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Five old open clusters: NGC 2682, NGC 2243, Berkeley 39, NGC 188 and NGC 6791

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Abstract. — The color-magnitude diagrams (CMD) of five old open clusters, namely NGC 2682, NGC 2243, Berkeley 39, NGC 188, and NGC 6791 are examined in detail with the aid of the synthetic color-magnitude diagram technique described by Carraro et al. (1993), and the color excess E_{B-V}, distance modulus $(m-M)$, and age of each cluster are derived. The goal is to provide an homogeneous ranking of the cluster ages. The analysis is made using two types of stellar models in which different prescriptions are adopted for the extension of the convective cores, i.e. either the classical or the overshoot scheme. The stellar models in use are from Alongi et al. (1993), Bressan et al. (1993), and Fagotto et al. (1993a,b). The evolutionary tracks go from the main sequence up to the start of the thermally pulsing regime of the asymptotic giant branch (TP-AGB) phase. The purpose is to discriminate between the two evolutionary scenarios. It turns out that all the clusters have turn-off masses either slightly greater or equal to the value of 1.1 $M_{\odot}$, at which we expect the transition from convective to radiative core H-burning to occur. As far as the mixing mechanism is concerned, the results are not very conclusive. For the youngest clusters of our sample, namely NGC 2682 and NGC 2243, models with convective overshoot seem to provide a slightly better fit of the observed CMD and luminosity function. For the oldest cluster of our sample, namely NGC 6791 with age of 8 10^9 yr, there is no appreciable difference passing from one scheme to another. Finally, for the remaining clusters, namely Berkeley 39 and NGC 188, the situation is ambiguous. This trend is not surprising, because we expect the convective core and hence associated overshoot to vanish at decreasing turn-off mass and hence increasing age. As expected the ages derived from overshoot models are slightly older than those obtained from standard models. The five clusters on consideration can be ranked as function of the age as follows (the first value refers to classical models, the second one to models with overshoot): NGC 2682 (4.3 10^9 yr and 4.8 10^9 yr), NGC 2243 (3.5 10^9 yr and 3.9 10^9 yr), Berkeley 39, (6.0 10^9 yr and 6.5 10^9 yr), and NGC 188 (7.0 10^9 yr and 7.5 10^9 yr). No age difference is found in the case of NGC 6791 for which both types of stellar models yield 8.0 10^9 yr.

Key words: open clusters and associations — stars: evolution — stars: interiors

1. Introduction

The study of the old Galactic open clusters is basic to understanding the past history of star formation and metal enrichment in the galactic Disk, the structure of this, and the theory of stellar evolution in that particular range of masses in which the core H-burning phase changes from radiative to convective. Unfortunately, those tasks are hampered by several difficulties. First of all, the number of known open clusters older than NGC 2682 (M 67) is very small, their structure is much more dispersed and less rich in stars (several orders of magnitude) than globular clusters, and thereupon much more subject to disruption by dynamical interactions with the surrounding medium. Second, the low number of stars in each cluster, together with the significant contamination by foreground stars, makes difficult the acquisition of CMDs with the desired degree of completeness in all evolutionary stages. In general, in these clusters the main sequence is well traced, whereas the subgiant and red giant branches (SGB and RGB, respectively) are much less evident, and finally the red clump (likely core He-burning stars) consists of a handful of objects (see below).

As far as the theory of stellar models is concerned, since the turn-off mass of these clusters is at about 1.1 $M_{\odot}$, namely the value at which the inner core of a star switches from radiative to convective, the detailed analysis of their CMDs bears very much on the efficiency and extension of central convection, a long debated subject (see Chiosi et al. 1992 for a recent review).

Previous studies of the CMD for this type of clusters have pointed out the existence of detailed features, such as the curvature of the turn-off, the morphology of the so-called blue hook, and presence or absence of gaps, whose full understanding is still unsettled. These topics have been recently examined at same extent by Carraro et al. (1993) and Bertelli et al. (1992) to whom we refer for more details. In this paper we examine the CMDs of five open clusters, namely NGC 2682 (M 67), NGC 2243, Berkeley 39, NGC 188, and NGC 6791, for which sufficiently good photometry and homogeneous determinations of the metallicity are available. For each cluster, adopting the metallicity contained in the compilation by Friel & Janes (1993), we derive the color excess E_{B-V}, the distance modulus, and the age. The method in use is the synthetic CMD technique which allows the simultaneous fit of several important features at the CMD (turn-off, RGB, and clump) at the same time. The synthetic CMDs are generated imposing that the number of main sequence (MS) stars counted in suitable magnitude ranges of the observational CMDs are matched. Two grids of stellar models are used, in which either the classical (semiconvective) or the overshoot scheme are adopted. The models are by Alongi et al. (1993), Bressan et al. (1993), and Fagotto et al. (1993a,b), to whom the reader should refer for all details about the input physics and the evolutionary results. Suffice to recall that these models are calculated with the most recent release of the radiative opacity by the Livermore Group (Iglesias et al. 1992 and references therein) and the abundances by Grevesse (1991). The mixing length of the external convection is calibrated by imposing that the effective temperature and luminosity of the Sun are matched by a model with the solar composition and age of 4.7 10^9 yr.

The metallicity [Fe/H] is converted to the metallicity Z characterizing the set of stellar models in use by means of the following relation

$$\mathrm{Log}\ Z = 1.03\ [\mathrm{Fe/H}] - 1.698. \quad (1)$$

Finally, the conversion from effective temperatures and luminosities to colors and magnitudes is from the Kurucz (1992, private communication) grids of model atmospheres, bolometric corrections, and colors as a function of metallicity, effective temperatures, and gravities.

In order to lend support to the age derived from the fit of the CMDs we also look either at the luminosity function of the MS stars or, when this is not feasible, we define and compare suitable ratios of star counts that are meant to grossly represent the luminosity functions in selected evolutionary stages. The ratios in use are σ_1 and σ_2 which are defined as follows. The ratio σ_1 between the number of MS stars in a suitable magnitude interval to the stars beyond the MS is related to the ratio of core H- to He-burning lifetimes, while the ratio σ_2 between the red giant stars brighter and fainter than the clump luminosity is related to the lifetimes in the clump (He-burning) and along the RGB.

The above ratios are good indicators of the core H- and He-burning lifetimes when they are derived from ideal CMDs in which only member stars are plotted. This requires the careful correction of the CMDs for contaminations by foreground and background stars. The lack of such an ideal CMD forced on us an empirical correction of the CMD. The procedure is as follows. All of the CMDs we are going to examine show broad MS bands with stars scattered at both sides likely by the presence of field stars. At each magnitude interval, first we look at star frequency as a function of the color, and draw two boundaries of the MS band within which are comprised the majority of stars, and remove all the stars outside this region. But for NGC 6791 and Berkeley 39, this correction turns out to be small. The errors affecting the above ratios are evaluated according to the relation

$$\Delta\sigma_i = \sigma_i \times (N_1^{-0.5} + N_2^{-0.5}) \quad (2)$$

by Buzzoni et al. (1983), where N_1 and N_2 stand for the numerator and denominator in the definition of σ_1 and σ_2.

In addition to this, all the CMDs have MS bands (after correction) that are broader than expected on the basis of photometric errors quoted in each source of the CMD. As explicitly mentioned discussing each CMD, previous studies have indicated that the MS band of the clusters on consideration are broadened toward the red by the existence of a certain percentage of unresolved binary stars (Mermilliod & Mayor 1989). The appearance of an unresolved binary on the CMD would be equivalent to that of a single star brighter and redder depending on the mass ratio of the two component stars (Maeder 1974). In order to take into account the probable presence of unresolved binaries, the synthetic CMDs are obtained by including a certain percentage of these objects with suitable mass ratios and by adding to the population of truly single stars a number of binaries whose magnitudes and colors are obtained by summing up the fluxes of the component stars. The percentage and mass ratio distribution are tuned till the desired broadening of the MS band is obtained.

In this paper not only we assign each cluster the age resulting from the adoption of one of the two evolutionary schemes, but also looking at the overall morphology of their CMDs, we seek to infer which of the two schemes for central mixing ought to be preferred.

We find that models with overshoot provide a slightly better fit of the global properties of the CMD for the youngest clusters. Indeed, the gaps on the main sequence are better reproduced by this type of models (see the paradigmatic case of NGC 2243).

As expected the ages derived from overshoot models are slightly older than those obtained from classical models. The five clusters on consideration can be ranked as a function of the age as follows (the first value refers to classical models, the second one to models with overshoot): NGC 2682 (4.3 10^9 yr and 4.8 10^9 yr), NGC 2243 (3.5 10^9 yr and 3.9 10^9 yr), Berkeley 39, (6.0 10^9 yr and 6.5 10^9 yr), and NGC 188 (7.0 10^9 yr and 7.5 10^9 yr). No age difference is found in the case of NGC 6791 for which both types of stellar models yield 8.0 10^9 yr.

2. NGC 2682 (M 67)

NGC 2682 ($l = 216$, $b = 32$) by virtue of its height above the galactic plane (about 400 pc), low reddening, and proximity to the Sun (about 800 pc), is perhaps the most studied open cluster older than the Iades.

The metallicity of NGC 2682 goes from [Fe/H]=0.0 (Norris et al. 1980) to the extreme value of [Fe/H]=0.50 (Peterson 1981). Intermediate, more recent values are [Fe/H]=−0.07 (Janes & Smith 1984), [Fe/H]=−0.05 (Canterna et al. 1986), [Fe/H]=−0.10 (Burstein et al. 1986), [Fe/H]=0.06 (Nissen et al. 1987), [Fe/H]=−0.07 (Anthony-Twarog 1987), [Fe/H]=−0.08 (Friel & Janes 1991), and [Fe/H]=−0.09±0.07 (Friel & Janes 1993). This latter value is adopted in the present study. The color excess E_{B-V} is small, going from the old estimates of $E_{B-V} = 0.05 - 0.06$ (Johnson & Sandage 1955; Eggen & Greenstein 1965; Eggen 1983), $E_{B-V} = 0.09$ (Racine 1971) to the more recent values of $E_{B-V} = 0.06$ (Janes 1984), $E_{B-V} = 0.015$ (Burstein et al. 1986), and $E_{B-V} = 0.05$ (Nissen et al. 1987).

Finally, the following estimates of the distance modulus can be found in the literature: $(m - M) = 9.70$ (Johnson 1965), $(m-M) = 9.38$ (Eggen 1964), $(m-M) = 9.28$ (Eggen 1981), $(m-M) = 9.57$ (Racine 1971), $(m-M) = 9.61$ (Nissen et al. 1987), and $(m - M) = 9.63$ (Anthony-Twarog 1987).

CMDs of the stellar content of NGC 2682 are by Johnson & Sandage (1955), Eggen & Sandage (1964), Racine (1971), Sanders (1987), Anthony-Twarog (1987), Nissen et al. (1987), and Gilliland et al. (1991). In this study we have adopted the CMD of Gilliland et al. (1991) shown in Fig. 1.

Outstanding features of this CMD are the well defined MS band and RGB, and the scarcely populated clump of red stars at $V = 10.5$ and $(B - V) = 1.1$. The MS band shows a gap at $V = 12.9$, that was already present in all the photometric studies cited above, and a parallel sequence. The gap is likely to be a real feature resulting from the evolutionary behaviour of stars at the end core H-burning and overall contraction stages, while the parallel sequence is likely the signal of a substantial population of binary stars.

Finally, we notice the relatively large number of stars brighter than the gap, compared to the number of stars fainter than this. This feature was not clearly evident in the older CMDs by Eggen & Sandage (1964), Racine (1971), and Sanders (1987). In order to clarify at what extent this feature depends on the particular CMD in use we define the following ratio of star counts.

$$\gamma = N_{\rm MS}(12 \le V \le 13)/N_{\rm MS}(13 \le V \le 14) \quad (3)$$

where $N_{\rm MS}$ are the numbers of stars in the indicated magnitude intervals. The ratio γ is found to vary among the various sources of the CMD. The situation is summarized in Table 1.

Table 1. γ ratios in NGC 2682

Reference	γ
Eggen & Sandage (1964)	0.33±0.12
Racine (1971)	0.31±0.15
Sanders (1987)	0.33±0.07
Gilliland *et al* (1991)	0.68±0.30

Mathieu & Latham (1986) argue that spectroscopic binaries and blue stragglers tend to concentrate toward the cluster center. Considering that the CMD by Gilliland et al. (1991) refers to the central region of M 67, the high value of γ could reflect an higher proportions of binary stars (recall that the MS of unresolved binaries is about 0.7 mag brighter than the MS of single stars). The binary star hypothesis will be examined at some extent below.

The synthetic CMDs are constructed adopting the stellar models with metallicity $Z = 0.02$. This in fact is the closest value to the metallicity $Z = 0.016$ that we derive from the estimate by Friel & Janes (1993) once converted to Z by means of the relation (1) above. The helium content of the models is $Y = 0.280$.

The synthetic CMDs are normalized to the observed number of stars in the magnitude interval $13.5 \le V \le 14.5$ and are calculated assuming a certain percentage of binary stars with suitable mass ratios, namely 25%, and 0.5−1.5, respectively.

Finally, the ratios σ_1 and σ_2 are defined as follows: σ_1 is the ratio of the MS stars in the range $13.5 \le V \le 14.5$ (28 stars) and the evolved stars above $V = 13.0$ (16 stars), while σ_2 is the ratio between the evolved stars above and below $V = 11.0$, 5 and 11 stars respectively.

The magnitude interval for MS stars is chosen in such a way that firstly the problems caused by incompleteness and evaporation of main sequence stars from the cluster that might arise from including the lower tail of the MS band, and secondly both evolutionary and binary effects on the definition of the turn-off luminosity are minimized.

With the classical stellar models we obtain the color excess $E_{B-V} = 0.02$, the apparent distance modulus $(m-M) = 9.60$, and the age of 4.3 10^9 yr.

The classical models well reproduce the gap along the MS and the relative proportions of stars above and below this. However, there are some difficulties with the detailed morphology of the turn-off region as the classical models predict a too hot blue hook, contrary to what observed. Other choices of the age aimed at providing a good fit of the turn-off region inevitably lead to an excessively red color of the MS termination.

Checking for the agreement between theory and observations as far as the star counts defined above are concerned, we get the results presented in Tables 2 and 3, which contain the ratios γ, and σ_1 and σ_2, respectively. The ratio γ is also calculated for other choices of percentage of binary stars as indicated. Table 3 also contains the data for the other clusters to be examined below.

Table 2. γ ratios in M 67 vs binary fraction

γ ratio	Standard models	Overshoot models
γ_{25}	0.44±0.21	0.33±0.22
γ_{50}	0.43±0.18	0.38±0.21
γ_{75}	0.43±0.23	0.43±0.20

It turns out that γ (see the value for γ_{25} in Table 2), σ_1 and σ_2 are in satisfactory agreement with the observational data.

Repeating the same analysis with the stellar models incorporating convective overshoot we get the color excess $E_{B-V} = 0.02$, the apparent distance modulus $(m-M) = 9.50$, and the age of 4.8 10^9 yr. The overall fit of the CMD is now better than with classical models. However, while the ratios $\sigma_1 = 2.53\pm0.81$ and $\sigma_2 = 0.86\pm0.84$ are close to the observational values, the ratio γ is too low compared to that from star counts in the CMD (see Table 2). The reason for it is the duration of the overall contraction phase at the end of central H-burning which is much shorter (about a factor of five) than in classical model. Finally, it can be noticed that σ_1 derived from the overshoot models is somewhat lower than the corresponding value obtained from the classical models.

Looking for a better agreement between theory and observations we have considered the possibility that a larger fraction of binaries are present in the cluster and hence in the CMD. With this aim, we have assumed percentages of binary stars as high as 50% and 75% and repeated the analysis both for classical and overshoot models.

With classical models there is no appreciable variation of γ at increasing binary percentage, whereas with the overshoot models γ increases from 0.33 to 0.43 as the percentage goes from 25% to 75% (see the data in Table 2). This is caused by the different evolutionay rate of the two types of stellar models during the core H-exhaustion and overall contraction stages. On the contrary, there is no significant effect on the ratios σ_1 and σ_2.

The sensitivity of γ to the fraction of binary stars indirectly confirms that the suspected larger fraction of binary stars in the Gilliland et al. (1991) sample ought to be real.

As examples of the CMD fit, we show in Figs. 2 and 3 two simulations for the classical and overshoot models, respectively, in which the above ages and the percentage of binary stars amounting to 50% have been assumed. Finally, Fig. 4 compares the CMD of Gilliland et al. (1991) with the isochrones of 4.3 10^9 yr (classical) and 4.8 10^9 yr (overshoot). The superposition of the isochrones on the observational CMD is made assuming the distance modulus and color excess found for each type of model (see above).

Finally, in Fig. 5 we present the Integrated MS Luminosity Function (ILF) and compare it with the observational data. The theoretical ILFs refer to simulations in which the fraction of binary stars has been supposed to amount to 50%. Lower percentages of binary stars would not yield ILFs in agreement with the data. The vertical bars show the dispersion in the ILF expected to be caused by stochastic effects due to the small total number of stars in question. The dispersion has been evaluated by performing 50 simulations for the same value of the age and percentage of binary stars.

The main result of this analysis of the CMD and ILF is that clearly disentangling between the two types of stellar models is not possible on the basis of the available data.

3. NGC 2243

NGC 2243 ($l = 240$ $b = -18$) is a rich, old open cluster of rather low metallicity according to the current estimates. The exact value is however controversial going from [Fe/H]=0.30 of van den Bergh (1977) to [Fe/H]=−1.2 of Hardy (1981) and Gratton (1982). Other determinations are [Fe/H]=−0.63 (Janes 1979), [Fe/H]=−0.93 (Geisler 1987), [Fe/H]=−0.56 (Friel & Janes 1991), and [Fe/H]=−0.56±0.17 (Friel & Janes 1993).

Depending on the assumed metallicity, both the color excess E_{B-V} and distance modulus greatly vary. The color excess E_{B-V} goes from 0.01 (van den Bergh 1977) to 0.05 (Janes 1979) and 0.08 Hawarden (1975). The accompanying distance modulus is $(m - M) = 12.80$ in Hawarden (1975), $(m - M) = 13.30$ in van den Bergh (1977), and $(m - M) = 12.77$ in Janes (1979).

The age of this cluster has been first obtained by Hawarden (1975) who, assuming $Y = 0.30$ and the above metallicity, derived 5.0 10^9 yr. It has been subsequently confirmed by van den Bergh (1977) who however argued for a higher metallicity. The most recent detailed studies of NGC 2243 are by Bonifazi et al. (1990) and Bergbusch et al. (1991) based on accurate BV CCD-photometry of a large number of stars. In short, Bonifazi et al. (1990) suggest the presence of a fraction of binary stars up to about 30% of the total population, and confirm that three additional features in the CMD are significant: a gap just below the top of the MS, six blue straggler candidates, and an incipient red horizontal branch that seems to split in two almost parallel substructures. Comparing the CMD with the synthetic ones derived from various models of stellar evolution (either classical or overshoot) they conclude that the metallicity is $0.003 \leq Z \leq 0.006$, the color excess is $0.06 \leq E_{B-V} \leq 0.8$, the true distance modulus is $12.7 \leq (m-M)_{\rm o} \leq 12.8$, and finally the age is in the range $3-5\ 10^9$ yr. They also argue that large differences are to be expected according to whether or not convective overshoot is taken into account in stellar interiors. Finally, they examine the nature of the gap on the main sequence noting that overshoot models (those by Bertelli et al. 1990) do not predict a gap and that even classical models do not match it exactly.

Bergbusch et al. (1991) arrive to similar conclusions. Specifically, from the comparison of the CMD of NGC 2243 with the fiducial one of 47 Tuc by Hesser et al. (1987) they argue that the open cluster has a slightly higher metal abundance. Assuming the two clusters to have the same metallicity, they get a distance modulus $(m-M)_{\rm o} = 13.05$ and finally from the comparison with theoretical isochrones in the metallicity range $-0.78\leq$[Fe/H]≤ -0.47 computed with oxygen enhancement they conclude that only the isochrones with [Fe/H]=-0.47 and [O/Fe]=0.23 reproduce the turn-off and the location of the RGB at the same time. The resulting age is $5\pm1\ 10^9$ yr. However they argue that the gap on the main sequence, which should be associated with the developments of the convective hook following central H-exhaustion, cannot adequately be explained. Because the inclusion of binary stars does not seem to solve the problem, they suggest that convective overshoot is the probable cause.

In our analysis we adopt the observational CMD of Bergbusch et al. (1991). This is shown in Fig. 6. The overall morphology of this CMD resembles that of NGC 188 (see below). Specifically, the RGB is scarcely populated and the clump is detectable at $V = 13.65$ and $B-V = 0.90$. As already mentioned, the MS band is characterized by a gap near its top at about $V = 16.20$. Finally, the broad scatter of the MS band is attributed to the presence of a significant fraction of binary stars (see Bonifazi et al. 1990).

The synthetic CMDs are based on evolutionary models with composition $Z = 0.008$ and $Y = 0.25$, which in light of the above discussion is suited to this cluster. The simulations are constrained to impose that the observed number of stars in the magnitude interval $16.5 \leq V \leq 17.5$ is matched, thus avoiding all uncertainties related to the existence of the gap at brighter magnitudes. Binary stars are included and supposed to amount to 20% of the total with mass ratios between the component stars in the range 0.5$-$1.5.

With the standard models we get the color excess $E_{B-V} = 0.02$, the apparent distance modulus $(m-M) = 13.00$, and the age of $3.5\ 10^9$ yr. With the overshoot models we get the color excess $E_{B-V} = 0.01$, the apparent distance modulus $(m-M) = 12.90$, and the age of $3.9\ 10^9$ yr. The synthetic CMDs for the above composition and ages are shown in Figs. 7 and 8 for the classical and overshoot models, respectively. For better understanding we show in Fig. 9 the CMD of NGC 2243 together with the isochrones for the above ages. The general fit of the CMD is good, but it is not possible to reproduce the color of the RGB. This is probably caused by the adopted metallicity, which perhaps is slightly higher than the value suited to this cluster.

Like in the case of NGC 2682 we check for the fit of the CMD by means of the star counts and ILF. The following magnitude intervals are used to construct the ratios σ_1 and σ_2. In σ_1 the MS and post-MS stars are those between $V = 16.5$ and $V = 17.5$ (110 stars), and brighter than $V = 16.0$ (22 stars), respectively. In σ_2, the dividing magnitude is $V = 14.5$ (approximately at the bottom of the red star clump). We count 7 stars brighter and 15 fainter than the above limit. The observational and theoretical ratios are given in Table 3. While the comparison of σ_1 is not very helpful, the observational value is indeed compatible with the prediction both from classical and overshoot models, the ratio σ_2 for overshoot models is in closer agreement with its observational counterpart. The same conclusion can be drawn from the comparison of the ILF shown in Fig. 10. Also in this case the vertical bars show the dispersion resulting from stochastic effects caused by the limited number of stars in the sample. The dispersion is evaluated performing 50 simulations of the CMD at assigned age, chemical composition, and fraction of binary stars.

4. Berkeley 39

Berkeley 39 ($l = 223$ $b = 10$) is a rich cluster, considered to be old by Janes (1988). Very little is known about this cluster.

Friel & Janes (1991) derive the metallicity [Fe/H]= -0.29, while Friel & Janes (1993) estimate [Fe/H]=-0.31 ±0.08, or $Z = 0.009$ according to relation (1). The first photometric study of this cluster is by Kaluzny & Richtler (1989), whose CMD is shown in Fig. 11. With the aid of the method proposed by Anthony-Twarog & Twarog (1985), and the stellar models calculated by VandenBerg (1985), Kaluzny & Richtler (1989) suggest the age of $10\ 10^9$ yr.

Clearly, the CMD of Berkeley 39 is more contaminated by field stars. Nevertheless the main features are evident, namely the broad MS band suggesting the presence of a significant number of binary stars, the scarcely populated RGB and clump of red stars, the latter occurring in the magnitude interval $14.0 \leq V \leq 14.5$, and finally the few stars above the turn-off which are probable candidates to being blue stragglers.

In the analysis of the CMD we assume the chemical composition $Z = 0.008$ and $Y = 0.25$ and normalize the synthetic CMDs to the number of stars counted on the MS in the magnitude interval $18.0 \leq V \leq 19.0$. In addition to this, we adopt a fraction of binary stars amounting to about 25% with mass ratios in the range 0.5 to 1.5.

The classical models yield the color excess $E_{B-V} = 0.11$, the apparent distance modulus $(m - M) = 13.60$, and the age of 6.0 10^9 yr. The overshoot models give the color excess $E_{B-V} = 0.10$, the apparent distance modulus $(m - M) = 13.50$, and the age of 6.5 10^9 yr. The CMDs corresponding to these ages are shown in Figs. 12 and 13 for the classical and overshoot models, respectively, whereas the corresponding isochrones are compared to the observational CMD in Fig. 14.

Although both types of models lead to a good reproduction of the observed CMD, the detailed morphology of the turn-off region is better matched by the classical models.

Specifically, models with overshoot give rise to the so-called blue hook for which there is no evidence in the CMD (compare Fig. 11 with Fig. 13). Since the turn-off mass corresponding to the above ages is in the range $1.1 - 1.2\ M_{\odot}$ and therefore has a convective core, this means that our models with convective overshoot tend to overestimate the efficiency of this in the mass range on consideration. Finally, looking at the usual ratios of star counts, we calculate σ_1 by using as MS stars those in the magnitude interval $18.0 \leq V \leq 19.0$ (125 objects) and as evolved stars those brighter than $V = 17.25$ (46 objects), and the ratio σ_2 by counting the RGB stars brighter and fainter than $V = 15.0$, which amount to 17 and 29 respectively. The results are summarized in Table 3.

The same remarks already made for NGC 2243 also apply to Berkeley 39.

5. NGC 188

NGC 188 ($l = 123$, $b = 22$) is a well studied open cluster, long considered to be the oldest object of this family.

Its color excess, distance modulus, and metallicity are quite controversial. The color excess has varied in the range 0.04 (Jennens & Helfer 1975) to 0.12 (Twarog 1978; Demarque et al. 1991), whereas the metal content has changed from the Spinrad & Taylor (1969) super-solar estimate to the lower than solar determination ([Fe/H]=−0.50) by Jennens & Helfer (1975). Similar variations have been quoted for the distance modulus. Specifically, the metallicity and distance modulus go from [Fe/H]=0.07 and $(m - M) = 10.95$ (Sandage 1962), [Fe/H]=0.02 and $(m - M) = 10.83$ (Eggen 1969), [Fe/H]=−0.25 and $(m - M) = 12.00$ (Jennens & Helfer 1975), [Fe/H]=−0.18 and $(m - M) = 10.90$ (Janes 1979), [Fe/H]=−0.05 (Canterna et al. 1986), [Fe/H]=−0.06 (Lynga 1987) to [Fe/H]=0.0 and $(m-M) = 11.50$ (Twarog & Anthony-Twarog 1989; Demarque et al. 1991).

As a consequence of the above uncertainties in color excess and distance modulus, the age assigned by the various authors to NGC 188 has considerably changed over the years. The situation is summarized in Table 4.

Table 4. Age determinations of NGC 188

Reference	Age(Gyr)
Sandage (1962)	14-16
Demarque & Larson (1964)	9-10
Iben (1967)	12
Aizenman *et al* (1969)	3.5-5.5
Sandage & Eggen (1969)	8-10
Torres-Peimbert (1971)	3.6
Twarog (1978)	4.3±1.0
Patenaude (1978)	8.0
Vandenberg (1985)	10.0
Twarog & Anthony-Twarog (1989)	6.5±0.5
Caputo *et al* (1990)	6.0
Hobbs *et al* (1990)	7.7±1.4
Demarque *et al* (1992)	6.5
This work (1993)	7.5

Recent determinations of the CMD are by McClure & Twarog (1976), and Caputo et al. (1990). In this study we prefer to use the CMD by McClure & Twarog (1976), as the more recent CMD by Caputo et al. (1990) does not contain the evolved stars, the importance of which is basic to the correct determination of the age. The CMD by McClure & Twarog (1976) is shown in Fig. 15. Also in this case foreground contamination is significant. However the MS band and turn-off luminosity are visible, whereas the clump of red stars is hardly detectable. Likely the clump consists of a few stars located at $V = 12.5$ and $B - V = 1.2$. The MS band appears to be larger than allowed by uncertainties in the photometry, perhaps suggesting that like in other clusters of this type a significant fraction of binary stars is present.

The synthetic CMDs are generated assuming the composition $Z = 0.02$ and $Y = 0.28$, which seems to be suited to NGC 188. Indeed, adopting the metallicity [Fe/H] by Linga (1987), which is homogeneous with the Friel & Janes (1993) scale, and converting it to the model metallicity (Eq. (1)) we get $Z = 0.016$.

The CMD simulations are normalized to impose the number of MS stars in the magnitude interval $15.5 \leq V \leq$

16.5. Finally, they also include a certain fraction of binary stars amounting to 25% with mass ratios in the range 0.5–1.5.

With the classical models we get the color excess $E_{B-V} = 0.04$, the apparent distance modulus $(m-M) = 11.30$, and then the age of 7.0 10^9 yr. With the overshoot models we obtain the color excess $E_{B-V} = 0.03$, the apparent distance modulus $(m-M) = 11.25$, and the age of 7.5 10^9 yr. The synthetic CMDs for these ages are shown in Figs. 16 and 17 for the classical and overshoot models, respectively, while the corresponding isochrones are mounted on the observational CMD assuming the color excess and distance modulus corresponding to each type of stellar models. This is shown in Fig. 18.

It can be noticed that, independently of the stellar models in use, the turn-off luminosity, the magnitude difference between the clump and turn-off, and the color difference between the turn-off and the bottom of the RGB are well matched, whereas the slope of the fainter part of the MS band is not. Precisely, the theoretical MS tends to be steeper than the observed one.

The ratios σ_1 and σ_2 are presented in Table 3. The ratio σ_1 is derived from counting stars of the MS falling in the magnitude interval $15.5 \leq V \leq 16.5$ and evolved stars brighter than $V = 15.5$. These are 101 and 29 stars, respectively. The ratio σ_2 is obtained by considering the red giant stars brighter (8 stars) and fainter (21 stars) than $V = 13.0$. It turns out that while the theoretical value of σ_1 is consistent with the observational value within the uncertainty, the ratio σ_2 is perhaps larger. The reason for the discrepancy is not understood.

6. NGC 6791

NGC 6791 ($l = 70,\ b = 11$) is a rich, compact and old open cluster approximately located at the solar distance from the Galactic Center and 1 Kpc above the Galactic plane, in a rather crowded region (Harris & Canterna 1981).

NGC 6791 has often been considered to be a very old cluster, perhaps as old as the globular cluster 47 Tuc, the age of which is currently estimated to be in the range $10 - 13$ 10^9 yr. Therefore, the correct determination of the age of NGC 6791 bears on the problem of a suspected age gap between the family of globular clusters and the oldest clusters of the Galactic Disk. In short, Kinman (1965) suggested that NGC 6791 has an age comprised between that of NGC 188 and that of globular clusters. Anthony-Twarog & Twarog (1985) using the isochrones by Ciardullo & Demarque (1977) get the age of 6 10^9 yr, whereas adopting the isochrones by VandenBerg (1985) get the age of 12.5 10^9 yr. Janes (1988) estimates the age to be 12.5 10^9 yr. Kaluzny (1990), assuming $Y = 0.25$ and $Z = 0.02$, suggests the age of 7.5 10^9 yr. Finally, Demarque et al. (1991), arbitrarily assuming that the metallicity of the cluster is $Z = 0.04$ and adopting different laws for the helium to heavy elements enrichment ratio $\Delta Y/\Delta Z$, get the age of 7.5 10^9 yr for $Y = 0.32$ ($\Delta Y/\Delta Z = 2$), and 6.5 10^9 yr for $Y = 0.36$ ($\Delta Y/\Delta Z = 4$).

As far as the metallicity is concerned, Kinman (1965) and Anthony-Twarog & Twarog (1985), comparing the CMD's, pointed out that the metallicity of NGC 6791 ought to be similar to that of NGC 188 for which the solar composition is customarily assumed. This is also supported by the photometric observations of the cluster giants by Janes (1984) and Canterna et al. (1986). Finally, Friel & Janes (1991) estimate [Fe/H]=+0.23, while Friel & Janes (1993) get [Fe/H] = $+0.19 \pm 0.19$. We adopt the metallicity [Fe/H] = 0.19 by Friel & Janes (1993), to which according to Eq. (1) $Z = 0.031$ corresponds.

The CMD of NGC 6791 was first analyzed by Kinman (1965), and more recently by Anthony-Twarog & Twarog (1985), Janes (1988), and Kaluzny (1990). The CMD of this latter is adopted here and it is shown in Fig. 19. Although significant, the contamination of field stars is less of a problem in that the bulk population of cluster stars can be singled out with reasonable confidence. In fact, the CMD of NGC 6791 resembles those of NGC 188 or Berkeley 39. However, because of its richness, the CMD of NGC 6791 clearly shows the MS band down to $V = 20$, the subgiant branch and RGB, and finally the distinct clump of red stars at $V = 14.6$ and $(B-V) = 1.3$, these latter with a little dispersion in the color.

The synthetic CMDs are calculated assuming the chemical composition $Y = 0.305$ and $Z = 0.030$ as suggested by the above discussion of the metallicity. Furthermore, the theoretical CMDs are normalized by imposing that the number of MS stars counted in the magnitude interval of the observational CMD, $17.5 \leq V \leq 18.5$, is matched. Finally, considering the width of the MS band, we suspect that a significant population of binary stars is present. The percentage of these is estimated to be 33% with mass ratios in the range 0.3 to 3.3.

A preliminary analysis of the CMD indicates that the turn-off mass of this cluster is below the limit for the existence of a convective core on the main sequence. Therefore no difference exists in the age determination due to the type of mixing. Convective overshoot remains during the core He-burning phase (stars in the red clump) in our case. However, the difference with respect to the classical scheme is very small. Since we have used models calculated with convective overshoot during the core He-burning phase, the results for NGC 6791 will be listed under the heading "overshoot" even if there is no actual dependence on this phenomenon. We get the color excess $E_{B-V} = 0.10$, the distance modulus $(m-M) = 13.50$, and the age of 8 10^9 yr. Figure 20 shows the synthetic CMDs for this value of the age. Our age determination agrees with that by Demarque et al. (1991) provided that the effect of the different chemical abundances is considered.

Finally, in order to check for the above age, we calculate the ratios σ_1 and σ_2. These are derived assuming the following limiting magnitudes. In σ_1, the MS stars are those in the magnitude interval $18.0 \leq V \leq 19.0$ whereas the evolved stars are those brighter than $V = 17.5$. These are 700 and 121 stars, respectively. Care is paid to exclude by as much as possible foreground stars and blue straggler candidates. In σ_2, the limit magnitude separating the bright red giants (42 objects) from the faint ones (69 objects) is $V = 15.5$ (just below the red clump luminosity). The results are presented in Table 3. Both theoretical ratios agree with their observational counterparts.

7. Discussion and conclusions

We have analyzed the CMD of five old open clusters for which good photometric data are available.

The main goal of this study and of the previous one on the same subject by Carraro et al. (1993) is to provide homogeneous determinations of the age for a selected sample of old open clusters, using both classical and overshoot models.

It goes without saying that an homogeneous age ranking is the preliminary step toward the study of more complex problems such as the properties of the solar vicinity as well as the history of star formation and chemical enrichment in the Galactic disk.

In addition to this, since the properties of the CMD are deeply related to the structure of the component stars, the study of old open clusters would help us to better understand the question of the extension of the convective cores in that particular mass range around 1 $M_{\odot}$, in which the transition from radiative to convective cores occurs, and in particular to clarify whether the overshoot scheme ought to be preferred to the classical one.

The main results of our study, namely adopted metallicity, and resulting color excess, distance modulus, and age both for classical and overshoot models are summarized in Table 5.

It must be pointed out that the age difference arising from the adoption of one or the other evolutionary scheme vanishes with NGC 6791 thus indicating that the turn-off mass is at the limit of, if not lower than, the critical value at which core H-burning switches from convective to radiative.

The remaining clusters have turn-off masses for which overshoot is expected to occur. Although the results are not very conclusive, both the overall morphology (gaps in particular) of the CMD and the ILF (see the cases of NGC 2682 and NGC 2243) seem to suggest that models with convective overshoot ought to be preferred.

Our attempt to better discriminate between the two types of models by means of the ratios of star counts, σ_1 and σ_2 has not let to clear results. It is worth recalling that σ_1 is related to the ratio of core H- to He-burning lifetimes, while σ_2 is related to the lifetimes along the RGB and in the clump. In principle, both should significantly vary with the type of stellar models. Unfortunately, the information contained in those ratios is masked and biased by many collateral effects such as contamination by foreground stars, stochastic effects caused by the low number of stars in total, binary stars, etc. The vexed question of the mixing scheme to be adopted in modeling central convection in stars cannot be answered by studying the CMDs of this sample of clusters. Besides the fact that better observational data would allow us to perform a more accurate analysis, perhaps the family of Galactic open clusters is not the best laboratory where this question can be tackled. Indeed, much richer clusters in the age range on consideration ought to be preferred. Unfortunately, clusters of this type seem to be rare even among those of the Large Magellanic Cloud.

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Table 3. Star counts in CMDs

	Observed	Classical Models	Overshoot Models
NGG 2682			
σ_1	1.75±0.71	2.64±0.69	2.53±0.81
σ_2	0.45±0.34	1.00±0.94	0.86±0.84
NGG 2243			
σ_1	5.00±1.54	3.47±0.94	3.15±0.85
σ_2	0.46±0.29	0.70±0.25	0.46±0.16
Berkeley 39			
σ_1	2.71±0.49	4.92±1.44	4.11±1.12
σ_2	0.58±0.25	0.71±0.34	0.65±0.30
NGG 188			
σ_1	3.48±0.99	5.29±1.17	5.01±2.34
σ_2	0.38±0.22	0.82±0.54	0.71±0.37
NGG 6791			
σ_1	5.78±0.74	......	6.90±1.14
σ_2	0.61±0.17	......	0.87±0.30

Table 5. Basic parameters of the clusters

		Standard	Models		Overshoot	Models	
Cluster	Z	Age (Gyr)	E_{B-V}	(m-M)	Age(Gyr)	E_{B-V}	(m-M)
M67	0.020	4.3	0.01	9.70	4.8	0.01	9.50
NGC2243	0.008	3.5	0.02	13.00	3.9	0.01	12.90
Berkeley39	0.008	6.0	0.11	13.60	6.5	0.10	13.50
NGC188	0.020	7.0	0.04	11.30	7.5	0.03	11.25
NGC6791	0.030	...	...	...	8.0	0.10	13.50

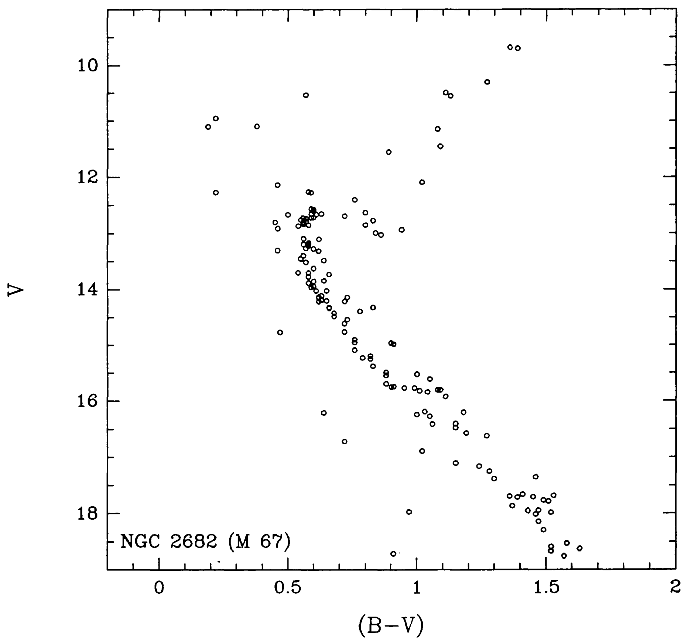

Fig. 1. The CMD of NGC 2682 (M 67) by Gilliland et al. (1991)

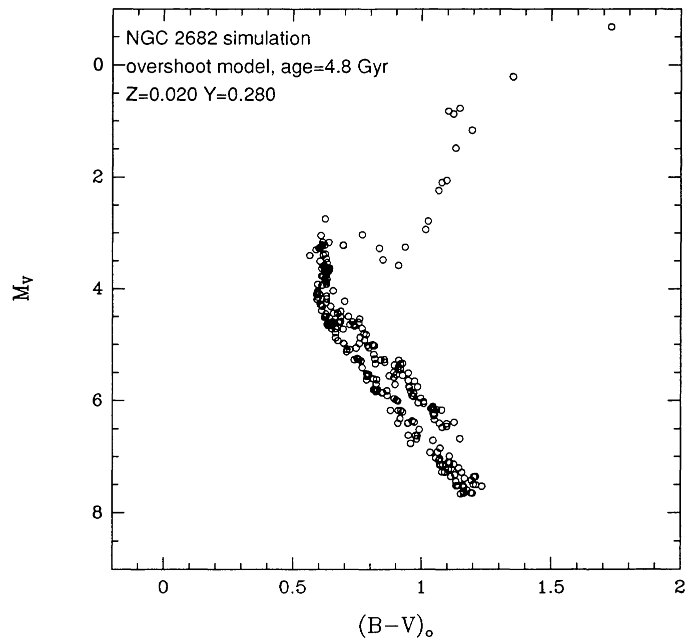

Fig. 3. A synthetic CMD derived from overshoot models which best reproduces the observed CMD of Fig. 1. The chemical composition of the models is $Z = 0.02$ and $Y = 0.28$. The age is 4.8 10^9 yr. In this simulation we have assumed that binary stars are present (50%)

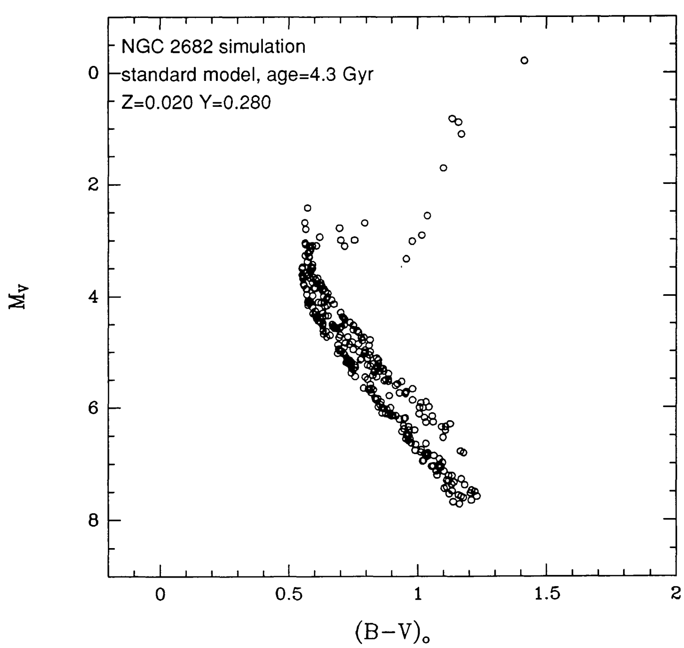

Fig. 2. A synthetic CMD derived from standard models which best reproduces the observed CMD of Fig. 1. The chemical composition of the models is $Z = 0.02$ and $Y = 0.28$. The age is 4.3 10^9 yr. In this simulation we have assumed that binary stars are present (50%). This is indicated by the few stars running parallel to the main sequence. See the test for more details

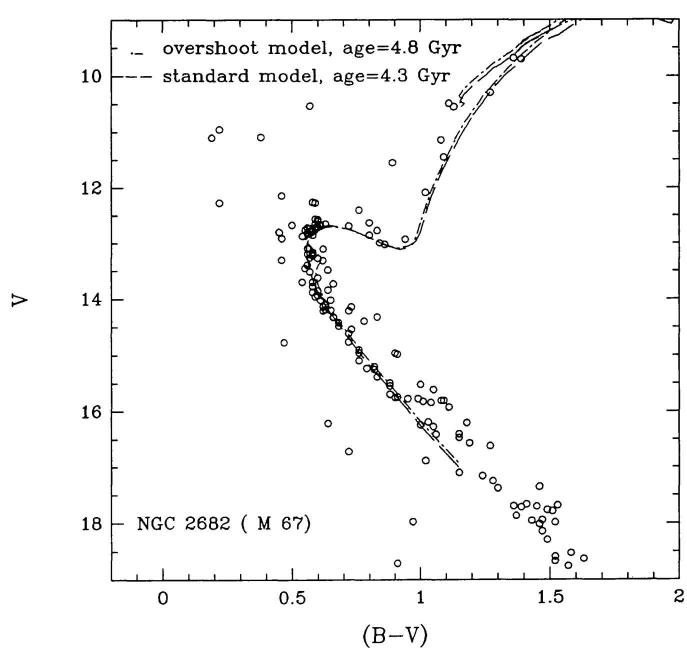

Fig. 4. The CMD of Fig. 1. Superposed are isochrones for standard and overshoot models. The isochrone of 4.3 10^9 yr for the standard models (dashed line) is drawn using $E_{B-V} = 0.02$ and $(m - M) = 9.60$. The isochrone of 3.0 10^9 yr for the overshoot models (dashed-dotted line) is drawn using $E_{B-V} = 0.02$ and $(m - M) = 9.50$. The chemical composition of the models is $Z = 0.020$ and $Y = 0.28$. See the text for more details

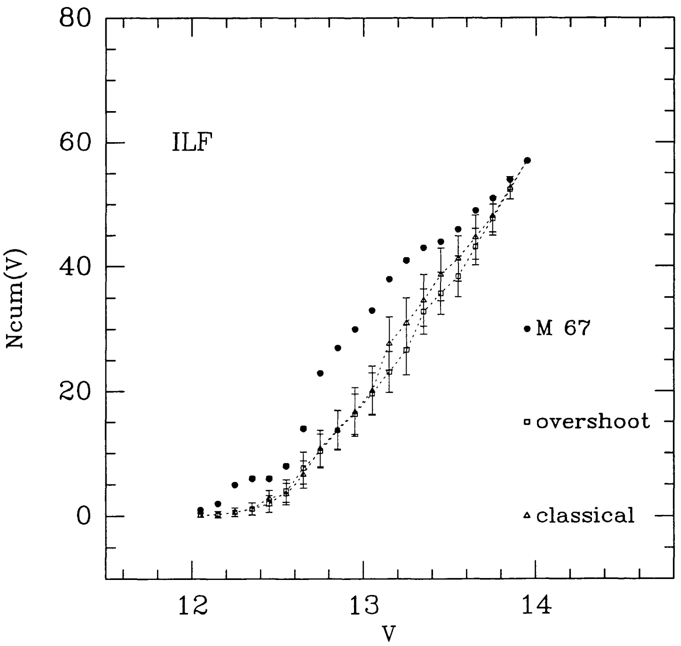

Fig. 5. Observed integrated MS luminosity function of M 67. Superposed are the luminosity function for standard and overshoot models

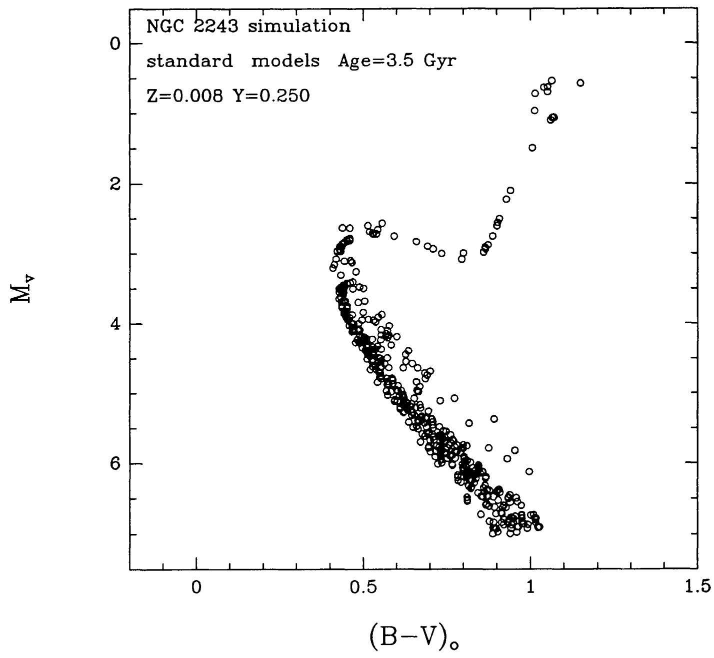

Fig. 7. A synthetic CMD derived from standard models which best reproduces the observed CMD of Fig. 6. The chemical composition of the models is $Z = 0.008$ and $Y = 0.25$. The age is 3.5 10^9 yr. In this simulation we have assumed that binary stars are present (20%). This is indicated by the few stars running parallel to the main sequence. See the text for more details

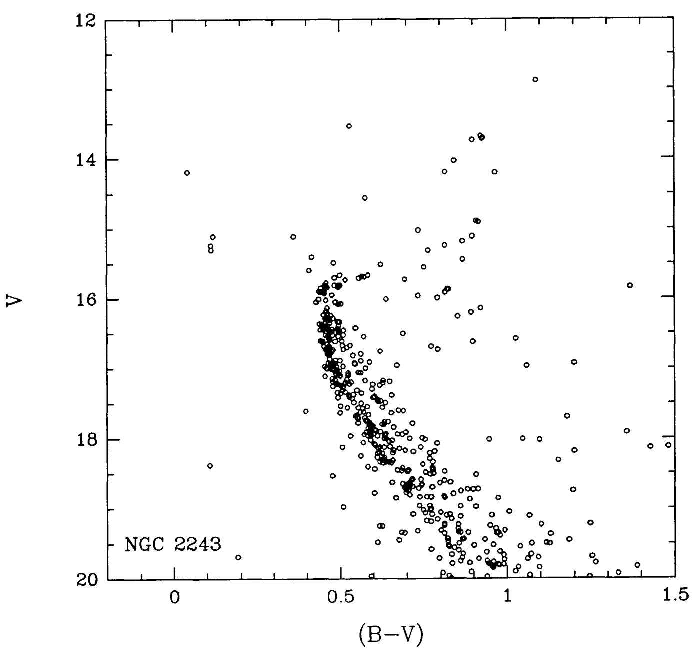

Fig. 6. The CMD of NGC 2243 by Bergbusch et al. (1991)

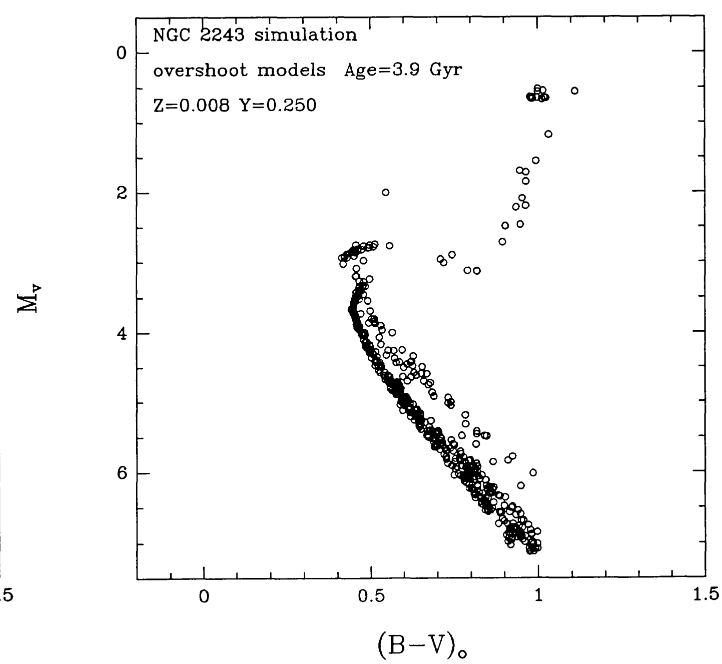

Fig. 8. A synthetic CMD derived from overshoot models which best reproduces the observed CMD of Fig. 6. The chemical composition of the models is $Z = 0.008$ and $Y = 0.25$. The age is 3.9 10^9 yr. In this simulation we have assumed that binary stars are present (20%)

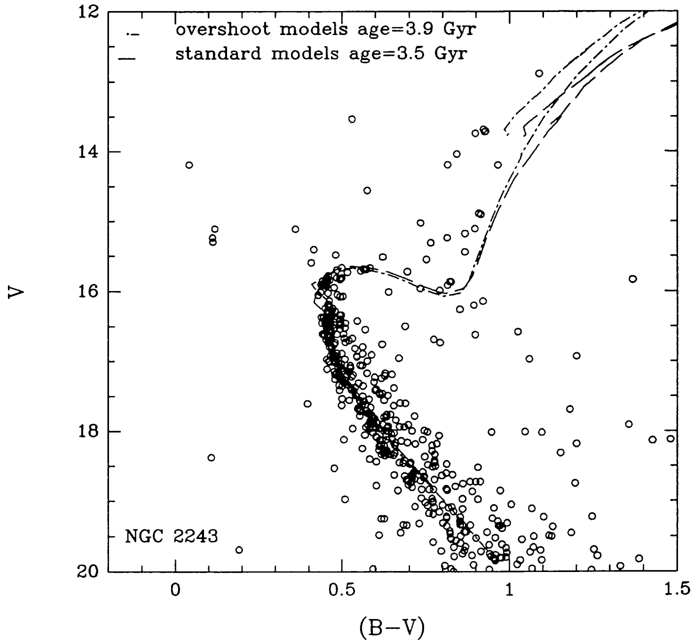

Fig. 9. The CMD of Fig. 6. Superposed are isochrones for standard and overshoot models. The isochrone of 3.5 10^9 yr for the standard models (dashed line) is drawn using $E_{B-V} = 0.01$ and $(m - M) = 13.00$. The isochrone of 3.9 10^9 yr for the overshoot models (dashed-dotted line) is drawn using $E_{B-V} = 0.01$ and $(m - M) = 12.90$. The chemical composition of the models is $Z = 0.008$ and $Y = 0.25$. See the text for more details

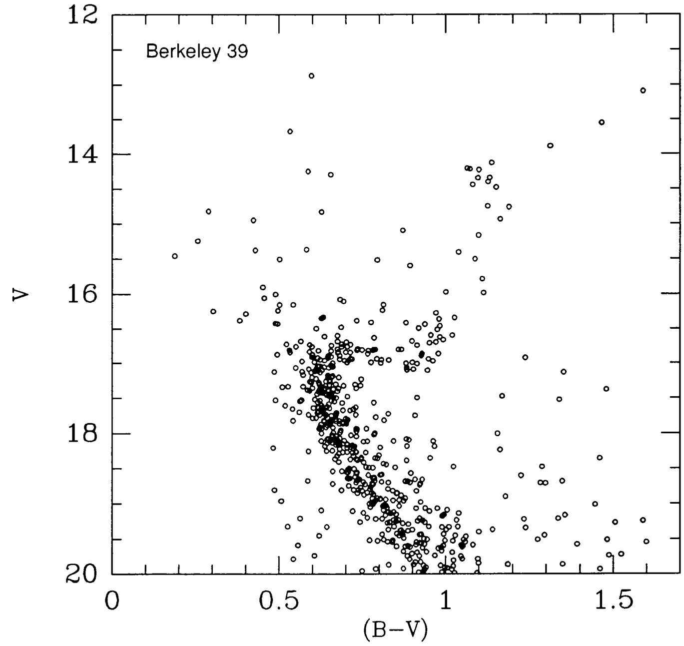

Fig. 11. The CMD of Berkeley 39 by Kaluzny & Richtler (1989)

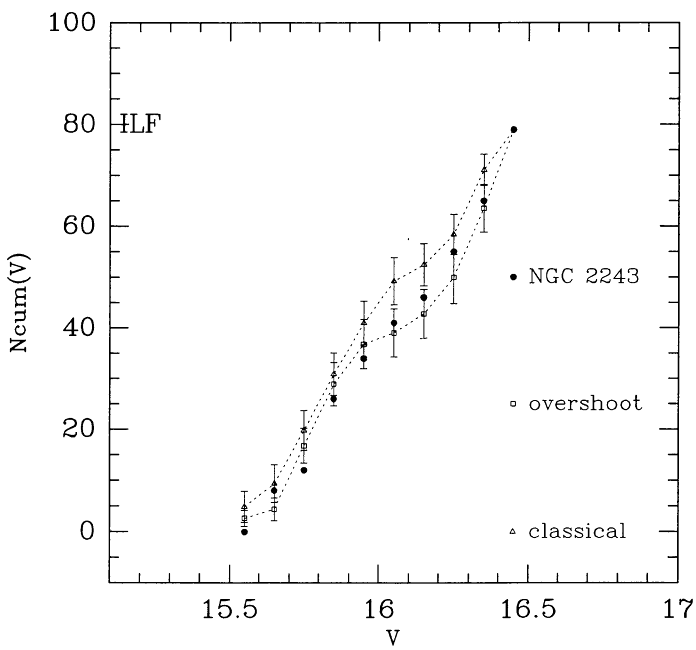

Fig. 10. Observed integrated MS luminosity function of NGC 2243. Superposed are the luminosity function for standard and overshoot models

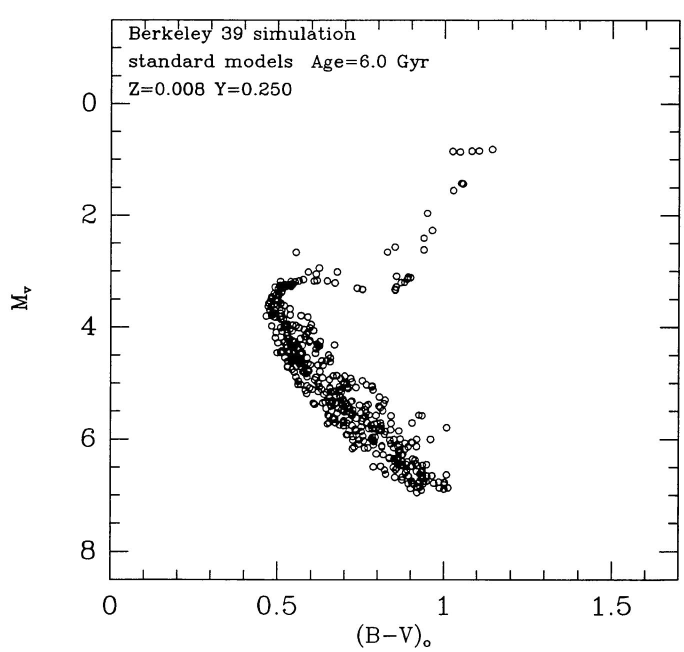

Fig. 12. A synthetic CMD derived from standard models which best reproduces the observed CMD of Fig. 11. The chemical composition of the models is $Z = 0.008$ and $Y = 0.25$. The age is 6.0 10^9 yr. In this simulation we have assumed that binary stars are present (25%). This is indicated by the few stars running parallel to the main sequence. See the text for more details

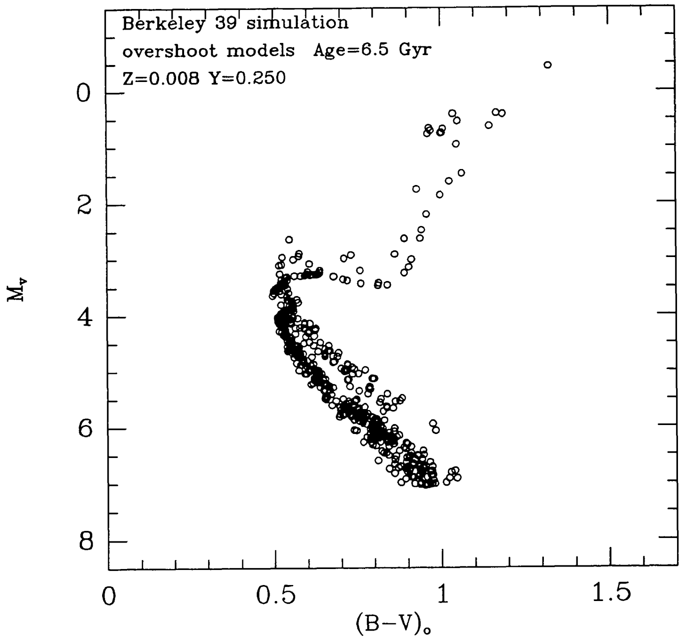

Fig. 13. A synthetic CMD derived from overshoot models which best reproduces the observed CMD of Fig. 11. The chemical composition of the models is $Z = 0.008$ and $Y = 0.25$. The age is 6.5 10^9 yr. In this simulation we have assumed that binary stars are present (25%)

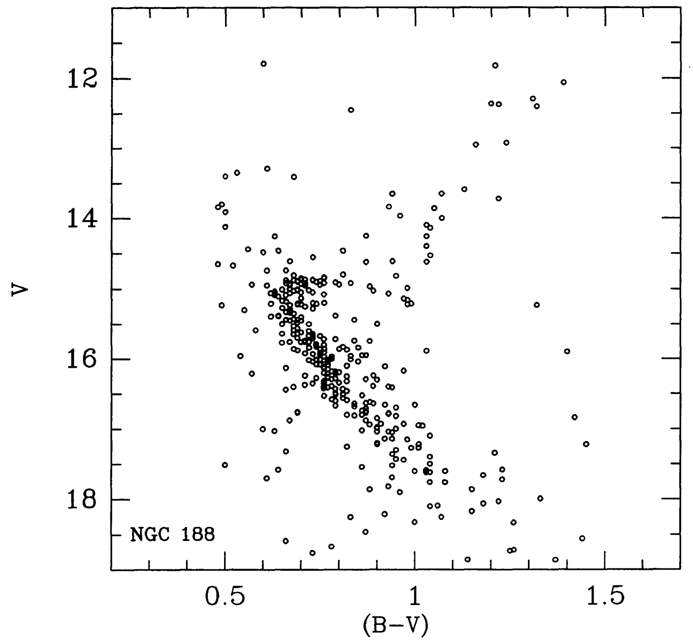

Fig. 15. The CMD of NGC 188 from McClure & Twarog (1976)

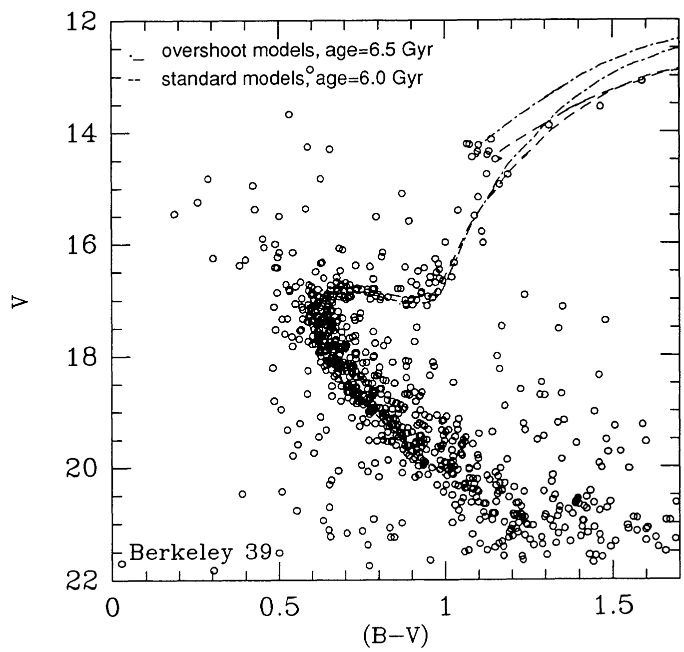

Fig. 14. The CMD of Fig. 11. Superposed are isochrones for standard and overshoot models. The isochrone of 6.0 10^9 yr for the standard models (dashed line) is drawn using $E_{B-V} = 0.11$ and $(m - M) = 13.60$. The isochrone of 6.0 10^9 yr for the overshoot models (dashed-dotted line) is drawn using $E_{B-V} = 0.10$ and $(m - M) = 13.50$. The chemical composition of the models is $Z = 0.008$ and $Y = 0.25$. See the text for more details

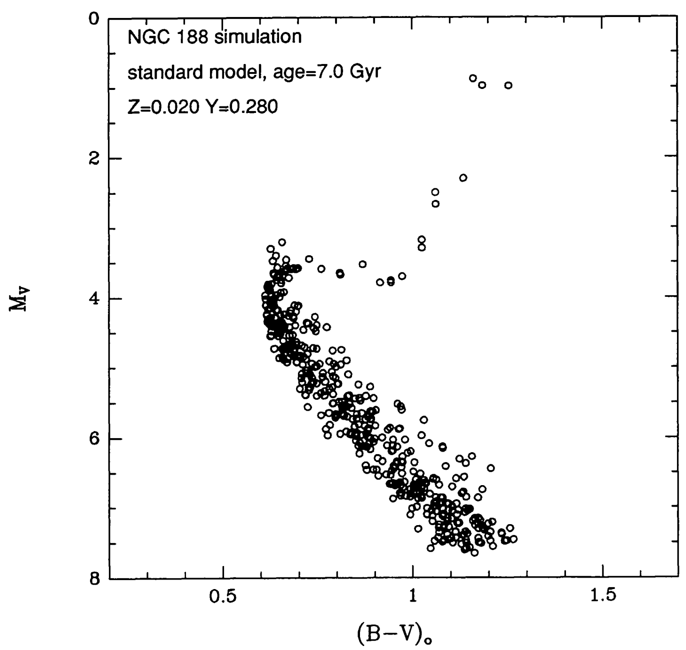

Fig. 16. A synthetic CMD derived from standard models which best reproduces the observed CMD of Fig. 15. The chemical composition of the models is $Z = 0.02$ and $Y = 0.28$. The age is 7.0 10^9 yr. In this simulation we have assumed that binary stars are present (25%). This is indicated by the few stars running parallel to the main sequence. See the text for more details

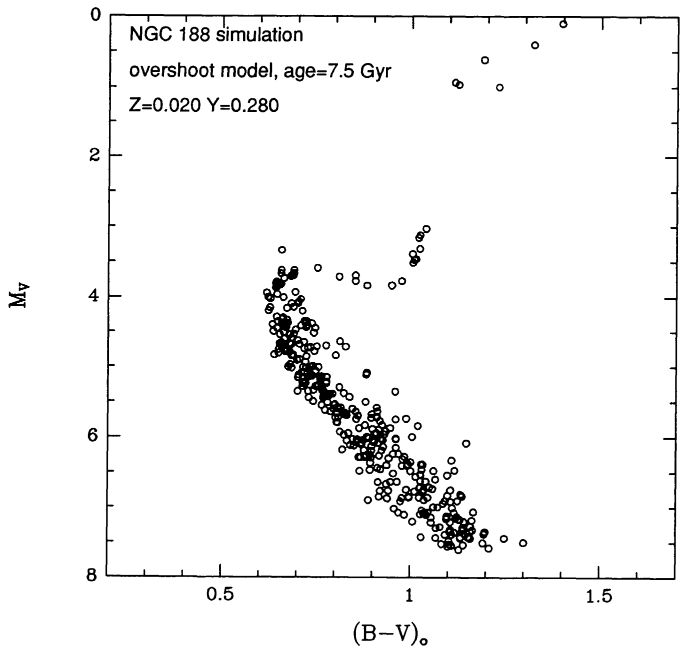

Fig. 17. A synthetic CMD derived from overshoot models which best reproduces the observed CMD of Fig. 15. The chemical composition of the models is $Z = 0.02$ and $Y = 0.28$. The age is 7.5 10^9 yr. In this simulation we have assumed that binary stars are present (25%)

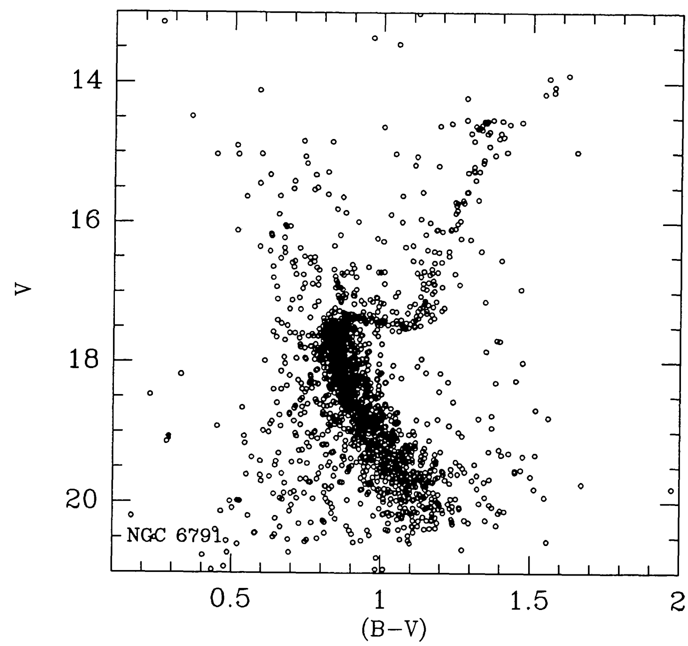

Fig. 19. The CMD of NGC 6791 by Kaluzny (1990)

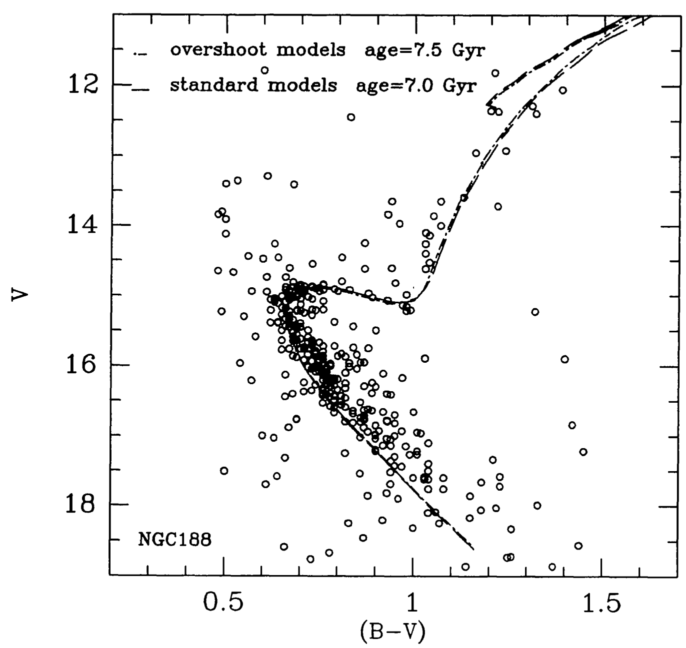

Fig. 18. The CMD of Fig. 15. Superposed are isochrones for standard and overshoot models. The isochrone of 7.0 10^9 yr for the standard models (dashed line) is drawn using $E_{B-V} = 0.04$ and $(m - M) = 11.30$. The isochrone of 7.5 10^9 yr for the overshoot models (dashed-dotted line) is drawn using $E_{B-V} = 0.03$ and $(m - M) = 11.25$. The chemical composition of the models is $Z = 0.020$ and $Y = 0.28$. See the text for more details

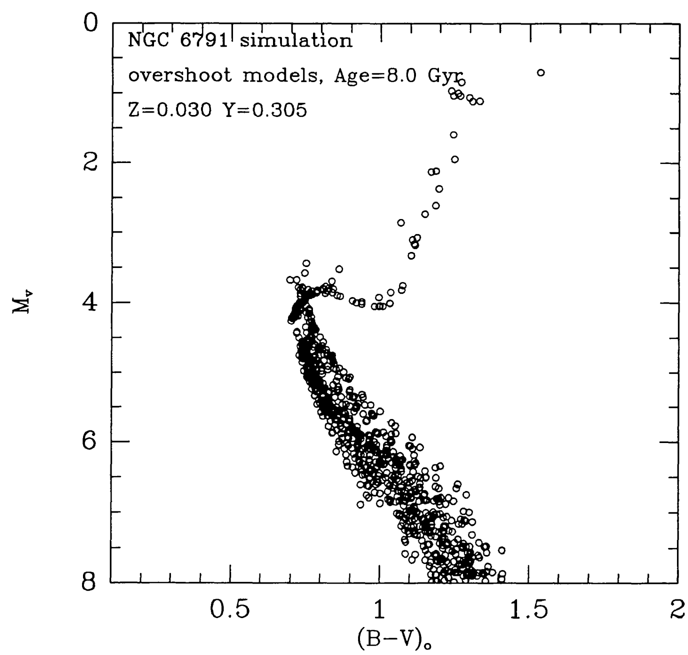

Fig. 20. A synthetic CMD derived from standard models which best reproduces the observed CMD of Fig. 19. The chemical composition of the models is $Z = 0.030$ and $Y = 0.305$. The age is 8.0 10^9 yr. In this simulation we have assumed that binary stars are present (33%). This is indicated by the few stars running parallel to the main sequence. See the text for more details