Precoat Filtration with Body-feed and Variable

Pressure. Part II: Experimental Tests and

Optimization of Filtration Cycles

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Abstract

The classical theory of the precoat filtration with body-feed suggests the Carman equation obtained by integration of the Darcy ODE with constant pressure. Hereinafter the equation obtained in Part I by mean a mathematical modelling of the precoat filtration with body-feed in more realistic conditions of variable pressure was re-called and an experimental validation was done. A pilot filter equipped with a small centrifugal pump was used. The experimental results of the curve of filtrate volume vs. filtration time were compared both with the new equation curve and the Carman equation curve. In the specific conditions of laboratory testing, the estimation error in the filtration time with the new equation is -3.7%, while the estimation error with the classical Carman equation is -21.6%. But with higher permeability conditions of the filtration layer the error with Carman equation can greatly increase, as it was contemplate in Part I.

The precoat filtration with body-feed must be followed by the filter cleaning. The research of an optimization of the filtration-cleaning cycle using the classic theory

of filtration, and therefore the Carman equation, leads to the well known condition to have equal both filtration time and cleaning time. With the proposal of the new equation, in this work a new modelling of the cycle optimization was elaborated. It provided a ratio between filtration time and cleaning time always greater than 1 with values also up to 16 for the higher permeability of the filtering layer (reduced $\mu \cdot c \cdot \alpha$). Therefore, the use of the previous optimization with a rate equal 1 is unacceptable with higher permeability, leading to a substantial halving of the filter productivity.

Keywords: Precoat filtration with body-feed; Mathematical modelling; ODE; Optimization; Filterability; Agricultural and food engineering

1. Introduction

In many fields of the agricultural and food engineering the precoat filtration with body-feed tends to be replaced by the Micro-filtration (MF) with membranes. In favor of the latter there is the nonuse of filter aid, with the corresponding reduction in operating costs and problem of waste treatment [1 and 2]. However the MF with membranes has high planting costs, also because, after 2-3 years of continuous use, the expensive membranes irreversibly clog and consequently they must be substituted. Therefore MF isn't always winning in the economic balance. This is true especially for small to medium production activities in which the incidence of investment costs of MF can be a serious obstacle. Some examples in Italy are the wine-making and the cheese-making. The consequence of this situation is that the study of the precoat filtration with body-feed deserves still attention.

For this reason, in the Part I [3] the problem of the error associated with the use of the Carman equation was faced. This equation, obtained by integrating the Darcy ODE [4 and 5] under the condition of constant pressure during the filtration, is a relationship among the volume of the filtrate, the filtration time, the filter area, the characteristics of the slurry and the cake, and the pressure of filtration. Due to the assumption of a constant pressure, the Carman equation is too much approximate, so in the Part I a replacement equation was proposed, also obtained by integration of the Darcy ODE but with the variable pressure according to the curve of the centrifugal pumps used in this type of filtration.

The goals of the present work are two. The first one is to verify experimentally the new equation presented in the Part I [3] to replace the Carman equation. The second goal is to develop an extension of the optimization procedure [6, 7 and 8] of the filtration cycles. In fact, it is known that the precoat filtration with body-feed is a discontinuous operation that needs the cleaning of the filter for which it was proposed an optimization to maximize the average flow rate of the full cycle. The result of this optimization was a relationship between the filtration time and the cleaning time with constant pressure [9]. So, when the filtration pressure is variable, a new relationship between the two times is needed to

optimized the filtration-cleaning cycle, that is always to maximize the average flow rate of the full cycle and consequently the productivity of the filter.

2. Materials and methods

A pilot plant of filtration to simulate the precoat filtration with body-feed was prepared. The plant was set up to carry out the filtration with variable pressure by mean a centrifugal filter feed pump (fig. 1); the hearth of the system was the filter-press with area $A = 0.24$ m², with a centrifugal pump to create a maximum pressure $P_{pmax} = 2.06$ bar and a maximum flow rate $\dot{V}_{max} = 0.84$ m³/h. The pump curve had the experimental values of $\dot{V} - P_p$, provided by the pump manufacturer, as in figure 2.

It can be noted (fig. 1) the presence of two manometers: the first one before the valve measuring the pump pressure P_p, and the other one after the valve measuring the filter pressure $\varDelta p$.
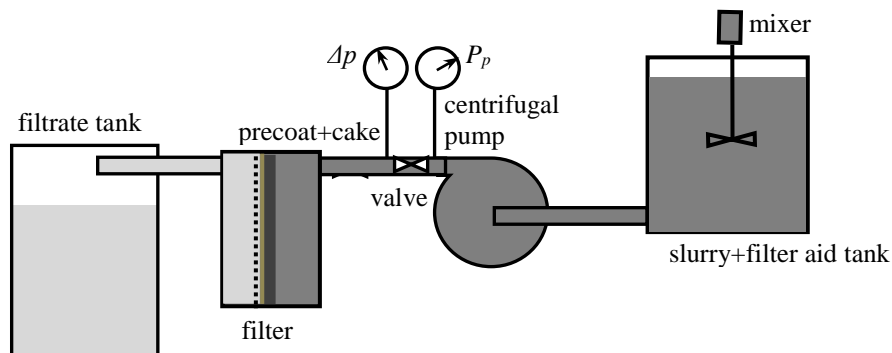

Fig. 1 - Layout for filtration with variable pressure due to centrifugal pump. The body-feed was simulated by premixed slurry with filter aid in the feed-tank

The first step was the creation of the precoat. To do this, in the tank with mixer (fig. 1) filled with water, the filter aid was also introduced, consisting of 0.2 kg of diatomaceous earth Celatom FW-6. The suspension in the tank was maintained homogenous by the mixer. The pump pushed the suspension in the filter to obtain a precoat with depth equal to 2 mm, by operating the system with the valve, between the pump and the filter, fully open. In this way the pilot plant began to operate with a filter pressure $\varDelta p$ (equal to the pump pressure P_p) equal 0 and a flow rate $\dot{V}_{max} = 0.84$ m³/h.

The second step was to simulate filtration with body-feed. To do this, the tank with mixer (fig. 1) was filled with slurry (grapes must) along with the filter aid

consisting of 2 kg/m³ₛₗᵤᵣᵣᵧ of diatomaceous earth Celatom FW-6. The suspension in the tank was maintained homogenous by the mixer. At the beginning of the filtration the valve (fig. 1) was partially closed until reaching a pressure before the valve P_{p0} equal to 1 bar. At this pressure the pump provided a flow $\dot{V}_0$ equal to 0.60 m³/h (fig. 2). So a value equal to $\beta_0 = \dfrac{\dot{V}_0}{A} = 2.5 \dfrac{m^3}{h \cdot m^2}$ was obtained [3].

The filtrate was accumulated in a tank with a measuring system of the liquid level and consequently of the filtrate volume. The value of the filtrate volume V was recorded every 5 minutes.

The third and final step was to take into account the data of the volume V and the time t experimentally observed starting from the filter pressure Δp_1 equal to 1.90 bar up to Δp_2 equal to 1.94 bar. As the variation of the corresponding pressure was only equal to 2% the pressure could be assumed practically constant and it was possible to use the V_1, V_2 and t_1, t_2 experimental values to calculate the quantity $\mu \cdot c \cdot \alpha$ (filtrate viscosity · solids concentration · specific resistance of the cake) using Carman equation [3] opportunely here below re-proposed:

$$\mu c \alpha = \frac{2A^2 \Delta p}{V_2^2 - V_1^2}(t_2 - t_1) \tag{1}$$

3. Results

In Part I [3], for the centrifugal pump used in industrial filters, it was possible to represent with high accuracy ($R^2 = 0.999$) the flow rate-pressure $\dot{V} - P_p$ experimental points of the pump curve with a parabola:

$$P_p = P_{p\,max} - B \cdot \dot{V}^2 \tag{2}$$

The figure 2 shows the parabola (2) able to better fit the experimental points of the small centrifugal pump used in the pilot filter to carry out the tests. In this case a $R^2 = 0.979$ was obtained, that is still good but lower probably due to the lower efficiency of the small pump compared to medium-large pumps of the industrial filters. As discussed in Part I [3] $B = \dfrac{P_{p\,max}}{\dot{V}_{max}^2}$, where $\dot{V}_{max} = 0.84$ m³/h and $P_{p\,max} = 2.06$ bar.
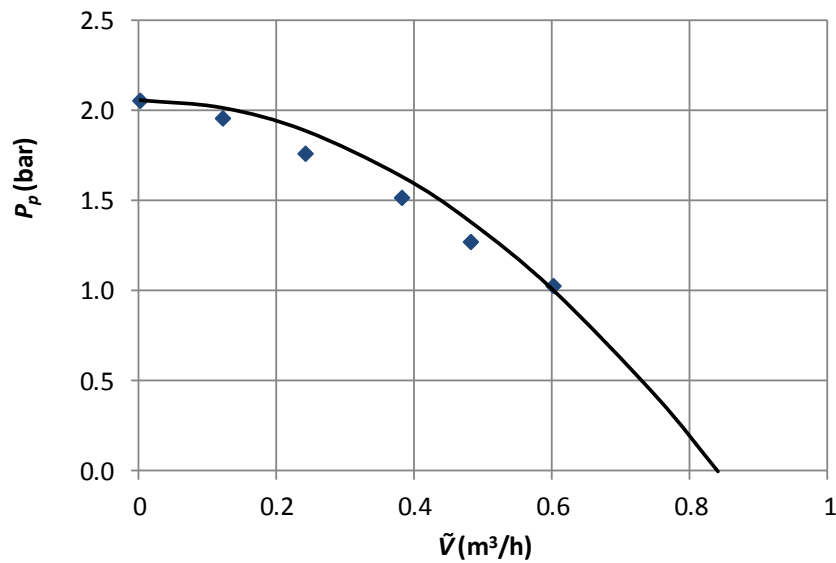
Fig. 2 – Experimental characteristic points (■) $\dot{V}$ -P_p of the small centrifugal pump of the pilot plant. Pump curve calculated (———) using the eq. (2)

The figure 3 shows the filtrate volume V vs. the filtration time t measured during the tests of the filtration with body-feed, starting from the initial flow rate $\dot{V}_0$ equal to 0.60 m³/h and the pressure P_{p0} equal to 1 bar.
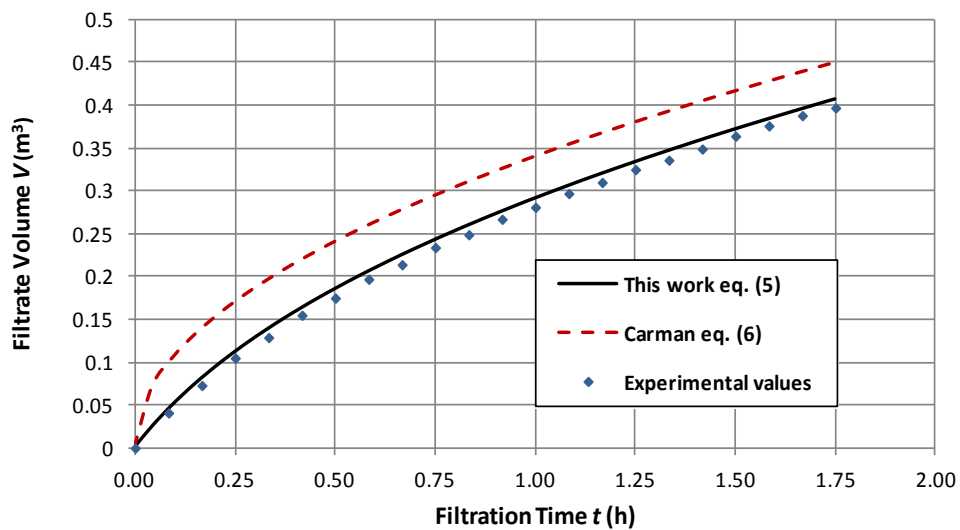

Fig. 3 – Filtrate volume vs. filtration time: experimental values (■); equation (5) of this work (———); Carman equation (6) (- - - - -)

By introducing in Carman equation (1) the measured values of V_1=0.320, V_2=0.372 m^3 and t_1=4200, t_2=5400 s, obtained when the filter pressure Δp varies from 1.90 to 1.94 bar, the value of $\mu \cdot c \cdot \alpha$ =7.38·10^8 Pa·s/m^2 was calculated. By using this value, the quantities V_0 e t_0 were calculated by [3]:

$$V_0 = \frac{2P_{p\max}A}{\mu c \alpha \cdot \beta_0} = 0.192 \text{ m}^3 \tag{3}$$

$$t_0 = \frac{2P_{p\max}}{\mu c \alpha \cdot \beta_0^2} = 1154 \text{ s} \tag{4}$$

So the curve of filtrate volume vs. filtration time *V-t,* shown in the figure 3 (———), was obtained by the equation of the part I [3]:

$$\frac{t}{t_0} = \frac{1}{2}\left(\frac{V^2}{V_0^2} + \frac{V}{V_0} \cdot \sqrt{1 + \frac{V^2}{V_0^2}} \right) + \frac{1}{2}\ln\left[\frac{V}{V_0} + \sqrt{1 + \frac{V^2}{V_0^2}} \right] \tag{5}$$

The figure 3 shows also the curve *V-t* (- - - - -) calculated by the Carman equation [3]:

$$\frac{t}{t_0} = \frac{V^2}{V_0^2} \tag{6}$$

To filtrate the experimental volume $V_{\max}$ di 0.398 m^3, the experimental time $t_{\max}$ was equal to 6300 s (1.75 h). In the face of this value the Carman equation (6) gave a time equal to 4939 s with an underestimation of -21.6%, while the equation of this work (5) gave a time equal to 6064 s with an error equal only to -3.7%, probably due to the imprecision of the parabola (2) comparing to the flow rate – pressure $\dot{V} - P_p$ experimental points (fig. 2) of the pump curve (R^2=0.979).

4. Optimization of filtration cycles

The filters with precoat and body-feed are discontinuous. In fact the filtration phase have to be followed by a cleaning to achieve a filtration cycle (fig. 4).
The problem is to optimize the filtration cycle, that is to maximize the average flow rate Q_a defined as the filtrate volume divided by the full time of a cycle (cleaning time θ + filtration time *t*):

$$Q_a = \frac{V}{\theta + t} = \frac{V_{opt}}{\theta + t_{opt}} \tag{7}$$

Thus, the solution is to find the optimum volume V_{opt} which passes through the optimum filtration time t_{opt} by maximizing the flow rate Q_a.

In the constant pressure filtration, the quicker procedure [9] is to insert the Carman equation (6):

$$V_{opt} = \frac{V_0 \sqrt{t_{opt}}}{\sqrt{t_0}}$$

(6')

in the equation (7) and set the first derivative of (7), with respect to time, equal to 0:

$$\frac{dQ_a}{dt} = \frac{V_0}{2\sqrt{t_0}\sqrt{t_{opt}}} - \frac{V_0\sqrt{t_{opt}}}{\sqrt{t_0}\left(\theta + t_{opt}\right)} = 0$$

(8)

At last by solving the (8) for optimum time t_{opt}, it is immediate to obtain:
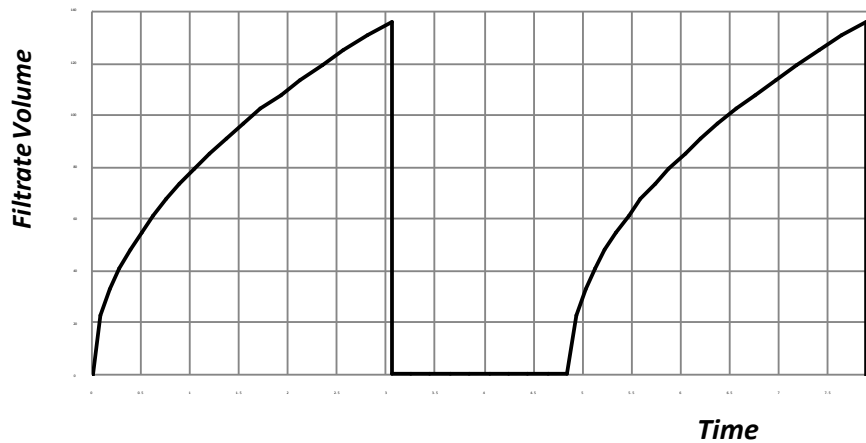
$$t_{opt} = \theta$$

(9)

Fig. 4 – Filtration cycles produced by the alternation between filtration time and cleaning time

However the same result can be obtained by zeroing the first derivative of the equation (7) with respect to volume. The path seems longer but now it is possible to better understand the previous result (9):

$$\frac{dQ_a}{dV} = 1 - \frac{V_{opt}\left(\theta + t_{opt}\right)^{-1}}{\dot{V}_{opt}} = 0$$

(10)

So it is obtained the expression:

$$\left(\theta + t_{opt}\right) = \frac{V_{opt}}{\dot{V}_{opt}} \tag{11}$$

from which, recalling also the (7), it is immediate the equation:

$$Q_a = \dot{V}_{opt} \tag{12}$$

where, recalling the (6'), the filtrate flow rate $\dot{V}_{opt}$ at the optimized time t_{opt} is:

$$\dot{V}_{opt} = \frac{d}{dt}\left(\frac{V_0\sqrt{t_{opt}}}{\sqrt{t_0}}\right) = \frac{V_{opt}}{2 \cdot t_{opt}} \tag{13}$$

By combining the (11) with the (13), the (9) is obtained again: $t_{opt} = \theta$.

But above all the (12) allows to give a graphic interpretation about the volume-time plane V-$(\theta + t)$ of the optimization of the filtration cycles. In the plane V-$(\theta + t)$, the average optimized flow rate, $Q_a = \dfrac{V_{opt}}{\theta + t_{opt}}$, represents the slope of the line outgoing from the origin. This slope, according to the (12), must be equal to the flow rate of the filtrate $\dot{V}_{opt}$, that is it must be equal to the slope of the tangent of the Carman curve (6) in the optimization point (V_{opt}-$(\theta + t_{opt})$).

For this to happen, the only way is to admit that the line outgoing from the origin is tangent to the Carman curve (fig. 5). So the point of tangency defines the coordinate (V_{opt}-$(\theta + t_{opt})$), in particular the optimized filtration time t_{opt}, in the above-mentioned volume-time plane. In fact the figure 5 shows that $t_{opt} = \theta = 1.5\,\text{h}$.

Definitely, while the equation of the optimization (9) is peculiar of the constant pressure filtration, the equations (11) and (12) are instead general and therefore they are also valid for the filtration with variable pressure, provided the Carman curve V-t is substituted by the curve of new equation. As regards this type of filtration the (11) can be re-written by inserting the expression of the ODE, that is

the $\dot{V}_{opt}$ obtained in the Part I [3] $\dot{V}_{opt} = -\dfrac{V_{opt}}{t_0} + \dfrac{V_0}{t_0}\sqrt{1 + \dfrac{V_{opt}^2}{V_0^2}}$:

$$\frac{t_{opt}}{t_0} = \frac{1}{-1 + \sqrt{\dfrac{V_0^2}{V_{opt}^2}}} - \frac{\theta}{t_0} \tag{14}$$
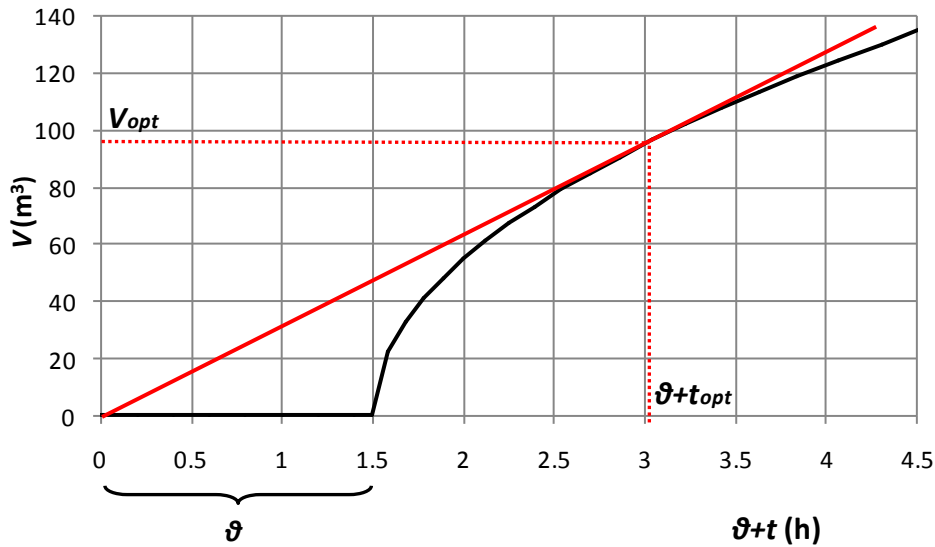
Fig. 5 – Graphic representation of the filtrate volume-time plane V-$(\theta+t)$ of the equation (12) (————)

By equating the dimensionless time t_{opt}/t_0 of the previous (14) with the time provided in the Part I [3] integrating the ODE:

$$\frac{t_{opt}}{t_0} = \frac{1}{2}\left(\frac{V_{opt}^2}{V_0^2} + \frac{V_{opt}}{V_0}\cdot\sqrt{1+\frac{V_{opt}^2}{V_0^2}}\right) + \frac{1}{2}\ln\left[\frac{V_{opt}}{V_0} + \sqrt{1+\frac{V_{opt}^2}{V_0^2}}\right] \qquad (15)$$

we obtain an equation with only one unknown V_{opt} and therefore with the (14) we obtain t_{opt}:

$$\frac{1}{2}\left(\frac{V_{opt}^2}{V_0^2} + \frac{V_{opt}}{V_0}\cdot\sqrt{1+\frac{V_{opt}^2}{V_0^2}}\right) + \frac{1}{2}\ln\left[\frac{V_{opt}}{V_0} + \sqrt{1+\frac{V_{opt}^2}{V_0^2}}\right] - \frac{1}{-1+\sqrt{\frac{V_0^2}{V_{opt}^2}+1}} + \frac{\theta}{t_0} = 0 \quad (16)$$

The solution of the (16) and the (14) gives the optimized filtration time t_{opt}. The ratio of these times with the cleaning times t_{opt}/θ vs. the quantity $\mu c\alpha$ is shown in figure 6. The lower values of $\mu c\alpha$ are the more permeable filtering layer. It is possible to note that the more permeable is the filtering layer the higher is the optimized filtration time comparing to the cleaning time, by moving more and more away from the condition of optimization of the constant pressure filtration ($t_{opt}=\theta$). The figure 6 shows also the reduced influence of the cleaning time θ on the ratio of the times t_{opt}/θ.
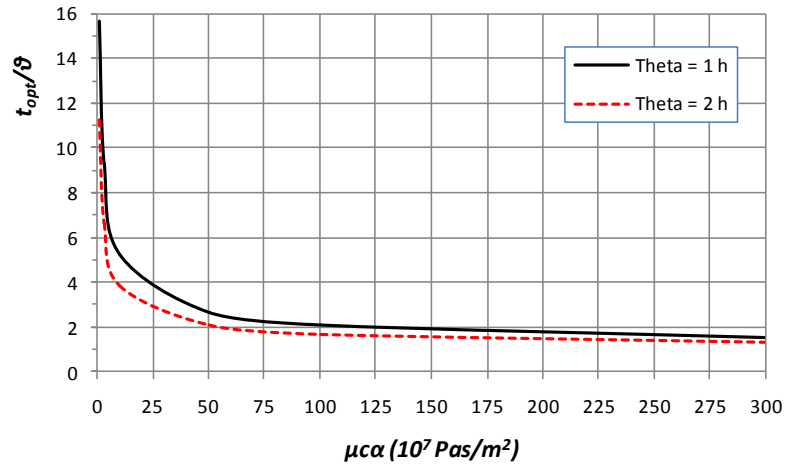
Fig. 6 – Ratio between the optimized filtration time and the cleaning time t_{opt}/θ vs. the quantity $\mu c \alpha$ and the value of the cleaning time θ. The values are obtained for $P_{pmax}=6.5$ bar and $\beta_0 = 2$

The error committed by using the optimization that simplifies the equation (9) ($t_{opt}/\theta = 1$) in place of the optimization of the equations (16) and (14) is shown in figure 7. The values are unacceptable especially for the more permeable layers, which lead to achieve average flow rates of filtrate Q_a also equal to less of half than those obtainable with the correct optimization through the (16) and the (14).
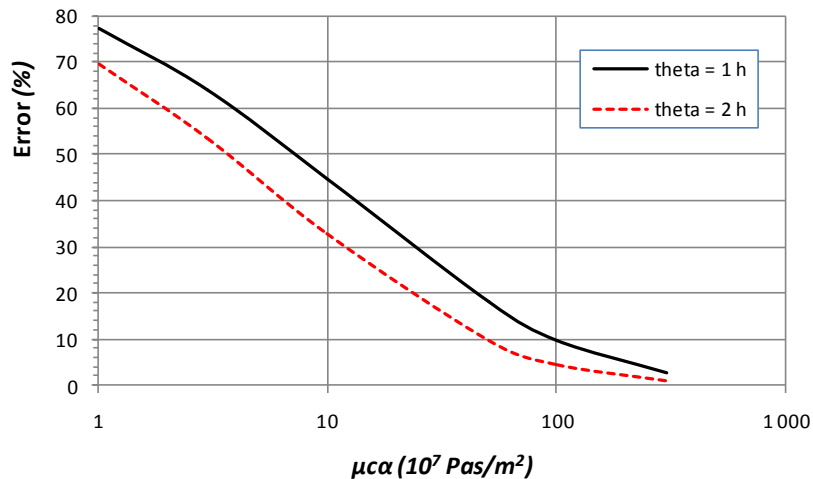

Fig. 7 – Error due to the use of the simplified optimization ($t_{opt}/\theta = 1$) in place of the correct optimization of the equations (16) and (14) and shown in figure 6

5. Conclusions

After proposing in Part I [3] a mathematical modelling of the precoat filtration with body-feed and variable pressure, in this Part II the modelling represented by a

new equation substituting the classical Carman equation was experimentally tested. The obtained result, under the specific conditions of the laboratory experimentation, can be synthesized by the error between the filtration time under the new equation and the experimental filtration time of -3.7%. For a comparison, the error committed by applying the classical Carman equation is equal to -21.6%. But, as it was foreseen in Part I, under higher permeability conditions of the filtering layer, the error with the Carman equation can greatly increase.

Besides, as the precoat filtration with body-feed is a discontinuous operation that needs the cleaning of the filter and hence a cleaning cycle, we tried to extend the concept of cycle optimization already known in the case of modelling with constant pressure. In fact, by using the related Carman equation the optimization provides a filtration time t_{opt} equal to cleaning time θ ($t_{opt}/\theta = 1$).

With the new equation proposed in the Part I the new modelling of the optimization developed in this Part II gives a ratio between filtration time and cleaning time always greater to 1 ($t_{opt}/\theta > 1$), with values even much higher, up to 16, for the higher permeability of the filtering layer. Consequently, the error committed with the old optimization ($t_{opt}/\theta = 1$) is unacceptable with the higher permeability values, leading to a reduction of less than half of the average flow rate in the full cycle and therefore to a filter productivity also more than halved.

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