



# Strategic maritime container service design in oligopolistic markets



Panagiotis Angeloudis<sup>a,\*</sup>, Luciano Greco<sup>b</sup>, Michael G.H. Bell<sup>c</sup>

<sup>a</sup> Centre for Transport Studies, Imperial College London, United Kingdom

<sup>b</sup> Department of Economics and Management, University of Padua, Italy

<sup>c</sup> University of Sydney Business School, Australia

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## ABSTRACT

This paper considers the maritime container assignment problem in a market setting with two competing firms. Given a series of known, exogenous demands for service between pairs of ports, each company is free to design liner services connecting a subset of the ports and demand, subject to the size of their fleets and the potential for profit. The model is designed as a three-stage complete information game: in the first stage, the firms simultaneously invest in their fleet; in the second stage, they individually design their services and solve the route assignment problem with respect to the transport demand they expect to serve, given the fleet determined in the first stage; in the final stage, the firms compete in terms of freight rates on each origin–destination movement. The game is solved by backward induction. Numerical solutions are provided to characterize the equilibria of the game.

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## 1. Introduction

Over the last 50 years, containerization has grown to account nowadays for roughly 70% of total deep sea trade (by value) and it is now a key component of the global economy. The resulting standardization of freight handling processes has enabled quick and efficient cargo movements across transport modes and has relied on the concept of transshipment to provide global coverage. This involves the use of multiple services to establish connectivity between ports that are not otherwise served by direct maritime links. As such, the global liner shipping network has adopted a hub-and-spoke structure, whereby there is a strong incentive for liner services to include visits to heavily connected ports (such as Singapore, Rotterdam and Hong Kong) in their rotations.

This structure, combined with the volatile and competitive nature of the ports and shipping markets, has led to intricate flows patterns that can often be difficult to predict. The development of accurate flow models would not only assist shipping lines in the deciding what services will be provided, but could also prove useful to other relevant stakeholders. These include port operators, governments and international organizations, who could use such models to allocate investments, perform strategic/operational planning, and design future policies.

One of the earliest works to model maritime container flows is the Container World project (reviewed in Newton, 2008), while Perrin et al. (2008) developed one of the earlier macroscopic container assignment models. At the same time, much

\* Corresponding author. Tel./fax: +4420 7594 5986.

E-mail address: [p.angeloudis@imperial.ac.uk](mailto:p.angeloudis@imperial.ac.uk) (P. Angeloudis).

research has focused on the empty container management problem, which occurs due to the effects of trade imbalances, predominately (e.g., between Western and Asian markets). Previous work by Bell et al. (2011, 2013) focused on the development of a container flow assignment framework that acknowledges transshipment operations and capacity constraints in the ports and vessels involved. The resulting optimization algorithms remain linear in nature, are built around the frequency-based structure of liner shipping services and seek to minimize aggregate container travel durations or costs, respectively. Both techniques are capable of simultaneously addressing full and empty container flows – in the past the latter have mostly been examined in isolation and as part of the empty container repositioning problem (Song and Carter, 2009). Given their linear nature they have modest computational requirements, therefore making them particularly attractive for application in large problem settings that involve hundreds of ports and services.

Following the seminal contributions by Agarwal and Ergun (2008) and Shintani et al. (2007), in the last few years a number of studies have jointly addressed the issues of network design and container flow assignment, taking into account cargo routing and empty container repositioning (e.g., Imai et al., 2009; Song and Dong, 2013; Mulder and Decker, 2014). Meng et al. (2014) provide an interesting overview of this fast-growing literature in the general context of studies on containership routing and scheduling. Brouer et al. (2014) also design a benchmark suite for liner shipping network design. However, all these studies keep the perspective of a single shipping line or alliance and do not deal with the consequences for freight rates or routes of competition between shipping lines or alliances.

The aforementioned work provides important insights into the forces that drive the global flow of containers. However, the existing models still fail to take into account some crucial features of the global shipping market such as: the elasticity of freight demand with respect to the economic conditions (e.g., travel time and fees) that prevail in the shipping industry at any given moment; and, on the supply side, the effects of strategies adopted by groups of players (e.g., shipping lines or alliances) in the same shipping market.

As regards the effects of competition on service network design, the economic theory of industrial organization highlights two possible outcomes (Tirole, 1988). If competing firms provide services of similar quality (e.g., service reliability) and features (e.g., planned delivery time), competition would take the form of strong price war, reducing profits and, in some cases, preventing the market from finding a stable configuration (or medium-term equilibrium). On the other hand, if competing shipping firms are able to determine some form of service differentiation they may reduce the strength of competition and, in this way, increase profits (which nevertheless will remain lower than in the monopolistic case). This case is potentially relevant for the shipping industry, given that firms are able to offer different service networks that, in turn, imply diverse delivery times and service features<sup>1</sup>. The policy implications of the two scenarios are quite different: if competition is strong, no regulation of the shipping sector is desirable, given that the market tends to determine the lowest possible prices for shippers; conversely, if competition is weak, prices and profits would tend to increase, and a regulatory remedy could be desirable if the barriers to market entry prevent new shipping lines filling unmet consumer demands.

The model introduced in this paper seeks to address this gap in academic literature, by developing an algorithm that could be used to determine an optimal set of liner services, given the presence of a competing shipping firm. The resulting model has game-theoretic elements, whereby the choice of services to be offered by each shipping line or alliance reacts to the actions of its competitors. Container flow assignment is integral part of this process, as it is used to establish how the market would respond to the simultaneous provision of routes by competing parties.

The types of service considered in the model follow established liner shipping trends, where vessel services comprise a looped sequence of port calls (commonly referred to as a port rotation). The resulting model takes into account revenues deriving from a demand for transport services that is distributed between firms in accordance to service costs. The latter include fees charged by the firm operating the chosen service as well as the opportunity cost of travel times. Empty container repositioning is retained in the container assignment model.

The key actors of the model are two firms (i.e., shipping lines or alliances) that seek to maximize their weekly profits by operating in a given region with known transport demands for full containers among a set of ports. To meet this objective, they both aim to operate sets of liner services, each being a circular tour of ports with a given frequency and capacity. Respective fleet sizes constrain the services that each firm is capable of offering.

The model acknowledges the practice of transshipment in the maritime industry, as the possibility exists for a firm to satisfy demand between two ports that are not served by the same tour. This is achieved by identifying two services that include the origin and destination port, respectively, that also intersect at some intermediate location. In modeling terms, this is achieved by simultaneously representing each service in three forms:

- as a sequence of port visits (in a closed loop form);
- as a sequence of links (journeys between two adjacent ports in the service);
- as a sequence of legs (a pair of any two ports in a service, between which travel is possible).

In our analysis set  $N$  contains a series of possible container routes. Each route  $n$  consists of a set of links  $L_n$ . Each link  $l$  within  $L_n$  represents a trip between two consecutive ports in the same route. For each route  $n$  it is possible to obtain a set of legs  $A_n$ , representing every single container journey that could be made using this route. The relationships among ports, links and legs are illustrated in Fig. 1.

<sup>1</sup> We can interpret the scenarios forms of monopolistic competition where the elasticity of substitution between services provided by different firms is infinite (first scenario) or finite (second scenario).

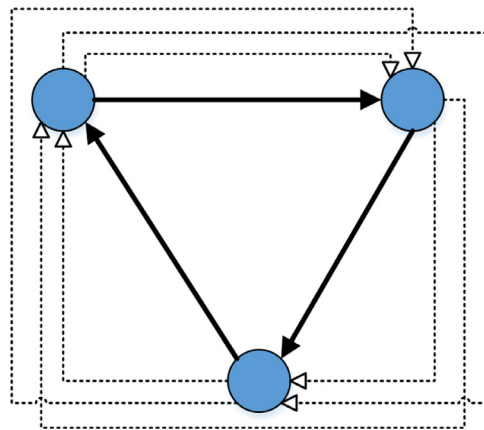


Fig. 1. A single service loop that consists of three port visits. Links are illustrated using solid arrows while legs are illustrated using dotted arrows.

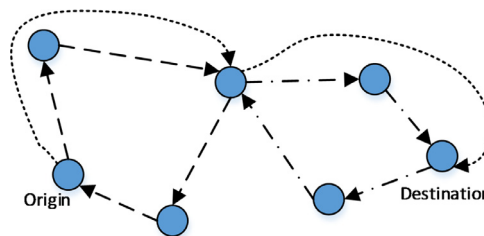


Fig. 2. A multi-service container trip (from origin to destination) that utilizes two intersecting service loops (and therefore consists of two legs).

Ideally these routes will intersect with at least some ports being served by several trips, therefore providing opportunities for transshipment of containers. This would be essential for the shipment of containers between two ports that are not directly connected. Furthermore, it might still be a viable option if direct routes exist, should they include large detours. Having said that, an ideal assignment algorithm would not merely compare journey times, but should also take into account the additional handling costs that are incurred in intermediate ports. An example of a multi-leg container trip with transshipment is provided in Fig. 2. Storage limitations in transshipment ports may impose constraints to the amount of containers that may be temporarily stored between journey legs. The model is designed as a complete-information sequential game, taking place over the following stages:

- Stage 1: The firms simultaneously invest in their fleet;
- Stage 2: The firms decide which routes they will operate and solve the flow assignment problem with respect to the transport demand that they expect to serve, given the fleet sizes determined in the first stage;
- Stage 3: The firms compete in terms of freight rates on each origin–destination movement, acknowledging the share of overall transport demand to be served by the competing party.

The game is solved using backward induction (i.e., incorporating the solution of later stages in previous ones), relying on the concept of Subgame Perfect Equilibrium (Gibbons, 1992)<sup>2</sup>. The model is solved for two cases: monopoly and duopoly. The main difference between these cases is the expression for revenues that captures the monopolistic or competitive behavior of each firm, respectively, and is worked out as the solution of the third stage of the game. Given the complexity of the problem, we rely on numerical examples to characterize the equilibria of the sequential game.

The contribution of the paper is both theoretical and methodological. First, we address the issue of the effect of competition on service provision in the shipping industry. From a methodological point of view, the main innovations are the use of game-theoretic concepts to address the issue of competition, while keeping the model tractable, as well as the introduction of innovation in the manipulation of variables to address the resulting mixed linear-integer program.

Our theoretical and methodological setting considers a shipping industry where only one or two firms operate. However, the algorithm structure implemented in this study could be used as the basis for a future model that can analyze oligopolistic shipping industries where more than two firms operate in a single region. Furthermore, the monopolistic and duopolistic

<sup>2</sup> Backward induction can be considered a generalization of Bellman's optimization principle (Fudenberg and Tirole, 1991). The latter is applied in dynamic optimization problems (hence, also in dynamic games where multiple agents choose their intertemporal strategies), while the first is applied also in static optimization problems.

cases provide appropriate frameworks to analyze competition in the real-world regional shipping markets where we observe only one or two alliances (Ducruet and Nottenboom, 2012).

The remaining of the paper is organized as follows. Section 2 describes the basic structure of the model; Sections 3 and 4 analyze the monopolistic and duopolistic cases; Section 5 draws conclusions.

## 2. The model

We consider a strategic model of service design with two competing firms (i.e., shipping lines or alliances); the list of all variables and parameters is in the Appendix. The geography and operational parameters of the ports sets are considered to be exogenous to the model. This includes the distances between port pairs as traversed by the services and the maximum throughput capacity  $MQP_i$  for each port  $i \in I$  (where  $I$  is the set of existing ports or nodes). To keep the analysis as simple as possible, distances between any two ports connected by a link  $l \in L$  (where  $L$  is the set of all links included in the services considered) are expressed in terms of the exogenous sailing time ( $STL_l$ ).

The demand for maritime transport services between any pair of ports is assumed to be exogenous. Having said that, it is assumed that market appetite for service between a port of origin  $r$  and a port of destination  $s$  would decrease as the charges to be borne by shippers increase. These are determined as the sum of fees  $q_{rs}^f$  charged by shipping firms for the service between  $r$  and  $s$  plus the opportunity cost of travel (to include interest and product depreciation). The latter is determined by the expression  $COT \times u_{rs}$ , where  $COT$  is the average opportunity cost per travel day for the shipper and  $u_{rs}$  is the travel time in days involved by the provided service<sup>3</sup>. At this point we also introduce a parameter  $MCT_{rs}$  for the maximum cost of travel between a port pair  $rs$ . This is exogenous to the problem, related to overall market conditions and acts as a reserve price (in economic terms), above which an average shipper would lose interest for transport services.

Continuously decreasing functions, commonly used in transport economics to represent market demand, would introduce non-linearities requiring a Mixed Integer Quadratically Constrained Quadratic Program (MIQCQP) formulation that is difficult to solve for larger problem instances. Thus, for the sake of tractability, we represent the container transport demand from  $r$  to  $s$  using a simple step-function:

- service demand will be equal to  $MQF_{rs}$ , for all  $r \in O$  and  $s \in D$  (where  $O, D \subset I$  are the set of origin and destination ports, respectively), if the total transport cost borne by shippers is lower or equal than a given threshold  $MCT_{rs}$ <sup>4</sup>;
- service demand will be equal to zero if the total transport cost is above the reserve price  $MCT_{rs}$ .

Each firm would then seek to maximize its total weekly profit  $\Pi$ , using the following objective function:

$$\Pi = TRP - \sum_{n \in N} (TCS_n + TCL_n + TCD_n) - TCC \quad (1)$$

The aggregate weekly profit function  $\Pi$  is defined a composite of the various components of operational costs and revenues:

$$TRP = \sum_{r \in O} \sum_{s \in D} d_{rs} \quad (1.1)$$

$$TCS_n = \sum_{l \in L} LRR_{nl} p_n (STL_l CSR_l + CSP_l) \quad (1.2)$$

$$TCL_n = CHC_n p_n \sum_{a \in A_n} (x_{a+}^f + x_a^e) \quad (1.3)$$

$$TCD_n = CCD \left( \sum_{a \in A_n} \left( (x_{a+}^f + x_a^e) \sum_{l \in L} (LLR_{al} STL_l) \right) + w_{++}^f + w_+^e \right) \quad (1.4)$$

$$TCC = CRC CFS \quad (1.5)$$

In Eq. (1.1),  $TRP$  is the total weekly revenue earned across all container transport demands, with  $d_{rs}$  being the specific revenue obtained by the firm for the transport of full containers between  $r$  and  $s$ . The remaining terms in Eq. (1) relate to various operational cost aspects, incurred by each potential route  $n \in N$  (where  $N$  is the set of all possible routes). Service costs for each route  $n$  are provided by Eq. (1.2). Only services that are actually offered are considered (therefore  $p_n = 1$ ). These can be decomposed into two components:

- A travel cost  $STL_l \times CSR_l$  for each link  $l$  that belongs to the route (therefore  $LRR_{nl} = 1$ ), that would include bunker fuel, vessel maintenance, crew salaries and other costs that would be a function of link sailing time.

<sup>3</sup> In case at the optimum a firm offers the service through different paths, we consider the average travel time, weighted in accordance to container flow allocations. This assumes that shippers bear the risk of different actual delivery times.

<sup>4</sup> It is assumed that  $MQF_{rr} = 0$  for all  $r \in O$ .

- Port visit costs  $CSP_l$  for the destination port of each link  $l$  in the route.

To simplify the presentation of the model, we introduce a custom notation that indicates a summation over all values of a specific index. In the above model this is used with the decision variables  $x_{as}^f$ ,  $w_{is}^f$  and  $w_l^e$  and operates as follows:

$$x_{a+}^f = \sum_{s \in D} x_{as}^f \quad w_{+}^e = \sum_{i \in L} w_i^e$$

A container handling cost  $CHC_n$  would be incurred by containers travelling through each journey legs  $a$ , and would include the cost of loading and unloading operations on leg endpoints (ports). The presence of such costs would deter unnecessary transshipment whenever a feasible and direct service exists between two ports and is captured by Eq. (1.3). This cost would apply to both full and empty container flows ( $x_{a+}^f$  and  $x_a^e$ , respectively), and is assumed to be the same for either class for the purposes of this study.

Container rental and cargo depreciation are captured by Eq. (1.4); these are incurred by the periods  $STL_l$  that a container spends aboard vessels over a leg  $a$ , as well as while waiting in a port before loading and in the case of transshipment ( $w_{++}^f$  and  $w_{+}^e$  for full and empty containers, respectively). The parameter  $LLR_{al}$  is used to identify links in set  $L$  that belong to a leg  $a$  in subset  $A_n$ .

Fixed vessel operation costs (such as loan repayments, vessel insurance and administrative costs) are captured by Eq. (1.5), where  $CFS$  is the number of vessels that compose the firm's fleet, and  $CRC$  is the average weekly vessel ownership cost. As such, the following remarks are in order:

- When a given route  $n$  is not operated (i.e.,  $p_n = 0$ ), only fixed costs associated with the capacity deployed on that route are borne by the firm;
- Each firm is assumed to operate a fleet of vessels that share a common vessel size (measured in TEUs); this assumption is made in order to simplify the presentation and discussion of the model in this manuscript, but would be easy to overcome using further model parameters;
- $FSL_n$  is the frequency of weekly sailings on route  $n$ , and  $RVR_n = FSL_n \sum_{a \in A_n} \sum_{l \in L} LLR_{al} STL_l$  is the number of vessels required to achieve it;<sup>5</sup>
- $MRC_n = RVR_n NCS$  is the maximum capacity (measured in terms of number of containers per week) that the firm deployed on route  $n$ , with  $NCS$  being the number of containers that can be transported on each ship;
- The total vessel requirements across the routes operated (any route  $n$  where  $p_n = 1$ ) shall not excel the size of the fleet ( $CFS$ ).<sup>6</sup>

The functional structure of the maritime container shipping market is represented by the following sequential game:

- Each firm deploys its maximum transport capacity for each route  $n$ , that in turn determines firm's fixed costs in the following stages of the game;
- given firms' capacity constraints and predictions<sup>7</sup> of the other firm's service offer and the outcome of downstream (i.e., third stage) competition, each firm decides which services to operate and solves its container assignment problem;
- as an outcome of the second stage, container travel times and potential transport capacity limits (with respect to the potential demand)<sup>8</sup> between each pair of ports are determined for each firm.

At this point, firms determine their fees for full containers either while competing à la Bertrand<sup>9</sup> between pair of ports that are served by both networks or operating as monopolist between pairs of ports that are served by their own network only.

### 3. Maritime container transport design in a monopoly market

In this section we consider (as a benchmark) the case of a single firm that is free to accommodate the entire transport demand in all ports. As usual in complete-information sequential games, the solution has to be found by backward induction (i.e., incorporating the solution of later stages in previous ones).

<sup>5</sup> For example, if the total duration of links for a complete tour of route  $n$  is equal to 2 weeks and there are 4 vessels assigned to the route, then the route would have a frequency of 2 (vessels per week).

<sup>6</sup> In practice this suggests that, for example, the fleet cannot operate 4 routes, each operated by one vessel, with a frequency of one sailing per week if it only has a fleet of 3 vessels.

<sup>7</sup> An initial estimate of the service fees to be set by the competing firms. These are used to commence the iterative solution process and are replaced with actual values in later stages of the game.

<sup>8</sup> Remark that capacity limits in the transport of containers between any two ports may arise because of the overall capacity on involved routes (that is fixed at the first stage of the game) is insufficient, given the solution of the assignment problem. In this case, firms just ration their services with respect to potential market demand.

<sup>9</sup> The underlying assumption of Bertrand's competition in the last stage of the game is that the service provided by competing firms is homogeneous. This is realistic for the maritime container transport industry, though some form of service quality differentiation may arise, which could lead to imperfect substitutability and monopolistic competition between the two firms.

### 3.1. Theoretical analysis

Once deployed capacity, operated routes and flow assignment have been chosen by the monopolistic firm, it is possible to determine the maximum admissible flow of full containers  $t_{rs}^f$ , and transport time,  $u_{rs}$ , for each port pair  $rs$  with origin  $r$  and destination  $s$ . The demand for transport from  $r$  to  $s$  will be considered as equal to  $MQF_{rs}$  only if  $MCT_{rs} \geq q_{rs}^f + COT u_{rs}$ . As such if transport time is too high ( $u_{rs} > \frac{MCT_{rs}}{COT}$ ) no containers would be shipped from  $r$  to  $s$ , independently of the level of transport fee. Conversely, if  $u_{rs} \leq \frac{MCT_{rs}}{COT}$ , the monopolistic firm would charge the maximum travel fee  $q_{rs}^f$  for container flows between ports  $r, s$  that is compatible with a positive demand such that  $q_{rs}^f = MCT_{rs} - COT u_{rs}$ . As such, the revenue that the monopolist earns on the (potential) service from  $r$  to  $s$  can be characterized as:

$$d_{rs} = \begin{cases} (MCT_{rs} - COT u_{rs})t_{rs}^f, & u_{rs} \leq \frac{MCT_{rs}}{COT} \\ 0, & u_{rs} > \frac{MCT_{rs}}{COT} \end{cases} \quad (2)$$

where  $t_{rs}^f \leq MQF_{rs}$ . Anticipating the impact of the market reaction on firm's revenue in Eq. (2), and given the deployed capacity on each route, the monopolist optimizes its route provision and assignment problem by solving a mixed integer linear program (MILP) that is characterized by the maximization of the profit function (1) subject to the following set of constraints (see the Appendix for notation):

$$\sum_{a \in A_i^+} x_{as}^f - \sum_{a \in A_i^-} x_{as}^f = b_{is}^f \quad \text{for all } i \in I, s \in D \quad (3.1)$$

$$\sum_{a \in A_i^+} x_a^e - \sum_{a \in A_i^-} x_a^e = -b_i^e \quad \text{for all } i \in I \quad (3.2)$$

$$x_{as}^f \leq \sum_{n \in N} FSL_n LR_{an} p_n w_{is}^f \quad \text{for all } a \in A_i^-, i \neq s \in I, s \in D \quad (3.3)$$

$$x_a^e \leq \sum_{n \in N} FSL_n LR_{an} p_n w_i^e \quad \text{for all } a \in A_i^-, i \in I \quad (3.4)$$

$$MQP_i \geq \sum_{a \in A_i^+} (x_{a+}^f + x_a^e) + \sum_{a \in A_i^-} (x_{a+}^f + x_a^e) \quad \text{for all } i \in I \quad (3.5)$$

$$MRC_n p_n \geq \sum_{a \in A_n} \sum_{s \in D} LLR_{al} (x_{a+}^f + x_a^e) \quad \text{for } n \in N, l \in L_n \quad (3.6)$$

$$b_{is}^f = \begin{cases} -t_{rs}^f & \text{if } i = r \in O \\ t_{rs}^f & \text{if } i = s \in D \\ 0 & \text{otherwise} \end{cases} \quad (3.7)$$

$$b_i^e = \begin{cases} t_{+i}^f & \text{if } i = r \in \bar{O} \cap D \\ -t_{+i}^f & \text{if } i = r \in \bar{D} \cap O \\ t_{+i}^f - t_{i+}^f & \text{if } i = r \in O \cap D \\ 0 & \text{otherwise} \end{cases} \quad (3.8)$$

$$d_{rs} = (MCT_{rs} - COT u_{rs})t_{rs}^f \geq 0 \quad (3.9)$$

$$t_{rs}^f \leq MQF_{rs} \quad (3.10)$$

$$\sum_{n \in N} p_n RVR_n \leq CFS \quad (3.11)$$

$$x_{as}^f \geq 0 \quad \text{for all } a \in A, s \in D \quad (3.12)$$

$$x_a^e \geq 0 \quad \text{for all } a \in A \quad (3.13)$$



$$d_{rs} \geq 0 \text{ for all } r \in O, s \in D \quad (3.14)$$

$$t_{rs}^f \geq 0 \text{ for all } r \in O, s \in D \quad (3.15)$$

$$p_n \in \{0, 1\} \text{ for all } n \in N \quad (3.16)$$

Flow conservation is maintained through constraints (3.1) and (3.2), for full and empty containers respectively. As empty containers are considered to be interchangeable, flow destinations are not important – as such the constraint definition iterates only across the set of ports  $i \in I$ .

The constraints (3.3) and (3.4) are used to obtain the expected dwell time of containers per unit time. For each active leg that carries container flows to port  $i$ , the expected delay is at least as large as the inverse of the combined service frequency (in the case of full containers we also differentiate for each destination  $s$ ). As written above, both of these constraints feature a multiplication between decision variables. However, it is possible to linearize this constraint using the following transformations:

$$b_i^e = x_{as}^f \leq \sum_{n \in N} FSL_n LR_{an} g_{nis} \text{ for all } a \in A_i^-, i \neq s \in I, s \in D \quad (3.3^*)$$

$$g_{nis} \leq MDT_i^f p_n \quad (3.3.1)$$

$$g_{nis} \leq w_{is}^f \quad (3.3.2)$$

$$g_{nis} \geq w_{is}^f - MDT_i^f (1 - p_n) \quad (3.3.3)$$

$$g_{nis} \geq 0 \quad (3.3.4)$$

where  $MDT_i$  is the maximum dwell time of full containers at port  $i$ . Similarly, the constraint (3.4) can be linearized as follows:

$$x_{as}^e \leq \sum_{n \in N} FSL_n LR_{an} h_{ni} \text{ for all } a \in A_i^-, i \neq s \in I, s \in D \quad (3.4^*)$$

$$h_{ni} \leq MDT_i^e p_n \quad (3.4.1)$$

$$h_{ni} \leq w_i^e \quad (3.4.2)$$

$$h_{ni} \geq w_i^e - MDT_i^e (1 - p_n) \quad (3.4.3)$$

$$h_{ni} \geq 0 \quad (3.4.4)$$

where  $MDT_i^e$  is the maximum dwell time of empty containers at port  $i$ .

Each port has been assigned a maximum throughput parameter, whose value is linked to the port's ability to load and unload vessels quickly as well as the storage capacity of its yard. Eq. (3.5) ensures that this is not violated by active container flows for each port  $i$ .

Whenever a vessel is traveling between two consecutive ports in a route  $n \in N$ , it might be carrying container flow that belongs to several legs  $a \in A_n$ . Eq. (3.6) is used to enforce an upper bound on the total volume of container flows to be allocated on legs that use any link  $l$  of the route. The Boolean parameter  $LLR_{al}$  is used to indicate whether a link  $l$  is used by a leg  $a$ . Constraints (3.7) and (3.8) are used to enforce the demand or supply of container flows for each port  $i$  – this is used in conjunction with the flow conservation rules (3.1) and (3.2).

The revenue for flows between a pair of ports  $r, s$  is determined by the relationship in (3.9), as discussed earlier in this section. Finally, (3.9) ensures that the actual flow of containers between a port pair  $r, s$  never exceeds the market demand  $MQF_{rs}$ , while (3.10) ensures that a shipping line will not operate a combination of routes that necessitates the deployment of more vessels than the size of their fleet  $CFS$ .

The distinction between empty and full containers throughout the above set of constraints demonstrates how the model accommodates the two types of flows using distinct decision variables. Should the need arise, the representation of container flows could be further decomposed into distinct container sizes and types (refrigerated, liquid-bulk etc.) with the introduction of additional demand parameters, decision variables and constraints, without otherwise changing the overall structure and premise of the problem.

Given the solution of the second and third stages of the optimization, the total profit of the firm depends on the maximum capacity deployed on each route. Thus, the firm chooses the capacity deployment to maximize its profit. This model possesses a sufficient amount of operational detail that would facilitate its applications to realistic scenarios, as illustrated by the numerical example presented in the following section.

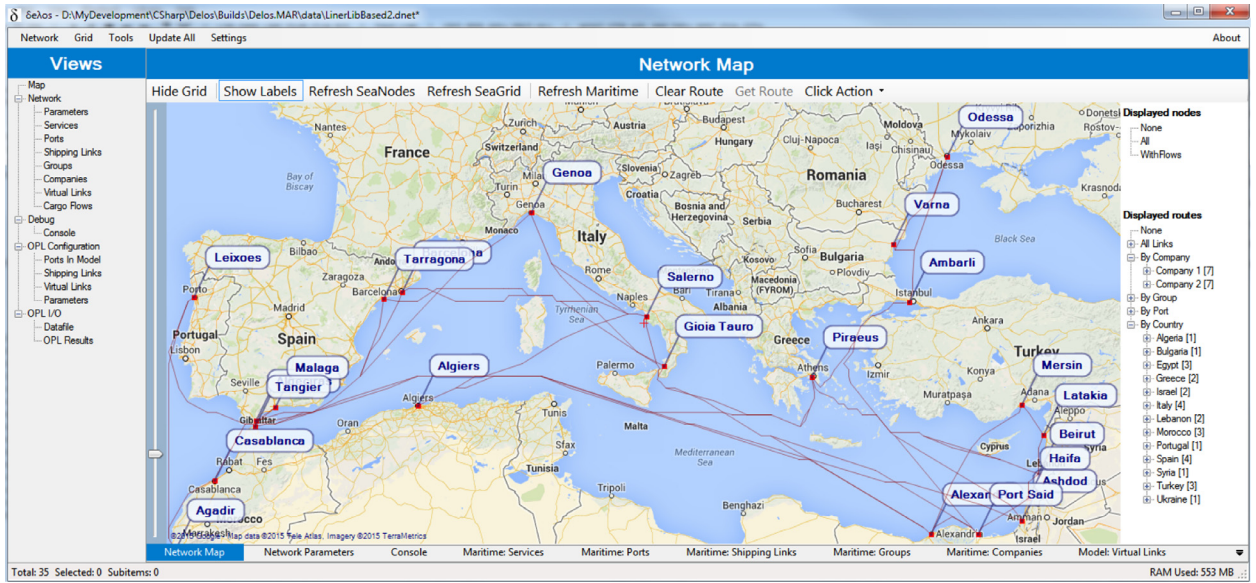


Fig. 3. Screenshot of the Delos software – the area covered and the routes included in the numerical example (background map data courtesy of Google Maps).

Table 1

Container flows considered in numerical tests (source: LINER-LIB 2012).

Origin	Destination	TEU/wk	Origin	Destination	TEU/wk	Origin	Destination	TEU/wk
Algeciras	Ambarli	532	Leixoes	Malaga	176	Genoa	Poti	124
Algeciras	Alexandria	380	Port Said	Beirut	172	Ambarli	Haifa	122
Algeciras	Casablanca	360	Haifa	Casablanca	168	Salerno	Malaga	118
Port Said	Casablanca	296	Ambarli	Algeciras	160	Algeciras	Agadir	116
Genoa	Tangier	292	Algeciras	Piraeus	152	Port Said	Mersin	114
Genoa	Port Said	250	Barcelona	Salerno	144	Algeciras	Ashdod	112
Tangier	Alexandria	224	Port Said	Ambarli	144	Varna	Mersin	110
Odessa	Port Said	210	Port Said	Latakia	142	Algeciras	Haifa	110
Gioia Tauro	Casablanca	208	Ambarli	Port Said	142	Ashdod	Varna	102
Tarragona	Salerno	206	Agadir	Algeciras	140	Genoa	Algeciras	102
Beirut	Port Said	198	Gioia Tauro	Ambarli	132	Leixoes	Algiers	100

### 3.2. Numerical analysis

Perhaps the most challenging step in the deployment of the above model would be the definition of the route, link and leg sets  $N$ ,  $L$  and  $A$ , as well as the parameter arrays  $LRR_{lm}$ ,  $LLR_{al}$  and  $LR_{an}$  that describe the relationships among them. A custom tool (Delos) was developed (using the C# programming language) to simplify this process. The key features of this tool include a geospatial data management interface that facilitates the visual definition of maritime transport networks, and a database that captures operational aspects of the maritime transport market environment (including parameters that relate to companies, services, ports and market demands). A screenshot of this software is provided in Fig. 3.

The actual MILP model that was defined in the previous section was implemented using the OPL modeling language and solved using the CPLEX optimization engine. A custom C#/OPL interface was developed to link Delos with the final OPL model and manage the algorithm workflow. Given the solution of the second and third stage, the total profit of the firm depends on the capacity deployed on each route. Thus, the firm chooses the capacity deployments to maximize its profit, taking into account the financial feasibility of service provision and limitations imposed by the available fleet. At this point, it should be pointed out that the vessels would incur fixed ownership costs (CRC) regardless of whether the firm chooses to operate a combination of services that would utilize them – as such, there is a strong incentive to do so.

To test the monopolistic version of the model we created a sample scenario focused on the Mediterranean feeder market (Fig. 3) and some of the busiest ports in the region. The largest potential container flow demands among ports in the region (Table 1) were extracted from the LINER-LIB benchmark dataset (Brouer et al., 2014). In our problem setting, these demands are conditional on the prices of the services offered and the delivery time realized by the network configuration that is selected by the firms. As these price and time values increase, it is more likely that shippers will become disinterested in the service. This threshold could be modified using the  $MCT_{rs}$  parameter for each flow between ports  $r$ ,  $s$ .



**Table 2**

Services included in the numerical example/order of port calls.

Service 1		Service 2		Service 3		Service 4		Service 5		Service 6		Service 7	
Port	Day	Port	Day	Port	Day	Port	Day	Port	Day	Port	Day	Port	Day
Leixoes	1(7)	Alexandria	1(7)	Beirut	1(7)	Algeciras	1(7)	Salerno	1(7)	Port Said	1(7)	Alexandria	1(7)
Agadir	2	Port Said	2	Ambarli	2	Barcelona	2	Genoa	2	Salerno	3	Genoa	3
Casablanca	3	Haifa	3	Odessa	3	Gioia Tauro	4	Tangier	4	Algiers	4	Algeciras	4
Tangier	4	Beirut	4	Varna	4	Piraeus	5	Tarragona	5	Ashdod	5	Casablanca	5
Malaga	5	Latakia	5	Piraeus	5	Algeciras	7(1)	Salerno	7(1)	Ambarli	6	Alexandria	7(1)
Algeciras	6	Mersin	6	Beirut	7(1)					Port Said	7(1)		
Leixoes	7(1)	Alexandria	7(1)										

**Table 3**

Model assumptions.

Model property	Parameter	Value
Vessel class	N/A	Feeder
Vessel capacity	MRC	400 TEU
Vessel ownership costs	CRC	~8000 \$/week
Vessel operating costs (per week of travel)	CSR <sub>i</sub>	~8000 \$/week
Opportunity cost (per week of TEU travel)	COT	200 \$/week
Maximum cost of travel (per TEU)	MCT	Duration · 700\$/week
Port visit cost (fixed)	CSP <sub>i</sub>	500 \$
Port container handling cost (fixed)	CHC	50 \$/TEU
Container operating costs(per week of TEU travel)	CCD	30 \$/TEU/week

Sources: [Stephens \(2013\)](#), [UNCTAD \(2011\)](#), [NILIM \(2006\)](#).**Table 4**

Results for the monopolistic problem instances.

Scenario	Overall profit	Services offered	Container flows
A: 1 vessel	\$52,183	Service 1	Leixoes → Malaga (176 TEU)
B: 2 vessels	\$72,465	Services 1, 2	Genoa → Tangier (200 TEU) Leixoes → Malaga (176 TEU)
C: 3 vessels	\$67,289	Services 1, 4, 7	Genoa → Tangier (16 TEU) Gioia Tauro → Casablanca (200 TEU) Leixoes → Malaga (176 TEU)
D: 4 vessels	\$88,673	Service 1, 4, 5, 7	Genoa → Tangier (224 TEU) Algeciras → Casablanca (200 TEU) Gioia Tauro → Algeciras (200 TEU) Leixoes → Malaga (176 TEU)
E: 5 vessels	\$80,673	As in scenario D	As in scenario D
F: 6 vessels	\$72,673	As in scenario D	As in scenario D
G: 7 vessels	\$64,673	As in scenario D	As in scenario D

A service network comprising seven weekly feeder services was designed using the Delos tool, featuring realistic travel times ([Table 2](#)). While each service only covers a subset of the ports encountered in [Table 1](#), the overall network is structured in a way that any of the O–D flow pairs would be feasible using direct journeys, or transshipment.

The number of services that visit each port is roughly proportional to the volume of container flows, and ranges between 1 and 4 calls. As such, transshipment is possible in 10 of the 23 ports considered. In this example, it is assumed that the firm seeks to operate in this market alone, seeking to serve potentially profitable flows using their small fleet of feeder vessels (see [Table 3](#)).

The analysis is conducted on the basis of an iterative process, where the number of available vessels – i.e., the transport capacity to be deployed on different routes measured in terms of TEUs per week – is pre-determined. In the first iteration, we run the mixed linear integer program of the monopolist firm assuming, as our first guess, that the delivery time between any two ports is the average of all services potentially linking them weighted by the demand. The firm, in particular, chooses on which possible services to deploy the available vessels (or transport capacity). Having observed the solution of this stage, we correct the second-iteration guess of delivery time between any two ports taking into consideration only the services that are actually offered by the monopolistic firm. Thus, we run again the program for as many iterations as needed to observe convergence of solutions.

Even though the model is represented as a MILP problem, individual instances were solved in less than 1200 ms using the CPLEX optimizer on a relatively modern computer workstation (Quad core Intel i7 2.8GHz processor and 16GB RAM). Problem instances were generated for all possible fleet compositions, with the results summarized in [Table 4](#) – converged solutions were obtained in less than three iterations for all instances.

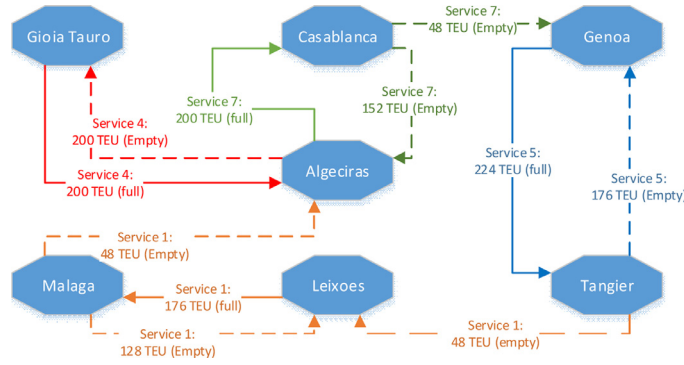


Fig. 4. Breakdown of container flows in optimal case – scenario D.

The highest profit is obtained in Scenario D, where the firm is equipped with 4 vessels. In the case of larger fleet sizes, the firm serves exactly the same routes and container flows as in Scenario D, with its profit decreased by the ownership costs of the inactive vessels. Using the backward induction procedure, we determine that in the first stage of the game, and given the structure of the market and potential services, the firm should choose to operate only four vessels. A comprehensive diagram of container flows across service legs in Scenario D is provided in Fig. 4.

The outcome of this process can be justified by the fact that further service provision would diminish the potential profit of the firm; given the limited set of potential services and the amount of transport service demanded (as defined within this problem instance), there was no requirement for more complicated transshipment arrangements than would have been necessary to serve port pairs without any direct service links. A different outcome would have been observed by expanding the set of potential services that the firm could select, thus making it more able to control the operating cost it faces to satisfy market demand.

#### 4. Maritime container transport design in duopolistic market

In this section we consider the case of two firms competing in the maritime container market. We solve the game relying on the concept of Subgame Perfect Equilibrium. Hence, we proceed by backward induction.

##### 4.1. Theoretical analysis

Once deployed capacity, route operation and assignment are chosen by the competing firms, we obtain (similarly to the benchmark case) the maximum admissible flow of full containers,  $t_{rs}^f$ , and transport time,  $u_{rs}$ , for each firm and for its rival (i.e.,  $t_{rs}^{f*}$  and  $u_{rs}^*$ ) and any pair of origin and destination ports. As before, the demand for transport from  $r$  to  $s$  is positive (and equal to  $MQF_{rs}$ ) only if any of the two competing firms is able to offer a sufficiently low total cost of transport (i.e., below  $MCT_{rs}$ ). Two kinds of equilibria may arise at this stage of the game on market of transport services from  $r$  to  $s$ :

- (a) if  $t_{rs}^f + t_{rs}^{f*} < MQF_{rs}$  there is an excess of demand over the supply of transport services offered by both firms;
- (b) if  $t_{rs}^f + t_{rs}^{f*} \geq MQF_{rs}$  there is an excess of supply over demand.

Let us consider a given firm (the same analysis holds true also for the rival firm). Any type of equilibrium involving non-zero demand flows for the firm may arise only if  $MCT_{rs} \geq COT u_{rs}$ . The revenue function of the firm is  $d_{rs} = q_{rs}^f t_{rs}^f$ . The following proposition characterizes the equilibrium of type (a):

**Proposition 1.** *If equilibrium flows from  $r$  to  $s$  are such that  $t_{rs}^f + t_{rs}^{f*} < MQF_{rs}$ , then the equilibrium firm's fee necessarily is  $q_{rs}^f = MCT_{rs} - COT u_{rs}$ .*

**Proof.** Remark that  $q_{rs}^f > MCT_{rs} - COT u_{rs}$  would imply a zero demand for the firm. Assume, by contradiction, that  $q_{rs}^f < MCT_{rs} - COT u_{rs}$ . The rival firm cannot absorb any excess of demand, because it already operates at its optimally planned flow (as determined in stage two). Thus, the firm could increase its revenue  $d_{rs} = q_{rs}^f t_{rs}^f$  (and hence its total profit) by raising the transport fee  $q_{rs}^f$ . And the proposition follows.  $\square$

The implication of the previous argument is that the revenue function for each firm is exactly as in the monopolistic case analyzed in Section 3. Let us now consider the equilibrium of type (b):

**Proposition 2.** *If equilibrium flows from  $r$  to  $s$  are such that  $t_{rs}^f + t_{rs}^{f*} \geq MQF_{rs}$ , then the equilibrium firm's fee necessarily is:*

$$q_{rs}^f = \begin{cases} COT (u_{rs}^* - u_{rs}), & u_{rs}^* \geq u_{rs} \\ 0, & u_{rs}^* < u_{rs} \end{cases} \quad (4)$$

**Proof.** By  $t_{rs}^f > MQF_{rs} - t_{rs}^{f*}$  the actual flow of demand for transport from  $r$  to  $s$  could be lower than the optimally planned flow  $t_{rs}^f$ . The same argument applies to the rival firm. Given the excess of supply over demand, the equilibrium fees necessarily should imply that the total cost of transport from  $r$  to  $s$  is equal across them, i.e.,  $q_{rs}^f = q_{rs}^{f*} + COT(u_{rs}^* - u_{rs})$ . Moreover, if  $u_{rs}^* < u_{rs}$  the firm offers a worse service in terms of travel (opportunity cost of) time, however the minimum transport fee that it can charge is  $q_{rs}^f = 0$  (earning a zero revenue, whatever the actual transport flow). The reverse holds if  $u_{rs}^* \geq u_{rs}$ , in particular  $q_{rs}^{f*} = 0$ . In this case, the firm's equilibrium fee cannot be higher than  $COT(u_{rs}^* - u_{rs}) \geq 0$  (otherwise it would lose demand flows to the benefit of the rival) nor lower than that (otherwise it could increase its revenue and profit by a small increase of the fee, given that the service provided by the rival is worse in terms of travel time). Thus, the proposition follows.  $\square$

A key feature of this model is that the most time-efficient firm can always attract all the traffic that it is able to serve (i.e.,  $t_{rs}^f$ ) by fixing its transport fee in such a way that the total transport cost for the shipper is slightly lower than for the rival firm. Having observed this we can conclude that the revenue function of the firm can be represented as follows:

$$d_{rs} = \begin{cases} COT(u_{rs}^* - u_{rs})t_{rs}^f, & u_{rs}^* \geq u_{rs} \\ 0, & u_{rs}^* < u_{rs} \end{cases} \quad (5)$$

where  $t_{rs}^f \leq MQF_{rs}$ . Taking as given the service network design and assignment solution of the rival, the firm anticipates the impact of the market reaction on its revenue, given the deployed capacity on each route. In particular, the firm will anticipate if it will be able to operate on each service from  $r$  to  $s$  as a monopolist (in which case revenue is given by Eq. (2)) or as a duopolist (in which case revenue is given by Eq. (5)). The firm would thus optimize its service network and flow assignment using a mixed integer linear program whose objective is to maximize the same profit function (1) subject to the following set of constraints (refer to the Appendix for notation):

$$\sum_{a \in A_i^+} x_{as}^f - \sum_{a \in A_i^-} x_{as}^f = b_{is}^f \text{ for all } i \in I, s \in D \quad (6.1)$$

$$\sum_{a \in A_i^+} x_a^e - \sum_{a \in A_i^-} x_a^e = -b_i^e \text{ for all } i \in I \quad (6.2)$$

$$x_{as}^f \leq \sum_{n \in N} FSL_n LR_{an} p_n w_{is}^f \text{ for all } a \in A_i^-, i \neq s \in I, s \in D \quad (6.3)$$

$$x_a^e \leq \sum_{n \in N} FSL_n LR_{an} p_n w_i^e \text{ for all } a \in A_i^-, i \in I \quad (6.4)$$

$$MQP_i \geq \sum_{a \in A_i^-} (x_{a+}^f + x_a^e + x_{a+}^{f*} + x_a^{e*}) + \sum_{a \in A_i^+} (x_{a+}^f + x_a^e + x_{a+}^{f*} + x_a^{e*}) \text{ for all } i \in I \quad (6.5)$$

$$MRC_n p_n \geq \sum_{a \in A_n} \sum_{s \in D} LLR_{al} (x_{a+}^f + x_a^e) \text{ for } n \in N, l \in L_n \quad (6.6)$$

$$b_{is}^f = \begin{cases} -t_{rs}^f & \text{if } i = r \in O \\ t_{rs}^f & \text{if } i = s \in D \\ 0 & \text{otherwise} \end{cases} \quad (6.7)$$

$$b_i^e = \begin{cases} t_{+i}^f & \text{if } i = r \in \bar{O} \cap D \\ -t_{+i}^f & \text{if } i = r \in \bar{D} \cap O \\ t_{+i}^f - t_{i+}^f & \text{if } i = r \in O \cap D \\ 0 & \text{otherwise} \end{cases} \quad (6.8)$$

$$d_{rs} = \begin{cases} COT(u_{rs}^* - u_{rs})t_{rs}^f & \text{if } t_{rs}^f + t_{rs}^{f*} \geq MQF_{rs} \\ (MCT_{rs} - COT(u_{rs}^* - u_{rs}))t_{rs}^f & \text{if } t_{rs}^f + t_{rs}^{f*} < MQF_{rs} \end{cases} \text{ for all } r \in O, s \in D \quad (6.9)$$

$$t_{rs}^f \leq MQF_{rs} \quad (6.10)$$

$$\sum_{n \in N} p_n RVR_n \leq CFS \quad (6.11)$$

$$x_{as}^f \geq 0 \text{ for all } a \in A, s \in D \quad (6.12)$$

$$x_a^e \geq 0 \text{ for all } a \in A \quad (6.13)$$

$$d_{rs} \geq 0 \text{ for all } r \in O, s \in D \quad (6.14)$$

$$t_{rs}^f \geq 0 \text{ for all } r \in O, s \in D \quad (6.15)$$

$$p_n \in \{0, 1\} \text{ for all } n \in N \quad (6.16)$$

The above problem formulation shares many constraints with the monopolistic version of the problem. Constraints (6.5) and (6.6) here incorporate the presence of competitor flows ( $x_{a+}^{f*}$  and  $x_a^{e*}$ ), while the revenue function (6.9) has been updated to reflect the duopolistic revenue model that was introduced earlier in this section. As written in the above model, (6.9) features a conditional constraint. This would violate the intended linear nature of the model, which we can maintain by adopting an additional constraint transformation.

This would require the introduction of two indicator decision variables  $da_{rs}$  and  $db_{rs}$ . Using these, we can suggest that  $da_{rs} = 1$  and  $db_{rs} = 0$  if  $t_{rs}^f + t_{rs}^{f*} \geq MQF_{rs}$ . Alternatively,  $da_{rs} = 0$  and  $db_{rs} = 1$  if  $t_{rs}^f + t_{rs}^{f*} < MQF_{rs}$ . The above would hold for all  $r \in O, s \in D$ . Constraint (6.9) can therefore be substituted by:

$$da_{rs} = (1 - db_{rs}) \quad (6.9.1)$$

$$t_{rs}^f + t_{rs}^{f*} \leq MQF_{rs} (2 - db_{rs}) \quad (6.9.2)$$

$$t_{rs}^f + t_{rs}^{f*} \geq MQF_{rs} da_{rs} \quad (6.9.3)$$

$$d_{rs} = MCT_{rs}t_{rs}^f + COT u_{rs}^*t_{rs}^f - COT u_{rs}t_{rs}^f - MCT_{rs}t_{rs}^f da_{rs} - COT u_{rs}^*t_{rs}^f db_{rs} \quad (6.9.4)$$

However, the constraint (6.9.4) is affected by multiplication between decision variables. Hence, we linearize it as follows:

$$d_{rs} = MCT_{rs}t_{rs}^f + COT u_{rs}^*t_{rs}^f - COT u_{rs}t_{rs}^f - MCT_{rs}dc_{rs} - COT u_{rs}^*dd_{rs} \quad (6.9.4^*)$$

$$dc_{rs} \leq MQF_{rs}^f da_{rs} \quad (6.9.4.1)$$

$$dc_{rs} \leq t_{rs}^f \quad (6.9.4.2)$$

$$dc_{rs} \geq t_{rs}^f - MQF_{rs}^f (1 - da_{rs}) \quad (6.9.4.3)$$

$$dc_{rs} \geq 0 \quad (6.9.4.4)$$

$$dd_{rs} \leq MQF_{rs}^f db_{rs} \quad (6.9.4.5)$$

$$dd_{rs} \leq t_{rs}^f \quad (6.9.4.6)$$

$$dd_{rs} \geq t_{rs}^f - MQF_{rs}^f (1 - db_{rs}) \quad (6.9.4.7)$$

$$dd_{rs} \geq 0 \quad (6.9.4.8)$$

As before, constraints (6.3) and (6.4) feature multiplication between decision variables that are linearized as follows:

$$x_{as}^f \leq \sum_{n \in N} FSL_n LR_{an} g_{nis} \text{ for all } a \in A_i^-, i \neq s \in I, s \in D \quad (6.3^*)$$

$$g_{nis} \leq MDT_i^f p_n \quad (6.3.1)$$

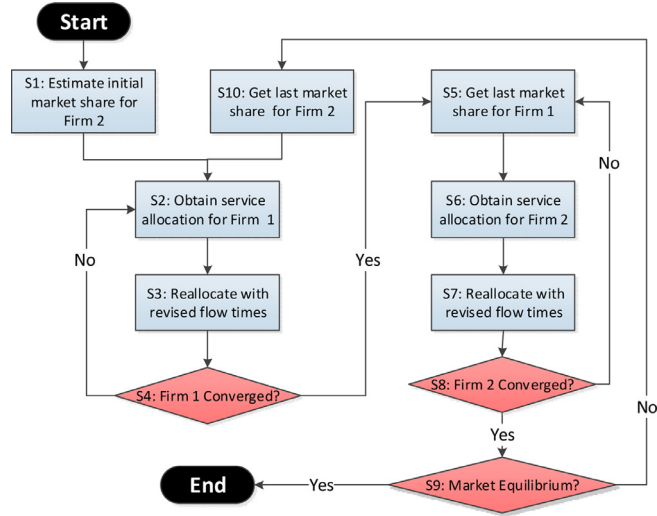


Fig. 5. Logic flow of the duopolistic assignment algorithm.

$$g_{nis} \leq w_{is}^f \quad (6.3.2)$$

$$g_{nis} \geq w_{is}^f - MDT_i(1 - p_n) \quad (6.3.3)$$

$$g_{nis} \geq 0 \quad (6.3.4)$$

where  $MDT_i$  is the maximum dwell time of full containers at port  $i$ . Similarly, constraint (6.4) is transformed as follows:

$$x_{as}^f \leq \sum_{n \in N} FSL_n LR_{an} h_{ni} \text{ for all } a \in A_i^-, i \neq s \in I, s \in D \quad (6.4*)$$

$$h_{ni} \leq MDT_i^e p_n \quad (6.4.1)$$

$$h_{ni} \leq w_i^e \quad (6.4.2)$$

$$h_{ni} \geq w_i^e - MDT_i^e(1 - p_n) \quad (6.4.3)$$

$$h_{ni} \geq 0 \quad (6.4.4)$$

where  $MDT_i^e$  is the maximum dwell time of empty containers at port  $i$ . At the first stage of the game, both firms are able to anticipate the effect of subgame equilibria (stages 2 and 3) on their outcome, depending on their strategic choices in terms of capacity limits. Again, because of the complexity of the sequential game, we rely on a numerical example to characterize the equilibrium.

#### 4.2. Numerical example

The analysis is conducted on the basis of an iterative process. The algorithm, represented in Fig. 5, was implemented again using a combination of the Delos maritime network design tool (used for data management and algorithm flow control) and IBM OPL CPLEX. The duopolistic version of the algorithm was tested on the same market environment that was used earlier in this study, with the key difference being that in this case two symmetrical shipping firms being allowed to operate and compete. In this context, Firm 2 is referred to as the rival firm (represented with an asterisk in Section 4.1) and Firm 1 the other firm – both have the ability to allocate up to seven vessels to accommodate the various transport demands in the market. The algorithm would then allocate flows among firms, taking into account the effects of competition in the revenue functions as discussed in Section 4.1.

A series of iterations is necessary to determine the convergence of the solutions of the mixed linear integer programs of the Firms 1 and 2 to the market equilibrium. The first iteration starts assuming a market share for the Firm 2 (stage S1 in Fig. 5); in particular, we assume that the rival firm behaves as a monopolist. Then, we run the mixed linear integer program of Firm 1 assuming – as first-iteration guess – that the delivery time between any two ports is the average of all services

**Table 5**

Results for the duopolistic problem instances.

Scenario	Firm 1 profit	Firm 2 profit	Firm 1 services	Firm 2 services	Firm 1 flows	Firm 2 flows
A: 1 vessel	\$52,184	\$20,657	S1	S2	Leixoes → Malaga (176)	Genoa → Tangier (200)
B: 2 vessels	\$72,465	\$14,800	S1, S5	S4, S7	Genoa → Tangier (200)	Gioia Tauro → Casablanca (200)
C: 3 vessels	\$67,172	\$6800	S1, S5 S6	S4, S7	Leixoes → Malaga (176) Genoa → Tangier (188) Leixoes → Malaga (176) Leixoes → Algiers (24)	Gioia Tauro → Casablanca (200)
D: 4 vessels	\$56,465	\$16,857	S1, S5	S1, S3 S4, S6	Genoa → Tangier (200) Leixoes → Malaga (176)	Gioia Tauro → Casablanca (200) Leixoes → Algiers (100)
E: 5 vessels	\$80,673	−\$40,000	S1, S4 S5, S7	N/A	Genoa → Tangier (200) Leixoes → Malaga (176) Gioia Tauro → Casablanca (200)	N/A

potentially linking them weighted by the demand (S2 in Fig. 5). Having observed the solution of the previous stage, we correct the second-iteration guess of delivery time between any two ports taking into consideration only the services that are actually offered by the Firm 1, and we run again the program (S3 in Fig. 5) for as many iterations as needed to observe convergence of solutions (S4 in Fig. 5).

We are now in the position to run the mixed linear integer problem of the Firm 2, taking as given the market share determined for Firm 1 (S5 in Fig. 5), applying the same iterative procedure described for the Firm 1 (S6, S7, S8 in Fig. 5). Once we obtain the convergence of solutions of the program of Firm 2 (S8 in Fig. 5) we use the output market share as input of new iterations regarding the program of Firm 1 (S9 in Fig. 5). We run as many iterations of the described procedure as necessary to determine a stable market equilibrium (S10 in Fig. 5), that is defined by the fact that the solutions worked out by the mixed linear integer programs of Firm 1 and 2 produce market shares that are mutually consistent.

A similar set of experiments as with the monopolistic case was carried out to test the behavior of the algorithm, where two firms of equal fleet size (ranging between 1 and 7 vessels for each case) were competing in the same market. No more than 10 steps were required to obtain converged solutions. Unique solutions were obtained for instances with fleet sizes below 5 vessels. As was the case with the monopolistic version of the algorithm, for larger fleet sizes, container flows were the same as Case E (5 vessels per firm), but with decreased profits (given the ownership costs of inactive vessels).

The final market allocations were found to not be affected by the initial market share assumption for the competing firms. As seen in Table 5, the largest overall container flow is observed in Case D, with a total of 676 full TEUs/week. These were accompanied by equal, but reverse empty container flows across the ports served. It is worth remarking that while one of the potential outcomes of the competitive iteration process would have been an unstable assignment of the market share among the two firms, with an oscillating set of allocations for the firms, that prohibits the algorithm from terminating, such an outcome was not observed in any of the scenarios studied.

The results show that the competing firms tend to choose different services and serve different markets (i.e., flows between pair of ports). Of particular interest in the instances attempted are Cases D and E. The former is the only case across the instances studied where both firms operate a common service (Service 1) in the converged solution. Even so, they do not seem to compete in servicing any of the flows. As highlighted in simpler contexts in the study of industrial organization economics (Tirole, 1988, Chapter 7), firms tend to invest in “product differentiation” to relax competition among them and, by this strategy, to increase their profits. In our model and this numerical example, the firms pursue such a service differentiation strategy by choosing services and container flow assignments that imply a complete separation of served markets.

In the extreme Case E, Firm 2 opts to not enter in the market altogether, as all potentially profitable cargo flows have been absorbed by Firm 1. Instead, covering the ownership costs associated by their idle vessels is a least costly outcome. This behavior is also consistent with the practices of real firms during troughs in the market cycles, when many vessels are laid up, in anticipation of better market conditions in the future (Stopford, 2009). While it might not be possible to predict which of the two firms will have two lay up its vessels in this scenario, the results of this analysis might urge both to pursue a less aggressive strategy. Therefore, even though maximum possible individual gains diminish, extremely unfavorable individual outcomes would be eliminated.

## 5. Conclusions and future work

This paper presents a sequential game-theoretic model to analyze service network design, container assignment and service provision of shipping firms (or alliances) both when they operate in a set of ports as a monopolist and when they compete with a rival firm. The model takes into account exogenous demands for container transport among ports of origin and destination which reacts to the total cost of transport, including the travel fee that is paid to the shipping firm and the opportunity cost of time (e.g., depreciation of shipped commodities) for the shipper. Because of the complexity of the theoretical model, we rely on a numerical algorithm to characterize the equilibrium of the game. We find that:



- the monopoly firm does not cover all possible market demand, because of the high cost of available services (i.e., possible routes that it can activate) mainly linked to transshipment; moreover, the monopoly never satisfies all the existing demand in ports that are served through the chosen network;
- when a duopoly is considered, the scope for demand satisfaction improves; the firms tend to choose different service networks in order to curb the competitive pressure; only rarely do they opt to operate common routes – this happens once other options have been exhausted and under the pressure of vessel ownership and operation costs; again, the existing market demand is not fully satisfied, because of transshipment and network operation costs (e.g., container repositioning), as is the case also under monopoly.

The introduction of competitive behavior adds a new level of complexity to container flow assignment and liner service network design problems, therefore bringing them closer to real life settings. Duopoly (and monopoly) is a good approximation of the behavior of the shipping market in several world regions. The model described in this paper can be used as a framework for future research addressing the case of oligopolistic shipping markets with multiple players.

Several other issues need to be investigated in future research, such as what is the effect of widening the scope for service line selection and assignment. One potential research direction would be to explore the possibility that firms differentiate their networks and thus relax competition among themselves, and the possibility for each firm to optimize the cost structure of its network. The assessment of these issues in alternative set of ports can also shed some light on the role of the vessel capacity limit (and investment costs). The theoretical model and the numerical algorithm presented in this paper are sufficiently powerful to address these issues, so which will be explored as part of future research.

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## Appendix. Variable definitions

Indices		Sets	Subsets
$a$	For legs	$A$	Legs
$b$	For companies	$O$	Origin ports
$l$	For links	$D$	Destination ports
$n$	For routes	$I$	All ports
$i$	For ports	$N$	Set of routes
$r$	For origin ports	$L$	Set of links
$s$	For destination ports		

Parameters		
$CSR_l$	Operating cost for link $l$ , per unit time	\$/Week <sup>2</sup>
$CSF_l$	Port visit cost for link $l$	\$/Week
$CHC_n$	Container handling cost for route $n$ offered	\$/TEU
$CCD$	Container operational costs per unit time	\$/ (TEU · Week)
$COT$	Opportunity cost of time per unit time	\$/ (TEU · Week)
$CRC$	Fixed vessel ownership costs	\$/ (vessel · Week)
$MCT_{rs}$	Maximum cost of travel (from port $r$ to port $s$ )	\$/TEU
$MRC_n$	Capacity of route $n$	TEU/Week
$MQF_{rs}$	Maximum flow of full containers (from port $r$ to port $s$ )	TEU/Week
$MQP_i$	Maximum throughput of port $i$	TEU/Week
$STL_l$	Sailing time for link $l$	Week
$LRR_{nl}$	1 if link $l$ uses route $n$ , and 0 otherwise	–
$LR_{an}$	1 if leg $a$ is on route $n$ , 0 otherwise	–
$LLR_{al}$	1 if leg $a$ contains link $l$ , 0 otherwise	–
$CFS$	Fleet size for current firm	Vessels
$RV R_n$	Route vessel requirement	Vessels
$MDT_i^f / MDT_i^e$	Max empty and full dwell times at $i$	TEU · Week/Week
$FSL_n$	Frequency of sailings at route $n$	Week <sup>-1</sup>

(continued on next page)

Decision variables		
$t_{rs}^f$	Flow of full containers (from port $r$ to port $s$ )	TEU/Week
$p_n$	1 if route $n$ is offered; 0 otherwise	–
$u_{rs}$	Travel time for full containers (from port $r$ to port $s$ )	Week
$d_{rs}$	Revenue for full containers from (from port $r$ to port $s$ )	\$/Week
$x_{as}^f$	Flow of full containers on leg $a$ en route to port $s$	TEU/Week
$x_a^e$	Flow of empty containers on leg $a$	TEU/Week
$w_{is}^f$	Dwell time at port $i$ for full cont. en route to port $s$	TEU•Week/Week
$w_i^e$	Dwell time at port $i$ for all empty containers	TEU•Week/Week
Composite expressions		
$TRP$	Total revenue across all OD pairs	\$/Week
$TCS_n$	Service operational costs for route $n$	\$/Week
$TCL_n$	Leg operational costs for route $n$	\$/Week
$TCD_n$	Container deployment costs for route $n$	\$/Week
$TCC_n$	Vessel deployment costs for route $n$	\$/Week

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