)}(K) = \sum_{j=0}^{K}\tilde{r}_j\,. \qquad (44)$$

For $b \ge 1$ the approximation can be further improved as described in [19].

VI. Definition and characterization of reference scenarios

To exemplify possible applications of the developed analysis, we consider three reference scenarios, namely Path Loss (PL), Rayleigh Fading (RF), and Lognormal (LN), which are detailed in the following.[14]

A. Path Loss model

For the sake of comparison with the previous literature, the first scenario included in our analysis is the so-called Path Loss (PL) model and refers to the case proposed in Section II.E of [11] and in other recent works [24]. The scenario consists of n users uniformly distributed in a circle of radius $\mathcal{D}$ centered at the common receiver (e.g., a Base Station of a cellular network, or an Access Point in a WLAN), so that the PDF of the distance r_j from the jth user to the common receiver is given by

$$f_{r_j}(a) = \frac{2a}{\mathcal{D}^2}, \quad \text{for} \quad 0 \le a \le \mathcal{D}\,,$$

and zero otherwise. The radio propagation is governed by a simple deterministic path-loss law, with neither fading nor shadowing. As in [11], we assume that the received power at a distance r from the transmitter is equal to $P(r) = (1+r)^{-\eta}$, where η is the path loss coefficient whereas the constant unit term is added to avoid a physically absurd behavior for $r \to 0$. Note that, for the sake of notation simplicity, we omit any multiplicative constant in the path-loss equation that, in any case, would be irrelevant in our analysis. Hence, the power P received from a generic node can be modeled as a random variable that takes values in the interval $[(\mathcal{D}+1)^{-\eta}, 1]$, with PDF and CDF given by[15]

$$f_P(x) = \frac{2}{\mathcal{D}^2\eta}\left(x^{-\frac{2}{\eta}-1} - x^{-\frac{1}{\eta}-1}\right),\; F_P(x) = 1 - \frac{\left(1 - x^{-\frac{1}{\eta}}\right)^2}{\mathcal{D}^2} \qquad (45)$$

for $(\mathcal{D}+1)^{-\eta} \le x \le 1$. From (45), it is then easy to derive the PDF of the auxiliary random variable $\alpha(u,v)$ that turns out to be equal to

$$f_{\alpha(u,v)}(a) = \frac{2\left(a^{-\frac{2}{\eta}-1} - a^{-\frac{1}{\eta}-1}\right)}{\mathcal{D}^2\eta(F_P(v) - F_P(u))}, \qquad (46)$$

for $\max((\mathcal{D}+1)^{-\eta}, u) < a \le \min(v, 1)$, and zero otherwise.

The FT of the PDF of $\alpha(u,v)$ depends on η. For space constraints, we report here only the result for $\eta = 2$ that, for $(\mathcal{D}+1)^{-2} \le u \le v \le 1$, turns out to be given as in (47) (see next page) where $[g(x)]_u^v = g(v) - g(u)$, and $\mathrm{Ei}(m,z) = \int_1^\infty \frac{\exp(-za)}{a^m}da$ is the exponential integral function.

The statistical mean and power of $\alpha(u,v)$, still for $\eta = 2$, are given respectively by

$$m_{\alpha(u,v)} = \frac{\log(v) - 2\sqrt{v} - \log(u) + 2\sqrt{u}}{\mathcal{D}^2(F_P(v) - F_P(u))}\,, \qquad (48)$$
$$M_{\alpha(u,v)} = \frac{3(v-u) - 2(\sqrt{v} - \sqrt{u})}{3\mathcal{D}^2(F_P(v) - F_P(u))}\,.$$

[14] In [18] we considered an additional scenario, where we combine the classical path loss model with the Rayleigh fading model.

[15] We observe that, for $\mathcal{D} \ge 10$, the PDF obtained with this model is basically equivalent to that returned by the classical path loss model $r^{-\eta}$ when user locations are constrained to be at least one meter apart from the transmitter, i.e., $r \in [1, \mathcal{D}]$.

$$\Psi_{\alpha(u,v)} = \frac{\left[2\,\frac{e^{-i\,2\pi x f}}{\sqrt{x}} + \frac{i2\pi^{3/2} f\sqrt{2}\,\mathrm{erf}\left(\sqrt{i2\pi f x}\right)}{\sqrt{i\pi f}} - \frac{e^{-i\,2\pi x f}}{x} + i\,2\pi f \mathrm{Ei}\left(1, i\,2x\pi f\right)\right]_u^v}{\mathcal{D}^2(F_P(v) - F_P(u))} \tag{47}$$

B. Rayleigh Fading model

In the Rayleigh Fading (RF) scenario, we assume that the path loss attenuation is the same for all users, as if all transmitters were at the same distance from the receiver or they used a power control mechanism that is capable of compensating the long term path loss attenuation. Therefore, the received signals have equal mean power, that we normalize to one. However, signals are affected by multi-path fading, so that the received power for the jth user will be an exponentially distributed random variable, with PDF and CDF given by

$$f_P(a) = e^{-a}\,, \quad F_P(a) = 1 - e^{-a}\,, \quad a \geq 0. \tag{49}$$

In this case, the FT of the PDF of $\alpha(u,v)$ is given by

$$\Psi_{\alpha(u,v)}(\xi) = \frac{e^{-u(i\,2\pi\,\xi+1)} - e^{-v(i\,2\pi\,\xi+1)}}{(1+i\,2\pi\,\xi)(e^{-u}-e^{-v})}\,,$$

whereas the statistical mean and power are

$$m_{\alpha(u,v)} = \frac{(u+1)e^{-u} - (v+1)e^{-v}}{e^{-u}-e^{-v}}\,, \tag{50}$$
$$\mathrm{M}_{\alpha(u,v)} = \frac{(2+2u+u^2)e^{-u} - (2+2v+v^2)e^{-v}}{e^{-u}-e^{-v}}\,.$$

We observe that, due to the memoryless property of the exponential distribution, the PDF of the auxiliary random variable $\bar{\Gamma}_h(u,\infty)$, defined in (28), admits the following closed form expression:

$$f_{\bar{\Gamma}_h(u,\infty)}(a) = \frac{(a-ur_h)^{r_h-1}}{(r_h-1)!} e^{-(a-ur_h)}\,; \quad a \geq ur_h\,.$$

It is thus possible to avoid the numerical computation of the inverse FT of $\left[\Psi_{\alpha(\lambda_0,\infty)}(\xi)\right]^{r_h}$ in (32). In particular, it is possible to get a closed form expression of the full-capture probability, which is $C_n(n) = (1-nb')^{n-1}$ for $nb' < 1$ and zero otherwise.

C. Lognormal power distribution model

Another interesting statistical distribution of the received signal power P is the lognormal model (LN), according to which the received signals have iid powers with PDF and CDF

$$f_P(x) = \frac{1}{x\sqrt{2\pi}\sigma} e^{\left\{\frac{-[\ln(x)-\mu]^2}{2\sigma^2}\right\}}\,, \tag{51}$$
$$F_P(x) = \frac{1}{2}\,\mathrm{erfc}\left[-\frac{\ln(x)-\mu}{\sigma\sqrt{2}}\right] = \Phi\left(\frac{\ln(x)-\mu}{\sigma}\right),$$

where $\ln(\cdot)$ is the natural logarithm, $\mathrm{erfc}(\cdot)$ is the complementary error function, and $\Phi(\cdot)$ is the standard normal CDF. The parameters μ and σ are the mean and standard deviation, respectively, of the associated normal distribution.

The LN model may be considered, for instance, when the terminals are at the same distance from the receiver, but the received signals are affected by independent shadowing fading terms. In this case, the parameters of the PDF can be expressed as $\mu = \mu_{dB}/A$ and $\sigma = \sigma_{dB}/A$ where $A = 10/\ln(10)$, μ_{dB} is the mean received power, in dB, whereas σ_{dB} is the standard deviation of the shadowing fading terms that ranges from 4 dB to 13 dB in outdoor environments [25, Ch. 2, p. 50]. The LN model can also be applied to CDMA systems that adopt power-control schemes to harmonize the power of the different received signals. Considering for simplicity the homogeneous-rate case, an ideal power control scheme shall make all the signals be received with equal target power $\bar{P}$, so that the capture condition (3) is satisfied by all signals. In practice, however, the actual power received from node $j = 1, 2, \ldots, n$, in dB, will be equal to $P_j[dB] = \bar{P}[dB] + \varepsilon_j$, where ε_j are iid zero-mean gaussian random variables, with standard deviation σ_ε. Perfect power control corresponds to $\sigma_\varepsilon = 0$ dB, whereas imperfect power control usually yields σ_ε from 1 dB to 4 dB [26]. Thus, P_j has lognormal distribution with $\mu = \bar{P}[\mathrm{dB}]/A = \ln(\bar{P})$, $\sigma = \sigma_\varepsilon/A$.

Unfortunately, the FT of the PDF of $\alpha(u,v)$ cannot be expressed in closed form in this case and, hence, needs to be computed numerically, e.g., using the recursive method proposed by Leipnik in [27] and based on the double Taylor expansion of $e^{-(\ln x - \mu)^2/(2\sigma^2)}$. Conversely, the mean and statistical power of the auxiliary random variable can be expressed in closed form as $m_{\alpha(u,v)} = I(u,v;1)\,;\ M_{\alpha(u,v)} = I(u,v;2)\,,$ where $I(u,v;h) = \mathtt{E}\left[\alpha(u,v)^h\right] = \frac{e^{h\mu+h^2\sigma^2/2}}{2(F_P(v)-F_P(u))}\left[\mathrm{erfc}\left(\frac{\mu+h\sigma^2-\ln v}{\sqrt{2\sigma^2}}\right) - \mathrm{erfc}\left(\frac{\mu+h\sigma^2-\ln u}{\sqrt{2\sigma^2}}\right)\right]\,.$

VII. Performance analysis

Here we present only a selection of the results obtained in the three scenarios above, with the purpose of illustrating how the method proposed in this paper can be used. We first discuss the case of pure MPR systems and, later, we address the SIC case. Unless otherwise specified, all the figures presented in this section have been obtained by setting $\mathcal{D} = 10$, $\eta = 2$, $\sigma = 3/A$, $\mu = 0$, and $b = 0.02$ or $b = 0.1$. Changing the values of these parameters will clearly yield different results that, however, preserve the fundamental behavior and properties illustrated in the following.

In Figs. 1, 2 and 3 we report the capture probability $C_n(r)$ for the PL, RF, and LN scenarios, respectively, as a function of r. We plot a set of curves obtained by varying n, with $b = 0.02$. We observe that in the PL scenario, when $n = 15$, which is well below the upper bound $\lfloor 1/b \rfloor = 50$ on the maximum number of signals that can be potentially captured, the capture probability presents a spike in $r = n$ because the most likely event is that all the n signals are captured (full capture). The full capture probability in the RF and LN cases is, instead, lower. The reason is that the full capture requires all the signals to be received with similar powers, an event that, with the parameters considered in this analysis, is unlikely

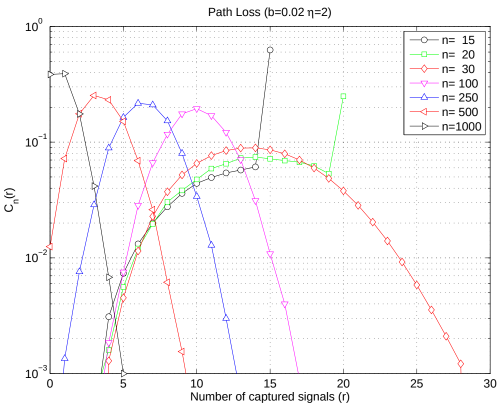


Fig. 1. Capture probability distributions $C_n(r)$ vs. r in the PL scenario when varying the collision size n ($b = 0.02, \eta = 2$).

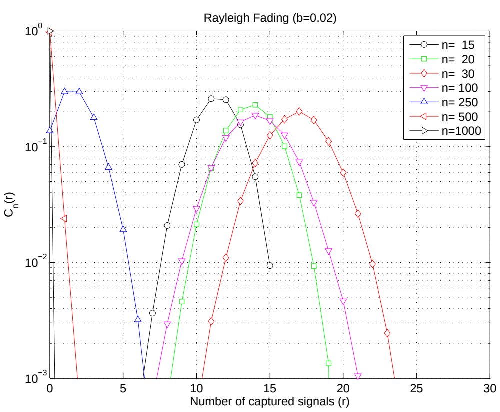


Fig. 2. Capture probability distributions $C_n(r)$ vs. r in the RF scenario when varying the collision size n ($b = 0.02$).

to occur in the RF and LN scenarios, since the range of the received signal power is larger than in the PL case.[16]

When n increases, the full capture probability decreases and the distribution roughly assumes a bell-shaped form in all three scenarios, with mean and variance that progressively decrease. Finally, for very large values of n, $C_n(0)$ tends to increase, and the system can capture fewer and fewer signals.

Fig. 4 reports the normalized throughput for the PL and RF scenarios, when varying the number of simultaneous transmissions n and with $b = 0.1$. (To reduce clutter, we do not report the results for the LN scenario that, with the parameters considered in this analysis, fall in-between the PL and RF results.) It is interesting to note that increasing the reception capability beyond a certain point yields diminishing returns. For example, in the case shown, $R = 6$ already provides a normalized throughput very close to the maximum possible, though the number of signals that can be potentially decoded with capture threshold $b = 0.1$ is $\lfloor 1/b \rfloor = 10$. This result suggests that it is possible to design radio systems with partial reception capability that practically attain the same performance as systems with infinite-reception capability.

Fig. 5 compares the normalized throughput $S_n(1) = 1 - C_n(0)$ of systems with single reception capability $R = 1$ (the metric considered in most of the previous literature) for different values of the capture threshold b. Solid curves refer to the PL case, whereas dashed curves are used for the RF scenario (once again, the LN case is omitted here). The exact results (lines) are compared with the approximate values (markers) obtained using $\tilde{C}_n(0)$ given by (36) in place of $C_n(0)$ in (17). As can be noted, the accuracy of the approximation is very good in all cases considered. Although not shown in the figure, we have also verified that using the approximate value $\hat{C}_n(0)$ given by (38) still yields excellent results for $b \leq 0.1$, whereas for larger values of b the approximation becomes less accurate.

For SIC systems we only show the results obtained in the RF scenario, due to space constraints. In Fig. 6 we report the capture probability $C_n^{(s)}(r; K)$ as a function of r, for $n = 20$, which is twice the maximum number of signals $\lfloor 1/b \rfloor$ that could be captured by a pure MPR system. The different curves have been obtained by varying K, as indicated in the legend of the figure, and setting $z = 0.1$. The solid lines with white markers refer to the exact solution given by (32), whereas the black markers correspond to the approximate capture probability values obtained as described in Sec. V-A.

First of all, we can appreciate that the Gaussian approxima-

[16] This conclusion depends on the relationship between the coverage range $\mathcal{D}$ in the PL scenario and the variance σ in LN, and different choices can be expected to yield different results.

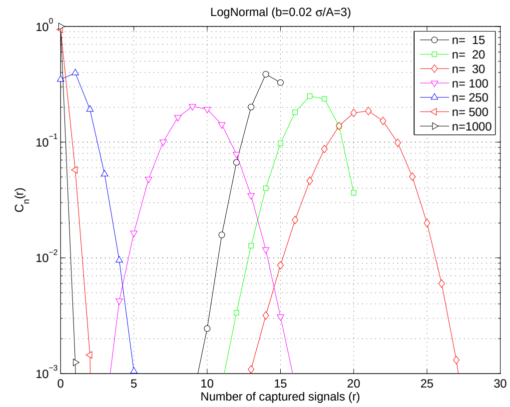


Fig. 3. Capture probability distributions $C_n(r)$ vs. r in the LN scenario when varying the collision size n ($b = 0.02, \sigma/A = 3$).

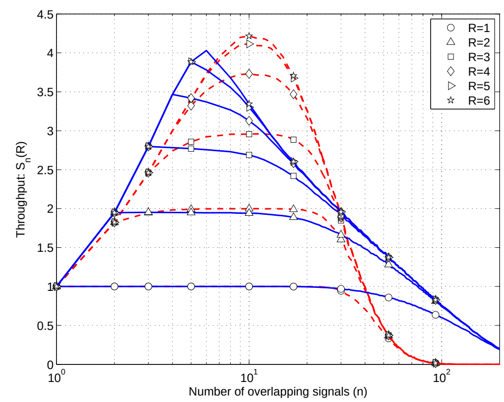


Fig. 4. Throughput $S_n(R)$ for different capture capabilities R vs. n ($b = 0.1$) in the PL (solid) and RF (dashed) scenarios. Note that the normalized throughput curve for $R = \infty$ (infinite-reception system) is superimposed to that obtained with $R = 6$.

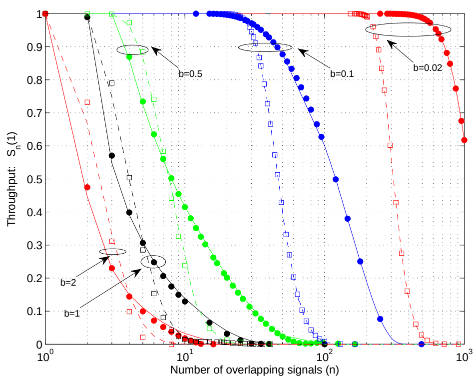


Fig. 5. Throughput $S_n(1)$ vs. n for single reception systems in the PL (solid lines) and RF (dashed lines) cases, for different values of the capture threshold b. Markers correspond to the approximate values obtained by using (36) in place of (6): empty squares for PL, and filled circles for RF.

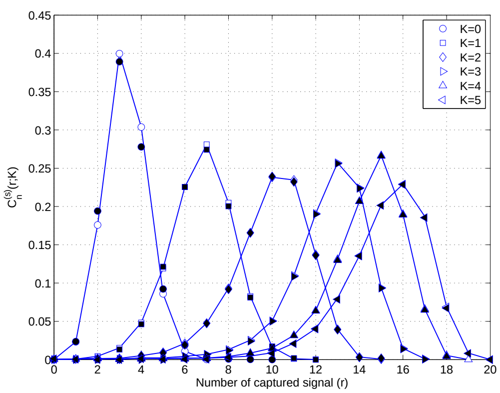


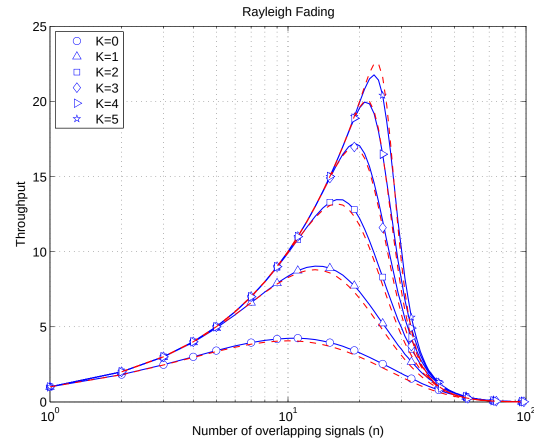
Fig. 6. Capture probability distributions $C_n^{(s)}(r; K)$ vs. r for $n = 20$ in the RF scenario, when varying K ($b = 0.1$). Solid line with white markers: exact values. Black markers: approximate values.

tion (37) already yields very good results when the number of captured signals is just a few. In fact, for $K \geq 2$ the approximate results are basically overlapping with the exact ones.

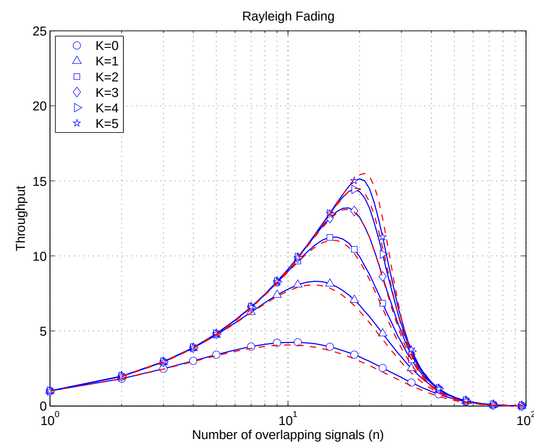
Second, we can clearly see that, in this case, SIC is very effective, since the maximum number of signals that can be captured with non negligible probability increases with K. However, the gain becomes progressively less significant as K grows, because of the imperfect interference cancellation.

Although we do not report here the results for space constraints, we observed that SIC is less effective in highly congested scenarios, i.e., when $n \gg 1/b$. In this case, the probability of early ending of the decoding process is much larger, so that it is likely that only a few SIC iterations can be performed before the reception process ends.

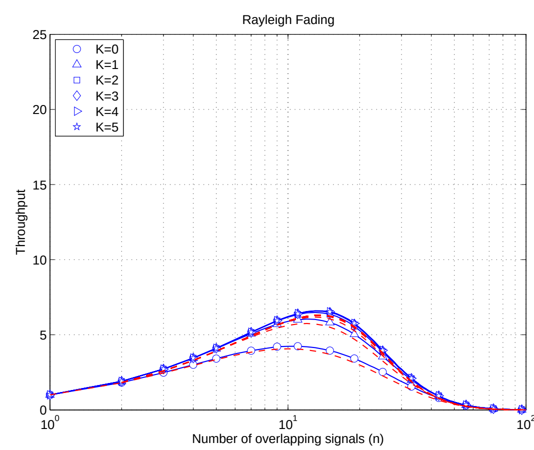
Fig. 7 shows the normalized throughput results in the RF scenario. Solid lines with markers have been obtained using the exact equation (17), whereas dashed lines report the approximate normalized throughput given by (44). The plot above in the figure refers to the case of perfect SIC, i.e., $z = 0$, whereas the plots below report the normalized throughput for $z = 0.1$ and $z = 0.5$, respectively. First of all, the rather close match between exact and approximate results confirms the accuracy of the proposed approximation method, at least for reasonable values of K. Comparing the different plots, furthermore, we note that, for a certain $K > 0$, decreasing z yields two benefits: first, the normalized throughput $S_n(K)$ increases for any n; second, the system becomes more robust to multi-signal collision events, as indicated by the growth of the value of n beyond which the normalized throughput starts decreasing. Nonetheless, we observe that, irrespective of the



(a) Perfect SIC: $z = 0$



(b) Imperfect SIC: $z = 0.1$



(c) Imperfect SIC: $z = 0.5$

Fig. 7. Normalized throughput $S_n(K)$ vs. n, when varying K, with $b = 0.1$: exact (solid) vs approximate (dashed) results.

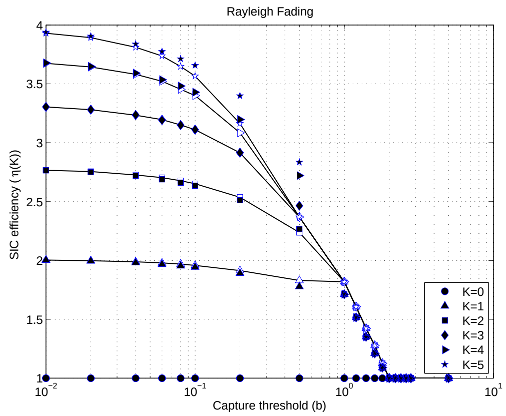


Fig. 8. SIC efficiency $\eta(K)$ vs. b, when varying K and with $z = 0.1$.

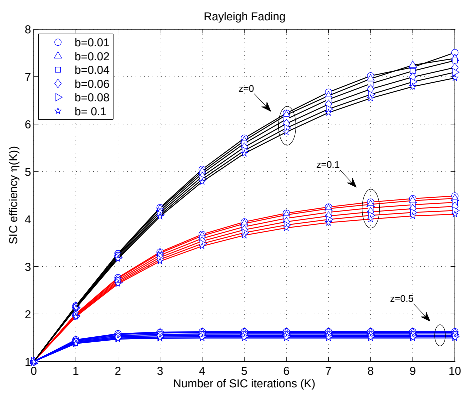


Fig. 9. SIC efficiency $\eta(K)$ vs. K, when varying b and z.

value of z, the normalized throughput gain tends to reduce after a few SIC iterations. In the case of ideal SIC ($z = 0$), for example, increasing the SIC capability from $K = 4$ to $K = 5$ yields less than 10% of peak normalized throughput gain.

As known from the literature, SIC is effective in wideband systems, i.e., when $b \ll 1$ [16]. To better appreciate the impact of the capture threshold b on the SIC performance, we introduce the *SIC efficiency* metric, defined as

$$\eta(K) = \frac{\max_n\{S_n^{(s)}(K)\}}{\max_n\{S_n^{(s)}(0)\}} .$$

In practice, $\eta(K)$ is the ratio between the peak throughput of a SIC system capable of performing up to K interference-cancellation iterations, over the peak throughput of the same system without SIC, and thus provides the maximum relative throughput gain that can be obtained by performing SIC. Fig. 8 shows the SIC efficiency $\eta(K)$ as a function of the capture threshold b, for different values of K. White markers connected by solid lines refer to exact results, whereas black markers have been obtained with the approximate method described in Sec. V-B. The figure clearly shows that the benefit of SIC decreases rapidly as b grows beyond 0.1 and, for narrowband systems, i.e., when $b > 1$, the SIC benefit is almost negligible.

An intuitive explanation of this behavior is the following. First of all, we observe that the larger b, the higher the probability $C_n(0)$ of capturing no signals at the first decoding cycle and, in turn, the probability that SIC is not performed at all. If (at least) one signal is decoded at the first iteration, then SIC might be effective to capture some other signals after interference cancellation. However, if the captured signal has power P, then according to (3), the aggregate power of the *other* signals must be lower than P/b. The larger b, the smaller the range of power values that the remaining received signals can take and, in turn, the less effective the SIC mechanism, which instead requires very disparate signal strengths [15].

In Fig. 9 we analyze the impact of the capture threshold b and the residual interference factor z in SIC systems by reporting $\eta(K)$ *vs* K when varying b and for three values of z, as indicated in the figure. The curves have been actually obtained using the approximate expression (44), which allows for a much faster, though still accurate, computation of the results. We see that, for a certain z, the curves obtained with different values of b are bundled. This means that the peak normalized throughput achievable with and without SIC scales almost linearly with $\lfloor 1/b \rfloor$. Moreover, $\eta(K)$ grows with K but the gain becomes negligible beyond a certain number of SIC iterations, which depends on z.

To exemplify a possible utilization of the proposed analysis, we consider the problem of dimensioning an MPR-enabled access point in a hot-spot scenario. We assume that users are uniformly distributed around the access point, within a cell of radius $\mathcal{D}$, and that they all use the same modulation scheme, which yields a capture threshold b. The number of users that simultaneously transmit in a certain slot is modeled as a spatial Poisson process, with density δ, so that the number n of overlapping signals is a Poisson random variable with average $\mu_{\mathcal{D}} = \delta\pi\mathcal{D}^2$. Note that the population of users served by that access point and, in turn, the mean collision size grow quadratically with the cell radius $\mathcal{D}$. The aim is to dimension the coverage range $\mathcal{D}$, MPR capability R, and SIC capability K of the access point in order to provide good performance, while avoiding costly and useless over-provisioning of the receiver capabilities. To begin with, we define the mean uplink throughput as

$$\bar{S}_{\mathcal{D}}(K) = \sum_{n=1}^{\infty} S_n^{(s)}(K)\frac{\mu_{\mathcal{D}}^n}{n!}\exp(-\mu_{\mathcal{D}}) .$$

We then use (40) (or (41)) to determine the mean throughput for a pure (no SIC) MPR system with infinite reception capability $R = \infty$, when varying the cell radius $\mathcal{D}$. The result is represented by the curve with white markers in Fig. 10, in three cases, namely i) all nodes transmit with equal and fixed power and the received radio signals are affected by path loss attenuation only (PL), ii) nodes apply long-term power control to compensate path loss attenuation, but signals are still affected by Rayleigh fading (RF), iii) nodes apply short-term power control that perfectly compensates path loss and fading, except for a residual zero-mean Gaussian error ε with standard deviation $\sigma_\varepsilon = 1.5$ dB (LN). Curves refer to density of $\delta = 1/200$ transmissions per square meter per slot.

From the figure, we can see that there exists an optimal value of the cell radius, which depends on the power control capabilities of the users. For smaller values of the coverage range, the MPR capability of the receiver is not fully exploited because the number of parallel transmissions is low. Conversely, when the coverage range is greater than the optimal value, the mutual interference will decrease the throughput.

We also observe that power control yields higher peak throughput, but less robustness to variations of the transmission density. For example, the optimal performance in the LN scenario with $\delta = 1/200$ is obtained with a cell radius of approximately 50 meters. However, doubling the cell radius that, in the LN case, is equivalent to fourfold increase on the user density, the throughput drops almost to zero. When users do not apply power control and signals are affected by path loss attenuation only, the peak throughout is significantly lower, but less sensitive to the variations of the transmission density. Therefore, this configuration is suitable when the focus is on the minimization of the infrastructure complexity (and cost) rather than on throughput maximization.

In the following, we consider the latter scenario and fix the coverage range to $\mathcal{D} = 50$ m. Therefore, the expected number of simultaneous transmissions per slot is $\mu_{\mathcal{D}} = 40$ with $\delta = 1/200$. From (17), we can then determine the minimum MPR capability R_m of the receiver beyond which the performance gain becomes negligible. For instance, we can set $R_m = \arg\min_R \{S_n(R)/S_n(\infty) \geq 1 - \rho\}$ where ρ is the maximum acceptable performance loss. In our example, setting $R_m = 15$ yields less than 10% of throughput loss for transmission density larger than $\delta = 1/200$. Note that the use of SIC largely compensates the throughput loss incurred by limiting R, since signals exceeding the MPR capability of the receiver at a certain decoding cycle will be decoded at the successive SIC iteration. The mean throughput $\bar{S}_{\mathcal{D}}(K)$ when varying the number K of admissible SIC iterations can be estimated using (33), or the approximation (44). The lines with filled markers in Fig. 17 report the throughout $\bar{S}_{\mathcal{D}}(1)$ after a single SIC iteration, whereas the results for larger values of K are omitted to reduce clutter. As already noted, the first SIC iteration brings along a significant performance gain, reaching about 50% of the peak throughput that is attained with $K \geq 5$ iterations.

VIII. Discussion and final remarks

In this paper, we proposed a novel approach for the computation of the probability that r out of n overlapping signals are received with SINR above the capture threshold and, hence, can be correctly decoded. Different from previous approaches presented in the literature, our method deals with the interdependency among the SINR values experienced by the different transmitters in a simple and scalable manner. Furthermore, it is rather flexible and can be used, with marginal adjustments, in a variety of scenarios.

We first applied the proposed mathematical framework to a multi-receiver system in a homogeneous scenario, where all users have the same capture threshold and received signal power distribution, whereas an extension to heterogeneous systems is presented in [19]. We then generalized the model to systems with SIC capabilities. Besides the exact analysis,

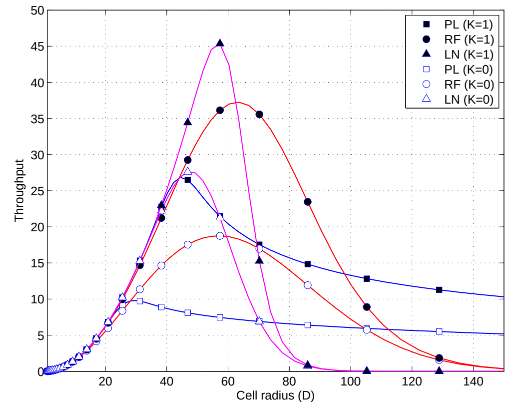


Fig. 10. Mean throughput $\bar{S}_{\mathcal{D}}(K)$ vs. cell radius $\mathcal{D}$ in PL ($\diamond$), RF ($\circ$), and LN ($\triangle$) scenarios, for an access point with pure (no SIC) MPR capabilities (empty markers) and single SIC (filled markers). The other parameters are: spatial density of transmitting nodes $\delta = 1/200$ users per square meter, capture threshold $b = 0.02$, residual interference factor is $z = 0.1$.

we also derived approximate expressions for the capture probability and the system normalized throughput that prove to be in very good agreement with the exact results, while being much easier to compute.

The results provided in this paper can be used to better assess the performance of a large variety of MAC and ARQ techniques under more realistic channel models, and to investigate the effect of a number of parameters, such as the capture threshold b, the multi-packet reception capability R and the SIC capability K, thus offering a valuable tool for the design, dimensioning and optimization of multi-receiver systems. For example, in our case-study analysis, we observed that, without SIC, the achievable normalized throughput is generally well below the maximum theoretical value $\lfloor 1/b \rfloor$ allowed by a capture threshold b, so that increasing R beyond a certain level yields marginal benefits. Furthermore, the SIC mechanism brings significant performance gains, in particular for wide band systems (with $b \ll 1$), though the normalized throughput increment rapidly diminishes at each SIC iterations.

Furthermore, the proposed approach makes it possible to analyze the asymptotic behavior of the zero-capture probability $C_n(0)$ and, in turn, of the single-receiver throughput $S_n(1) = 1 - C_n(0)$, for large values of n. In fact, using (38), the zero-capture probability $C_n(0)$ can be approximated as $\hat{C}_n(0) \simeq \sum_{y^\star \in Y^\star} F_P(y^\star)^n \geq F_P(y^*)^n$ where y^* is the largest value in $Y^\star$. From (39) and (35), it is now easy to realize that, for large n, we have $y^* \simeq nb' m_{\alpha(0,\infty)}$, so that we can write $C_n(0) \simeq F_P(nb' m_{\alpha(0,\infty)})^n$. Applying the one-side variant of the Chebishev inequality, we get

$$C_n(0) \geq \left[1 - \frac{\sigma^2_{\alpha(0,\infty)}}{\sigma^2_{\alpha(0,\infty)} + m^2_{\alpha(0,\infty)}(nb'-1)^2}\right]^n \qquad (52)$$

$$\geq 1 - \frac{n\sigma^2_{\alpha(0,\infty)}}{\sigma^2_{\alpha(0,\infty)} + m^2_{\alpha(0,\infty)}(nb'-1)^2} \simeq 1 - \frac{W^2}{nb'^2},$$

where the second inequality follows from $(1-x)^n \geq 1 - nx$, whereas in the last step we introduced $W = \frac{\sigma_{\alpha(0,\infty)}}{m_{\alpha(0,\infty)}}$, which is the coefficient of variation of the received signal power distribution, and simplified the negligible terms under the assumption $nb' \gg 1$. Therefore, in congested scenarios (i.e., when nb' is large), we have that the normalized throughput for single-receiver systems is upper bounded by $S_n(1) = 1 - C_n(0) \leq \frac{W^2}{nb'^2}$ which is larger in the case of high-variance signal power distributions. This observation is in line with the results reported in Fig. 10 for the hot-spot scenario, according to which the throughput for large values of the cell radius $\mathcal{D}$ is better in the PL case than in the RF or LN cases. In fact, increasing $\mathcal{D}$, the mean collision size n grows as quickly as $\mathcal{D}^2$, whereas the coefficient of variation W of the received signal power increases as quickly as $\mathcal{D}/\log(\mathcal{D})$ in the PL case, or remains constant in the RF and LN cases. Therefore, the probability of capturing at least one signal decreases as $\log(\mathcal{D})^{-2}$ in the PL case, and as $\mathcal{D}^{-2}$ in the other cases.

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